21. Logarithm

Let us do the sum 21

1. Question

Let us evaluate:

(i) $\log_4\left(\frac{1}{64}\right)$

(ii) $\log_{0.01} 0.000001$

(iii) $\log_{\sqrt{6}} 216$

(iv) $\log_{\sqrt{3}} 27$

Answer

(i) $\log_4\left(\frac{1}{64}\right) = \log_4(64^{-1})$

$= \log_4 4^{3^{-1}} = \log_4 4^{-3}$

As, $\log_a M^c = c \log_a M$

$= (-3)\log_4 4 = (-3) \times 1 = -3$

(ii) $\log_{0.01}(0.000001) = \log_{\left(\frac{1}{100}\right)}\left(\frac{1}{1000000}\right)$

$= \log_{10^{-2}} 10^{-6} = \log_{10^{-2}} 10^{-2^3}$

As, $\log_a M^c = c \log_a M$

$= (3)\log_{10^{-2}} 10^{-2} = (3) \times 1 = 3$

[As, $\log_a a = 1$]

(iii) $\log_{\sqrt{6}}(216) = \log_{\sqrt{6}}(6^3)$

$= \log_{\sqrt{6}}(\sqrt{6}^{2^3}) = \log_{\sqrt{6}} \sqrt{6}^6$

As, $\log_a M^c = c \log_a M$

$= (6)\log_{\sqrt{6}}\sqrt{6} = (6) \times 1 = 6$

[As, $\log_a a = 1$]

(iv) $\log_{\sqrt{3}}(1728) = \log_{\sqrt{3}}(3^3)$

$= \log_{\sqrt{3}}\left(\sqrt{3}^2\right)^3$

$= \log_{\sqrt{3}}(\sqrt{3})^6$

$= 6(\log_{\sqrt{3}}\sqrt{3})$ [As, $\log_a M^c = c \log_a M$]

$= 6(1)$ [As, $\log_a a = 1$]

$= 6$

2 A. Question

Let us write by calculating, find its base if logarithm of 625 is 4

Answer

Let the base be a.

$\log_a 625 = 4$

We know that if $\log_a M = p$

Then $a^p = M$

So,$\log_a 625 = 4$

$\therefore a^4 = 625$

$a^4 = 5 \times 5 \times 5 \times 5$

$a^4 = 5^4$

We know that if the powers are same on both the sides then the bases must also be same.

Therefore, a=5

2 B. Question

Let us write by calculating, find its base if logarithm 5832 is 6

Answer

Let the base be a.

$\log_a 5832 = 6$

We know that if $\log_a M = p$

Then $a^p = M$

So, $\log_a 5832 = 6$

$\therefore a^6 = 5832$

$a^6 = 6 \times 6 \times 6 \times 3 \times 3 \times 3$

${a^2}^3 = (6 \times 3)^3$

${a^2}^3 = (18)^3$

$a^2 = 18$

$a = \sqrt{18}$

3 A. Question

If $1 + \log_{10}a = 2\log_{10}b$, then express a by b

Answer

Given expression :

$1 + \log_{10} a = 2\log_{10} b$

$\Rightarrow 1 = 2\log_{10} b - \log_{10} a$

$\Rightarrow 1 = \log_{10} b^2 - \log_{10} a$

$\Rightarrow 1 = \log_{10} \frac{b^2}{a}$

$\Rightarrow \log_{10} 10 = \log_{10} \frac{b^2}{a}$

$\Rightarrow 10 = \frac{b^2}{a} \Rightarrow 10a = b^2$

$\therefore \frac{a}{b^2} = \frac{1}{10}$

3 B. Question

If $3 + \log_{10}x = 2\log_{10}y$, then express x by y

Answer

Given expression :

$$3 + \log_{10} x = 2\log_{10} y$$

$$\Rightarrow 3 = 2\log_{10} y - \log_{10} x$$

$$\Rightarrow 3 = \log_{10} y^2 - \log_{10} x$$

$$\Rightarrow 3 = \log_{10} \frac{y^2}{x}$$

$$\Rightarrow \log_{10} 10^3 = \log_{10} \frac{y^2}{x}$$

$$\Rightarrow 10^3 = \frac{y^2}{x} \Rightarrow 1000x = y^2$$

$$\therefore \frac{x}{y^2} = \frac{1}{1000}$$

4 A. Question

Let us evaluate:

