Sample Paper - 4

GENERAL INSTRUCTIONS

All questions are compulsory.
The question paper consist of 30 questions divided into four sections A, B, C and D. Section A comprises of 6 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 8 questions of 4 marks each.
There is no overall choice.
Use of calculator is not allowed.

SECTION-A

(1 mark each)

1. Suppose that the division $x \div 5$ leaves a remainder 4 and the division $x \div 2$ leaves a remainder 1. Find the ones digit of x.

2. Find the number of digits in the square root of 4489. (Without any calculation).

3. Express 16^{-2} as a power with the base 2.

4. Factonse : $\frac{x^2}{9} - \frac{y^2}{25}$

5. How many vertices are there of a sphere?

6. Find the product of $(-3x^2y) \times (4x^2y - 3xy^2 + 4x - 5y)$.

SECTION-B

(2 marks each)

7. 160 m^3 of water is to be used to irrigate a rectangular field whose area is 800 m^2 . What will be the height of the water level in the field?

8. If $117\frac{1}{3}$ m long rope is cut into equal pieces measuring $7\frac{1}{3}$ m each. How many such small pieces are there?

9. Factorise the following :

(a) $\frac{x^2}{4} + 2x + 4$

(b) $16x^2 + 40x + 25$

10. Find the side of a square whose area is equal to the area of a rectangle with sides 6.4 m and 2.5 m.

11. A colour TV is available for Rs. 26880 inclusive of VAT. If the original cost of the TV is Rs. 24,000, find the rate of VAT.

12. The length and breadth of a rectangle are $3x^2 - 2$ *and* $2x + 5$ respectively. Find its area.

SECTION-C

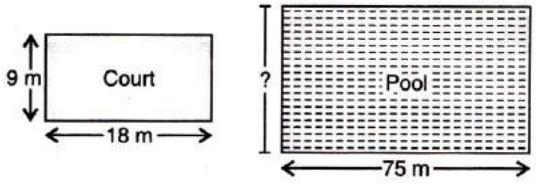
(3 marks each)

13. (a) Find the area of rectangular park which is $36\frac{3}{5}$ m long and $16\frac{2}{3}$ m broad.

(b) Write the name of property for any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have $\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$

14. If 756 x is divisible by 11, where x is a digit find the value of x.

15. A volleyball court is in a rectangular shape and its dimensions are directly proportional to the dimensions of the swimming pool given below. Find the width of the pool.

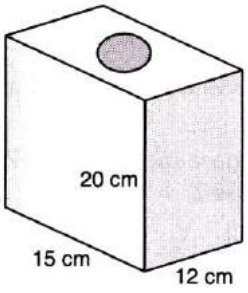


16. The denominator of a rational number is greater then its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

17. If $\frac{5^m\times5^3\times5^{-2}}{5^{-5}}=5^{12}$, find m.

18. Construct a quadrilateral ABCD in which AB = 5-6 cm, BC = 4.1 cm, CD = 4.4 cm, AD = 3.3 cm and $\angle A = 75°$.

19. Find the area to be painted in the following block with a cylindrical hole. Given that length is 15 cm, weight 12 cm, and radius of the hole 2.8 cm.



20. Prove that if x number is doubled then its cube is 8 times cube of the given number.

21. Factorise the following :

(a) $a^3-4a^2+12-3a$ (b) $4x^2-20x+25$

22. Vishakha offers a discount of 20% on all the items at her shop and still makes a profit of 12%. What is the cost price of an article marked at Rs. 280?

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SECTION-D

(4 marks each)

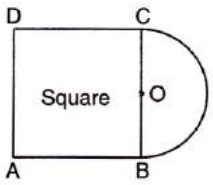
23. Factorise and divide the following :

(a) $\left(x^2-22x+117\right)\div\left(x-13\right)$ (b) $\left(9x^2-4\right)\div\left(3x+2\right)$

24. It is given that I varies directly as m.
(a) Write an equation which relates 1 and m.
(b) Find the constant of proportion (k), when 1 is 6 then m is 18.
(c) Find 1, when m is 33.
(d) Find m when 1 is 18.

