22.GRAVITATION

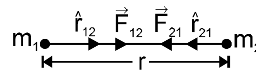
GRAVITATION : **Universal Law of Gravitation**

$$F \quad \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = G\frac{m_1 m_2}{r^2}$$

where G = 6.67 10 Nm kg is the universal gravitational constant.

Newton's Law of Gravitation in vector form :

$$F_{12} = \frac{Gm_1m_2}{r^2} \; r_{12} \qquad \& \qquad F_{21} = \frac{Gm_1m_2}{r^2} \; r_{21}$$



Now $r_{12} = -r_{21}$, Thus $F_{21} = \frac{-G\,m_1\,m_2}{r^2} r_{12}$. Comparing above, we get $\vec{F}_{12} = -\vec{F}_{21}$

Gravitational Field $E = \frac{F}{m} = \frac{GM}{r^2}$

Gravitational potential : gravitational potential, $V = \frac{GM}{r}$. $E = \frac{dV}{dr}$.

1. **Ring.** $V = \frac{-GM}{x \text{ or } (a^2 + r^2)^{1/2}}$ & $E = \frac{-GMr}{(a^2+r^2)^{3/2}} r$ or $E = \frac{GM\cos\theta}{x^2}$

   Gravitational field is maximum at a distance, $r = \pm\, a/\sqrt{2}$ and it is $2GM/3\sqrt{3}\,a^2$

2. **Thin Circular Disc.**

$$V = \frac{-2GM}{a^2}\left[\left[a^2 + r^2\right]^{\frac{1}{2}} - r\right] \quad \& \quad E = \frac{2GM}{a^2}\left[1 - \frac{r}{\left[r^2 + a^2\right]^{\frac{1}{2}}}\right] = \frac{2GM}{a^2}\left[1 - \cos\theta\right]$$

3. **(a) Point P inside the sphere.** $r \le a$, then

   $V = -\frac{GM}{2a^3}(3a^2 - r^2)$ & $E = \frac{GMr}{a^3}$, and at the centre $V = \frac{3GM}{2a}$ and E = 0

   (b) Point P outside the sphere . $r \ge a$, then $V = -\frac{GM}{r}$ & $E = \frac{GM}{r^2}$

4. **Uniform Thin Spherical Shell**

   (a) Point P Inside the shell. $r \le a$, then $V = \frac{-GM}{a}$ & E = 0

   (b) Point P outside shell. $r \ge a$, then $V = \frac{-GM}{r}$ & $E = \frac{GM}{r^2}$

VARIATION OF ACCELERATION DUE TO GRAVITY :

1. **Effect of Altitude** $g_h = \frac{GM_e}{(R_e + h)^2} = g\left(1 + \frac{h}{R_e}\right)^{-2} \simeq g\left(1 - \frac{2h}{R_e}\right)$ when h << R.

2. **Effect of depth** $g_d = g\left(1 - \frac{d}{R_e}\right)$

3. **Effect of the surface of Earth**

   The equatorial radius is about 21 km longer than its polar radius.

   We know, $g = \frac{GM_e}{R_e^2}$ Hence $g_{pole} > g_{equator}$.

SATELLITE VELOCITY (OR ORBITAL VELOCITY)

$$v_0 = \left[\frac{GM_e}{(R_e + h)}\right]^{\frac{1}{2}} = \left[\frac{gR_e^2}{(R_e + h)}\right]^{\frac{1}{2}}$$

When $h << R_e$ then $v_0 = \sqrt{gR_e}$

$\therefore \quad v_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \times 10^3\ ms^{-1} = 7.92\ km\ s^{-1}$

Time period of Satellite $\quad T = \dfrac{2\pi(R_e+h)}{\left[\dfrac{gR_e^2}{(R_e+h)}\right]^{\frac{1}{2}}} = \dfrac{2\pi}{R_e}\left[\dfrac{(R_e+h)^3}{g}\right]^{\frac{1}{2}}$

Energy of a Satellite $U = \dfrac{-GM_e m}{r}$ $\quad$ K.E. $= \dfrac{GM_e m}{2r}$; then total energy $\quad E = \dfrac{GM_e m}{2R_e}$

Kepler's Laws

Law of area :

The line joining the sun and a planet sweeps out equal areas in equal intervals of time.