$\log_2[\log_2\{\log_3 (\log_3 27^3)\}]$

Answer

$$\log_2[\log_2\{\log_3 (\log_3 27^3)\}]$$

$$= \log_2[\log_2\{\log_3 (\log_3 3^{3^3})\}]$$

$$= \log_2[\log_2\{\log_3 (\log_3 3^9)\}]$$

$$= \log_2[\log_2\{\log_3 9\}]$$

$$= \log_2[\log_2\{\log_3 3^2\}]$$

$$= \log_2[\log_2\{2\}]$$

$$= \log_2[1] = 0$$

4 B. Question

Let us evaluate:

$$\frac{\log \sqrt{27} + \log 8 - \log \sqrt{1000}}{\log 1.2}$$

Answer

$\frac{\log\sqrt{27}+\log 8-\log\sqrt{1000}}{\log 1.2}$

$$= \frac{\log\sqrt{3^3} + \log 2^3 - \log\sqrt{10^3}}{\log 1.2}$$

$$= \frac{\log 3^{\frac{3}{2}} + \log 2^3 - \log 10^{\frac{3}{2}}}{\log\frac{12}{10}}$$

$$= \frac{\frac{3}{2}\log 3 + 3\log 2 - \frac{3}{2}\log 10}{\log\frac{12}{10}}$$

$$= \frac{\frac{3}{2}(\log\frac{3}{10}) + \frac{3}{2}\log 2^2}{\log\frac{12}{10}}$$

$$= \frac{\frac{3}{2}(\log\frac{3}{10} + \log 4)}{\log\frac{12}{10}}$$

$$= \frac{\frac{3}{2}\log\frac{12}{10}}{\log\frac{12}{10}} = \frac{3}{2}$$

4 C. Question

Let us evaluate:

$\log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 3$

Answer

$\log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 3$

$$= \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 3}{\log 7}$$

$= \frac{\log 3}{\log 3} = 1$

4 D. Question

Let us evaluate:

$$\log_{10}\frac{384}{5} + \log_{10}\frac{81}{32} + 3\log_{10}\frac{5}{3} + \log_{10}\frac{1}{9}$$

Answer

$\log_{10}\frac{384}{5}+\log_{10}\frac{81}{32}+3\log_{10}\frac{5}{3}+\log_{10}\frac{1}{9}$

$=\log_{10}\frac{384}{5}+\log_{10}\frac{81}{32}+\log_{10}\frac{5^3}{3}+\log_{10}\frac{1}{9}$

$=\log_{10}\frac{384}{5}+\log_{10}\frac{81}{32}+\log_{10}\frac{125}{27}+\log_{10}\frac{1}{9}$

$=\log_{10}(\frac{384}{5}\times\frac{81}{32}\times\frac{125}{27}\times\frac{1}{9})$

$=\log_{10}100$

$=\log_{10}10^2$

$=2\log_{10}10=2\times1=2$

5 A. Question

Let us prove:

$$\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$$

Answer

LHS $=\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}$

$=\log\frac{5^2\times 3}{2^4}-2\log\frac{5}{3^2}+\log\frac{2^5}{3^5}$

$=\log 5^2+\log 3-\log 2^4-2\log 5+2\log 3^2+\log 2^5-\log 3^5$

$=2\log 5+\log 3-4\log 2-2\log 5+4\log 3+5\log 2-5\log 3$

$=5\log 2-4\log 2=\log 2=$ RHS

5 B. Question

Let us prove:

$\log_{10}15(1+\log_{15}30)+\frac{1}{2}\log_{10}16(1+\log_4 7)-\log_{10}6(\log_6 3+1+\log_6 7)=2$

