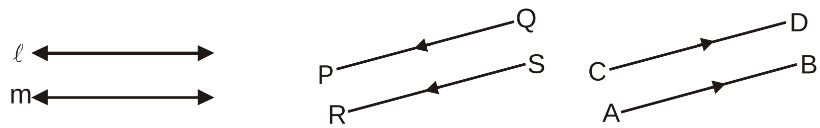
PARALLEL LINES

PARALLEL LINES

If two lines AB and CD lie in the same plane and do not intersect when produced on either side, then such lines are said to be parallel to each other and we write AB||CD.

(i) ℓ||m (ii) PQ||RS (iii) AB||CD



Parallel lines are indicated by arrow heads drawn in the same direction.

The angles of any pair of interior angles on the some side of transversal are supplementary then the lines are parallel.

INTERSECTING LINES AND NON-INTERSECTING LINES

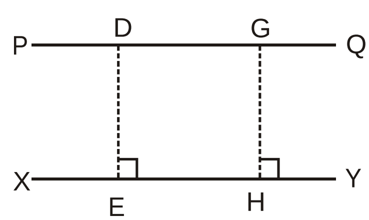
Lines PQ and RS in fig (i) are intersecting lines and in fig (ii) are parallel lines that the lengths of the common perpendicular at different points on these parallel line is the same.

This equal length is called the distance between two parallel lines.

(i) Intersecting Lines (ii) Non -Intersecting lines

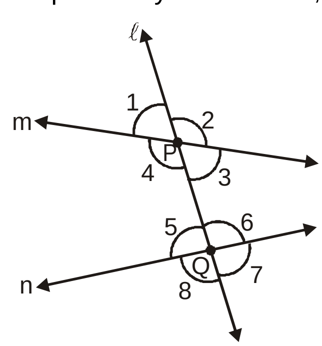
- The distance between two parallel lines always remains the same.

This distance is given by the length of perpendicular drawn from any point on either line to the other line. Thus, in the adjoining figure, if PQ||XY, then DE = GH.



PARALLEL LINES AND A TRANSVERSAL

- Transversal : A line which intersects two or more lines at distinct points is called a transversal, line ℓ intersects lines m and n at points P and Q respectively. Therefore, line ℓ is a transversal for lines m and n.



we observe that four angles are formed at each of the points P and Q. Let us name these angles as $\angle 1$, $\angle 2$,.....$\angle 8$ as shown.

These eight angles can be classified into following groups :

(a) Exterior Angles :- In above figure. $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are called exterior angles.

(b) Interior Angles: In above fig. $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$ are called interior angle.

(c) Corresponding Angles : Two angles on the same side of transversal are known as corresponding angles, if both lie either above the two lines or below the two lines. The following pairs of angles are the pairs of corresponding angles :

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$ (iii) $\angle 4$ and $\angle 8$ (iv) $\angle 3$ and $\angle 7$

è Each pair of corresponding angles are equal

(d) Alternate Interior Angles : The following pairs of angles are the pairs of alternate interior angles :

(i) $\angle 4$ and $\angle 6$ (ii) $\angle 3$ and $\angle 5$

è Each pair of alternate interior angles are equal.

(e) Alternate Exterior Angles : The following pairs of angles are the pairs of alternate exterior angles :

(i) $\angle 1$ and $\angle 7$ (ii) $\angle 2$ and $\angle 8$

è Each pair of alternate exterior angles are equal.

(f) Consecutive Interior Angles or Co-interior Angles : The pairs of angles on the same side of the transversal are called pairs of consecutive interior angles. The following pairs of angles are the pairs of consecutive interior angles :

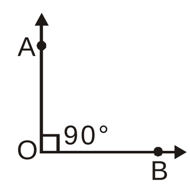
(i) $\angle 4$ and $\angle 5$ (ii) $\angle 3$ and $\angle 6$

è Each pair of consecutive interior angles are supplementary.

