CGPET 2014

Solved Paper

Question 1

The work done in increasing the size of a rectangular soap film with dimensions $8\,\text{cm} \times 3.75\,\text{cm}$ to $10\,\text{cm} \times 6\,\text{cm}$ is $2 \times 10^{-4}\text{J}$. The surface tension of the film in Nm^{-1} is

Options:

A. 3.3×10^{-2}

B. 1.65×10^{-2}

C. 8.25×10^{-2}

D. 6.6×10^{-2}

Answer: A

Solution:

Solution:

Surface tension, $S = \dfrac{\text{Work done}}{\text{Increase in area}}$

$= \dfrac{2 \times 10^{-4}}{2(10 \times 6 - 8 \times 3.75) \times 10^{-4}}$

$= 0.033$

$= 3.3 \times 10^{-2}\text{Nm}^{-1}$

Question 2

A body of mass m rises to a height $h = \dfrac{R}{5}$ from the earth's surface, where R is the earth's radius. If g is acceleration due to gravity at the earth's surface, the increase in potential energy will be

Options:

A. mgh

B. $\dfrac{4}{5}mgh$

C. $\dfrac{5}{6}mgh$

D. $\dfrac{6}{7}mgh$

Answer: D

Solution:

Solution:
Increase in potential energy
= Final potential energy − Initial potential energy
$= -\frac{GMm}{(R+\frac{R}{5})} - (-\frac{GMm}{R})$
$= -\frac{5}{6}\frac{GMm}{R} + \frac{GMm}{R}$
$= \frac{1}{6}\frac{GMm}{R} = \frac{1}{6}\frac{GMmR}{R^2}$
$= \frac{1}{6}mgR \left[\because g = \frac{GM}{R^2}\right]$
$= \frac{5}{6}mgh \ [\because h = \frac{R}{5}]$

Question 3

Two springs of spring constant 2000Nm^{-1} and 1000Nm^{-1} are stretched with same force. They will have potential energy in the ratio of

Options:

A. 4:1

B. 2:1

C. 1:2

D. 1:4

Answer: A

Solution:

Solution:
Potential energy, $U = \frac{1}{2}\frac{F^2}{K}$
$\Rightarrow \ U \propto \frac{1}{K}$ [$\because F$ is same]
$\therefore \ \frac{U_1}{U_2} = \frac{K_2}{K_1}$
$= \frac{1000}{2000}$
$= \frac{1}{2}$
= 1:2

Question 4

The energy required to break the covalent bond in a semiconductor is

Options:

A. always $1\,eV$

B. equal to the forbidden energy gap of semiconductor

C. equal to fermi energy

D. much less than fermi energy

Answer: B

Solution:

Solution:
To break the covalent bond in a semiconductor, energy equal to forbidden energy gap is required.

Question 5

Consider the following statements.
I. Random and excess exposure to X-rays may induce diseases.
II. X-rays has a damaging effect on the living cells of a body which may lead to cell death.

Options:

A. Both I and II are true

B. I is true but II is not true

C. II is true but I is not true

D. Both I and II are not true

Answer: A

Solution:

Solution:
Random and excess exposure to X-rays may induce diseases and has a damaging effect on the living cells of a body.

Question 6

In the potentiometer experiment, if deflection in galvanometer is measured zero, then the current will become zero in

Options:

A. potentiometer wire

B. galvanometer circuit

C. main circuit

D. cell

Answer: B

Solution:

Solution:
In a potentiometer experiment, if deflection in galvanometer is measured zero, then current in galvanometer circuit will also become zero.

Question 7

When a glass prism of refracting angle 60° is immersed in a liquid its angle of minimum deviation is 30°. The critical angle of glass with respect to the liquid medium is

Options:

A. 45°

B. 42°

C. 50°

D. 52°

Answer: A

Solution:

Solution:
Given, $A = 60^\circ$ and $\delta_m = 30^\circ$

So, $\mu = \frac{sin\left(\frac{A+\delta_m}{2}\right)}{sin\ \frac{A}{2}} = \frac{sin\ (\frac{60^\circ + 30^\circ}{2})}{sin\ \frac{60^\circ}{2}}$

$= \frac{sin\ 45^\circ}{sin\ 30^\circ} = \frac{1}{\sqrt{2}} \times 2 = \frac{2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

We have, $\mu = \frac{1}{sin\ C}$

or $\sqrt{2} = \frac{1}{sin\ C}$ or $C = sin^{-1}(\frac{1}{\sqrt{2}}) = 45^\circ$

Question 8

If one penetrates a uniformly charged conducting spherical shell, the electric field E is

Options:

A. increases

B. decreases

C. remains same as it is on surface

D. zero at all points

Answer: D

Solution:

Solution:
The electric field (E) inside a uniformly charged conducting spherical shell is zero at all points.

Question 9

Faraday constant

Options:

A. depends on the amount of the electrolyte

B. depends on the current in the electrolyte

C. is a universal constant

D. depends on the amount of charge passed through the electrolyte

Answer: C

Solution:

Solution:
Faraday constant is a universal constant.

Question 10

Interference effect is observed in

Options:

A. only transverse wave

B. only longitudinal wave

C. Both (a) and (b)

D. None of the above

Answer: C

Solution:

Solution:
Interference effect can be observed in both transverse wave and longitudinal wave.

Question 11

A comb is run through wet hair on a rainy day, then

Options:

A. it will attract large number of small bits of paper

B. it will not go through the hair

C. it will not attract small bits of paper

D. None of the above

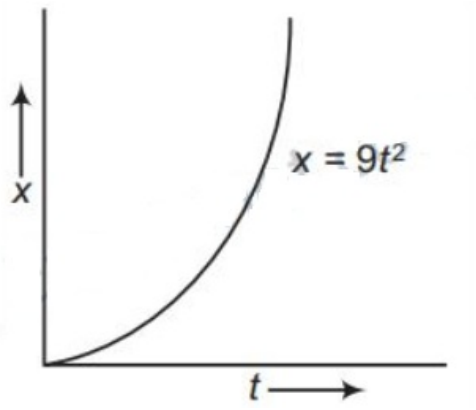
Answer: C

Solution:

Solution:
If a comb is run through wet hair on rainy day, then it will not attract small bits of paper.

Question 12

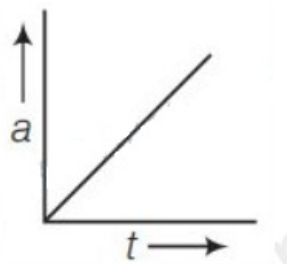
The displacement-time graph of a particle moving along a straight line is shown in the figure.



The acceleration-time graph of this particle is.

Options:

A.



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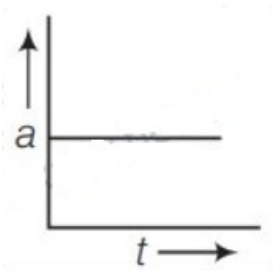
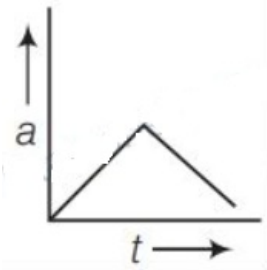
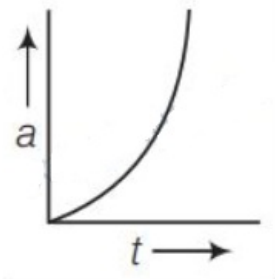


C.



D.



Answer: B

Solution:

Solution:
Given, $x = 9t^2$
$\therefore\ v = \frac{dx}{dt} = \frac{d}{dt}(9t^2) = \frac{9d}{dt}(t^2) = 18t$
also, $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{d}{dt}(18t) = 18$
So, the graph between acceleration and time is a line parallel to time axis as acceleration is constant.

Question 13

Calculate the heat required to increase the temperature of 1 mole of one atomic gas from $0^\circ C$ to $150^\circ C$, when no work is done. [$C_p = 2.5R$. and $R = 8.3 J mol^{-1} K^{-1}$]

Options:

A. 867.5J

B. 186.5J

C. 1867.5J

D. 86.7J

Answer: C

Solution:

Solution:
From first law of thermodynamics,
$dQ = dU + dW$
For a isochoric process, $dW = 0$

$\therefore \quad dQ = dU$
$\therefore$ Amount of heat required, $dQ = nC_V(T_2 - T_1)$
$= n\dfrac{C_V}{C_p}C_p(T_2 - T_1)$
$= 1 \times \gamma \times 2.5 \times 8.3 \times (423 - 273)$
$= \dfrac{3}{5} \times 2.5 \times 8.3 \times 150$
$= 1867.5\text{J}$

Question 14

The maximum kinetic energy of photoelectrons coming out of a metal surface is 10 eV. The minimum voltage required to stop, the emission of electrons from this metal surface is

Options:

A. 10V

B. 5V

C. -5V

D. -10V

Answer: A

Solution:

Solution:
The minimum negative potential V_0 is called stopping potential.
Here, $K_{\max} = 10\,\text{eV}$
So, the stopping potential, $eV_0 = K_{\max}$
$\Rightarrow \text{eV}_0 = 10\,\text{eV}$
or $V_0 = -10\text{V}$

Question 15

Tangent galvanometer is not useful to measure current, because

Options:

A. this is not directly readable

B. same current will give different readings at different places

C. near by magnetic material will effect the readings

D. All of the above

Answer: D

Solution:

Solution:
Tangent galvanometer is not useful to measure current as it is not directly readable, at different places different readings are obtained and if there is any magnetic material near it, it will effects its readings.

Question 16

Thermal radiation exist in which part of electromagnetic spectrum?

Options:

A. Ultraviolet

B. Infrared

C. Visible

D. Violet

Answer: B

Solution:

Solution:
Thermal radiations exists in infrared part of the electromagnetic spectrum.

Question 17

Two similar cells are connected first in series and then in parallel, the ratio of balancing length on the potentiometer wire will be

Options:

A. 1:2

B. 2:1

C. 1:4

D. 4:1

Answer: B

Solution:

Solution:
When two similar cells are connected in series.
The effective emf, $E_1 = E + E = 2E$
When two similar cells are connected in parallel.
Then, the effective emf $E_2 = E$
According to potentiometer wire,

$\frac{E_1}{E_2} = \frac{I_1}{I_2} \Rightarrow \frac{2E}{E} = \frac{I_1}{I_2}$
Hence, $I_1 : I_2 = 2:1$

Question 18

The dimensions of $\frac{a}{b}$ in the equation $p = \frac{a - t^2}{bx}$, where p is pressure, x is distance and t is time, is

Options:

A. $[\mathrm{M}^2 \,\mathrm{corner}\, \mathrm{T}^{-3}]$.

B. $[\mathrm{MT}^{-2}]$

C. $[\mathrm{ML}^3\mathrm{T}^{-1}]$

D. $[M^2L^{-3}T^2]$

Answer: B

Solution:

Solution:
Given, $p = \frac{a - t^2}{bx}$
$\Rightarrow \quad p = \frac{a}{bx} - \frac{t^2}{bx}$
By the principle of homogeneity
$p = \frac{a}{bx} \Rightarrow \frac{a}{b} = px$
$[\frac{a}{b}] = [p][x]$
$= [\mathrm{ML}^{-1}\mathrm{T}^{-2}][L] = [\mathrm{MT}^{-2}]$

Question 19

A rod of length L is composed of a uniform length $\frac{L}{2}$ of wood whose mass in m_w and a uniform length $\frac{L}{2}$ of brass whose mass is m_b. The moment of inertia I of the rod about an axis perpendicular to the rod and through its centre is equal to

Options:

A. $(m_w + m_b)\,\frac{L^2}{6}$

B. $(m_w + m_b)\,\frac{L^2}{2}$

C. $(m_w + m_b)\ \frac{L^2}{12}$

D. $(m_w + m_b)\ \frac{L^2}{3}$

Answer: C

Solution:

Solution:
For a thin uniform rod, moment of inertia about an axis through its centre perpendicular to length of rod, $I = \frac{1}{12}ML^2$
Here, $M = (m_w + m_b)$
$\therefore\ I = \frac{1}{12}(m_w + m_b)L^2$

Question 20

A particle, doing simple harmonic motion, at a distance $3\,\text{cm}$ from mean position has acceleration $12\,\text{cm}/s^2$. What is its time period?

Options:

A. $0.5\,s$

B. $1\,s$

C. $2\,s$

D. $3.14\,s$

Answer: D

Solution:

Solution:
The time period of a particle executing SHM is given as
$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$
$= 2\pi\sqrt{\frac{3\times10^{-2}}{12\times10^{-2}}} = 2\pi\times\frac{1}{2}$
$= \pi = 3.14\,s$

Question 21

If the earth did not rotate on its axis, the magnitude of the gravitational acceleration at the equator would be about

Options:

A. $0.003\,\%$ larger

B. 0.3% larger

C. 0.3% smaller

D. 0.003% smaller

Answer: B

Solution:

Solution:
The value of gravitational acceleration at equator due to rotation of the earth is
$g_{eq} = g - R\omega^2$
If the earth stops rotating, then $g_{eq} = g$
So, change in g, $\Delta g = R\omega^2 = 3.41 \times 10^{-2} ms^{-2}$
Hence % increase in $g = \frac{\Delta g}{g} \times 100$
$= \frac{3.41 \times 10^{-2}}{9.8} \times 100$
$= 0.3\%$

Question 22

A charge particle enters a magnetic field B with its initial velocity v making an angle of 45° with B. The path of the charge particle will be

Options:

A. a straight line

B. a circle

C. an ellipse

D. a helix

Answer: D

Solution:

Solution:
If the moving charged particle enters in a magnetic field making some angle with its initial direction of motion, then it will describes a helical path.

Question 23

A step up transformer has turn ratio 10:1. A cell of emf 2V is fed to the primary, then the secondary voltage developed is

Options:

A. 20V

B. 10V

C. 2V

D. zero

Answer: D

Solution:

Solution:
A transformer is essentially an AC device, it does not work on DC. So, voltage developed across secondary is zero.

Question 24

When yellow light is refracted by a prism in minimum deviation state, then

Options:

A. angle of incidence is equal to angle of refraction

B. angle of incidence is greater than the angle of refraction

C. angle of incidence is less than angle of refraction

D. sum of angle of incidence and angle of refraction is 90°

Answer: C

Solution:

Solution:
In the state of minimum deviation the refracted ray becomes parallel to the base of the prism. So, angle of incidence is less than angle of refraction.

Question 25

If in a Young double slit experiment maximum intensity is I_{max}, then intensity at $\lambda/2$ path difference, is

Options:

A. I_{max}

B. $I_{max\,/\,2}$

C. $I_{max\,/\,4}$

D. zero

Answer: D

Solution:

Solution:
Here path difference $= \frac{\lambda}{2}$
$\therefore$ Phase difference $= \frac{2\pi}{\lambda} \times$ path difference
i.e., $\varphi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$
So, maximum intensity,
$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\pi$
$I_{max} = I + I + 2\sqrt{II}(-1)$ [$\because I_1 = I_2$]
$I_{max} = 0$

Question 26

A man at a distance 11 km from two pillars wants to see two pillars separately. What will be the approximate distance between the pillars?

Options:

A. 3m

B. 1m

C. 0.25m

D. 0.5m

Answer: A

Solution:

Solution:
Resolving power of eye $= (\frac{1}{60})^{\circ} = \frac{1}{60} \times \frac{\pi}{180}$
Let the minimum distance between the poles be d
then, $\frac{d}{11000} = \frac{1}{60} \times \frac{\pi}{180}$
or
$d = 11000 \times \frac{1}{60} \times \frac{\pi}{180}$
$= 3m$

Question 27

Dimensions of Stefan's constant is

Options:

A. $[\mathrm{MLT}^{-3}\theta^{-4}]$

B. $[\mathrm{MT}^{-3}\theta^{-4}]$

C. $[M^2T^{-3}\theta^{-4}]$

D. $[M^2T^{-2}\theta^{-4}]$

Answer: B

Solution:

Solution:
Stefan's constant, $\sigma = \frac{E}{T^4}$ (where, E is energy/s area)
$= \frac{[ML^2T^{-2}]}{[T][L^2][\theta^4]}$
$= [MT^{-3}\theta^{-4}]$

Question 28

When a current changes from 2A to 4A in 0.05 s in a coil, induced emf is 8V. The self-inductance of coil is

Options:

A. 0.1H

B. 0.2H

C. 0.4H

D. 0.8H

Answer: B

Solution:

Solution:
Here, $e = 8\mathrm{V}$
$dI = (4-2)A = 2A$
$\mathrm{dt} = 0.05\,s$
As,
$\therefore\ L = \frac{e(dt)}{dI}$
$= \frac{8 \times (0.05)}{5}$
$= 0.2\mathrm{H}$

Question 29

A force $F = Ay^2 + By + C$ acts on a body in the y-direction. Find the work

done by this force during a displacement $y = -a$ to $y = +a$.

Options:

A. $\frac{2Aa^3}{3}$

B. $\frac{2Aa^3}{3} + 2Ca$

C. $\frac{2Aa^3}{3} + \frac{Ba^3}{2} + Ca$

D. None of these

Answer: B

Solution:

Solution:
Here, the force is acting along y-direction.
$\therefore$ The work done in displacing a body from $y = -a$ to $y = +a$

$$W = \int_{-a}^{+a} F \cdot dy = \int_{-a}^{+a} F dy$$
$$= \int_{-a}^{+a} (Ay^2 + By + C)\, dy$$
$$= \int_{-a}^{+a} (Ay^2)\, dy + \int_{-a}^{+a} (By)\, dy + \int_{-a}^{+a} C dy$$
$$= \frac{2Aa^3}{3} + 2Ca$$

Question 30

Time period of oscillation of mass m suspended from a spring is T. What is the time period when the spring is cut in half and the same mass is suspended from one of the halves?

Options:

A. $T/2$

B. $T/\sqrt{2}$

C. $\sqrt{2}T$

D. $2T$

Answer: B

Solution:

Solution:
Time period of oscillation of mass m suspended from a spring

$T = 2\pi\sqrt{\frac{m}{k}}$

If the spring is cut into two halves, then the new time period.

$T' = 2\pi\sqrt{\frac{m}{2k}} = 2\,\frac{\pi}{\sqrt{2}}\sqrt{\frac{m}{k}} = \frac{T}{\sqrt{2}}$

Question 31

An electric current of 2A passes through a wire of resistance 25Ω. How much heat will be generated in 1 min?

Options:

A. 6×10^3J

B. 3.6×10^3J

C. 0.6×10^3J

D. 0.36×10^3J

Answer: A

Solution:

Solution:

Given, $I = 2.0A$

$R = 25\Omega$

$t = 1\,\text{min} = 60\,s$

$\therefore$ Heat produced, $H = I^2Rt$

$= (2)^2 \times 25 \times 60$

$= 4 \times 25 \times 60$J

$= 6 \times 10^3$J

Question 32

- Specific resistance of a conductor increases with

Options:

A. increase in temperature

B. increase in cross-section area

C. increase in cross-section area and decrease in length

D. decrease in cross-section area

Answer: A

Solution:

Solution:
Specific resistance depends on the temperature. For metals $\rho_t = \rho_0(1 + \alpha \Delta t)$ i.e., specific resistance of a conductor increases with rise in temperature.

Question 33

For hydrogen like ions with z protons, the radius of nth orbit is given by r_n (where, a_0 is Bohr radius)

Options:

A. $n^2 a_0 z^2$

B. $\frac{n^2 a_0}{z}$

C. $\frac{n^2 a_0}{z^2}$

D. $\frac{n^3 a_0^2}{z^4}$

Answer: B

Solution:

Solution:
The radius of nth orbit of hydrogen like an ion,
$r_n = \frac{n^2 h^2}{4\pi^2 k m z e^2}$
where, $\frac{h^2}{4\pi^2 k m e^2} = a_0$
(Bohr's radius)
then,
$r_n = \frac{a_0 n^2}{z}$

Question 34

A mass m hanging from a spring is doing simple harmonic motion with frequency f. If the mass is increased by 4 times, then frequency will be

Options:

A. $2f$

B. $f/2$

C. $4f$

D. $f/4$

Answer: B

Solution:

Solution:
Frequency for a mass m executing SHM
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
When the mass is increased by 4 times, then the new frequency
$f' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}} = \frac{1}{2}\cdot\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{f}{2}$

Question 35

The SI unit of ε_0 in the formula of capacitance is given by

Options:

A. microfarad/meter

B. farad/meter

C. meter ^{2} / farad

D. farad/centimeter

Answer: B

Solution:

Solution:
The capacitance of a capacitor,
$C = \frac{\varepsilon_0 A}{d}$
$\Rightarrow \varepsilon_0 = \frac{Cd}{A} = \frac{\text{farad} \times \text{meter}}{(\text{meter})^2}$
$= \text{farad} / \text{meter}$

Question 36

Distance between objective and eye-piece of a microscope is 20.6 cm. Consider both/lens are thin and focal length of each lens is 6 mm. If last imane is formed at infinity, then linear magnification of the objective is

Options:

A. -1347

B. -6.19

C. -32.3 times

D. -3.23 times

Answer: C

Solution:

Solution:
As, the final image in formed at infinity by the eye lens, so the object distance for eye lens is focal length of eye lens.
So, $u_e = 6\,\text{mm} = 0.6\,\text{cm}$
The distance between the objective and eye lens is $20.6\,\text{cm}$.
$\therefore \quad L = v_0 + u_e \Rightarrow v_0 = L - u_e$
$= 20.6 - 0.6$
$= 20\,\text{cm}$
Let u be the object distance form objective, then from lens formula for objective.
$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0} \Rightarrow \frac{1}{u_0} = -\frac{1}{f_0} + \frac{1}{v_0} = \frac{1}{20} - \frac{1}{0.6}$
$= -\frac{19.4}{12}$
or $u_0 = -\frac{12}{19.4} = -0.62\,\text{cm}$.
So, the magnification of objective,
$m = \frac{v_0}{u_0} = \frac{20}{-0.62}$
$= -32.3$

Question 37

Internal energy of a gas remains unchanged in
I. an isothermal process
II. an adiabatic process
III. a reversible process
IV. a cyclic process
Which of these are true?

