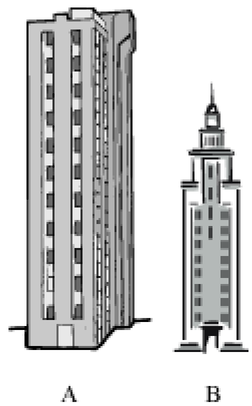
Understanding Elementary Shapes

Comparison Of Lengths Of Line Segments By Observation

Consider two buildings A and B in the following figure.

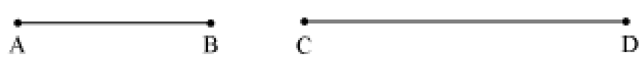


Suppose you are standing on one side of the road and these two buildings are on the other side. **Can you tell which building is taller?**

Yes, you are right. Simply by looking at the buildings, you can say that the building on the left (i.e. building A) is taller. This is called the **method of observation**, where we look at two objects and say that which of them is taller or which is shorter.

Now, let us apply this concept of observation in geometry. You already know what a line segment is. Remember that the **length of a line segment is the distance between its end points.**

Now consider the line segments AB and CD drawn in the following figure.



Can you tell which line segment is longer?

Surely, you will not face any problem in answering this question. Simply by observing the two line segments, we can say that the length of CD is more than AB. We can also say the length of line segment AB is less than CD.

This method of comparison of lengths of line segments is the method of observation.

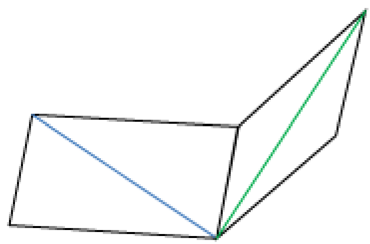
Comparison Of Lengths Of Line Segments Using Tracing Paper Method

Consider the line segments AB and CD drawn in the following figure.

Can you tell which line segment is longer?

Yes, simply by observing the line segments, we can say that the length of CD is more AB.

However, we cannot always tell which line segment is longer, merely by looking at them. For example: consider the following figure.



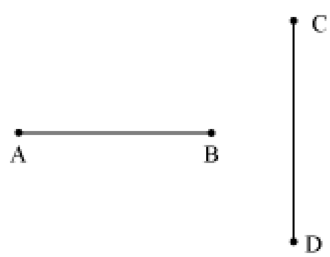
Can you say which line segment is longer – the blue one or the green one?

Here, you might be a little confused. Actually, in this figure, both line segments are of same length, even though, the green line segment seems longer than the blue one. This is an example of optical illusion.

So, is there any other way, by which we can compare the lengths of line segments without having such observation errors?

One such method is **tracing**. Let us now look at this method.

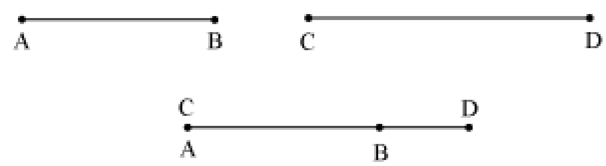
Consider the two line segments AB and CD given in the following figure.



Here, the line segments $\overline{AB}$ and $\overline{CD}$ seem to be of equal lengths. However, we cannot say this for sure. So, we can use **the method of tracing** in such cases, which is as follows.

- We start by tracing one line segment, say AB, using a tracing paper.
- We then place the traced line segment AB on the other line segment(CD) which needs to be compared.

This can be done as follows.



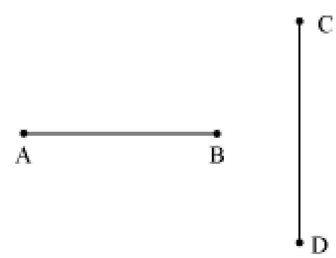
Now, we can say for sure that line segment AB is shorter than the line segment CD.

This technique of comparison of line segments is known as method of tracing.

SCROLL DOWN FOR THE NEXT TOPIC

Comparison Of Lengths Of Line Segments Using Ruler And Divider

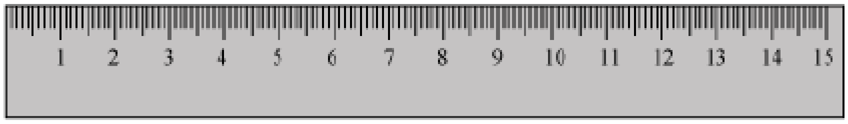
Look at the line segments AB and CD shown in the following figure.



Can you say which of the two line segments is longer?

We know a method of comparing the lengths of the line segments i.e., the method of tracing. However, this technique is time consuming, since we have to trace the segment each time to compare it with another segment.

There is an even easier way to compare the lengths of two or more line segments with the help of a ruler. A ruler looks like:



Ruler

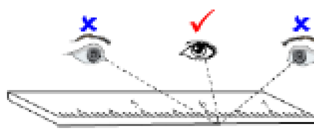
The ruler is evenly marked up to 15 units, where the length of each unit is 1 cm.

Each unit of 1 cm is further subdivided into 10 smaller units, where the length of each smaller unit is 1 mm. Thus, there are 10 mm in each centimetre.

Now, let us learn how to use a ruler to compare the lengths of two line segments.

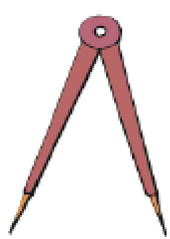
When you are working with a ruler, some errors might creep into your observations. The positioning of your eye while reading the observation is very important, especially, while using a thick ruler.

See the figure given below.



When you are marking the length of the line segment, the eye should be directly above the mark on the ruler. In the above figure, the position of the eye in the middle shows the correct way of taking the reading. The positions of the eye on the left and right lead to wrong observations. These errors are called **positioning errors** or **parallax errors**.

Now, you might be wondering if there is an even better and an easier way than this, to measure the length of a line segment. Actually, there is one more method. We can also use a divider. It looks like:

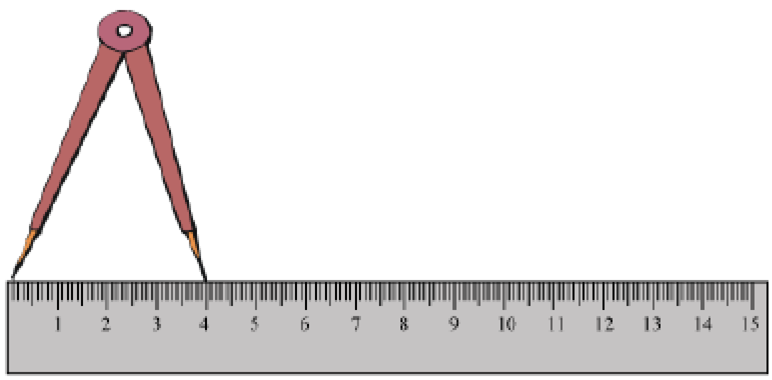

Divider

If we use this divider alongside a ruler, then our task will become much easier. Let us see how to use the divider to compare the lengths of the line segments.

