Chapter – 6

Gravitation

Multiple Choice Questions

Question 1.
The linear momentum and position vector of the planet is perpendicular to each other at
(a) perihelion and aphelion
(b) at all points
(c) only at parihelion
(d) no point

Answer:
(a) perihelion and aphelion

Question 2.
If the masses of the Earth and Sun suddenly double, the gravitational force between them will
(a) remain the same
(b) increase 2 times
(c) increase 4 times
(d) decrease 2 times

Answer:
(c) increase 4 times

Solution:

The gravitation force of attraction is given by F = $F = \frac{Gm_1m_2}{r^2}$
If the masses are doubled then the force will be

$$F = \frac{G(2m_1)(2m_2)}{r^2}$$

$$F = 4\frac{Gm_1m_2}{r^2}$$

The gravitational force between them will be increase 4 times.

Question 3.
A planet moving along an elliptical orbit is closest to the Sun at distance r_1 and farthest away at a distance of r_2 If V_1 and v_2 are linear speeds at these points respectively. Then the ratio $\frac{v_1}{v_2}$ is …….
[NEET 2016]

Answer:
(a) $\frac{r_2}{r_1}$

Solution:
According to the Law of conservation of angular momentum
$$mv_1r_1 = mv_2r_2 \ ; \ v_1r_1 = v_2r_2$$
$$\frac{v_1}{r_2} = \left(\frac{r_2}{r_1}\right)$$

Question 4.
The time period of a satellite orbiting Earth in a circular orbit is independent of ……..
(a) Radius of the orbit
(b) The mass of the satellite
(c) Both the mass and radius of the orbit
(d) Neither the mass nor the radius of its orbit

Answer:
(b) The mass of the satellite

Question 5.
If the distance between the Earth and Sun were to be doubled from its present value, the number of days in a year would be
(a) 64.5
(b) 1032
(c) 182.5
(d) 730

Answer:
(b) 1032

Solution:
By Kepler's law

$T^2 \propto a^3$

$\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3}$

If the distance between the Earth and Sun were to be doubled from its present Value

$$T_2^2 = \left(\frac{a_2}{a_1}\right)^3 \times T_1^2 = \left(\frac{2a_2}{a_1}\right)^3 \times (1\ \text{year})^2 = 8 \times (365)^2$$

Present distance = a_1; Doubled distance $a_2 = 2a_1$

$$T_2 = \sqrt{8 \times (365)^2} = \sqrt{8} = 365$$

$$T_2 = 1032 \text{ days in a year}$$

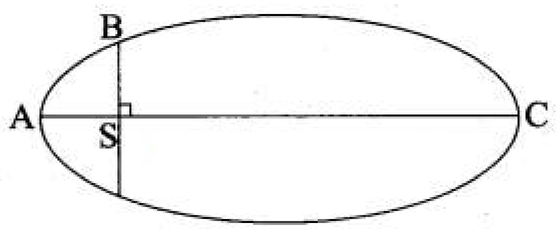
Question 6.
According to Kepler's second law, the radial vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of
(a) conservation of linear momentum
(b) conservation of angular momentum
(c) conservation of energy
(d) conservation of kinetic energy

Answer:
(b) conservation of angular momentum

Question 7.
The gravitational potential energy of the Moon with respect to Earth is
(a) always positive
(b) always negative
(c) can be positive or negative
(d) always zero.

Answer:
(b) always negative

Question 8.
The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then ….. [NEET 2018]



(a) $K_A > K_B > K_C$
(b) $K_B < K_A < K_C$
(c) $K_A < K_B < K_C$
(d) $K_B > K_A > K_C$

Answer:
(a) $K_A > K_B > K_C$

Solution:
The kinetic energy of a planet becomes

$$E_K = \frac{1}{2}\frac{GMm}{(R+h)}$$

So, kinetic energy is inversly proportional to the orbital distance. Where the distance will be closer E_K is larger and the distance will be larger E_K is less.

Question 9.
The work done by the Sun's gravitational force on the Earth is ….
(a) always zero
(b) always positive
(c) can be positive or negative
(d) always negative

Answer:
(c) can be positive or negative

Question 10.
If the mass and radius of the Earth are both doubled, then the acceleration due

to gravity g1
(a) remains same
(b) $\frac{g}{2}$
(c) 2g
(d) 4g

Answer:
(b) g/2

Solution:
Acceleration due to gravity $g' = \frac{GM}{R^2}$

If the mass and radius of the Earth are both doubled
$$g' = \frac{G(2M)}{(2R)^2} = \frac{2GM}{4R^2} = \frac{1}{2}\left(\frac{GM}{R^2}\right) \ ; g' = \frac{g}{2}$$

Question 11.
The magnitude of the Sun's gravitational field as experienced by Earth is .....
(a) same over the year
(b) decreases in the month of January and increases in the month of July
(c) decreases in the month of July and increases in the month of January
(d) increases during day time and decreases during night time.

Answer:
(c) decreases in the month of July and increases in the month of January

Question 12.
If a person moves from Chennai to Trichy, his weight .....
(a) increases
(b) decreases
(c) remains same
(d) increases and then decreases

Answer:
(b) decreases

Question 13.
An object of mass 10 kg is hanging on a spring scale which is attached to the roof of a lift. If the lift is in free fall, the reading in the spring scale is ……..
(a) 98 N
(b) zero
(c) 49 N
(d) 9.8 N

Answer:
(b) zero

Solution:
The lift is in freefall. It and its contents will experience apparent weightlessness just like astronauts.
The spring balance reading will change from 100 N to zero.

Question 14.
If the acceleration due to gravity becomes 4 times its original value, then escape speed
(a) remains same
(b) 2 times of original value
(c) becomes halved
(d) 4 times of original value

Answer:
(b) 2 times of original value

Solution:
Escape speed 4 times of 'g'

$$v_e' = \sqrt{2(4g) R_E} = \sqrt{4} \times \sqrt{2g R_E}$$

$$v_e = \sqrt{2g R_E} \; ; \; v_e' = 2v_e$$

Question 15.
The kinetic energy of the satellite orbiting around the Earth is
(a) equal to potential energy
(b) less than potential energy
(c) greater than kinetic energy

(d) zero

Answer:
(b) less than potential energy

Short Answer Questions

Question 1.
State Kepler's three laws.

Answer:

1. Law of Orbits: Each planet revolves moves around the Sun in an elliptical orbit with the Sun at one of the foci of the ellipse.

2. Law of area: The radial vector line joining the Sun to a planet sweeps equal areas in equal intervals of time.

3. Law of period: The square of the time period of revolution of a planet around the Sun in its elliptical orbit is directly proportional to the cube of the semi-major axis of the ellipse.

$$T^2 \propto a^3$$

$$\frac{T^2}{a^3} = \text{constant}$$

Question 2.
State Newton's Universal law of gravitation.

Answer:
Newton's law of gravitation: States that the gravitational force between two masses is directly proportional to product of masses and inversely proportional to square of the distance between the masses.

In the mathematical form, it can be written as,

$$\vec{F} = \frac{GM_1M_2}{r^2}\hat{r}$$

Question 3.
Will the angular momentum of a planet be conserved? Justify your answer.

Answer:
The triumph of the law of gravitation is that it concludes that the mango that is falling down and the Moon orbiting the Earth are due to the same gravitational force.

Question 4.
Define the gravitational field. Give its unit.

Answer:

The gravitational field intensity $\vec{E}_1$ at a point is defined as the gravitational force experienced by unit mass at that point. It's unit N kg^{-1}.

Question 5.
What is meant by superposition of gravitational field?