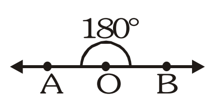
TYPES OF ANGLES

(i) Zero Angle : If the initial and the final positions of a ray coincide without making a revolution, then the angle formed is a zero angle. In the adjoining figure, $\angle AOB$ is a zero angle i.e. $\angle AOB = 0°$.

(ii) Right Angle : The angle formed at the corner of a rectangular blackboard is a right angle. Measure of a right angle is 90°. In the adjoining figure, $\angle AOB$ = 1 right angle = 90°.

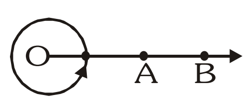


(iii) Straight Angle : An angle formed by two opposite rays is called a straight angle. Measure of a straight angle is 180°. In the adjoining figure, the angle formed by two opposite rays OA

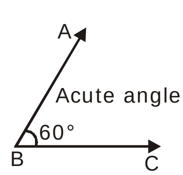


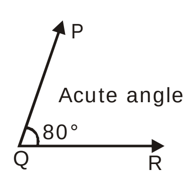
and OB is a straight angle.

(iv) Complete Angle : If a rotating ray, after completing one rotation coincides with the initial position, then the angle formed is called a complete angle. Measure of a complete angle is 360°. In the adjoining figure, arms OA and OB coincide after making a complete revolution

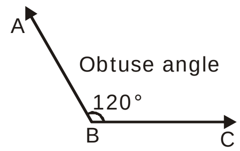


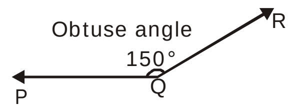
(v) Acute Angle : An angle which is greater than 0° but less than 90° is known as an acute angle.



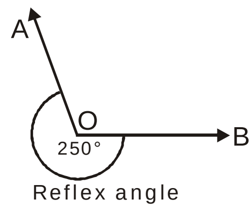


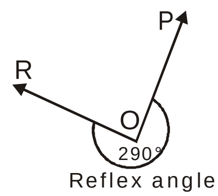
(vi) Obtuse Angle : An angle which is greater than a right angle (i.e., 90°) but less than a straight angle (i.e., 180° is called on obtuse angle.



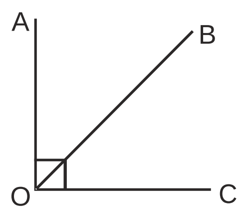


(vii) Reflex Angle : An angle which is more than 180° but less than 360° is called a reflex angle.

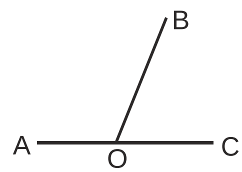




COMPLEMENTARY ANGLES AND SUPPLEMENTARY ANGLES



(i)



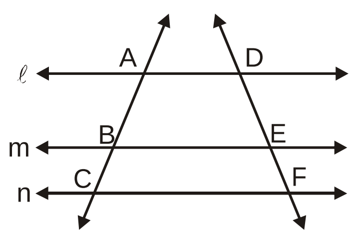
(ii)

- Complementary angles : Two acute angles are called complementary angles or complements of each other if the sum of their measures is a right angle i.e. 90°.

Here fig-(i) $\angle AOB + \angle BOC = 90°$. Therefore, $\angle AOB$ and $\angle BOC$ are complementary angles.

- Supplementary angles : Two angles are said to be supplementary if the sum of their measures is two right angles i.e. 180°

fig- (ii) $\angle AOB + \angle BOC = 180°$

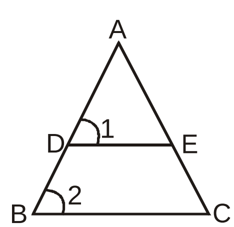


LINE JOINING MID POINTS OF SIDES OF A TRIANGLES

The line segment joining mid points of two sides of a triangles is parallel to the third side and is half of it.