The steps involved are as follows.

- Place the two end points of the arms of a divider at the two end points of the line segment to be measured.

- Then, without disturbing the divider, place it on the ruler with one of its arms at the 0 mark of the ruler.
- Now, the measurement of the two arms of the divider is the measurement of the length of the line segment.

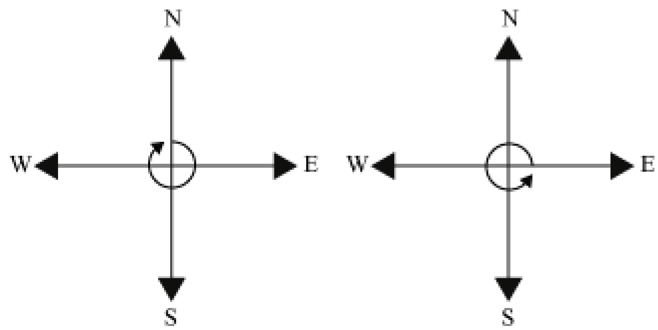


Similarly, we can measure the other line segment using divider and scale. Now that we have the lengths of the two line segments, we can easily conclude which line segment is longer or shorter.

Now, you will definitely agree that to measure a line segment using a ruler and a divider, is one of the most convenient, easy, and appropriate method.

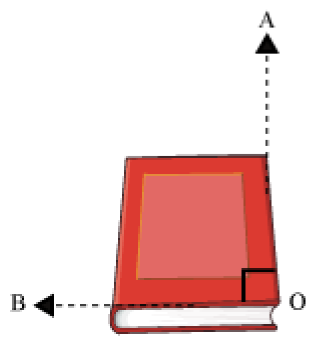
Classification of Angles

Let us consider a situation. Suppose that you are facing North. You turn around making a complete revolution such that you again face North. The angle you turned by is a **complete angle.** Try the same for different directions and you will obtain the same results.



Now, suppose while facing the North direction, if you turn around making a half turn, then the angle you turned by is a **straight angle,** and if you turn around making a quarter turn, then the angle you turned by is a **right angle.**

Now, consider a notebook and look at the angle made by the adjacent sides.



The adjacent sides OB and OA of the notebook form an angle AOB. This angle is a **right angle**.

Similarly, the sides of the alphabet ‘**L**’ form a right angle.

Now, consider the hour and minute hands of a clock when the time is 8’o clock as shown in the figure below.

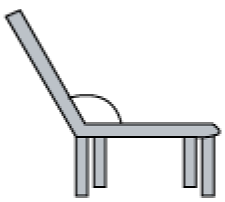
What angle is formed between the hands of the clock?

It is clear from the above discussions that the angle shown in the figure is not among right angle, straight angle, and complete angle. Therefore, how can we classify this angle?

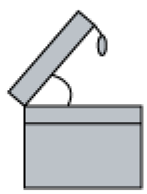
Let us first look at different types of angles.

1. An angle smaller than a right angle is called an **acute angle** i.e., acute angle < 90°.
2. An angle larger than a right angle but smaller than a straight angle is called an **obtuse angle** i.e., 90° < Obtuse angle < 180°.
3. An angle larger than a straight angle but smaller than a complete angle is called a **reflex angle** i.e., 180° < Reflex angle < 360°.

The following figures are examples of the different types of angles.



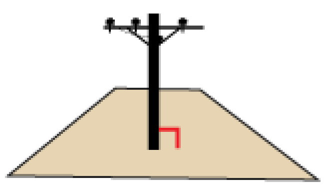
Obtuse angle



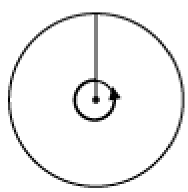
Acute angle

Reflex angle

Straight angle



Right angle



Complete angle

We cannot always say which angle is the greatest out of the given two angles merely by looking at the angles. To compare the angles, we have to measure the angles individually.

An angle is measured in terms of degrees. One complete revolution is equivalent to 360° (° is the symbol of degrees). Thus, the measure of a complete angle is 360°.

The measure of a straight angle is 180° and the measure of a right angle is 90°.

Now, observe the ray OP.

If this ray does not rotate from this position, then the directed angle so formed by the ray is known as the **zero angle**. In other words, it can be said that the amount of rotation of the ray OP about the point O is 0.

The measure of zero angle is 0°.

Now, let us go through the following video to understand the relation between angle measure of a complete angle, straight angle, and a right angle.

Thus, the angle measure of one complete revolution is 360°.

Let us now look at some examples to understand this concept better.

Example 1:
Does the hour hand of a clock make a straight angle when it moves from

1. **12 noon to 6 pm**
2. **5 pm to 10 pm**
3. **2 am to 8 am**
4. **3 pm to 9 pm**
5. **11 am to 7 pm**
6. **3 am to 6 am**

Solution:

We know that two straight angles make a complete angle. A complete angle means a complete revolution. Thus, to make a complete angle, the hour hand has to cross 12 hour marks. To make a straight angle, the hour hand of a clock has to cross 6 hour marks.

1. When the hour hand of a clock moves from 12 noon to 6 pm, it covers 6 hour marks and thus forms a straight angle.
2. When the hour hand of a clock moves from 5 pm to 10 pm, it covers 5 hour marks. Thus, the angle formed is less than a straight angle.
3. When the hour hand of a clock moves from 2 am to 8 am, it covers 6 hour marks and thus forms a straight angle.
4. When the hour hand of the clock moves from 3 pm to 9 pm, it covers 6 hour marks and thus forms a straight angle.
5. When the hour hand of the clock moves from 11 am to 7 pm, it covers 8 hour marks. Thus, the angle formed is more than a straight angle.
6. When the hour hand of the clock moves from 3 am to 6 am, it covers 3 hour marks. Thus, the angle formed measures less than a straight angle.

Example 2:
Does the hour hand of a clock make a complete angle when it goes from

1. **3 am to 3 pm**
2. **9 am to 8 pm**
3. **5 pm to 6 am**
4. **7 pm to 7 am**

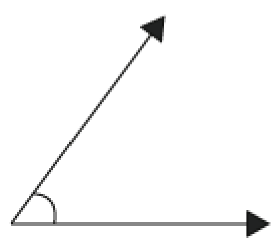
Solution:

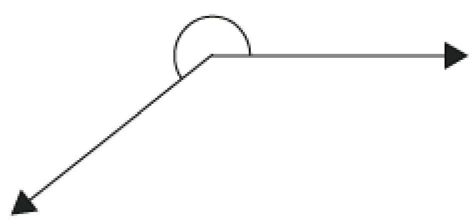
We know that a complete angle means a complete revolution. Thus, the hour hand will make a complete angle when it moves from a mark and comes back to the same mark again. By looking at the above four cases, we observe that the hour hand of a clock forms a complete angle in the following cases.

i) 3 am to 3 pm

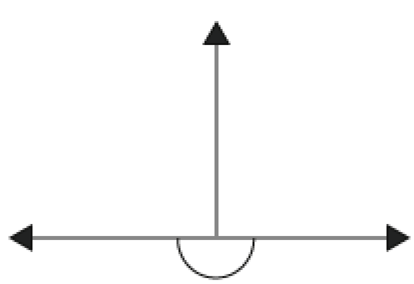
iv) 7 pm to 7 am

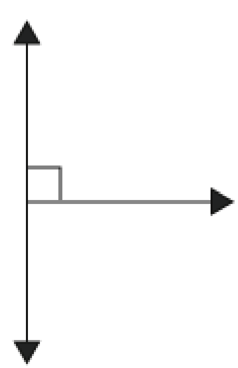
Example 3:
Classify each of the following angles as acute, right, obtuse, straight, or reflex.



