CBSE Board
Class IX Mathematics

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. If $\left(\sqrt{5}+\sqrt{6}\right)^2$ = a + b $\sqrt{30}$, then find the values of a and b.

2. What is the value of a polynomial f (x) = $8x^2 - 3x + 7$ at x = –1?

3. In quadrilateral PQRS, PQ = QR and RS = SP, then what you can say about the quadrilateral?

4. Comment on the graph of the linear equation 3x = 4.

5. Find the Number of classes, if the class size is 15 and maximum and minimum values are 159 and 69 respectively.

6. The sides of the given triangle are 6 cm, 8 cm and 10 cm, then what is the value of semi-perimeter of a triangle?

Section B
(Questions 7 to 12 carry 2 marks each)

7. Evaluate: $\sqrt[3]{(343)^{-2}}$

8. Draw the graph of y – 4x = 8.

9. Find the value of k, if x = 1, y = 1 is a solution of the equation 9kx + 12ky = 63.

10. A right triangle with its sides 3 cm, 4 cm and 5 cm is rotated about its side of 4 cm to form a cone having base radius as 3 cm. Find the volume of the solid so generated. (π = 3.14)

11. How many litres of water flow out through a pipe having 5 cm^2 area of cross section in one minute, if the speed of water in the pipe is 30 cm/sec?

12. Factorise: $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$

Section C
(Questions 13 to 22 carry 3 marks each)

13. Simplify:

$$\frac{(25)^{\frac{3}{2}}\times(343)^{\frac{3}{5}}}{16^{\frac{5}{4}}\times8^{\frac{4}{3}}\times7^{\frac{3}{5}}}$$

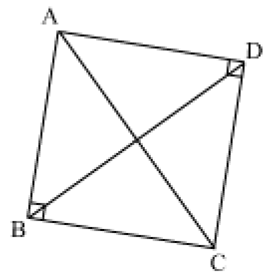
14. Which of the following expressions are polynomial in one variable? State reasons for your answers:

(i) $\frac{(x+1)(x+2)}{x}$ (ii) $t^2(t^2-3)$

(iii) $\frac{1}{2}(x^2+4x+5)$ (iv) $\sqrt{3}x^2+6\sqrt{x}$

(v) $z+\frac{1}{z}$

15. ΔABC and ΔADC are two right triangles with common hypotenuse AC. Prove that ABCD is a cyclic quadrilateral and hence prove that $\angle CAD = \angle CBD$.



16. Factorize: $(x - 3y)^3 + (3y - 7z)^3 + (7z - x)^3$.

17. Draw a line segment of length 8 cm and bisect it.

18. A bag contains 12 balls out of which x are white. If one ball is taken out from the bag, find the probability of getting a white ball. If 6 more white balls are added to the bag and the probability now for getting a white ball is double the previous one, find the value of x.

19. Draw the graph of $2x + 3y = 11$. From graph, find the value of x, if $y = 5$.

20. The polynomials $p(x) = ax^3 + 3x^2 - 3$ and $q(x) = 2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders R_1 and R_2. Find 'a' if $R_1 + R_2 = 0$.

21. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

22. Fifty seeds each were selected at random from 5 bags of seeds, and were kept under standardised conditions favorable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Bags	1	2	3	4	5
Number of germinated seeds	40	48	42	39	41

What is the probability of

i. More than 40 seeds germinating in a bag?
ii. 49 seeds germinating in a bag?
iii. More than 35 seeds germinating in a bag?

Section D
(Questions 23 to 30 carry 4 marks each)

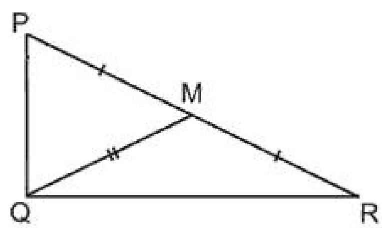
23. Find the value of:

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

24. Construct ΔXYZ in which $m\angle Y = 30°$, $m\angle Z = 90°$ and $XY + YZ + ZX = 11$ cm.

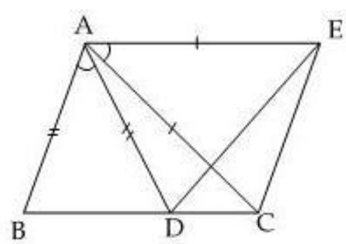
25. Simplify: $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

26. If M is the mid-point of the hypotenuse PR of a right-angled triangle PQR, prove that $QM = \frac{1}{2}PR$



27. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm. The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.