Draw any ΔABC, mark the mid points D & E of sides AB & AC respectively. Join D to E measure BC and DE

$\overline{DE}$ $\quad$ $\overline{BC}$ and $\overline{DE} = \frac{1}{2}\overline{BC}$



Measure $\angle 1$ and $\angle 2$

that they are equal and form a fair of corresponding angles $\overline{DE}$ and $\overline{BC}$

PROPORTIONAL INTERCEPT PROPERTY

If a line is drawn parallel to one side of a triangle, intersecting the other two sides, then it divides the other two sides in the same ratio.

The intercepts made by three or more parallel lines on two or more transversals are always proportional in figures $\ell \parallel m \parallel n$ then

$$\frac{AB}{BC} = \frac{DE}{EF}$$

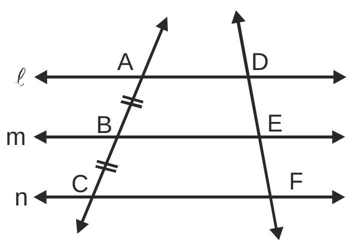
'EQUAL INTERCEPT PROPERTY

If three or more parallel lines make equal intercepts on one transversal, they make equal intercepts on any other transversal.

Draw three lines $\ell \parallel m \parallel n$ and transversals p and q as shown in the fig.

The transversal 'p' intercepts lines ℓ, m and n in such a manner that AB = BC.

Now measure the intercepts $\overline{DE}$ and $\overline{EF}$ on q.

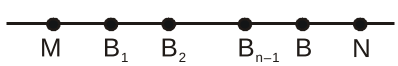


What do you observe?

$\overline{DE} = \overline{EF}$. Thus, we conclude that

DIVISION OF A LINE SEGMENT INTO EQUAL PARTS

We conclude that if MN is any line segment & 'n' is a positive integer, AB is divided into n equal parts of point B_1, $B_2 \ldots B_{n-1}$.

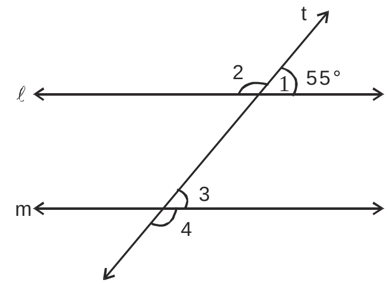


or $MB_1 = B_1B_2 = B_2B_3 = B_3B_4 = \ldots. = B_{n-1}B_n = B_nN$

Let us now learn to divide a given line segment into 'n' equal parts.

SOLVED EXAMPLES

Ex.1 In the figure ℓ||m, If Ď1 = 55°, find Ď2, Ď3 and Ď4.



Sol. $\angle 1 + \angle 2 = 180°$ (supplementary angles as they form a linear pair)

$\angle 2 = 180° - 55° = 125°$

$\angle 1 = \angle 3$ [pair of corresponding angles]

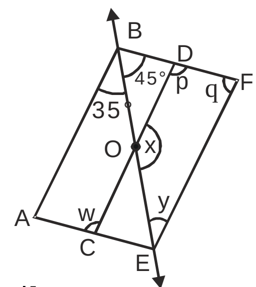
$\angle 3 = 55°$

$\angle 2 = \angle 4$

$\angle 4 = 125°$

Thus $\angle 2$, $\angle 3$ and $\angle 4$ are 125°, 55° and 125° respectively.

Ex.2 In the figure, $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$ and $\overline{AE}$ ⊔ $\overline{BF}$. Find Ďx, Ďy, Ďp, Ďq and Ďw.



Sol. We have, $\overline{AB}$ ⊔ $\overline{CD}$ and $\overline{BF}$ is a transversal.

$\therefore \angle ABD = \angle CDF$ [Corresponding angles]

$\therefore 35° + 45° = \angle CDF$

or $\angle p = 80°$

Now, $\overline{CD}$ ⊔ $\overline{EF}$ and $\overline{DF}$ is a transversal.

$\therefore \angle p + \angle q = 180°$ [a pair of interior angles on the same side of the transversal]

$\therefore \angle q = 180° - 80° = 100°$

Also, $\overline{AB}$ ⊔ $\overline{CD}$ and $\overline{BF}$ is transversal.

