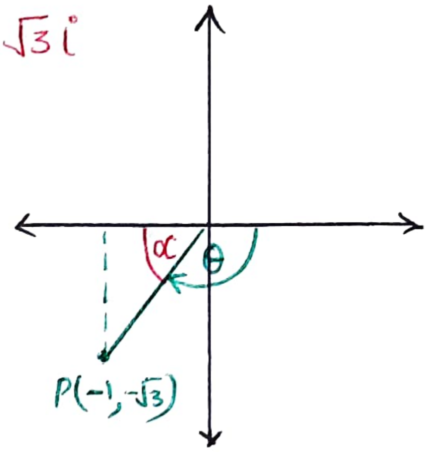
EXERCISE 5.2

QNo. 1. Find the modulus and argument of $z = -1-\sqrt{3}i$



Sol: Let $z = -1-\sqrt{3}i = r(\cos\theta + i\sin\theta)$

$\Rightarrow \quad x = r\cos\theta = -1$

$\quad\quad y = r\sin\theta = -\sqrt{3}$

$\therefore$ Modulus of $z = |z| = \sqrt{x^2+y^2}$

$= r. = \sqrt{(-1)^2+(-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$ $[\because$ conventionally $r>0]$

$\arg z = -\pi+\alpha, \ \tan\alpha = \left|\frac{y}{x}\right|$

and Argument of $z = \arg z = \theta$

$\therefore \quad \cos\theta = \frac{-1}{2}$ and $\sin\theta = \frac{-\sqrt{3}}{2}$

Now θ lies in third quadrant and $-\pi < \theta \le \pi$

$\therefore$ from fig $\tan\alpha = \left|\frac{-\sqrt{3}}{-1}\right|$ ie $\tan\alpha = \sqrt{3}$

$\Rightarrow \quad \alpha = \pi/3 \quad [\because \tan\pi/3 = \sqrt{3}]$

$\therefore \quad \arg z = \theta = -\pi+\alpha = -\pi+\frac{\pi}{3} = \frac{-3\pi+\pi}{3} = -\frac{2\pi}{3}$

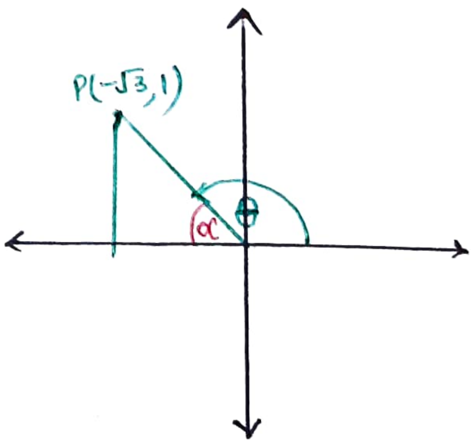
$\therefore$ Modulus of $z = 2$ and Argument of $z = -\frac{2\pi}{3}$

QNo 2. Find the modulus and the arguments of $z = -\sqrt{3}+i$

Sol. Let $z = -\sqrt{3}+i = r(\cos\theta + i\sin\theta)$; r is Modulus and θ is argumen

$\Rightarrow \quad x = r\cos\theta = -\sqrt{3}$

$\quad\quad y = r\sin\theta = 1$



$\therefore$ Modulus of $z = |z| = r = \sqrt{x^2+y^2}$

$= \sqrt{(-\sqrt{3})^2+(1)^2}$

$= \sqrt{3+1} = \sqrt{4}$

$= 2 \quad [\because r>0]$

$\arg z = \pi-\alpha, \ \tan\alpha = \left|\frac{y}{x}\right|$

Now since θ lies is second quadrant and $-\pi < \theta \leq \pi$.

$\therefore$ Argument of $z = \theta = \pi - \alpha$ where $\tan\alpha = \left|\frac{y}{x}\right|$

Now $\tan\alpha = \left|\frac{1}{-\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$

$\Rightarrow \alpha = \frac{\pi}{6} \quad \because \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\therefore$ Argument of $z = \theta = \pi - \alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

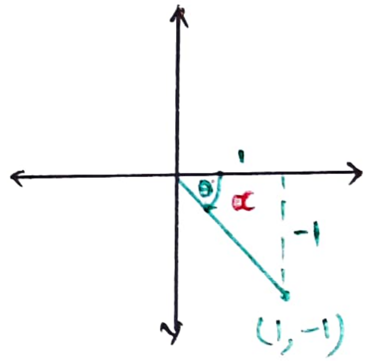
$\therefore -\sqrt{3} + i = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$\therefore$ Modulus of $z = 2$ and $\arg z = \frac{5\pi}{6}$.

