Chapter 5 Quadratic Functions

Ex 5.8

Answer 1e.

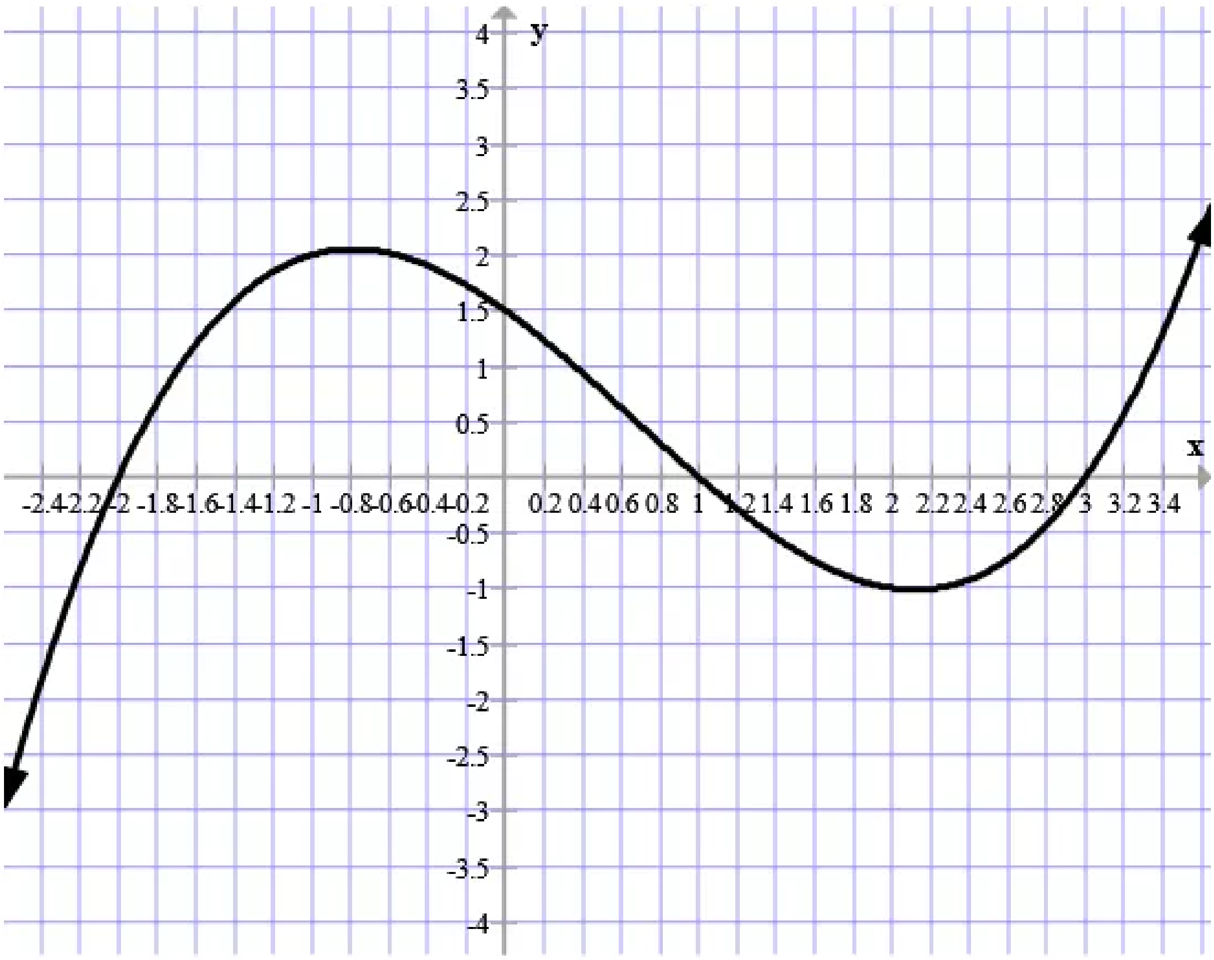
One of the important characteristics of the graphs of polynomial functions is that they have turning points. These points can be higher or lower than all near by points.

When the turning point is higher than all other near by points, the y-coordinate of that point is the local maximum. On the other hand, if the turning point is lower than all other near by points, then its y-coordinate is the local minimum.

Therefore, the statement can be completed as "A local maximum or local minimum of a polynomial function occurs at a **turning** point of the function's graph.

Answer 1gp.

The graph of the function $f(x)=0.25(x+2)(x-1)(x-3)$ is shown below:



From the graph we can see that the function f have 3 x-intercepts. It can be noticed that the graph has two turning points.

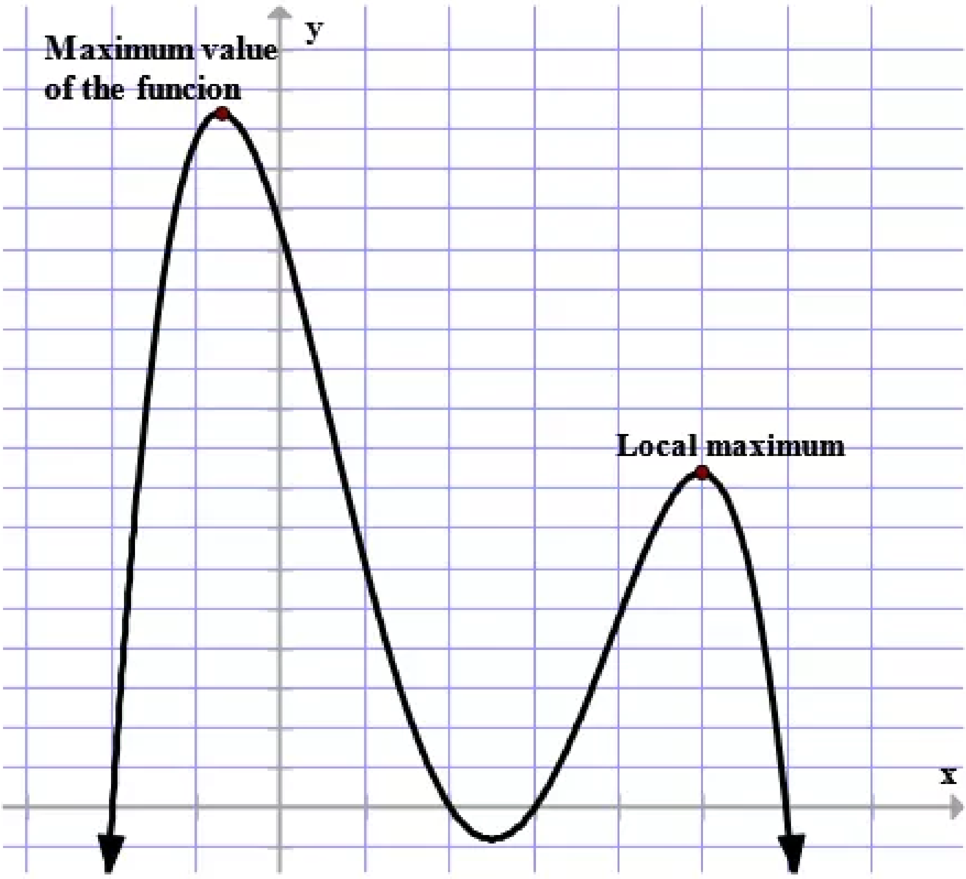
The x-intercepts of the function $f(x)=0.25(x+2)(x-1)(x-3)$ are $\boxed{-2,1 \text{ and } 3}$.

Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(2.2,-1)}$ and has a local maximum at $\boxed{(-0.8,2)}$.

Answer 2e.

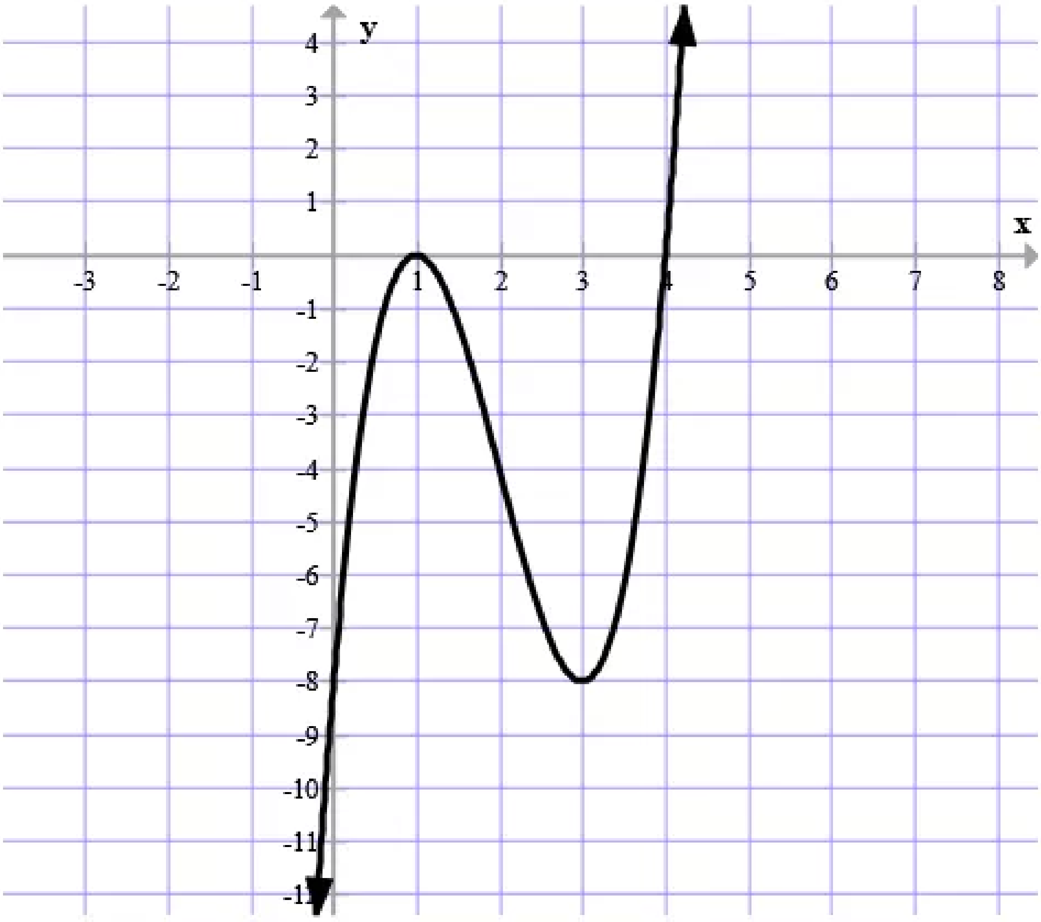
A local maximum of a function is the turning point which is higher than all nearby points. And the maximum value of a function is the point which is greater than all the points.

A graph of local maximum and the height value of a function is shown below:



Answer 2gp.

The graph of the function $g(x)=2(x-1)^2(x-4)$ is shown below:



From the graph we can see that the function f have 2 x-intercepts. It can be noticed that the graph has two turning points.

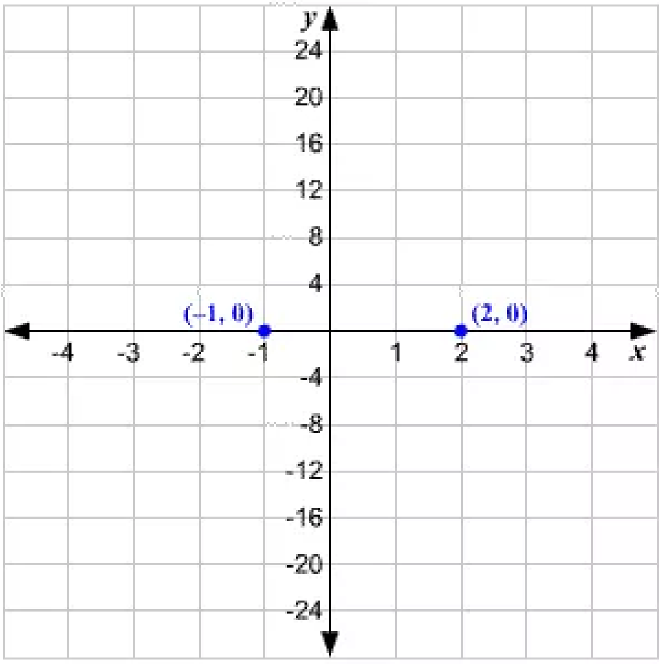
The x-intercepts of the function $g(x)=2(x-1)^2(x-4)$ are $\boxed{1 \text{ and } 4}$.

Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(3,-8)}$ and has a local maximum at $\boxed{(1,0)}$.

Answer 3e.

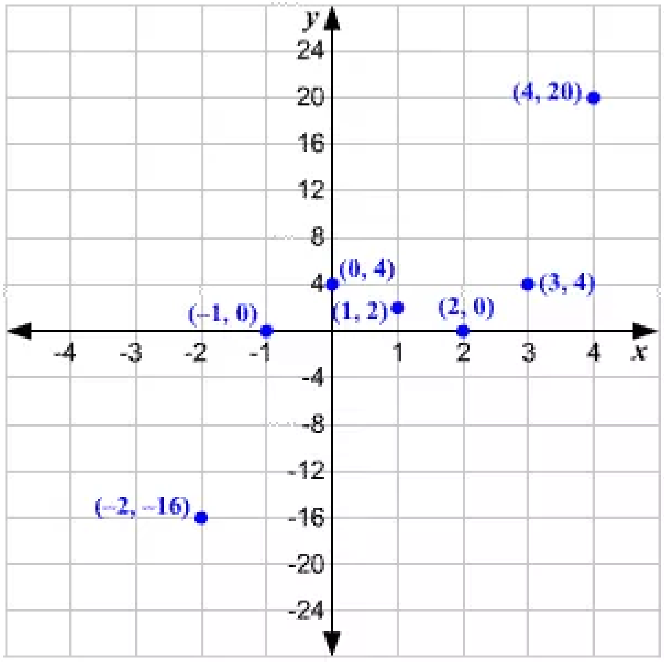
STEP 1 **Plot** the intercepts.

From the given function, we get the zeros, which are the x-intercepts of the graph of the function, as 2 and -1. Plot the points (2, 0) and $(-1, 0)$.



STEP 2 **Plot** points between and beyond the x-intercepts.

x	-2	0	1	3	4
y	-16	4	2	4	20

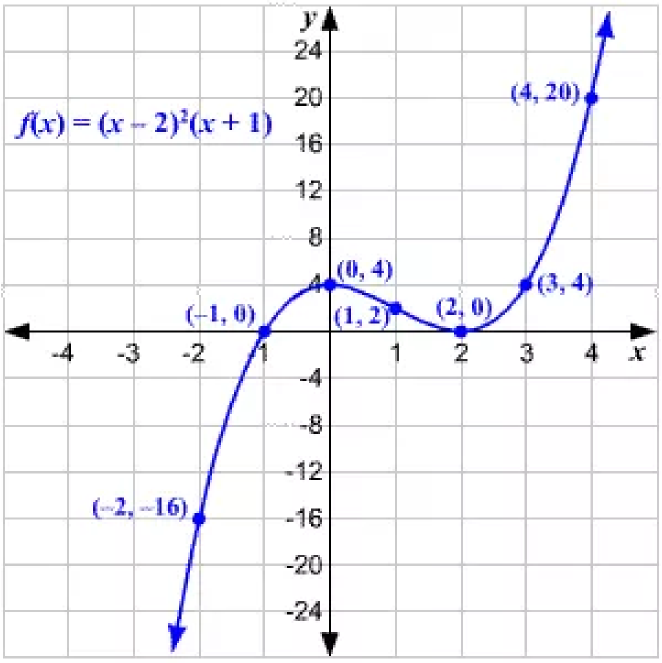


STEP 3 **Determine** end behavior.

Since $f(x)$ has three factors of the form $x - k$, the function is cubic. The leading coefficient is 1, which is positive.

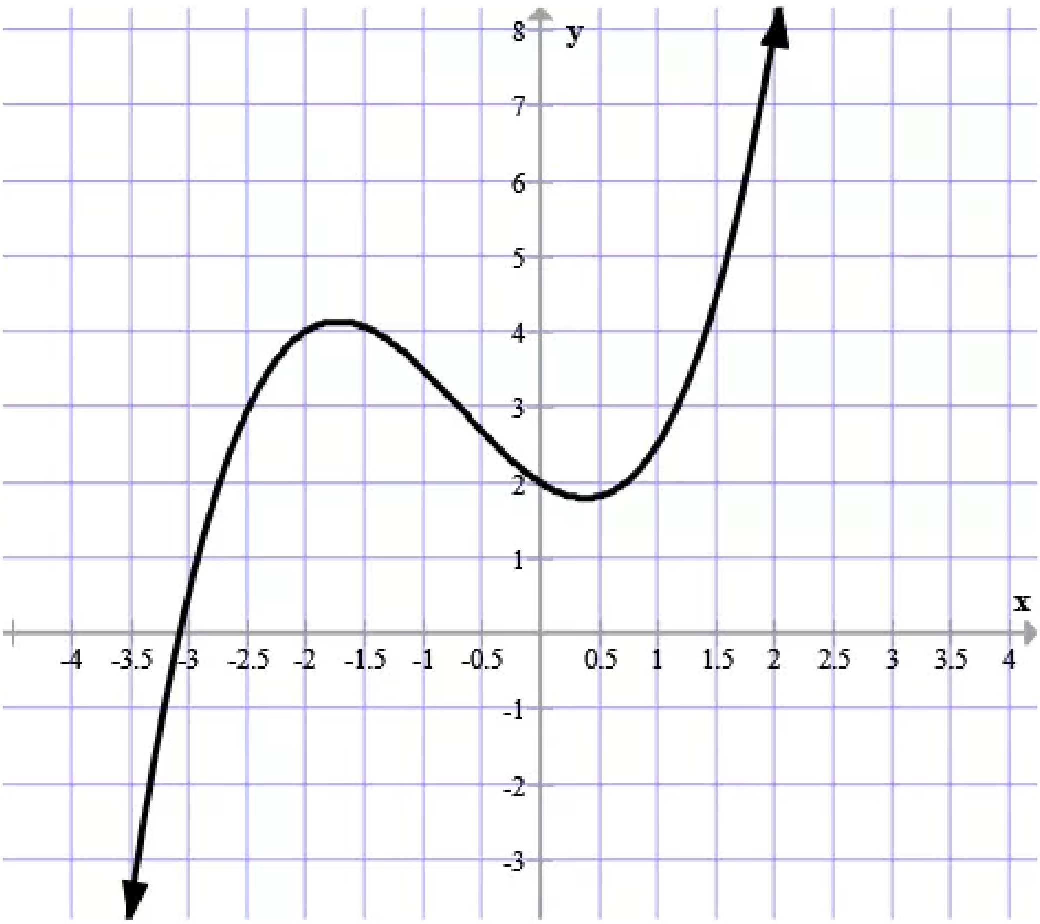
For a function with odd degree and positive leading coefficient, the end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.



Answer 3gp.

The graph of the function $h(x)=0.5x^3+x^2-x+2$ is shown below:



From the graph we can see that the function *f* has 1 *x*-intercept. It can be noticed that the graph has two turning points.
The *x*-intercept of the function $h(x)=0.5x^3+x^2-x+2$ is approximately $\boxed{-3.1}$.

Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(0.4,\ 1.8)}$ and has a local maximum at $\boxed{(-1.75,\ 4.1)}$.

Answer 4e.

Considering polynomial function is $f(x)=(x+1)^2(x-1)(x-3)$. We need to draw the graph of the given function.

Here, -1, 1 and 3 are the zeros of function *f*. Therefore we plot the *x*-intercepts $(-1,0)$, $(1,0)$ and $(3,0)$ in the graph.

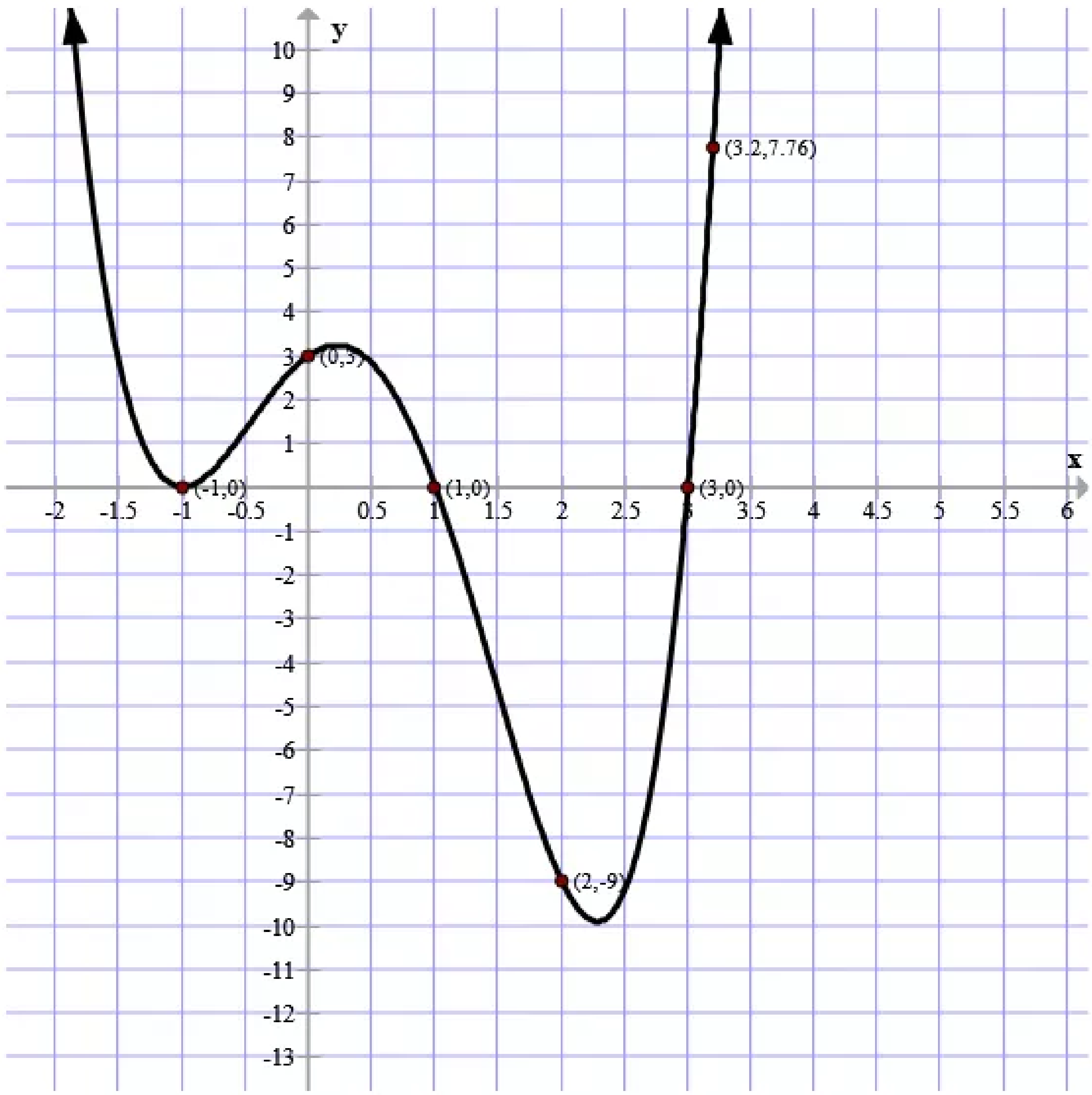
The table of points between and beyond the x-intercepts are shown below:

x	-1	0	1	2	3	3.2
y	0	3	0	-9	0	7.76

These points are plotted in the graph.