28. In t he figure, AC = AE, AB = AD and $\angle BAD = \angle EAC$ show that BC = DE.



29. How does Euclid's fifth postulate imply the existence of parallel lines? Give a mathematical proof.

30. The polynomials $x^3 + 2x^2 - 5ax - 8$ and $x^3 + ax^2 - 12x - 6$ when divided by $(x - 2)$ and $(x - 3)$ leave remainders p and q, respectively. If $q - p = 10$, then find the value of a.

CBSE Board
Class IX Mathematics
Solution

Time: 3 hrs **Total Marks: 80**

Section A

1.

$\left(\sqrt{5}+\sqrt{6}\right)^2 = 5+6+2\sqrt{30}$ .....$\because (a+b)^2 = a^2 + 2ab + b^2$

$= 11+2\sqrt{30}$

On compairing $a + b\sqrt{30}$ and $11 + 2\sqrt{30}$, we get

a = 11 and b = 2

2. At x = −1,

$f(x) = 8(-1)^2 - 3(-1) + 7 = 8 + 3 + 7 = 18$

$\therefore$ The value of f(x) at x = −1 is 18.

3. We know that a quadrilateral with two separate pairs of equal adjacent sides is called a kite.

$\therefore$ □PQRS is a Kite.

4. The given equation is 3x = 4.

$\therefore x = \frac{4}{3}$

As the Graph of equation x = k (k is any constant) is parallel to the y-axis.

$\therefore$ The graph of the linear equation 3x = 4 is parallel to y-axis.

5. Here, class size = 15, maximum value = 159 and minimum value = 69.

$\therefore$ Range = 159 – 69 = 90

Number of classes = $\frac{\text{range}}{\text{class size}} = \frac{90}{15} = 6$

Therefore, the number of classes is 6.

6. In triangle, a = 6 cm, b = 8 cm, c = 10 cm

$\therefore$ semi-perimeter $= s = \dfrac{a + b + c}{2} = \dfrac{6+8+10}{2} = 12$ cm