25. The product of two rational numbers is $\frac{-28}{75}$. If one of the numbers is $\frac{14}{25}$. Find the other.

26. (a) A cylindrical tank has a capacity of $5632\,m^3$. If the diameter of its base is 16 m. Find its depth.
(b) If side of square is 14 cm, then find the area of semi-circle as shown in the figure.



27. Lakshmi is a cashier in a bank. She has currency notes of denominations Rs.100,Rs. 50 and Rs. 10 respectively. The ratio of the number of these notes is 2: 3: 5. The total cash with Lakshmi is Rs. 400,000. How many notes of each denomination does she have?

28. In a quadrilateral ABCD, DO and CO are the bisectors of $\angle D$ and $\angle C$ respectively.

Prove that $\angle COD = \frac{1}{2}[\angle A + \angle B]$.

29. A shopkeeper bought two TV sets at Rs. 10,000 each. He sold one at a profit 10% and the other at a loss of 10%. Find whether he made an overall profit or loss.

30. Divide 63 $\left(p^4 + 5p^3 - 24p^2\right)$ *by* $9p(p+8)$

Solutions

Section 'A'

1. Since, $x \div 5$ leaves a remainder 4, so ones digit of x can be 4 or 9. Also, since $x \div 2$ leaves-a remainder 1, so ones digit must be 9 only. **1**

2. For 4489, n = 4 [Even number] **½**

$\therefore$ Number of digits in its square root $= \frac{n}{2} = \frac{4}{2} = 2$

½

3. $16^{-2} = \frac{1}{(16)^2} = \frac{1}{(2^4)^2} = \frac{1}{2^8}$ $[\because (a^m)^n = a^{mn}]$

$= 2^{-8}$ **1**

4. $\frac{x^2}{9} - \frac{y^2}{25} = \left(\frac{x}{3}\right)^2 - \left(\frac{y}{5}\right)^2$

$= \left(\frac{x}{3} + \frac{y}{5}\right)\left(\frac{x}{3} - \frac{y}{5}\right)$ **1**

5. There are 0 vertices of a sphere. **1**

6. $(-3x^2 y) \times (4x^2 y - 3xy^2 + 4x - 5y) = -12x^4 y^2 + 9x^3 y^3 - 12x^3 y + 15x^2 y^2$

Section 'B'

(2 marks each)

7. Volume of water s= 160 m^3

Area of rectangular field = 800 m^2

Let h be the height of water level in the field.

Now, volume of water = volume of cuboid formed on the field by water.

160 = Area of base × height

= 800 × h

$h = \frac{160}{800} = 0.2$ **2**

So, required height = 0.2 m.

8. Given, Length of rope $= 117\frac{1}{3} = \frac{352}{3} m$

Length of each small piece $= 7\frac{1}{3} m = \frac{22}{3} m$ **1**

Number of such small pieces

$= \frac{\frac{352}{3}}{\frac{22}{3}} = \frac{352 \times 3}{3 \times 22}$

= 16 **1**

9. (a) $\frac{x^2}{4}+2x+4=\frac{1}{4}[x^2+8x+16]$

$=\frac{1}{4}[x^2+4x+4x+16]$

$=\frac{1}{4}[x(x+4)+4(x+4)]$

$=\frac{1}{4}[x(x+4)(x+4)]$

$=\frac{1}{4}(x+4)^2$ **1**

(b) $163x^2+40x+25=16x^2+(20+20)x+25$

$=16x^2+20x+20x+25$

$=4x(4x+5)+5(4x+5)$ **½**

$=(4x+5)(4x+5)$

$=(4x+5)^2$ **½**

10. Given, Area of square = Area of rectangle

$a^2=1\times b$ **1**

$a^2=6.4\times 2.5$

$a^2=16$

a =4

So, the side of square is 4 m. **1**

11. Let the rate of VAT = x%

Then $24000+\ x\%\ of\ \ 24000=26880$

or $\frac{x}{100}\times 24000=26880-24000$

or $240x=2880$

$x=\frac{2880}{240}=12$ **1**

Therefore, Rate of VAT = 12%

12. Here, length $3x^2-2$

breadth = 2x + 5

Area = (length) × (breadth)