Answer:
The total gravitational field at a point due to all the masses is given by the vector sum of the gravitational field due to the individual masses. This principle is known as superposition of gravitational fields.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n = -\frac{Gm_1}{r_1^2}\hat{r}_1 - \frac{Gm_2}{r_2^2}\hat{r}_2 - \ldots - \frac{Gm_n}{r_n^2}\hat{r}_n$$

$$\vec{E}_{total} = -\sum_{i=1}^{n}\frac{Gm_i}{r_i^2}\hat{r}_i$$

Question 6.
Define gravitational potential energy.

Answer: Potential energy of a body at a point in a gravitational field is the work done by an external agent in moving the body from infinity to that point.

Question 7.
Is potential energy the property of a single object? Justify.

Answer:
There is no potential energy for a single object. The gravitational potential energy depends upon the two masses and the distance between them.

Question 8.
Define gravitational potential.

Answer:
The gravitational potential is defined as the amount of work required to bring unit mass from infinity to that point.

Question 9.
What is the difference between gravitational potential and gravitational potential energy?

Answer:

Gravitational Potential	**Gravitational potential Energy**
The amount of work required to bring unit mass from infinity to that point in point an gravitational field.	The work done by an external agent in moving the body from infinity to that in an gravitational field.
The unit of V(r) is J kg^{-1}	The unit of U(r) is J (or) joule.

Question 10.
What is meant by escape speed in the case of the Earth?

Answer:
The escape speed is independent of the direction in which the object is thrown. Irrespective of whether the object is thrown vertically up, radially outwards or tangentially it requires the same initial speed to escape Earth's gravity force. This can be written as, $v_e = \sqrt{2gR_E}$.

Question 11.
Why is the energy of a satellite (or any other planet) negative?

Answer:
The energy of satellite is negative. Because the energy implies that the satellite is bound to the Earth by means of the attractive gravitational force.

Question 12.
What are geostationary and polar satellites?

Answer:

Geostationary Satellite: It is the satellite which appears at a fixed position and at a definite height to an observer on earth.

Polar Satellite: It is the satellite which revolves in polar orbit around the earth.

Question 13.
Define weight.

Answer:
The weight of an object $\overrightarrow{W}$ is defined as the downward force whose magnitude W is equal to that of upward force that must be applied to the object to hold it at rest or at constant velocity relative to the earth. The direction of weight is in the direction of gravitational force. So the magnitude of weight of an object is denoted as, $W = N = mg$.

Question 14.
Why is there no lunar eclipse and solar eclipse every month?

Answer:
If the orbits of the Moon and Earth lie on the same plane, during full Moon of every month, we can observe lunar eclipse. If this is so dining new Moon we can observe solar eclipse. But Moon's orbit is tilted 5° with respect to Earth's orbit. Due to this 5° tilt, only during certain periods of the year, the Sun, Earth and Moon align in straight line leading to either lunar eclipse or solar eclipse depending on the alignment.

Question 15.
How will you prove that Earth itself is spinning?

Answer:
The Earth's spinning motion can be proved by observing star's position over a night. Due to Earth's spinning motion, the stars in sky appear to move m circular motion about the pole star.

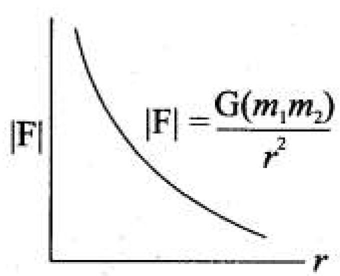
Long Answer Questions

Question 1.
Discuss the important features of the law of gravitation.

Answer:

1. As the distance between two masses increases, the strength of the force tends to decrease because of inverse dependence on r^2. Physically it implies that the planet Uranus experiences less |F| gravitational force from the Sun than the Earth since Uranus is at larger distance from the Sun compared to the Earth.

2. The gravitational forces between two particles always constitute an action-reaction pair. It implies that the gravitational force exerted by the Sim on the Earth is always towards the Sun. The reaction-force is exerted by the Earth on the Sun. The direction of this reaction force is towards Earth.

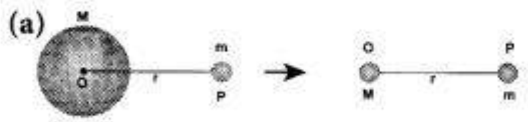


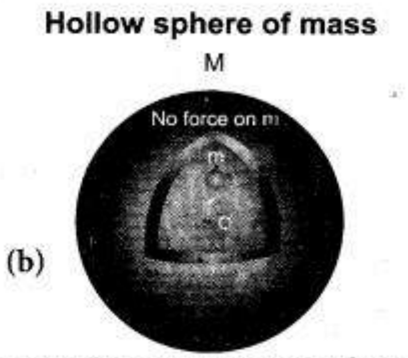
3. The torque experienced by the Earth due to the gravitational force of the Sun is given by

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \left[-\frac{GM_S M_E}{r^2} \hat{r} \right] = 0$$

Since $\vec{r} = r\,\hat{r}, (\hat{r} \times \hat{r}) = 0$

So $\vec{\tau} = \frac{d\vec{L}}{dt} = 0.$ It implies that angular momentum $\vec{L}$ is a constant vector.





A mass placed in a hollow sphere.

4. The expression $\vec{F} = -\frac{GM_1M_2}{r^2}\hat{r}$ has one inherent assumption that both M_1, and M_2 are treated as point masses. When it is said that Earth orbits around the Sun due to Sun's gravitational force, we assumed Earth and Sun to be point masses.

5. Point masses holds even for small distance.

6. There is also another interesting result. Consider a hollow sphere of mass M. If we place another object of mass 'm' inside this hollow sphere the force experienced by this mass 'm' will be zero.

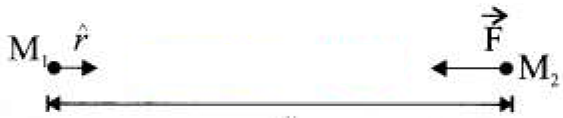
Question 2.
Explain how Newton arrived at his law of gravitation from Kepler's third law.

Answer:
Newton law of gravitation states that a particle of mass M_1 attracts any other

particle of mass M_2 in the universe with an attractive force. The strength of this force of attraction was found to be directly proportional to the product of their masses and is inversely proportional to the square to the distance between them. In mathematical form, it can be written as:

$$\vec{F} = \frac{GM_1M_2}{r^2}\hat{r}$$

where $\hat{r}$ is the unit vector from M_1 towards M_2 as shown in given figure, and G is the Gravitational constant that has the value of 6.67×10^{-11} N m^2 kg^{-2}, and r is the distance between the two masses M_1 and M_2.



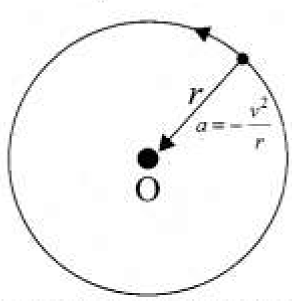
Attraction of two masses towards each other

In given figure the vector $\vec{F}$ denotes the gravitational force experienced by M_2 due to M_1. Here the negative sign indicates that the gravitational force is always attractive in nature and the direction of the force is along the line joining the two masses. In Cartesian coordinates, the square of the distance is expressed as $r^2 = (x^2 + y^2 + z^2)$

Question 3.
Explain how Newton verified his law of gravitation.