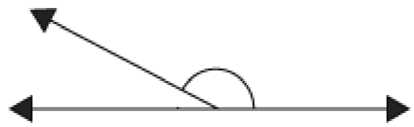


(i) **(ii)**





(iii) **(iv)**



(v)

Solution:

1. The given angle is smaller than a right angle. Hence, it is an acute angle.
2. The given angle is more than a straight angle and less than a complete angle. Hence, it is a reflex angle.
3. The given angle is formed on a straight line and is thus a straight angle.
4. The given angle is a right angle.
5. The given angle is more than a right angle and less than a straight angle. Hence, it is an obtuse angle.

Example 4:
The measures of some angles are given below. Classify these angles based on their measures.

1. **280° (iii) 90° (v) 56°**
2. **180° (iv) 360° (vi) 108°**

Solution:

1. The measure of a reflex angle lies between 180° (straight angle) and 360° (complete angle). Since the given measure is 280°, the angle is a reflex angle.
2. The measure of a straight angle is 180°. Since the given measure is also 180°, the angle is a straight angle.
3. The measure of a right angle is 90°. Since the given measure is also 90°, the angle is a right angle.
4. The measure of a complete angle is 360°. Since the given measure is also 360°, the angle is a complete angle.
5. The measure of an acute angle is less than 90° (right angle). Since the given measure is 56°, the angle is an acute angle.
6. The measure of an obtuse angle lies between 90° (right angle) and 180° (straight angle). Since the given measure is 108°, the angle is an obtuse angle.

Example 5:
Where will the hour hand of a clock stop, if it starts from

1. **6 and turns through two right angles**
2. **5 and turns through two straight angles**
3. **4 and turns through one right angle**

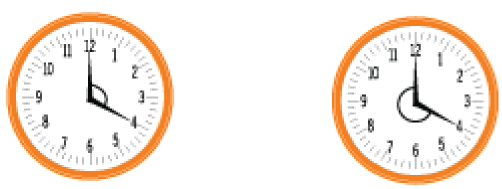
Solution:

1. Since the hour hand turns through two right angles, it crosses 6 hour marks. Therefore, the hour hand will stop at 12.
2. Since the hour hand turns through two straight angles, it makes a complete revolution. Therefore, the hour hand will stop at 5.
3. Since the hour hand turns through one right angle, it crosses 3 hour marks. Therefore, the hour hand will stop at 7.

Example 6:
Find in degrees the angle between the hands of a clock at 4'o clock.

Solution

At 4'o clock, we notice that there are two angles formed by the hands of a clock.



In 4 hours, the hour hand of a clock rotates $\frac{4}{12}$ turn. Hence, the smaller angle between the hands at 4'o clock is $\frac{4}{12}$ turn.

We know that,
1 turn = 360°

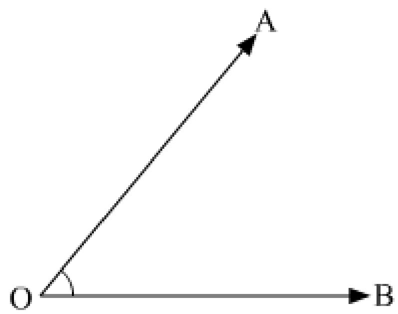
$$\therefore \frac{4}{12}\text{turn} = \frac{4}{12} \times 360° = \frac{1}{3} \times 360° = 120°$$

Now, since the larger and the smaller angle add up to 1 complete turn, the larger angle between the hands at 4'o clock is $\left(1 - \frac{4}{12}\right)$turn $= \frac{8}{12}$ turn

$\frac{8}{12}$ turn $= \frac{8}{12} \times 360° = \frac{2}{3} \times 360° = 240°$

Measurement and Construction of Angles Using Protractor

Look at the following figure in which ∠AOB is shown.



∠AOB is an acute angle. **Can we know the exact measure of ∠AOB?**

Now, suppose you have to construct a particular angle (say an angle of measure 50°). **How will you construct it?**

We can construct different angles from 0 to 180° using a protractor.

Let us now look at some examples to understand the concept better.

Example 1:

Construct the following angles with the help of a protractor.

1. **150°**
2. **Right angle**

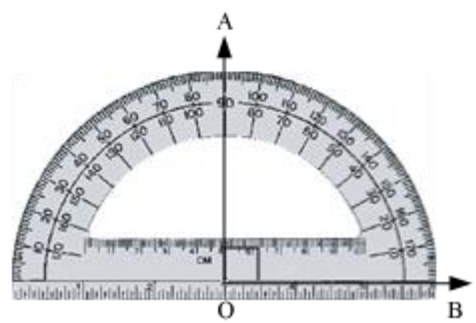
Solution:

1. To construct an angle of measure 150°, we first draw a ray $\overrightarrow{OB}$. Then, we place the protractor such that the straight edge of the protractor lies along the ray and the mid-point of the protractor lies on the starting point of the ray i.e., on the point O.

   Then, a point is marked on the required measurement of the curved edge and a ray is drawn from the point O to the marked point A.

The angle ∠AOB is of the required measure i.e., ∠AOB is of measure 150°.

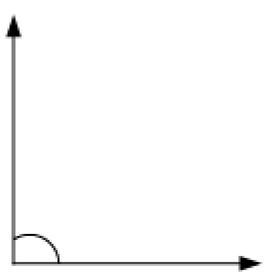
2. Firstly, we draw a ray $\overrightarrow{OB}$. Then, the protractor is placed such that the mid-point of the protractor lies on point O. Then, a point is marked on the required measurement of the curved edge and a ray is drawn from point O to the marked point A. ∠AOB is of the required measure i.e., ∠AOB is of measure 90°.

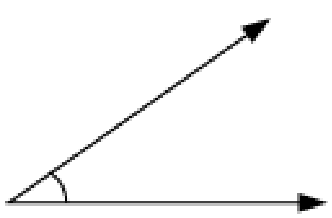


Example 2:

Find the measure of the angles shown in the following figures.

1. 

