Sample Question Paper - 40
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours | *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Solve the following quadratic equation for x : $x^2 - 4ax - b^2 + 4a^2 = 0$.

2. Find the ratio between their surface areas, if the ratio of the volumes of two spheres is 8 : 27.

3. If $\dfrac{3+5+7+\ldots\text{to } n \text{ terms}}{5+8+11+\ldots\text{to 10 terms}} = 7$, then find the value of n.

OR

Find the middle term of the A.P. : 0.6, 1.7, 2.8, 3.9,..., 27.

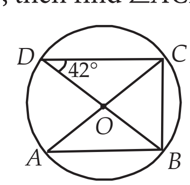
4. If the mean of a, b, c is M and $ab + bc + ca = 0$, the mean of a^2, b^2, c^2 is KM^2, then find $5K$.

5. If roots of quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal.

OR

Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46, find the integers.

6. In the given circle, O is a centre and $\angle BDC = 42°$, then find $\angle ACB$?



SECTION - B

7. A tree breaks due to the storm and the broken part bends such that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 m. Then find the height of the tree.

OR

The angles of elevation of an artificial satellite measured from two earth stations are 30° and 60° respectively. If the distance between the earth stations, which are in straight line with the point directly below the satellite, is 4000 km, then find the height of the satellite. (Use $\sqrt{3} = 1.732$)

8. Find the median of the following frequency distribution :

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	7	15	20	12	8	10

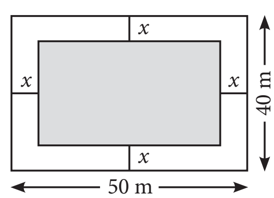
9. Draw a circle of radius 3 cm. From a point *P*, 7 cm away from its centre draw two tangents to the circle. Measure the length of each tangent.

10. The mean of the following frequency table is 50. Find f_1 and f_2.

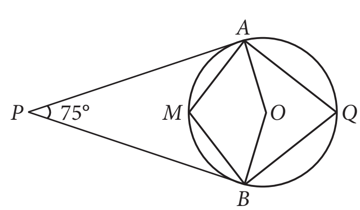
Class	0-20	20-40	40-60	60-80	80-100	Total
Freq-uency	17	f_1	32	f_2	19	120

SECTION - C

11. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2 [see figure]. Find the length and breadth of the pond.

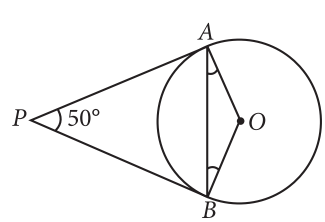


12. In the given figure, *O* is the centre of the circle. If *PA* and *PB* are tangents and $\angle APB = 75°$, then find $\angle AQB$ and $\angle AMB$.



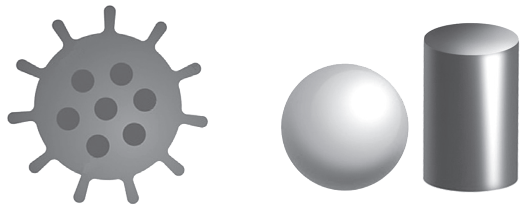
OR

In the given figure, if *PA* and *PB* are tangents to the circle with centre *O* such that $\angle APB = 50°$, then find $\angle OAB$.



Case Study- 1

13. Arun a 10^{th} standard student makes a project on corona virus in science for an exhibition in his school. In this project, he picks a sphere which has volume 38808 cm^3 and 11 cylindrical shapes, each of volume 1540 cm^3 with length 10 cm.



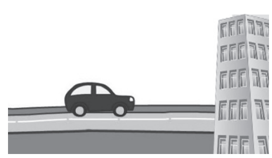
Based on the above information, answer the following questions.

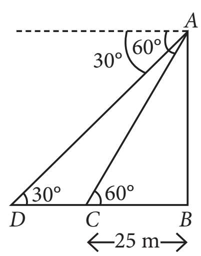
(i) Find the total volume of the shape formed.

(ii) Find the curved surface area of the one cylindrical shape.

Case Study- 2

14. Rohit is standing at the top of the building observes a car at an angle of 30°, which is approaching to the foot of the building with a uniform speed. 6 seconds later, angle of depression of car formed to be 60°, whose distance at that instant from the building is 25 m.





Based on the above information, answer the following questions.

(i) Find the distance between two positions of the car.