Answer

LHS = $\log_{10}15(1+\log_{15}30) + \frac{1}{2}\log_{10}16(1+\log_4 7) - \log_{10}6(\log_6 3+1+\log_6 7)$

$$= \log_{10}15 + \log_{10}15\log_{15}30 + \frac{1}{2}\log_{10}16 + \frac{1}{2}\log_{10}16\log_4 7 - \log_{10}6\log_6 3 - \log_{10}6 - \log_{10}6\log_6 7$$

$$= \frac{\log 15}{\log 10} + \frac{\log 15}{\log 10} \times \frac{\log 30}{\log 15} + \frac{1}{2}\frac{\log 16}{\log 10} + \frac{1}{2}\frac{\log 16}{\log 10} \times \frac{\log 7}{\log 4} - \frac{\log 6}{\log 10} \times \frac{\log 3}{\log 6} - \frac{\log 6}{\log 10} - \frac{\log 6}{\log 10} \times \frac{\log 7}{\log 6}$$

$$= \frac{\log 15}{\log 10} + \frac{\log 30}{\log 10} + \frac{1}{2}\frac{\log 16}{\log 10} + \frac{2}{2}\frac{\log 4}{\log 10} \times \frac{\log 7}{\log 4} - \frac{\log 3}{\log 10} - \frac{\log 6}{\log 10} - \frac{\log 7}{\log 10}$$

$$= \frac{\log 15}{\log 10} + \frac{\log 30}{\log 10} + \frac{1}{2}\frac{\log 16}{\log 10} + \frac{\log 7}{\log 10} - \frac{\log 3}{\log 10} - \frac{\log 6}{\log 10} - \frac{\log 7}{\log 10}$$

$$= \frac{\log 15 + \log 30 + \log 4 + \log 7 - \log 3 - \log 6 - \log 7}{\log 10} = \frac{2\log 10}{\log 10} = 2$$

5 C. Question

Let us prove:

$$\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2 = 5$$

Answer

LHS $= \log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$

$$= \log_2 \log_2 \log_4 4^4 + 2\log_{\sqrt{2}} \sqrt{2}^2$$

$$= \log_2 \log_2 4 + 2 \times 2$$

$$= \log_2 \log_2 2^2 + 4$$

$$= \log_2 2 + 4$$

$= 1 + 4 = 5 =$ RHS

5 D. Question

Let us prove:

$$\log_{x^2} x \times \log_{y^2} y \times \log_{z^2} z = \frac{1}{8}$$

Answer

$$\text{LHS} = \log_{x^2} x \times \log_{y^2} y \times \log_{z^2} z$$

$$= \frac{\log x}{\log x^2} \times \frac{\log y}{\log y^2} \times \frac{\log z}{\log z^2}$$

$$= \frac{\log x}{2\log x} \times \frac{\log y}{2\log y} \times \frac{\log z}{2\log z}$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = \text{RHS}$$

5 E. Question

Let us prove:

$$\log_{b^3} a \times \log_{c^3} b \times \log_{a^3} c = \frac{1}{27}$$

Answer

$$\text{LHS} = \log_{a^3} a \times \log_{b^3} b \times \log_{c^3} c$$

$$= \frac{\log a}{\log a^3} \times \frac{\log b}{\log b^3} \times \frac{\log c}{\log c^3}$$

$$= \frac{\log a}{3\log a} \times \frac{\log b}{3\log b} \times \frac{\log c}{3\log c}$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27} = \text{RHS}$$

5 F. Question

Let us prove:

$$\frac{1}{\log_{xy}(xyz)} + \frac{1}{\log_{yz}(xyz)} + \frac{1}{\log_{zx}(xyz)} = 2$$