$=(3x^2-2)\times 2(2x+5)$

$=3x^2(2x+5)+(-2)(2x+5)$ **1**

$=6x^3+15x^2-4x-10$

Thus, the required area of the rectangle is

$6x^3-15x^2-4x-10$ sq. units. **1**

Section 'C'

(3 marks each)

13. (a) Since length of rectangular park $=36\frac{3}{5}$ m $=\frac{183}{5}$ m

and breadth of rectangular park $= 16\frac{2}{3}m = \frac{50}{3}m$

1

Then area of park = $I \times b$

$= \frac{183}{5}m \times \frac{50}{3}m$

$= 61 \times 10m^2 = 610m^2$

(b) $\left(\frac{a}{b} \times \frac{c}{d}\right) = \left(\frac{c}{d} \times \frac{a}{b}\right)$, It is commutative law of property. **1**

14. Sum of digits of odd place = 7 + 6 = 13

Sum of digit of even place = 5 + x

Difference $(x+5) - 13 = x - 8$ (i)

Now $(x-8)$ should be equal 0 or a multiple of 11 (i.e., 11, 22, 33,... etc) **1**

$x - 8 = 0$

$\Rightarrow \quad x = 8$ *or* $x - 8 = 11$

x = 11 + 8 = 19 **1**

Since, x is a digit, so it can take value from 0-9

Hence, x = 8

Required number is 7568. **1**

15. Here, a volleyball court and a swimming pool both are in rectangular shape and their dimensions are directly proportional. Let I and b are length and breadth, respectively.

If I = b **1**

$\Rightarrow \quad \frac{l_1}{b_1} = \frac{l_2}{b_2}$

Here, $l_1 = 18m, b_1 = 9m, l_2 = 75m, b_2 = ?$

Then, $\frac{18}{9} = \frac{75}{b_2}$ **1**

$2b_2 = 75$

$b_2 = \frac{75}{2}$

$b_2 = 37.5m$

So, the width of the pool is 37.5 m. **1**

16. Let the numerator of rational number be x

and denominator = x + 8

Therefore rational number $= \frac{x}{x+8}$ **1**

According to question,

$\frac{x+17}{(x+8)-1} = \frac{3}{2}$

or, $\frac{x+17}{x+7} = \frac{3}{2}$

or, $2(x+17) = 3(x+7)$

or, $2x + 34 = 3x + 21$

or $2x - 3x = 21 - 34$

[Transposing 3x to LHS and 34 to RHS]

or, $-x = -13$

or, $x = 13$

Hence, the required rational number

$= \frac{13}{13+8} = \frac{13}{21}$ **1**

17. $\frac{5^m \times 5^3 \times 5^{-2}}{5^{-5}} = 5^{12}$

$5^m \times 5^3 \times 5^{-2} \times 5^5 = 5^{12}$

$5^m \times 5^{3-2+5} = 5^{12}$

$5^m \times 5^6 = 5^{12}$ **1**

$m+6=12$

$m = 12-6$

$M=6$ **1**

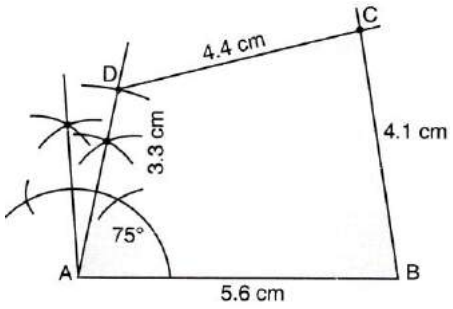
18. Steps of Construction:

(a) Draw a line segment AB = 5-6 cm. **1**

(b) Making $\angle A = 75°$ with centre A.

(c) Draw two arcs of radius 4-1 cm and 4-4 cm with centre B and D both arcs intersects each other at point C.