(ii) Find the total time taken by the car to reach the foot of the building from starting point.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Given, $x^2 - 4ax - b^2 + 4a^2 = 0$

$\Rightarrow x^2 + (-4a)x + (4a^2 - b^2) = 0$

Using quadratic formula,

$$x = \frac{4a \pm \sqrt{16a^2 - 16a^2 + 4b^2}}{2}$$

$$\Rightarrow x = \frac{4a \pm 2b}{2} \Rightarrow x = 2a \pm b$$

$\therefore x = 2a + b$ or $x = 2a - b$

2. **(d) :** $\dfrac{\text{Volume of sphere with radius } R}{\text{Volume of sphere with radius } r}$

$$= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{8}{27} \Rightarrow \frac{R^3}{r^3} = \frac{8}{27} \Rightarrow \left(\frac{R}{r}\right)^3 = \left(\frac{2}{3}\right)^3 \Rightarrow \frac{R}{r} = \frac{2}{3}$$

Ratio between their surface areas

$$= \frac{4\pi R^2}{4\pi r^2} = \frac{R^2}{r^2} = \left(\frac{R}{r}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

3. Given, $\dfrac{3+5+7+\ldots \text{ to } n \text{ terms}}{5+8+11+\ldots\text{to}10 \text{ terms}} = 7$

$$\Rightarrow \frac{\frac{n}{2}\{2(3)+(n-1)2\}}{\frac{10}{2}\{2(5)+(10-1)3\}} = 7$$

$$\Rightarrow \frac{n\{6+2n-2\}}{10\{10+27\}} = 7 \Rightarrow \frac{2n^2+4n}{370} = 7$$

$\Rightarrow 2n^2 + 4n - 2590 = 0 \Rightarrow n^2 + 2n - 1295 = 0$

$\Rightarrow (n + 37)(n - 35) = 0 \Rightarrow n = 35$ or $n = -37$,

$\because$ Number of terms cannot be negative.

$\therefore n = 35$

OR

Given A.P. is 0.6, 1.7, 2.8, 3.9, ..., 27

Here, first term, $a = 0.6$ and

common difference, $d = 1.7 - 0.6 = 1.1$

Let there be n terms in the A.P.

Then, last term $a_n = 27$

$a + (n - 1)d = 27 \Rightarrow 0.6 + (n - 1)(1.1) = 27$

$\Rightarrow (n - 1)(1.1) = 26.4 \Rightarrow (n - 1) = 24 \Rightarrow n = 25$

Clearly, n is odd, so, $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ term, *i.e.*, 13^{th} term is the middle term.

$\therefore a_{13} = a + 12d = 0.6 + 12(1.1) = 0.6 + 13.2 = 13.8$

4. We have $\dfrac{a+b+c}{3} = M$

$\Rightarrow a + b + c = 3M$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow a^2 + b^2 + c^2 = 9M^2$

$$\Rightarrow \frac{a^2+b^2+c^2}{3} = 3M^2 = KM^2$$

$\Rightarrow K = 3 \Rightarrow 5K = 15$

5. $x^2 + 2px + mn = 0$ ...(i)

and $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ ...(ii)

Equation (i) have equal roots.

$\therefore D = 0 \Rightarrow (2p)^2 - 4mn = 0$

$\Rightarrow 4p^2 = 4mn \Rightarrow p^2 = mn$ ...(iii)

Now for equation (ii), discriminant D

$= (-2(m + n))^2 - 4(m^2 + n^2 + 2p^2)$

$= 4(m^2 + n^2 + 2mn) - 4(m^2 + n^2 + 2mn)$ (Using (iii))

$= 0$

Thus, equation (ii) also has equal roots.

OR

Let the three consecutive positive integers be x, $x + 1$ and $x + 2$.

According to question, $x^2 + (x + 1)(x + 2) = 46$

$\Rightarrow x^2 + x^2 + 3x + 2 = 46$

$\Rightarrow 2x^2 + 3x - 44 = 0$

Using quadratic formula,

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-44)}}{2(2)} = \frac{-3 \pm \sqrt{9+352}}{4}$$

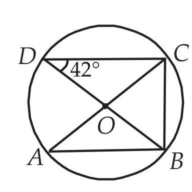
$$= \frac{-3 \pm \sqrt{361}}{4} = \frac{-3 \pm 19}{4}$$

$$\Rightarrow x = \frac{-3+19}{4} \text{ or } \frac{-3-19}{4} \Rightarrow x = 4 \text{ or } -\frac{22}{4}$$

Since x is a positive integer $\therefore x = 4$

Thus, the required consecutive positive integers are 4, 5 and 6.