Answer

$$\text{LHS} = \frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$$

$$= \frac{1}{\frac{\log xyz}{\log xy}} + \frac{1}{\frac{\log xyz}{\log yz}} + \frac{1}{\frac{\log xyz}{\log zx}}$$

$$= \frac{\log xy}{\log xyz} + \frac{\log yz}{\log xyz} + \frac{\log zx}{\log xyz}$$

$$= \frac{(\log xy + \log yz + \log zx)}{\log xyz}$$

$$= \frac{\log x^2y^2z^2}{\log xyz}$$

$$= \frac{\log(xyz)^2}{\log xyz}$$

$$= \frac{2\log xyz}{\log xyz} = 2 = \text{RHS}$$

5 G. Question

Let us prove:

$$\log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab} = 0$$

Answer

$$\text{LHS} = \log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab}$$

$$= \log a^2 - \log bc + \log b^2 - \log ca + \log c^2 - \log ab$$

$$= 2\log a - \log b - \log c + 2\log b - \log c - \log a + 2\log c - \log a - \log b$$

$$= 0 = \text{RHS}$$

5 H. Question

Let us prove:

$x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y} = 1$

Answer

$\text{LHS} = x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y}$

$$= x^{\log\frac{y}{z}} \times y^{\log\frac{z}{x}} \times z^{\log\frac{x}{y}}$$

Now, taking log we have,

$$= \log x^{\log\frac{y}{z}} + \log y^{\log\frac{z}{x}} + \log z^{\log\frac{x}{y}}$$

$$= (\log y - \log z)\log x + (\log z - \log x)\log y + (\log x - \log y)\log z$$

$$= \log y \log x - \log z \log x + \log z \log y - \log x \log y + \log x \log z - \log y \log z$$

$= 0 = \log 1$

Now, RHS = 1

Taking log we have, log 1= 0 =LHS

Therefore, LHS=RHS

Hence, proved.

6 A. Question

If $\log\frac{x+y}{5} = \frac{1}{2}(\log x + \log y)$, then let us show that $\frac{x}{y} + \frac{y}{z} = 23$.

Answer

$\log\frac{x+y}{5} = \frac{1}{2}(\log x + \log y)$

$\Rightarrow \log\frac{x+y}{5} = \frac{1}{2}(\log xy)$

$\Rightarrow \log\frac{x+y}{5} = (\log xy^{\frac{1}{2}})$

$\Rightarrow \frac{x+y}{5} = \sqrt{xy}$

$\Rightarrow \left(\frac{x+y}{5}\right)^2 = xy$

$\Rightarrow \frac{x^2+y^2+2xy}{xy} = 25$

$\Rightarrow \frac{x^2}{xy} + \frac{y^2}{xy} + \frac{2xy}{xy} = 25$

$\Rightarrow \frac{x}{y} + \frac{y}{x} + 2 = 25$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = 25 - 2 = 23$

$\therefore \frac{x}{y} + \frac{y}{x} = 23$

6 B. Question

If $a^4 + b^4 = 14a^2b^2$, then let us show that $\log(a^2 + b^2) = \log a + \log b + 2\log 2$.

Answer

Given that $a^4 + b^4 = 14a^2b^2$

$a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$

$\Rightarrow 14a^2b^2 = (a^2 + b^2)^2 - 2a^2b^2$

$\Rightarrow (a^2 + b^2)^2 = 16a^2b^2$

$= \log 4 + \log a + \log b \Rightarrow (a^2 + b^2) = \sqrt{16a^2b^2}$

$\Rightarrow (a^2 + b^2) = 4ab$.....eq(1)

$\text{LHS} = \log(a^2 + b^2)$

$= \log 4ab$ (from eq(1))

$= \log 2^2 + \log a + \log b$

$= 2\log 2 + \log a + \log b$ = RHS

Therefore, LHS =RHS

Hence , proved.

7. Question

If $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y}$, then let us show that xyz = 1

Answer

Given that $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y}$,

Let $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$

$\log x = ky - kz$.....eq(1)

$\log y = kz - kx$.....eq(2)

$\log z = kx - ky$.....eq(3)

Now, adding eq(1) , eq(2) and eq(3)

$\log x + \log y + \log z = ky - kz + kz - kx + kx - ky$

$\Rightarrow \log xyz = 0$

$\Rightarrow \log xyz = \log 1$

$xyz = 1$

Hence, proved.