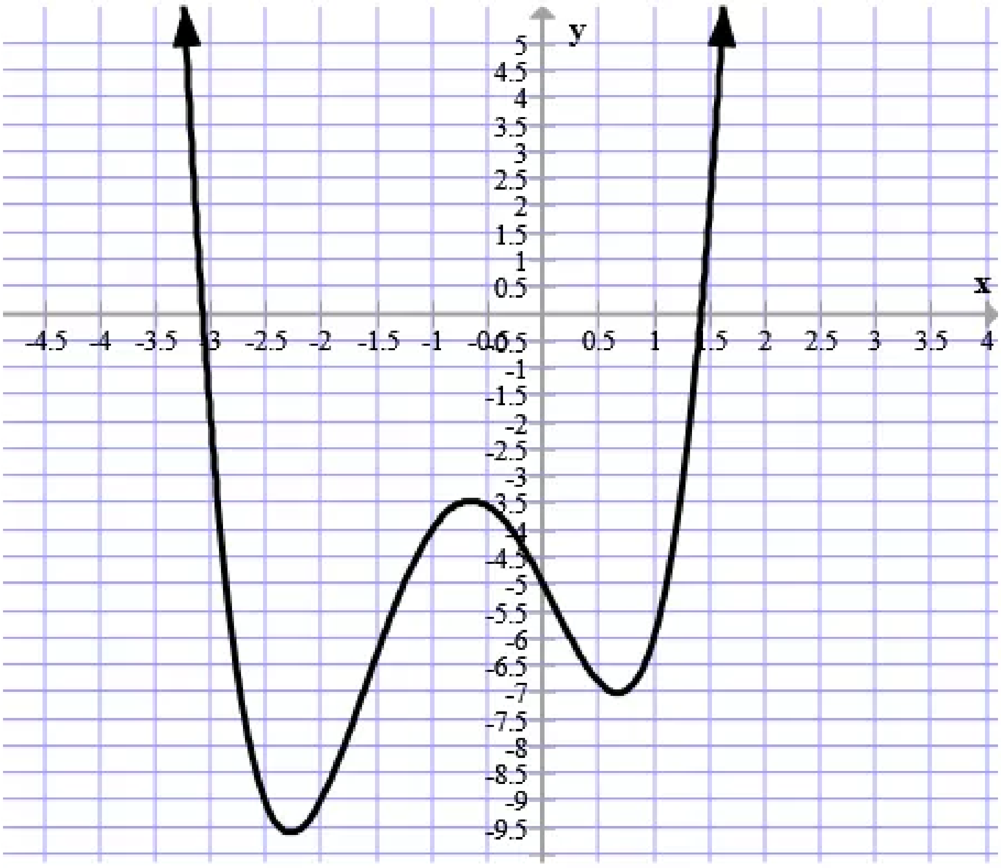
To determine the end behavior, we consider the function and it is seen that the function f has four factor of the form $x-k$ is a quadric function, and is a positive leading co-efficient. So $f(x)\to+\infty$ as $x\to-\infty$ and $f(x)\to+\infty$ as $x\to+\infty$

The graph of the function $f(x)=(x+1)^2(x-1)(x-3)$ which passes through the plotting points is shown below:



Answer 4gp.

The graph of the function $f(x)=x^4+3x^3-x^2-4x-5$ is shown below:



From the graph we can see that the function f has 2 x-intercepts. It can be notice that the graph has three turning points.

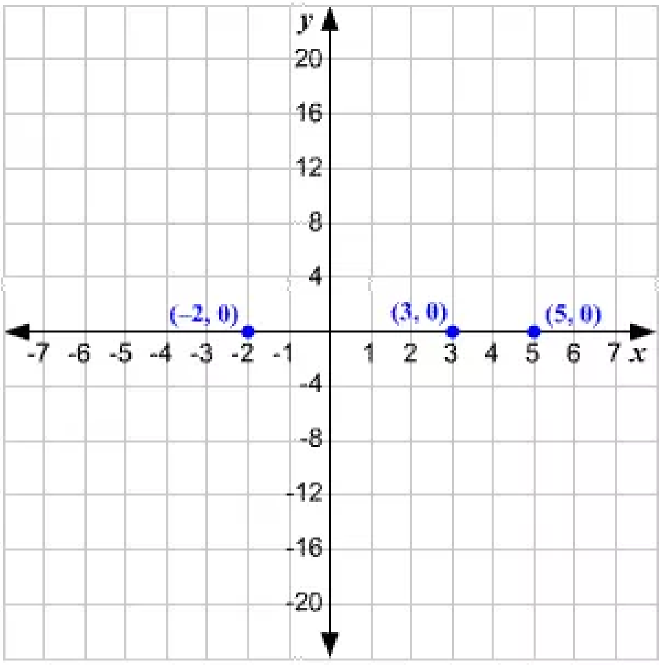
The x-intercept of the function $f(x)=x^4+3x^3-x^2-4x-5$ is approximately $x \approx \boxed{-3.1}$ and $x \approx \boxed{1.4}$

Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(-2.3, -9.5)}$, $\boxed{(-0.65, -7)}$ and has a local maximum at $\boxed{(-0.65, -3.5)}$.

Answer 5e.

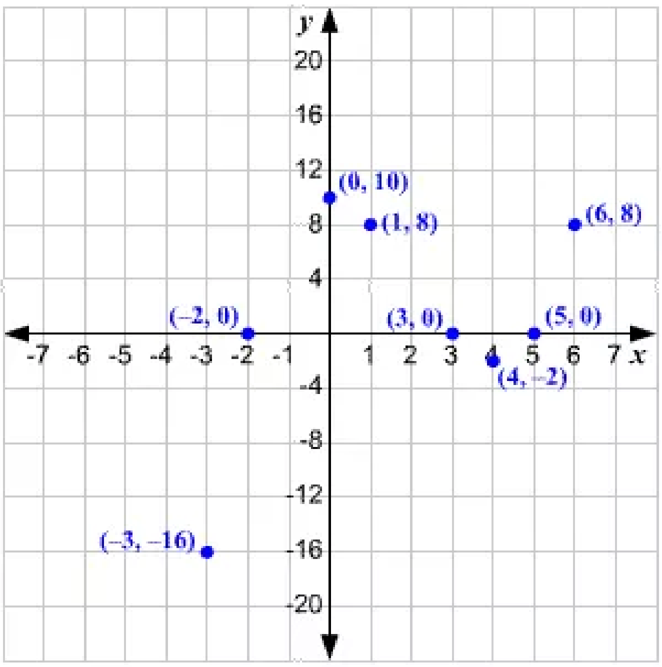
STEP 1 **Plot** the intercepts.

From the given function, we get the zeros, which are the x-intercepts of the graph of the function, as 5, −2, and 3. Plot the points (5, 0), (−2, 0), and (3, 0).



STEP 2 **Plot** points between and beyond the x-intercepts.

x	−3	0	1	4	6
y	−16	10	8	−2	8

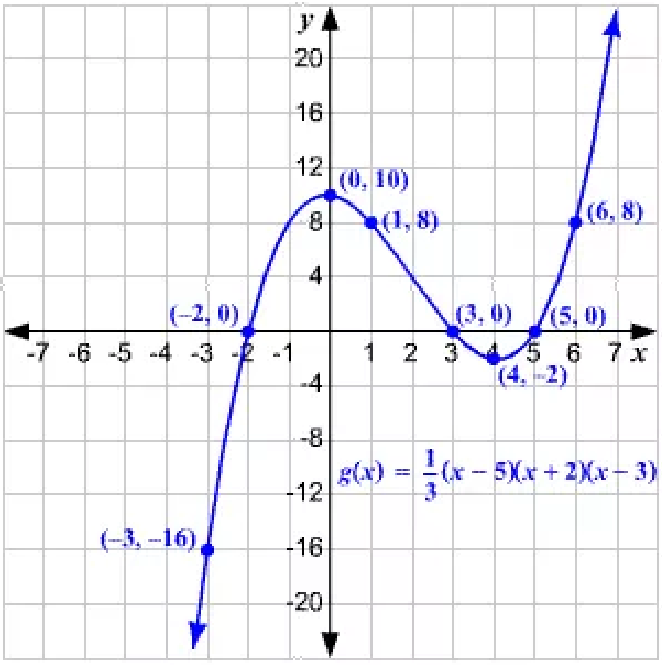


STEP 3 **Determine** end behavior.

Since $g(x)$ has three factors of the form $x-k$, the function is cubic. The leading coefficient is $\frac{1}{3}$, which is positive.

For a function with odd degree and positive leading coefficient, the end behavior is $g(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $g(x) \rightarrow +\infty$ as $x \rightarrow +\infty$.

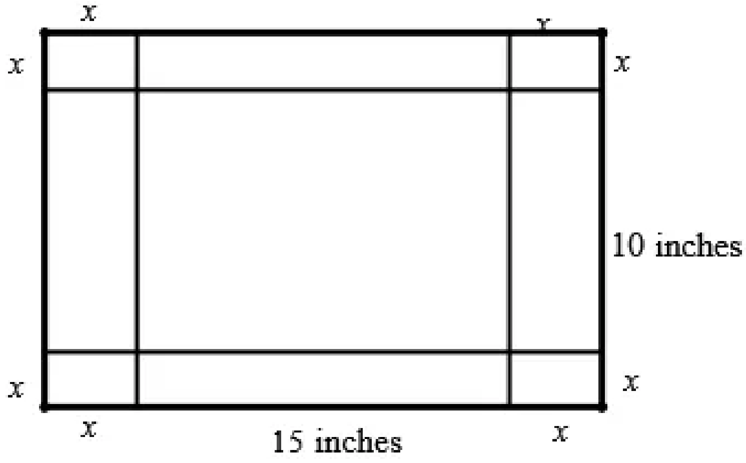
STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.



Answer 5gp.

To make a box of cardboard that is 10 inches by 15 inches we need to find the dimension of the box with maximum volume and the maximum volume of the box.

A verbal model for the volume is shown below:



We write the function from this figure as,

$$\text{Volume} = \text{Length} \cdot \text{width} \cdot \text{Height}$$

Suppose the volume is V and height x. Therefore,

$$\text{Volume} = \text{Length} \cdot \text{width} \cdot \text{Height}$$

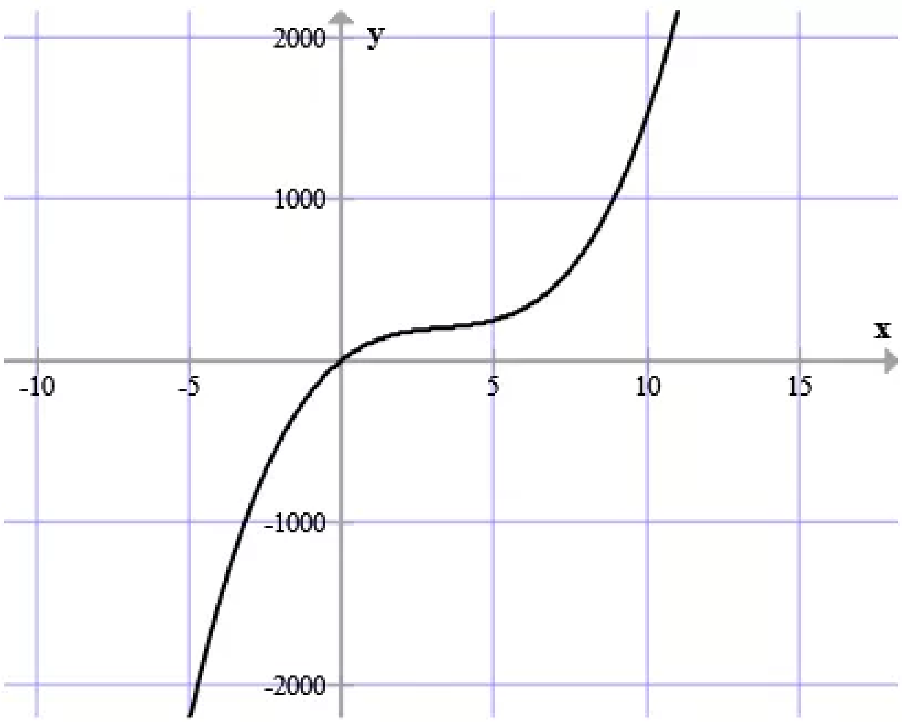
$$V = (15-2x)\cdot(10-2x)\cdot x$$

$$= (150-40x+4x^2)x \qquad \text{[Multiplying binomial]}$$

$$= 4x^3 - 40x^2 + 150x \qquad \text{[Standard form]}$$

The maximum volume can be found if we draw the graph of the function

$V = 4x^3 - 40x^2 + 150x$ as follows:



From the graph, we can see that we cannot determined the maximum volume .

Answer 6e.

Considering polynomial function is $h(x)=\frac{1}{12}(x+4)(x+8)(x-1)$, We need to draw the graph of the given function.

Here, -4, -8 and 1 are the zeros of function f. Therefore we plot the x-intercepts $(-4,0)$, $(-8,0)$ and $(1,0)$ in the graph.

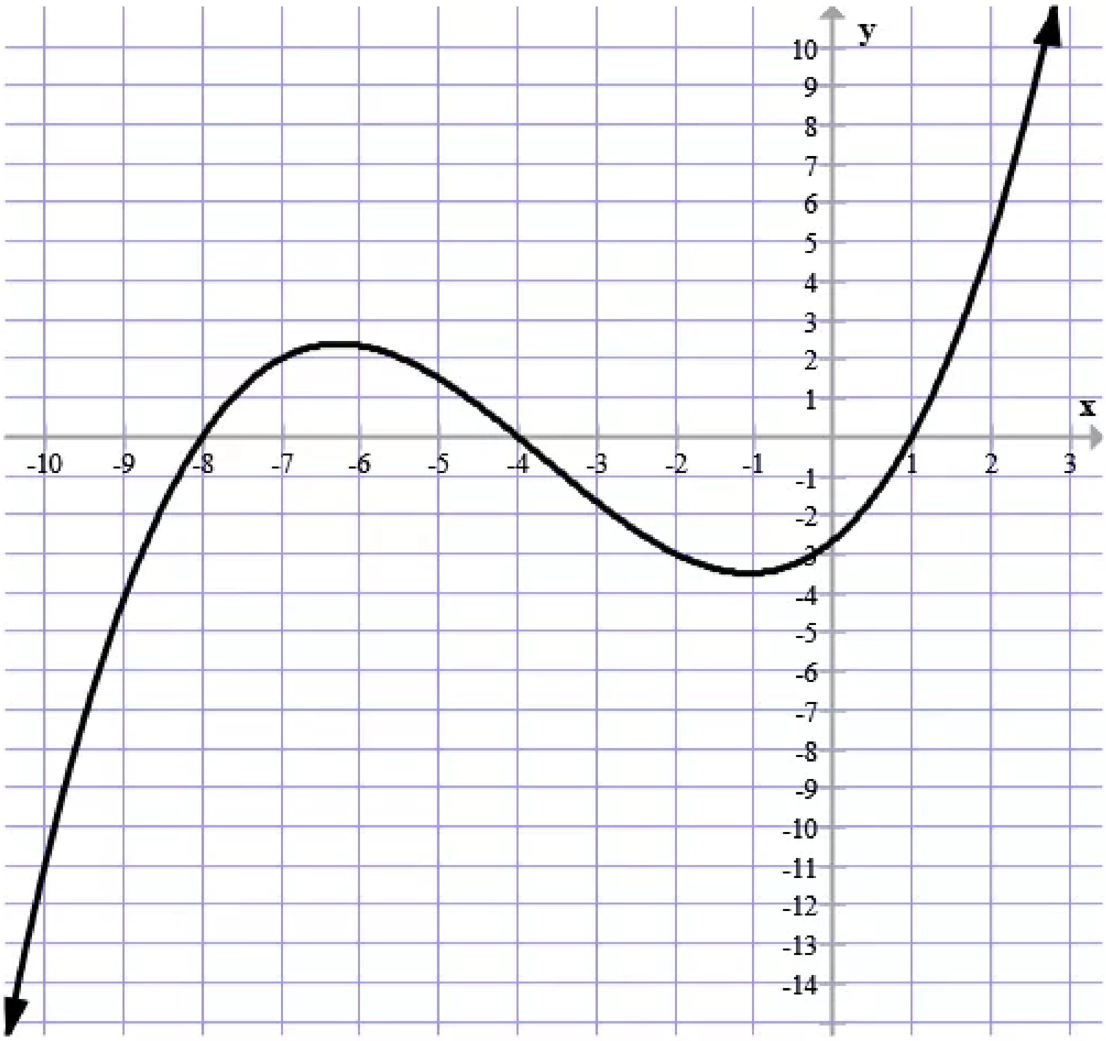
The table of points between and beyond the x-intercepts are shown below:

x	-8	-7	-4	-1	1	2
y	0	2	0	3.5	0	5

These points are plotted in the graph.

To determine the end behavior , we consider the function and it is seen that the function f has three factor of the form $x-k$ is a cubic function, and is a positive leading co-efficient $\frac{1}{12}$. So $f(x)\to-\infty$ as $x\to-\infty$ and $f(x)\to+\infty$ as $x\to+\infty$

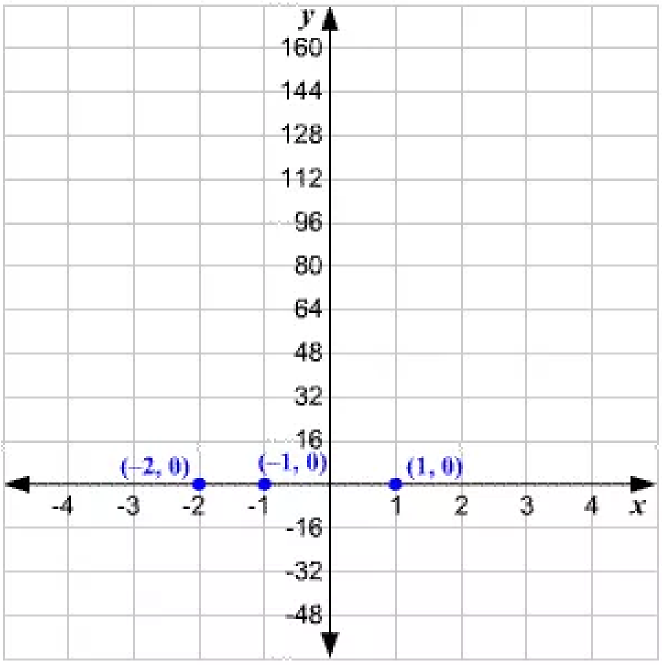
The graph of the function $h(x)=\frac{1}{12}(x+4)(x+8)(x-1)$ which passes through the plotting points is shown below:



Answer 7e.

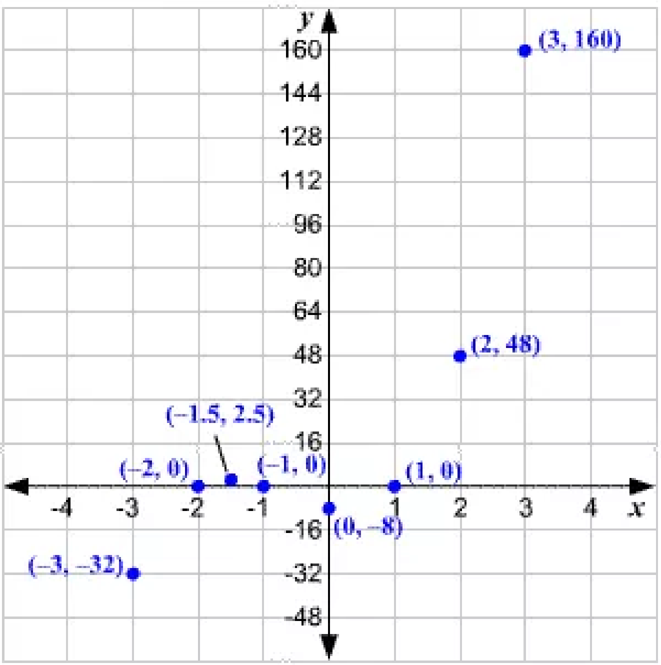
STEP 1 **Plot** the intercepts.

From the given function, we get the zeros, which are the x-intercepts of the graph of the function, as -1, -2, and 1. Plot the points $(-2, 0)$, $(-1, 0)$, and $(1, 0)$.



STEP 2 **Plot** points between and beyond the x-intercepts.

x	-3	-1.5	0	2	3
y	-32	2.5	-8	48	160

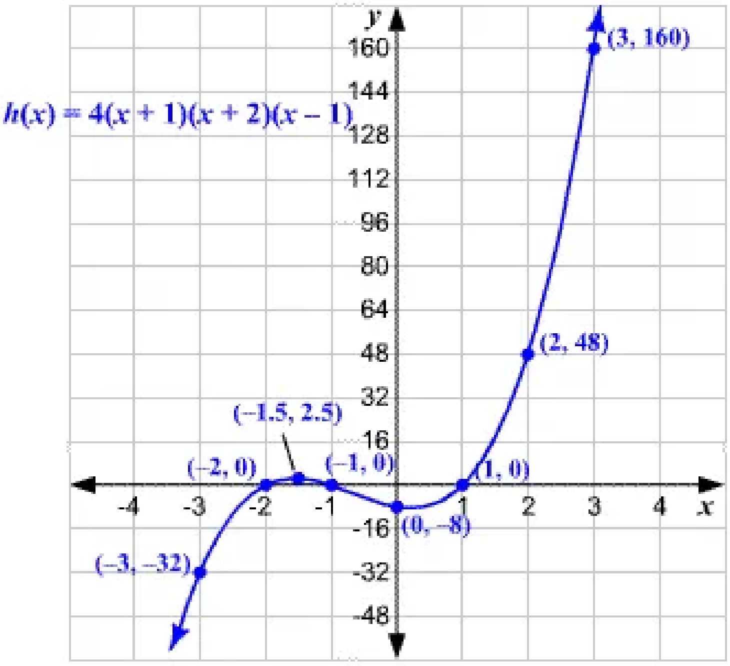


STEP 3 **Determine** end behavior.

Since $h(x)$ has three factors of the form $x-k$, the function is cubic. The leading coefficient is 4, which is positive.

For a function with odd degree and positive leading coefficient, the end behavior is $h(x) \to -\infty$ as $x \to -\infty$ and $h(x) \to +\infty$ as $x \to +\infty$.

STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.



Answer 8e.

Considering polynomial function is $f(x)=0.2(x-4)^2(x+1)^2$, We need to draw the graph of the given function.

Here, 4 and -1 are the zeros of function f. Therefore we plot the x-intercepts $(4,0)$ and $(-1,0)$ in the graph.