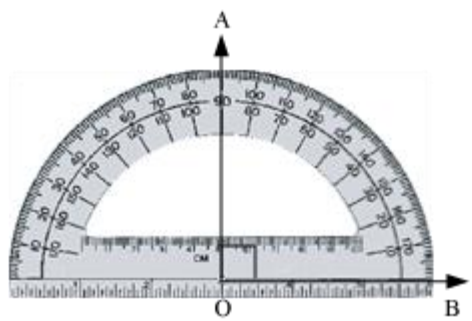
2. 

3. 

Solution:

(i) When we place the protractor along one side of the angle given in figure (i) such that the middle point M of the protractor lies on the vertex of the angle, we find that the measure of ∠AOB is 170°.

(ii) Similarly, when we place the protractor along one side of the angle given in figure (ii), we find that the measure of ∠AOB is 90°.



(iii) When we place the protractor along one side of the angle given in figure (iii), we find that the measure of ∠AOB is 35°.

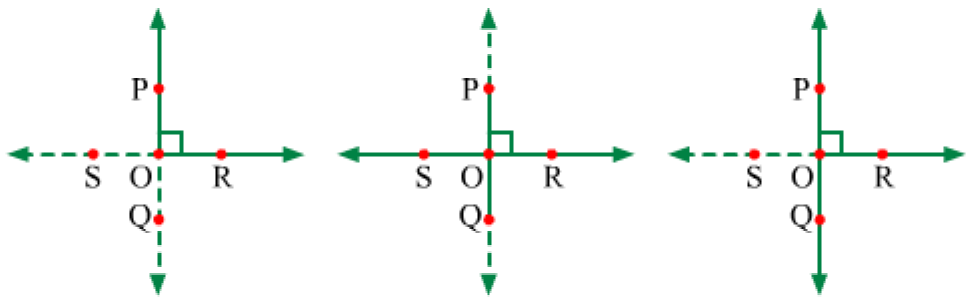
Perpendicular Lines and Perpendicular Bisectors

Two intersecting lines can have any angle between them, but perpendicular lines are special lines, which intersect each other at an angle of ninety degrees.

Perpendicularity of segments and rays:

Two rays or two segments or a segment and a ray are said to be perpendicular to each other if the lines containing them are perpendicular.

Observe the following figure.



In figure (i), ray OP is perpendicular to ray OR i.e., ray OP ⊥ ray OR.

In figure (ii), segment PQ is perpendicular to segment RS i.e., seg PQ ⊥ seg RS.

In figure (iii), line PQ is perpendicular to ray OR i.e., line PQ ⊥ ray OR.

For each case, we have $m\angle POR = 90°$.

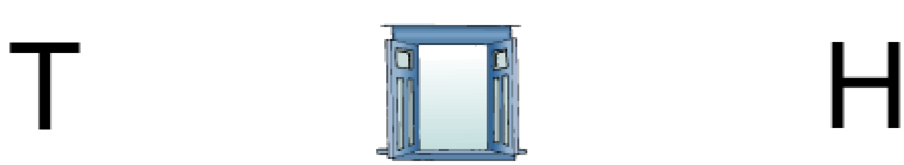
Foot of the perpendicular:

In the figure (ii), line OP intersects line SR at point O at right angle. Thus, line OP ⊥ line SR. Here, the point of intersection O is known as the foot of the perpendicular.

Let us now look at some examples to understand the concept better.

Example 1:

Identify the perpendicular lines and show them by dotted extended lines in the following figures.

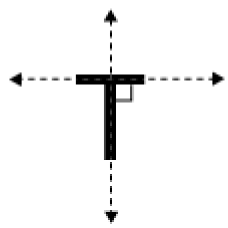


(i) **(ii)** **(iii)**

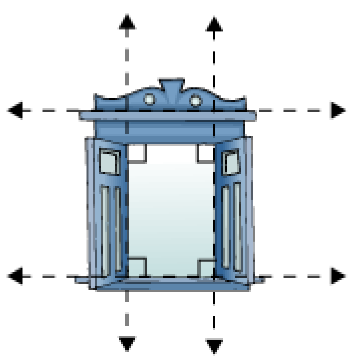
Solution:

The dotted extended lines shown below are perpendicular lines.

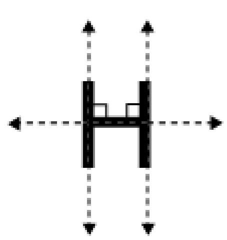
(i)



(ii)

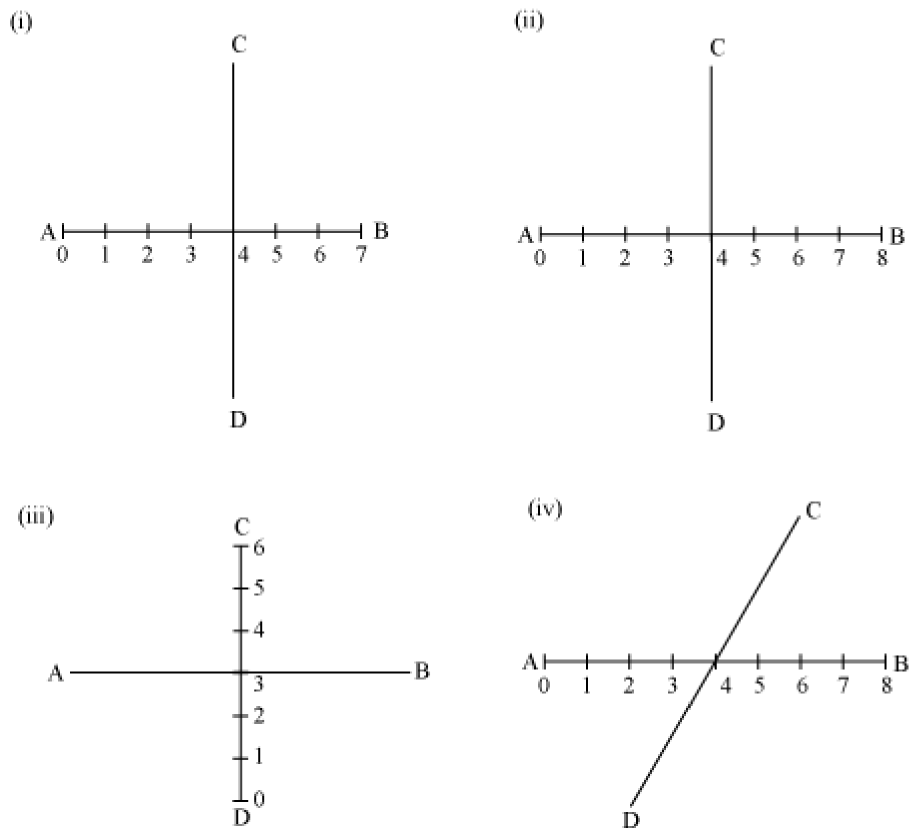


(iii)



Example 2:

Which two among the following figures represent a perpendicular bisector?