Answer:
Newton inverse square law: Newton considered the orbits of the planets as circular. For circular orbit of radius r, the centripetal acceleration towards the centre is

$$a = -\frac{v^2}{r} \quad ...(1)$$



Point mass orbiting in a circular orbit

The velocity in terms of known quantities r and T, is

$$v = \frac{2\pi r}{T} \qquad ...(2)$$

Here T is the time period of revolution of the planet. Substituting this value of v in equation (1) we get,

$$a = -\frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \qquad ...(3)$$

Substituting the value 'a' from (3) in Newton's second law, F = ma, where 'm' is the mass of the planet.

$$F = -\frac{4\pi^2 mr}{T^2} \qquad ...(4)$$

From Kepler's third law, $\frac{r^3}{T^2} = k$ (constant) ...(5)

$$\frac{r}{T^2} = \frac{k}{r^2} \qquad ...(6)$$

By substituting equation (6) in the force expression, we can arrive at the law of gravitation.

$$F = -\frac{4\pi^2 mk}{r^2} \qquad ...(7)$$

Here negative sign implies that the force is attractive and it acts towards the center. But Newton strongly felt that according to his third law, if Earth is attracted by the Sun, then the Sim must also be attracted by the Earth with the same magnitude of force. So he felt that the Sun's mass (M) should also occur explicitly in the expression for force (7). From this insight, he equated the constant $4\pi^2k$ to GM which turned out to be the law of gravitation.

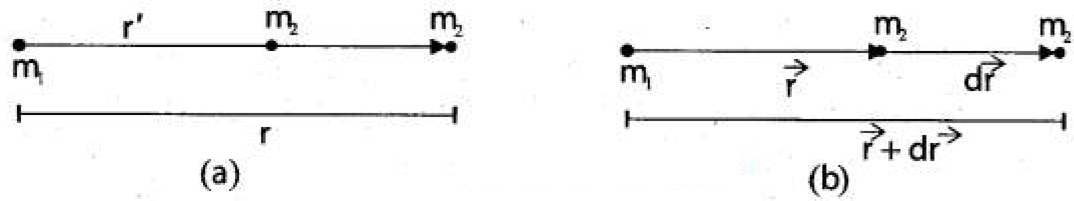
$$F = -\frac{GMm}{r^2}$$

Again the negative sign in the above equation implies that the gravitational force is attractive.

Question 4.
Derive the expression for gravitational potential energy.

Answer:
The gravitational force is a conservative force and hence we can define a gravitational potential energy associated with this conservative force field.

Two masses m_1 and m_2 are initially separated by a distance r'. Assuming m_1 to be fixed in its position, work must be done on m_2 to move the distance from r' to r.



Two distant masses changing the linear distance

To move the mass m_2 through an infinitesimal displacement $d\vec{r}$ from $\vec{r}$ to $\vec{r} + d\vec{r}$, work has to be done externally. This infinitesimal work is given by

$$dW = \vec{F}_{ext} \cdot d\vec{r} \qquad \text{...(1)}$$

The work is done against the gravitational force, therefore,

$$\vec{F}_{ext} = \frac{Gm_1m_2}{r^2}\vec{F}_G \qquad \text{...(2)}$$

Substituting equation (2) in (1), we get

$$dW = \frac{Gm_1m_2}{r^2}\hat{r} \cdot d\vec{r} \qquad \text{...(3)}$$

$$d\vec{r} = dr\,\hat{r} \Rightarrow dW = \frac{Gm_1m_2}{r^2}\hat{r}\cdot(dr\,\hat{r})$$

$$\hat{r}\cdot\hat{r} = 1 \text{ (Since both are unit vectors)}$$

$$\therefore \quad dW = \frac{Gm_1m_2}{r^2}dr \qquad \text{...(4)}$$

Thus the total work for displacing the particle from r' to r is

$$W = \int_{r'}^{r}\frac{Gm_1m_2}{r^2}dr \qquad \text{...(5)}$$

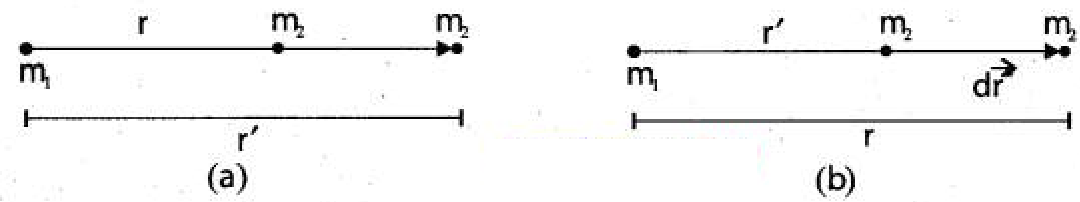
$$W = -\left(\frac{Gm_1m_2}{r}\right)_{r'}^{r}$$

$$W = -\frac{Gm_1m_2}{r} + \frac{Gm_1m_2}{r'} \qquad \text{...(6)}$$

$$W = U(r) - U(r')$$

where $$U(r) = \frac{-Gm_1m_2}{r} \qquad \text{...(7)}$$

This work done W gives the gravitational potential energy difference of the system of masses m_1 and m_2 when the seperation between them are r and r' respectively.



Case for calculation of workdone by gravity

Case 1: If r < r' : Since gravitational force is attractive, m_2 is attracted by m_1. Then m_2 can move from r' to r without any external Work. Here work is done by the system spending its internal energy and hence the work done is said to be negative.

Case 2: If r > r': Work has to be done against gravity to move the object from r' to r. Therefore work is done on the body by external force and hence work done is positive.

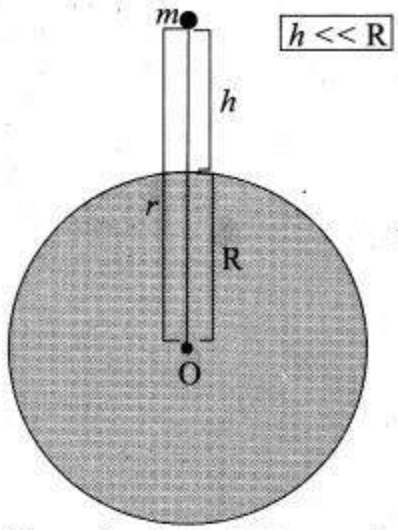
Question 5.
Prove that at points near the surface of the Earth, the gravitational potential energy of the object is U = mgh.

Answer:
When an object of mass m is raised to a height h, the potential energy stored in the object is mgh. This can be derived using the general expression for gravitational potential energy.

Consider the Earth and mass system, with r, the distance between the mass m and the Earth's centre. Then the gravitational potential energy.

$$U = -\frac{GM_e m}{r} \quad \text{...(1)}$$

Here $r = R_e + h$, where R_e is the radius of the Earth, h is the height above the Earth's surface



Mass placed at a distance r from the center of the Earth

$$U = -G\frac{M_e m}{(R_e + h)} \qquad ...(2)$$

If $h << R_e$, equation (2) can be modified as

$$U = -G\frac{M_e m}{R_e\left(1+\frac{h}{R_e}\right)}$$

$$U = -G\frac{M_e m}{R_e}\left(1+\frac{h}{R_e}\right)^{-1} \qquad ...(3)$$

By using Binomial expansion and neglecting the higher order terms, we get

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ... + \infty$$

Here, $x = \frac{h}{R_e}$ and $n = -1$

$$\left(1+\frac{h}{R_e}\right)^{-1} = \left(1-\frac{h}{R_e}\right)$$

Replace this value and we get,

$$U = -\frac{GM_e m}{R_e}\left(1-\frac{h}{R_e}\right) \qquad ...(4)$$

We know that, for a mass m on the Earth's surface,

$$G\frac{M_e m}{R_e} = mgR_e \qquad ...(5)$$

Substituting equation (4) and (5) we get,
$U = -mg_e + mgh$ …….. (6)

It is clear that the first term in the above expression is independent of the height h. For example, if the object is taken from height h_1 to h_2, then the potential energy at h_1 is

$U(h_1) = - mgR_e + mgh_1$ …(7)
and the potential energy at h_2 is

$U(h_2) = - mgR_e + mgh_2$ …(8)
The potential energy difference between h_1 and h_2 is

$U(h_2) - U(h_1) = mg(h_2 - h_1)$ …(9)

The term mgR_e in equation (7) and (8) plays no role in the result. Hence in the equation (6) the first term can be omitted or taken to zero. Thus it can be stated that the gravitational potential energy stored in the particle of mass m at a height h from the surface of the Earth is U = mgh. On the surface of the Earth, U = 0, since h is zero.