(d) Join BC and CD, then ABCD is a required quadrilateral. **1**



19. Here, l = 15 cm, b = 12 cm, h = 20 cm

and r = 2.8 cm

Surface area of cuboid = 2(lb + bh + hl)

$= 2(15\times12+12\times20+20\times15)$

$= 2(180+240+300)$

$= 2\times720$

$= 1440$ cm^2 **1**

Area of two holes $= 27\pi r^2$

$= 2\times\frac{22}{7}\times2.8\times2.8$

$= 49.28$ cm^2 **1**

So,

Required area for paint = S.A. of cuboid - area of holes

$= 1440 - 49.28$

$= 1390.72$ cm^2 **1**

20. Let y be the double of x

i.e. $y = 2x$ **1**

By using on both sides

$y^3 = (2x)^3$

$\Rightarrow\ y^3 = 2^3 \times x^3 = 2 \times 2 \times 2 \times x^3$ **1**

$\Rightarrow\ \ y^3 = 8x^3$ **1**

21. (a) $a^3 - 4a^2 + 12 - 3a = a^2(a-4) - 3a + 12$ **½**

$= a^2(a-4) - 3(a-4)$

$= (a-4)(a^2-3)$ **1**

(b) $4x^2 - 20x + 25 = (2x)^2 - 2 \times 2x \times 5 + (5)^2$ **½**

$= (2x-5)^2$

[Since, $a^2 - 2ab + b^2 = (a-b)^2$]

$= (2x-5)(2x-5)$ **1**

22. Marked Price =Rs. 280

Discount = 20% of Rs.280 **1**

$= \frac{20}{100} \times 280 = Rs.56$

So, selling price $= Rs.(280-56)$

=Rs. 224

Let the cost price be Rs. 100

Profit =12% of Rs. 100

=Rs. 12 **1**

So

Selling price = Rs. (100 + 12) = Rs. 112

If the selling price is Rs. 112, cost price = Rs. 100

If the selling price is Rs. 224, cost price = Rs.

$\frac{100 \times 224}{112}$

= Rs. 200. **1**

Section 'D'

(4 marks each)

23. (a) $(x^2 - 22x + 117) \div (x-13)$

$\because\ x^2 - 22x + 117 = x^2 - (13+9)x + 117$

$= x^2 - 13x - 9x + 117$

$= x(x-13) - 9(x-13)$

$= (x-13)(x-9)$ **1**

$\therefore\ \frac{x^2 - 22x + 117}{(x-13)} = \frac{(x-13)(x-9)}{(x-13)} = x-9$ **1**

(b) $(9x^2 - 4) \div (3x+2)$

$\because\ \ 9x^2 - 4 = (3x)^2 - (2)^2$

$= (3x+2)(3x-2)$ **1**

$\therefore \quad \frac{9x^2-4}{(3x+2)} = \frac{(3x+2)(3x-2)}{(3x+2)} = (3x-2)$ **1**

24. Given I $\propto$ m

(a) I = km, where k is a constant **1**

(b) l = 6, m = 18

l =km

$6 = k \times 18$

$\Rightarrow \qquad k = \frac{6}{18}$

$\therefore \qquad k = \frac{1}{3}$ **1**

(c) m =33, l=?

$\because$ I = km

$l = \frac{1}{3} \times 33 \qquad \left[\because k = \frac{1}{3}\right]$

$\therefore$ l=11 **1**

(d) l = 8,m=?

l=km

$\Rightarrow \quad 8 = \frac{1}{3} \times m$

M=24 **1**

25. $\because$ Product of two numbers $= -\frac{28}{75}$

Any one of the rational number $= \frac{14}{25}$ **1**

$\therefore$ The other number $= \left[\frac{-28}{75}\right] \div \frac{14}{25}$ **1**

$= \frac{-28}{75} \times \frac{25}{14}$ **1**

$= \frac{-2 \times 1}{3 \times 1} = -\frac{2}{3}$

Thus, the required rational number is $-\frac{2}{3}$. **1**

26. (a) Let the depth of the cylindrical tank = h

and Radius of its base (r) = 8 m

Then, the capacity of the tank = Volume of the tank

$= \pi r^2 h$

$= \frac{22}{7} \times 8 \times 8h$ **1**

Thus, $\frac{22}{7} \times 8 \times 8 \times h = 5632$

or $h = \frac{5632 \times 7}{22 \times 8 \times 8}$ **1**

or h =28m

Hence, the depth of the cylindrical tank = 28 m

(b) Since side of square = 14 cm

Then, diameter of semi-circle = 14 cm

Therefore, radius of semi-circle (r) $= \frac{14}{2}$

$= 7\text{cm}$

Thus, Area of semi-circle $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7^2$