6. BD is a diameter of circle



$\therefore \angle BCD = 90°$

[angle in a semicircle]

In ΔOCD,

$OD = OC$ [radii of circle]

$\Rightarrow \angle ODC = \angle OCD = 42°$

$\Rightarrow \angle OCD + \angle OCB = 90°$

$\Rightarrow \angle OCB = 90° - 42° = 48°$

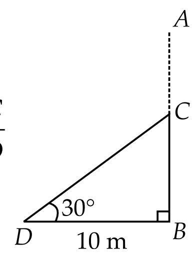
$\Rightarrow \angle ACB = \angle OCB = 48°$

7. Let height of tree $AB = h$ metre. It broke at C such that, its top A touches the ground at D.

Now, $AC = CD$

and $AB = AC + BC = h$

$BD = 10$ m



Now, in ΔBCD, we have $\tan 30° = \frac{BC}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{10} \Rightarrow BC = \frac{10}{\sqrt{3}}$

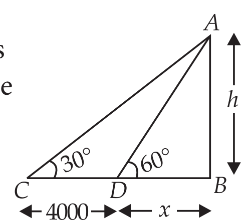
and $\cos 30° = \frac{BD}{CD} \Rightarrow \frac{\sqrt{3}}{2} = \frac{10}{CD} \Rightarrow CD = \frac{20}{\sqrt{3}}$

$\therefore$ Height of tree $= BC + CD$

$$= \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

OR

Let C and D be the earth stations and AB be the height of the satellite.



In ΔABC, $\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{4000 + x} \Rightarrow 4000 + x = \sqrt{3}h$ ...(1)

In ΔABD, $\tan 60° = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(2)

From (1) and (2), we get

$4000 + \frac{h}{\sqrt{3}} = \sqrt{3}h \Rightarrow 4000\sqrt{3} = 3h - h$

$\Rightarrow 2h = 4000\sqrt{3} \Rightarrow h = 2000\sqrt{3}$

$\Rightarrow h = 2000(1.732) = 3464$ km

8. The frequency distribution table for the given data can be drawn as :

Class	Class marks (x_i)	Frequency (f_i)	Cumulative frequency (*c.f.*)
0-10	5	8	8
10-20	15	7	15
20-30	25	15	30
30-40	35	20	50
40-50	45	12	62
50-60	55	8	70
60-70	65	10	80
		$N = \Sigma f_i = 80$	

Now, Median $= l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$

Here, $N = 80$. So, $\frac{N}{2} = \frac{80}{2} = 40$, which lies in the class 30-40.

$\therefore$ Median class is 30-40.

Here, $l = 30$, $c.f. = 30$, $f = 20$, $h = 10$

$\therefore$ Median $= 30 + \left(\frac{40 - 30}{20}\right) \times 10$

$$= 30 + \frac{10}{20} \times 10 = 30 + 5 = 35$$

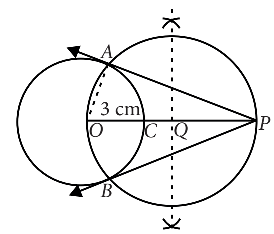
9. Steps of construction :

Step-I : Draw a circle of radius 3 cm, taking O as centre and OC be its radius.

Step-II : Produce OC to P such that $OP = 7$ cm.

Step-III : Draw perpendicular bisector of OP that meets OP at Q.

Step-IV : Taking Q as centre and radius QP draw a circle which intersect previous circle at points A and B.



Step-V : Join P to A and P to B. Now, PA and PB are the required tangents.

Now, join OA to find PA.

In ΔAOP, $\angle OAP = 90°$ [Angle in semicircle]

$\therefore AP^2 = OP^2 - OA^2$

$= 7^2 - 3^2 = 40$

$\Rightarrow AP = 6.32$ cm

$\therefore$ Length of each tangent $= 6.32$ cm

10.

Class	f_i	x_i	$f_i x_i$
0-20	17	10	170
20-40	f_1	30	$30f_1$
40-60	32	50	1600
60-80	f_2	70	$70f_2$
80-100	19	90	1710
	$\Sigma f_i = 68 + f_1 + f_2$		$\Sigma f_i x_i = 3480 + 30f_1 + 70f_2$

We have $\Sigma f_i = 120$

$\Rightarrow 68 + f_1 + f_2 = 120 \Rightarrow f_1 + f_2 = 52$ ...(1)

Now, mean = 50

$\Rightarrow 50 = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 50 = \frac{3480 + 30f_1 + 70f_2}{120}$

$\Rightarrow 6000 = 3480 + 30f_1 + 70f_2$

$\Rightarrow 30f_1 + 70f_2 = 2520$ ...(2)