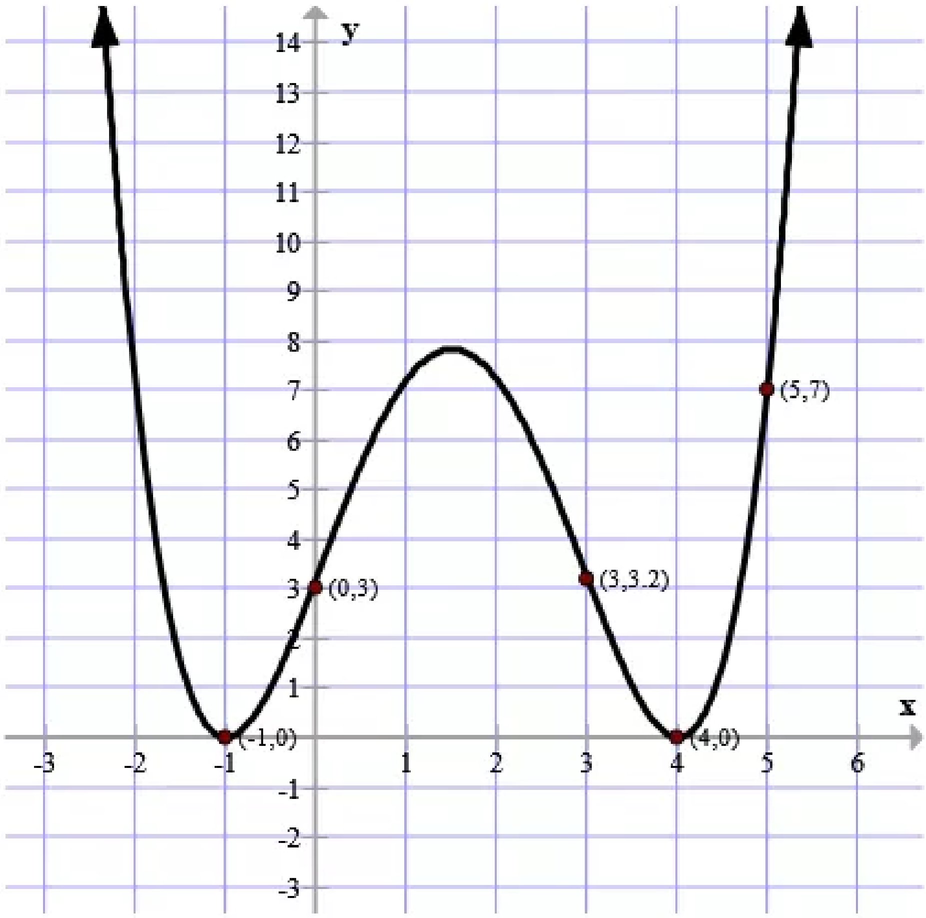
The table of points between and beyond the x-intercepts are shown below:

x	-1	0	3	4	5
y	0	3	3.2	0	7

These points are plotted in the graph.

To determine the end behavior , we consider the function and it is seen that the function f has four factor of the form $x-k$ is a quadric function, and is a positive leading co-efficient 0.2 . So $f(x)\to+\infty$ as $x\to-\infty$ and $f(x)\to+\infty$ as $x\to+\infty$

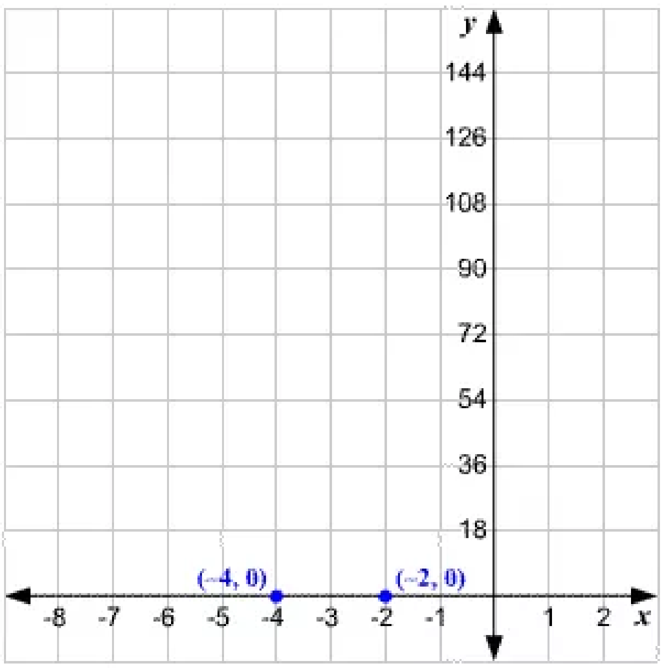
The graph of the function $f(x) = 0.2(x-4)^2(x+1)^2$ which passes through the plotting points is shown below:



Answer 9e.

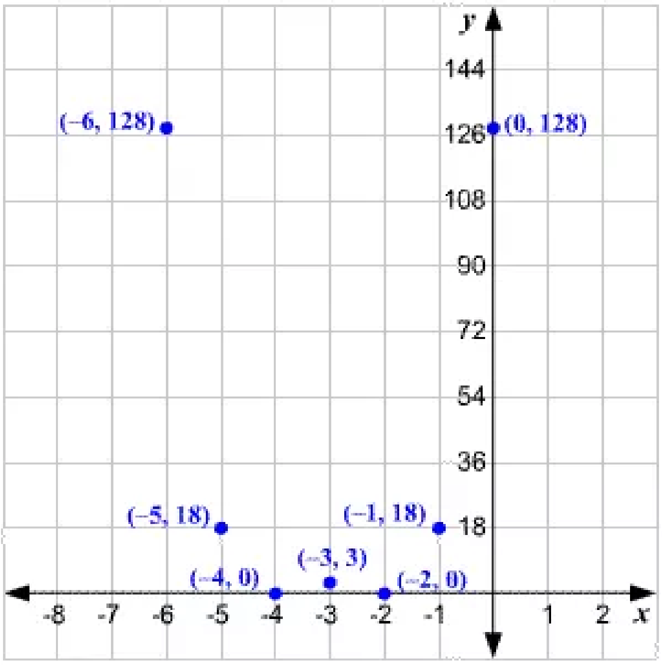
STEP 1 **Plot** the intercepts.

From the given function, we get the zeros, which are the x-intercepts of the graph of the function, as –2 and –4. Plot the points (–4, 0) and (–2, 0).



STEP 2 **Plot** points between and beyond the x-intercepts.

x	−6	−5	−3	−1	0
y	128	18	3	18	128

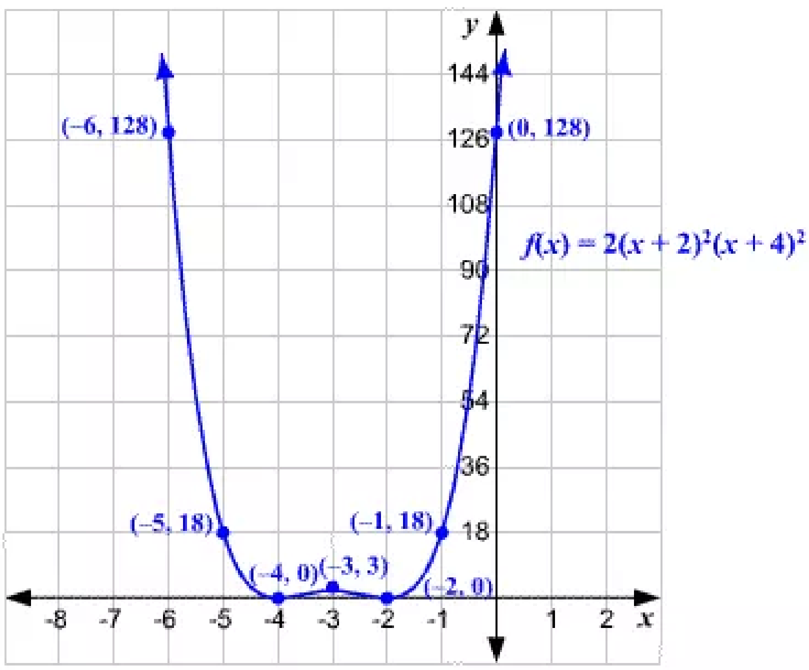


STEP 3 **Determine** end behavior.

Since $f(x)$ has four factors of the form $x - k$, the function is quadratic. The leading coefficient is 2, which is positive.

For a function with even degree and positive leading coefficient, the end behavior is $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.



Answer 10e.

Considering polynomial function is $h(x)=5(x-1)(x-2)(x-3)$. We need to draw the graph of the given function.

Here, 1, 2 and 3 are the zeros of function *f*. Therefore we plot the *x*-intercepts $(1,0)$, $(2,0)$ and $(3,0)$ in the graph.

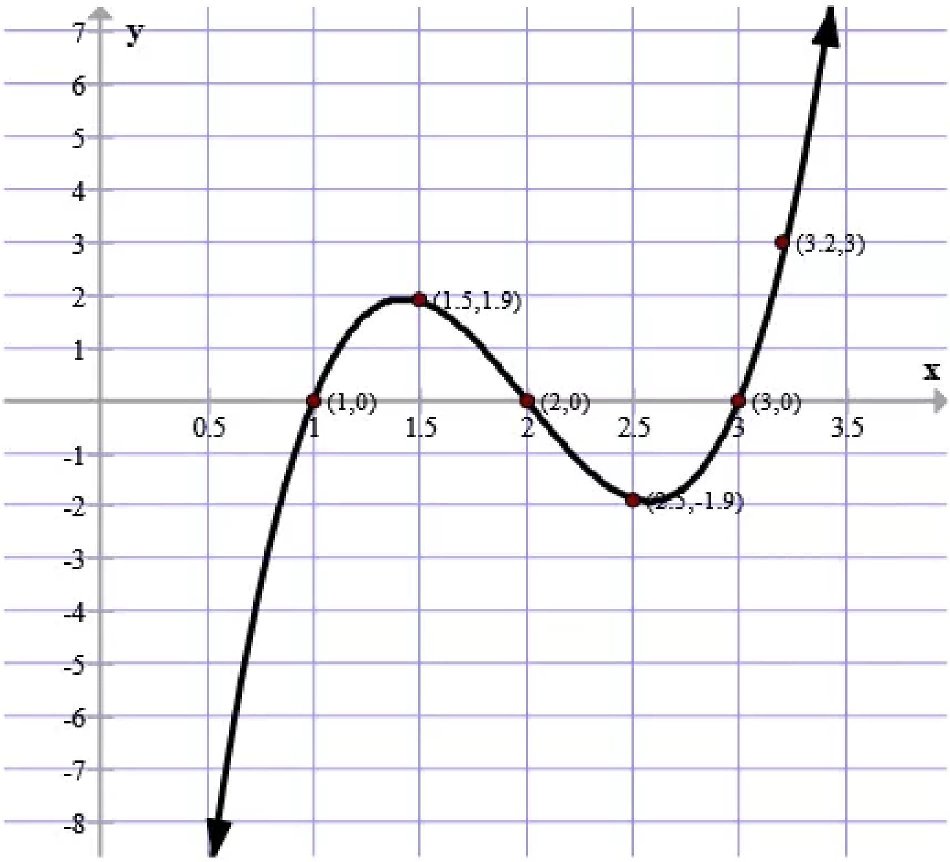
The table of points between and beyond the *x*-intercepts are shown below:

x	1	1.5	2	2.5	3	3.2
y	0	1.9	0	-1.9	0	3

These points are plotted in the graph.

To determine the end behavior , we consider the function and it is seen that the function *f* has four factor of the form $x-k$ is a cubic function, and is a positive leading co-efficient 5. So $f(x)\to-\infty$ as $x\to-\infty$ and $f(x)\to+\infty$ as $x\to+\infty$

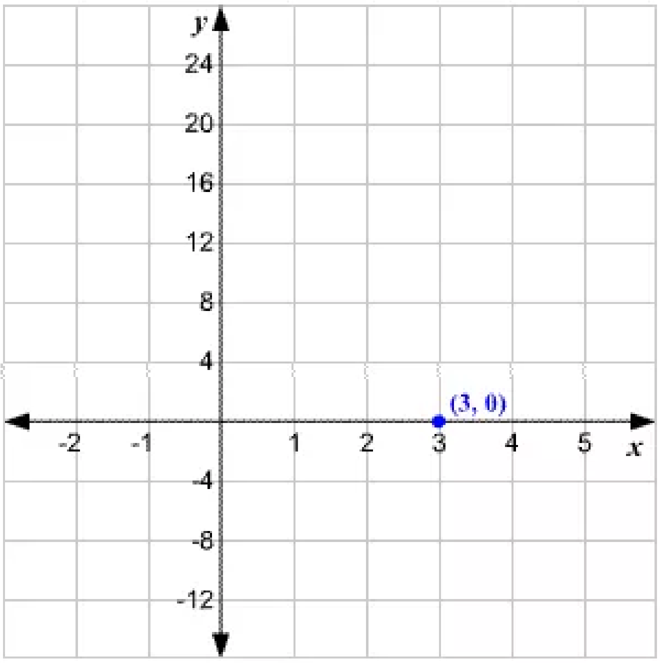
The graph of the function $h(x)=5(x-1)(x-2)(x-3)$ which passes through the plotting points is shown below:



Answer 11e.

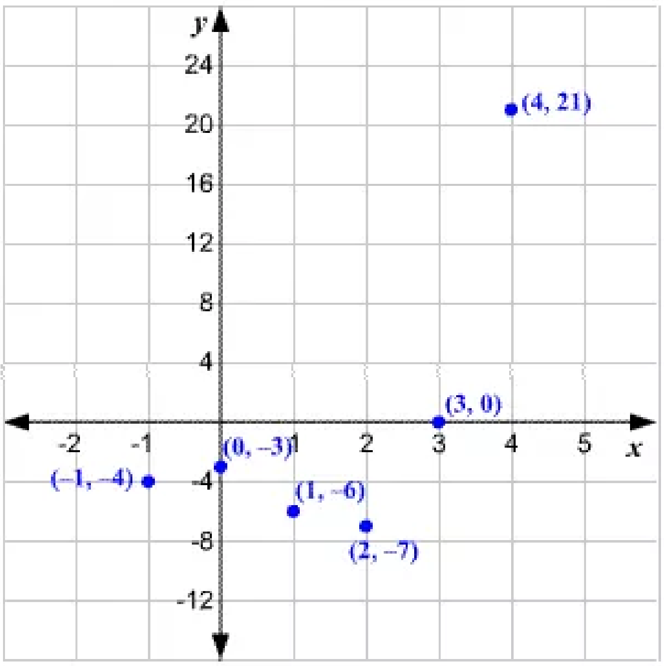
STEP 1 **Plot** the intercepts.

From the given function, we get the zero, which is the x-intercept of the graph of the function, as 3. Plot the point (3, 0).



STEP 2 **Plot** points beyond the x-intercept.

x	-1	0	1	2	4
y	-4	-3	-6	-7	21

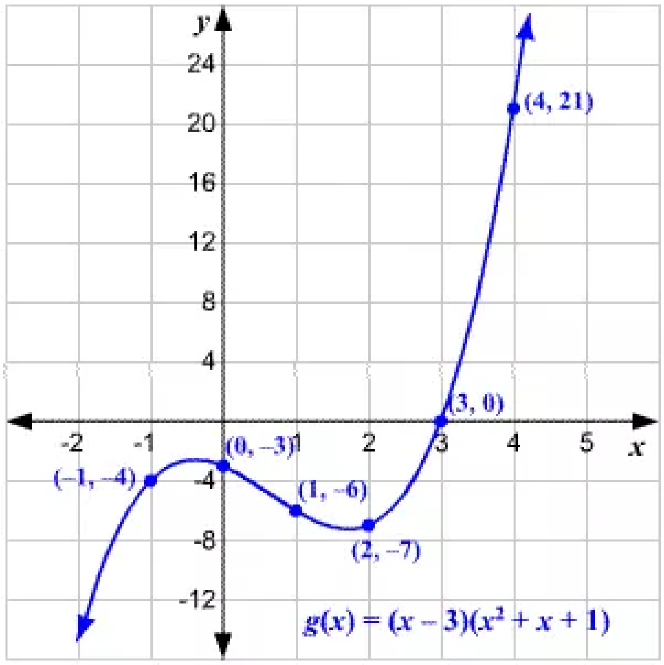


STEP 3 **Determine** end behavior.

Since g(x) has a factor of the form $x - k$, and another factor of the form $ax^2 + bx + c$, the function is cubic. The leading coefficient is 1, which is positive.

For a function with odd degree and positive leading coefficient, the end behavior is $g(x) \to -\infty$ as $x \to -\infty$ and $g(x) \to +\infty$ as $x \to +\infty$.

STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.



Answer 12e.

Considering polynomial function is $h(x)=(x-4)(2x^2-2x+1)$. We need to draw the graph of the given function.

Here, $x=4$ is the zeros of function *f*. Therefore we plot the *x*-intercepts $(4,0)$ in the graph.

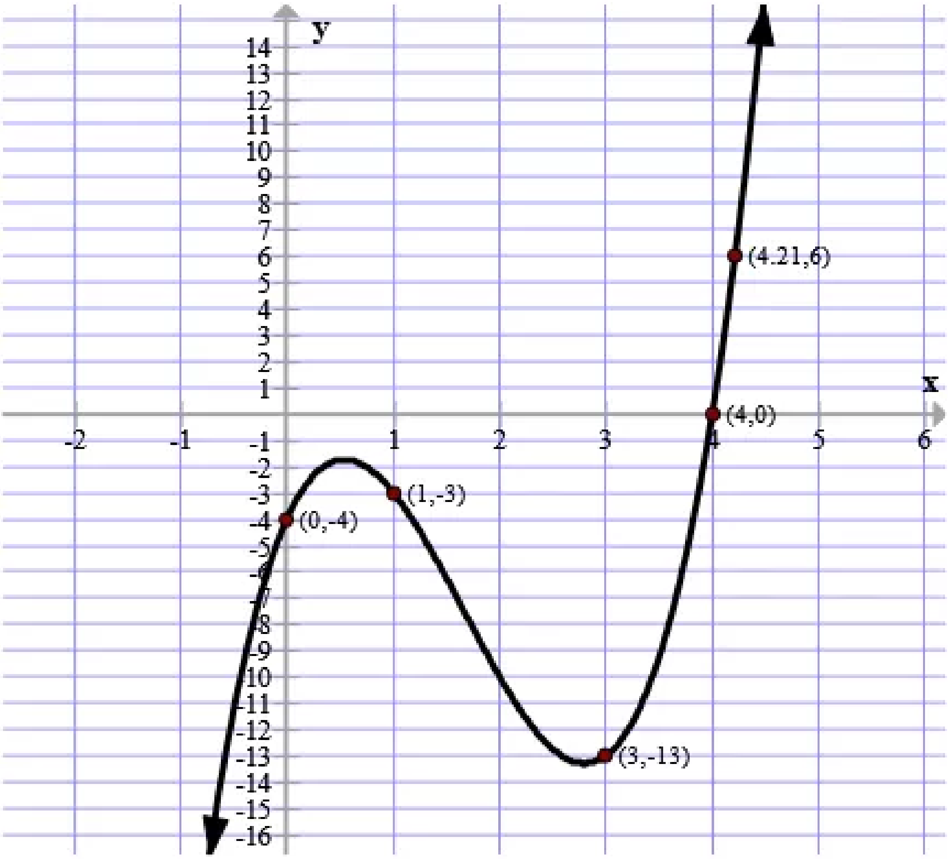
The table of points between and beyond the *x*-intercepts are shown below:

x	0	1	3	4	4.21
y	-4	-3	-13	0	6

These points are plotted in the graph.

To determine the end behavior , we consider the function and it is seen that the function *f* has the form of $x-k$ and (ax^2+bx+c), and is a positive leading co-efficient 1. So $f(x)\to-\infty$ as $x\to-\infty$ and $f(x)\to+\infty$ as $x\to+\infty$

The graph of the function $h(x)=(x-4)(2x^2-2x+1)$ which passes through the plotting points is shown below:



Answer 13e.

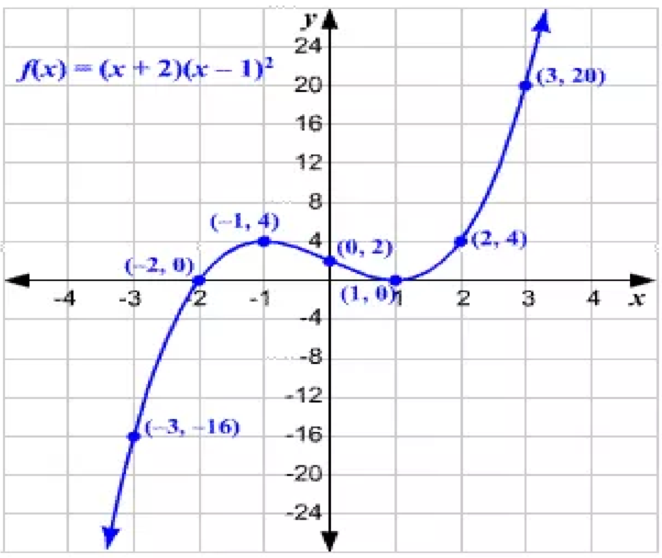
A polynomial function can be graphed by first plotting the zeros or the intercepts of the function. For the given $f(x)$, the intercepts are -2 and 1.

If we observe the graph given, we can see that the intercepts are plotted wrongly as -1 and 2.

Let us graph the function correctly. Select some values for x that are between and beyond the intercepts, and find the corresponding values of the function.

x	-3	-1	0	2	3
y	-16	4	2	4	20

Plot the intercepts and the points from the table and draw a smooth curve passing through them.



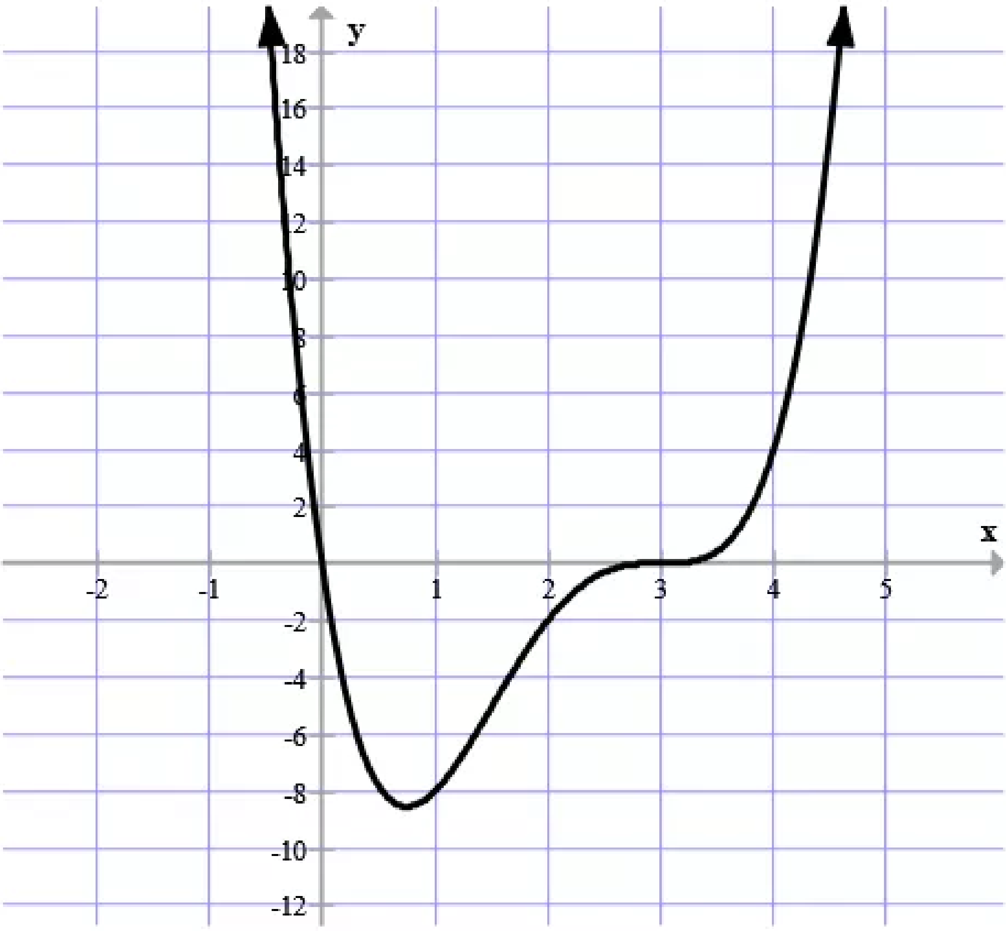
Answer 14e.