Question 6.
Explain in detail the idea of weightlessness using lift as an example.

Answer:
When a man is standing in the elevator, there are two forces acting on him.
1. Gravitational force which acts downward. If we take the vertical direction as positive y direction, the gravitational force acting on the man is
$\overrightarrow{\mathrm{F}}_{\mathrm{G}} = -m\hat{g}\hat{j}$

2. The normal force exerted by floor on the man which acts vertically upward,
$\overrightarrow{\mathrm{N}} = \mathrm{N}\hat{j}$ Weightlessness of freely falling bodies: Freely falling objects experience only gravitational force. As they fall freely, they are not in contact with any surface (by neglecting air friction). The normal force acting on the object is zero. The downward acceleration is equal to the acceleration due to the gravity of the Earth, i.e., (a = g)

Newton's 2nd law acting on the man N = m(g – a)
a = g ∴ N = m(g – g) = 0.

Question 7.
Derive an expression for escape speed.

Answer:
Consider an object of mass M on the surface of the Earth. When it is thrown up with an initial speed v_i, the initial total energy of the object is

$$E_i = \frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} \qquad ...(1)$$

where, M_E is the mass of the Earth and R_E the radius of the Earth. The term $-\frac{GMM_E}{R_E}$ is the potential energy of the mass M.

When the object reaches a height far away from Earth and hence treated as approaching infinity, the gravitational potential energy becomes zero [U(∞) = 0] and the kinetic energy becomes zero as well. Therefore the final total energy of the object becomes zero. This is for minimum energy and for minimum speed to escape. Otherwise kinetic energy can be nonzero.
$E_f = 0$

According to the law of energy conservation,
$E_i = E_f$ .... (2)

Substituting (1) in (2) we get,

$$\frac{1}{2}Mv_i^2 - \frac{GMM_E}{R_E} = 0$$

$$\frac{1}{2}Mv_i^2 = \frac{GMM_E}{R_E} \qquad ...(3)$$

Consider the escape speed, the minimum speed required by an object to escape Earth's gravitational field, hence replace v_i with v_e, i.e.,

$$\frac{1}{2}Mv_e^2 = \frac{GMM_E}{R_E}$$

$$v_e^2 = \frac{GMM_E}{R_E} \cdot \frac{2}{M}$$

$$v_e^2 = \frac{2GM_E}{R_E}$$

Using $g = \frac{GM_E}{R_E^2}$,

$$v_e^2 = 2gR_E$$

$$v_e = \sqrt{2gR_E} \quad \text{...(4)}$$

From equation (4) the escape speed depends on two factors acceleration due to gravity and radius of the Earth. It is completely independent of the mass of the object. By substituting the values of g ($9.8\ ms^{-2}$) and $R_e = 6400$ km, the escape speed of the Earth is $v_e = 11.2\ kms^{-1}$.

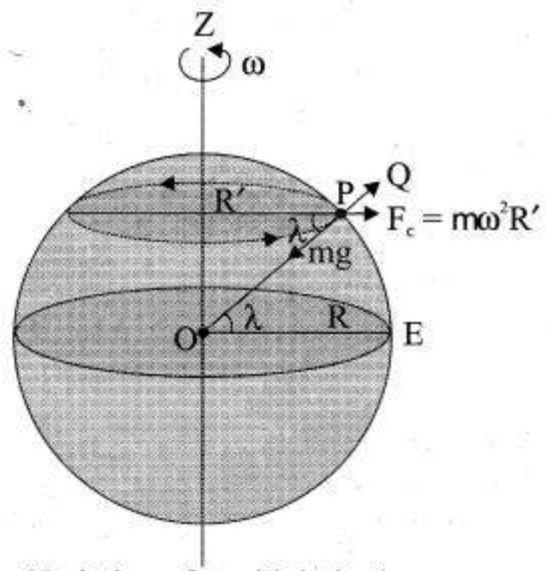
The escape speed is independent of the direction in which the object is thrown. Irrespective of whether the object is thrown vertically up, radially outwards or tangentially it requires the sarfte initial speed to escape Earth's gravity.

Question 8.
Explain the variation of g with latitude.

Answer:
When an object is on the surface for the Earth, it experiences a centrifugal force that depends on the latitude of the object on Earth. If the Earth were not spinning, the force on the object would have been mg. However, the object experiences an additional centrifugal force due to spinning of the Earth.
This centrifugal force is given by $m\omega^2R'$.

$$OP_z, \cos\lambda = \frac{PZ}{OP} = \frac{R'}{R}$$

$$R' = R\cos\lambda$$



Variation of g with latitude

where λ is the latitude. The component of centrifugal acceleration experienced by the object in the direction opposite to g is

$a_{PQ} = \omega^2 R' \cos\lambda = \omega^2 R \cos^2\lambda$

Since $R' = R\cos\lambda$

Therefore, $g' = g - \omega^2 R \cos^2\lambda$

From the above expression, we can infer that at equator, $\lambda = 0$, $g' = g - \omega^2 R$. The acceleration due to gravity is minimum. At poles $\lambda = 90$; $g' = g$, it is maximum. At the equator, g' is minimum.

Question 9.
Explain the variation of g with altitude from the Earth's surface.

Answer:
Consider an object of mass m at a height h from the surface of the Earth. Acceleration experienced by the object due to Earth is

$$g' = \frac{GM}{(R_e + h)^2}\ ;\ g' = \frac{GM}{R_e^2\left(1 + \frac{h}{R_e}\right)^2}$$

$$g' = \frac{GM}{R_e^2}\left(1 + \frac{h}{R_e}\right)^{-2}$$

If $h << R_e$: We can use Binomial expansion. Taking the terms up to first order

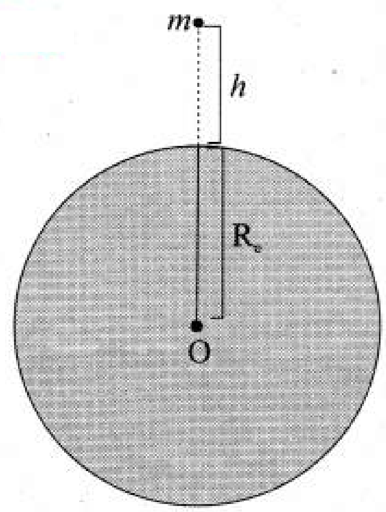
$$(1 + x)^n = 1 + nx$$

$$\left[1 + \frac{h}{R_e}\right]^{-2} = \left[1 - 2\left(\frac{h}{R_e}\right)\right]$$

Replacing this value and we get

$$g' = \frac{GM}{R_e^2}\left[1 - \frac{2h}{R_e}\right]$$

$$g' = g\left(1 - 2\frac{h}{R_e}\right)$$



Mass at a height h from the center of the Earth

We find that g' < g. This means that as altitude h increases the acceleration due to gravity g decreases.