Options:

A. I and IV

B. I, III and IV

C. III and IV

D. II and III

Answer: A

Solution:

Solution:
For isothermal process, $dU = 0$,
i.e., internal energy remains unchanged.
For a cyclic process, $dU = 0$,
i.e., internal energy remains unchanged.

Question 38

A projectile is thrown with an initial velocity of $u = (a\hat{i} + b\hat{j})$ m / s. If the range of the projectile is double the maximum height reached by it, then

Options:

A. $a = 2b$

B. $b = 2a$

C. $a = b$

D. None of these

Answer: B

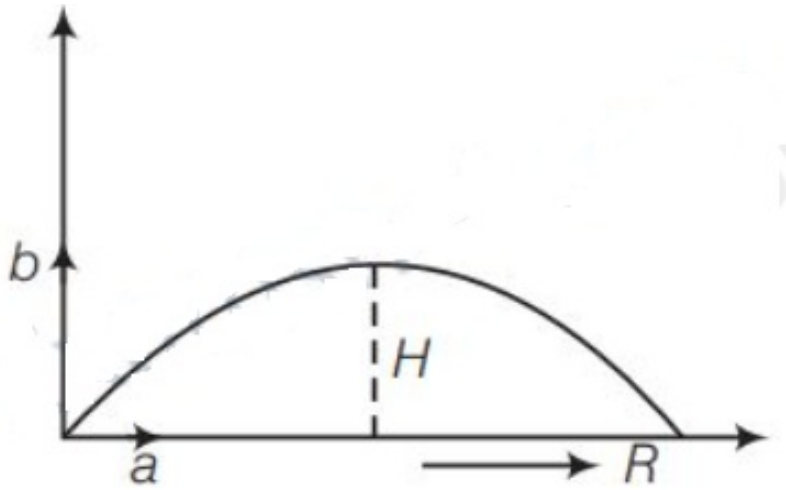
Solution:

Solution:

Given, $u = (a\hat{i} + b\hat{j})$ m / s

So, $u\cos\theta = a$

and $u\sin\theta = b$



According to the question,

$R = 2H$

$\Rightarrow \frac{u^2 \sin 2\theta}{g} = \frac{2 \times u^2 \sin^2\theta}{2g}$

$\Rightarrow \frac{u^2 \cdot 2\sin\theta \cdot \cos\theta}{g} = \frac{2u^2 \sin^2\theta}{2g}$

$\Rightarrow 2 = \frac{b}{a}$

$\Rightarrow b = 2a$

Question 39

An electric charge in uniform motion produces

Options:

A. only electric field

B. only magnetic field

C. Both electric and magnetic field

D. Neither electric nor magnetic field

Answer: C

Solution:

Solution:
An electric charge in uniform motion produces both electric field and magnetic field.

Question 40

Time constant of a series R - C circuit is

Options:

A. $+RC$

B. $-RC$

C. R/C

D. C/R

Answer: A

Solution:

Solution:
During charging and discharging of a capacitor through resistor the time constant, $\tau = RC$.

Question 41

A magnet makes 25 oscillations in 5 min at one place, where as it takes 9 s to complete one oscillation at another place. The ratio of horizontal components of the earths magnetic field at these places $\frac{H_1}{H_2}$ =

Options:

A. $\frac{2}{7}$

B. $\frac{81}{274}$

C. $\frac{1}{8}$

D. $\frac{9}{16}$

Answer: D

Solution:

Solution:
Time period,
$T = 2\pi\sqrt{\frac{I}{MH}}$
$\therefore \frac{H_1}{H_2} = \frac{T_2^2}{T_1^2}$
Here, $T_1 = \frac{5 \times 60}{25}$
$= 12s$
and $T_2 = 9s$
$\therefore \frac{H_1}{H_2} = \frac{(9)^2}{(12)^2} = \frac{9}{16}$

Question 42

If λ is the incident wavelength and λ_0 is the threshold wavelength for a metal surface, photoelectric effect takes place only, if

Options:

A. $\lambda \leq \lambda_0$

B. $\lambda \geq \lambda_0$

C. $\lambda \geq 2\lambda_0$

D. None of the above

Answer: A

Solution:

Solution:
For photoelectric effect to take place, the incident wavelength (λ) should be equal to or smaller than the threshold wavelength (λ_0) i.e.,
$\lambda \leq \lambda_0$

Question 43

The mass number of an atom is 15 and its atomic number is 7 . Now, this atom absorbs an α-particle and emits a proton. What will be the mass number of changed atom?

Options:

A. 16

B. 18

C. 17

D. 15

Answer: B

Solution:

Solution:

$_{7}X^{15} + {}_{2}He^{4} \longrightarrow {}_{9}Y^{19}$

$_{9}Y^{19} \longrightarrow {}_{8}Z^{18} + {}_{1}H^{1}$

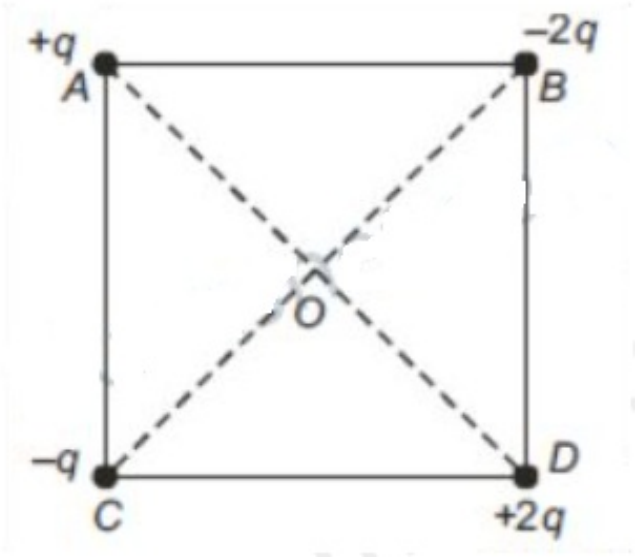
According to the conservation of mass number

$19 = 18 + 1$

So, the mass number of changed atom = 18

Question 44

What is the direction of the electric field at the centre O of the square in the figure shown below? Given that, q = 10 nC and the side of the square is 5 cm.



Options:

A. at 45° to OA upward

B. at 135° to OA towards BD

C. no direction, because $E = 0$

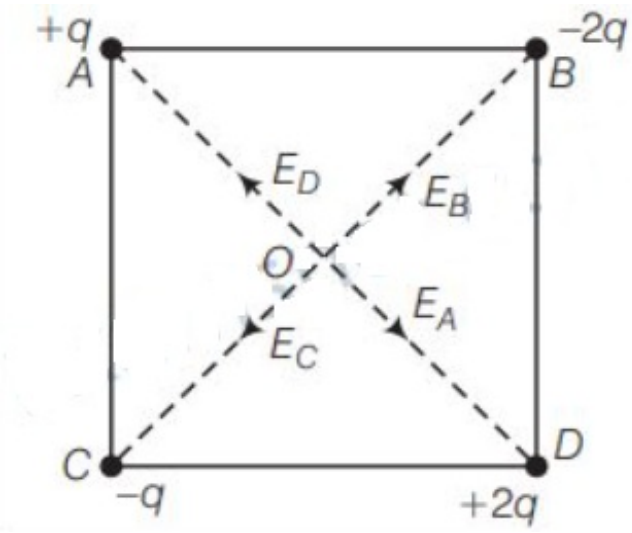
D. None of the above

Answer: A

Solution:

Solution:

$AD = BC = \sqrt{(5)^2 + (5)^2}$
$= \sqrt{25 + 25}$
$= \sqrt{2} \times 5\,\text{cm}$



$\Rightarrow AO = BO = CO = OD = \frac{5\sqrt{2}}{2}\,\text{cm}$

The electric field,

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

So, $E_A = \frac{9 \times 10^9 \times 10 \times 10^{-9} \times 4}{25 \times 2}$
$= 7.2\text{NC}^{-1}$ along OD

$E_B = \frac{9 \times 10^9 \times 2 \times 10 \times 10^{-9} \times 4}{25 \times 2}$
$= 14.4\text{NC}^{-1}$ along OB

$E_C = \frac{9 \times 10^9 \times 10 \times 10^{-9} \times 4}{25 \times 2}$
$= 7.2$ along OC

$E_D = \frac{9 \times 10^9 \times 2 \times 10 \times 10^{-9} \times 4}{25 \times 2}$
$= 14.4$ along OA

Resultant of E_A and E_D, $E_1 = (14.4 - 7.2)$
$= 7.2\text{NC}^{-1}$ along OA.

Resultant of E_B and E_C, $E_2 = (14.4 - 7.2)$
$= 7.2\text{NC}^{-1}$ along OB.

Since, E_1 and E_2 are perpendicular to each other.
$\therefore E = \sqrt{E_1^2 + E_2^2}$ is along 45° to OA upward.

Question 45

Which equation is valid for adiabatic process?

Options:

A. $TV^{\gamma-1} =$ constant

B. $pV^{\gamma-1} =$ constant

C. $T^{\gamma}V^{\gamma-1} =$ constant

D. $\frac{p^{\gamma-1}}{T^{\gamma-1}} =$ constant

Answer: A

Solution:

Solution:
For an adiabatic process, $TV^{\gamma-1} =$ constant.

Question 46

For changing the range of a galvanometer with G ohm resistance from V volt to n volt, what will be the value of resistance connected in series to it?

Options:

A. $(n-1)G$

B. G/n

C. nG

D. $\frac{G}{n-1}$

Answer: A

Solution:

Solution:
The value of resistance connected in series to galvanometer $= (n-1)G$

Question 47

Which of the following relation correct? (v_{rms} .-root mean square velocity, $\overline{v}$-mean velocity and $v_{\mp}$-most probable velocity)

Options:

A. $v_{rms} > \overline{v} < v_{mp}$

B. $v_{rms} < \overline{v} > v_{mp}$

C. $v_{rms} > \overline{v} > v_{mp}$

D. None of the above

Answer: C

Solution:

Solution:
The correct relation is $v_{rms} > \overline{v} > v_{mp}$
As $\overline{v} = 1.6\sqrt{\frac{RT}{M}}$, $v_{rms} = 1.73\sqrt{\frac{RT}{M}}$ and $v_{mp} = 1.41\sqrt{\frac{RT}{M}}$

Question 48

The effect of reverse bias in a junction diode on its potential barrier is

Options:

A. increases

B. decreases

C. remains same

D. None of the above

Answer: A

Solution:

Solution:
Due to reverse biasing, in a junction diode the potential barrier increases.

Question 49

Which of the following is the Biot-Savart's law in vector form?

Options:

A. $d\mathrm{B} = \frac{\mu_0}{4\pi}\frac{\mathrm{Idl}\sin\theta}{r}\hat{n}$

B. $d\mathrm{B} = \frac{\mu_0}{4\pi}\frac{\mathrm{Idl}\sin\theta}{r^3}\hat{n}$

C. $d\mathrm{B} = \frac{\mu_0}{4\pi}\frac{\mathrm{Idl}\sin\theta}{r^2}\hat{n}$

D. None of the above

Answer: C

Solution:

Solution:
Biot-Savart's law in vector form
$dB = \frac{\mu_0}{4\pi}\frac{\mathrm{Idl}\sin\theta}{r^2}\hat{n}$

Question 50

During an experiment, an ideal gas is found to obey an additional law $Vp^2 =$ constant. The gas is initially at temperature T and volume V. The temperature of the gas will be following, when it expands to a volume $2V$?

Options:

A. $\sqrt{2}T$

B. $\sqrt{4}T$

C. $\sqrt{6}T$

D. $\sqrt{5}T$

Answer: A

Solution:

Solution:
Given, $Vp^2 =$ constant
or $V[\frac{RT}{V}]^2 =$ constant or $\frac{T^2}{V} =$ constant
or $\frac{T'^2}{T^2} = \frac{V'}{V}$
$\Rightarrow \frac{T'^2}{T^2} = \frac{2V}{V} \Rightarrow T' = \sqrt{2}T$

Question 51

Correct basicity of the following compounds are
I. Aniline
II. Pyridine
III. Pyrrole
IV. Guanidine

Options:

A. I > II > III > IV

B. III > I > II > IV

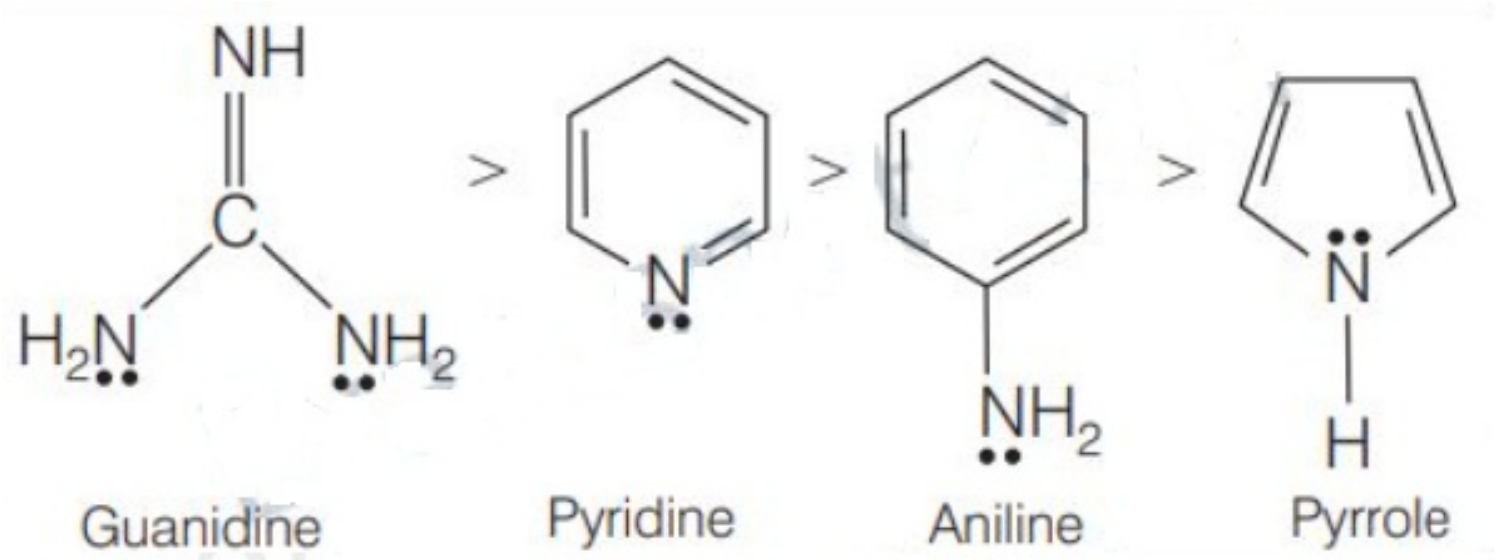
C. IV > II > I > III

D. II > IV > III > I

Answer: A

Solution:

Solution:



From the structures of the four compounds, It is clear that in pyrrole, the electron pair on nitrogen is involved in delocalisation of the ring. So, it is least basic.
In aniline, the electron pair involved in conjugation with the π^- electrons of the ring while in pyridine it is relatively free.
Guanidine is a strong base.

Question 52

Lithium is the strongest reducing agent because of

Options:

A. its greater hydration energy

B. its high ionization energy

C. its high electron affinity

D. its low electronegativity

Answer: A

Solution:

Solution:
Lithium is the strongest reducing agent. However IE of lithium is highest among alkali metals. So, its reducing nature should be minimum.
The greatest reducing nature of Li in aqueous medium is due to the maximum hydration energy of Li^+ ion.

Question 53

Which of the following is not basic amino acid?

Options:

A. Leucine

B. Lysine

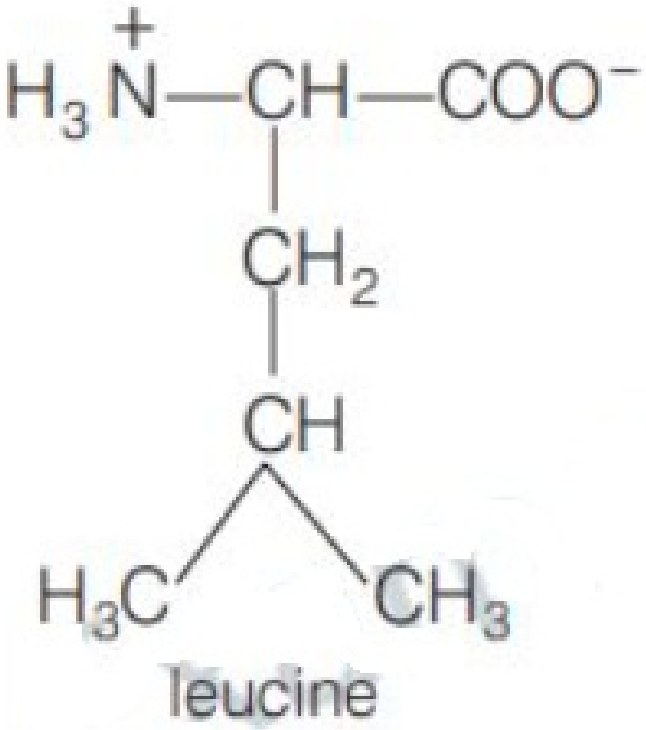
C. Arginine

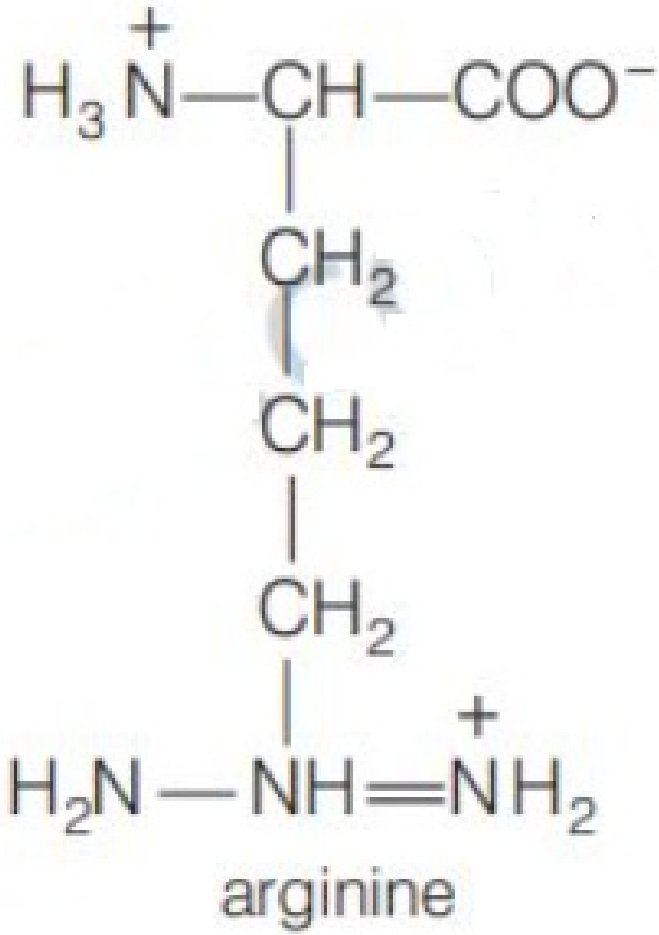
D. Histidine

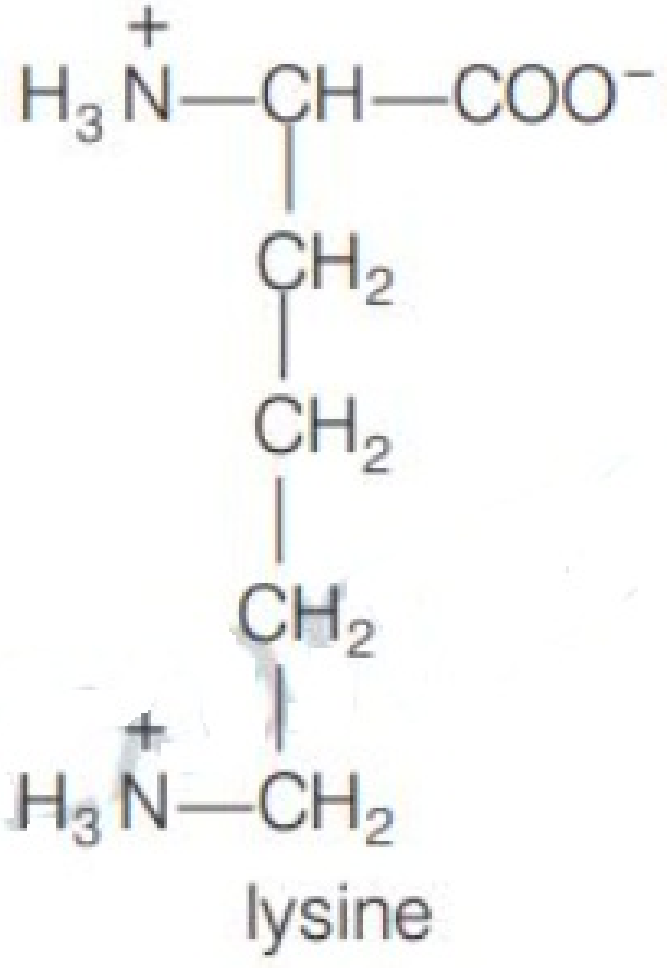
Answer: A

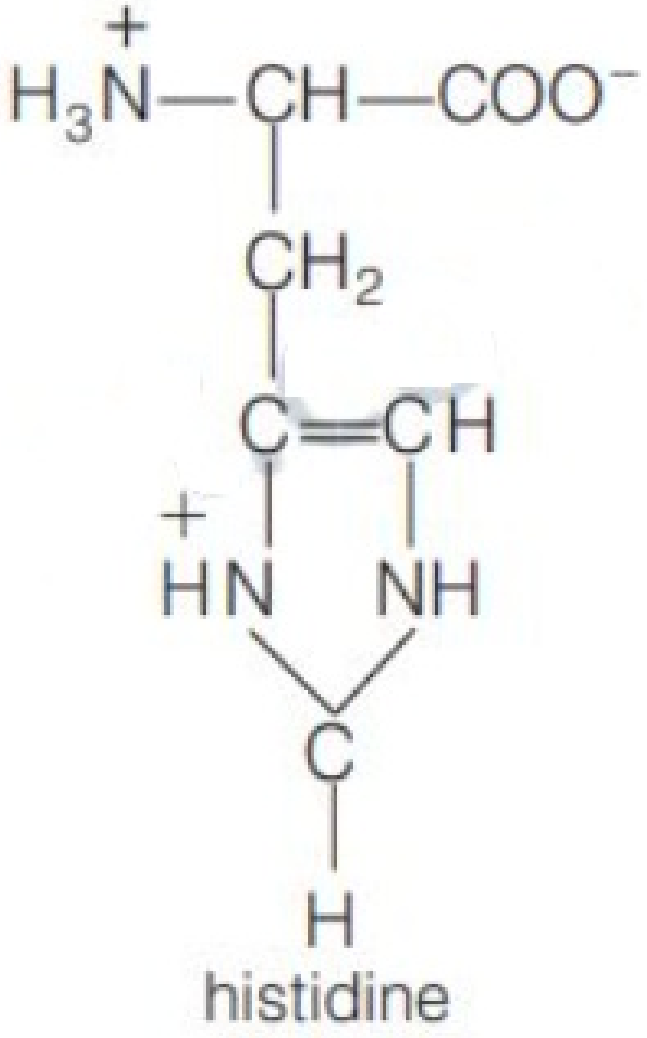
Solution:

Solution:









Leucine is not a basic amino acid while others three i.e., arginine, lysine and histidine are basic amino acids as they have more number of amino groups than carboxyl groups.

Question 54

Finely divided powder of charcoal adsorbs the substance X to a large extent. This is because

Options:

A. the surface area of charcoal is increased

B. charcoal powder can be spread over X homogeneously

C. X can be spread over charcoal homogeneously

D. charcoal is highly reactive

Answer: A

Solution:

Solution:
The extent of adsorption increases with the increase of surface area of adsorbent. Finely divided charcoal is a good adsorbent.

Question 55

In qualitative analysis, NH_4Cl is added before NH_4OH

Options:

A. to increase $[OH^-]$ concentration

B. for making HCl

C. to decrease $[OH^-]$ concentration

D. statement is wrong

Answer: C

Solution:

Solution:
In qualitative analysis, NH_4Cl is added before NH_4OH to decrease $[OH^-]$ concentration. NH_4Cl suppresses the ionisation of NH_4OH due to common ion effect. Otherwise basic radicals of group V and group VI will be precipitated along with IIIrd group basic radicals.

Question 56

This method is

Options:

A. Wurtz synthesis

B. Kolbe synthesis

C. Corey House synthesis

D. Friedel-Craft synthesis

Answer: A

Solution:

Solution:

$$C_2H_5Cl \xrightarrow[CuI]{Li} (C_2H_5)_2CuLi \xrightarrow{C_2H_5Cl} \underset{n-butane}{CH_3CH_2CH_2CH_3}$$

The reaction is known as Corey House synthesis for alkanes. (symmetrical and unsymmetrical both)

Question 57

Certain electric current for half an hour can collect 11.2L of hydrogen at NTP. Same current when passed through an electrolytic solution for one hour, can deposite how much silver?

Options:

A. 216g

B. 108g

C. 47g

D. 60g

Answer: D

Solution:

Solution:

$11.2L\,H_2$ gas at NTP $= 0.5$ mole of H_2 gas $= 1g\,of_2g$

$\frac{W_1}{w_2} = \frac{E_1}{E_2} = \frac{1}{W_2} = \frac{1}{108}$ or $w_2 = 108g$ silver is deposited in half an hour.

Therefore, $108 \times 2 = 216g$ silver will be deposited in one hour.

Question 58

Which of the following complexes is an outer orbital complex?

Options:

A. $[Co(NH_3)_6]^{3+}$

B. $[Fe(CN)_6]^{4-}$

C. $[Ni(NH_3)_6]^{2+}$

D. $[Mn(CN)_6]^{4-}$

Answer: B

Solution:

Solution:
A. $[Co(NH_3)_6]^{+3}$ inner orbital (d^2sp^3) complex
B. $[Fe(CN)_6]^{4-}$ inner orbital (d^2sp^3) complex
C. $[Ni(NH_3)_6]^{2+}$ outer orbital (sp^3d^2) complex
D. $[Mn(CN)_6]^{2-}$ inner orbital (d^2sp^3) complex

Question 59

IUPAC name for the complex compound $K_3[Fe(CN_6)]$ is

Options:

A. Potassium hexacyanoferrate [II]

B. Potassium cynohexaferrate [II]

C. Potassium hexacyanoferrate [IIII]

D. Potassium ferrocyanide iron [III]

Answer: A

Solution:

Solution:
$K_3[Fe(CN)_6]$ Potassium hexacyanoferrate (III)

Question 60

Which is true for a cyclic process?

Options:

A. $\Delta E = 0$

B. $\Delta E = q - W$

C. $q = W$

D. All of these

Answer: B

Solution:

Solution:
For a cyclic process, $\Delta E = 0$. In cyclic process, system returns to its original state after a number of reactions.

Question 61

In the cannizzaro reaction given below

$$2C_6H_5CHO \xrightarrow{OH^-} C_6H_5CH_2OH + C_6H_5COO^-$$

The slowest step is

Options:

A. The attack of OH^- at the carbonyl group

B. The transfer of hydride ion to the carbonyl group

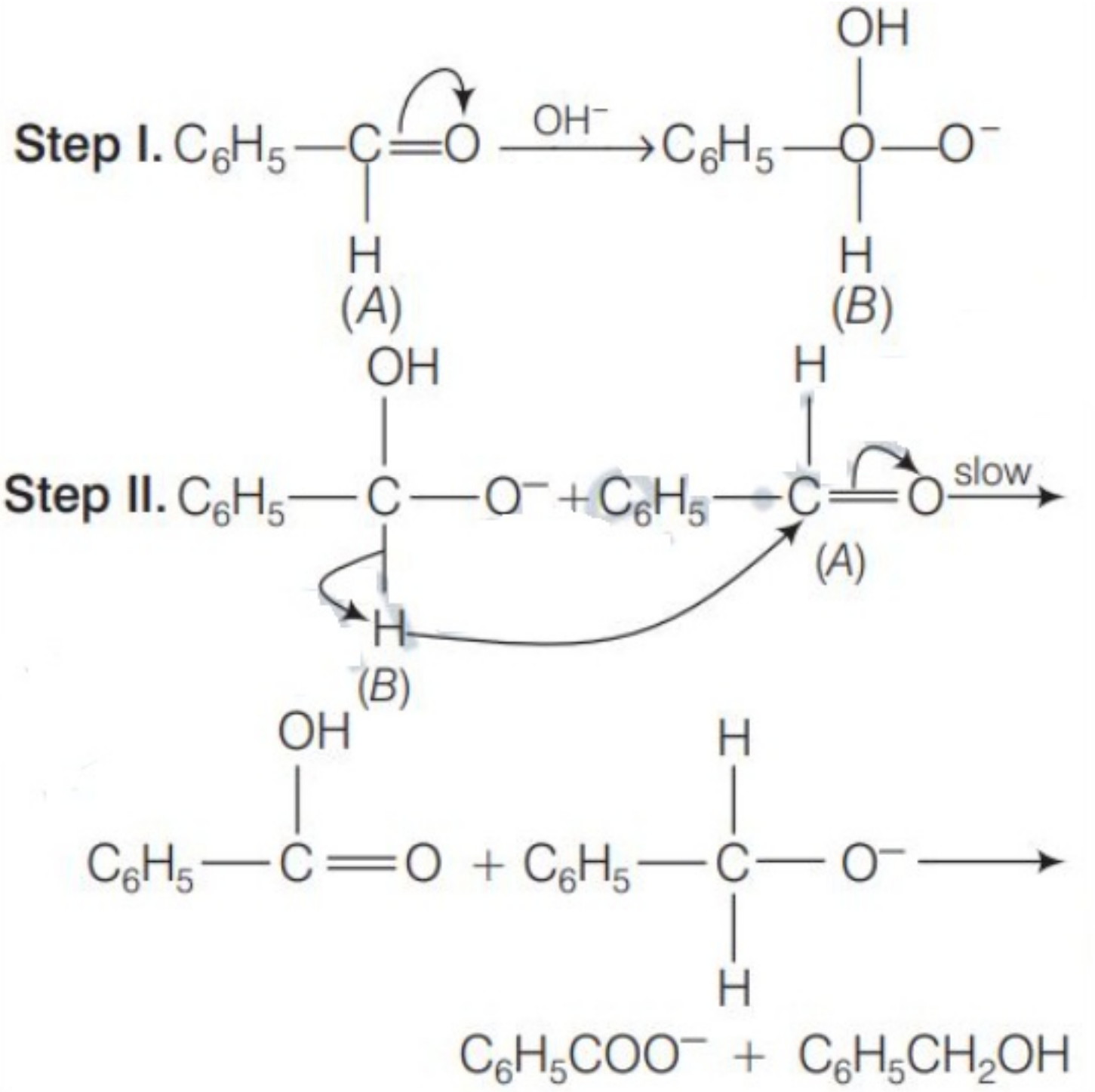
C. The abstraction of proton from the carboxylic acid

D. The deprotonation of benzyl alcohol

Answer: B

Solution:

Solution:



H^- is obtained by fission of $C-H$ bond of (B). It is transferred to (II). This is the slowest step.

Question 62

The rate of a reaction doubles when the initial concentration of the reactant is made four fold. If the initial concentration is made 400 fold, then the rate will become

Options:

A. 400 times

B. 200 times

C. 40 times

D. 20 times

Answer: D

Solution:

Solution:
$R = k[A]^x$ . . . (i)
$2R = k[4A]^x$ . . . (ii)
Divide Eq. (ii) by (i)
$2 = [4]^x$
$R = k[A]^{1/2}$
$x = \frac{1}{2}$
$= 0.5$
$R = k(400A)^{1/2}$
Rate $= 20k[A]^{1/2}$
Rate becomes 20 times.

Question 63

Bisphenol and epichlorohydrin condensed in presence of $NaOH$ forming

Options:

A. Resins

B. Rubber

C. Foam

D. Polyester

Answer: A

Solution:

Solution:
Epoxy resins are obtained by copolymeristation of epichlorohydrin and bisphenol A. These resins have good adhesive strength.

Question 64

Which compound present in diesel?

Options:

A. Cetane

B. $TiCl_4$

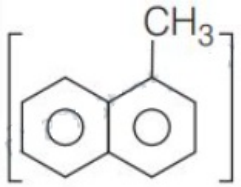
C. Cyclo pentandienyl manganese carbonyl

D. Iso octane

Answer: A

Solution:

Solution:
Diesel is fraction of petroleum having $C_{15} - C_{18}$ composition and boiling point in the range of $573 - 673K$. Generally cetane $[CH_3 - (CH_2)_{14} - CH_3]$ and α-methyl naphthalene



are its main constituents.

Question 65

The correct order of electron gain enthalpy with negative sign is

Options:

A. $S < O < Cl < F$

B. $O < S < F < Cl$

C. $Cl < F < S < O$

D. $F < Cl < O < S$

Answer: B

Solution:

Solution:
Order of electron gain enthalpy is as follows $O < S < F < Cl$. Because of the compact nature of oxygen and fluorine atoms, they have less negative electron gain enthalpy than S and Cl respectively .
Due to the small size of oxygen and fluorine atoms, there are strong interelectronic repulsions in the relatively small $2p$-orbitals of oxygen and fluorine and thus incoming electron does not experience much attraction.

Question 66

Which of the following is an organometallic compound?

Options:

A. Lithium acetate

B. Methyl lithium

C. Lithium dimethyl amide

D. Lithium methoxide

Answer: B

Solution:

Solution:
Organometalllic compounds must contain, at least one metal-carbon bond. Methyl lithium, CH_3Li is an organometallic compound.

Question 67

The total number of optical isomers possible in aldohexose is

Options:

A. 2

B. 16

C. 8

D. 4

Answer: A

Solution:

Solution:
Aldohexose has 4 chiral carbon atoms.
No. of optical isomers $= 2^n = 2^4 = 16$

Question 68

Acetylene and $HCHO$ react in presence of copper acetylide catalyst to form

Options:

A. But-2-yne-1, 2-diol

B. But-1-yne-1, 4-diol

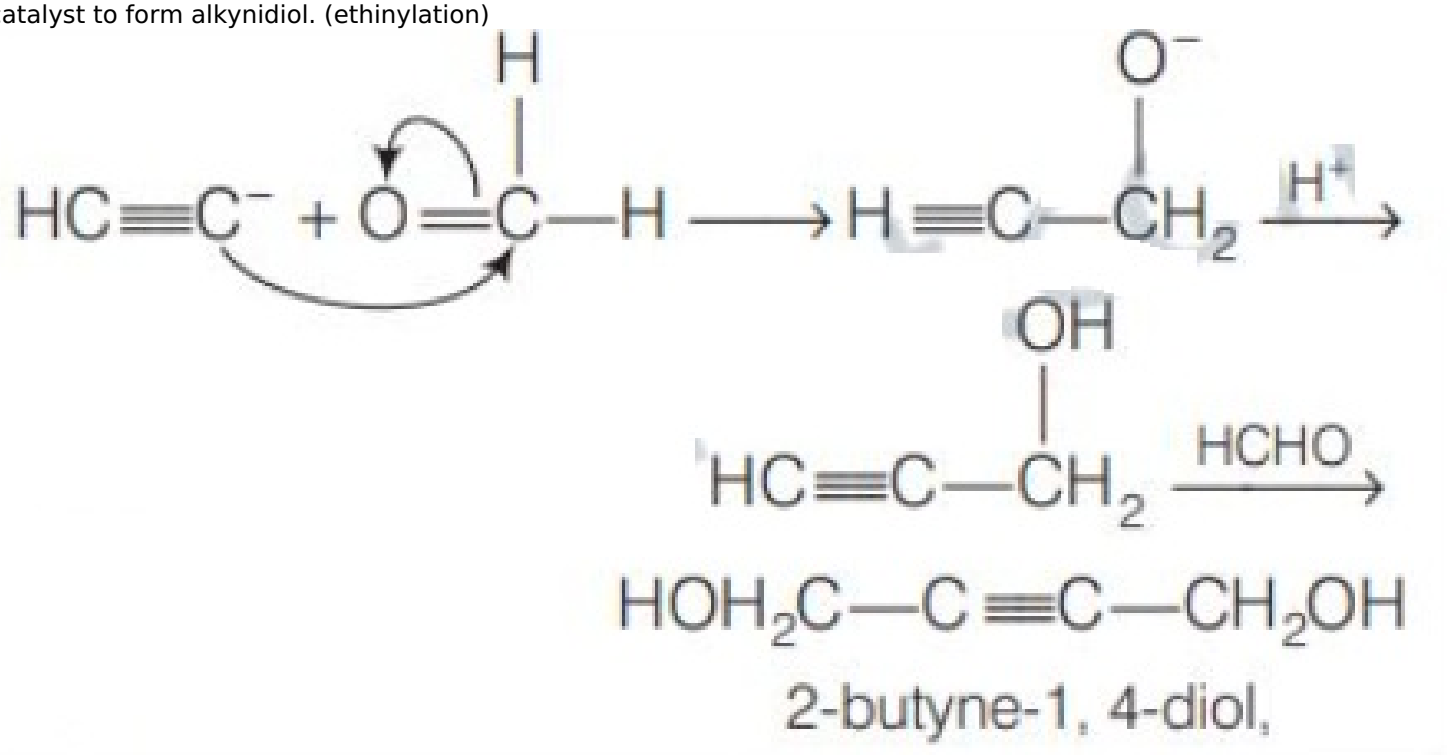
C. But-2-ene-1, 4-diol

D. 2-butyne-1, 4-diol

Answer: B

Solution:

Solution:
Terminal alkynes (Containing C ≡ CH, i.e., a methine hydrogen atom) can add on to carbonyl group in the presence of catalyst to form alkynidiol. (ethinylation)



Question 69

If in the reaction $N_2O_4 \rightleftharpoons 2NO_2$; α is the degree of dissociation of N_2O_4, then total number of moles at equilibrium is

Options:

A. $(1-\alpha)$

B. $(1+\alpha)$

C. $(1-\alpha)^2$

D. $(1-\alpha)^2$

Answer: B

Solution:

Solution:
Initial
$N_2O_4 \rightleftharpoons 2NO_2$
Total number of moles at equilibrium
$= (1-\alpha) + 2\alpha$
$= 1+\alpha$

Question 70

To convert 94g of ice at 0°C into 94g of vapour at 100°C, the quantity of coal (90\% carbon) required is : (ΔH_c. of carbon = −94k cal mol^{-1}); latent heat of fusion = +80 cal per gm; latent heat of vaporisation = +540 cal per gm; specific heat of water = 1 cal per gm per °C)

Options:

A. 94g

B. 80g

C. 9.4g

D. 9.6g

Answer: D

Solution:

Solution:
$q = ms\Delta t$
$0^\circ C$ Ice (s) $\longrightarrow 0^\circ C$ water (I)
$q_1 = 94 \times 80 = 7520$ cal
$0^\circ C$ water (l) $\longrightarrow 100^\circ C$ water (l)
$q_2 = 94 \times 1 \times 100 = 9400$ cal
$100^\circ C$ (water) $100^\circ C$ water vapours
$q_3 = 94 \times 540 = 50760$ cal
Total (q) = 67680 cal
= 67.680 kcal
$\because$ 94 kcal = 12 gcoal
$\therefore$ 67.68 kcal = $\frac{67.68 \times 12}{94}$
= 8.64g cal
90% of x = 8.64
or x = 9.6g cal

Question 71

Correct order of paramagnetism is

Options:

A. Mn > Cr > Zn

B. Fe > Zn > Cr

C. Cr > Fe > Zn

D. Zn > Mn > Fe

Answer: C

Solution:

Solution:
Paramagnetic character $\propto$ number of unpaired electrons
$Cr = 3d^5 4s^1;\ 6e^-$
$Mn = 3d^5 4s^2;\ 5e^-$
$Zn = 3d^{10} 4s^2;$ No
$Fe = 3d^6 4s^2;\ 4e^-$
Hence, the correct order of paramagnetism is Cr > Mn > Fe > Zn.