Section B

7. $\sqrt[3]{(343)^{-2}} = (343)^{-2/3}$

$= [(7)^3\}^{-2/3}$

$= 7^{3 \times -2/3}$

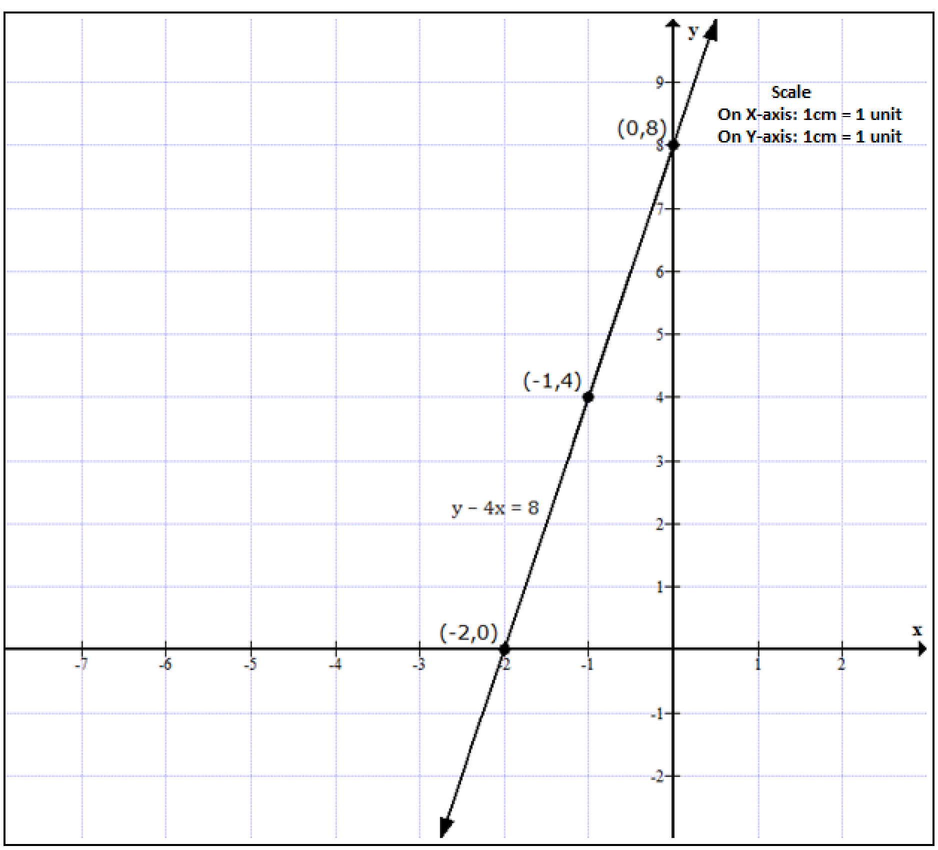
$= 7^{-2}$

$= \dfrac{1}{49}$

8. The given equation is y – 4x = 8 ⇒ y = 8 + 4x

x	0	–1	–2
y	8	4	0

Plot the points (0, 8), (–1, 4) and (–2, 0). Draw a line passing through these points.



9. Since x = 1, y = 1 is the solution of 9kx + 12ky = 1, it will satisfy the equation.

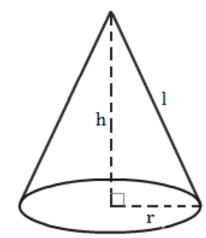
$\therefore$ 9k(1) + 12k(1) = 63

$\therefore$ 9k + 12k = 63

$\therefore$ 21k = 63

$\therefore$ k = 3

10. When rotated about the side of 4 cm.



Given,

r = 3 cm, h = 4 cm, l = 5 cm

$$\text{Volume of solid} = \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times (3)^2 \times 4\right) cm^3 = 37.68\ cm^3$$

11. Area of cross section of pipe = 5 cm^2

Speed of water flowing out of the pipe = 30 cm/sec

Volume of water that flows out in 1 sec = 5 × 30 = 150 cm^3

Volume of water that flows out in 1 minute = 150 × 60 = 9000 cm^3 = 9 litres.

12.

$$x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$$

$$= \left(x^2 + \frac{1}{x^2} + 2\right) - 2\left(x + \frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 2\right)$$

Section C

13. $\dfrac{(25)^{\frac{3}{2}} \times (343)^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$

$$= \frac{(5^2)^{\frac{3}{2}} \times (7^3)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}} \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^5 \times 2^4 \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{9}{5}}}{2^9 \times 7^{\frac{3}{5}}}$$

$$= \frac{5^3 \times 7^{\frac{6}{5}}}{2^9}$$

14.

i. No.

$\dfrac{x^2 + 3x + 2}{x} = x + 3 + 2x^{-1}$ has a negative power of x.

ii. Yes

$t^2(t^2 - 3)$

$t^4 - 3t^2$

iii. Yes

$\dfrac{(x^2 + 4x + 5)}{2} = \dfrac{x^2}{2} + \dfrac{4x}{2} + \dfrac{5}{2} = \dfrac{x^2}{2} + 2x + \dfrac{5}{2}$

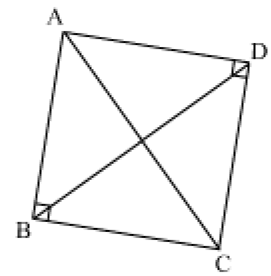
iv. No

$\sqrt{3}x^2 + 6\sqrt{x} = \sqrt{3}x^2 + 6(x)^{1/2}$ has fractional power of x.

v. No

$z + \dfrac{1}{z}$ i.e. $z + z^{-1}$ has a negative power of x.