8. Question

If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$, then let us show that xyz = 1

(a)$x^{b+c} . y^{c+a} . z^{a+b} = 1$

(b) $x^{b^2+bc+c^2} \cdot y^{c^2+ca+a^2} \cdot z^{a^2+ab+b^2} = 1$

Answer

Given that $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$,

Let $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k$

$\log x = kb - kc$.....eq(1)

$\log y = kc - ka$.....eq(2)

$\log z = ka - kb$.....eq(3)

Now, adding eq(1) , eq(2) and eq(3)

$\log x + \log y + \log z = kb - kc + kc - ka + ka - kb$

$\Rightarrow \log xyz = 0$

$\Rightarrow \log xyz = \log 1$

$xyz = 1$

(a) LHS = $x^{b+c} . y^{c+a} . z^{a+b}$

Taking log we have,

$= \log x^{b+c} . y^{c+a} . z^{a+b}$

$= \log x^{b+c} + \log y^{c+a} + \log z^{a+b}$

$= (b + c) \log x + (c + a) \log y + (a + b) \log z$

$= b \log x + b \log z + c \log x + c \log y + a \log y + a \log z$

$= b(\log x + \log z) + c(\log x + \log y) + a(\log y + \log z)$

$= b(kb - kc + ka - kb) + c(kb - kc + kc - ka) + a(kc - ka + ka - kb)$

$= b(ka - kc) + c(kb - ka) + a(kc - kb)$

$= bka - bkc + ckb - cka + akc - akb \;= 0$

$= \log 1 = RHS$

Therefore, $x^{b+c} . y^{c+a} . z^{a+b} = 1$

(b) LHS $= x^{b^2+bc+c^2} . y^{c^2+ca+a^2} . z^{a^2+ab+b^2}$

Taking log we have,

$= \log x^{b^2+bc+c^2} . y^{c^2+ca+a^2} . z^{a^2+ab+b^2}$

$= \log x^{b^2+bc+c^2} + \log y^{c^2+ca+a^2} + \log z^{a^2+ab+b^2}$

$= (b^2 + bc + c^2)\log x + (c^2 + ca + a^2)\log y + (a^2 + ab + b^2)\log z$

$= b^2(\log x + \log z) + c^2(\log x + \log y) + a^2(\log y + \log z) + bc\log x + ca\log y + ab\log z$

$= b^2(kb - kc + ka - kb) + c^2(kb - kc + kc - ka) + a^2(kc - ka + ka - kb) + bc(kb - kc) + ca(kc - ka) + ab(ka - kb)$

$= b^2(ka - kc) + c^2(kb - ka) + a^2(kc - kb) + bc(kb - kc) + ca(kc - ka) + ab(ka - kb)$

$= b^2ka - b^2kc + c^2kb - c^2ka + a^2kc - a^2kb + b^2kc - bkc^2 + akc^2 - cka^2 + bka^2 - akb^2 \;= 0$

$= \log 1 = RHS$

Therefore, $x^{b^2+bc+c^2} . y^{c^2+ca+a^2} . z^{a^2+ab+b^2} = 1$

9. Question

If, $a^{3-x} . b^{5x} = a^{5+x} . b^{3x}$, then let us show that, $x\log\left(\frac{b}{a}\right) = \log a$

Answer

Given that $a^{3-x} . b^{5x} = a^{5+x} . b^{3x}$

Taking log on both sides

$\log a^{3-x} . b^{5x} = \log a^{5+x} . b^{3x}$

$\Rightarrow \log a^{3-x} + \log b^{5x} = \log a^{5+x} + \log b^{3x}$

$\Rightarrow (3-x)\log a + 5x\log b = (5+x)\log a + 3x\log b$

$\Rightarrow \log a\ (3-x-5-x) + \log b\ (5x-3x) = 0$

$\Rightarrow 2x\log b = (2+2x)\log a$

$\Rightarrow 2x\log b - 2x\log a = 2\log a$

$\Rightarrow 2x\log\frac{b}{a} = 2\log a$

$\Rightarrow x\log\frac{b}{a} = \log a$

Hence ,proved .