The polynomial function is $f(x)=x(x-3)^3$.

The function f will zeros when $x=0$ and 3.

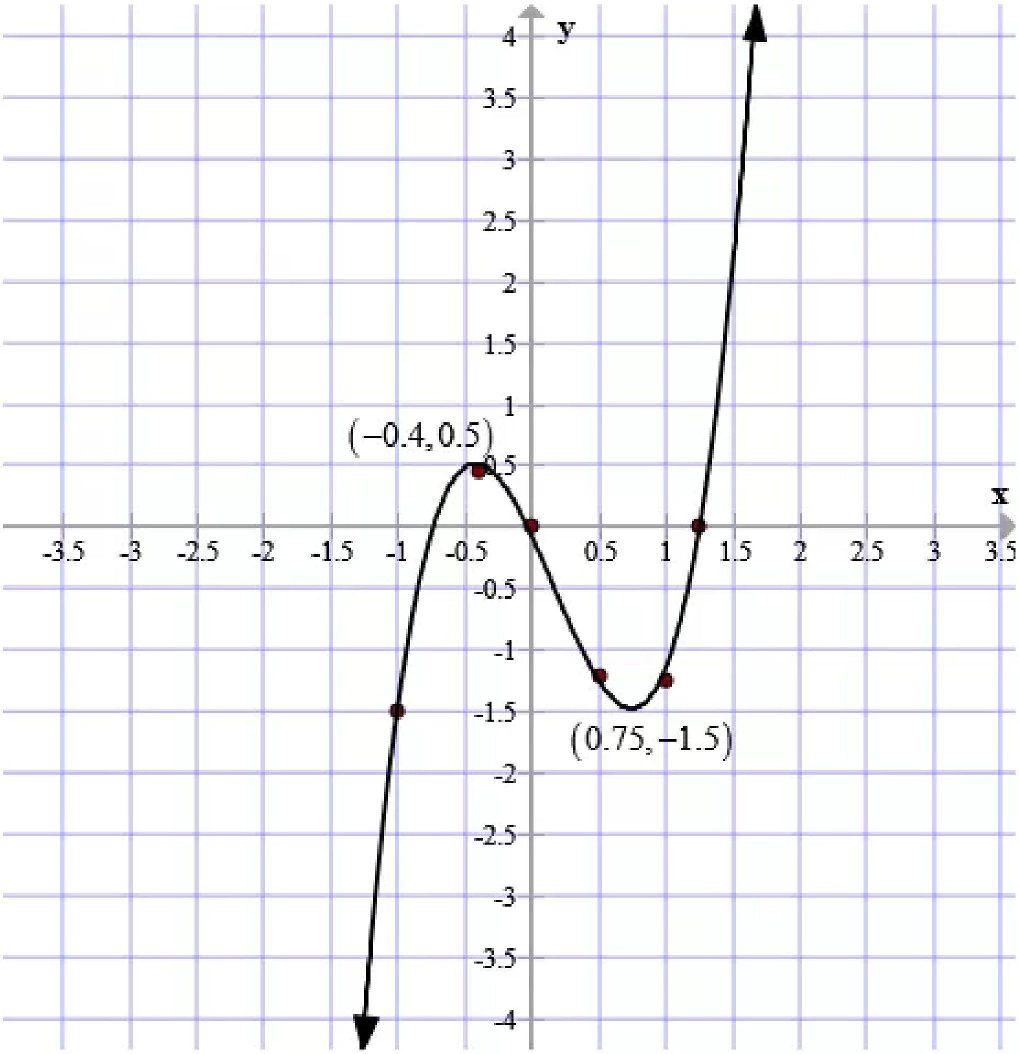
In the function f, x is multiplied with a cubic function. Therefore $f(x)\to+\infty$ when $x\to+\infty$ and $f(x)\to+\infty$ when $x\to-\infty$.

The correct graph of the function $f(x)=x(x-3)^3$ is shown below:



Answer 15e.

Considering the graph we have to find the turning point, and the approximate a local maximum point and a local minimum point. We need to estimate the real zeroes and the least degree the function can have.



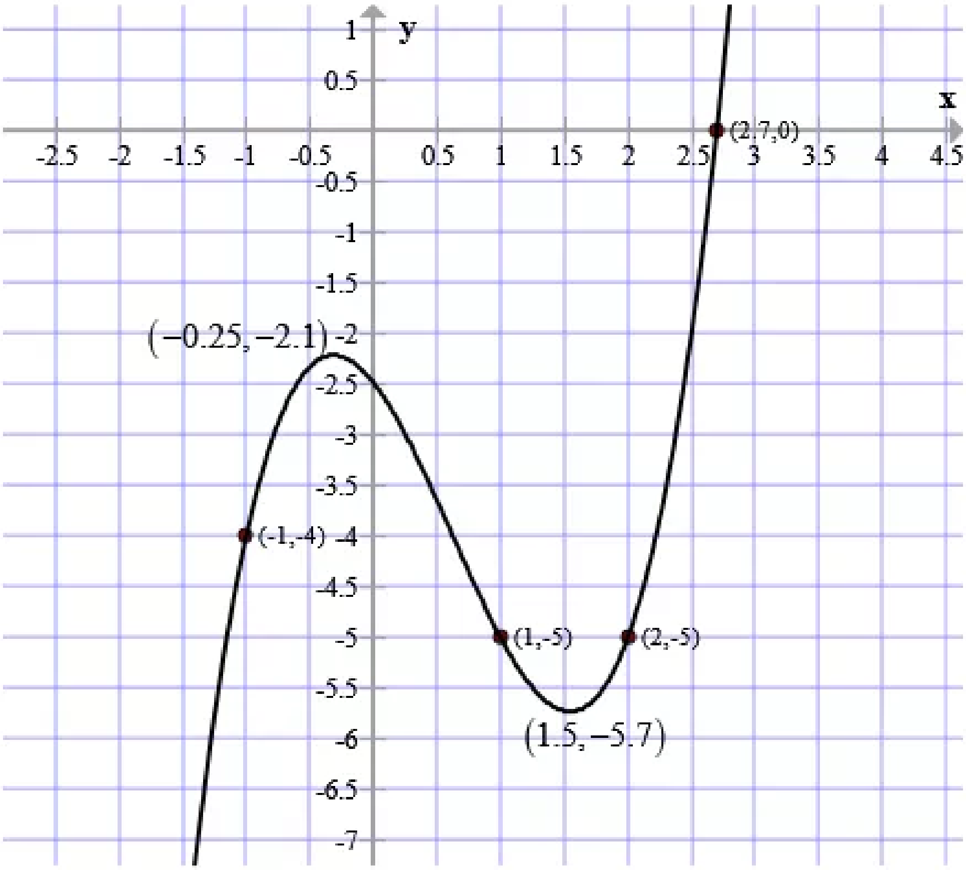
The graph has two turning point. And the estimated point of local maximum is $\boxed{(-0.4, 0.5)}$ and local minimum is $\boxed{(0.75, -1.5)}$.

The graph cut the x-axis at the estimated points at $(-0.75, 0)$, $(0, 0)$ and $(1.25, 0)$.

The least degree of the function is $\boxed{2}$, as the graph has two turning point.

Answer 16e.

Considering the graph we have to find the turning point, and the approximate a local maximum point and a local minimum point. We need to estimate the real zeroes and the least degree the function can have.



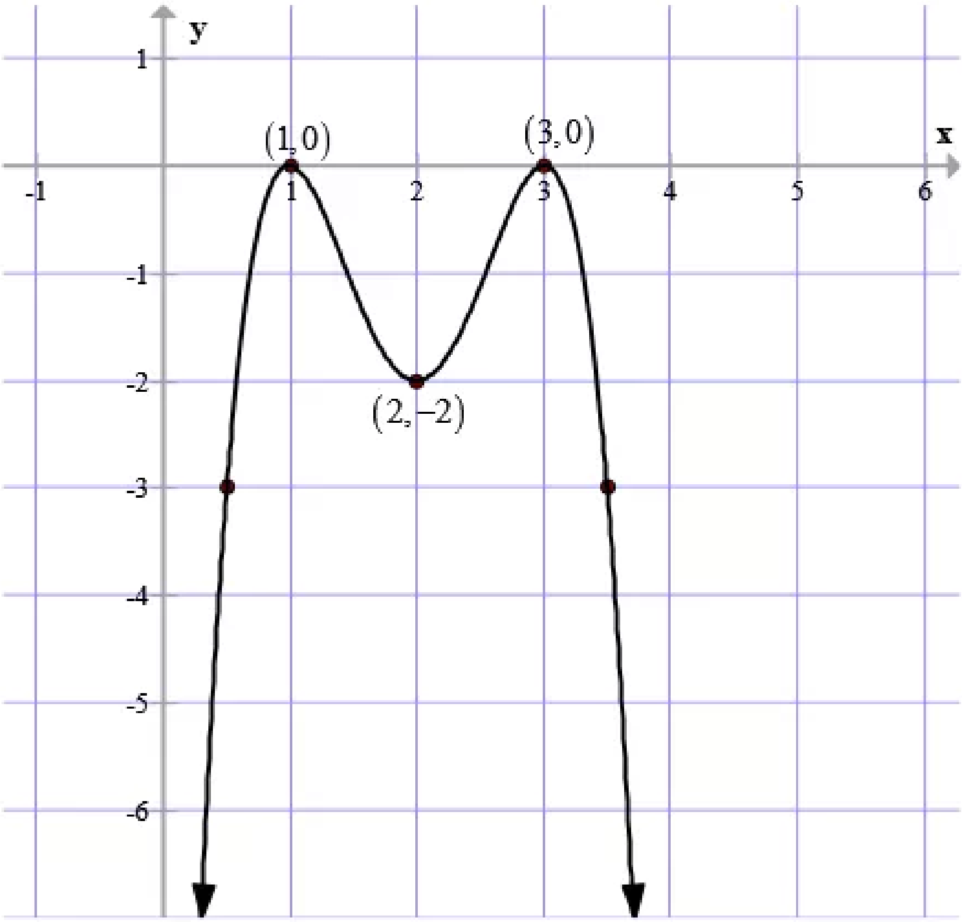
The graph has two turning point. And the estimated point of local maximum is $\boxed{(-0.25,-2.1)}$ and local minimum is $\boxed{(1.5,-5.7)}$.

The graph cut the x-axis at the estimated point $(2.7,0)$.

The least degree of the function is $\boxed{3}$, as the graph has two turning point.

Answer 17e.

Considering the graph we have to find the turning point, and the approximate a local maximum point and a local minimum point. We need to estimate the real zeroes and the least degree the function can have.



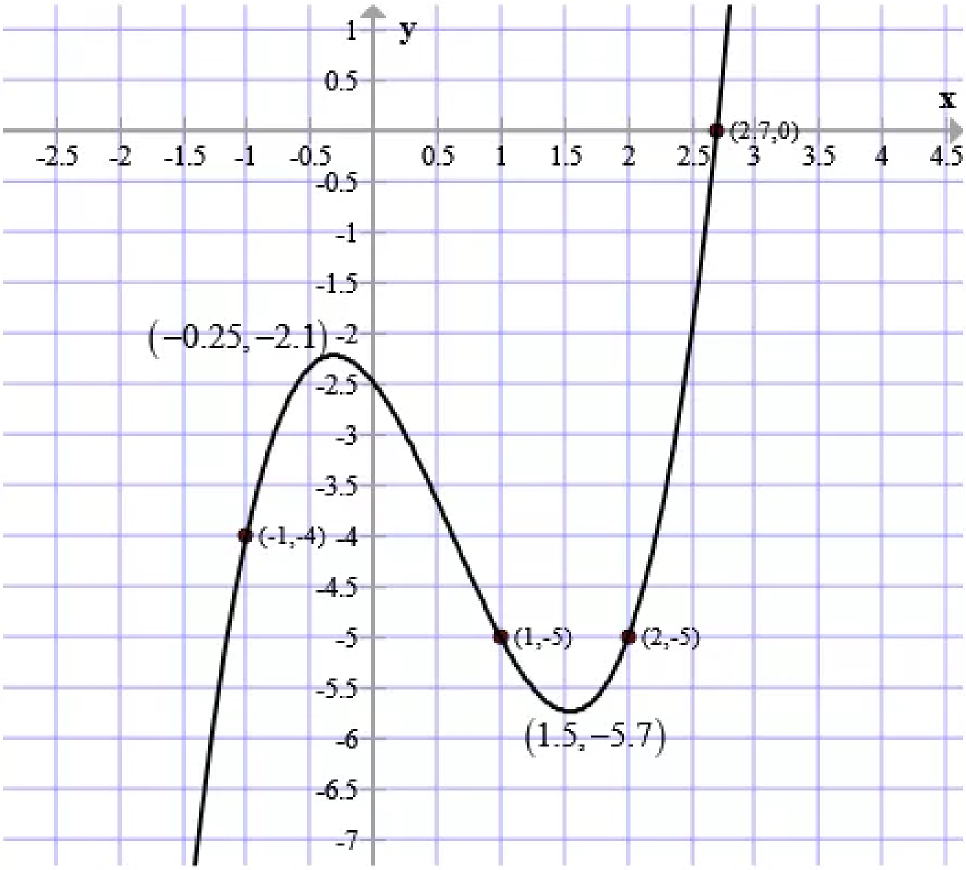
The graph has three turning point. And the estimated points of local maximum are $\boxed{(1,0)}$ and $\boxed{(3,0)}$. The local minimum is at the point $\boxed{(2,-2)}$.

The graph cut the x-axis at the estimated points at $\boxed{(1,0) \text{ and } (3,0)}$.

The least degree of the function is $\boxed{4}$, as the graph has three turning points.

Answer 18e.

Considering the graph we have to find the turning point, and the approximate a local maximum point and a local minimum point. We need to estimate the real zeroes and the least degree the function can have.



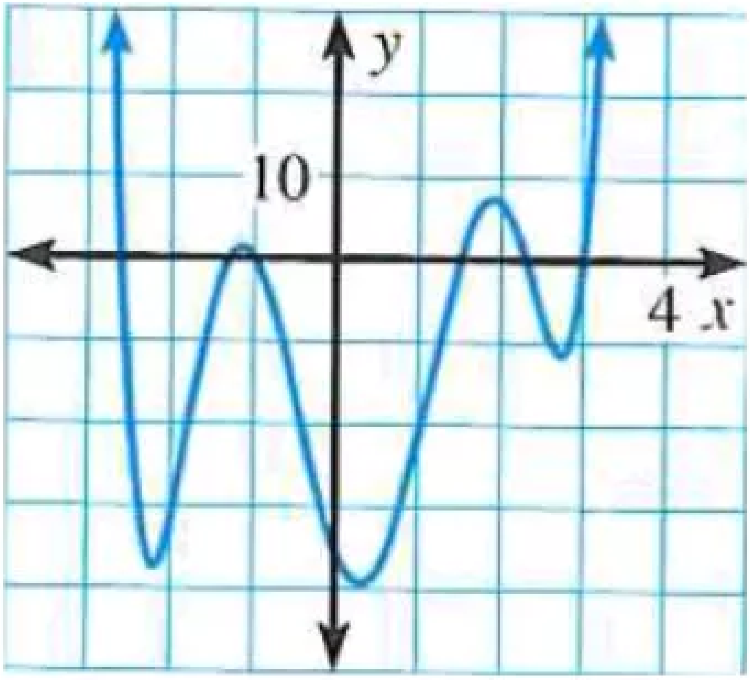
The graph has two turning point. And the estimated point of local maximum is $\boxed{(-0.25,-2.1)}$ and local minimum is $\boxed{(1.5,-5.7)}$.

The graph cut the x-axis at the estimated point $(2.7,0)$.

The least degree of the function is $\boxed{3}$, as the graph has two turning point.

Answer 19e.

Considering the graph we have to find the turning point, and the approximate a local maximum point and a local minimum point. We need to estimate the real zeroes and the least degree the function can have.



The graph has five turning point. And the estimated points of local maximum are $\boxed{(-1.1,1)}$ and $\boxed{(2.9,8)}$. The local minimum are at the points $\boxed{(-2.2,-37)}$, $\boxed{(0.3,-40)}$ and $\boxed{(2.8,-11)}$.

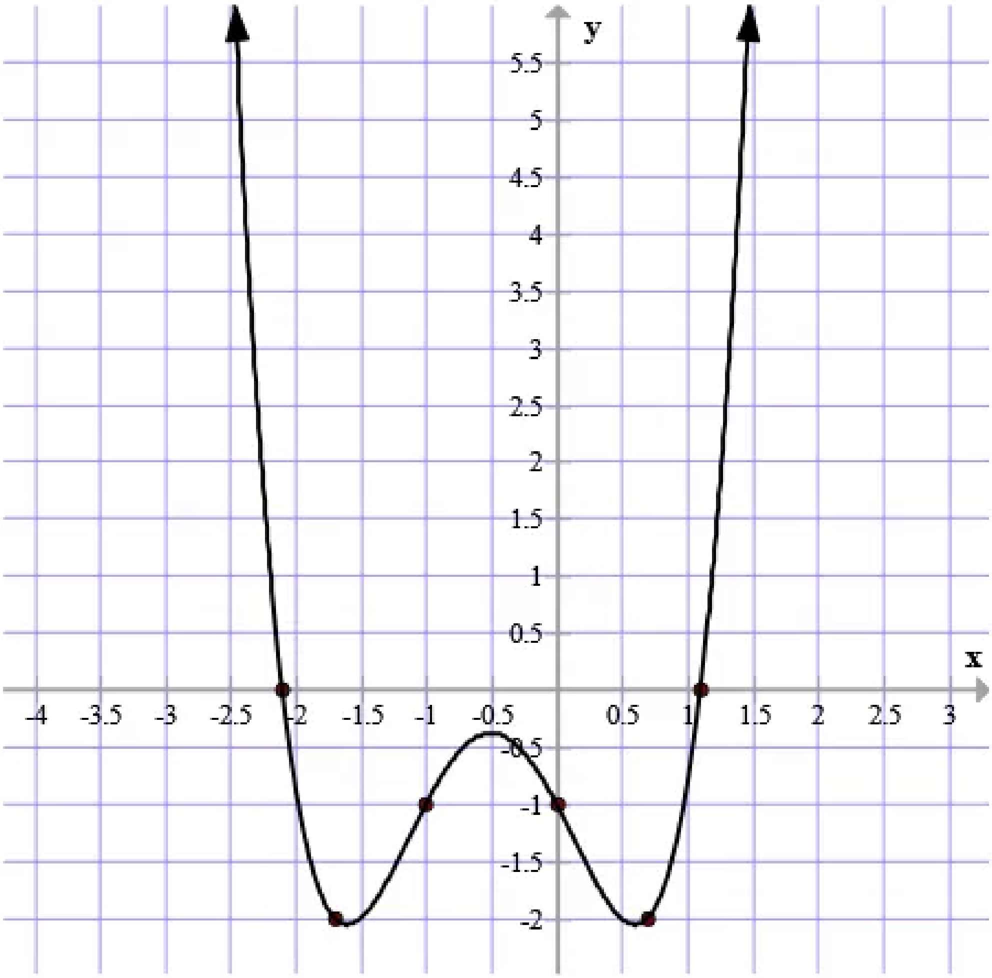
The graph cut the x-axis at the estimated points

$$\boxed{(-2.6,0)}, \boxed{(-1.2,0)}, \boxed{(1,0)}, \boxed{(2.5,0)} \text{ and } \boxed{(3,0)}$$

The least degree of the function is $\boxed{6}$, as the graph has five turning points.

Answer 20e.

Considering the graph we have to find the turning point, and the approximate a local maximum point and a local minimum point. We need to estimate the real zeroes and the least degree the function can have.



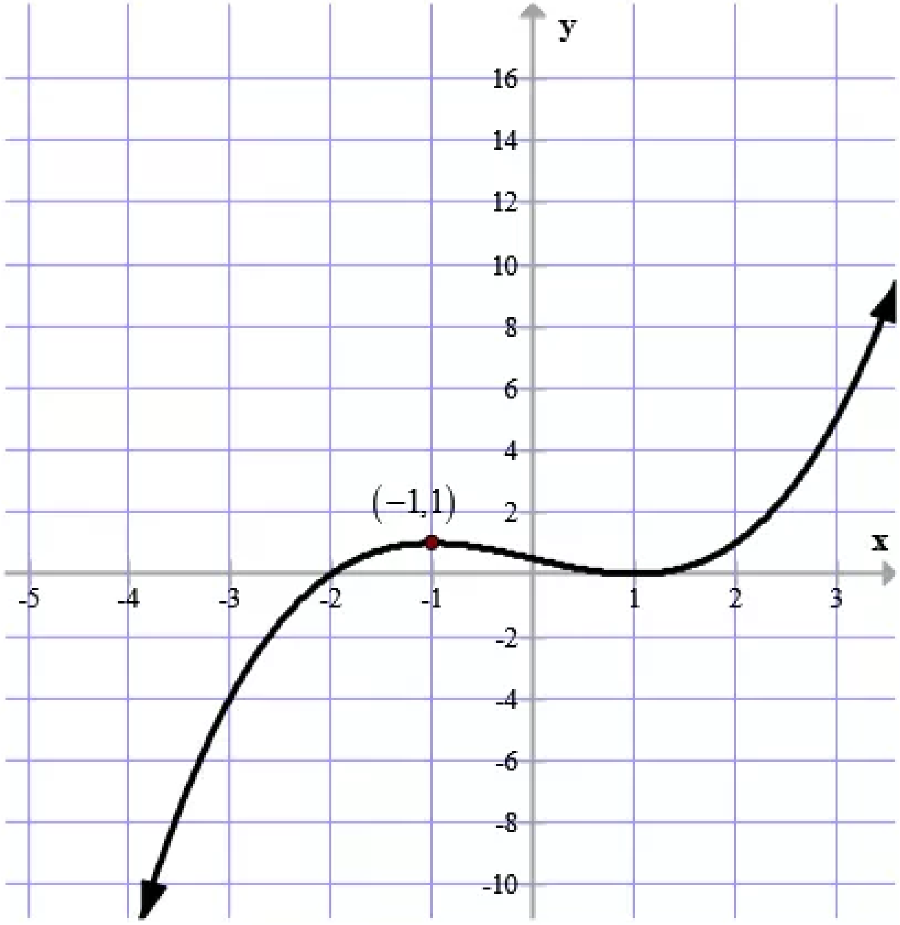
The graph has three turning points. And the estimated point of local maximum is $\boxed{(-0.5,-0.4)}$ and local minimum are $\boxed{(-1.65,-2)}$ and $\boxed{(0.65,-2)}$.