Convert each of the complex nos given in Exercise 3 to 8 in polar form.

Q No. 3 $\quad 1 - i$

Sol. Let $1 - i = r(\cos\theta + i\sin\theta)$



$\Rightarrow r\cos\theta = 1$

$r\sin\theta = -1$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = (1)^2 + (-1)^2 = 1 + 1 = 2$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

$\therefore \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{-1}{\sqrt{2}}$.

$\arg z = \theta = -\alpha$
where $\tan\alpha = |y/x|$

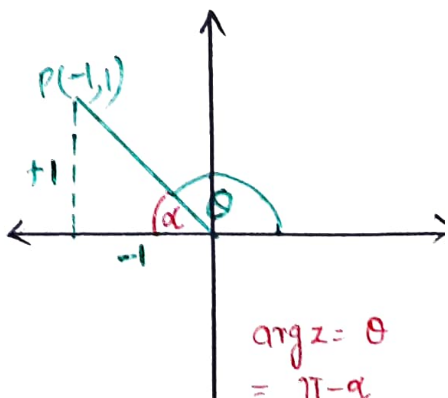
As θ lies in 4th quadrant and $-\pi < \theta \leq \pi$

$\therefore \theta = -\alpha = -\frac{\pi}{4}$ where $\tan\alpha = \left|\frac{y}{x}\right| = 1$.

$\therefore 1 - i = \sqrt{2}\left(\cos(-\pi/4) + i\sin(-\pi/4)\right)$

Q No 4: $\quad -1 + i$

Let $-1 + i = r(\cos\theta + i\sin\theta)$



$\Rightarrow r\cos\theta = -1$

$r\sin\theta = 1$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = (-1)^2 + (1)^2 = 1 + 1 = 2$

$\arg z = \theta$
$= \pi - \alpha$

$\Rightarrow \quad r = \sqrt{2}$

Also $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$.

Now as. θ lies in Second quadrant and $-\pi < \theta \leq \pi$

$\therefore$ From fig $\tan\alpha = \left|\frac{y}{x}\right| = \left|\frac{1}{-1}\right| = 1$

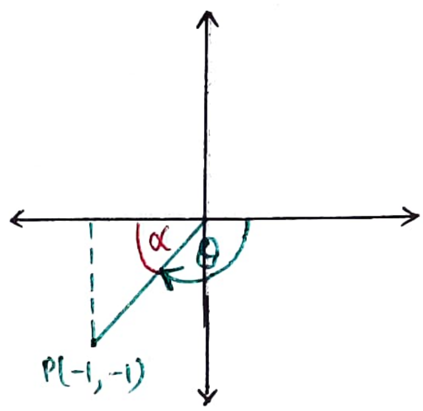
$\Rightarrow \alpha = \frac{\pi}{4} \quad \because \tan\frac{\pi}{4} = 1$

$\therefore \arg z = \theta = \pi - \alpha = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\therefore \quad -1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

QNo 5: $-1-i$

Let $z = -1-i = r(\cos\theta + i\sin\theta)$ where r is modulus and θ is $\arg z$.



$\arg z = \theta = -\pi + \alpha$

Then $r\cos\theta = -1$

$r\sin\theta = -1$

$\Rightarrow r^2[\cos^2\theta + \sin^2\theta] = (-1)^2 + (-1)^2 = 1+1 = 2$

$\Rightarrow \quad r^2 = 2 \Rightarrow r = \sqrt{2} \quad [\because r > 0]$

and $\therefore \quad \cos\theta = -\frac{1}{\sqrt{2}}; \ \sin\theta = -\frac{1}{\sqrt{2}}$

Now as θ lies in 3rd quadrant

from fig $\theta = -\pi + \alpha$ where $\tan\alpha = \left|\frac{-1}{-1}\right| = 1$