Question 72

$$CHCl_3 \xrightarrow[h\nu]{[0]} A \xrightarrow{NH_3} B \xrightarrow[PoCl_3]{\text{Malonic acid}} C$$

The end product C in the following reaction is used as

Options:

A. explosive

B. hypnotic

C. tear gas

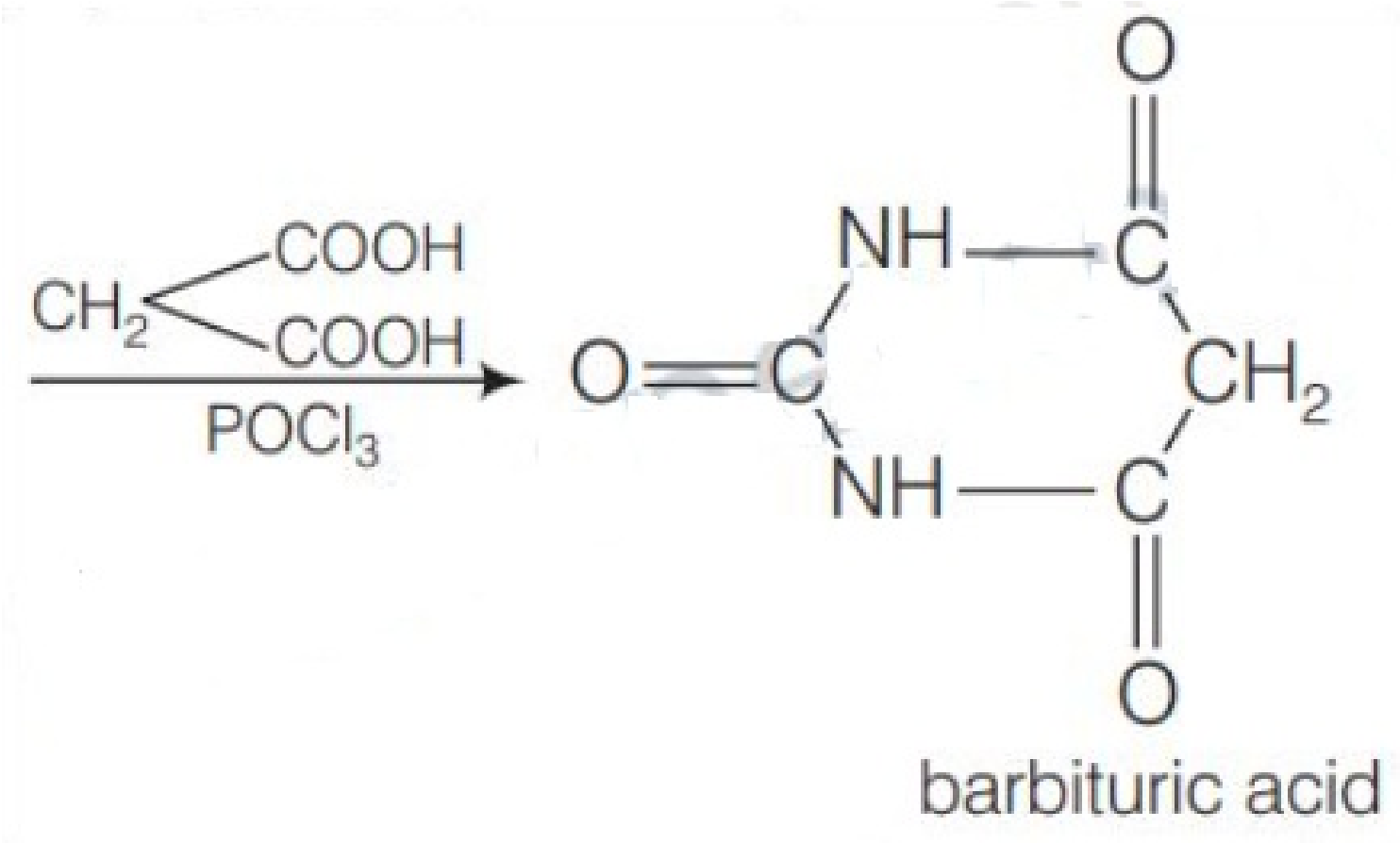
D. analgesic

Answer: B

Solution:

Solution:

$$CHCl_3 \xrightarrow[h\nu]{[0]} COCl_2 \xrightarrow{NH_3} NH_2CONH_2$$



Barbituric acid and its derivatives are used in medicines as hypnotics and sedatives.

Question 73

The rate constant of a reaction is given by $\frac{N_t}{N_o} = e^{-kt}$. This represents for reaction of

Options:

A. zero order

B. second order

C. first order

D. None of these

Answer: C

Solution:

Solution:
$N_t = N_o e^{-Kt}$ for first order reactions.

Question 74

Which statement is false for white phosphorus (P_4)?

Options:

A. It has six $P-P$ single bonds

B. It has four $P-P$ single bonds

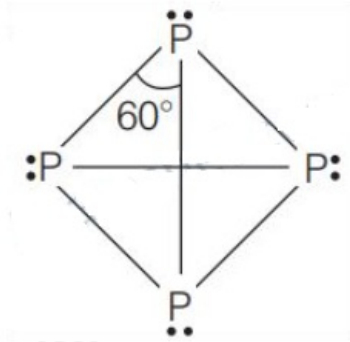
C. It has four lone pairs of electrons

D. It has PPP angle 60°

Answer: B

Solution:

Solution:



White phosphorus (P_4)
It has six $P-P$ single bonds.

Question 75

Compound A on ozonolysis gives acetone and glyoxal. The compound A is

Options:

A. 2, 5-dimethyl hex-2, 4-diene

B. 2, 5-dimethyl hex-1, 5-diene

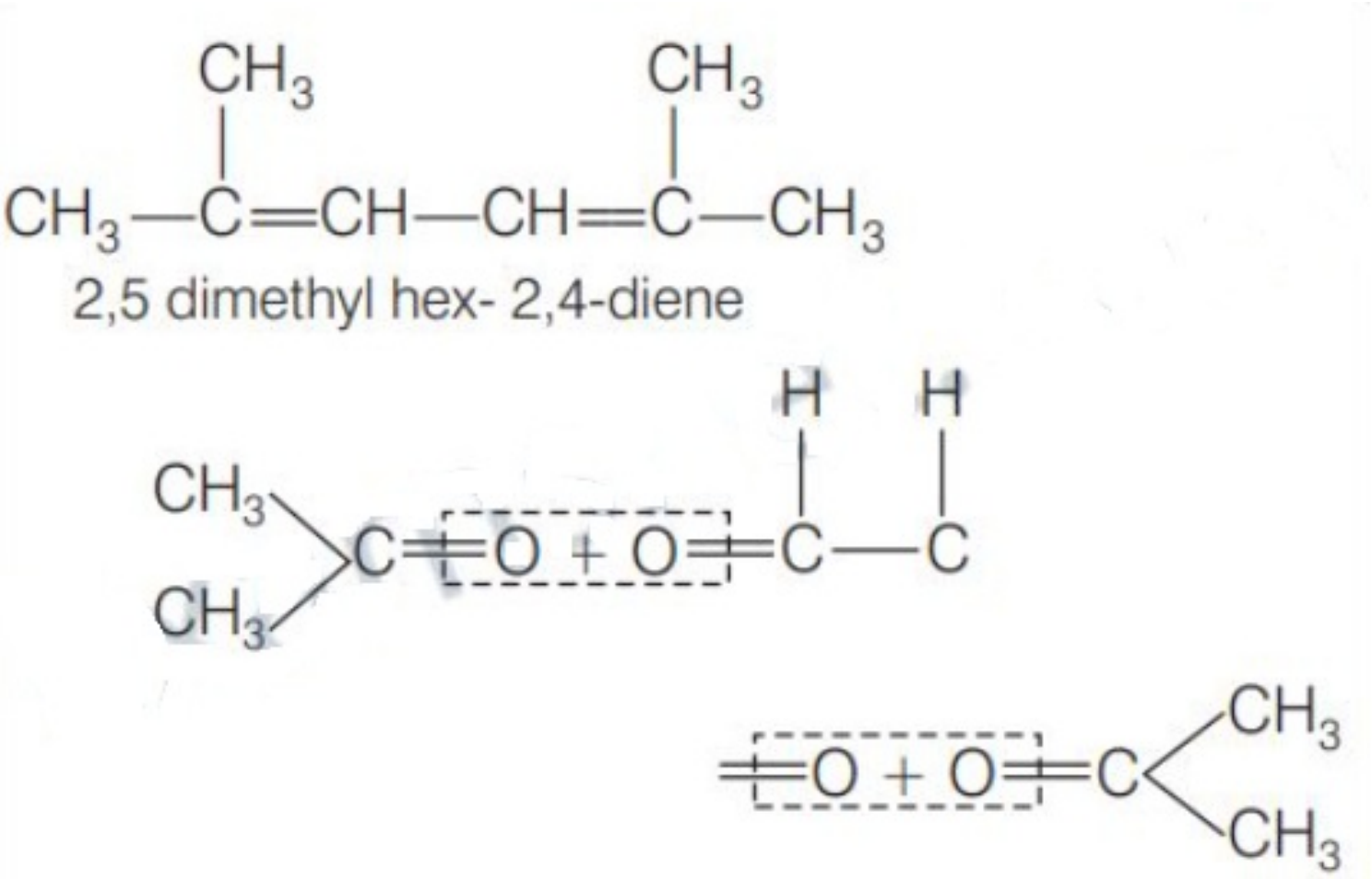
C. 2, 5-dimethyl hex-3, 4-diene

D. 2, 5-dimethyl but-2, ene

Answer: A

Solution:

Solution:



To decide the structure of alkane from ozonolysis products bring the products together in such a way that ' O ' atoms are face to face, replace O by C = C bond.

Question 76

Natural rubber is a polymer of monomer isoprene. During polymerisaton

Options:

A. 1, 4 addition takes place

B. 1, 2 addition takes place

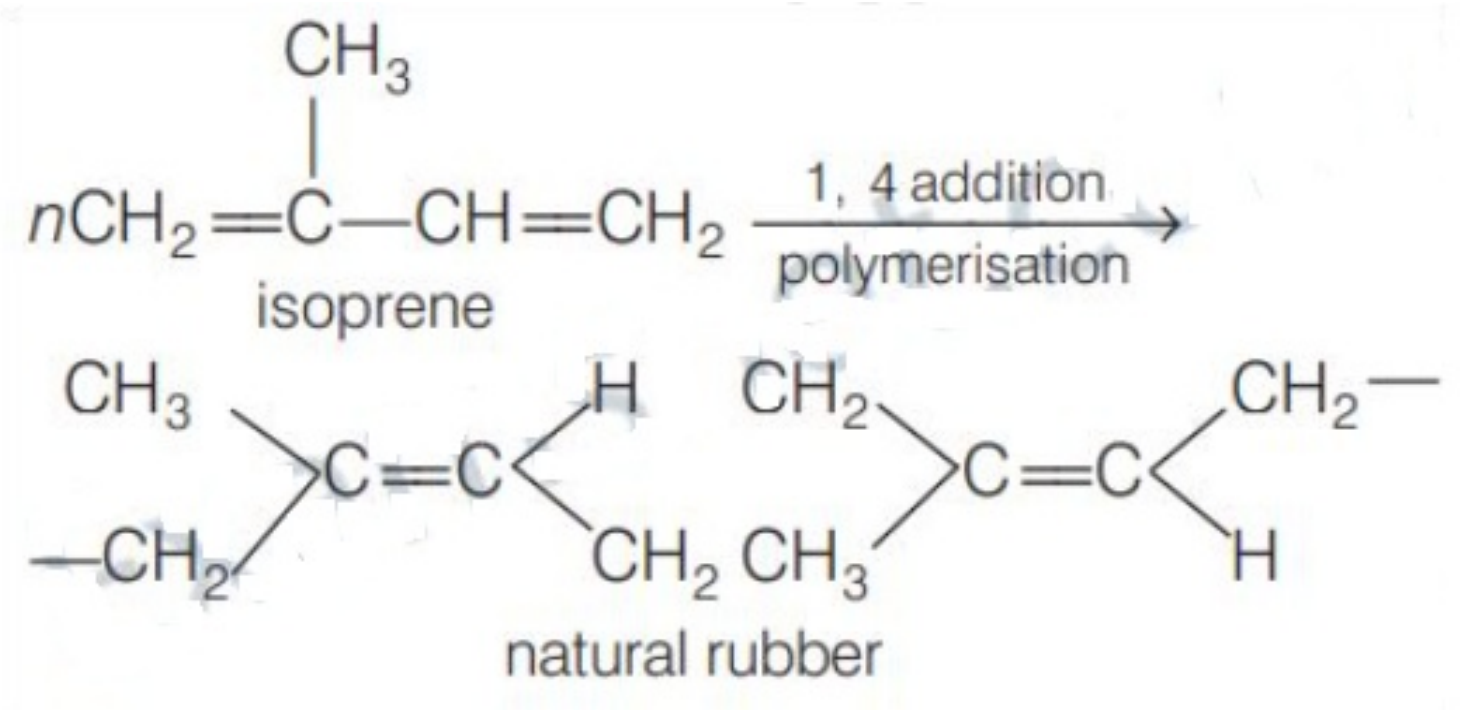
C. 1, 3 addition takes place

D. both double bonds are converted into single bond

Answer: A

Solution:

Solution:
Natural rubber is a linear polymers of isoprene and is also called as cis-1, 4-polyisoprene.
It is a linear 1,4-polymer of isoprene (2-methyl-1,3-butadiene).



Question 77

Select the correct statement.

Options:

A. Orlon is an addition polymer

B. Dacron is an addition polymer

C. Orlon is a condensation polymer

D. The monomers of orlon is styrene

Answer: A

Solution:

Solution:
Orlon is an addition polymer.

Question 78

Match List I (species) with List II (hybridization) and select the correct code given below

List I	List II
A. XeF_4	(i) dsp^2
B. H_2O	(ii) sp^3
C. PCl_5	(iii) sp^3d^2
D. $\left[Pt(NH_3)_4\right]^{2+}$	(iv) sp^3d

Options:

A. (A-iii), (B-ii), (C-iv), (D-i)

B. (A-i), (B-iv), (C-ii), (D-iii)

C. (A-iii), (B-iv), (C-ii), (D-i)

D. (A-i), (B-ii), (C-iv), (D-iii)

Answer: A

Solution:

Solution:

A. XeF_4	sp^3d^2
B. H_2O	sp^3
C. PCl_5	sp^3d
D. $\left[Pt\left(NH_3\right)_4\right]^{2+}$	dsp^2

Question 79

In acidic medium, the equivalent weight of $K_2Cr_2O_7$ (Mol. wt. = M) is

Options:

A. M

B. $M/2$

C. $M/3$

D. $M/6$

Answer: D

Solution:

Solution:
In acidic medium
$Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$
$\text{eq. wt} = \frac{\text{molecular weight}}{\text{no. of electrons transferred}}$
$= \frac{M}{6}$

Question 80

Consider the following reaction $Me_2C = CH_2 + H^+ \overset{H_2SO_4}{\rightleftharpoons} (X) \overset{EtOH}{\rightleftharpoons} (Y) \underset{(Z)}{\longrightarrow} Me_3COEt$.
In this reaction which is not correct?

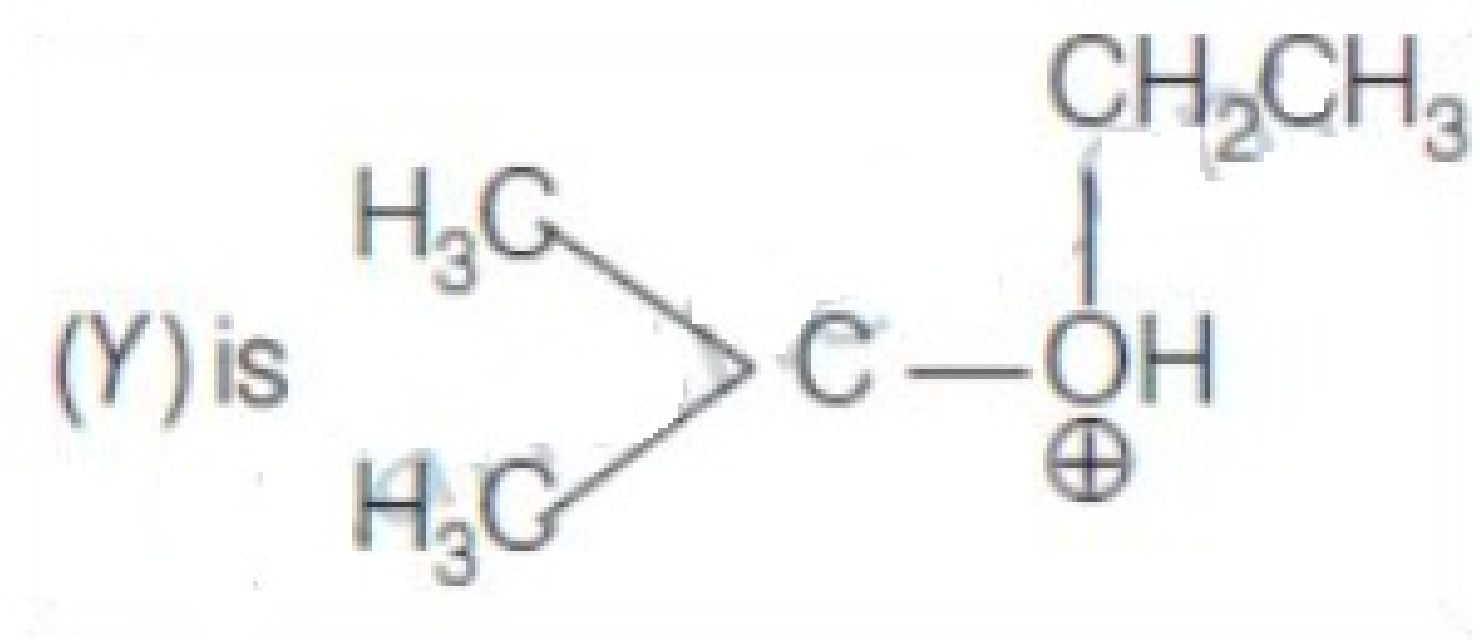
Options:

A. (X) is Me_3C^+

B. (X) is $Me_3CH - C^+H_2$

C. (Z) is H^+

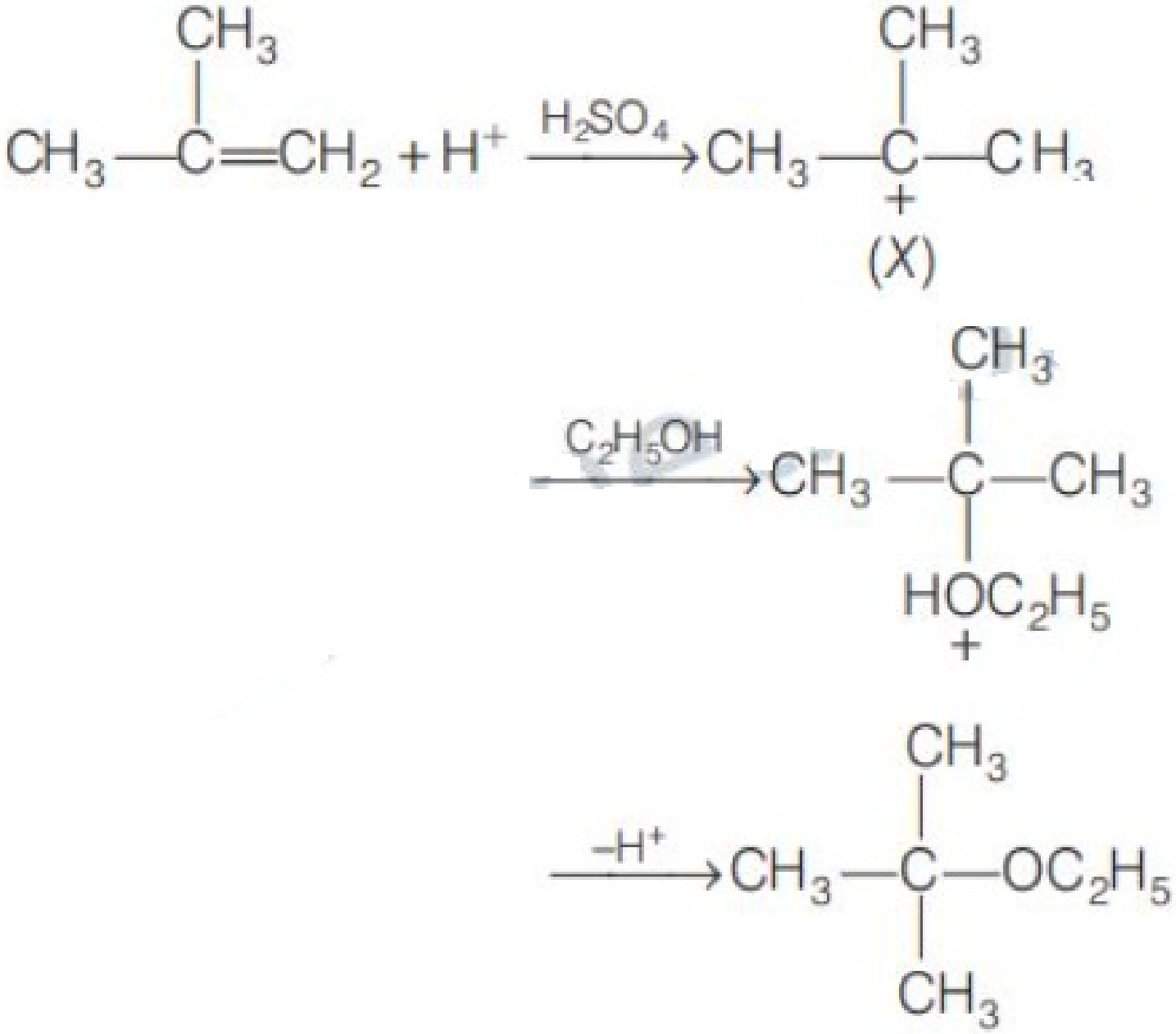
D.