10 A. Question

Let us solve:

$$\log_8\left[\log_2\left\{\log_3\left(4^x+17\right)\right\}\right] = \frac{1}{3}$$

Answer

$$\log_8\left[\log_2\left\{\log_3\left(4^x+17\right)\right\}\right] = \frac{1}{3}$$

$\Rightarrow \log_8[\log_2\{\log_3(4^x+17)\}] = \frac{1}{3}$

$\Rightarrow \log_8[\log_2\{\log_3(4^x+17)\}] = \frac{1}{3} \times \log_8 8$

$\Rightarrow \log_8[\log_2\{\log_3(4^x+17)\}] = \log_8 8^{\frac{1}{3}}$

$\Rightarrow \log_2\{\log_3(4^x+17)\} = 2^{3\times\frac{1}{3}}$

$\Rightarrow \log_2\{\log_3(4^x+17)\} = 2 = 2\log_2 2$

$\Rightarrow \log_2\{\log_3(4^x+17)\} = \log_2 2^2$

$\Rightarrow \log_3 4^x + 17 = 4$

$\Rightarrow \log_3 4^x + 17 = 4\log_3 3$

$\Rightarrow \log_3 4^x + 17 = \log_3 3^4$

$\Rightarrow 4^x + 17 = 81$

$\Rightarrow 4^x = 81 - 17 = 64$

$\Rightarrow 4^x = 4^3$

Therefore, x = 3.

10 B. Question

Let us solve:

$\log_8 x + \log_4 x + \log_2 x = 11$

Answer

$\log_8 x + \log_4 x + \log_2 x = 11$

$\Rightarrow \frac{\log x}{\log 8} + \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 11$

$\Rightarrow \frac{\log x}{\log 2^3} + \frac{\log x}{\log 2^2} + \frac{\log x}{\log 2} = 11$

$\Rightarrow \frac{\log x}{3\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 11$

$\Rightarrow \frac{6}{6}\left(\frac{\log x}{\log 2}\right) = 11$

$\Rightarrow \log x = 11 \log 2$

$\Rightarrow \log x = \log 2^{11}$

$\Rightarrow x = 11$

11. Question

Let us show that the value of $\log_{10} 2$ lies between $\frac{1}{4}$ and $\frac{1}{3}$.

Answer

Let $\log_{10} 2 = x$

$\therefore 10^x = 2$

Now, LCM of denominator of $\frac{1}{4}$ and $\frac{1}{3}$ $= 4 \times 3 = 12$

$10^{x^{12}} = 2^{12} = 8192$

$\Rightarrow 10^{12x} = 8192$

$\therefore 1000 < 8192 < 10000$

Or $10^3 < 10^{12x} < 10^4$

Or $3 < 12x < 4$

Or $\frac{1}{4} < x < \frac{1}{3}$

So, $\frac{1}{4} < \log_{10} 2 < \frac{1}{4}$

12 A. Question

If $\log_{\sqrt{x}} 0.25 = 4,$ then the value of x

A. 0.5

B. 0.25

C. 4

D. 16

Answer

$\log_{\sqrt{x}} 0.25 = 4$

We know that if $\log_a M = p$

Then $a^p = M$

So, $\log_{\sqrt{x}} 0.25 = 4$

$\therefore \sqrt{x}^4 = 0.25$

$x^{4 \times \frac{1}{2}} = \frac{25}{100}$

$x^2 = \frac{5}{10}^{2} \Rightarrow x = \frac{5}{10} = 0.5$

We know that if the powers are same on both the sides then the bases must also be same.

12 B. Question

If $\log_{10}(7x - 5) = 2,$ then the value of x

A. 10

B. 12

C. 15

D. 18

Answer

$\log_{10}(7x - 5) = 2$

We know that if $\log_a M = p$

Then $a^p = M$

So, $\log_{10}(7x - 5) = 2$

$\therefore 10^2 = 7x - 5$

$7x - 5 = 100$

$7x = 105$

$x = \frac{105}{7} = 15$

12 C. Question

If $\log_2 3$ = a, then the value of $\log_8 27$ is

A. 3a

B. $\frac{1}{a}$

C. 2a

D. a

Answer

We know,

$\left[\log_b M = \frac{\log_a M}{\log_a b}\right]$

$\therefore \log_8 27 = \frac{\log_2 27}{\log_2 8}$

$\Rightarrow \log_8 27 = \frac{\log_2 3^3}{\log_2 2^3}$

Now, we know that $\log_a M^c = c \log_a M$

$$\Rightarrow \log_8 27 = \frac{3\log_2 3}{3\log_2 2}$$

$= \frac{a}{1} = a$ [As, $\log_a a = 1$]