Question 10.
Explain the variation of g with depth from the Earth's surface.

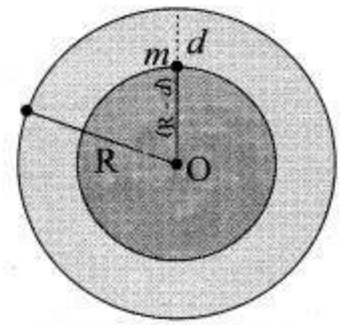
Answer:
Consider a particle of mass m which is in a deep mine on the Earth. (Example: coal mines in Neyveli). Assume the depth of the mine as d. To calculate g' at a depth d, consider the following points.

The part of the Earth which is above the radius (R_e – d) do not contribute to the acceleration. The result is proved earlier and is given as

$$g' = \frac{GM'}{(R_e - d)^2}$$

Here M' is the mass of the Earth of radius (R_e – d)

Assuming the density of Earth ρ to be constant, $\rho = \frac{M}{V}$
where M is the mass of the Earth and V its volume, thus,



Particle in a mine

$$\rho = \frac{M}{V}$$

$$\frac{M'}{V'} = \frac{M}{V} \text{ and } M' = \frac{M}{V}V'$$

$$M' = \left(\frac{M}{\frac{4}{3}\pi R_e^3}\right)\left(\frac{4}{3}\pi(R_e - d)^3\right)$$

$$M' = \frac{M}{R_e^3}(R_e - d)^3$$

$$g' = G\frac{M}{R_e^3}(R_e - d)^3 \cdot \frac{1}{(R_e - d)^2}$$

$$g' = GM\frac{R_e\left(1 - \frac{d}{R_e}\right)}{R_e^3}$$

$$g' = GM\frac{\left(1 - \frac{d}{R_e}\right)}{R_e^2}$$

Thus, $$g' = g\left(1 - \frac{d}{R_e}\right)$$

Here also g' < g. As depth increase, g' decreases. It is very interesting to know that acceleration due to gravity is maximum on the surface of the Earth but decreases when we go either upward or downward.

Question 11.
Derive the time period of satellite orbiting the Earth.

Answer:
Time period of the satellite: The distance covered by the satellite during one rotation in its orbit is equal to $2\pi(R_E + h)$ and time taken for it is the time period, T. Then,

$$\text{Speed, } v = \frac{\text{Distance travelled}}{\text{Time taken}} = \frac{2\pi(R_E + h)}{T} \quad ...(1)$$

$$\text{Speed of the satellite, } V = \sqrt{\frac{GM_E}{(R_E + h)}}$$

$$\sqrt{\frac{GM_E}{(R_E + h)}} = \frac{2\pi(R_E + h)}{T}$$

$$T = \frac{2\pi}{\sqrt{GM_E}}(R_E + h)^{\frac{3}{2}} \quad ...(2)$$

Squaring both sides of the equation (2) we get

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$$

$$\frac{4\pi^2}{GM_E} = \text{constant say } c$$

$$T^2 = c(R_E + h)^3 \quad ...(3)$$

Equation (3) implies that a satellite orbiting the Earth has the same relation between time and distance as that of Kepler's law of planetary motion. For a satellite orbiting near the surface of the Earth, h is negligible compared to the radius of the Earth R_E. Then,

$$T^2 = \frac{4\pi^2}{GM_E}R_E^3$$

$$T^2 = \frac{4\pi^2}{\left(GM_E / R_E^2\right)}R_E \Rightarrow T^2 = \frac{4\pi^2}{g}R_E$$

Since $\frac{GM_E}{R_E^2} = g$

$$T = 2\pi\sqrt{\frac{R_E}{g}}$$

By substituting the values of $R_E = 6.4 \times 10^6$ m and $g = 9.8 ms^{-2}$, the orbital time period is obtained as $T \cong 85$ minutes.

Question 12.
Derive an expression for energy of satellite.

Answer:
The total energy of the satellite is the sum of its kinetic energy and the gravitational potential energy. The potential energy of the satellite is,

$$U = -\frac{GM_sM_E}{(R_E + h)} \quad \text{...(1)}$$

Here M_s – mass of the satellite, M_E -mass of the Earth, R_E – radius of the Earth.
The Kinetic energy of the satellite is

$$K.E = \frac{1}{2}M_s v^2 \quad \text{...(2)}$$

Here v is the orbital speed of the satellite and is equal to

$$v = \sqrt{\frac{GM_E}{(R_E + h)}}$$

Substituting the value of v in (2) the kinetic energy of the satellite becomes,

$$K.E = \frac{1}{2}\frac{GM_EM_s}{(R_E + h)}$$

Therefore the total energy of the satellite is

$$E = \frac{1}{2}\frac{GM_EM_s}{(R_E + h)} - \frac{GM_sM_E}{(R_E + h)}$$

$$E = -\frac{GM_sM_E}{2(R_E + h)}$$

The negative sign in the total energy implies that the satellite is bound to the Earth and it cannot escape from the Earth.

Note: As h approaches ∞, the total energy tends to zero. Its physical meaning is that the satellite is completely free from the influence of Earth's gravity and is not bound to Earth at large distance.

Question 13.
Explain in detail the geostationary and polar satellites.

Answer:
The satellites orbiting the Earth have different time periods corresponding to different orbital radii. Kepler's third law is used to find then radius of the orbit.

$$T^2 = \frac{4\pi^2}{GM_E}(R_E + h)^3$$

$$(R_E + h)^3 = \frac{GM_E T^2}{4\pi^2}$$

$$R_E + h = \left(\frac{GM_E T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

Substituting for the time period (24 hours = 86400 seconds), mass, and radius of the Earth, h turns out to be 36,000 km. Such satellites are called "geo-stationary satellites", since they appear to be stationary when seen from Earth.

India uses the INSAT group of satellites that are basically geo-stationary satellites for the purpose of telecommunication. Another type of satellite which is placed at a distance of 500 to 800 krn from the surface of the Earth orbits the Earth from north to south direction. This type of satellite that orbits Earth from North Pole to South Pole is called a polar satellite.

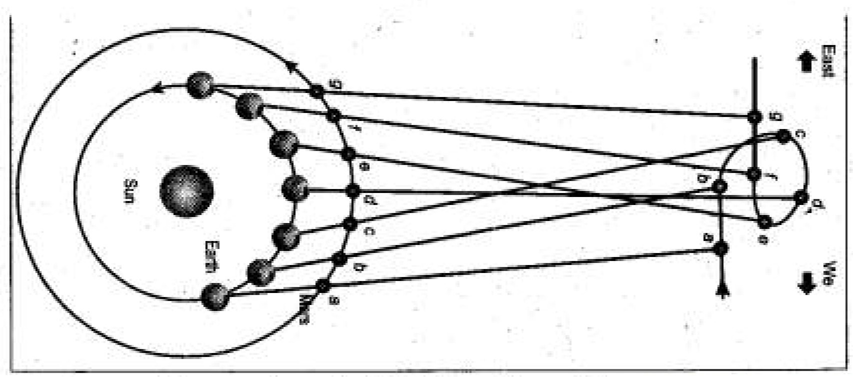
The time period of a polar satellite is nearly 100 minutes and the satellite completes many revolutions in a day. A polar satellite covers a small strip of area from pole to pole during one revolution. In the next revolution it covers a different strip of area since the Earth would have moved by a small angle. In this way polar satellites cover the entire surface area of the Earth.

Question 14.
Explain how geocentric theory is replaced by heliocentric theory using the

idea of retrograde motion of planets.