15. In ΔABC



$m\angle ABC + m\angle BCA + m\angle CAB = 180°$ (Angle sum property of a triangle)

$\Rightarrow 90° + m\angle BCA + m\angle CAB = 180°$

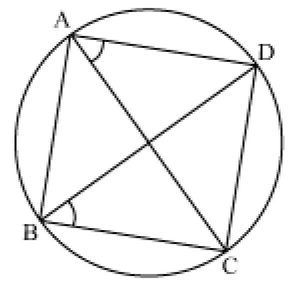
$\Rightarrow m\angle BCA + m\angle CAB = 90°$ ... (1)

In ΔADC

$m\angle CDA + m\angle ACD + m\angle DAC = 180°$ (Angle sum property of a triangle)

$\Rightarrow 90° + m\angle ACD + m\angle DAC = 180°$

$\Rightarrow m\angle ACD + m\angle DAC = 90°$ ... (2)



Adding equations (1) and (2), we have

$m\angle BCA + m\angle CAB + m\angle ACD + m\angle DAC = 180°$

$\Rightarrow (m\angle BCA + m\angle ACD) + (m\angle CAB + m\angle DAC) = 180°$

$m\angle BCD + m\angle DAB = 180°$ ... (3)

But it is given that

$m\angle B + m\angle D = 90° + 90° = 180°$ ... (4)

From equations (3) and (4), we can see that quadrilateral ABCD is having sum of measures of opposite angles as 180°.

So, it is a cyclic quadrilateral.

Consider chord CD.

Now, $\angle CAD = \angle CBD$ (Angles in same segment)

16. We know that if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Here,

$x - 3y + 3y - 7z + 7z - x = 0$

i.e. $a + b + c = 0$

$\therefore\ (x-3y)^3 + (3y-7z)^3 + (7z-x)^3$

$= 3(x-3y)(3y-7z)(7z-x)$

So factors are 3, $(x-3y)$, $(3y-7z)$, $(7z-x)$.

17. Steps of construction:

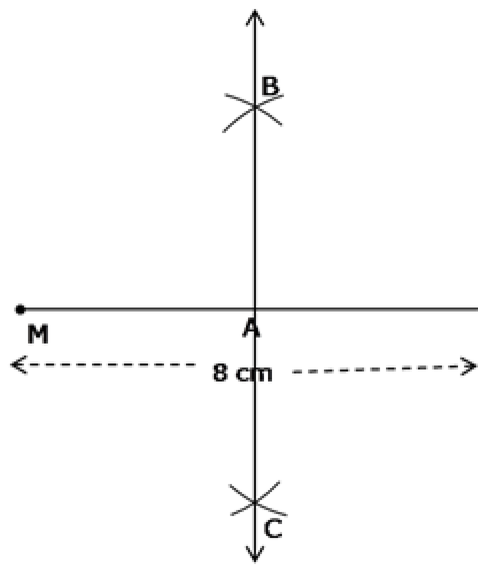
i. Draw a line segment MN = 8 cm

ii. Taking M as the centre and radius more than half the length MN, draw two arcs in the upper and lower portion of MN.

iii. Taking N as the centre and the same radius, draw two arcs which cut the previous arcs at B and C.

iv. Join BC which cuts MN at A.

BC is the required perpendicular bisector of MN.



18. Number of white balls = x

Total no. of balls $= 12$

$P(\text{white ball}) = \frac{x}{12}$

If 6 white balls are added,

Total no. of balls $= 18$

White ball $= x + 6$

$\therefore P(\text{white ball}) = \frac{x+6}{18}$

If 6 more white balls are added to the bag then probability for getting a white ball is doubled