The graph cut the x-axis at the estimated point $(-2.1,0)$ and $(1.1,0)$.

The least degree of the function is $\boxed{4}$, as the graph has three turning point.

Answer 21e.

The graph of the function $f(x)=0.25(x+2)(x-1)^2$ is shown below:

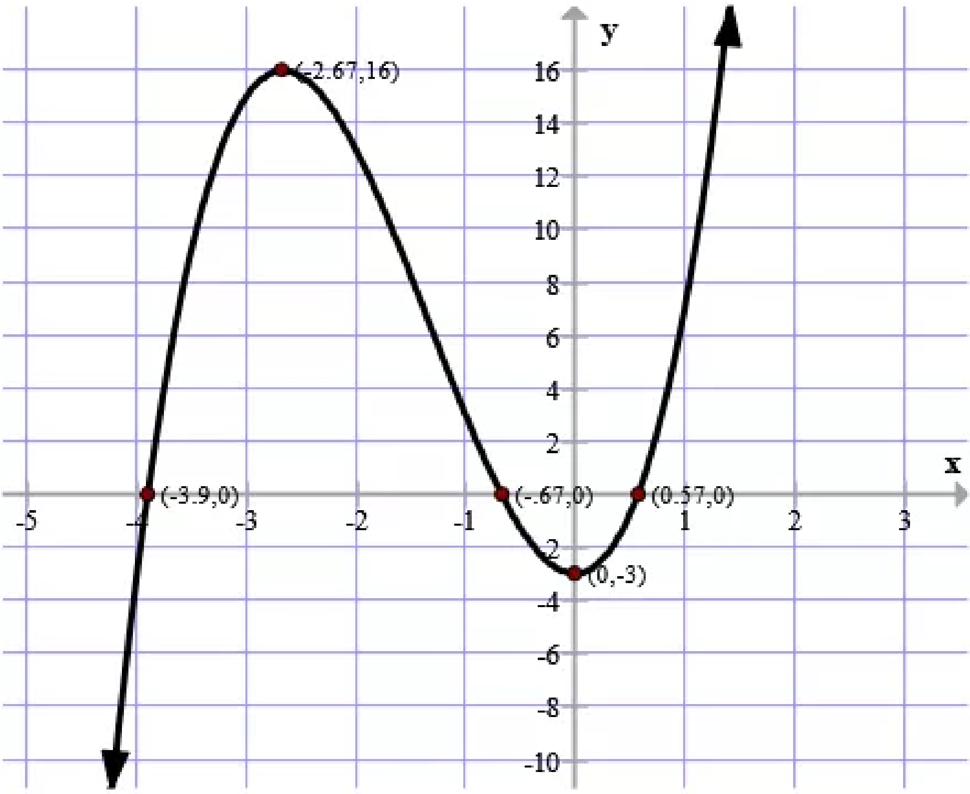


From the graph we can see that the function f has a local maximum at the point $(-1,1)$.

Therefore the correct answer is $\boxed{\text{B. } (-1,1)}$.

Answer 22e.

The graph of the function $f(x)=2x^3+8x^2-3$ is shown below:

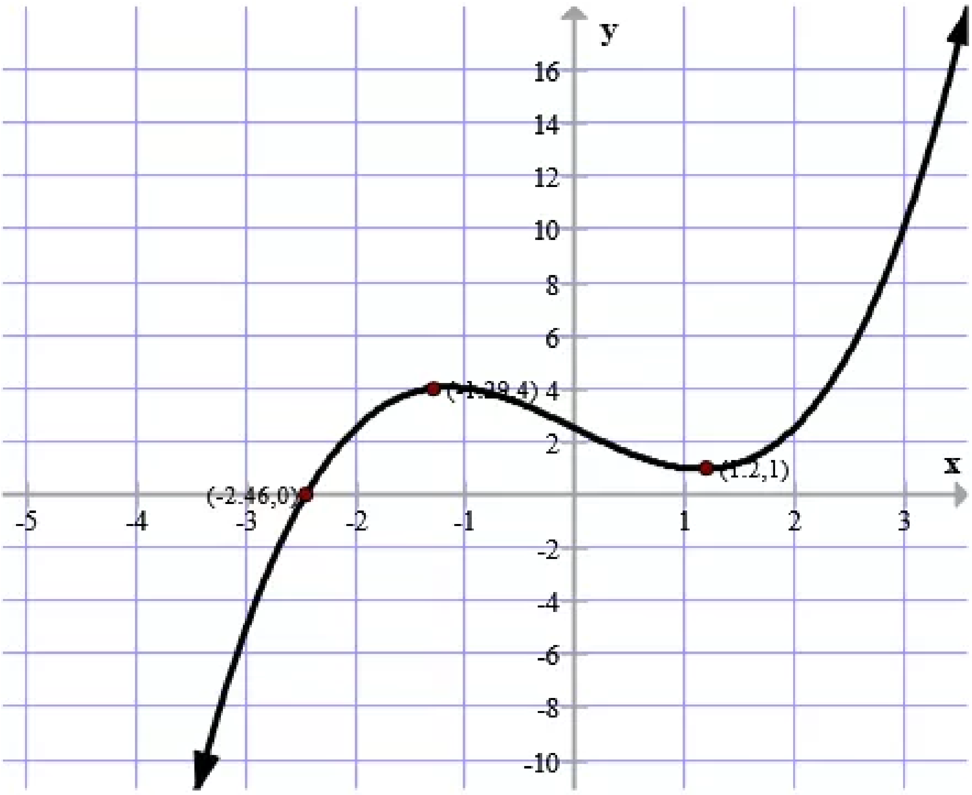


From the graph we can see that the function f has three x-intercepts. The x-intercepts of the function $f(x)=x^4+3x^3-x^2-4x-5$ is approximately $x\approx\boxed{-3.9}$, $x\approx\boxed{-0.67}$ and $x\approx\boxed{0.57}$

It can be notice that the graph has two turning points.
Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(0,-3)}$ and has a local maximum at $\boxed{(-2.67,16)}$.

Answer 23e.

The graph of the function $g(x)=0.5x^3-2x+2.5$ is shown below:

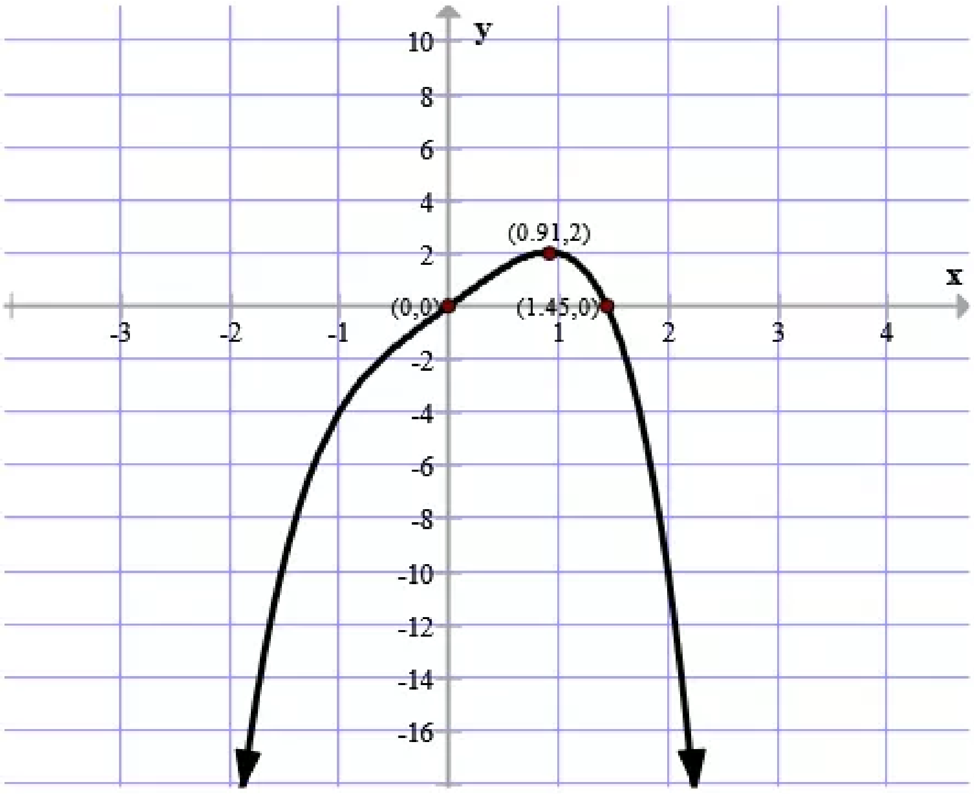


From the graph we can see that the function *f* has one *x*-intercept. The *x*-intercepts of the function $g(x)=0.5x^3-2x+2.5$ is approximately $x \approx \boxed{-2.46}$.

It can be notice that the graph has two turning points.
Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(1.2,1)}$ and has a local maximum at $\boxed{(1.29,\ 4)}$.

Answer 24e.

The graph of the function $h(x) = -x^4 + 3x$ is shown below:

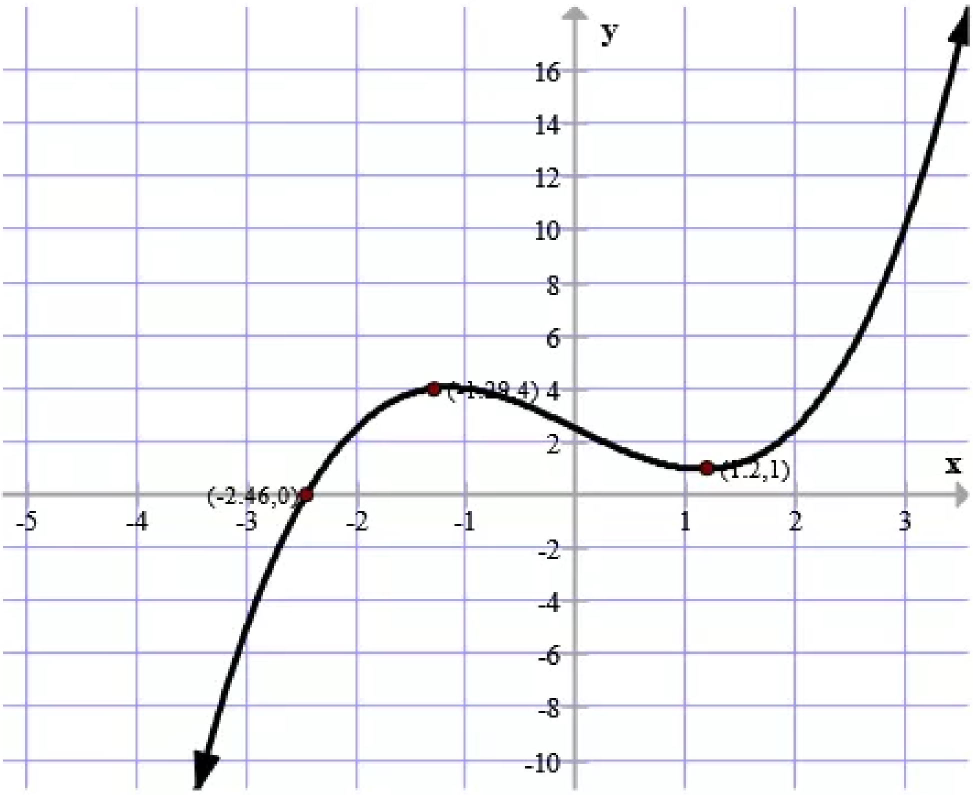


From the graph we can see that the function f has one x-intercept. The x-intercepts of the function $h(x) = -x^4 + 3x$ is $x \approx \boxed{0}$ and $x \approx \boxed{1.45}$

It can be notice that the graph has one turning point.
Using graphing calculator we can approximate that the function has no local minimum has a local maximum at $\boxed{(0.91,\ 2)}$.

Answer 25e.

The graph of the function $g(x) = 0.5x^3 - 2x + 2.5$ is shown below:

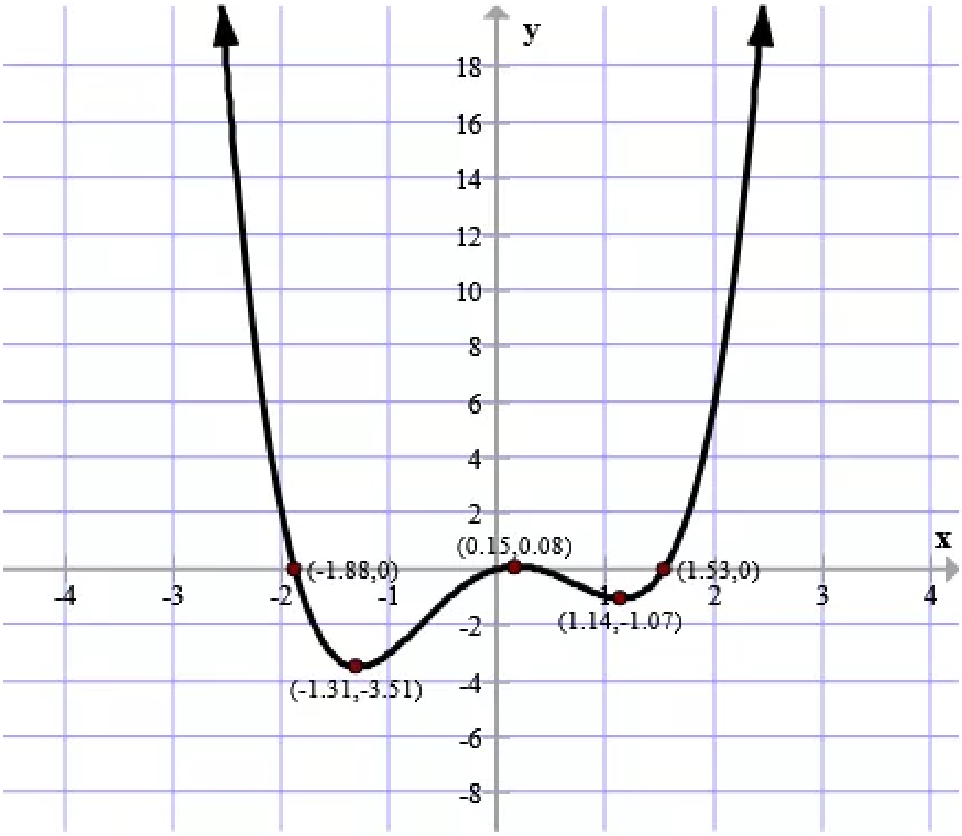


From the graph we can see that the function f has one x-intercept. The x-intercepts of the function $g(x) = 0.5x^3 - 2x + 2.5$ is approximately $x \approx \boxed{-2.46}$.

It can be notice that the graph has two turning points.
Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(1.2, 1)}$ and has a local maximum at $\boxed{(-1.29,\ 4)}$.

Answer 26e.

The graph of the function $g(x)=x^4-3x^2+x$ is shown below:



From the graph we can see that the function f has one x-intercept. The x-intercepts of the function $g(x)=x^4-3x^2+x$ is approximately $x\approx\boxed{-1.88}$, $x\approx\boxed{0}$ and $x\approx\boxed{1.53}$.

It can be notice that the graph has three turning points.
Using graphing calculator we can approximate that the function has a local minimum at $\boxed{(-1.31,-3.51)}$ and $\boxed{(1.14,-1.07)}$. The local maximum at the point $\boxed{(0.15,0.08)}$.

Answer 27e.

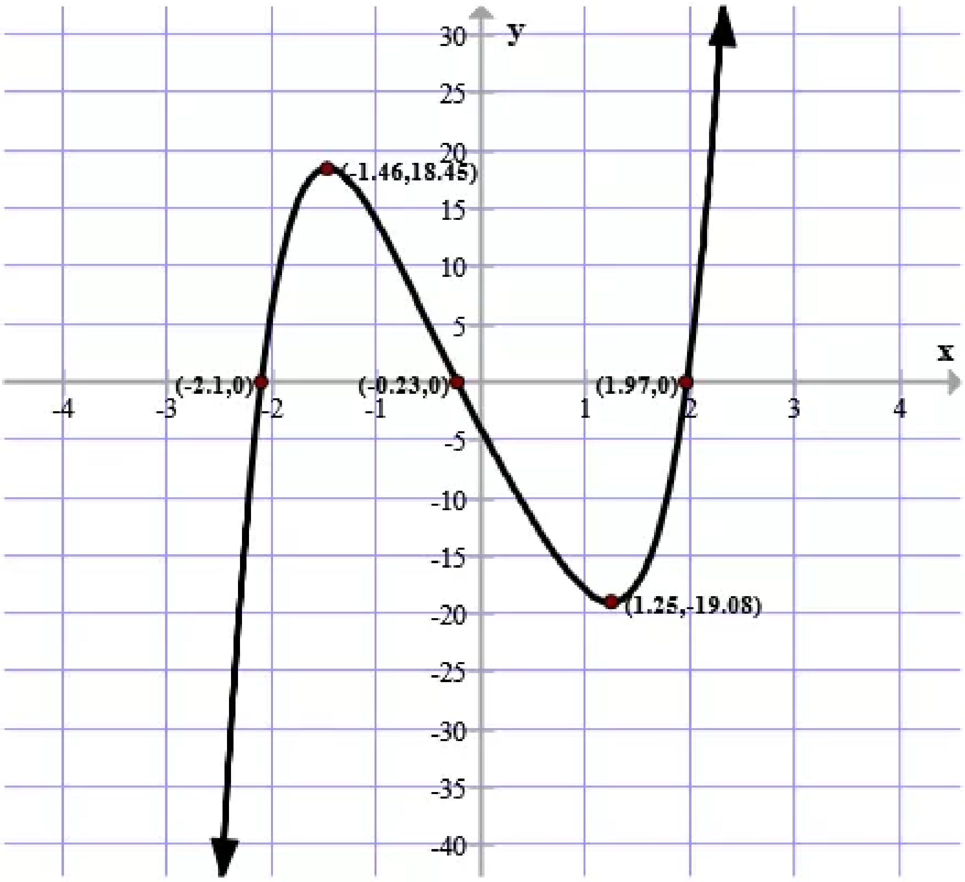
The graph of the function $h(x)=x^4-5x^3+2x^2+x-3$ is shown below:

From the graph we can see that the function f has two x-intercepts. The x-intercepts of the function $h(x)=x^4-5x^3+2x^2+x-3$ is approximately $x\approx\boxed{-0.766}$ and $x\approx\boxed{4.55}$.

It can be notice that the graph has three turning points.
Using graphing calculator we can approximate that the function has a local maximum at $\boxed{(0.46,-2.56)}$. The local minimum at the point $\boxed{(-0.15,-3.09)}$ and $\boxed{(3.42,-39.4)}$

Answer 28e.

The graph of the function $h(x)=x^5+2x^2-17x-4$ is shown below:

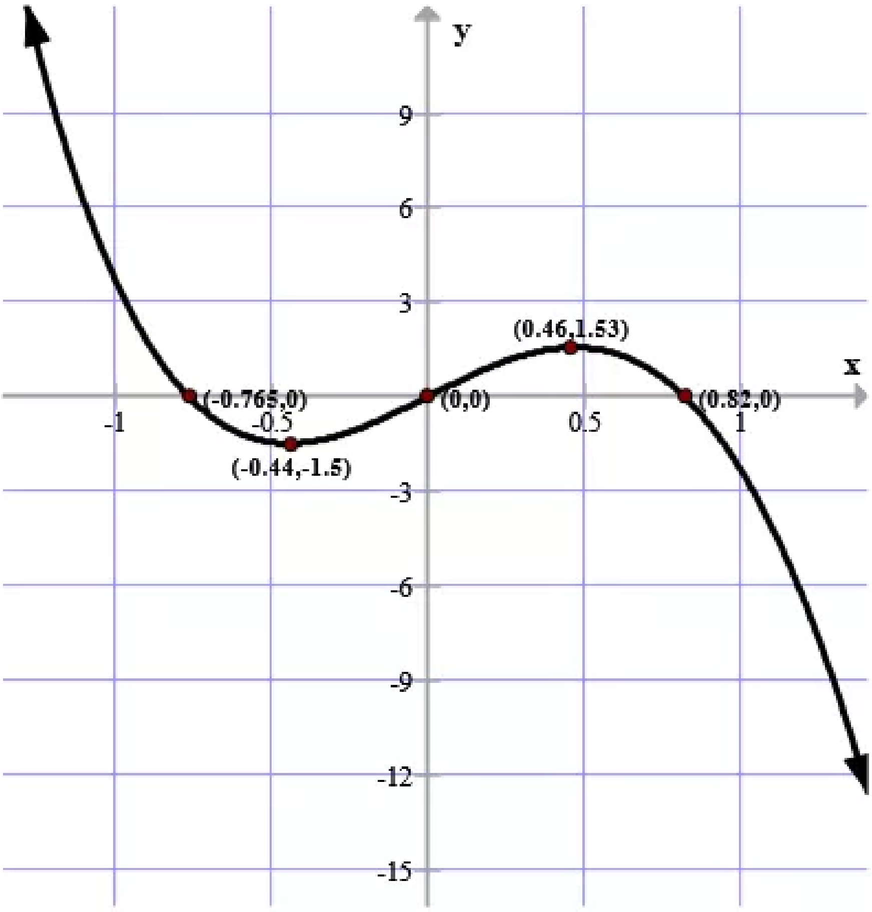


From the graph we can see that the function f has two x-intercepts. The x-intercepts of the function $h(x)=x^5+2x^2-17x-4$ is approximately $x \approx \boxed{-2.1}$, $x \approx \boxed{-0.23}$ and $x \approx \boxed{1.97}$.

It can be notice that the graph has three turning points.
Using graphing calculator we can approximate that the function has a local maximum at $\boxed{(-1.46, 18.45)}$ and a local minimum at the point $\boxed{(1.25, -19.08)}$.

Answer 29e.