Answer:
When the motion of the planets are observed in the night sky by naked eyes over a period of a few months, it can be seen that the planets move eastwards and reverse their motion for a while and return to eastward motion again.

This is called "retrograde motion of planets.



'Retrograde motion' in heliocentric model

Careful observation for a period of a year clearly shows that Mars initially moves eastwards (February to June), then reverses its path and moves backwards (July, August, September). It changes it direction of motion once again and continues its forward motion (October onwards). In olden days, astronomers recorded the retrograde motion of all visible planets and tried to explain the motion.

According to Aristotle, the other planets and the Sun move around the Earth in the circular orbits. If it was really a circular orbit it was not known how the planet could reverse its motion for a brief interval. To explain this retrograde motion, Ptolemy introduced the concept of "epicycle" in his geocentric model. According to this theory, while the planet orbited the Earth, it also underwent another circular motion termed as "epicycle'.

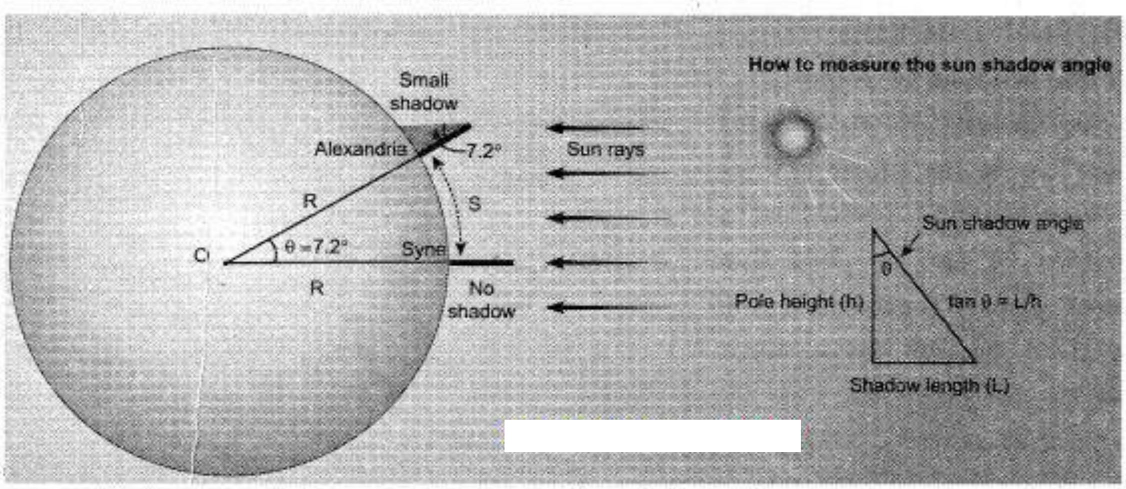
A combination of epicycle and circular motion around the Earth gave rise to retrograde motion of the planets with respect to Earth. Essentially Ptolemy retained the Earth centric idea of Aristotle and added the epicycle motion to it.

But Ptolemy's model became more and more complex as every planet was found to undergo retrograde motion. In the 15th century, the Polish astronomer Copernicus proposed the heliocentric model to explain this problem in a simpler manner. According to this model, the Sun is at the centre of the solar system and all planets orbited the Sun. The retrograde motion of planets with respect to Earth is because of the relative motion of the planet with respect to Earth.

Earth orbits around the Sun faster than Mars. Because of the relative motion between Mars and Earth, Mars appears to move backwards from July to October. In the same way the retrograde motion of all other planets was explained successfully by the Copernicus model. It was because of its simplicity, the heliocentric model slowly replaced the geocentric model.

Question 15.
Explain in detail the Eratosthenes method of finding the radius of Earth.

Answer:
Eratosthenes observed that during noon time of summer solstice the Sun's rays cast no shadow in the city Syne which was located 500 miles away from Alexandria. At the same day and same time he found that in Alexandria the Sun's rays made 7.2 degree with local vertical. He realized that this difference of 7.2 degree was due to the curvature of the Earth.



Measuring radius of The Earth

The angle 7.2 degree is equivalent to $\frac{1}{8}$ radian. So, $\theta = \frac{1}{8}$ rad;

If S is the length of the arc between the cities of Syne and Alexandria, and if R is radius of Earth, then

$$S = R\theta = 500 \text{ miles},$$

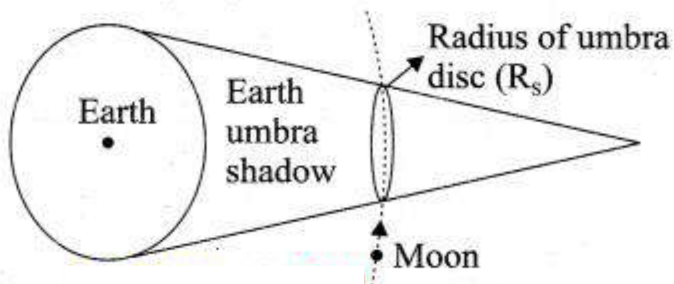
so radius of the Earth $R = \frac{500}{\theta} \text{ miles}$; $R = 500 \frac{\text{miles}}{\frac{1}{8}}$

$$R = 4000 \text{ miles}$$

1 mile is equal to 1.609 km. So, he measured the radius of the Earth to be equal to R = 6436 km, which is amazingly close to the correct value of 6378 km.

Question 16.
Describe the measurement of Earth's shadow (umbra) radius during total lunar eclipse.

Answer:
Lunar eclipse and measurements of shadow of Earth: On January 31, 2018 there was a total lunar eclipse which was observed from various place including Tamil Nadu. It is possible to measure the radius of shadow of the Earth at the point where the Moon crosses.



Schematic diagram of umbra disk radius

When the Moon is inside the umbra shadow, it appears red in colour. As soon as the Moon Schematic diagram of umbra disk radius exits from the umbra shadow, it appears in crescent shape.

By finding the apparent radii of the Earth's umbra shadow and the Moon, the ratio of the these radii can be calculated.

The apparent radius of Earth's umbra shadow = R_s = 13.2 cm
The apparent radius of the Moon = R_m = 5.15 cm

The ratio $\frac{R_s}{R_m} \approx 2.56$

The radius of Moon R_m = 1737 km

The radius of the Earth's umbra shadow is $R_s = 2.56 \times 1737$ km $\cong 4446$
The correct radius is 4610 km

$$\text{The percentage of error in the calculation} = \frac{4610 - 4446}{4610} \times 100 = 3.5\%$$

The error will reduce if the pictures taken using a high quality telescope are used.

Numerical Problems

Question 1.
An unknown a planet orbits the Sun with distance twice the semi major axis distance of the Earth's orbit. If the Earth's time period is Tp what is the time period of this unknown planet?

Answer:
By Kepler's 3rd law $T^2 \propto a^3$
Time period of unknown planet = T_2
Time period of Earth = T_1
Distance of unknown planet from the Sun = a_2
Distance of the Earth from the Sun = a_1

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

$$T_2 = \left(\frac{a_2}{a_1}\right)^{\frac{3}{2}} T_1 \qquad a_2 = 2a,$$

$$T_2 = \left(\frac{2a_1}{a_1}\right)^{\frac{3}{2}} T_1 \Rightarrow \quad T_2 = 2\sqrt{2}\, T_1$$

Question 2.
Assume that you are in another solar system and provided with the set of data given below consisting of the planets' semi major axes and time periods. Can you infer the relation connecting semi major axis and time period?