Solving (1) and (2), we get

$f_1 = 28, f_2 = 24$

11. Length of the rectangular lawn = L = 50 m
and breadth of the rectangular lawn = B = 40 m
Let width of area of the grass surrounding the pond = x m.
Then, length of rectangular pond = l
$= 50 - (x + x) = 50 - 2x$
and breadth of rectangular pond = b
$= 40 - (x + x) = 40 - 2x$
Now, area of grass surrounding the pond
$= L \times B - l \times b = 1184 \text{ m}^2$ [Given]
$\Rightarrow 50 \times 40 - [(50 - 2x)(40 - 2x)] = 1184$
$\Rightarrow 2000 - (2000 - 80x - 100x + 4x^2) = 1184$
$\Rightarrow 80x + 100x - 4x^2 = 1184 \Rightarrow x^2 - 45x + 296 = 0$
Here, $a = 1$, $b = -45$ and $c = 296$
$\therefore b^2 - 4ac = (-45)^2 - 4(1)(296) = 2025 - 1184 = 841 > 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-45) \pm \sqrt{841}}{2(1)} = \frac{45 \pm 29}{2}$$

$$\Rightarrow x = \frac{45+29}{2} \text{ or } x = \frac{45-29}{2} \Rightarrow x = \frac{74}{2} \text{ or } x = \frac{16}{2}$$

$\Rightarrow x = 37$ or $x = 8$
But $x = 37$ is not possible.
$\therefore x = 8$
$\therefore$ Length of pond $= 50 - 2x = 50 - 16 = 34$ m and breadth of pond $= 40 - 2x = 40 - 16 = 24$ m

12. In quadrilateral $APBO$, $AO \perp PA$ and $BO \perp PB$. [$\because$ Radius is perpendicular to tangent at the point of contact]
$\therefore \angle PAO = \angle PBO = 90°$
$\because \angle APB + \angle PAO + \angle PBO + \angle AOB = 360°$
[by angle sum property of a quadrilateral]
$\therefore 75° + 90° + 90° + \angle AOB = 360°$
$\Rightarrow \angle AOB = 105°$

$$\Rightarrow \angle AQB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 105° = 52\frac{1}{2}^\circ$$

[Angle subtended by an arc at the centre is double the angle subtended by it on remaining part of the circle]
In cyclic quadrilateral $AQBM$,
$\angle AQB + \angle AMB = 180°$

$$\Rightarrow 52\frac{1}{2}^\circ + \angle AMB = 180°$$

$$\Rightarrow \angle AMB = 180° - \frac{105°}{2} = \frac{255°}{2} = 127\frac{1}{2}^\circ$$

Hence, $\angle AQB = 52\frac{1}{2}^\circ$ and $\angle AMB = 127\frac{1}{2}^\circ$

OR

Given, PA and PB are tangents.
$\therefore PA = PB$ [$\because$ Lengths of tangents drawn from an external point to a circle are equal]
$\Rightarrow \angle PBA = \angle PAB$
Let $\angle PBA = \angle PAB = \theta$
In ΔPAB, $\angle APB + \angle PBA + \angle PAB = 180°$
[By angle sum property]
$\Rightarrow 50° + \theta + \theta = 180°$
$\Rightarrow 2\theta = 180° - 50° = 130° \Rightarrow \theta = 65°$
Also, $OA \perp PA$
[$\because$ Tangent drawn at any point of a circle is perpendicular to the radius through the point of contact]
$\therefore \angle PAO = 90°$
$\Rightarrow \angle PAB + \angle OAB = 90° \Rightarrow 65° + \angle OAB = 90°$
$\Rightarrow \angle OAB = 90° - 65° = 25°$

13. (i) Total volume of shape formed = Volume of cylindrical shapes + Volume of sphere
$= 11 \times 1540 + 38808 = 16940 + 38808 = 55748 \text{ cm}^3$
(ii) We know that, volume of cylinder $= \pi r^2 h$

$$\Rightarrow 1540 = \frac{22}{7} \times r^2 \times 10$$

$$\Rightarrow \frac{154 \times 7}{22} = r^2 \Rightarrow r^2 = 49 \Rightarrow r = 7 \text{ cm}$$

Curved surface area of one cylindrical shape $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 10 = 440 \text{ cm}^2$$

14. (i) In ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow AB = 25 \times \sqrt{3}$

$\therefore$ Height of building is $25\sqrt{3}$ m .

Now, in ΔABD, $\frac{AB}{BD} = \tan 30°$

$$\Rightarrow \frac{25\sqrt{3}}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 75 \text{ m}$$

$\therefore$ Distance between two positions of car = (75 – 25) m = 50 m.

(ii) Time taken to cover 50 m distance = 6 sec.
$\therefore$ Time taken to cover 25 m distance = 3 sec.
$\therefore$ Total time taken by car = 6 sec + 3 sec = 9 sec