The graph of the function $g(x) = 0.7x^4 - 8x^3 + 5x$ is shown below:

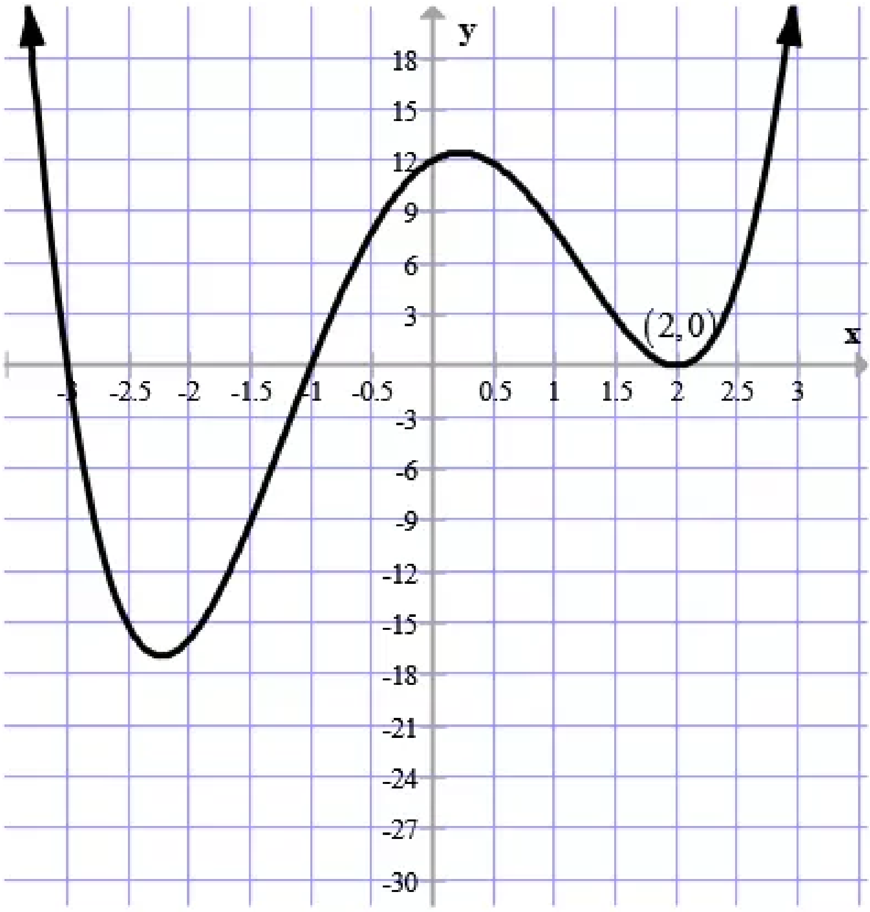


From the graph we can see that the function *f* has three *x*-intercepts. The *x*-intercepts of the function $g(x) = 0.7x^4 - 8x^3 + 5x$ is approximately $x \approx \boxed{-0.765}$, $x = \boxed{0}$ and $x \approx \boxed{0.82}$.

It can be notice that the graph has two turning points.
Using graphing calculator we can approximate that the function has a local maximum at $\boxed{(0.46, 1.53)}$ and a local minimum at the point $\boxed{-0.44, -1.5}$.

Answer 30e.

The graph of the function $g(x) = x^4 - 9x^2 + 4x + 12$ is shown below:



From the graph we can see that the function *f* has a turning point at $(2,0)$.

Therefore, the correct answer is $\boxed{\text{D. } (2,0)}$

Answer 31e.

The maximum and minimum values of a function are determined using the graph of the function.

In the case of a quadratic function, the graph is a parabola that opens upwards or downwards. The *y*-coordinate of the vertex of the parabola is the maximum or the minimum value of the function. Also, there can be only either of the values for a quadratic function. Since a parabola can have only one vertex, the value of the *y*-coordinate is simply called as a maximum or a minimum.

On the other hand, the graph of a cubic function is not a parabola. In this case, the maximum and minimum values are calculated at the tuning points of the graph of the function. If the y-coordinate of a turning point is higher than all other nearby points, it is the maximum value. Since the maximum value is obtained by comparison with other values, it is called as the local maximum.

If the y-coordinate of a turning point is lower than all other nearby points, it is the local minimum of the function. Thus, the adjective *local* is used to describe the maximums and minimums of cubic functions but not quadratic functions.

Answer 32e.

A cubic function always have a turning point, because for the domain of $(-\infty, +\infty)$

The range of the cubic function will be $(-\infty, +\infty)$. It means the value of the function will move from negative to positive. Therefore it has a turning point.

Answer 33e.

The x-intercepts of the graph of a function are the zeros of the function. Since -2, 0, and 4 are the intercepts, they are the zeros of the function.

Let the function be $f(x)$. Using factor theorem, we can say that $x + 2$, x, and $x - 4$ are factors of $f(x)$. Any of these factors can repeat twice, thrice, and so on.

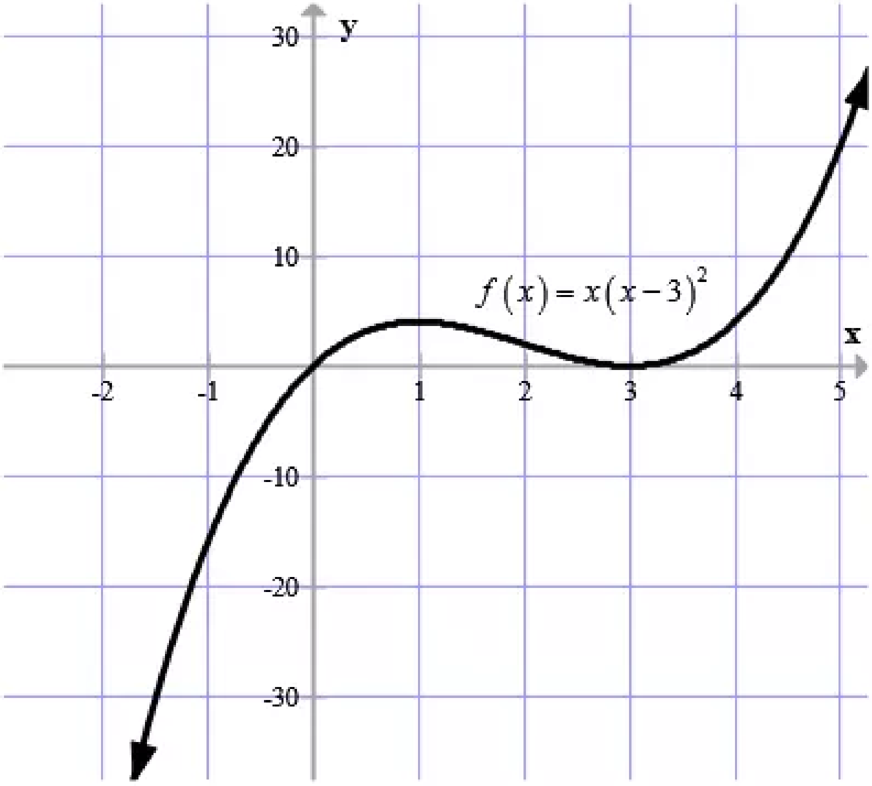
For a cubic function, there will be three factors. We can thus write the cubic function as $f(x) = x(x + 2)(x - 4)$.

A quadratic function will have four factors. To obtain a quadratic function, one of the three factors must repeat twice. Let x be the factor that is repeated twice.
The function can be written as $f(x) = x^2(x + 2)(x - 4)$.

Similarly, a fifth degree function has five factors. Let us take $x + 2$ as the factor repeating thrice. The fifth degree function will be $f(x) = x(x + 2)^3(x - 4)$.

Answer 34e.

The graph of the function $f(x) = x(x-3)^2$ is shown below:

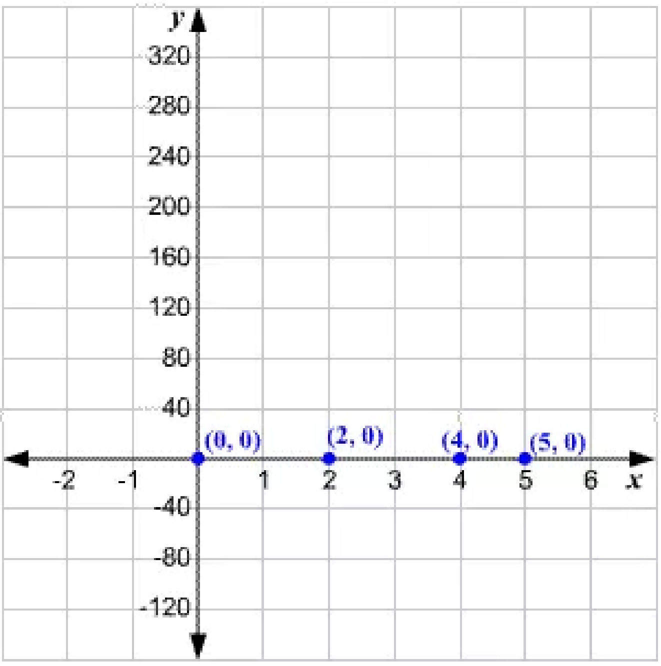


From the graph we can say that the domain of the function is $(-\infty, +\infty)$. And for this domain the range is $(-\infty, +\infty)$.

Answer 35e.

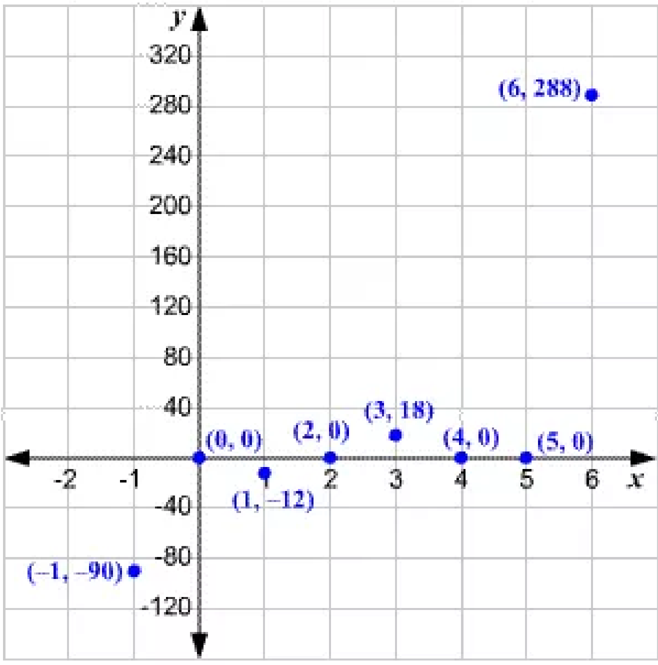
STEP 1 **Plot** the intercepts.

From the given function, we get the zeros, which are the x-intercepts of the graph of the function, as 0, 2, 4, and 5. Plot the points (0, 0), (2, 0), (4, 0), and (5, 0).



STEP 2 **Plot** points between and beyond the x-intercepts.

x	−1	1	3	6
y	−90	−12	18	288

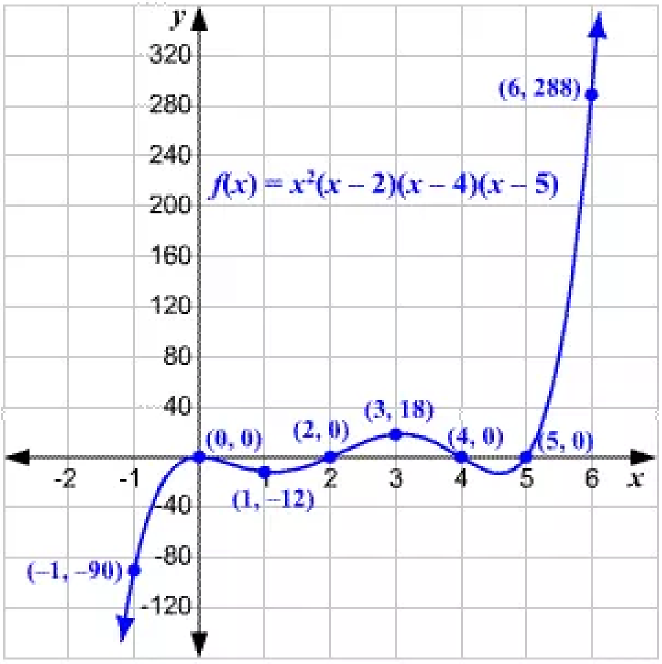


STEP 3 **Determine** end behavior.

Since $f(x)$ has five factors of the form $x - k$, the function is of fifth degree. The leading coefficient is 1, which is positive.

For a function with odd degree and positive leading coefficient, the end behavior is $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.

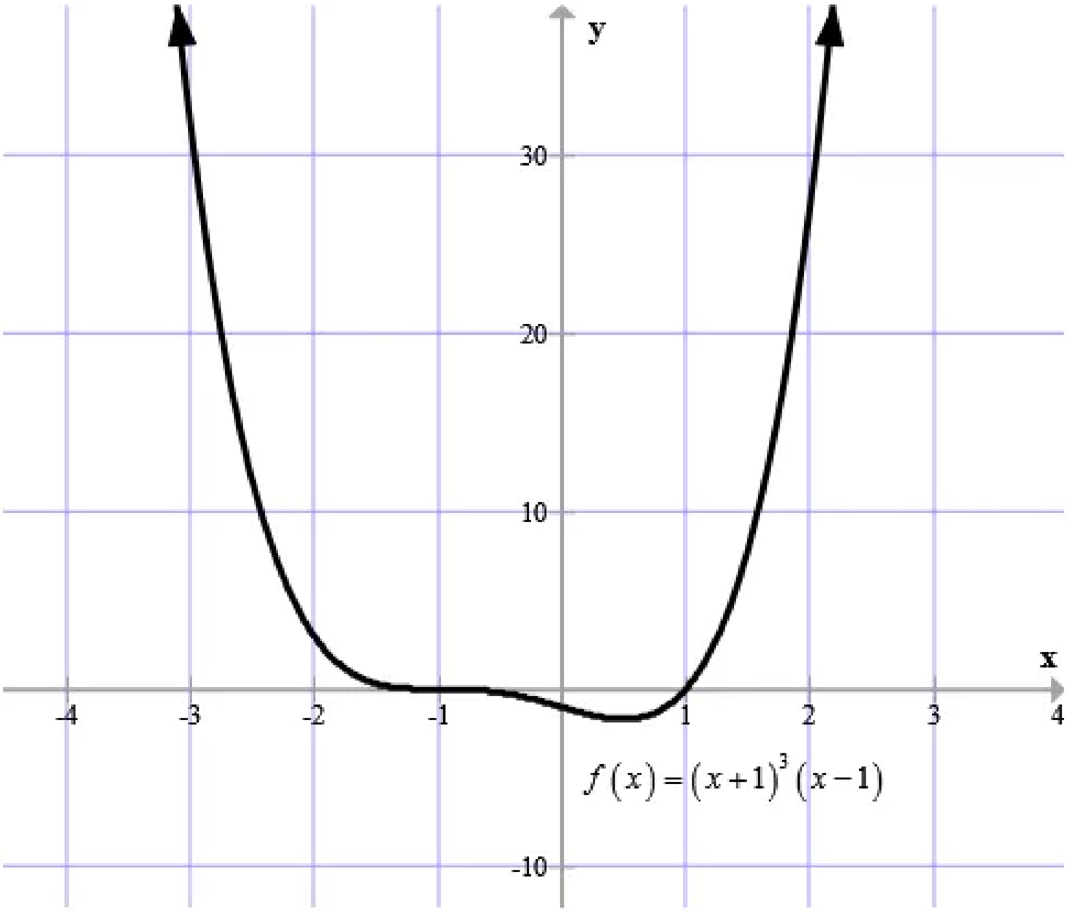


The domain of a function is the set of x-values and the range is the set of y-values of that function. The given function is valid for any real value of x in the interval $-\infty$ to ∞. In the graph, we can see that the y-values takes all real values in the interval $-\infty$ to ∞.

Thus, the domain and the range of $f(x)$ are the set of all real numbers.

Answer 36e.

The graph of the function $f(x) = (x+1)^3(x-1)$ is shown below:



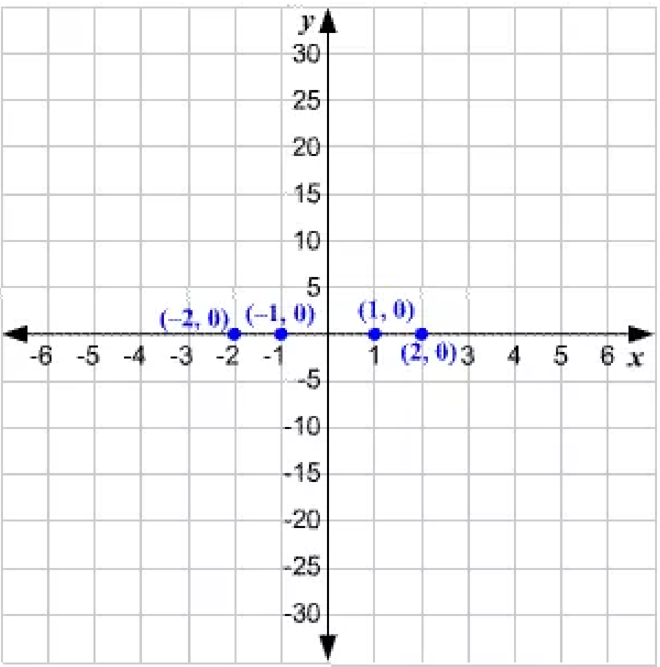
From the graph we can say that the domain of the function is $\boxed{(-\infty, +\infty)}$.

From the graph the minimum value of the function is -1.68.
Therefore range is $[-1.68, \infty)$.

Answer 37e.

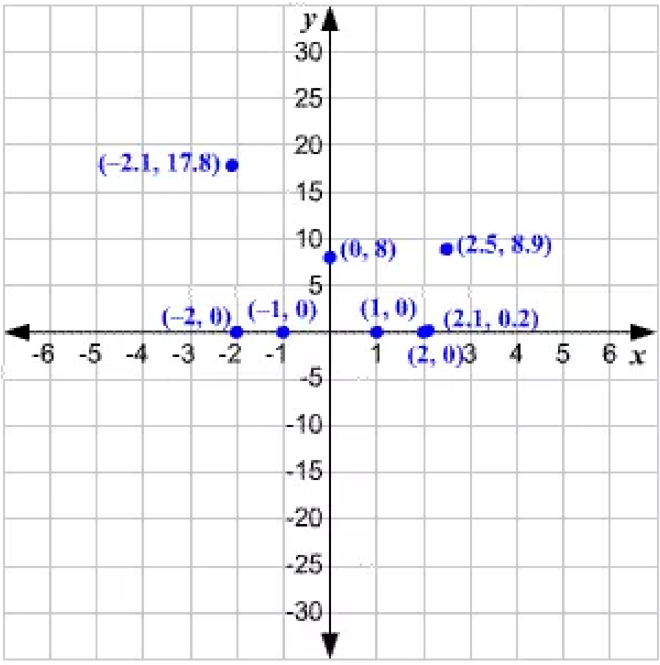
STEP 1 **Plot** the intercepts.

From the given function, we get the zeros, which are the x-intercepts of the graph of the function, as $-2, -1, 1,$ and 2. Plot the points $(-2, 0)$, $(-1, 0)$, $(1, 0)$, and $(2, 0)$.



STEP 2 **Plot** points between and beyond the x-intercepts.

x	−2.1	0	2.1	2.5
y	17.8	8	0.2	8.9

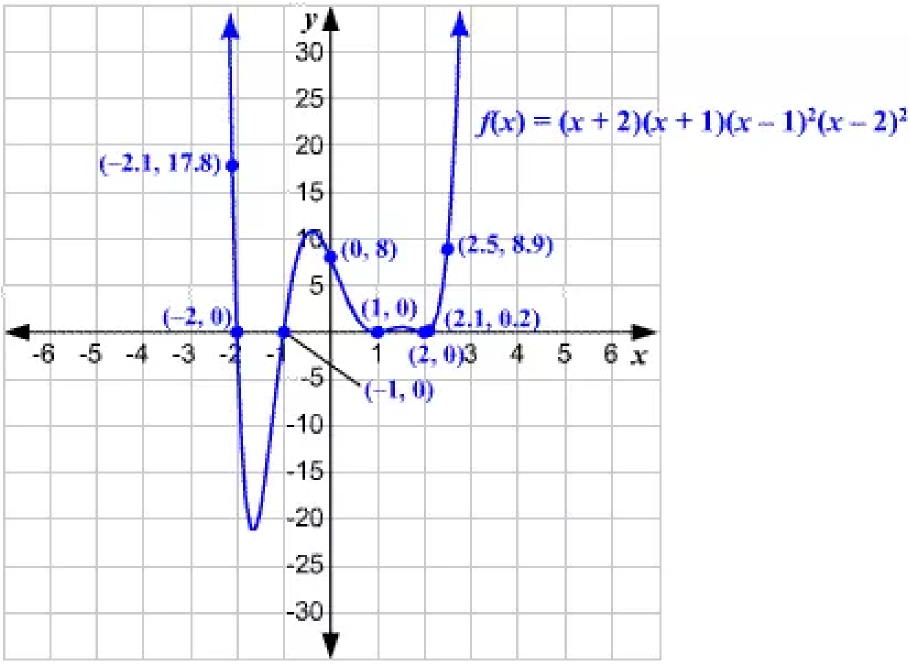


STEP 3 **Determine** end behavior.

Since $f(x)$ has six factors of the form $x - k$, the function is of sixth degree. The leading coefficient is 1, which is positive.

For a function with even degree and positive leading coefficient, the end behavior is $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$.

STEP 4 **Draw** the graph so that it passes through the plotted points and has the appropriate end behavior.

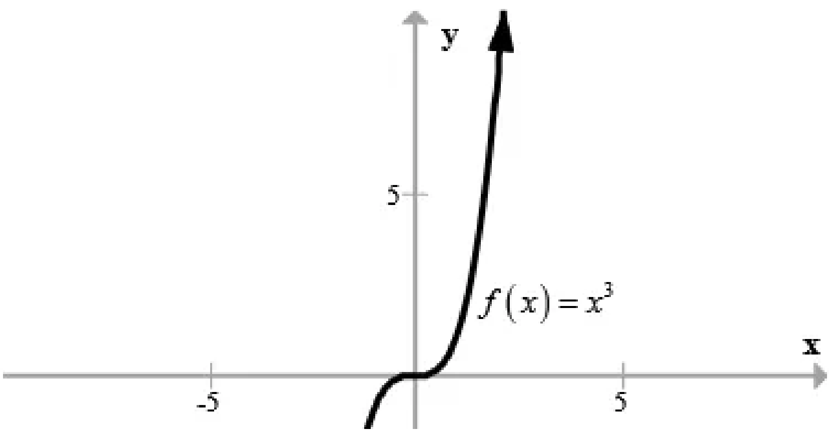


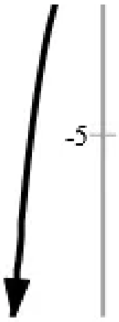
The domain of a function is the set of x-values and the range is the set of y-values of that function. The given function is valid for any real value of x in the interval $-\infty$ to ∞. In the graph, we can see that the y-values takes all real values in the interval -21.3 to ∞.

Thus, the domain of $f(x)$ is the set of all real numbers and the range is $y \geq -21.3$.

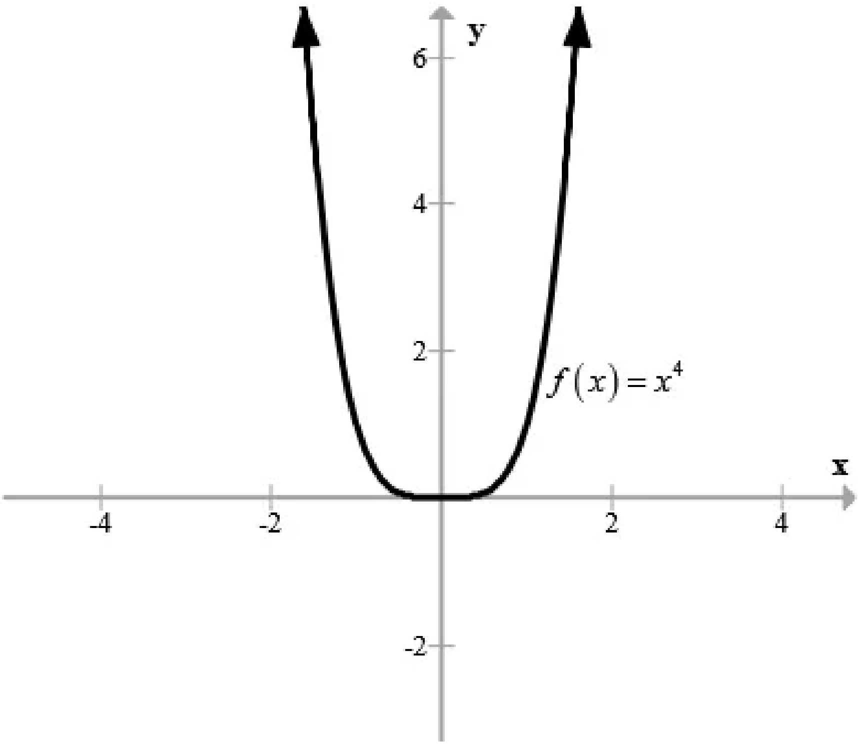
Answer 38e.