Solution:

(i) CD is perpendicular to AB. However, it does not bisect AB.

(ii) CD acts as a perpendicular bisector to AB.

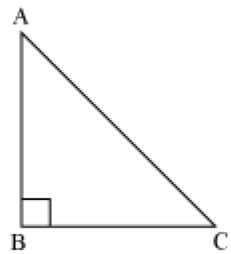
(iii) AB acts as a perpendicular bisector to CD.

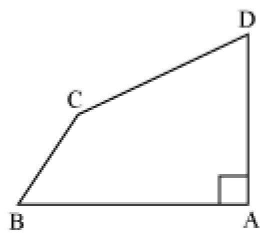
(iv) AB and CD are not perpendicular to each other. Therefore, neither AB nor CD is a perpendicular bisector.

Hence, figures **(ii)** and **(iii)** represent a perpendicular bisector.

Example 3:

Identify perpendicular lines in the following figures.





(i) **(ii)**

Solution:

1. Sides AB and BC form a right angle.

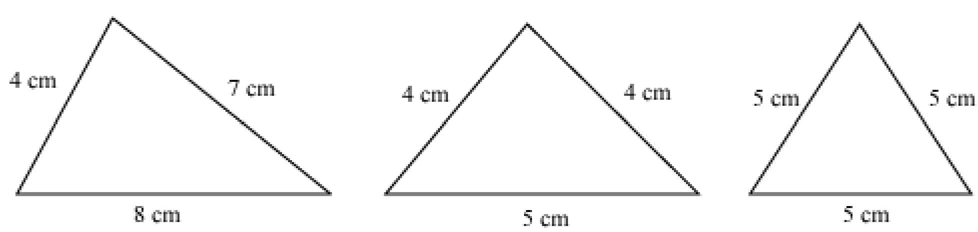
   Therefore, AB ⊥ BC

2. Sides AB and DA form a right angle.

   Therefore, AB ⊥ DA

Classification of Triangles

Consider the following triangles.



Do you observe any difference among the given triangles?

In the first triangle, all sides are of different lengths.

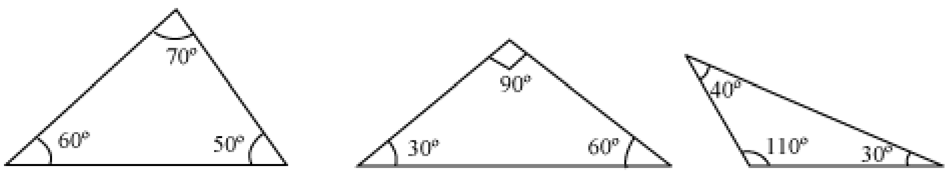
In the second triangle, two sides are equal and the third one is of a different length.

In the third triangle, all the sides are of equal lengths.

These are actually different types of triangles. One of the ways of classifying triangles is on the basis of the lengths of their sides.

Is the length of the sides the only criterion for the classification of triangles?

No. Let us first consider the following triangles.



What do you observe in these triangles?

Observe the following points.

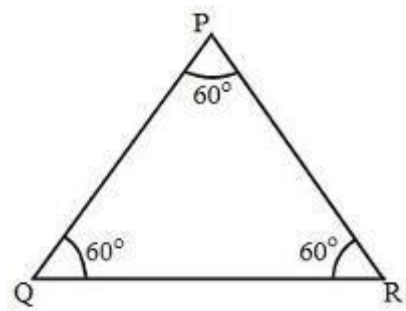
In the first triangle, all angles are less than 90°.

In the second triangle, only one angle is a right angle and the other two angles are less than 90°.

In the third triangle, one angle is more than the right angle and the other two angles are less than 90°.

These are all different types of triangles, which can be classified according to the measures of their angles.

Equiangular triangle:
An equiangular triangle is the triangle whose each angle is equal to each other, or we can say that whose each angle is equal to 60°.

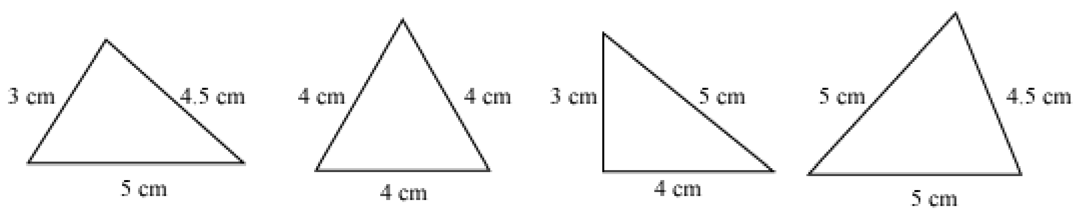


ΔPQR is an equiangular triangle as its each angle measures 60°.
i.e.,□ P = Q = R = 60°

Let us now look at some more examples to understand the concept better.

Example 1:

Classify the following triangles based on the nature of their sides.



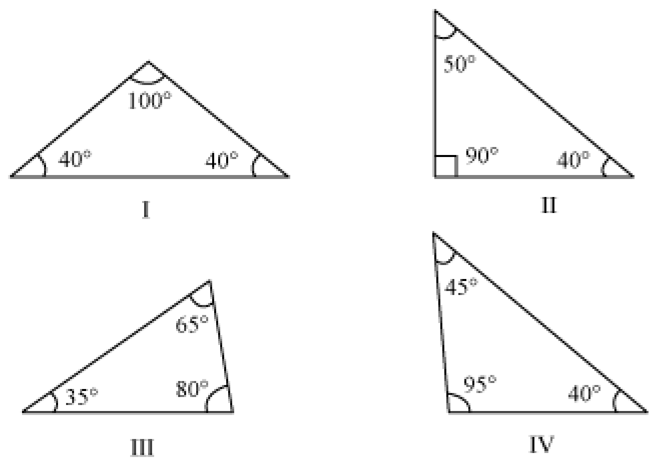
(i) **(ii)** **(iii)** **(iv)**

Solution:

1. Since all the sides are of different lengths, it is a scalene triangle.
2. Since all the sides are of same lengths, it is an equilateral triangle.
3. Since all the sides are of different lengths, it is a scalene triangle.
4. Since two sides of the given triangle are equal, it is an isosceles triangle.

Example 2:

Classify the following triangles based on the nature of their angles.

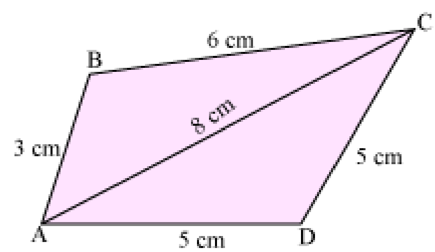


Solution:

1. Since one of the three angles is greater than a right angle, it is an obtuse-angled triangle.
2. Since one angle is a right angle, it is a right-angled triangle.
3. Since all the angles are less than 90°, it is an acute-angled triangle.
4. Since one of the angles is greater than a right angle, it is an obtuse-angled triangle.