Answer:

Planet (imaginary)	Time period (T) (in year)	Semi major axis (a) (in AU)
Kurinji	2	8
Mullai	3	18
Marutham	4	32
Neithal	5	50
Paalai	6	72

In a given data's tells us the relation connecting to the semi major axis is proportional to the two times of square of the time period.
$a \propto 2T^2$

Question 3.
If the masses and mutual distance between the two objects are doubled, what is the change in the gravitational force between them?

Answer:
By Newton's law of gravitation

$$F = \frac{GM_1M_2}{r^2}$$

Here, the masses and mutual distance between the two objects are doubled .

$$F = \frac{G(M_1)(2M_2)}{(2r)^2} = \frac{4GM_1M_2}{4r^2}$$

$$F = \frac{GM_1M_2}{r^2}$$

There is no change in the gravitational force between them.

Question 4.
Two bodies of masses m and 4m are placed at a distance r. Calculate the gravitational potential at a point on the line joining them where the gravitational field is zero.

Answer:
Let the point be the position when the gravitational field is zero.,

$$\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(r-x)^2}$$

Square root on both sides, $\frac{1}{x} = \frac{2}{(r-x)}$

$\therefore$ $$x = \frac{r}{3}$$

The point P is at a distance r/3 from mass 'm' and 2r/3 from mass '4m'

Gravitational potential $V = \frac{-Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(\frac{2r}{3}\right)}$

$$= \frac{-3Gm}{4} - \frac{3G(4m)}{2r} = \frac{-3Gm}{r} - \frac{12Gm}{2r} \Rightarrow V = \frac{-9Gm}{r}$$

Question 5.

If the ratio of the orbital distance of two planets d1/d2 = 2, what is the ratio of gravitational field experienced by these two planets?

Answer:

The gravitational field experienced by planets 1

$$E_1 = \frac{GM}{d_1^2}$$

The gravitational field experienced by planet 2

$$E_2 = \frac{GM}{d_2^2}$$

The ratio of their orbital distance $\frac{d_1}{d_2} = 2$

$\therefore$ $$d_1 = 2d_2$$

The ratio of gravitational field of two planets.

$$\frac{E_1}{E_2} = \frac{GM}{(2d_2)^2} \times \frac{d_2^2}{GM} = \frac{GM}{4d_2^2} \times \frac{d_2^2}{GM}$$

$$\frac{E_1}{E_2} = \frac{1}{4}$$

$$\therefore \quad E_2 = 4E_1$$

Question 6.
The moon I_o orbits Jupiter once in 1.769 days. The orbital radius of the Moon I_0 is 421700 km. Calculate the mass of Jupiter?

Answer:
Kepler's third law is used to find the mass of the planet

$$T^2 = \frac{4\pi^2}{GM}(R+h)^2$$

$$M = \frac{4\pi^2}{GT^2}(R+h)^2 = \frac{4(3.14)^2 \times (421700 \times 10^3)^2}{6.67 \times 10^{-11} \times (1.769 \times 86400)^2}$$

$$= \frac{39.4384 \times 7.499128631 \times 10^{25}}{6.67 \times 10^{-11} \times 2.336055469 \times 10^{10}}$$

$$= \frac{295.7536 \times 10^{25}}{1.5581} = 189.8169 \times 10^{25}$$

$$M = 1.898 \times 10^{27} \text{ kg}$$

Question 7.

If the angular momentum of a planet is given by $\vec{L} = 5t^2\hat{i} - 6t\hat{j} + 3\hat{k}.$

What is the torque experienced by the planet? Will the torque be in the same direction as that of the angular momentum?

Answer:
The torque experienced by the planet

$$\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \left[\frac{d}{dt}(t^n) = nt^{n-1}\right]$$

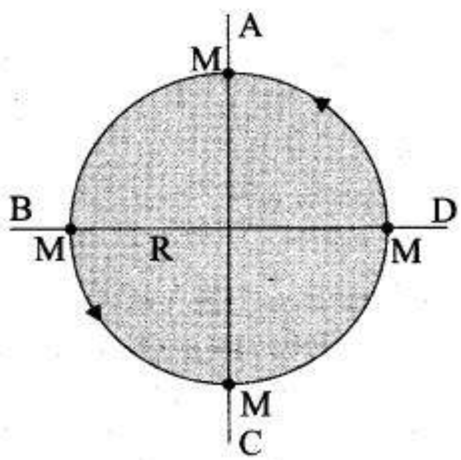
$$= \frac{d}{dt}(5t^2\hat{i} - 6t\hat{j} + 3\hat{k})$$

$$= 5\frac{d}{dt}(t^2)\hat{i} - 6\frac{d}{dt}(t)\hat{j} + 3\frac{d}{dt}(\hat{k}) = 10t\hat{i} - 6\hat{j} + 0$$

$$\vec{\tau} = 10t\hat{i} - 6\hat{j}$$

Question 8.
Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. Calculate the speed of each particle.

Answer:
The net gravitational force = Centripetal force



Force F_1 between A and B, $\quad F_1 = \frac{GMM}{(\sqrt{2}R)^2}$

Force F_2 between A and D, $\quad F_2 = \frac{GMM}{(\sqrt{2}R)^2}$

Force F_3 between A and C (or B and D)

$$F_3 = \frac{GMM}{(2R)^2}$$

The components F1 and F2 along the radius

$F_1 \cos 45$ and $F_2 \cos 45$ $F_1 = F_2 = F$

$$\therefore \quad \text{Net force} = 2F \cos 45 + F_3$$

$$= 2\frac{GM^2}{\left(\sqrt{2}R\right)^2} \times \left(\frac{1}{2}\right) + \frac{GM^2}{4R^2}$$

$$\frac{Mv_0^2}{R} = \frac{GM^2}{2R^2}\left(2\sqrt{2}+1\right)$$

$$v_0^2 = \frac{GM}{4R}\left[1+2\sqrt{2}\right]$$

$$v_o = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

Question 9.

Suppose unknowingly you wrote the universal gravitational constant value as $G = 6.67 \times 10^{11}$ instead of the correct value $G = 6.67 \times 10^{-11}$, what is the acceleration due to gravity g' for this incorrect G? According to this new acceleration due to gravity, what will be your weight W?

Answer:

Data: Incorrect Gravitational constant $G = 6.67 \times 10^{11}$ Nm^2 kg^{-2}

Mass of the Earth $M_e = 5.972 \times 10^{24}$ kg

Radius of the earth $R_e = 6371$ km (or) 6371×10^3 m

$$\text{Acceleration due to gravity } g' = \frac{GM_e}{R_e^2}$$

$$g' = \frac{6.67 \times 10^{11} \times 5.97 \times 10^{24}}{(6371 \times 10^3)^2}$$

$$= \frac{39.8332 \times 10^{35}}{4.0589641 \times 10^{13}} = 9.81 \times 10^{22}$$

$$g' = 10^{22} g$$

New weight $\quad W' = Mg' = M(10^{22}g) = 10^{22}\ mg$

$\therefore \quad W' = 10^{22}\ W$

Question 10.
Calculate, the gravitational field at point O due to three masses m_1, m_2, and m_3 whose positions are given by the following figure. If the masses m_1 and m_2 are equal what is the change in gravitational field at the point O?