Answer: B

Solution:

Solution:



Question 81

A new carbon-carbon bond formation is possible in :
I. Cannizzaro reaction
II. Friedel-Craft reaction
III. Clemmensen reduction
IV. Reimer-Tiemann reaction

Options:

A. I, II and III

B. II, III and IV

C. I and III

D. II and IV

Answer: D

Solution:

Solution:
A new carbon- carbon bond formation takes place in Friedel- Craft reaction and ReimerTiemann reaction.

Question 82

10g of glucose (π_1), 10g of urea (π_2) and 10g of sucrose (π_3) are dissolved in 250mL of water at 300K. Correct order of osmotic pressure of solutions is

Options:

A. $\pi_1 > \pi_2 > \pi_3$

B. $\pi_3 > \pi_1 > \pi_2$

C. $\pi_2 > \pi_1 > \pi_3$

D. $\pi_2 > \pi_3 > \pi_1$

Answer: C

Solution:

Solution:
$\because \quad \pi = \frac{n}{V}RT \quad \therefore \quad \pi \propto n$
n (glucose) $= \frac{10}{180} = 0.055$
n (urea) $= \frac{10}{60} = 0.16$
n (sucrose) $= \frac{10}{342} = 0.029$
π (urea) $> \pi$ (glucose) $> \pi$ (sucrose)

Question 83

Which of the following compounds formed during Perkin's reaction?

Options:

A. Resorcinol

B. Cinnamic acid

C. Benzaldehyde

D. Benzoin

Answer: B

Solution:

Solution:
Cinnamic acid

Question 84

$^{235}_{92}\mathrm{U}$ belongs to IIIB group of the Periodic When it loses one α-particle, the new element will belong to the group

Options:

A. I B

B. V B

C. I A

D. III B

Answer: D

Solution:

Solution:
$^{235}_{92}\mathrm{U} \xrightarrow{-\alpha} {}^{231}_{90}\mathrm{Th}$
The elements $_{89}Ac$ to $_{103}\mathrm{Lr}$ belong to IIIB group of periodic table (Actinoid series).

Question 85

The ratio of de-Broglie wavelengths for electron accelerated through 200V and 50V is

Options:

A. 1:2

B. 2:1

C. 3:10

D. 10:3

Answer: A

Solution:

Solution:

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m \times (\text{KE})}}$$

$$\lambda_1 = \frac{h}{\sqrt{2m \times 200}}$$

$$\lambda_2 = \frac{h}{\sqrt{2m \times 50}}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{1 \times \sqrt{50}}{\sqrt{200 \times 1}} = \frac{1}{2}$$

$$\lambda_1 : \lambda_2 = 1:2$$

Question 86

Which set of elements have nearly the same atomic radii ?

Options:

A. $\text{F}, \text{Cl}, \text{Br}, \text{I}$

B. $\text{Na}, \text{K}, \text{Rb}, \text{Cs}$

C. $\text{Li}, \text{Be}, \text{B}, \text{C}$

D. $\text{Fe}, \text{Co}, \text{Ni}, \text{Cu}$

Answer: D

Solution:

Solution:
$\text{Fe}, \text{Co}, \text{Ni}, \text{Cu}$ have almost same atomic radii [126, 125, 125, 128 (in pm) respectively]. This is due to the fact that successive addition of d - electrons screen the outer electrons (4s-) from invard pull of the nucleus. As a result of this, the size of the atom does not alter much in moving from chromium to copper.

Question 87

. $\text{CH}_3\text{CH}_2\text{C} \equiv \text{CH} \overset{A}{\underset{B}{\Rightarrow}} \text{CH}_3\text{C} \equiv \text{C} - \text{CH}_3$**,** A **and** B **are**

Options:

A. alc. KOH and SeO_2

B. NaNH_2 and Lindlar catalyst

C. alc. KOH and NaNH_2

D. Lindlar catalyst and NaNH_2

Answer: C

Solution:

Solution:

$$CH_3CH_2C \equiv CH \underset{B}{\overset{A}{\rightleftharpoons}} CH_3 - C \equiv C - CH_3$$

$$\underset{\text{but}-1-\text{yne}}{CH_3CH_2C \equiv CH} \xrightarrow{\text{Ethanoilic "KOH}} \left[\underset{\text{Intermediate}}{CH_3CH = C = CH_2}\right] \longrightarrow \underset{\text{but}-2-\text{yne}}{CH_3C \equiv CCH_3}$$

Acetylene homologues isomerise when heated with ethanolic KOH, the triple bond moving towards the centre of the chain.
When alkynes are heated with sodamide in an inert solvent (paraffin), triple bond shifts towards the end of the chain.

Question 88

Malachite decomposed to give $A + CO_2 + H_2O$ and compound A on reduction with carbon gives $CO + B$. Here, A and B are

Options:

A. CuO, Cu

B. Cu_2O, CuO

C. Cu_2O, Cu

D. $CuCO_3, Cu$

Answer: C

Solution:

Solution:

$$CuCO_3 \cdot Cu(OH)_2 \longrightarrow \underset{(A)}{Cu_2O} + CO_2 + H_2O$$

$$Cu_2O + C \longrightarrow \underset{(B)}{2Cu} + CO$$

Question 89

Match List I with List II and choose correct answer from the codes given below

List I	List II
A. $NaNO_3$	(i) Baking soda
B. $Na(NH_4)HPO_4$	(ii) Chile salt petre
C. $NaHCO_3$	(iii) Microcosmic salt
D. $Na_2CO_3 \cdot 10H_2O$	(iv) Washing soda

Options:

A. (A-i), (B-ii), (Ciii-), (D-iv)

B. (A-ii), (B-iii), (C-i), (D-iv)

C. (A-iii), (B-i), (C-ii), (D-iv)

D. (A-iv), (B-i), (C-ii), (D-iii)

Answer: B

Solution:

Solution:

List I	List II
A. $NaNO_3$	Chile salt petre
B. $Na(NH_4)HPO_4$	Microcosmic salt
C. $NaHCO_3$	Baking soda
D. $Na_2CO_3 \cdot 10H_2O$	Washing soda

Question 90

When MnO_2 is heated with PbO_2 and conc. HNO_3, pink colour is obtained due to formation of

Options:

A. $KMnO_4$

B. $HMnO_4$

C. $Pb(MnO_4)_2$

D. $PbMnO_4$

Answer: B

Solution:

Solution:
$2MnO_2 + 4HNO_3 \rightarrow 2Mn(NO_3)_2 + 2H_2O + O_2$
$2Mn(NO_3)_2 + 5PbO_2 + 6HNO_3 \longrightarrow$

$2HMnO_4 + 5Pb(NO_3)_2 + 2H_2O$
Permanganic
acid (violet pink)

Question 91

The nuclear reaction, $^{27}_{13}Al + ^{4}_{2}He \rightarrow ^{30}_{14}Si + ^{1}_{1}H$ is

Options:

A. nuclear fusion

B. nuclear fission

C. nuclear transmutation

D. artificial radioactivity

Answer: D

Solution:

Solution:
$^{27}_{13}Al + ^{4}_{2}He \longrightarrow ^{30}_{14}Si + ^{1}_{1}H$
Bombardment of stable elements with high energy α-particles, protons, neutrons, deutrons or γ-rays produce radioactive nuclides.
These radio-nuclides do not occur naturally and may be called man-made or artificial nuclides. The radioactivity exhibited by these artificial radio nuclides is referred to as artificial radioactivity.

Question 92

Which is mismatched for NaCl crystal ?

Options:

A. $\frac{r^+}{r^-} = 0.414$ to 0.732

B. Coordination number $= 6:6$

C. Edge of unit cell $= (r^+ + r^-)$

D. Crystal structure $=$ fcc

Answer: C

Solution:

Solution:
Edge length of (NaCl crystal) unit cell
$= 2(r^+_{Na} + r^-_{Br^-})$

Question 93

Which of the following ions has the highest magnetic moment?

Options:

A. Zn^{2+}

B. Ti^{3+}

C. Sc^{3+}

D. Mn^{2+}

Answer: D

Solution:

Solution:
Magnetic moment $\propto$ no. of unpaired electron.
$Zn^{2+} = 3d^{10}4s^{0}$ unpaired electron $= 0$;
$\mu = 0$
$Ti^{3+} = 3d^{1}4s^{0}$ unpaired electron $= 1$
$\mu = \sqrt{n(n+2)}$
$= 1.73$
$Sc^{3+} = 3d^{0}4s^{0}$ unpaired electron $= 0$;
$\mu = 0$
$Mn^{2+} = 3d^{5}4s^{0}$ unpaired electrons $= 5$
$\mu = \sqrt{5(5+2)}$
$= 5.91$

Question 94

"ELEKTRON" is an alloy of

Options:

A. Cu, Zn and Mg

B. Fe and Mg

C. Ni and Zn

D. Al and Zn

Answer: D

Solution:

Solution:
Elektron is an alloy of different metals such as aluminium, zinc, silver, maganese, yttrium, neodymium, gadolinium, zirconium and other rare earth metals.

Question 95

The structure of ionic compound A^+B^- is identical to NaCl. If the edge length is 400 pm and cation radius is 75 pm, the radius of anion will be

Options:

A. 100 pm

B. 125 pm

C. 250 pm

D. 325 pm

Answer: B

Solution:

Solution:

Edge length $= 400\,\text{pm}$

$= 2(r^+ + r^-)$

$= 2r^+ + 2r^-$

$400 = 2 \times 75 + 2r^-$

or $r^- = 125\,\text{pm}$

Question 96

Which of the following statements is true?

Options:

A. $SnCl_4$ is more stable than $SnCl_2$

B. $PbCl_2$ is more stable than $PbCl_4$

C. $GeCl_4$ is more stable than $GeCl_2$

D. $TlCl_3$ is more stable than TlCl

Answer: B

Solution:

Solution:

- In lead, +2 oxidation state becomes more stable due to inert pair effect.

This is also supported by the fact that the $Pb(+4)$ compounds act as strong oxidising agent.

Question 97

Correct order for solubility of alkaline earth metals in water is

Options:

A. $MgF_2 > CaF_2 > SrF_2 > BaF_2$

B. $MgF_2 < CaF_2 < SrF_2 < BaF_2$

C. $MgF_2 > CaF_2 < SrF_2 < BaF_2$

D. $BaF_2 > MgF_2 > SrF_2 > CaF_2$

Answer: A

Solution:

Solution:
Order of solubility of fluorides of alkaline earth metals in water is
Halides of alkaline earth metals (except Be) are ionic solids and are therefore are water soluble and their solubility in water decreases from Mg to Ba due to the decrease in the hydration energy.
However, fluorides of alkanline earth metals excepts BeF_2 are almost insoluble in water.

Question 98

An organic compound A contains 20% C, 46.66% N and 6.66% H. It gives NH_3 gas on heating with NaOH. A can be

Options:

A. CH_3CONH_2

B. $C_6H_5CONH_2$

C. NH_2CONH_2

D. $CH_3NHCONH_2$

Answer: C

Solution:

Solution:
$\therefore$ Empirical formula $= CN_2H_4O$

Question 99

A metal reacts with dil acid and liberates hydrogen. If the reduction potential of hydrogen be considered zero, the reduction potential of that metal will be

Options:

A. equal to its oxidation potential

B. positive

C. zero

D. negative

Answer: D

Solution:

Solution:
Metals having negative values of E° (reduction) will displace H_2 with acids.

Question 100

In the equilibrium mixture, $KI + I_2 \rightleftharpoons KI_3$; the concentration of KI and I_2 is made two fold and three fold respectively . The concentration of Kb_3 becomes

Options:

A. two fold

B. three fold

C. five fold

D. six fold

Answer: A

Solution:

Solution:
$KI + I_2 \longrightarrow KI_3$ or $\underset{\text{from KI}}{I^-} + I_2 \rightarrow I_3$
On increasing concentration of KI and I_2 two fold and three fold respectively, the concentration of KI_3 becomes two fold.

Question 101

The angle between planes $2x - y + z = 6$ and $x + y + 2z = 3$ is

Options:

A. 30°

B. 60°

C. $\cos^{-1}\sqrt{\frac{3}{2}}$

D. $sin^{-1}\sqrt{\frac{3}{2}}$

Answer: B

Solution:

Solution:
Given equations of plane are
$2x - y + z = 6$ and $x + y + 2z = 3$
On comparing with $ax + by + cz = d$, we get
and $a_1 = 2, b_1 = -1, c_1 = 1$
$a_2 = 1, b_1 = 1, c_1 = 2$
$\therefore$ Angle between two planes is

$$\theta = \cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$$

$$= \cos^{-1}\left(\frac{2 \times 1 + (-1) \times 1 + 1 \times 2}{\sqrt{2^2 + (-1)^2 + (1)^2}\sqrt{1^2 + 1^2 + (2)^2}}\right)$$

$$= \cos^{-1}\left(\frac{2 - 1 + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}\right)$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{6} \times \sqrt{6}}\right) = \cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right)$$

$$= 60^\circ$$

Question 102

The maximum value of $3\cos\theta + 4sin\ \theta$ is

Options:

A. 3

B. 4

C. 5

D. None of these

Answer: 0

Solution:

Solution:

$\because$ Maximum value of $a\cos\theta + b\sin\theta = \sqrt{a^2 + b^2}$

$\therefore$ Maximum value of $3\cos\theta + 4\sin\theta = \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$

Question 103

If A and B are two such events that $P(A \cup B) = P(A \cap B)$, then which of the following is true?

Options:

A. $P(A) + P(B) = 0$

B. $P(A) + P(B) = P(A)\,P(B/A)$

C. $P(A) + P(B) = 2P(A)\,P(B/A)$

D. None of the above

Answer: C

Solution:

Solution:
Given, $P(A \cup B) = P(A \cap B)$
$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A \cap B)$
$\Rightarrow \quad P(A) + P(B) = 2P(A \cap B)$
$\Rightarrow \quad P(A) + P(B) = 2 \times P(A)\,P(B/A)$
$\left[\because P(B/A) = \dfrac{P(A \cap B)}{P(A)}\right]$

Question 104

The y-coordinate of a point P on the line joining $A(7, 2, 1)$ and $B(10, 5, 7)$ is 4 . Then, x and z-coordinates of the point are

Options:

A. $x = 9, z = 5$

B. $x = 3, z = 7$

C. $x = 2, z = 3$

D. None of these

Answer: A

Solution:

Solution:
Suppose P divides the line joining points $A(7, 2, 1)$ and $B(10, 5, 7)$ in the ratio $\lambda : 1$. Then,

Coordinates of P are
$(\frac{10\lambda+7}{\lambda+1}, \frac{5\lambda+2}{\lambda+1}, \frac{7\lambda+1}{\lambda+1})$
Since, y-coordinate of P is 4 .
$\therefore \frac{5\lambda+2}{\lambda+1} = 4$
$\Rightarrow 5\lambda+2 = 4\lambda+4$
$\Rightarrow \lambda = 2$
$\therefore$ x-coordinate of $P = (\frac{10\times 2+7}{2+1})$
$= (\frac{20+7}{3})$
$= \frac{27}{3}$
$\Rightarrow x = 9$
and z-coordinate of $P = (\frac{7\times 2+1}{2+1})$
$= (\frac{14+1}{3})$
$= \frac{15}{3}$
$\Rightarrow z = 5$

Question 105

If point D divides the base BC of a $\triangle ABC$ in the ratio $n:m$, then the value of $mBD^2 + nCD^2 + (m+n)AD^2$ is

Options:

A. $mAC^2 + nAB^2$

B. $(m+n)(AC^2 + AB^2)$

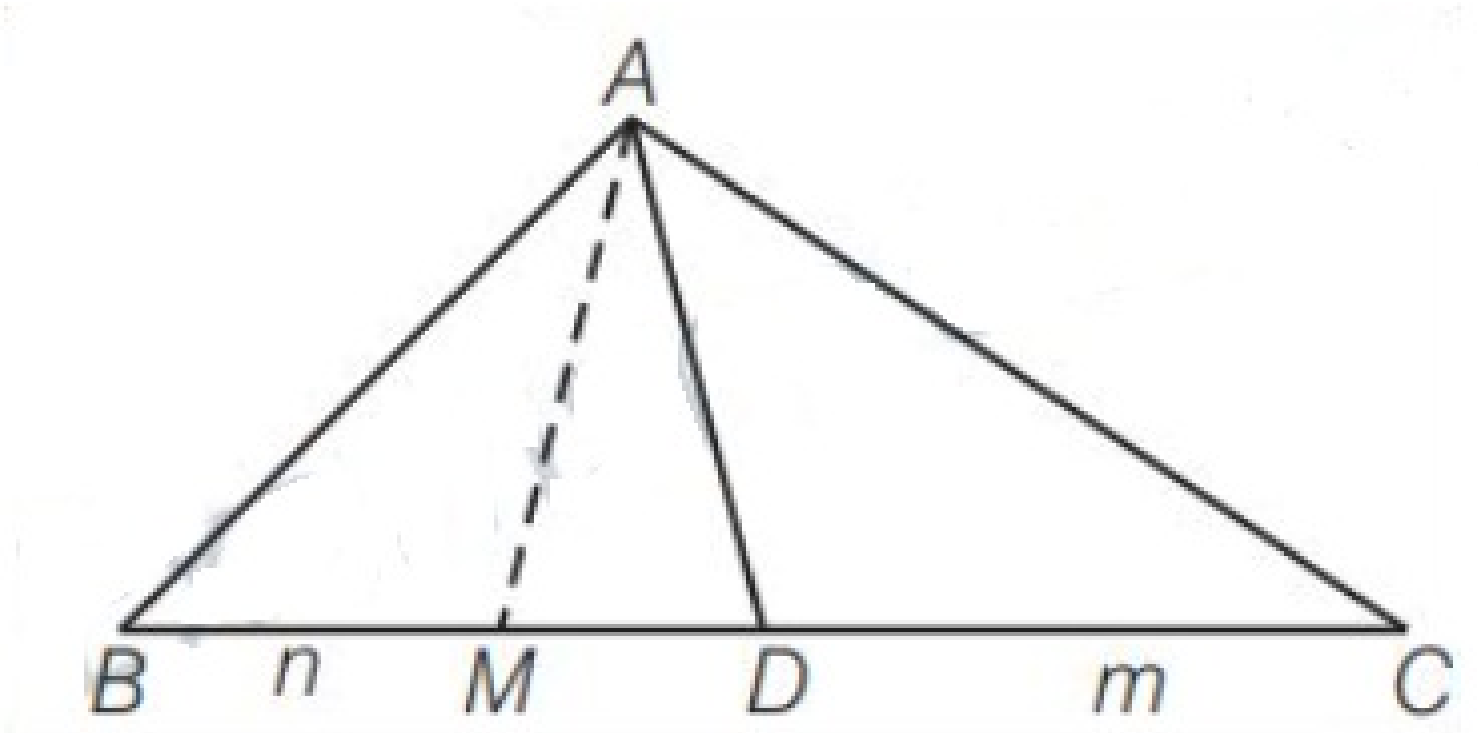
C. $nAC^2 + mAB^2$

D. None of these

Answer: C

Solution:

Solution:
$mBD^2 + nCD^2 + (m+n)AD^2$
$= mBD^2 + nCD^2 + mAD^2 + nAD^2$



$= m(BD^2 + AD^2) + n(CD^2 + AD^2)$
$= m(AB^2 + 2DM \cdot DB) + n(AC^2 - 2DM \cdot DC)$
$= mAB^2 + m \cdot 2DM \cdot DB + nAC^2 - n \cdot 2DM \cdot DC$
$= mAB^2 + nAC^2 + 2DM(mDB - nDC)$
$= mAB^2 + nAC^2 + 2DM(mn - nm)$
$= mAB^2 + nAC^2$

Question 106

A man is standing on the horizontal plane. The angle of elevation of top of the pole is α. If he walks a distance double the height of the pole, then the elevation of the pole is 2α. The value of α is

Options:

A. $\frac{\pi}{12}$

B. $\frac{\pi}{4}$

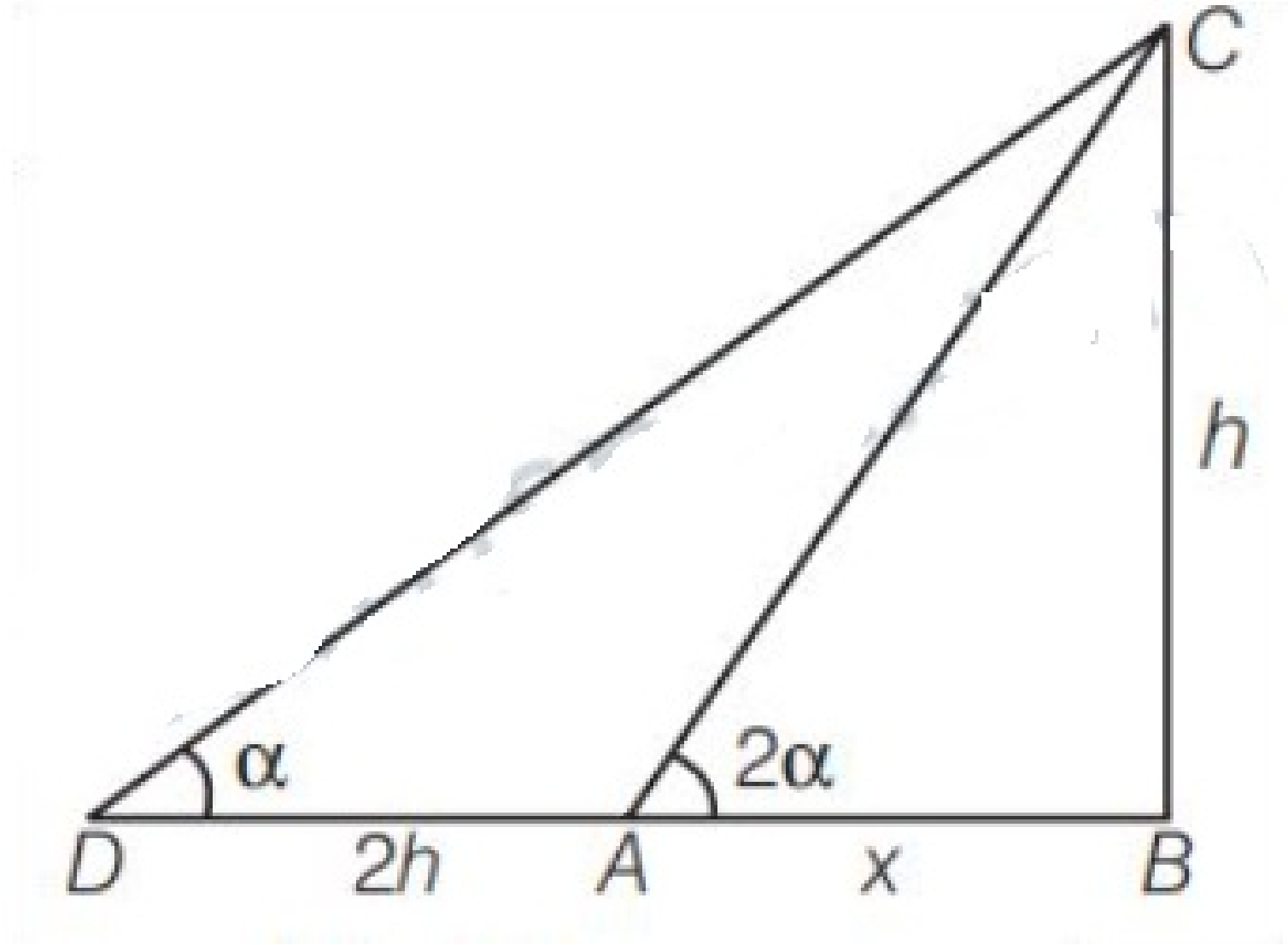
C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: A

Solution:

Solution:
Let the height of the pole be $BC = h$m.
In $\triangle ABC$,
$\tan 2\alpha = \frac{h}{x}$ . . . (i)



and in $\triangle DBC$,

$\tan\alpha = \dfrac{h}{2h+x}$

$\Rightarrow \tan\alpha = \dfrac{\frac{h}{x}}{2(\frac{h}{x})+1}$

$\Rightarrow \tan\alpha = \dfrac{\tan 2\alpha}{2\tan 2\alpha + 1}$

$\Rightarrow \tan\alpha(2\tan 2\alpha + 1) = \tan 2\alpha$

$\Rightarrow \tan\alpha\left[2 \times \dfrac{2\tan\alpha}{1-\tan^2\alpha} + 1\right] = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$\Rightarrow [4\tan\alpha + 1 - \tan^2\alpha] = 2$

$\Rightarrow \tan^2\alpha - 4\tan\alpha \pm 1 = 0$

$\Rightarrow \tan\alpha = \dfrac{+4 \pm \sqrt{16-4}}{2\times 1}$

$= \dfrac{4\pm\sqrt{12}}{2} = \dfrac{4\pm 2\sqrt{3}}{2}$

$\Rightarrow \tan\alpha = 2 \pm \sqrt{3}$

$\Rightarrow \tan\alpha = 2 - \sqrt{3}$

$\Rightarrow \tan\alpha = \tan 15^\circ$

$\Rightarrow \alpha = 15^\circ \quad \text{or} \quad \dfrac{\pi}{12}$

Question 107

If

$$f(x) = \begin{cases} \dfrac{\sin[x]}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$$

where, $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x \to 0} f(x)$ is equal to

Options:

A. 1

B. -1

C. 0

D. Does not exist

Answer: D

Solution:

Solution:
Given,

$$f(x) = \begin{cases} \dfrac{\sin[x]}{[x]} & \text{for} [x] \neq 0 \\ 0 & \text{for} [x] = 0 \end{cases}$$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{sin\,(1 + [x])}{[x]}$$
$$= \frac{sin\,[1 - 1]}{-1} = \frac{sin\,0}{-1} = 0$$
$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{sin\,(1 + [x])}{[x]}$$
$$= \frac{sin\,(1 + 0)}{0} = \infty$$

Hence, limit does not exist.

Question 108

The value of the angle between two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$ is

Options:

A. 30°

B. 60°

C. 45°

D. 90°

Answer: B

Solution:

Solution:
Given equation of straight lines are
$y = (2-\sqrt{3})x + 5$
and $y = (2+\sqrt{3})x - 7$
On comparing with $y = mx + c$, we get
$m_1 = 2-\sqrt{3}$
and $m_2 = 2+\sqrt{3}$
$\therefore \tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$
$= \dfrac{2+\sqrt{3}-(2-\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}$
$= \dfrac{2\sqrt{3}}{1+(4-3)} = \dfrac{2\sqrt{3}}{1+4-3}$
$\Rightarrow \tan\theta = \dfrac{2\sqrt{3}}{2}$
$= \sqrt{3}$
$\Rightarrow \theta = 60^\circ$

Question 109

Which of the following is a universal gate?

Options:

A. NAND

B. OR

C. AND

D. NOT

Answer: A

Solution:

Solution:
NAND is a universal gate, because with the help of this gate, any logic gate can be designed.

Question 110

According to Simpson's rule, the value of $\int_1^7 \frac{dx}{x}$ is

Options:

A. 1.358

B. 1.958

C. 1.625

D. 1.458

Answer: B

Solution:

Solution:

Let $I = \int_1^7 \frac{dx}{x}$

$\because \quad h = \frac{b-a}{n}$

Let

$n = 12 \ \therefore \ h = \frac{7-1}{12} = \frac{6}{12}$

$= 0.5$

Then, we have the following table

x	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
$\frac{1}{x}$	1	0.67	0.5	0.4	0.33	0.29	0.25	0.22	0.2	0.18	0.17	0.15	0.14
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}

Now, by Simpson's rule,

$= \frac{0.5}{3}[(1 + 0.14) + 4(0.67 + 0.4 + 0.29 + 0.22 + 0.18 + 0.15) + 2(0.5 + 0.33 + 0.25 + 0.2 + 0.17)]$

$= \frac{5}{30}[1.14 + 4 \times 1.91 + 2 \times 1.45]$

$= \frac{5}{30}[1.14 + 7.64 + 2.9] = \frac{5 \times 11.68}{30} = 1.958$

Question 111

Equation of a plane passing through $(-1, 1, 1)$ and $(1, -1, 1)$ and perpendicular to $x + 2y + 2z = 5$ is

Options:

A. $2x + 3y - 3z + 3 = 0$

B. $x + y + 3z - 5 = 0$

C. $2x + 2y - 3z + 3 = 0$

D. $x + y + z - 3 = 0$

Answer: C

Solution:

Solution:
Equation of plane passing through $(-1, 1, 1)$ is
$a(x+1) + b(y-1) + c(z-1) = 0$ . . . (i)
Also, it is passing through $(1, -1, 1)$.
$\therefore\ a(1+1) + b(-1-1) + c(1-1) = 0$
$\Rightarrow 2a - 2b + 0c = 0$ . . . (ii)
Also, required equation of plane (i) is perpendicular to $x + 2y + 2z = 5$.
$\therefore\ a \times 1 + b \times 2 + c \times 2 = 0$
$\Rightarrow a + 2b + 2c = 0$ . . . (iii)
Eqs. (ii) and (iii) are identical.
$\therefore\ \frac{a}{-4-0} = \frac{-b}{4-0} = \frac{c}{4+2}$
$\Rightarrow \frac{a}{-4} = \frac{b}{-4} = \frac{c}{6}$
$\Rightarrow \frac{a}{-2} = \frac{b}{-2} = \frac{c}{3} = \lambda$ (say)
$\Rightarrow a = -2\lambda, b = -2\lambda, c = 3\lambda$
On putting the values of a, b and c in Eq. (i), we get
$-2\lambda(x+1) - 2\lambda(y-1) + 3\lambda(z-1) = 0$
$\Rightarrow\ \lambda[-2x - 2 - 2y + 2 + 3z - 3] = 0$
$\Rightarrow\ -2x - 2y + 3z - 3 = 0$
$\Rightarrow\ 2x + 2y - 3z + 3 = 0$

Question 112

The value of $sin\,50^\circ \cos 10^\circ + \cos 50^\circ\ sin\ 10^\circ$ is

Options:

A. $\frac{1}{2}$

B. $\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. 1

Answer: C

Solution:

Solution:
$sin\,50^\circ \cos 10^\circ + \cos 50^\circ\ sin\ 10^\circ$
$= sin\,(50^\circ + 10^\circ)$
$= sin\,60^\circ = \frac{\sqrt{3}}{2}$

Question 113

If $f'(x) > 0\ \forall x \in R, f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4), 0 < x < \frac{\pi}{2}$, then $g(x)$ is increasing in

Options:

A. $(0, \frac{\pi}{4})$

B. $(\frac{\pi}{6}, \frac{\pi}{3})$

C. $(0, \frac{\pi}{3})$

D. $(\frac{\pi}{4}, \frac{\pi}{2})$

Answer: D

Solution:

Solution:
Given, $g(x) = f(\tan^2 x - 2\tan x + 4)$
On differentiating w.r.t. x, we get
$g'(x) = f'(\tan^2 x - 2.\tan x + 4)$
$\times (2\tan x \ sec^2 x - 2sec^2 x)$
$= f'(\tan^2 x - 2\tan x + 4)2sec^2 x(\tan x - 1)$
$\because \ f'(x) > 0 \ \forall x \in (0, \frac{\pi}{2})$
Also, $2sec^2 x > 0 \ \forall x \in (0, \frac{\pi}{2})$
[$\because x \in (0, \frac{\pi}{2})$. , given]
But $\tan x - 1 > 0 \ \forall x \in (\frac{\pi}{4}, \frac{\pi}{2})$
$\therefore \ g'(x) > 0 \ \forall x \in (\frac{\pi}{4}, \frac{\pi}{2})$
Hence, $g(x)$ is increasing in $(\frac{\pi}{4}, \frac{\pi}{2})$.

Question 114

Probability of getting a total of 7 or 9 in a single throw of two dice is

Options:

A. $\frac{5}{18}$

B. $\frac{1}{6}$

C. $\frac{1}{9}$

D. None of these

Answer: A

Solution:

Solution:
The total number of sample points in a sample space of single throw of two dice, $n(S) = 36$
Let $E_1 =$ Event of getting a sum 7

E_2 = Event of getting a sum 9
$\therefore \quad E_1 = \{(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)\}$
$\Rightarrow \quad n(E_1) = 6$
and $\quad E_2 = \{(3,6),(6,3),(4,5),(5,4)\}$
$\Rightarrow \quad n(E_2) = 4$
$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$
and $\quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$
Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$
$= \frac{1}{6} + \frac{1}{9} = \frac{6+4}{36} = \frac{10}{36} = \frac{5}{18}$

Question 115

What is compiler?

Options:

A. Application software

B. System software

C. Utility software

D. All of these

Answer: B

Solution:

Solution:
Compiler is a system software which transforms source code written in a programming language into another computer language.

Question 116

If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x + x^3}{1 + 3x^2}$, then $fog(x)$ is equal to

Options:

A. $-f(x)$

B. $3f(x)$

C. $[f(x)]^3$

D. None of these

Answer: B

Solution:

Solution:

Given, $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \left(\frac{3x+x^3}{1+3x^2}\right)$

$\therefore \quad f \circ g(x) = f\{g(x)\} = f\left(\frac{3x+x^3}{1+3x^2}\right)$

$$= \log\left[\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right]$$

$$= \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$$

$$= \log\left[\frac{(1+x)^3}{(1-x)^3}\right] = \log\left(\frac{1+x}{1-x}\right)^3$$

$$= 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$$

Question 117

Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then, $f(x)$ is equal to

Options:

A. $\frac{1}{3x} + \frac{2}{3}x^2$

B. $-\frac{x}{3} + \frac{4x^2}{3}$

C. $-\frac{1}{x}$

D. $-\frac{1}{x} + \frac{2}{x^2}$

Answer: A

Solution:

Solution:

Given, $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Using L' Hospital's rule, we get

$$\lim_{t \to x} \frac{t^2 f'(x) - 2xf(t)}{-1} = 1$$

$$\Rightarrow x^2 f'(x) - 2xf(x) + 1 = 0$$

$$\Rightarrow \frac{x^2 f'(x) - 2xf(x)}{(x^2)^2} + \frac{1}{x^4} = 0$$

$$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = -\frac{1}{x^4}$$

On integrating both sides, we get

$$\frac{f(x)}{x^2} = +\frac{1}{3x^3} + c \Rightarrow f(x) = \frac{1}{3x} + cx^2$$

Also, $f(1) = 1 \Rightarrow 1 = \frac{1}{3 \times 1} + c(1)^2$

$$\Rightarrow 1 = \frac{1}{3} + c \Rightarrow \frac{2}{3} = c$$

$$\therefore f(x) = \frac{1}{3x} + \frac{2}{3x^2}$$

Question 118

The position vectors of three non-collinear points A, B and C are a, b and c, respectively. The perpendicular distance of point C from the straight line AB is

Options:

A. $\frac{|b \times c|}{|b - c|}$

B. $\frac{|a \times b|}{|b - a|}$

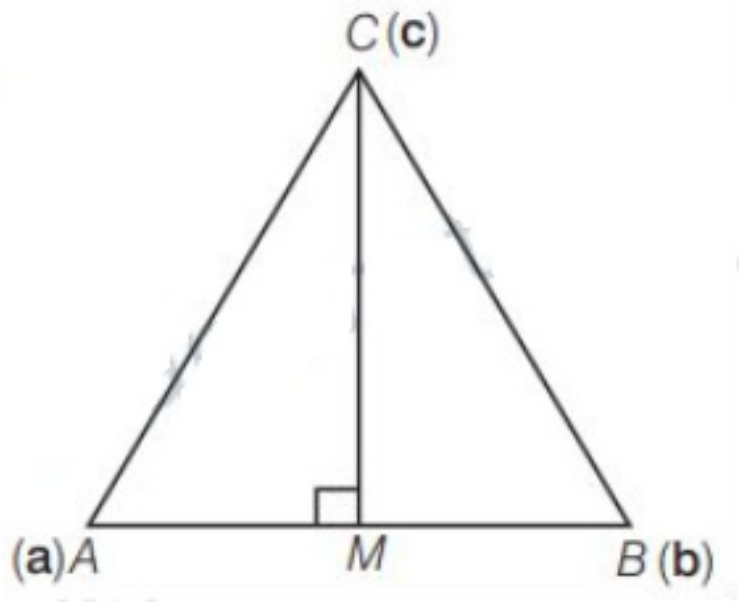
C. $\frac{|c \times a|}{|c - a|}$

D. $\frac{|b \times c + c \times a + a \times b|}{|b - a|}$

Answer: D

Solution:

Solution:
Given, position vectors of three non-collinear points A, B, C are a, b and c.
Let CM be the perpendicular from C on AB. Then,



Area of $\triangle ABC = \frac{1}{2}AB \cdot CM = \frac{1}{2}\left|AB \cdot CM\right|$

But area of $\triangle ABC = \frac{1}{2}\left|a \times b + b \times c + c \times a\right|$

$\therefore \quad \frac{1}{2}\left|AB \cdot CM\right| = \frac{1}{2}\left|a \times b + b \times c + c \times a\right|$

$\Rightarrow \quad CM = \frac{|a \times b + b \times c + c \times a|}{|b - a|}$

Question 119

If $\int sin\ \{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\}\,dx$ $= Asin^{-1}x + Bx\sqrt{1-x^2} + C$, **then** $A + B$ **is equal to**

Options:

A. 10

B. $\frac{1}{2}$

C. 1

D. $-\frac{1}{2}$

Answer: C

Solution:

Solution:
We have,
$\int sin\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\}\,dx = Asin^{-1}x + Bx\sqrt{1-x^2} + C$
Let $I = \int sin\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\}\,dx$
$= -\int sin\{2\tan^{-1}\sqrt{\frac{2sin^2\theta}{2\cos^2\theta}}\}2sin\,2\theta d\theta$
$= -\int sin\{2\tan^{-1}\sqrt{\tan^2\theta}\}2sin\,2\theta d\theta$
$= -\int sin\{2\tan^{-1}\tan\theta\}2sin\,2\theta d\theta$
$= -\int[sin(2\theta)]2sin\,2\theta d\theta$
$= -\int 2sin^2 2\theta d\theta$
$= -\int(1-\cos 4\theta)\,d\theta$
$= -[\theta - \frac{sin\,4\theta}{4}] + C$
$= [-\frac{1}{2}\cos^{-1}x + \frac{1}{4}\times 2sin\,2\theta\cos 2\theta] + C$
$= [-\frac{1}{2}(\frac{\pi}{2} - sin^{-1}x) + \frac{1}{2}\sqrt{1-x^2}\times x] + C$
$= \frac{1}{2}sin^{-1}x + \frac{x}{2}\sqrt{1-x^2} + (C - \frac{\pi}{4})$
But $I = Asin^{-1}x + Bx\sqrt{1-x^2} + C$
$\therefore\ A = \frac{1}{2}$ and $B = \frac{1}{2}$
Hence, $A + B = \frac{1}{2} + \frac{1}{2} = 1$

Question 120

The coefficient of x^4 in $(1 + x + x^3 + x^4)^{10}$ is

Options:

A. 210

B. 100

C. 310

D. 110

Answer: C

Solution:

Solution:
Let $E = (1 + x + x^3 + x^4)^{10}$
$= [1 + x + x^3(1 + x)]^{10} = (1 + x)^{10}(1 + x^3)^{10}$
$= [{}^{10}C_0 + {}^{10}C_1x^1 + {}^{10}C_2x^2 + \ldots + {}^{10}C_9x^9 + {}^{10}C_{10}x^{10}]$
$\times [{}^{10}C_0 1 + {}^{10}C_1 1 \times (x^3) + {}^{10}C_2(x^3)^{2.}$
$+ {}^{10}C_3(x^3)^3 + \ldots]$

$\therefore$ Coefficient of x^4 in $E = {}^{10}C_4 \times {}^{10}C_0 + {}^{10}C_1 \times {}^{10}C_1$
$= \frac{10!}{6! \times 4!} \times 1 + 10 \times 10$
$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + 100$
$= 210 + 100$
$= 310$

Question 121

If $A = \begin{bmatrix} 4 & 11 \\ 2 & 6 \end{bmatrix}$, then A^{-1} is equal to

Options:

A. $\begin{bmatrix} -1 & \frac{-11}{2} \\ 3 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 3 & \frac{-11}{2} \\ -1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 2 \\ -1 & -11 \\ 2 & \end{bmatrix}$

D. None of these

Answer: B

Solution:

Solution:
Given, $A = \begin{bmatrix} 4 & 11 \\ 2 & 6 \end{bmatrix}$
Now,
$|A| = \begin{vmatrix} 4 & 11 \\ 2 & 6 \end{vmatrix}$
$= 24 - 22 = 2$
$\text{adj } A = \begin{bmatrix} 6 & -11 \\ -2 & 4 \end{bmatrix}$
$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|}$
$= \frac{1}{2}\begin{bmatrix} 6 & -11 \\ -2 & 4 \end{bmatrix}$
$= \begin{bmatrix} 3 & \frac{-11}{2} \\ -1 & 2 \end{bmatrix}$

Question 122

The locus of centre of circles which cuts orthogonally the circle $x^2 + y^2 - 4x + 8 = 0$ and touches $x + 1 = 0$, is

Options:

A. $y^2 + 6x + 7 = 0$

B. $x^2 + y^2 + 2x + 3 = 0$

C. $x^2 + 3y + 4 = 0$

D. None of the above

Answer: A

Solution:

Solution:
Let equation of circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$
Given equation of circle is
$x^2 + y^2 - 4x + 8 = 0$
The centres of above circles are $(-g, -f)$ and $(2, 0)$.
Condition of orthogonality is
$2(g_1g_2 + f_1f_2) = c_1 + c_2$
$\therefore\ 2(g \times (-2) + (f) \times 0) = c + 8$
$\Rightarrow\ -4g = c + 8$ . . . (i)
Also, the assume circle touch the line $x + 1 = 0$.
$\therefore$ The perpendicular drawn from centre to the line is equal to radius.
$\therefore\ \frac{-g + 1}{\sqrt{1^2}} = \sqrt{g^2 + f^2 - c}$
$\Rightarrow -g + 1 = \sqrt{g^2 + f^2 - c}$
On squaring both sides, we get
$g^2 + 1 - 2g = g^2 + f^2 - c$
$\Rightarrow\ c = f^2 + 2g - 1$
Putting the value of c in Eq. (i), we get
$-4g = f^2 + 2g - 1 + 8$
$\Rightarrow f^2 + 2g + 4g + 7 = 0$
$\Rightarrow\ f^2 + 6g + 7 = 0$
$\therefore$ Locus of centre of circle is $y^2 + 6x + 7 = 0$.