Example 3:

Classify the triangles ABC and ADC based on the nature of their sides.



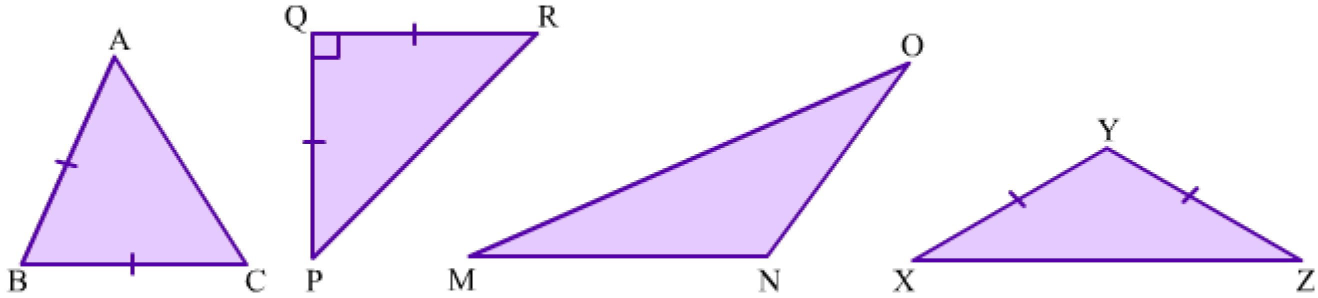
Solution:

ΔABC is a scalene triangle because all the three sides are of different lengths.

ΔACD is an isosceles triangle because two of its sides, AD and CD, are of equal lengths.

Example 4:

Classify the following triangles based on the nature of their angles and sides both.



Solution:

ΔABC has two equal sides and all of its angles are acute angles, so it is an acute angled isosceles triangle.

ΔPQR is a right angled triangle having two equal sides, so it is a right angled isosceles triangle.

ΔMNO is an obtuse angled triangle having three unequal sides, so it is an obtuse angled scalene triangle.

ΔXYZ is an obtuse angled triangle having two equal sides, so it is an obtuse angled isosceles triangle.

Various Types of Quadrilaterals and Their Properties

Quadrilaterals are four-sided polygons. There are various types of quadrilaterals such as parallelograms, trapeziums, rectangles, and squares. Let us look at this video to learn what it is that makes these types of quadrilaterals different from one another.

Let us now look at following examples.

Example 1:

State whether the following statements are true or false.

1. **Each angle of a parallelogram is a right angle.**
2. **All the sides of a rectangle are equal in length.**
3. **The opposite sides of a trapezium are parallel.**
4. **The diagonals of a rectangle are perpendicular to one another.**

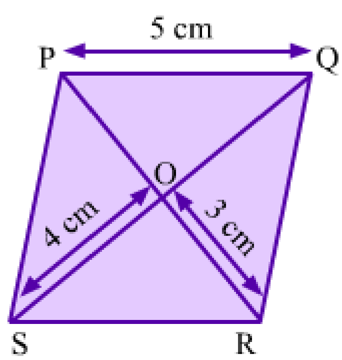
5. **All the sides of a square are of equal lengths.**
6. **Opposite angles of a parallelogram are equal.**

Solution:

1. False
2. False
3. False
4. False
5. True
6. True

Example 2:

Observe the figure of given rhombus.



Find the following components of this rhombus.

(1) $m\angle SOR$

(2) $l(SQ)$

(3) $l(PO)$

(4) $l(QR)$, $l(RS)$ and $l(SP)$

Solution:

(1) Diagonals of a rhombus intersect each other at the angle of 90°.

∴ $m\angle SOR = 90°$

(2) Diagonals of a rhombus bisect each other.

∴ $l(SQ) = 2 \times l(SO) = (2 \times 4)$ cm = 8 cm

(3) Diagonals of a rhombus bisect each other.

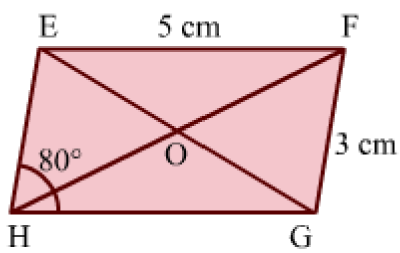
∴ $l(PO) = l(RO) = 3$ cm

(4) All sides of a rhombus are of equal length.

$l(QR) = l(RS) = l(SP) = l(PQ) = 5$ cm

Example 3:

Observe the figure of given parallelogram.



Find the following components of this parallelogram.

(1) $m\angle EFG$

(2) $l(GH)$

(3) $l(HE)$

Solution:

(1) Opposite angles of a parallelogram are equal.

∴ $m\angle EFG = m\angle GHE = 80°$

(2) Opposite sides of a parallelogram are equal.

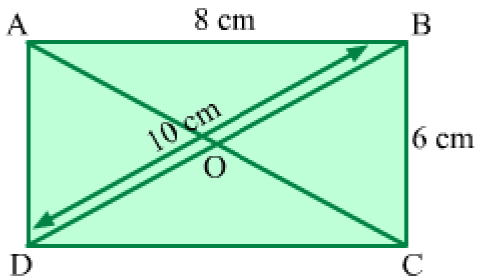
∴ $l(GH) = l(EF) = 5$ cm

(3) Opposite sides of a parallelogram are equal.

∴ l(HE) = l(FG) = 3 cm

Example 4:

Observe the figure of given rectangle.



Find the following components of this rectangle.

(1) *l*(AO)

(2) *m*∠ABC, *m*∠BCD, *m*∠CDA and *m*∠DAB

(3) *l*(CD)

(4) *l*(DA)

Solution:

(1) Diagonals of of rectangle are equal and bisect each other.

∴ l(AC) = l(BD) = 10 cm

∴ l(AO) = $\frac{1}{2}l(\text{AC})$ = 5 cm

(2) All angles of a rectangle are of 90°.

∴ m∠ABC = m∠BCD = m∠CDA = m∠DAB = 90°

(3) Opposite sides of a rectangle are equal.

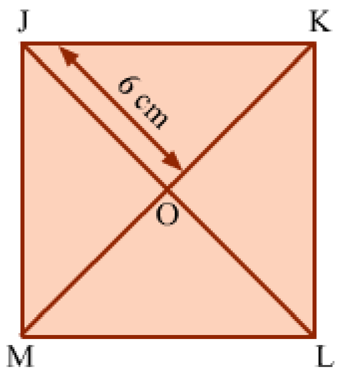
∴ l(CD) = l(AB) = 8 cm

(4) Opposite sides of a rectangle are equal.

∴ l(DA) = l(BC) = 6 cm

Example 5:

Observe the figure of given square.



Find the following components of this square.

(1) $m\angle$KOL

(2) l(LO)

(3) l(MK)

Solution:

(1) Diagonals of a square intersect each other at the angle of 90°.