$\therefore \frac{x+6}{18} = \frac{2x}{12}$

$\Rightarrow 6x + 36 = 18x$

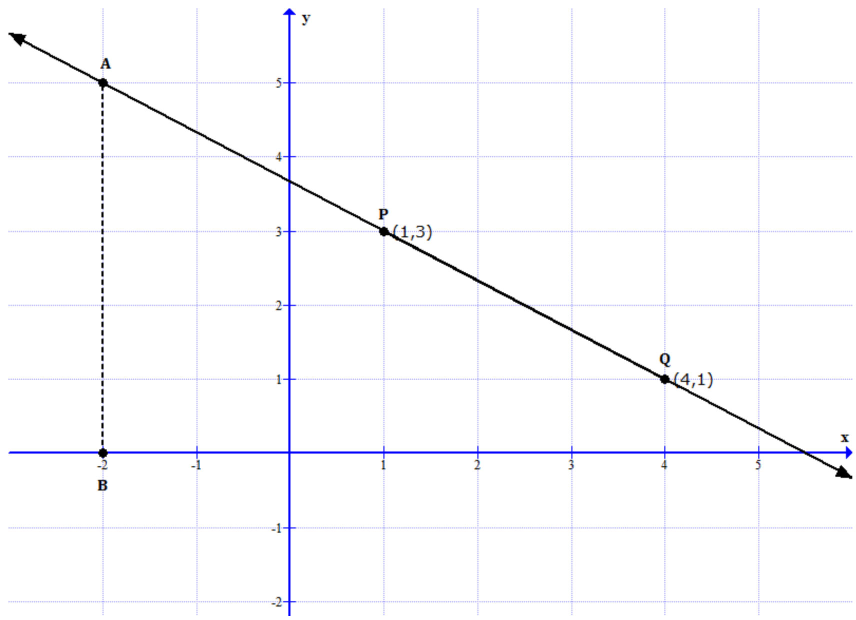
$\Rightarrow x = 3$

19. The given equation is 2x + 3y = 11.

$\Rightarrow y = \dfrac{11-2x}{3}$

x	1	4
y	3	1

Plot points (1, 3), (4, 1). Draw line passing through the points.



From the graph we can see that if y = 5 then the value of x is −2.

20. When $p(x) = ax^3 + 3x^2 - 3$ is divided by (x – 4), the remainder is given by

$R_1 = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

When $q(x) = 2x^3 - 5x + a$ is divided by (x – 4), the remainder is given by

$R_2 = 2(4)^3 - 5(4) + a = 108 + a$

Given: $R_1 + R_2 = 0$

$\Rightarrow 65a + 153 = 0 \Rightarrow a = \dfrac{-153}{65}$

By hit and trial we find x = 3 is factor of given polynomial, as

$2(3)^3 - 9 - 39 - 6 = 54 - 54 = 0$

By dividing $2x^3 - x^2 - 13x - 6$ by x – 3 we get $(2x^2 + 5x + 2)$ as quotient.

Factorising this further

$2x^2 + 5x + 2 = 2x^2 + 4x + x + 2$

$= 2x(x + 2) + 1(x + 2)$

$= (2x + 1)(x + 2)$

So, $2x^3 - x^2 - 13x - 6 = (2x + 1)(x + 2)(x - 3)$

21. The given tank is cuboidal in shape having its length (l) as 20 m, breadth (b) as 15 m and height (h) as 6m.

Capacity of tank = l × b × h

⇒ Capacity of tank = (20 × 15 × 6) m^3 = 1800 m^3

⇒ Capacity of tank = 1800000 litres

Water consumed by people of the village in 1 day = 4000 × 150 litres = 600000 litres

Let us assume that the water in the tank lasts for n days.

Water consumed by all people of the village in n days = capacity of tank

⇒ n × 600000 = 1800000

⇒ n = 3

Water in this tank will last for 3 days.

22. Total number of bags is 5.

i. Number of bags in which more than 40 seeds germinated out of 50 seeds is 3.

P(germination of more than 40 seeds in a bag) = $\frac{3}{5}$ = 0.6

ii. Number of bags in which 49 seeds germinated = 0

P(germination of 49 seeds in a bag) = $\frac{0}{5}$ = 0

iii. Number of bags in which more than 35 seeds germinated = 5.

The required probability = $\frac{5}{5} = 1$

Section D

23.

$$\frac{1}{3-\sqrt{8}}=\frac{1}{3-\sqrt{8}}\times\frac{3+\sqrt{8}}{3+\sqrt{8}}=\frac{3+\sqrt{8}}{9-8}=3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}}=\frac{1}{\sqrt{8}-\sqrt{7}}\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}=\frac{\sqrt{8}+\sqrt{7}}{8-7}=\sqrt{8}+\sqrt{7}$$

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$$

$$\frac{1}{\sqrt{6}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}+\sqrt{5}}{6-5}=\sqrt{6}-\sqrt{5}$$

$$\frac{1}{\sqrt{5}-2}=\frac{1}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}=\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$$

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

$$=3+\sqrt{8}-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}-\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$=5$$