Question 123

Let $f(x) = \begin{vmatrix} \sin 3x & 1 & 2(\cos\frac{3x}{2} + \sin\frac{3x}{2})^2 \\ \cos 3x & -1 & 2(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}) \\ \tan 3x & 4 & 1 + 2\tan 3x \end{vmatrix}$
Then, the value of $f'(x)$ **at** $x = (2n + 1)\pi, n \in I$ **(the set of integers) is equal to**

Options:

A. $(-1)^n$

B. $(-1)^{n+1}$

C. 3

D. 9

Answer: C

Solution:

Solution:

Given,

$$f(x)=\begin{vmatrix} sin\,3x & 1 & 2(\cos\frac{3x}{2}+sin\ \frac{3x}{2})^2 \\ \cos 3x & -1 & 2(\cos^2\frac{3x}{2}-sin^2\frac{3x}{2}) \\ \tan 3x & 4 & 1+2\tan 3x \end{vmatrix}$$

On differentiating w.r.t. x, we get

$$f'(x)=\begin{vmatrix} \frac{d}{dx}(sin\,3x) & 1 & 2(\cos\frac{3x}{2}+sin\ \frac{3x}{2})^2 \\ \frac{d}{dx}(\cos 3x) & -1 & 2(\cos^2\frac{3x}{2}-sin^2\frac{3x}{2}) \\ \frac{d}{dx}(\tan 3x) & 4 & 1+2\tan 3x \end{vmatrix}$$

$$+\begin{vmatrix} sin\,3x & \frac{d}{dx}(1) & 2(\cos\frac{3x}{2}+sin\ \frac{3x}{2})^2 \\ \cos 3x & \frac{d}{dx}(-1) & 2(\cos^2\frac{3x}{2}-sin^2\frac{3x}{2}) \\ \tan 3x & \frac{d}{dx}(4) & 1+2\tan 3x \end{vmatrix}$$

$$+\begin{vmatrix} sin\,3x & 1 & 2\frac{d}{dx}(\cos\frac{3x}{2}+sin\ \frac{3x}{2})^2 \\ \cos 3x & -1 & 2\frac{d}{dx}(\cos^2\frac{3x}{2}-sin^2\frac{3x}{2}) \\ \tan 3x & 4 & \frac{d}{dx}(1+2\tan 3x) \end{vmatrix}$$

$$=\begin{vmatrix} 3\cos 3x & 1 & 2(\cos\frac{3x}{2}+sin\ \frac{3x}{2})^2 \\ 3sin\,3x & -1 & 2(\cos^2\frac{3x}{2}-sin^2\frac{3x}{2}) \\ 3sec^2 3x & 4 & 1+2\tan 3x \end{vmatrix}$$

$$+\begin{vmatrix} sin\,3x & 0 & 2(\cos\frac{3x}{2}+sin\ \frac{3x}{2})^2 \\ \cos 3x & 0 & 2(\cos^2\frac{3x}{2}-sin^2\frac{3x}{2}) \\ \tan 3x & 0 & 1+2\tan 3x \end{vmatrix}$$

$$sin\,3x\ \ 1\ \ 2\times 2(\cos\frac{3x}{2}+sin\ \frac{3x}{2})$$

$$\times(\frac{-3}{2}sin\ \frac{3x}{2}+\frac{3}{2}\cos\frac{3x}{2})$$

$$+\cos 3x\ -1\ \ 2(-2\cos\frac{3x}{2}\times\frac{3}{2}sin\ \frac{3x}{2}.$$

$$-2sin\ \frac{3x}{2}\times\frac{3}{2}\cos\frac{3x}{2})$$

$$\tan 3x\ \ 4$$

$$(0+2\times 3sec^2 3x)$$

At $x=(2n+1)\pi$,

$$f'(x)=\begin{vmatrix} 3(-1) & 1 & 2(1) \\ 0 & -1 & 2(-1) \\ 3 & 4 & 1+0 \end{vmatrix}+0$$

$$+\begin{vmatrix} 0 & 1 & 4(0-1)\times[-\frac{3}{2}(-1)+\frac{3}{2}\times 0] \\ 0 & & \\ 0 & -1 & 0 \end{vmatrix}$$

$$=\begin{vmatrix} -3 & 1 & 2 \\ 0 & -1 & -2 \\ 3 & 4 & 1 \end{vmatrix}+0+\begin{vmatrix} 0 & 1 & -6 \\ -1 & -1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$=[-3(-1+8)-1(0+6)+2(0+3)]$$

$$+[0-1(0-0)-6(-4)]$$

$$=-21+24=3$$

Question 124

The condition for the line $lx + my + n = 0$ to be a normal to $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is

Options:

A. $\frac{l^2}{9} + \frac{m^2}{25} = \frac{n^2}{256}$

B. $\frac{9}{m^2} + \frac{25}{l^2} = \frac{256}{n^2}$

C. $\frac{p^2}{9} - \frac{m^2}{25} = \frac{n^2}{256}$

D. None of these

Answer: B

Solution:

Solution:

Given equation of line is $lx + my + n = 0$ and equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

$\therefore$ The equation of any normal to the ellipse is

$5x\sec\theta - 3y\,\text{cosec}\,\theta = 25 - 9$

$\left(\because \text{ the equation of any normal to the ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } ax\sec\theta - by\,\text{cosec}\,\theta = a^2 - b^2\right)$

$\Rightarrow \quad 5x\sec\theta - 3y\,\text{cosec}\,\theta - 16 = 0$

As the Eq. (i) is the normal to the ellipse.

$\therefore \quad \frac{5\sec\theta}{1} = \frac{-3\,\text{cosec}\,\theta}{m} = \frac{-16}{n}$

$\Rightarrow \cos\theta = \frac{5n}{-16l}$ and $\sin\theta = \frac{3n}{16m}$

$\because \quad \cos^2\theta + \sin^2\theta = 1$

$\therefore \quad \left(\frac{5n}{16l}\right)^2 + \left(\frac{3n}{16m}\right)^2 = 1$

$\Rightarrow \quad \frac{25n^2}{256l^2} + \frac{9n^2}{256m^2} = 1$

$\Rightarrow \quad \frac{25}{1^2} + \frac{9}{m^2} = \frac{256}{n^2}$

Question 125

The least value of a, for which the function $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a$ has atleast one solution in the interval $(0, \frac{\pi}{2})$, is

Options:

A. 9

B. 4

C. 5

D. 1

Answer: A

Solution:

Solution:

Given, $\frac{4}{sin\,x} + \frac{1}{1-sin\,x} = a$

$\Rightarrow\ 4(1-sin\,x) + sin\,x = asin\,x(1-sin\,x)$

$\Rightarrow\ 4-4sin\,x + sin\,x = asin\,x - asin^2x$

$\Rightarrow\ asin^2x - (3+a)sin\,x + 4 = 0$ . . . (i)

It is a quadratic equation in $sin\,x$, so

$D \geq 0$

$\Rightarrow (3+a)^2 - 4 \times 4a \geq 0$

$\Rightarrow 9 + a^2 + 6a - 16a \geq 0$

$\Rightarrow\ a^2 - 10a + 9 \geq 0$

$\Rightarrow\ (a-1)(a-9) \geq 0$

$\Rightarrow\ a \geq 9$ or $a \leq -9$

Now, at a $= 9$, Eq. (i) becomes

$9sin^2x - 12sin\,x + 4 = 0$

$\Rightarrow\ (3sin\,x - 2)^2 = 0$

$\Rightarrow\ sin\,x = \frac{2}{3} < 0$

$\Rightarrow\ x \in (0, \frac{\pi}{2})$

Hence, least value of a is 9 .

Question 126

If one line of regression coefficient is less than unity, then the other will be

Options:

A. less than unity

B. equal to unity

C. greater than unity

D. All of these

Answer: A

Solution:

Solution:

If one line of regression coefficient is less than unity, then other will be greater than unity.

Question 127

Three concurrent edges of a parallelopiped are given by

$a = 2\hat{i} - 3\hat{j} + \hat{k}$,
$b = \hat{i} - \hat{j} + 2\hat{k}$
$c = 2\hat{i} + \hat{j} - \hat{k}$
The volume of the parallelopiped is

Options:

A. 14 cu units

B. 20 cu units

C. 25 cu units

D. 60 cu units

Answer: A

Solution:

Solution:
Given edges of a parallelopiped are
$a = 2\hat{i} - 3\hat{j} + \hat{k}, b = \hat{i} - \hat{j} + 2\hat{k}$ and $c = 2\hat{i} + \hat{j} - \hat{k}$
$\therefore$ Volume of parallelopiped $= [abc]$

$$= \begin{vmatrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

$= |2(1-2) + 3(-1-4) + 1(1+2)|$
$= |-2 - 15 + 3| = 14$ cu units

Question 128

Roots of equation $x^3 - 6x + 1 = 0$ lie in the interval

Options:

A. $(2, 3)$

B. $(3, 4)$

C. $(3, 5)$

D. $(4, 6)$

Answer: A

Solution:

Solution:
Let $f(x) = x^3 - 6x + 1$
Now, $f(2) = (2)^3 - 6 \times 2 + 1$
$= 8 - 12 + 1 = -3$
and $f(3) = (3)^3 - 6 \times 3 + 1$
$= 27 - 18 + 1 = 10$

Here, we see that $f(2)$ and $f(3)$ have opposite signs, so one of the roots lies in $(2,3)$.

Question 129

RAM is a

Options:

A. volatile memory

B. non-volatile memory

C. cash memory

D. dynamic memory

Answer: A

Solution:

Solution:
RAM is a volatile memory, because when computer is turned off, the content of RAM is immediately lost.

Question 130

If

$$\lim_{n\to\infty} \sum \frac{\log(n+r)-\log n}{n} = 2\left(\log 2 - \frac{1}{2}\right)$$

, then $\lim_{n\to\infty} \frac{1}{n^{\lambda}}[(n+1)^{\lambda}(n+2)^{\lambda}...(n+n)^{\lambda}]^{1/n}$ **is equal to**

Options:

A. $\frac{4\lambda}{e}$

B. $\left(\frac{4}{e}\right)^{\lambda}$

C. $\left(\frac{4}{e}\right)^{\frac{1}{\lambda}}$

D. $\left(\frac{e}{4}\right)^{\lambda}$

Answer: B

Solution:

Solution:

Given,

$$\lim_{n\to\infty}\sum\frac{\log(n+r)-\log n}{n}=2(\log 2-\frac{1}{2})$$

or

$$\lim_{n\to\infty}\sum\frac{1}{n}\log(1+\frac{r}{n})=2(\log 2-\frac{1}{2})$$

Let $A=\lim_{n\to\infty}\frac{1}{n^{\lambda}}[(n+1)^{\lambda}(n+2)^{\lambda}\ldots(n+n)^{\lambda}]^{1/n}$

$=\lim_{n\to\infty}[(1+\frac{1}{n})^{\lambda}(1+\frac{2}{n})^{\lambda}\ldots(1+\frac{n}{n})^{\lambda}]^{1/n}$

On taking log both sides, we get

$\log A=\lim_{n\to\infty}\frac{1}{n}[\log(1+\frac{1}{n})^{\lambda}+\log(1+\frac{2}{n})^{\lambda}.$

$+\ldots+\log(1+\frac{n}{n})^{\lambda}]$

$$\Rightarrow\ \log A=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\lambda\log(1+\frac{r}{n})$$

$$\Rightarrow\ \log A=\lambda\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\log(1+\frac{r}{n})$$

$\Rightarrow\ \log A=2\lambda(\log 2-\frac{1}{2})$

$\Rightarrow\ \left[\text{from Eq. (i)}\right)$

$\Rightarrow\ \log A=\log 4^{\lambda}-\lambda$

$\Rightarrow\ \log A=\log 4^{\lambda}-\lambda\log e$

$\therefore\ \log A=\log\frac{4^{\lambda}}{e^{\lambda}}$

$\Rightarrow\ A=(\frac{4}{e})^{\lambda}$

Question 131

If $\lim_{x\to 0}\frac{sin\,(sin\,x)-sin\,x}{ax^3+bx^5+c}=\frac{-1}{12}$, then

Options:

A. $a=2, b\in R, c=0$

B. $a=-2, b\in R, c=0$

C. $a=1, b\in R, c=0$

D. $a=-1, b\in R, c=0$

Answer: B

Solution:

Solution:

Given, $\lim_{x\to 0}\frac{sin\,(sin\,x)-sin\,x}{ax^3+bx^5+c}=-\frac{1}{12}\ldots$ (i)

$\Rightarrow\ \frac{sin\,sin\,0-sin\,0}{a(0)^3+b(0)^5+c}=-\frac{1}{12}$

$\Rightarrow\ \frac{0}{c}=-\frac{1}{12}\Rightarrow c=0$

Applying L' Hospital's rule in Eq. (i), we get

$\lim_{x \to 0} \frac{\cos(sin\ x)\cos x - \cos x}{3ax^2 + 5bx^4 + 0} = -\frac{1}{12}$

$\Rightarrow \lim_{x \to 0} \frac{\cos x(\cos(sin\ x) - 1)}{x^2(3a + 5bx)} = -\frac{1}{12}$

$\Rightarrow \lim_{x \to 0} \cos x \frac{2sin^2(\frac{sin\ x}{2})}{x^2(3a + 5bx)} = -\frac{1}{12}$

$\Rightarrow \frac{\cos 0}{3a + 5b \times 0} \times \lim_{x \to 0} \frac{2sin^2(\frac{sin\ x}{2})}{4(\frac{sin\ x}{2})^2 \times (\frac{x}{sin\ x})^2}$

$\Rightarrow \frac{1}{3a + 0} \times \frac{1}{2} \times \frac{1}{1} = -\frac{1}{12}$ $(\because \lim_{\theta \to 0} \frac{sin\ \theta}{\theta} = 1)$

$\Rightarrow \frac{1}{6a} = -\frac{1}{12} \Rightarrow 6a = -12$

$\therefore a = -2$

Hence, $a = -2, b \in R$ and $c = 0$

Question 132

According to Newton-Raphson method, the value of $\sqrt{12}$ upto three places of decimal will be

Options:

A. 3.463

B. 3.462

C. 3.467

D. None of these

Answer: D

Solution:

Solution:

Let $x = \sqrt{12}$

$\Rightarrow x^2 = 12$

$\Rightarrow x^2 - 12 = 0$

Let $f(x) = x^2 - 12$

The first approximation in the Newton-Raphson method is given by

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{x_0^2 - 12}{2x_0}$

$= \frac{x_0^2 + 12}{2x_0}$

$\because 3 < \sqrt{12} < 3.5$

We can take $x_0 = 3.5$

$\therefore x_1 = \frac{(3.5)^2 + 12}{2 \times 3.5} = \frac{12.25 + 12}{7}$

$= \frac{24.25}{7}$

$= 3.464$

Question 133

If $\frac{(3-i)^2}{2+i} = A + iB$, where A and B are real numbers, then A and B are equal to

Options:

A. $A = -4, B = 2$

B. $A = 2, B = -4$

C. $A = 2, B = 4$

D. None of these

Answer: B

Solution:

Solution:
Given equation is
$\frac{(3-i)^2}{2+i} = A + iB$
$\Rightarrow \frac{9-1-6i}{2+i} = A + iB$
$\Rightarrow \frac{8-6i}{2+i} = A + iB$
$\Rightarrow \frac{2(4-3i)}{2+i} \times \frac{2-i}{2-i} = A + iB$
$\Rightarrow \frac{2[8-4i-6i-3]}{4+1} = A + iB$
$\Rightarrow \frac{2[5-10i]}{5} = A + iB$
$\Rightarrow 2 - 4i = A + iB$
On equating the real and imginary parts from both sides, we get
$A = 2$
$B = -4$
and

Question 134

The radical centre of the system of circles, and $x^2 + y^2 + 4x + 7 = 0$
$2(x^2 + y^2) + 3x + 5y + 9 = 0$
$x^2 + y^2 + y = 0$ is

Options:

A. $(-2, -1)$

B. $(1, -2)$

C. $(-1, -2)$

D. None of these

Answer: A

Solution:

Solution:
Given system of circles is
$x^2 + y^2 + 4x + 7 = 0$ . . . (i)
$2(x^2 + y^2) + 3x + 5y + 9 = 0$ . . . (ii)
or $x^2 + y^2 + \frac{3}{2}x + \frac{5}{2}y + \frac{9}{2} = 0$ . . . (iii)
and $x^2 + y^2 + y = 0$
The radical centre can be obtained by solving the Eqs. (i), (ii) and (iii).
On subtracting Eq. (ii) from Eq. (i), we get
$4x - \frac{3}{2}x - \frac{5}{2}y + 7 - \frac{9}{2} = 0$
$\Rightarrow \frac{5}{2}x - \frac{5}{2}y + \frac{5}{2} = 0$
$x - y + 1 = 0$ . . . (iv)
On subtracting Eq. (iii) from Eq. (i), we get
$4x - y + 7 = 0$ . . . (v)
On solving Eqs. (iv) and (v), we get and $y = -1$
Hence, radical centre is $(-2, -1)$.