∴ $m\angle$KOL = 90°

(2) Diagonals of a square bisect each other.

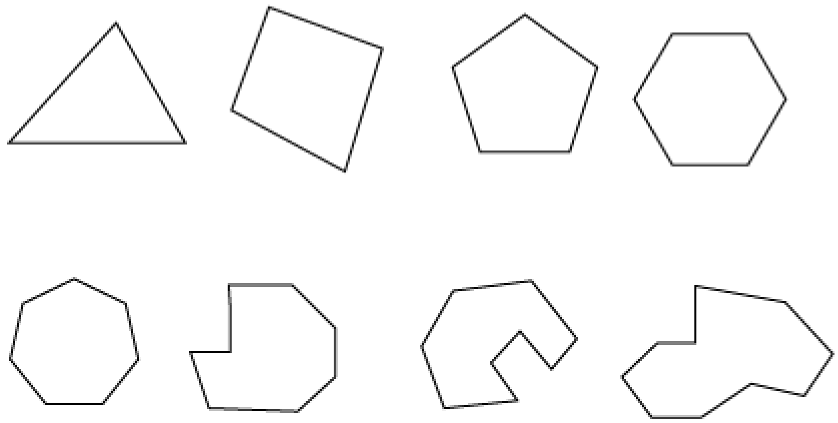
∴ l(LO) = l(JO) = 6 cm

(3) Diagonals of a square are equal.

∴ l(MK) = l(JL) = l(LO) + l(JO) = 6 cm + 6 cm = 12 cm

Classification of Polygons on the Basis of Their Sides

Look at the following figures.



What do we observe in these figures?

We observe that each figure is made up of line segments only and has different number of sides. All these figures are known as **polygons.** We know that polygons with three sides are known as ***triangles*** and polygons with 4 sides are known as ***quadrilaterals.***

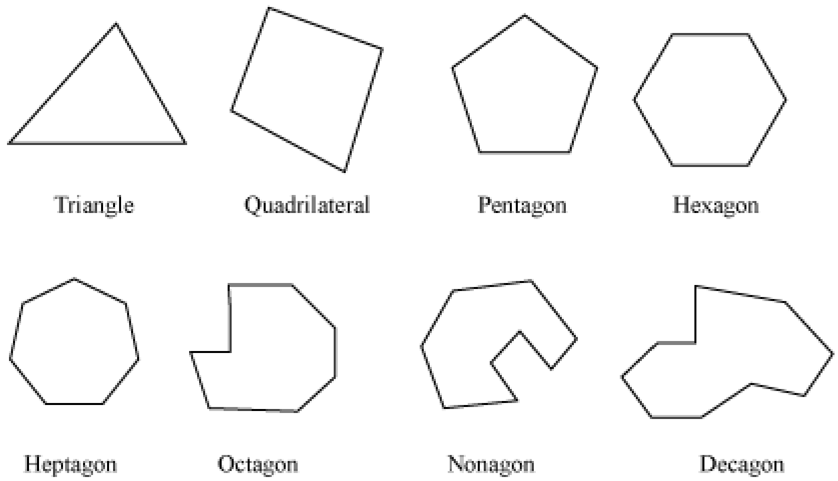
But how do we classify the polygons having more than four sides?

Let us see.

We can classify polygons on the basis of the number of sides as follows:

- **Pentagon:** Polygon having five sides
- **Hexagon:** Polygon having six sides
- **Heptagon:** Polygon having seven sides
- **Octagon:** Polygon having eight sides
- **Nonagon:** Polygon having nine sides
- **Decagon**: Polygon having ten sides

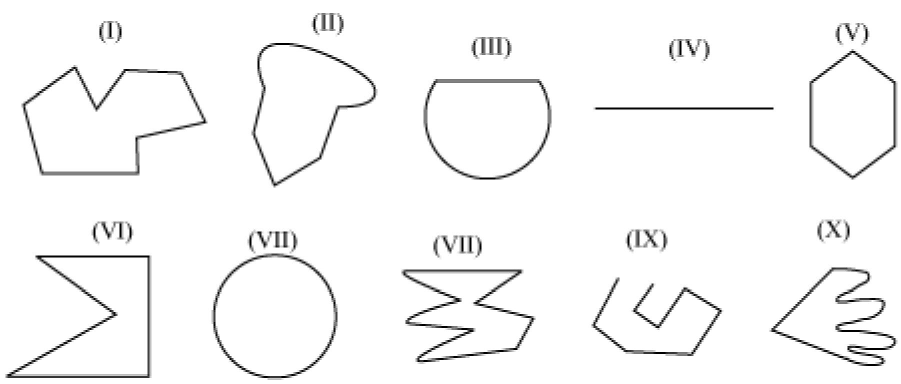
Therefore, now we can classify the polygons in the above given figures.



Let us now look at some more examples to understand this concept better.

Example 1:

Identify and name the polygons out of the following figures.



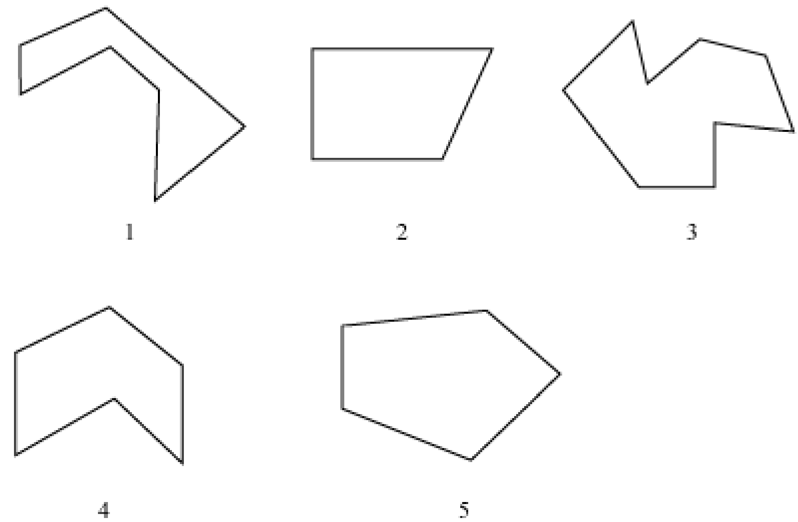
Solution:

1. The closed figure is made of nine line segments. Therefore, it is a nonagon.
2. The closed figure has a curve. Therefore, it is not a polygon.
3. The closed figure has a curve. Therefore, it is not a polygon.
4. The figure has only one line segment. A polygon should have at least three line segments. Therefore, it is not a polygon.
5. The closed figure is made of six line segments. Therefore, it is a hexagon.
6. The closed figure is made of five line segments. Therefore, it is a pentagon.
7. The closed figure is a curve. Therefore, it is not a polygon.
8. The closed figure is made of nine line segments. Therefore, it is a nonagon.
9. The figure is not closed. Therefore, it cannot be a polygon.