Answer:

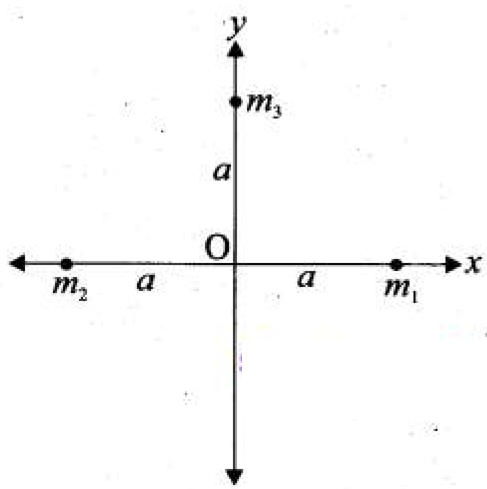
Gravitational field due to 'm_1' at a point 'O' is $\vec{E}_1 = \frac{Gm_1}{a^2}\hat{i}$

Gravitational field due to 'm_1' at a point 'O' is $\vec{E}_2 = -\frac{Gm_2}{a^2}\hat{i}$

(Negative sign indicates field acting along negative x direction)

Gravitational field due to 'm_3' at a point 'O' is $\vec{E}_3 = \frac{Gm_3}{a^2}\hat{j}$

(Negative sign indicates field acting along negative x direction)



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{Gm_1}{a^2}\hat{i} - \frac{Gm_2}{a^2}\hat{i} + \frac{Gm_3}{a^2}\hat{j}$$

$$\vec{E} = \frac{G}{a^2}\left[(m_1 - m_2)\hat{i} + m_3\hat{j}\right]$$

If the masses $m_1 = m_2$, then

$$\vec{E} = \frac{Gm_3}{a^2}\hat{j}$$

Question 11.
What is the gravitational potential energy of the Earth and Sun? The Earth to Sun distance is around 150 million km. The mass of the Earth is 5.9×10^{24} kg and mass of the Sun is 1.9×10^{30} kg.

Answer:
Mass of the Earth $M_E = 5.9 \times 10^{24}$ kg
Mass of the Sun $M_S = 1.9 \times 10^{30}$ kg
Distance between the Sun and Earth
r = 150 million km; $r = 150 \times 10^9$ m
Gravitational constant $G = 6.67 \times 10^{-11}$ $Nm^2\ kg^{-2}$
The gravitational potential energy

$$U = -\frac{GM_EM_S}{r} = \frac{-6.67 \times 10^{-11} \times 5.9 \times 10^{24} \times 1.9 \times 10^{30}}{150 \times 10^9}$$

$$= -\frac{74.7707}{150} \times 10^{34} = -0.4985 \times 10^{34}$$

$$U = -4.985 \times 10^{33} \text{ joule (or) J}$$

Question 12.
Earth revolved around the Sun at 30 km s^{-1}. Calculate the kinetic energy of the Earth. In the previous example you calculated the potential energy of the Earth. What is the total energy of the Earth in that case? Is the total energy positive? Give reasons.

Answer:
Mass of the Earth $M_E = 5.9 \times 10^{24}$ kg
Speed of the Earth rovolves around the Sun

$$V_E = 30 \text{ k ms}^{-1}$$
$$V_E = 30 \times 10^3 \text{ ms}^{-1}$$

Kinetic energy of Earth $\quad E_K = \frac{1}{2} M_E V_E^2$

$$E_K = \frac{1}{2} \times 5.9 \times 10^{24} \times (30 \times 10^3)^2$$
$$= \frac{1}{2} \times 5310 \times 10^{30} = 2655 \times 10^{30}$$
$$E_K = 2.655 \times 10^{33} \text{ joule (or) J}$$

Total energy of the Earth, $\quad E =$ Kinetic energy + Potential energy

$$= \frac{1}{2} MV^2 + \left(-\frac{GM_1M_2}{r^2}\right)$$

$= 2.655 \times 10^{33} + (- 4.985 \times 10^{33})$
$= 2.655 \times 10^{33} - 4.985 \times 10^{33}$
$E = -2.33 \times 10^{33}$ joule (or) J
'-Ve' implies that Earth is bounded with Sun.

Question 13.
An object is thrown from Earth in such a way that it reaches a point at infinity with non-zero kinetic energy $\left[\mathbf{K.E}(r = \infty) = \frac{1}{2}\mathbf{M}v_\infty^2\right]$, with that velocity should the object be thrown from Earth?

Answer:
An object is thrown up with an initial velocity is v_i, So Total energy of the object is

$$E_i = \frac{1}{2} Mv_i^2 - \frac{GMm_e}{R_E}$$

Now, the object reaches a height with a non-zero K.E.
K.E becomes infinity. P.E becomes zero.

So,
$$E_f = \frac{1}{2}Mv_\infty$$
$$E_i = E_f \Rightarrow v_i = \underline{V}_\infty$$
$$\frac{1}{2}Mv_\infty^2 = \frac{1}{2}Mv_e^2 - \frac{2GMM_E}{R_E}$$
$$\frac{1}{2}Mv_\infty^2 = \frac{1}{2}M\left[V_e^2 - \frac{2GM_E}{R_E}\right]$$
$$v_e^2 = v_\infty^2 + 2\left(\frac{GM_E}{R_E}\right)R_E$$
$$= v_\infty^2 + 2gR_E$$
$$v_e = \sqrt{v_\infty^2 + 2gR_E}$$

Question 14.

Suppose we go 200 km above and below the surface of the Earth, what are the g values at these two points? In which case, is the value of g small?

Answer:

Variation of g' with depth

$$g' = g\left(1 - \frac{d}{R_E}\right) \qquad \left[\begin{array}{l} d = 200\text{ km} = 200 \times 10^3\text{ m} \\ R_E = 6371 \times 10^3\text{ m} \end{array}\right]$$
$$= g\left(1 - \frac{200 \times 10^3}{6371 \times 10^3}\right) = g(1 - 0.0314) = g(0.9686)$$
$$g' = 0.96g$$

Variation of g' with altitude

$$g' = g\left(1 - \frac{2h}{R_E}\right) \qquad [h = 200\text{ km} = 200 \times 10^3\text{ m}]$$
$$= g\left(1 - \frac{2 \times 200 \times 10^3}{6371 \times 10^3}\right) = g(1 - 2(0.0314)) = g(0.9372)$$
$$g' = 0.93\text{ g}$$

Question 15.
Calculate the change in g value in your district of Tamil Nadu. (Hint: Get the latitude of your district of Tamil Nadu from the Google). What is the difference in g values at 'Chennai and Kanyakumari?

Answer:
Variation of 'g' value in the latitude to Chennai

$$g'_{Chennai} = g - \omega^2 R \cos^2\lambda$$

Here $$\omega^2 R = \left(\frac{2\pi}{T}\right)^2 \times R$$

Period of revolution (T) = 1 day = 86400 sec
Radius of the Earth (R) = 6400×10^3 m
Latitude of Chennai (λ) = 13° = 0.2268 rad

$$g'_{Chennai} = 9.8 - \left[\left(\frac{2 \times 3.14}{86400}\right)^2 \times 6400 \times 10^3\right] \times (\cos 0.2268)^2$$

$$= 9.8 - \left[(3.4 \times 10^{-2}) \times (0.9744)^2\right]$$

$$= 9.8 - [0.034 \times 0.9494] = 9.8 - 0.0323$$

$$g'_{Chennai} = 9.7677 \text{ ms}^{-2}$$

Variation of 'g' value in the latitude of Kanyakumari

$$\lambda'_{Kanyakumari} = 8°35' = 0.1457 \text{ rad}$$

$$g'_{Kanyakumari} = 9.8 - \left[3.4 \times 10^{-2} \times (\cos 0.1457)^2\right] = 9.8 - 0.0333$$

$$g'_{Kanyakumari} = 9.7667 \text{ ms}^{-2}$$

The difference of 'g' value $\Delta g = g'_{Chennai} - g'_{Kanyamukari}$

$$= 9.7677 - 9.7667$$

$$\Delta g = 0.001 \text{ ms}^{-2}$$