12 D. Question

If $\log_{\sqrt{2}} x = a$, then the value of $\log_{2\sqrt{7}} x$ is

A. $\frac{a}{3}$

B. a

C. 2a

D. 3a

Answer

Since $\log_{\sqrt{2}} x = a$

We know that if $\log_k M = p$

Then $k^p = M$

So, $\log_{\sqrt{2}} x = a$

$\therefore \sqrt{2}^a = x$.......eq(1)

Now, $\log_{2\sqrt{2}} x = \frac{\log x}{\log 2\sqrt{2}}$

$$= \frac{a \log \sqrt{2}}{\log 2\sqrt{2}}$$

$$= \frac{a \log 2^{\frac{1}{2}}}{\log 2^{\frac{3}{2}}}$$

$$= \frac{\frac{1}{2} a \log 2}{\frac{3}{2} \log 2}$$

$$= \frac{a}{3}$$

12 E. Question

If $\log_x \frac{1}{3} = -\frac{1}{3}$, then the value of x is

A. 27

B. 9

C. 3

D. $\frac{1}{27}$

Answer

$\log_x \frac{1}{3} = -\frac{1}{3}$

$\Rightarrow \log_x \frac{1}{3} = -\frac{1}{3} \log_x x$

$\Rightarrow \log_x \frac{1}{3} = \log_x x^{-\frac{1}{3}}$

$\Rightarrow \frac{1}{3} = x^{-\frac{1}{3}}$

$\Rightarrow \frac{1}{x^{\frac{1}{3}}} = \frac{1}{3}$

$\Rightarrow x^{\frac{1}{3}} = 3 \Rightarrow x = 3^3 = 27$

13 A. Question

Let us calculate the value of log4log4log4 256.

Answer

$[\log_4\{\log_4 (\log_4 256)\}]$

$= [\log_4\{\log_4 (\log_4 4^4)\}]$

$= [\{\log_4 \log_4 4\}]$

$= \log_4 [4] = 0$

13 B. Question

Let us calculate the value of $\log\frac{a^n}{b^n}+\log\frac{b^n}{c^n}+\log\frac{c^n}{a^n}$

Answer

$\log\frac{a^n}{b^n}+\log\frac{b^n}{c^n}+\log\frac{c^n}{a^n}$

$= \log a^n - \log b^n + \log b^n - \log c^n + \log c^n - \log a^n$

$= 0$

13 C. Question

Let us show that $a^{\log_a^x} = x$

Answer

LHS $= a^{\log_a x}$

Taking log we have,

$= \log a^{\log_a x}$

$= \log_a x \log a$

$= \frac{\log x}{\log a}\times \log a \;= \log x$

RHS $= x$

Taking log we have, $= \log x$

Therefore, LHS = RHS

Hence , proved .

13 D. Question

If loge2 . logx25 = log1016 . loge10, then let us calculate the value of x.

Answer

Given that $\log_e 2 . \log_x 25 = \log_{10} 16 . \log_e 10$

$\Rightarrow \frac{\log 2}{\log e}\times\frac{\log 25}{\log x} = \frac{\log 16}{\log 10}\times\frac{\log 10}{\log e}$

$\Rightarrow \frac{\log 2}{\log x} = \frac{\log 16}{\log 25}$

$$\Rightarrow \frac{\log 2}{\log x} = \frac{\log 2^4}{\log 5^2}$$

$$\Rightarrow \frac{\log 2}{\log x} = \frac{4 \log 2}{2 \log 5}$$

$$\Rightarrow \log x = \frac{1}{2} \log 5$$

$$\Rightarrow \log x = \log 5^{\frac{1}{2}}$$

Therefore, $x = 5^{\frac{1}{2}}$