10. The closed figure has curves. Therefore, it is not a polygon.

Example 2:

Name the following polygons.



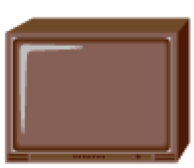
Solution:

1. The given figure has seven sides. Therefore, it is a heptagon.
2. The given figure has four sides. Therefore, it is a quadrilateral.
3. The given figure has nine sides. Therefore, it is a nonagon.
4. The given figure has six sides. Therefore, it is a hexagon.
5. The given figure has five sides. Therefore, it is a pentagon.

Example 3:

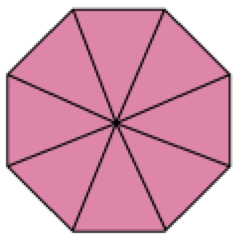
Write the number of sides and the types of polygons represented by the following figures.

(i)



(ii)

(iii)



Solution:

1. This figure has four sides. Therefore, it is a quadrilateral.
2. This figure has three sides. Therefore, it is a triangle.
3. This figure has eight sides. Therefore, it is an octagon.

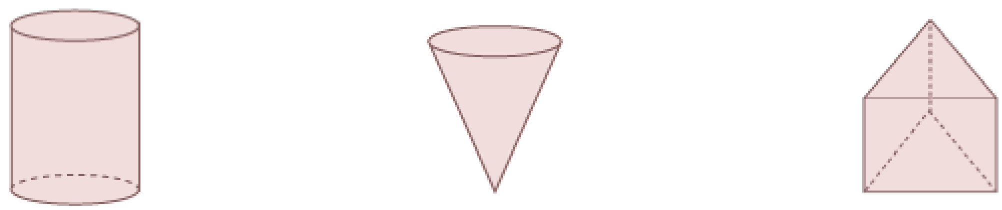
Identification of Three-dimensional Shapes

Similar to two-dimensional shapes, we have various types of three-dimensional objects, which are classified on the basis of the nature of arrangement and orientation of various faces of the shape.

Let us now look at some more examples to understand this concept better.

Example 1:

Identify the following shapes.



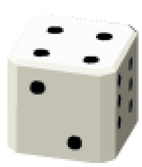
(i) **(ii)** **(iii)**

Solution:

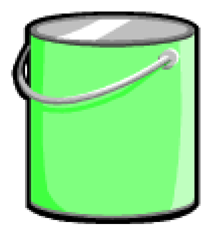
1. Cylinder
2. Cone
3. Prism
4. Cube

Example 2:

Identify the following three-dimensional shapes.



(a)



(b)

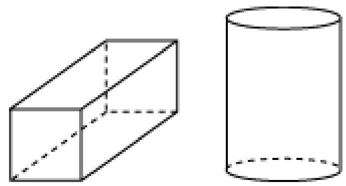
Solution:

1. This figure is cubical in shape. A dice has six sides and all of them are equal. Such types of shapes are known as cubes.

(b) This figure is cylindrical in shape.

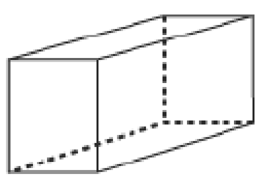
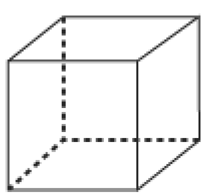
Attributes of Three-dimensional Shapes

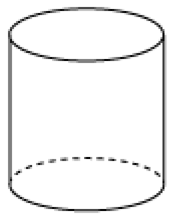
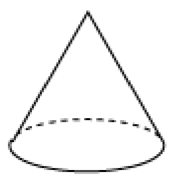
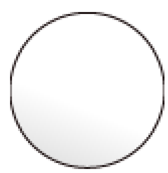
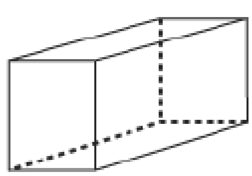
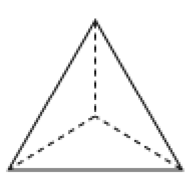
Consider the following figures.

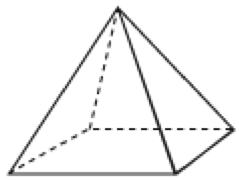
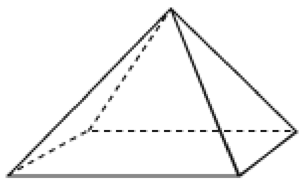


Both of the above are three-dimensional figures. The first one is a cuboid and the second one is a cylinder. Faces, vertices, and edges are the attributes of three-dimensional figures.

hree-dimensional figures.

Name	Shape	No. of straight edges	No. of faces	No. of Vertices	Example
Cuboid		12	6	8	Pencil box, notebook
Cube		12	6	8	Dice

Cylinder		None	Two flat faces and one curved surface	None	Can, cooking cylinder
Cone		None	One flat face and one curved surface	1	Softy cone, birthday cap
Sphere		None	None	None	Ball
Triangular prism		9	5	6	Laboratory prisms
Rectangular prism		12	6	8	A rectangular glass slab
Triangular pyramid		6	4	4	

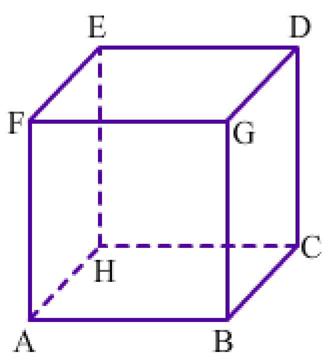
Square pyramid		8	5	5	The great pyramids of Egypt
Rectangular pyramid		8	5	5	

We know about the top and base of the solid, let us learn about its lateral face(s).

The faces that join the bases of a solid are called **lateral faces**.

We know that a cube has six square faces. Any face of the cube can be taken as its base.

Consider the cube shown below.



Here, ABCH is the base of the cube and EFGD is the top of the cube.

Rest four faces of the cube, namely ABGF, BGCD, CDEH and AHEF are the lateral faces of the cube as these faces meet

the base as well as the top of the cube.

Let us now look at some examples.

Example 1:

Find the number of faces and vertices of the following three-dimensional shapes.

(i) **Cuboid** **(ii)** **Cube** **(iii)** **Cylinder** **(iv)** **Cone**

(v) **Sphere**

Solution:

1. A cuboid has six faces and eight vertices.
2. A cube has six faces and eight vertices.
3. A cylinder has two flat faces and one curved surface. It has no vertices.
4. A cone has one flat face and one curved surface. It has one vertex.
5. A sphere has no flat face. Also, it has no vertex.

Example 2:

Find the number of faces and edges of the following three-dimensional shapes.

1. **Triangular prism**
2. **Rectangular prism**
3. **Triangular pyramid**
4. **Rectangular pyramid**

Solution:

1. A triangular prism has five faces and nine edges.
2. A rectangular prism has six faces and twelve edges.
3. A triangular pyramid has four faces and six edges.
4. A rectangular pyramid has five faces and eight edges.