24. The steps of construction for the required triangle are as follows:

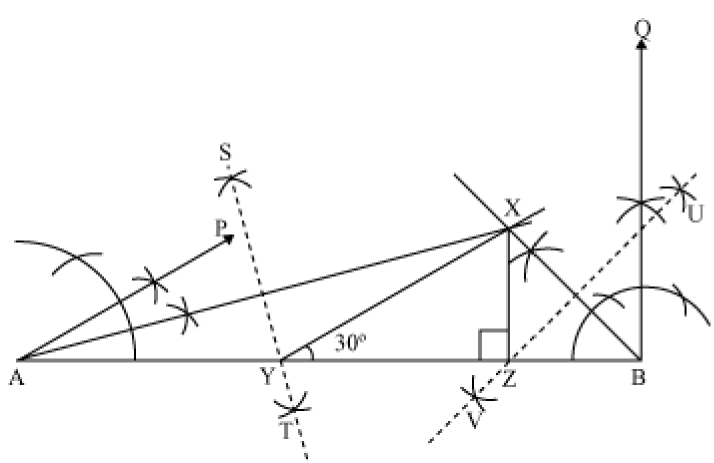
Step I: Draw a line segment AB of 11 cm (As XY + YZ + ZX = 11 cm).

Step II: Construct ∠PAB of 30° at point A and an angle ∠QBA of 90° at point B.

Step III: Bisect ∠PAB and ∠QBA. Let these bisectors intersect each other at point X.

Step IV: Draw perpendicular bisector ST of AX and UV of BX.

Step V: Let ST intersects AB at Y and UV intersects AB at Z. Join XY and XZ. ΔXYZ is the required triangle.



25. Consider $\frac{\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

We know that,

If x + y + x = 0 then $x^3 + y^3 + z^3 = 3xyz$

Now, $a^2 - b^2 + b^2 - c^2 - a^2 = 0$

And, a – b + b – c + c – a = 0

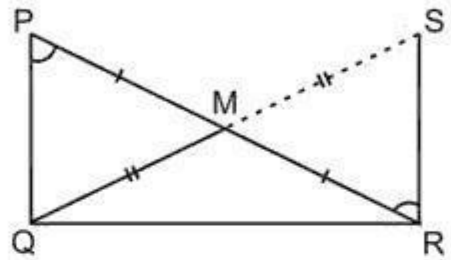
$$\therefore \frac{\left(a^2-b^2\right)^3+\left(b^2-c^2\right)^3+\left(c^2-a^2\right)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$=\frac{3\left(a^2-b^2\right)\left(b^2-c^2\right)\left(c^2-a^2\right)}{3(a-b)(b-c)(c-a)}$$

$$=\frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$$

$$=(a+b)(b+c)(c+a)$$

26.



Produce QM to S such that QM = MS. Join SR

In ΔPMQ and ΔRMS

PM = MR (M is the mid point)

QM = MS (By construction)

$\angle$PMQ = $\angle$RMS...vertically opposite angles

$\therefore$ ΔPMQ $\cong$ ΔRMS (SAS congruence criterion)

$\therefore$PQ = SR and $\angle$QPM = $\angle$SRM (c.p.c.t)

$\angle$QPM = $\angle$SRM (alternate angles) $\therefore$RS $\parallel$ PQ

$\angle$PQR + $\angle$QRS = 180° (Co–interior angles)

$\Rightarrow$ 90° + $\angle$QRS = 180°

$\Rightarrow$ $\angle$QRS = 90°

In ΔPQR and ΔQRS

QR = RQ (common)

PQ = SR

$\angle$PQR = $\angle$QRS (90° each)

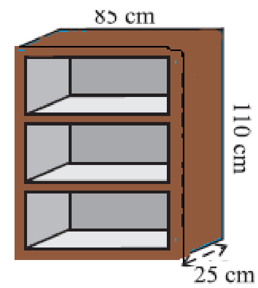
$\therefore$ ΔPQR $\cong$ ΔSRQ (SAS congruence criterion)

$\therefore$PR = QS $\Rightarrow \frac{1}{2}$SQ = $\frac{1}{2}$PR

$\Rightarrow \frac{1}{2}$SQ = QM = $\frac{1}{2}$PR

Hence, proved

27. Given,



External length (l) of bookshelf = 85 cm.

External breadth (b) of bookshelf = 25 cm.

External height (h) of bookshelf = 110 cm.