In the odd degree polynomial function for any domain, suppose the domain $(-\infty, +\infty)$, the range will be $(-\infty, +\infty)$. For example, a graph of odd degree function is shown below:





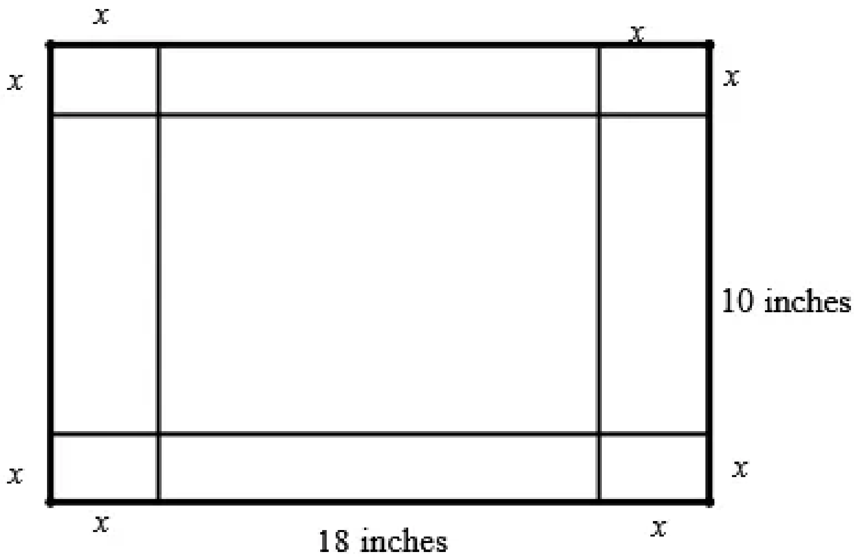
And for the even degree polynomial function for any domain, suppose the domain $(-\infty,+\infty)$, there will be minimum or maximum value. Therefore the range will be in general form $[a,\infty)$ or $(-\infty,a]$. For example, a graph of even degree function is shown below:



Answer 39e.

Marcie wants to make a box of cardboard that is 10 inches by 18 inches. We need to find the dimension of the box with maximum volume and the maximum volume of the box.

A verbal model for the volume is shown below:



We write the function from this figure as,

$$\text{Volume} = \text{Length} \cdot \text{width} \cdot \text{Height}$$

Suppose the volume is V and height x. Therefore,

$$\text{Volume} = \text{Length} \cdot \text{width} \cdot \text{Height}$$

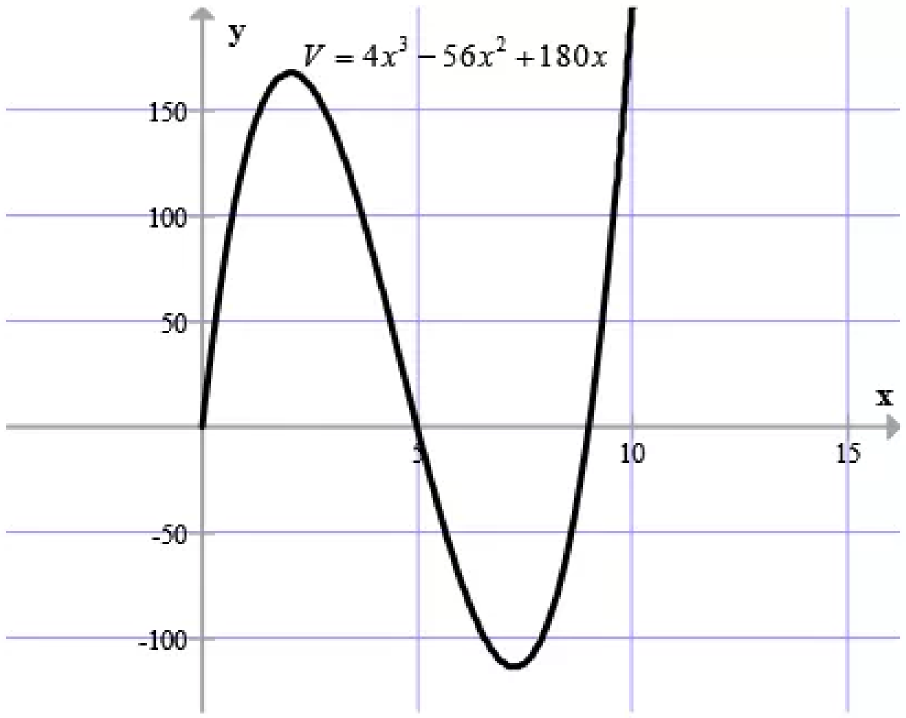
$$V = (18-2x)\cdot(10-2x)\cdot x$$

$$= (180-56x+4x^2)x \qquad \text{[Multiplying binomial]}$$

$$= 4x^3-56x^2+180x \qquad \text{[Standard form]}$$

The maximum volume can be found if we draw the graph of the function

$V = 4x^3-56x^2+180x$ as follows:



From the graph, we can see that the maximum volume is about 168 and occur when $x \approx 2.05$

Therefore Marcie should make the cuts about 2 inches long.

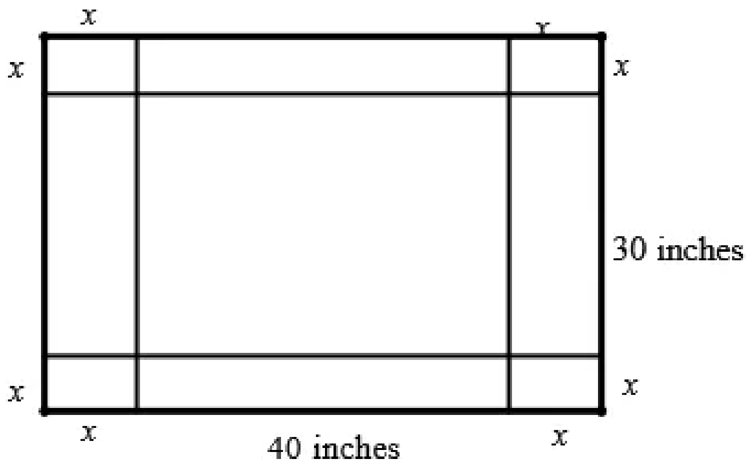
The maximum volume is about $\boxed{168 \text{ inches}^3}$.

The dimensions of the box with this volume will be about $\boxed{x = 2 \text{ inches}}$ by $\boxed{x = 6 \text{ inches}}$ by $\boxed{x = 14 \text{ inches}}$.

Answer 40e.

Jorge wants to make a box of cardboard that is 30 cm by 18 cm. We need to find the dimension of the box with maximum volume and the maximum volume of the box.

A verbal model for the volume is shown below:



We write the function from this figure as,

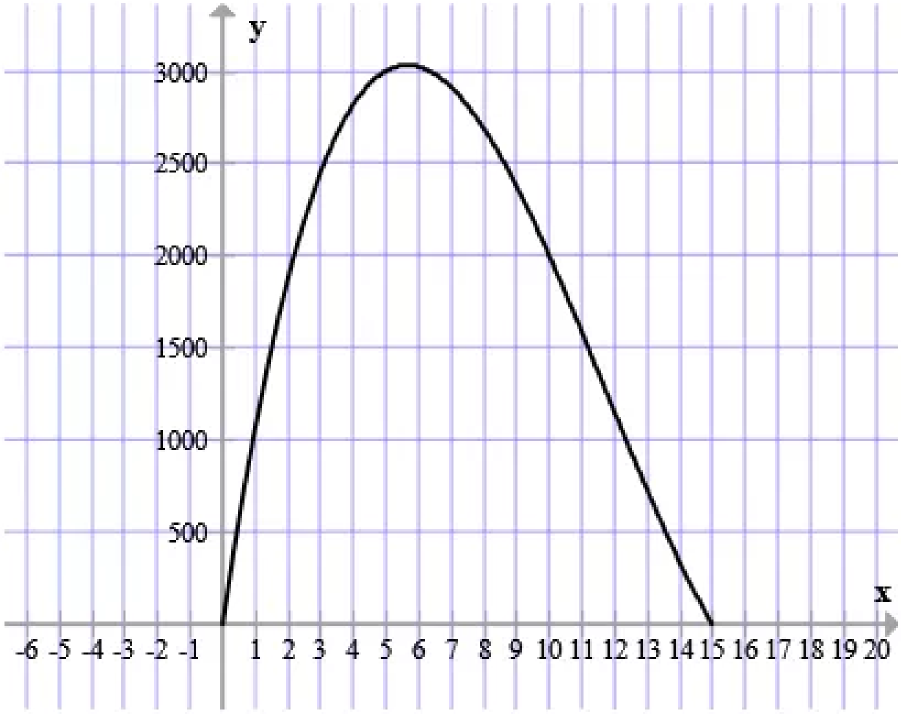
$$\text{Volume} = \text{Length} \cdot \text{width} \cdot \text{Height}$$

Suppose the volume is V and height x. Therefore,

$$\begin{aligned} \text{Volume} &= \text{Length} \cdot \text{width} \cdot \text{Height} \\ V &= (40-2x)\cdot(30-2x)\cdot x \\ &= \left(1200-140x+4x^2\right)x && [\text{Multiplying binomial}] \\ &= 4x^3-140x^2+1200x && [\text{Standard form}] \end{aligned}$$

The maximum volume can be found if we draw the graph of the function

$V = 4x^3-140x^2+1200x$ as follows:



From the graph, we can see that the maximum volume is about 3030 cm^3 and occur when

$$x \approx 6$$

Therefore Jorge should make the cuts about 6 cm long.

The maximum volume is about $\boxed{3030 \text{ cm}^3}$.

The dimensions of the box with this volume will be about $\boxed{x = 6 \text{ cm}}$ by $\boxed{x = 24 \text{ cm}}$ by $\boxed{x = 34 \text{ cm}}$.

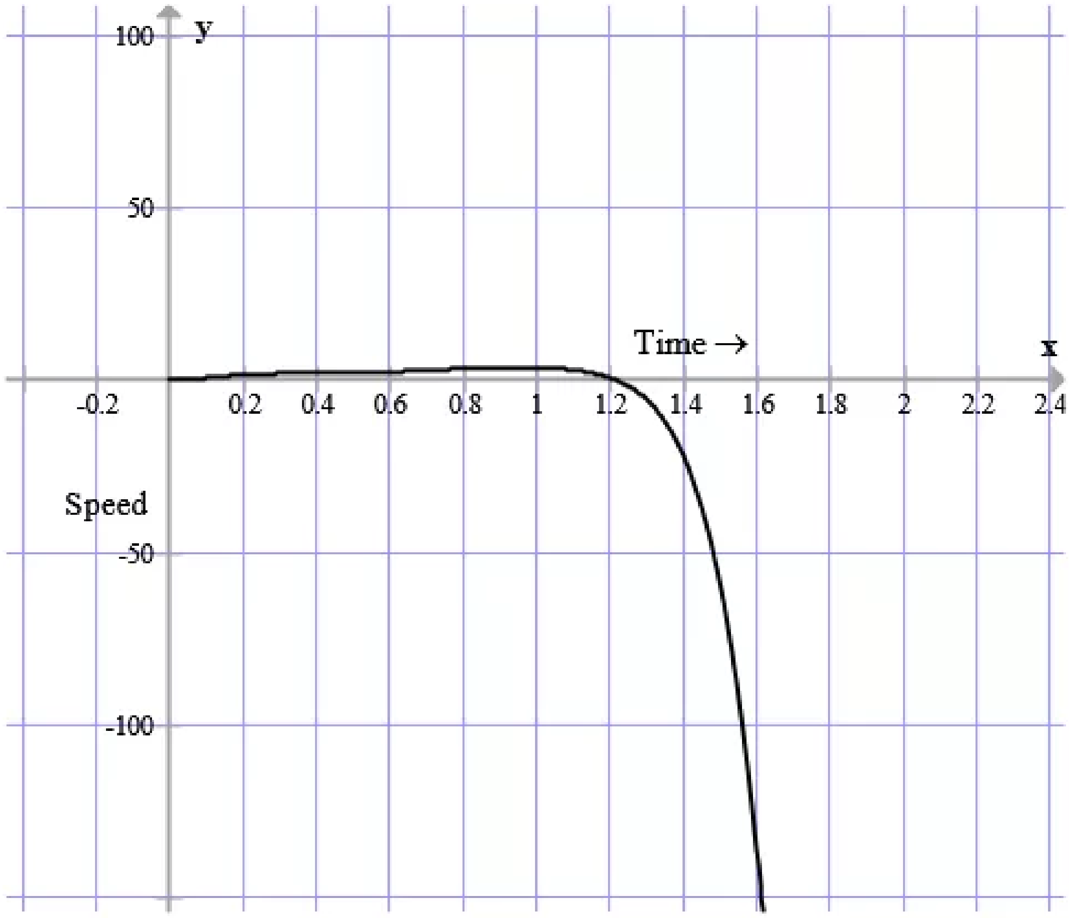
Answer 41e.

For a swimmer doing the breaststroke, the function can be found as

$$S = -241t^7 + 1060t^6 - 1870t^5 + 1650t^4 - 737t^3 + 144t^2 - 2.43t$$

We need to draw the graph of the function and have to find out at what time during the stroke the swimmer is going the fastest.

The graph of the function is shown below:



From the graph, we can see that at the time approximately $t \approx 0.96$ the swimmer can go fastest.

Therefore the time is $\boxed{t=1}$ where the swimmer take the breaststroke to go fastest.

Answer 42e.

Consider the graph

(a)

The building of a greenhouse that is shaped like half a cylinder is shown below:

The surface area of a green house is given by

$S = \pi r^2 + \pi r l$ ...... (1)

It is also given that the surface area is given as 600 square feet. We need to find the expression for l in terms of r.

Substituting $S = 600$ square feet in equation (1), we have

$$600 = \pi r^2 + \pi r l$$
$$600 - \pi r^2 = \pi r^2 + \pi r l - \pi r^2 \qquad \left[\text{Subtracting } \pi r^2 \text{ from both sides}\right]$$
$$\pi r l = 600 - \pi r^2$$

$\boxed{l = \frac{1}{\pi r}\left(600 - \pi r^2\right)}$ $\left[\text{Dividing both sides by } \pi r\right]$...... (2)

(b)

It is given that the volume of a green house is given by

$V = \frac{1}{2}\pi r^2 l$ ...... (3)

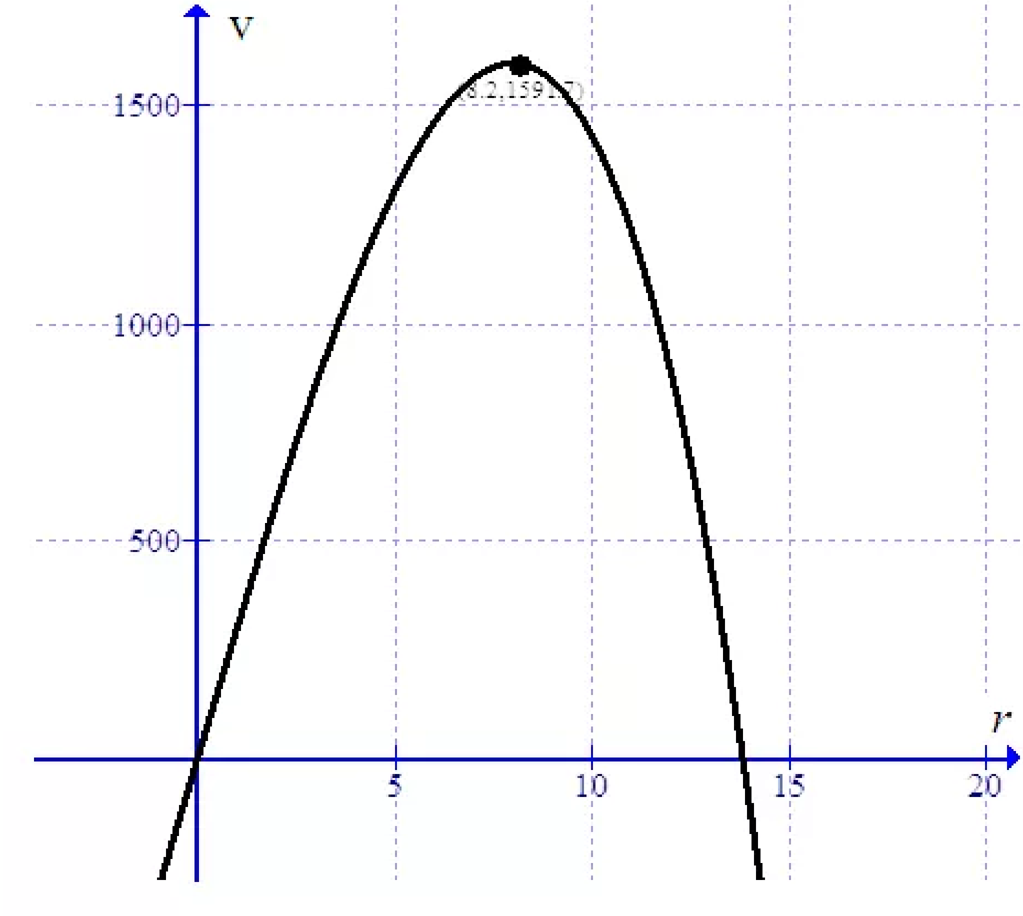
Substituting the value of l in equation (3) we have

$$V = \frac{1}{2}\pi r^2 \cdot \frac{1}{\pi r}\left(600 - \pi r^2\right)$$

$\boxed{V = 300r - \frac{1}{2}\pi r^3}$.

(c)

The graph of the volume function $V = 300r - \frac{1}{2}\pi r^3$ is shown below:



As seen from the graph, the maximum volume is $\approx \boxed{1592\,\text{cm}^3}$ and the radius is $\boxed{8.2\,\text{cm}}$

Now,

Substitute the volume of radius $r = 8.2\,\text{cm}$ in equation (2) we have

$$\begin{aligned} l &= \frac{1}{\pi r}\left(600 - \pi r^2\right) \\ &= \frac{1}{8.2\pi}\left(600 - \pi \times (8.2)^2\right) \\ &= 15.09 \end{aligned}$$

Thus,

$l \approx \boxed{15\,\text{cm}}$

$r = \boxed{8.2\,\text{cm}}$

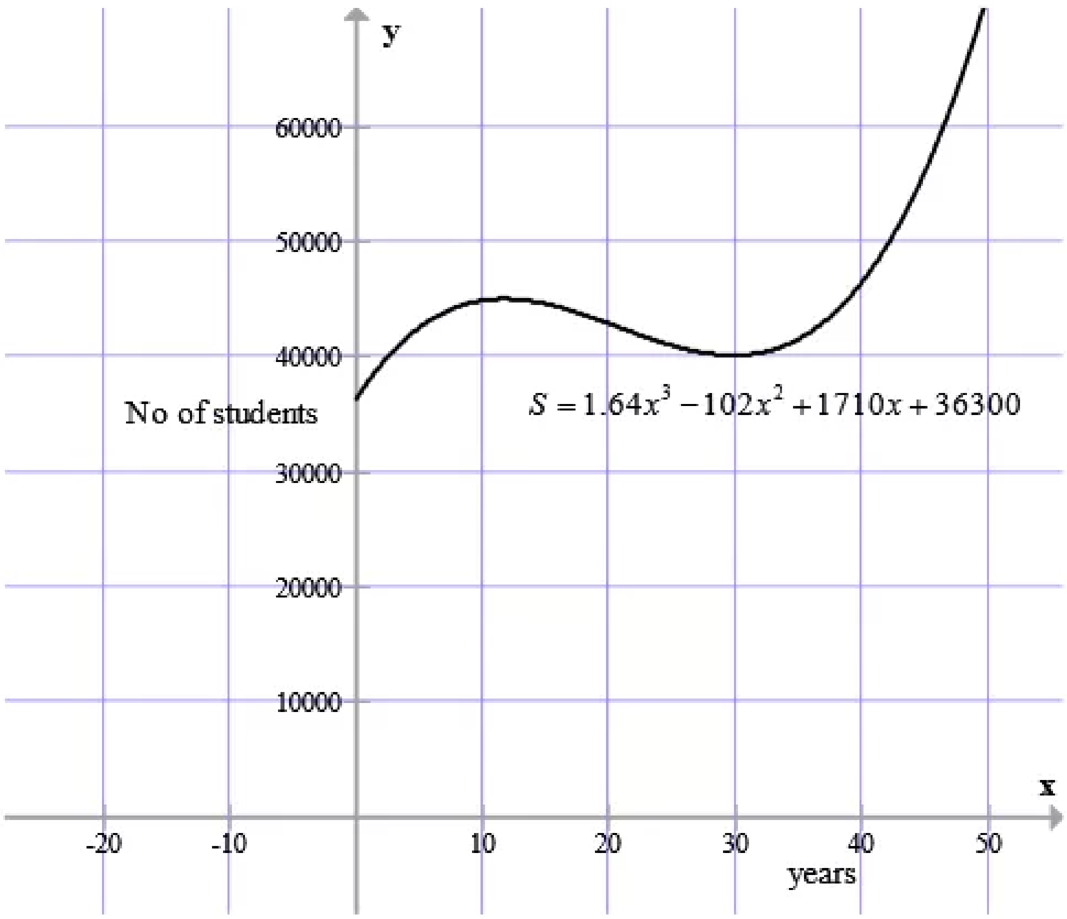
$V = \boxed{1592\,\text{cm}^3}$.

Answer 43e.

(a)

The number of students S enrolled in public schools in the United States can be modeled by $S = 1.64x^3 - 102x^2 + 1710x + 36300$, where x denote number of years, in the years from 1960 to 2001 years. We need to graph the function.

The graph of the function is shown below:



(b)

From the graph, we can see that there are two turning points in the domain $0 \le x \le 41$. At the turning point $x = 10$ for the function $S = 1.64x^3 - 102x^2 + 1710x + 36300$ the student enrolled is maximum, and at the point $x = 30$ the enrolled is minimum.

(c)

From the graph, we can find the range of the function for the domain $0 \le x \le 41$.