$= \frac{11}{7} \times 7 \times 7$

$= 77\ \text{cm}^2$ **1**

27. Let number of notes be 2x, 3x and 5x

According to question,

$2x \times 100 + 3x \times 50 + 5x \times 10 = 4,00,000$ **1**

or $200x + 150x + 50x = 4,00,000$

or $400x = 4,00,000$

or $x = \frac{4,00,000}{400}$ **1**

[Dividing both sides by 400]

or x = 1,000

Hence, Number of denomination of Rs. 100 notes

$= 2 \times 1,000$

=2,000 **1**

Number of denomination of Rs. 50 notes

$= 3 \times 1,000$

= 3,000

Number of denomination of notes Rs. 10 notes

$= 5 \times 1,000$

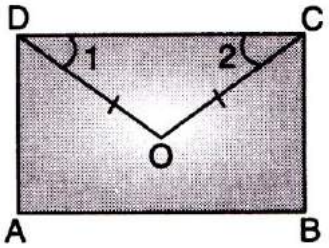
= 5,000

Hence, required denominations of notes of Rs. 100, Rs. 50 and Rs. 10 are 2,000, 3,000 and 5,000 respectively. 1

28. In ΔCOD, we have

$\angle COD + \angle 1 + \angle 2 = 180°$

$\Rightarrow \quad \angle COD = 180° - [\angle 1 + \angle 2]$



$\Rightarrow \angle COD = 180° - \left[\frac{1}{2}\angle D + \frac{1}{2}\angle C\right]$

$\Rightarrow \angle COD = 180° - \frac{1}{2}[\angle D + \angle C]$

But $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad \angle C + \angle D = 360° - (\angle A + \angle B)$

$\Rightarrow \quad \angle COD = 180° - \frac{1}{2}[360° - (\angle A + \angle B)]$ **1**

$= 180° - \frac{1}{2}\ [360°]\ +\ \frac{1}{2}\ [\angle A\ +\ \angle B]$

$= 180° - 180°\ +\ \frac{1}{2}\ [\angle A\ +\ \angle B]$ **1**

$= \frac{1}{2}(\angle A + \angle B)$

Thus, $\angle COD = \frac{1}{2}[\angle A + \angle B]$ **1**

29. In first case, when shopkeeper found a profit.

Cost Price (C.E) =Rs. 10,000

Profit% =10%

Selling Price (S.E) =?

We know that,

$S.P. = \frac{(100 + \text{Pr}\,ofit\%)}{100} \times C.P.$ **1**

$= \left(\frac{100+10}{100}\right) \times 10,000$

S.P. $= 110 \times 100 =$ Rs. 11,000

Selling price (S.E) of first case =Rs. 11,000 **1**

In second case,

When shopkeeper found a loss,

C.P. =Rs.10, 000

Loss% = 10%

S.P. =?

We know that, $S.P. = \frac{(100 - loss\%)}{100} \times C.P.$

$= \frac{(100-10)}{100} \times 10,000$

$= 90 \times 100$

S.P. =Rs. 9,000

S.P. of second case =Rs. 9,000 **1**

According to whole transaction,

Total C.E of both cases $= 10,000 \times 2 = Rs.20,000$

Total S.E of both cases = (11,000 + 9,000)

=Rs. 20,000

Hence, these is no profit or no loss. **1**

30. We have, $63(p^4+5p^3-24p^2)\div 9p(p+8)$

$$=\frac{63(p^4+5p^3-24p^2)}{9p(P+8)}$$ **1**

$$=\frac{63p^2(p^2+5p-24)}{9p(p+8)}$$

$$=\frac{63p^2}{9p}\left[\frac{(p^2+5p-24)}{(p+8)}\right]$$ **1**

$$=7p\left[\frac{p^2+8p-3p-24}{p+8}\right]$$

$$=7p\left[\frac{p(p+8)-3(p+8)}{(p+8)}\right]$$ **1**

$$=7p\left[\frac{(p+8)(p-3)}{(p+8)}\right]$$

$$=7p(p-3).$$ **1**