External surface area of shelf excluding its front face

$= lh + 2(lb + bh)$

External surface area of shelf excluding its front face

$= [85 \times 110 + 2(85 \times 25 + 25 \times 110)]$ cm^2

External surface area of shelf excluding its front face = 19100 cm^2

Area of front face = $[85 \times 110 - 75 \times 100 + 2(75 \times 5)]$ cm^2

$= 1850 + 750$ cm^2 $= 2600$ cm^2

Area to be polished = $(19100 + 2600)$ cm^2 = 21700 cm^2

Cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area = Rs. (21700×0.20) = Rs. 4340

Now, length (l), breadth (b) height (h) of each row of bookshelf is 75 cm, 20 cm, and 30 cm respectively.

Area to be painted in 1 row = $2(l + h)b + lh$

$= [2(75 + 30) \times 20 + 75 \times 30]$ cm^2

$= (4200 + 2250)$ cm^2

$= 6450$ cm^2

Area to be painted in 3 rows = (3×6450) cm^2 = 19350 cm^2

Cost of painting 1 cm^2 area = Rs. 0.10

Cost of painting 19350 cm^2 area = Rs. (19350×0.10) = Rs. 1935

Total expenses required for polishing and painting the surface of the bookshelf

= Rs. $(4340 + 1935)$ = Rs. 6275

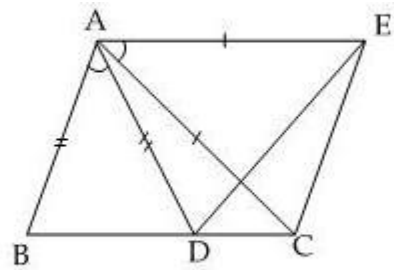
28. Given: AC = AE, AB = AD and ∠BAD = ∠EAC

To prove: BC = DE

Proof: ∠BAD = ∠EAC (given)

⇒∠BAD + ∠DAC = ∠EAC + ∠DAC

⇒∠BAC = ∠DAE



Now in ΔABC and ΔADE

AB = AD

∠BAC = ∠DAE

AC = AE

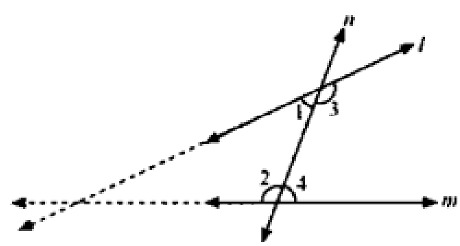
Thus, ΔABC ≅ ΔADE

⇒ BC = DE

29. Euclid's 5th postulate states that:

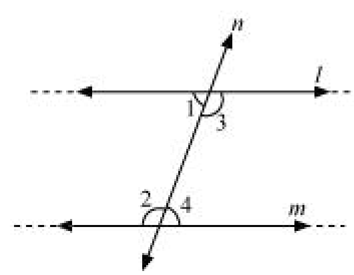
If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

This implies that if n intersects lines l and m and if $\angle 1 + \angle 2 < 180^0$, then $\angle 3 + \angle 4 > 180^0$. In that case, producing line l and further will meet in the side of ∠1 and ∠2 which is less than 180^o.



If $\angle 1 + \angle 2 = 180^0$, then $\angle 3 + \angle 4 = 180^0$

In that case, the lines l and m neither meet at the side of ∠1 and ∠2 nor at the side of ∠3 and ∠4 implying that the lines l and m will never intersect each other. Therefore, the lines are parallel.



30. Let $f(x) = x^3 + 2x^2 - 5ax - 8$ and $g(x) = x^3 + ax^2 - 12x - 6$

When divided by $(x - 2)$ and $(x - 3)$, $f(x)$ and $g(x)$ leave remainder p and q respectively

$f(x) = x^3 + 2x^2 - 5ax - 8$

$\therefore\ f(2) = 2^3 + 2 \times 2^2 - 5a \times 2 - 8$

$= 8 + 8 - 10a - 8$

$\therefore\ p = 8 - 10a \quad \text{....(1)}$

$g(x) = x^3 + ax^2 - 12x - 6$

$\therefore\ g(3) = 3^3 + a \times 3^2 - 12 \times 3 - 6$

$= 27 + 9a - 36 - 6$

$\therefore\ q = -15 + 9a \quad \text{....(2)}$

If $q - p = 10$

$\Rightarrow -15 + 9a - 8 + 10a = 10$

$\Rightarrow 19a - 23 = 10$

$\Rightarrow 19a = 33$

$\therefore a = \frac{33}{19}$