$\Rightarrow \quad \theta = -\pi + \frac{\pi}{4} \quad [\because \tan\pi/4 = 1]$

$= \frac{-4\pi + \pi}{4} = -\frac{3\pi}{4}$

$\therefore \quad -1-i = \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

QNo.6: $-3 = -3 + 0i$

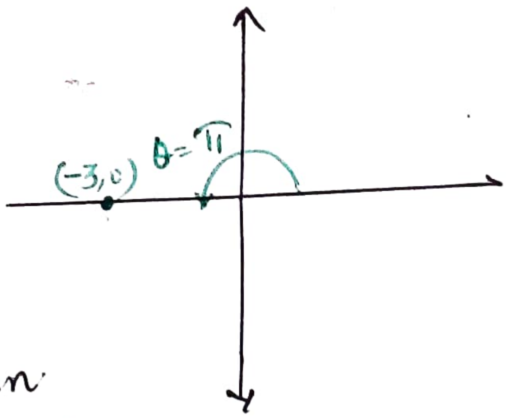
Let $z = -3 + 0i = r(\cos\theta + i\sin\theta)$, where r is modulus and θ is argument of complex No.

$\Rightarrow \quad r\cos\theta = -3$

$\quad\quad r\sin\theta = 0$

$\Rightarrow \quad r^2 = (-3)^2 + (0)^2 = 9 + 0 = 9$

$\Rightarrow \quad r = 3 \quad [\because r > 0]$



Also $\cos\theta = -1$

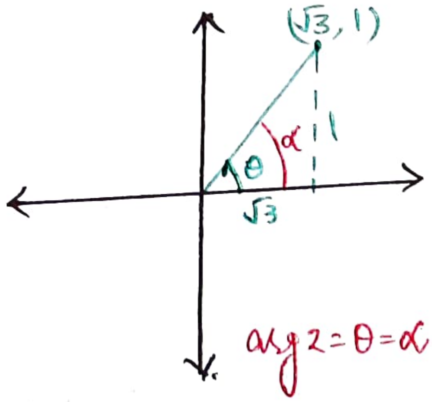
$\quad\quad \sin\theta = 0$

Since z is pure real no. $\therefore$ lies on x-axis on Left side of origin

$\therefore$ Arg z = Angle with +ve x-axis = π.

$\therefore \quad -3 = 3(\cos\pi + i\sin\pi)$

Q No. 7 $\sqrt{3} + i$



Let $\sqrt{3} + i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = \sqrt{3}$

$\quad\; r\sin\theta = 1$

$\Rightarrow r = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\therefore \quad \cos\theta = \frac{\sqrt{3}}{2} \;;\; \sin\theta = \frac{1}{2}$

As θ lies in first quadrant

$\therefore$ Argument $\theta = \alpha$ where $\tan\alpha = \frac{1}{\sqrt{3}}$ i.e. $\alpha = \frac{\pi}{6}$

$\therefore \quad \sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

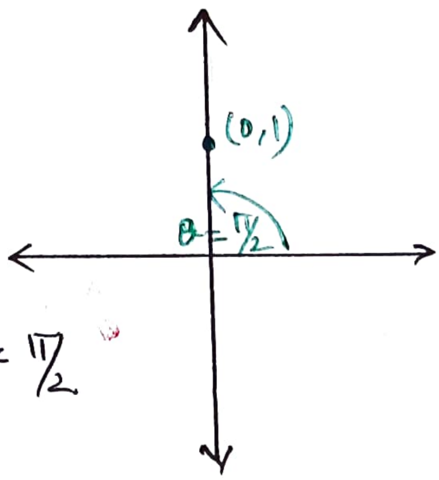
Q No. 8: $i = 0 + 1i$

Let $z = i = 0 + 1i = r(\cos\theta + i\sin\theta)$

$\Rightarrow \quad r\cos\theta = 0$

$\quad\quad r\sin\theta = 1$

$\Rightarrow \quad r = \sqrt{(0)^2 + (1)^2} = \sqrt{1} = 1$



Since z is purely imaginary complex. No. and lies above x-axis

$\therefore \quad \theta = \text{Arg } z = \text{Angle with +ve x-axis} = \pi/2$

$\therefore \quad i = 1(\cos\pi/2 + i\sin\pi/2)$

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