Putting $x = 0$ in the function $S = 1.64x^3 - 102x^2 + 1710x + 36300$, we get

$$S = 36300$$

And for $x = 41$ the maximum value of function is

$$S = 1.64(41)^3 - 102(41)^2 + 1710(41) + 36300$$
$$= 47978.44$$

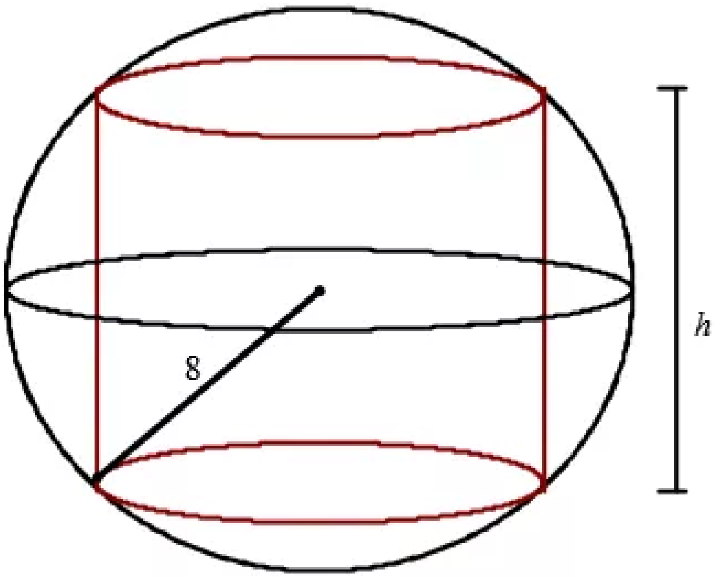
We can see that the graph is continuous in the domain $0 \le x \le 41$, therefore the range is

$$[36300, 47978.44]$$

Answer 44e.

A cylinder is inscribed in a sphere of radius 8. We need to write a equation of the volume of the cylinder as a function of h. the value of h that maximized the volume of the inscribed cylinder.
And have to find the maximum volume of the cylinder.

Considering the figure:



Suppose, the radius of circle of the cylinder is r. height of the cylinder is h.
We can find the radius by,

$$r = \sqrt{8^2 - \left(\frac{h}{2}\right)^2}$$
$$= \sqrt{64 - \frac{h^2}{4}}$$

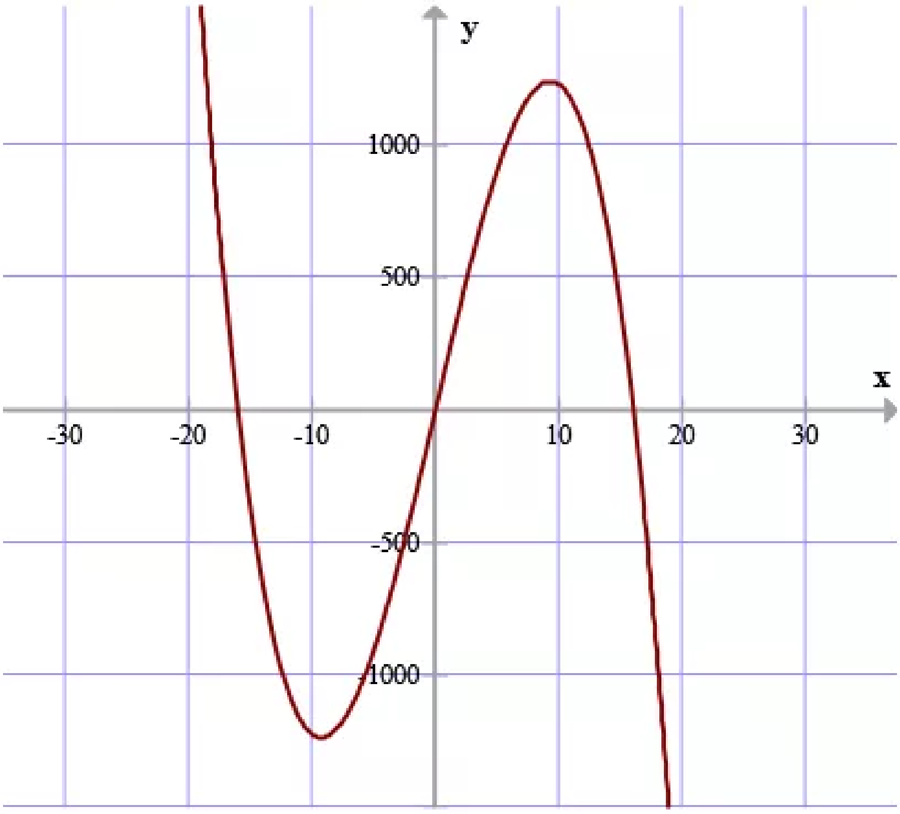
Therefore the volume of the cylinder can be found as,

$$V = \pi r^2 h$$
$$= \pi\left(\sqrt{64 - \frac{h^2}{4}}\right)^2 h$$
$$= \pi\left(64 - \frac{h^2}{4}\right) \cdot h$$

This volume can be write as a function of h, as

$$V(h) = \pi\left(64 - \frac{h^2}{4}\right) \cdot h$$

Drawing the graph of the function $V(h) = \pi\left(64 - \frac{h^2}{4}\right) \cdot h$ we get,



From the graph we can see that at the value $\boxed{h = 9}$ maximum the volume of inscribed cylinder.
The maximum volume of the cylinder is $\boxed{1238}$.

Answer 45e.

The direct variation equation for the variables x and y is $y = ax$.

Substitute 1 for x, and 5 for y in the equation.
$5 = a(1)$

Divide both the sides by 1 to solve for a.

$$\frac{5}{1} = \frac{a(1)}{1}$$
$$5 = a$$

Replace a with 5 in $y = ax$.
$y = 5x$

Thus, the direct variation equation that relates x and y is $y = 5x$.

Answer 46e.

The variable x and y vary directly. We need to write a equation to relates x and y.

As x and y vary directly so we can write,

$$x \propto y$$

If k is a constant than $x = ky$, therefore k will be

$$k = \frac{x}{y} \quad \text{...... (1)}$$

Now substituting $x = -2$ and $y = 8$, in equation (1) we have

$$k = \frac{-2}{8}$$
$$= -\frac{1}{4}$$

Substituting the value of k in equation (1) we have,

$$-\frac{1}{4} = \frac{x}{y}$$
$$y = -4x \quad \text{[Cross multiplicaton]}$$

Therefore the relation between x and y is $\boxed{y = -4x}$

Answer 47e.

The direct variation equation for the variables x and y is $y = ax$.

Substitute 3 for x, and -5 for y in the equation.
$3 = a(-5)$

Divide both the sides by -5 to solve for a.

$$\frac{3}{-5} = \frac{a(-5)}{-5}$$
$$-\frac{3}{5} = a$$

Replace a with $-\frac{3}{5}$ in $y = ax$.

$$y = -\frac{3}{5}x$$

Thus, the direct variation equation that relates x and y is $y = -\frac{3}{5}x$.

Answer 48e.

The variable x and y vary directly. We need to write a equation to relates x and y.

As x and y vary directly so we can write,

$$x \propto y$$

If k is a constant than $x = ky$, therefore k will be

$$k = \frac{x}{y} \qquad \ldots\ldots (1)$$

Now substituting $x = 4$ and $y = 6$, in equation (1) we have

$$k = \frac{4}{6}$$

$$= \frac{2}{3}$$

Substituting the value of k in equation (1) we have,

$$\frac{2}{3} = \frac{x}{y}$$

$$2y = 3x \qquad \text{[Cross multiplicaton]}$$

$$y = \frac{3}{2}x \qquad \text{[2 is divided in both side]}$$

Therefore the relation between x and y is $\boxed{y = \frac{3}{2}x}$

Answer 49e.

The direct variation equation for the variables x and y is $y = ax$.

Substitute 5 for x, and -2 for y in the equation.

$5 = a(-2)$

Divide both the sides by -2 to solve for a.

$$\frac{5}{-2} = \frac{a(-2)}{-2}$$

$$-\frac{5}{2} = a$$

Replace a with $-\frac{5}{2}$ in $y = ax$.

$$y = -\frac{5}{2}x$$

Thus, the direct variation equation that relates x and y is $y = -\frac{5}{2}x$.

Answer 50e.

The variable x and y vary directly. We need to write a equation to relates x and y.

As x and y vary directly so we can write,

$$x \propto y$$

If k is a constant than $x = ky$, therefore k will be

$$k = \frac{x}{y} \qquad \text{...... (1)}$$

Now substituting $x = -12$ and $y = -4$, in equation (1) we have

$$k = \frac{-12}{-4}$$
$$= 3$$

Substituting the value of k in equation (1) we have,

$$3 = \frac{x}{y}$$
$$\frac{3}{1} = \frac{x}{y}$$
$$3y = x \qquad \text{[Cross multiplicaton]}$$
$$y = \frac{1}{3}x \qquad \text{[3 is divided in both sides]}$$

Therefore the relation between x and y is $\boxed{y = \frac{1}{3}x}$

Answer 51e.

The vertex form of a quadratic function is $y = a(x-h)^2 + k$, where the vertex is (h, k).

It is given that the vertex is (5, 4). Replace h with 5, and k with 4 in the equation.
$y = a(x-5)^2 + 4$

Since the graph passes through (3, 12), the point must satisfy the equation,
$y = a(x-5)^2 + 4$.

Substitute 3 for x, and 12 for y.
$12 = a(3-5)^2 + 4$

Solve for a.

$$12 = 4a + 4$$
$$8 = 4a$$
$$2 = a$$

Replace a with 2 in $y = a(x-5)^2 + 4$.
$y = 2(x-5)^2 + 4$

The quadratic function having a graph with the given characteristics is $y = 2(x-5)^2 + 4$.

Answer 52e.

The equation of a quadratic function of vertex (h,k) is

$$y=a(x-h)^2+k \qquad \ldots\ldots (1)$$

Here the vertex is $(-4,-6)$. Therefore

Putting the vertex $(-4,-6)$ in equation (1) we have,

$$y=a(x-(-4))^2+(-6)$$

$$y=a(x+4)^2-6 \qquad \ldots\ldots (2)$$

Since the function passes through $(2,3)$, therefore

$$3=a(2+4)^2-6$$

$$3=a(6)^2-6$$

$$3+6=36a \qquad \left[+6 \text{ is addded in both side, } (6)^2=36\right]$$

$$36a=9 \qquad [\text{Side change}]$$

$$a=\frac{9}{36}$$

$$a=\frac{1}{4}$$

Substituting the value $a=\frac{1}{4}$ in equation (2), we have

$$y=\frac{1}{4}(x+4)^2-6$$

Therefore the quadratic function of the vertex $(-4,-6)$ and passes through the point $(2,3)$ is $\boxed{y=\frac{1}{4}(x+4)^2-6}$.

Answer 53e.

Step 1 Substitute the coordinates of each point into $y=ax^2+bx+c$ to obtain a system of three linear equations.

Substitute -3 for x, and 12 for y in the equation and simplify.
$12=a(-3)^2+b(-3)+c$

Simplify.
$12=9a-3b+c$ Equation 1

Now, substitute -2 for x, and 3 for y in $y=ax^2+bx+c$.
$3=a(-2)^2+b(-2)+c$

Simplify.
$3=4a-2b+c$ Equation 2

Replace x with 1, and y with 0.
$0 = a(1)^2 + b(1) + c$

Simplify.
$0 = a + b + c$ Equation 3

Step 2 Rewrite the system of three equations in Step 1 as a system of two equations.

Add −2 times Equation 1 to 3 times Equation 2 to eliminate b.

$$\begin{array}{lll} 12 = 9a - 3b + c & \times\ -2 & -36 = -18a + 6b - 2c \\ 3 = 4a - 2b + c & \times\ 3 & \underline{9 = 12a - 6b + 3c} \\ & & 27 = -6a + c \quad \text{Revised Equation 1} \end{array}$$

Add 2 times Equation 3 to Equation 2 to eliminate b again.

$$\begin{array}{lll} 3 = 4a - 2b + c & & 3 = 4a - 2b + c \\ 0 = a + b + c & \times\ 2 & \underline{0 = 2a + 2b + 2c} \\ & & 3 = 6a + 3c \quad \text{Revised Equation 2} \end{array}$$

Step 3 Now, solve the system consisting of Revised Equations 1 and 2. In order to eliminate a, add the two equations.

$$\begin{array}{l} 27 = -6a + c \\ \underline{3 = 6a + 3c} \\ 30 = 4c \end{array}$$

Solve for c.

$$\frac{30}{4} = a$$

$$\frac{15}{2} = a$$

Substitute $\frac{15}{2}$ for a in Revised Equation 1.

$$27 = -6\left(\frac{15}{2}\right) + c$$

Solve for c.

$$27 + 45 = c$$

$$72 = c$$

Replace a with $\frac{15}{2}$, and c with 72 in Equation 3 and solve for b.

$$0 = \frac{15}{2} + b + 72$$

$$-\frac{159}{2} = b$$

Substitute $\frac{15}{2}$ for a, $-\frac{159}{2}$ for b, and 72 for c in $y = ax^2 + bx + c$.

$$y = \frac{15}{2}x^2 - \frac{159}{2}x + 72$$

Therefore, the quadratic equation is $y = \frac{15}{2}x^2 - \frac{159}{2}x + 72$.

Answer 54e.

We need to find the equation of a quadratic function which passes through the points

$(-2,19)$, $(2,-5)$ and $(4,-11)$

The general form of quadratic function is $y = ax^2 + bx + c$

Substituting the co-ordinates of each point into $y = ax^2 + bx + c$, we get the three linear equation as ,

$$19 = a(-2)^2 + b(-2) + c \qquad [\text{Substituted } x = -2, y = 19]$$

$$19 = 4a - 2b + c$$

$$-5 = a(2)^2 + b(2) + c \qquad [\text{Substituted } x = 2, y = -5]$$

$$-5 = 4a + 2b + c$$

$$-11 = a(4)^2 + b(4) + c \qquad [\text{Substituted } x = 4, y = -11]$$

$$-11 = 16a + 4b + c$$

Now solving the system equations as shown below, we get

$$19 = 4a - 2b + c \quad \ldots\ldots (1)$$
$$-5 = 4a + 2b + c \quad \ldots\ldots (2)$$
$$-11 = 16a + 4b + c \quad \ldots\ldots (3)$$

Subtracting (1) from equation (2),

$$-5 - 19 = (4a - 4a) + (2b - (-2b)) + c - c$$
$$-24 = 4b$$
$$b = \frac{-24}{4}$$
$$= -6$$

Adding equation (1) and (2) we get,

$$14 = 8a + 2c$$
$$7 = 4a + c \quad \ldots\ldots (4)$$

Substituting, $b = -6$ in equation (3) we have,

$$-11 = 16a + 4b + c$$
$$-11 = 16a + 4(-6) + c$$
$$-11 = 16a - 24 + c$$
$$-11 + 24 = 16a + c$$
$$13 = 16a + c \quad \ldots\ldots (5)$$

Subtracting equation (4) from (5) we get,

$$7 = 4a + c$$
$$13 = 16a + c$$
$$6 = 12a$$
$$a = \frac{6}{12}$$
$$= \frac{1}{2}$$

Substituting $a = \frac{1}{2}$ in equation (4), we have

$$7 = 4a + c$$
$$7 = 4\left(\frac{1}{2}\right) + c$$
$$7 = 2 + c$$
$$c = 5$$

Substituting $a=\frac{1}{2}$, $b=-6$, $c=5$ in the general form of the quadratic equation we get,

$$y=\left(\frac{1}{2}\right)x^2+(-6)x+(5)$$

$$y=\frac{1}{2}x^2-6x+5$$

Therefore the general form of the quadratic equation is $\boxed{y=\frac{1}{2}x^2-6x+5}$

Answer 55e.

In the given expression, a product is raised to a power. We can simplify such expressions using the power of a product property, which states that $(ab)^m=a^mb^m$.

$$\left(3^2x^3\right)^5=\left(3^2\right)^5\left(x^3\right)^5$$

By the power of a power property, $(a^m)^n=a^{mn}$.
Raise each power to the power 5 using this property and simplify.

$$\left(3^2\right)^5\left(x^3\right)^5=3^{2\cdot5}x^{3\cdot5}$$
$$=3^{10}x^{15}$$
$$=59{,}049x^{15}$$

Therefore, the given expression simplifies to $59{,}049x^{15}$, and we used the power of a product property and the power of a power property.

Answer 56e.

We need to simplify the expression $\left(x^2y^4\right)^{-1}$

We can simplify the expression through the power of product rule,

$$(ab)^m=a^mb^m$$

Applying this rule in the expression we get.

$$\left(x^2y^4\right)^{-1}=\left(x^2\right)^{-1}\cdot\left(y^4\right)^{-1}$$

$$=x^{2\cdot(-1)}\cdot y^{4\cdot(-1)} \qquad \text{[Power of a power rule]}$$

$$=x^{-2}\cdot y^{-4} \qquad \text{[Power of a power rule]}$$

Therefore the simplification of the expression is $\left(x^2y^4\right)^{-1}=\boxed{x^{-2}\cdot y^{-4}}$.

Answer 57e.

In the given expression, there is a product is raised to the power −3. We can simplify such expressions using the power of a product property, which states that $(ab)^m = a^m b^m$.

$$\left(xy^3\right)\left(x^{-2}y\right)^{-3} = \left(xy^3\right)\left(x^{-2}\right)^{-3}(y)^{-3}$$

By the power of a power property, $(a^m)^n = a^{mn}$.

Simplify $\left(x^{-2}\right)^{-3}$ using this property.

$$\begin{aligned}\left(xy^3\right)\left(x^{-2}\right)^{-3}(y)^{-3} &= \left(xy^3\right)\left(x^{-2(-3)}y^{-3}\right)\\ &= \left(xy^3\right)\left(x^6y^{-3}\right)\end{aligned}$$

Use the properties of multiplication to rearrange the terms.

$$\left(xy^3\right)\left(x^6y^{-3}\right) = \left(x \cdot x^6\right)\left(y^3 \cdot y^{-3}\right)$$

Now, simplify using the product of powers property which states that $a^m \cdot a^n = a^{m+n}$.

$$\begin{aligned}\left(x \cdot x^6\right)\left(y^3 \cdot y^{-3}\right) &= x^{1+6} \cdot y^{3+(-3)}\\ &= x^7 \cdot y^0\end{aligned}$$

By the zero exponent property, $a^0 = 1$.

$$\begin{aligned}x^7 \cdot y^0 &= x^7 \cdot 1\\ &= x^7\end{aligned}$$

Therefore, the given expression simplifies to x^7, and we used the power of a product property, the power of a power property, product of powers property, and the negative exponent property.

Answer 58e.

We need to simplify the expression $-4x^{-3}y^0$

We can simplify the expression as follows,

$$\begin{aligned}-4x^{-3}y^0 &= -4 \cdot \frac{1}{x^3} \cdot y^0 && \left[\text{Negative Exponent, } a^{-m} = \frac{1}{a^m}, a \neq 0\right]\\ &= -4 \cdot \frac{1}{x^3} \cdot 1 && \left[\text{Zero Exponent rule, } a^0 = 1, a \neq 0\right]\\ &= -\frac{4}{x^3}\end{aligned}$$

Therefore the simplification of the expression is $-4x^{-3}y^0 = \boxed{-\frac{4}{x^3}}$.

Answer 59e.

The given expression is the quotient of two powers having like bases.
We can simplify expressions of such form using the quotient of powers property which states that $\frac{a^m}{a^n} = a^{m-n}$.

$$\frac{x^6}{x^{-2}} = x^{6-(-2)}$$
$$= x^8$$

Therefore, the given expression simplifies to x^8, and we used the quotient of powers property.

Answer 60e.

We need to simplify the expression $\frac{3x^2y}{12xy^{-1}}$

We can simplify the expression as follows,

$$\frac{3x^2y}{12xy^{-1}} = \frac{3}{12}\cdot\left(\frac{x^2}{x}\right)\cdot\left(\frac{y}{y^{-1}}\right) \qquad \text{[Separated the components]}$$

$$= \frac{1}{4}\cdot\left(x^{2-1}\right)\cdot\left(y^{1-(-1)}\right) \qquad \left[\text{Quotient of powers, } \frac{a^m}{a^n} = a^{m-n},\ a \neq 0\right]$$

$$= \frac{1}{4}\cdot x\cdot y^{1+1}$$

$$= \frac{xy^2}{4}$$

Therefore the simplification of the expression is $\frac{3x^2y}{12xy^{-1}} = \boxed{\frac{xy^2}{4}}$.

Answer 61e.

Begin by rewriting the given quotients as a product of fractions.

$$\frac{8xy}{7x^4}\cdot\frac{7x^5y}{4y^2} = \frac{8\cdot 7}{7\cdot 4}\cdot\frac{x\cdot x^5}{x^4}\cdot\frac{y\cdot y}{y^2}$$

Now, apply the product of powers property which states that $a^m\cdot a^n = a^{m+n}$, on the numerator.

$$\frac{8\cdot 7}{7\cdot 4}\cdot\frac{x\cdot x^5}{x^4}\cdot\frac{y\cdot y}{y^2} = \frac{8\cdot 7}{7\cdot 4}\cdot\frac{x^{1+5}}{x^4}\cdot\frac{y^{1+1}}{y^2}$$

$$= 2\cdot\frac{x^6}{x^4}\cdot\frac{y^2}{y^2}$$

Simplify using the quotient of powers property $\frac{a^m}{a^n} = a^{m-n}$.

$$2 \cdot \frac{x^6}{x^4} \cdot \frac{y^2}{y^2} = 2 \cdot x^{6-4} \cdot y^{4-2}$$
$$= 2 \cdot x^2 \cdot y^0$$

By the zero exponent property, $a^0 = 1$.

$$2 \cdot x^2 \cdot y^0 = 2 \cdot x^2 \cdot 1$$
$$= 2x^2$$

Therefore, the given expression simplifies to $2x^2$, and we used the product of powers, quotient of powers, and the zero exponent properties.

Answer 62e.

We need to simplify the expression $\left(\frac{5x^3y^7}{25x^2y^4}\right)^3$

We can simplify the expression as follows,

$$\left(\frac{5x^3y^7}{25x^2y^4}\right)^3$$

$$=\left(\frac{5}{25}\cdot\left(\frac{x^3}{x^2}\right)\cdot\left(\frac{y^7}{y^4}\right)\right)^3 \quad \text{[Separated the components]}$$

$$=\left(\frac{1}{5}\cdot\left(x^{3-2}\right)\cdot\left(y^{7-4}\right)\right)^3 \quad \left[\text{Quotient of powers, } \frac{a^m}{a^n}=a^{m-n},\ a\neq 0\right]$$

$$=\left(\frac{1}{5}\cdot x\cdot y^3\right)^3$$

$$=\left(\frac{1}{5}\right)^3 x^3\left(y^3\right)^3 \quad \left[\text{Power of Product, } (ab)^m = a^m b^m\right]$$

$$=\frac{1^3}{5^3}x^3\left(y^3\right)^3 \quad \left[\text{Power of Quotient, } \left(\frac{a}{b}\right)^m=\frac{a^m}{b^m},\ b\neq 0\right]$$

$$=\frac{1}{125}x^3y^{3\cdot 3} \quad \left[\text{Power of Power, } \left(a^m\right)^n = a^{mn}\right]$$

$$=\frac{1}{125}x^3y^9$$

Therefore the simplification of the expression is $\left(\frac{5x^3y^7}{25x^2y^4}\right)^3 = \boxed{\frac{1}{125}x^3y^9}$.