Question 135

The sum of n terms of the series $1 + 5 + 12 + 22 + 35 + \ldots$ is

Options:

A. $\frac{n^2(n+1)}{8}$

B. $\frac{n^2(n+1)}{6}$

C. $\frac{n^2(n+1)}{2}$

D. None of these

Answer: C

Solution:

Solution:
- Let
$S_n = 1 + 5 + 12 + 22 + 35 + \ldots + T_n$
$S_n = 1 + 5 + 12 + 22 + \ldots + T_n$
$0 = 1 + 4 + 7 + 10 + 13 + \ldots - T_n$
$\Rightarrow T_n = 1 + 4 + 7 + 10 + 13 + \ldots + n$ terms
This is an arithmetic series whose first term a is 1 and common difference d is 3 .
$\therefore T_n = \frac{n}{2}[2 \times 1 + (n-1)3]$
$\Rightarrow T_n = \frac{n}{2}[2 + 3n - 3]$
$= \frac{n}{2}[3n - 1]$
$= \frac{1}{2}[3n^2 - n]$
$S_n = \Sigma T_n$
$= \frac{1}{2}[\Sigma(3n^2 - n)]$
$= \frac{1}{2}[3\Sigma n^2 - \Sigma n]$
$= \frac{1}{2}[\frac{3n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}]$
$= \frac{n(n+1)}{2}[\frac{2n+1}{2} - \frac{1}{2}]$

$= \frac{n(n+1)}{2}[\frac{2n}{2}] = \frac{n^2(n+1)}{2}$

Question 136

The curve, for which the area of the triangle formed by X-axis, the tangent line at any point P and line OP is equal to a^2, is given by

Options:

A. $y = x - Cx^2$

B. $x = Cy \pm \frac{a^2}{y}$

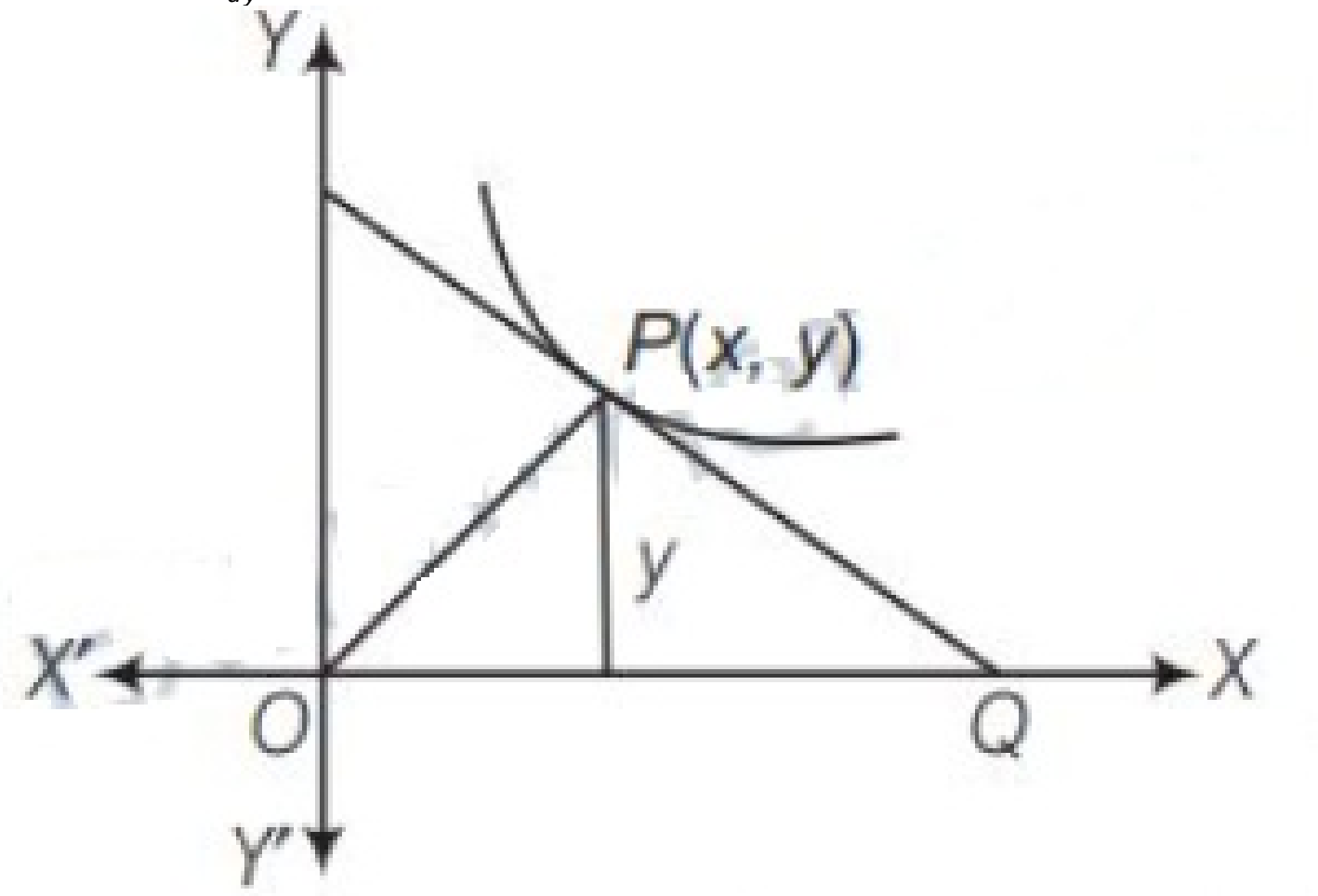
C. $y = Cx \pm \frac{a^2}{x}$

D. None of these

Answer: B

Solution:

Solution:
Tangent drawn at any point (x, y) is
$Y - y = \frac{dy}{dx}(X - x)$
When $Y = 0, X = x - y\frac{dx}{dy}$



$\because$ Area of $\triangle OPQ = a^2$ (given)

$\therefore \quad \left|\frac{1}{2}X \cdot y\right| = a^2$

$\Rightarrow \quad \left|(x - y\frac{dx}{dy})y\right| = 2a^2$

$\Rightarrow \quad xy - y^2\frac{dx}{dy} = \pm 2a^2$

$\Rightarrow \quad \frac{dx}{dy} - \frac{x}{y} = \pm\frac{2a^2}{y^2}$

Here, $P = -\frac{1}{y}$ and $Q = \pm\frac{2a^2}{y^2}$

$\therefore \quad F = e^{\int Pdy} = e^{\int -\frac{1}{y}dy}$

$= e^{-\log y} = e^{\log\frac{1}{y}} = \frac{1}{y}$

Question 137

If the function $f:[1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is defined by

Options:

A. $(\frac{1}{2})^{x(x-1)}$

B. $\frac{1}{2}(1 \pm \sqrt{1 + 4\log_2 x})$

C. $\frac{1}{2}(1 - \sqrt{1 - 4\log_2 x})$

D. None of these

Answer: C

Solution:

Solution:

Given, function $f[1, \infty) \to [1, \infty)$ is defined as $f(x) = 2^{x(x-1)}$. It is an exponential function, so it is continuous and increasing in their domain. Thus, f^{-1} exists.

Let

$\Rightarrow \log y = x(x-1)\log 2$

$\Rightarrow (x^2 - x)\log 2 - \log y = 0$

$\Rightarrow x^2 - x - \frac{\log y}{\log 2} = 0$

$\therefore \; x = \frac{+1 \pm \sqrt{(-1)^2 - 4(1)(-\frac{\log y}{\log 2})}}{2(1)}$

$= \frac{1 \pm \sqrt{1 + 4\log_2 y}}{2}$

Here, we see that range of $f(x)$ is $[1, \infty)$.

$\because \; x = \frac{1 + \sqrt{1 + 4\log_2 y}}{2}$

$\therefore \; f^{-1}(x) = \frac{1 + \sqrt{1 + 4\log_2 y}}{2}$

Question 138

Solution of the equation $\cos^2 x \frac{dy}{dx} - (\tan 2x)y = \cos^4 x, \ \ |x| < \frac{\pi}{4}$, where $y(\frac{\pi}{6}) = \frac{3\sqrt{3}}{8}$, is given by

Options:

A. $y \frac{\tan 2x}{1 - \tan^2 x} = 0$

B. $y(1 - \tan^2 x) = C$

C. $y = sin\ 2x + C$

D. $y = \frac{1}{2} \cdot \frac{sin\ 2x}{1 - \tan^2 x}$

Answer: C

Solution:

Solution:

Given differential equation can be written as

$\frac{dy}{dx} - (\frac{\tan 2x}{\cos^2 x}) y = \cos^2 x$

Here, $P = -\frac{\tan 2x}{\cos^2 x} = \frac{-sin\ 2x}{\cos 2x(\frac{\cos 2x + 1}{2})}$

and $Q = \cos^2 x$

$\because$ mathbb $F = e^{\int P dx} = e^{-\int \frac{2sin\ 2x}{\cos 2x(\cos 2x + 1)} dx}$

Put $\cos 2x = t \Rightarrow -2sin\ 2x\, dx = dt$

$\therefore$ IF $= e^{\int \frac{1}{t}(\frac{1}{t+1}) dt}$

$= e^{\int (\frac{1}{t} - \frac{1}{t+1}) dt}$

$= e^{[\log t - \log(t+1)]}$

$= e^{\log \frac{t}{t+1}}$

$= e^{\log \frac{\cos 2x}{\cos 2x + 1}}$

$= \frac{\cos 2x}{\cos 2x + 1}$

Now, solution is

$y \times \frac{\cos 2x}{\cos 2x + 1} = \int \frac{\cos 2x}{\cos 2x + 1} \times \cos^2 x\, dx + C$

$= \int \frac{\cos 2x}{2\cos^2 x} \times \cos^2 x\, dx + C$

$= \frac{1}{2} \int \cos 2x\, dx + C$

$= \frac{1}{2} \times \frac{sin\ 2x}{2} + C$

$\Rightarrow y \frac{\cos 2x}{\cos 2x + 1} = \frac{1}{4} sin\ 2x + C$

But $y(\frac{\pi}{6}) = \frac{3\sqrt{3}}{8}$

$\therefore \quad \frac{3\sqrt{3}}{8} \times \frac{\cos(2 \times \frac{\pi}{6})}{\cos(2 \times \frac{\pi}{6}) + 1} = \frac{1}{4} sin\ 2(\frac{\pi}{6}) + C$

$\Rightarrow \quad \frac{\frac{3\sqrt{3}}{8} \times \frac{1}{2}}{\frac{1}{2} + 1} = \frac{1}{4} \times \frac{\sqrt{3}}{2} + C$

$\Rightarrow \quad \frac{3\sqrt{3}}{2 \times 8 \times \frac{3}{2}} = \frac{\sqrt{3}}{8} + C$

$\Rightarrow \quad \frac{\sqrt{3}}{8} = \frac{\sqrt{3}}{8} + C \Rightarrow C = 0$

From Eq. (i), we get

$$\frac{\cos 2x}{\cos 2x+1} = \frac{1}{4}sin\,2x + 0$$
$$\Rightarrow \quad y = \frac{1}{4}\frac{sin\,2x}{\cos 2x}{\cos 2x+1}$$
$$= \frac{1}{4}\frac{sin\,2x}{\frac{\cos^2 x - sin^2 x}{2\cos^2 x}}$$
$$= \frac{1}{2}\cdot\frac{sin\,2x}{1-\tan^2 x}$$

Question 139

If lines of regression are $3x + 12y = 19$ and $3y + 9x = 46$, then value of r_{xy} will be

Options:

A. 0.289

B. -0.289

C. 0.209

D. None of these

Answer: B

Solution:

Solution:
Let the line of regression of y on x be
$3x + 12y = 19$
or $\quad y = \frac{-3x+19}{12}$
and x on y be
$x = \frac{-3y+46}{9}$
$\therefore$ Regression coefficient of y on x is
$b_{yx} = \frac{-3}{12}$
$= -\frac{1}{4}$
and regression coefficient of x on y is
$b_{xy} = -\frac{3}{9} = -\frac{1}{3}$
$\therefore$ Correlation $r_{xy} = -\sqrt{b_{yx} \times b_{xy}}$
(here, we take negative sign outside the square root, because both regression coefficients are negative)
$= -\sqrt{-\frac{1}{4} \times -\frac{1}{3}}$
$= -\sqrt{\frac{1}{12}}$
$= -\sqrt{0.083} = -0.289$

Question 140

If 1, ω and ω^2 are the cube roots of unity, then the value of $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$ is equal to

Options:

A. 4

B. 0

C. 2

D. 3

Answer: A

Solution:

Solution:
$(1-\omega+\omega^2)(1+\omega-\omega^2)$
$=(1+\omega^2-\omega)(1+\omega-\omega^2)$
$=(-\omega-\omega)(-\omega^2-\omega^2)$
$(\because 1+\omega+\omega^2=0)$
$=(-2\omega)(-2\omega^2)$
$=4(\omega^3)=4\times 1\ \ (\because \omega^3=1)$
$=4$

Question 141

If two events A and B are mutually exclusive events, then P (A / B) is equal to

Options:

A. 0

B. 1

C.

$\frac{P(A\cap B)}{P(A)}$

D.

$\frac{P(A\cap B)}{P(B)}$

Answer: A

Solution:

Solution:
$\therefore P(A\cap B)=0$

Then, $P\left(\frac{A}{B}\right)=0$

Question 142

The number of points, where $f(x) = [sin\ x + \cos x]$ (where ['] denotes the greatest integer function) and $x \in (0, 2\pi)$ is not continuous, is

Options:

A. 3

B. 4

C. 5

D. 6

Answer: C

Solution:

Solution:
Given, $f(x) = [sin\ x + \cos x]$
$= [\sqrt{2}(\frac{1}{\sqrt{2}} sin\ x + \frac{1}{\sqrt{2}} \cos x)]$
$= [\sqrt{2} sin\ (x + \frac{\pi}{4})]$
We know that, greatest integer function is discontinuous on integer values.
Function $\sqrt{2} sin\ (x + \frac{\pi}{4})$ will gives integer values at $x = 90^\circ, 135^\circ, 180^\circ, 270^\circ, 315^\circ,$
Hence, there are five points in the given interval, in which $f(x)$ is not continuous.

Question 143

The value of $\cot^{-1}\left(\frac{\sqrt{1 + sin\ x} + \sqrt{1 - sin\ x}}{\sqrt{1 + sin\ x} - \sqrt{1 - sin\ x}}\right)$ is equal to

Options:

A. $\frac{x}{3}$

B. $\frac{x}{4}$

C. 1

D. $\frac{x}{2}$

Answer: D

Solution:

Solution:
Let $I = \cot^{-1}(\frac{\sqrt{1 + sin\ x} + \sqrt{1 - sin\ x}}{\sqrt{1 + sin\ x} - \sqrt{1 - sin\ x}})$

$$= \cot^{-1}\left[\frac{\sqrt{(sin\ \frac{x}{2}+\cos\frac{x}{2})^2}+\sqrt{(\cos\frac{x}{2}-sin\ \frac{x}{2})^2}}{\sqrt{(sin\ \frac{x}{2}+\cos\frac{x}{2})^2}-\sqrt{(\cos\frac{x}{2}-sin\ \frac{x}{2})^2}}\right]$$
$$= \cot^{-1}\left(\frac{sin\ \frac{x}{2}+\cos\frac{x}{2}+\cos\frac{x}{2}-sin\ \frac{x}{2}}{sin\ \frac{x}{2}+\cos\frac{x}{2}-\cos\frac{x}{2}+sin\ \frac{x}{2}}\right)$$
$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2sin\ \frac{x}{2}}\right)$$
$$= \cot^{-1}(\cot\frac{x}{2}) = \frac{x}{2}$$

Question 144

If $A(-1,3,2), B(2,3,5)$ and $C(3,5,-2)$ are vertices of a $\triangle ABC$, then angles of $\triangle ABC$ are

Options:

A. $\angle A = 90^\circ, \angle B = 30^\circ, \angle C = 60^\circ$

B. $\angle A = \angle B = \angle C = 60^\circ$

C. $\angle A = \angle B = 45^\circ, \angle C = 90^\circ$

D. None of the above

Answer: D

Solution:

Solution:
Given vertices of a $\triangle ABC$ are $A(-1,3,2), B(2,3,5)$ and $C(3,5,-2)$.
Now DR's of $AB = (2+1, 3-3, 5-2)$
$= (3,0,3)$
DR's of $BC = (3-2, 5-3, -2-5)$
$= (1,2,-7)$
and DR's of $CA = (-1-3, 3-5, 2+2)$
$= (-4,-2,4)$
Now, the angle between AB and BC,
$$\cos B = \frac{|3\times1+0\times2+3\times(-7)|}{\sqrt{3^2+0^2+3^2}\sqrt{1^2+2^2+(-7)^2}}$$
$$= \frac{|3+0-21|}{\sqrt{9+0+9}\sqrt{1+4+49}}$$
$$= \frac{18}{3\sqrt{2}\times3\sqrt{6}}$$
$$= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$
angle between BC and CA,
$$\cos C = \frac{|1\times(-4)+2(-2)+(-7)(4)|}{\sqrt{1^2+2^2+(-7)^2}\sqrt{(-4)^2+(-2)^2+(4)^2}}$$
$$= \frac{|-4-4-28|}{\sqrt{1+4+49}\sqrt{16+4+16}}$$
$$= \frac{36}{\sqrt{54}\sqrt{36}} = \frac{36}{3\sqrt{6}\times6}$$
$$= \frac{2}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$
and angle between AC and AB,
$$\cos A = \frac{|-4\times3+(-2)\times0+4\times3|}{\sqrt{(-4)^2+(-2)^2+(4)^2}\sqrt{3^2+0^2+3^2}}$$
$$= |0|$$
$$\Rightarrow A = 90^\circ$$

Question 145

$\int_{1/e}^{1} \left| \log x \right| dx$ **is equal to**

Options:

Answer: 1

Solution:

Solution:

Let $I = \int_{1/e}^{1} \log x \, dx$

Here, we see that $\log x$ is negative for $x \in (1, \frac{1}{e})$.

$\therefore \quad I = -\int_{1/e|}^{1} 1 \times (\log x) \, dx$

$= -[\log x \times x - \int \frac{1}{x} \times x \, dx]_{1/e}^{1}$

$= -[x \log x - x]_{1/e}^{1}$

$= -[1 \log 1 - 1 - (\frac{1}{e} \log \frac{1}{e} - \frac{1}{e})]$

$= -[0 - 1 - \{\frac{1}{e}(\log 1 - \log e)\} - \frac{1}{e}]$

$= -[-1 - \{\frac{1}{e}(0 - 1) - \frac{1}{e}\}]$

$= 1 + (-\frac{1}{e} - \frac{1}{e}) = 1 - \frac{2}{e}$

Question 146

$\lim_{x \to 0} \frac{\int_{0}^{x^2} sin \sqrt{t} \, dt}{x^3}$ **is equal to**

Options:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. 0

D. ∞

Answer: A

Solution:

Solution:

$$\lim_{x \to 0} \frac{\int_0^{x^2} \sin \sqrt{t}\, dt}{x^3} \left(. \text{ form } \frac{0}{0} \right)$$
$$= \lim_{x \to 0} \frac{\sin \sqrt{x^2} \times 2x}{3x^2} \text{ (using L'Hospital's rule)}$$
$$= \lim_{x \to 0} \frac{2}{3} \frac{\sin x}{x} = \frac{2}{3} \times 1 = \frac{2}{3}$$

Question 147

If a, b and c are three non-coplanar vectors, then $[a \times bb \times cc \times a]$ is equal to

Options:

A. $[abc]^3$

B. $[\text{abc}]^2$

C. 0

D. None of these

Answer: B

Solution:

Solution:
$$[a \times bb \times cc \times a]$$
$$= (a \times b) \cdot [(b \times c) \times (c \times a)]$$
$$= (a \times b) \cdot [((b \times c) \cdot a)c - ((b \times c) \cdot c)a]$$
$$= (a \times b) \cdot ([bca]c - [bcc]a)$$
$$= (a \times b \cdot c)[bca] - [a \times b \cdot a]0$$
$$= [abc][abc] - 0$$
$$= [abc]^2$$

Question 148

If geometric mean and harmonic mean of two numbers a and b are 16 and 64/5 respectively, then the value of $a : b$ is

Options:

A. 4:1

B. 3:2

C. 2:3

D. 1:4

Answer: A

Solution:

Solution:
Geometric mean of a and $b = \sqrt{ab}$
$\Rightarrow \sqrt{ab} = 16$ (given)
$\Rightarrow ab = 256$ . . . (i)
And harmonic mean of a and $b = \frac{2ab}{a+b}$
$\therefore \quad \frac{2ab}{a+b} = \frac{64}{5}$
(given)
$\Rightarrow \frac{2 \times 256}{a+b} = \frac{64}{5}$ [from Eq. (i)]
$\Rightarrow a + b = 40$
Now, ...(ii)
$= \sqrt{(40)^2 - 4 \times 256}$
$= \sqrt{1600 - 1024}$
$= \sqrt{(a+b)^2 - 4ab}$
$\Rightarrow a - b = 24$
On solving Eqs. (ii) and (iii), we get
$a = 32$ and $b = 8$
$\therefore \quad a : b = 32 : 8$
$= 4 : 1$

Question 149

The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ **is**

Options:

A. 0

B. $a + b + c$

C. abc

D. 1

Answer: A

Solution:

Solution:
Let $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$
Applying $C_3 \to C_3 + C_2$, we get
$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$
$= (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$
$= (a+b+c) \times 0$ ($\because C_1$ and C_3 are identical)
$= 0$

Question 150

If the sum of four numbers in GP is 60 and the arithmetic mean of the first and last numbers is 18 , then the numbers are

Options:

A. $3, 9, 27, 81$

B. $4, 8, 16, 32$

C. $2, 6, 18, 54$

D. None of these

Answer: B

Solution:

Solution:
Let four terms in a GP be $ar^3, ar, \frac{a}{r}$ and $\frac{a}{r^3}$.
According to the given condition,
$ar^3 + ar + \frac{a}{r} + \frac{a}{r^3} = 60$ . . . (i)
and $\frac{ar^3 + \frac{a}{r^3}}{2} = 18$
$\Rightarrow \quad ar^3 + \frac{a}{r^3} = 36$ . . . (ii)
Now, from Eq. (i), we have
$(ar + \frac{a}{r}) + ar^3 + \frac{a}{r^3} = 60$
$\Rightarrow a(r + \frac{1}{r}) + 36 = 60$ [from Eq. (ii)]
$\Rightarrow a(r + \frac{1}{r}) = 24$ . . . (iii)

On dividing Eq. (iii) by Eq. (ii), we get
$\frac{a(r^3 + \frac{1}{r^3})}{a(r + \frac{1}{r})} = \frac{36}{24}$