Areal velocity $= \dfrac{\text{area swept}}{\text{time}} = \dfrac{\frac{1}{2}r(rd\theta)}{dt} = \dfrac{1}{2}r^2\dfrac{d\theta}{dt}$ = constant . Hence $\dfrac{1}{2}r^2\omega$ = constant.

Law of periods : $\quad \dfrac{T^2}{R^3}$ = constant

1 FLUID MECHANICS & PROPERTIES OF MATTER

FLUIDS, SURFACE TENSION, VISCOSITY & ELASTICITY :

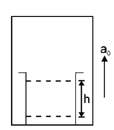
Hydraulic press. $\quad p = \dfrac{f}{a} = \dfrac{F}{A}$ or $F = \dfrac{A}{a} \times f$.

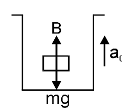
Hydrostatic Paradox $\quad P_A = P_B = P_C$

(i) Liquid placed in elevator : When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by,

$$p = \rho h\ [g + a_0]$$

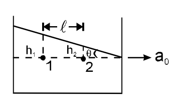
and force of buoyancy, $B = m\ (g + a_0)$





(ii) Free surface of liquid in horizontal acceleration :

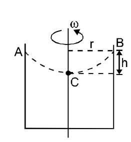
$$\tan\theta = \frac{a_0}{g}$$



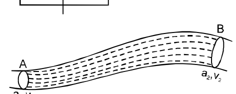
$p_1 - p_2 = \rho\ell\, a_0$ where p_1 and p_2 are pressures at points 1 & 2. Then $h_1 - h_2 = \dfrac{\ell a_0}{g}$

(iii) Free surface of liquid in case of rotating cylinder.

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$

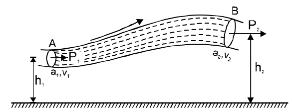


Equation of Continuity $\quad a_1v_1 = a_2v_2$



In general av = constant .

Bernoulli s Theorem



i.e. $\frac{P}{\rho} + \frac{1}{2} v^2 + gh = $ constant.

(vi) Torricelli s theorem (speed of efflux) $v=\sqrt{\frac{2gh}{1-\frac{A_2^2}{A_1^2}}}$,A_2 = area of hole A_1 = area of vessel.

ELASTICITY & VISCOSITY : stress = $\frac{\text{restoring force}}{\text{area of the body}} = \frac{F}{A}$

Strain, = $\frac{\text{change in configuration}}{\text{original configuration}}$

(i) Longitudinal strain = $\frac{\Delta L}{L}$

(ii) $_v$ = volume strain = $\frac{\Delta V}{V}$

(iii) **Shear Strain :** $\tan\phi$ or $\phi = \frac{x}{\ell}$

. **Young's modulus of elasticity** $Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$

Potential Energy per unit volume = $\frac{1}{2}$(stress strain) = $\frac{1}{2}$ (Y strain^2)

Inter-Atomic Force-Constant $k = Yr_0$.

1 Newton s Law of viscosity, F A $\frac{dv}{dx}$ or $F = \eta A \frac{dv}{dx}$

Stoke s Law $F = 6\pi\eta r v$. Terminal velocity = $\frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

SURFACE TENSION

Surface tension (T) = $\frac{\text{Total force on either of the imaginary line (F)}}{\text{Length of the line } (\ell)}$; $T = S = \frac{\Delta W}{A}$

Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area.

Inside a bubble : (p p_a) = $\frac{4T}{r}$ = p_{excess} ;

Inside the drop : (p p_a) = $\frac{2T}{r}$ = p_{excess}

Inside air bubble in a liquid :(p p_a) = $\frac{2T}{r}$ = p_{excess}

Capillary Rise $h = \frac{2T\cos\theta}{r\rho g}$