$\therefore \angle ABO = \angle BOD$ [alternate interior angles]

$\therefore \angle BOD = 35°$

$\angle x + \angle BOD = 180°$ [Linear pair

$\therefore \angle x = 180° - 35° = 145°$ [alternate interior angles]

$\angle ABE = \angle y$

$\therefore \angle y = 35°$.

$\overline{AB}$ ⊔ $\overline{BD}$ and $\overline{CD}$ is a transversal

$\angle w = \angle p = 80°$ [Alternate interior angles]

Hence, $\angle p = 80°$, $\angle q = 100°$, $\angle x = 145°$, $\angle y = 35°$ and $\angle w = 80°$

Ex.3 In fig AB||CD and CD||EF. Also EA {⫶AB. If ĎDBEF = 55°. find the values of x, y and z.

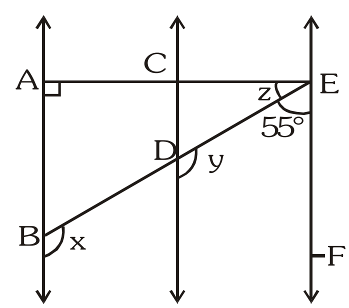
Sol. y = 55° = 180° (Interior angle on the same side of the transversal ED)

Therefore, y = 180° – 55° = 125°

Again x = y

(AB || CD, corresponding angles axiom)

Therefore x = 125°



Now, since AB||CD, corresponding angles axiom)

Therefore x = 125°

Now, since AB||CD and CD||EF, therefore, AB||EF.

So, $\angle EAB + \angle FEA = 180°$ (Interior angles on the same side of the transversal EA)

Therefore, 90° + z + 55° = 180°

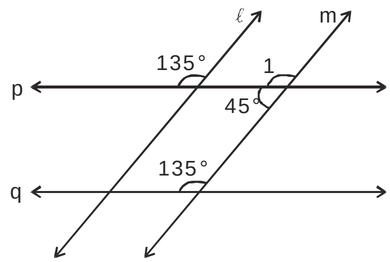
Where gives z = 35°

Ex.4 In the figure $\ell \| m$ and $p \| q$. Give reasons.

Sol. $p \| q$ (one pair interior angles on the same side of transversal is supplementary.

Now, $\angle 1 = 180° - 45° = 135°$ (Linear pair)

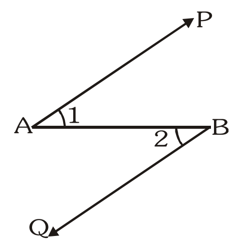
$\therefore \ell \| m$; p is transversal and one pair of corresponding angles is equal.



Ex.5 Divide a line-segment $\overline{AB}$ = 6 cm into 5 equal parts.

Sol. (i) Draw a line segment $\overline{AB}$ = 6 cm.

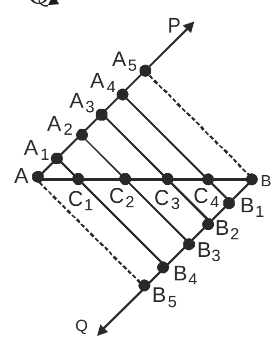
(ii) At A, draw ray $\overrightarrow{AP}$ making an acute angle with AB.



(iii) At B, draw a ray $\overrightarrow{BQ}$ parallel to $\overrightarrow{AP}$ on the opposite side of $\overline{AB}$ as shown in the figure by constructing $\angle 2 = \angle 1$

(iv) Using a compass, mark 5 points on $\overrightarrow{AP}$ at equal distances and 5 points on $\overrightarrow{BQ}$ with the same distance say,

A_1, A_2, A_3, A_4, A_5 $\overrightarrow{AP}$ and B_1, B_2, B_3, B_4, B_5 $\overrightarrow{BQ}$.



(v) Join A_5 to B and B_5 to A.

(vi) Join A_1B_4, A_2B_3, A_3A_2, and A_4B_1. Intersecting $\overline{AB}$ at C_1, C_2, C_3, C_4.

(vii) Measure $\overline{AC_1}, \overline{C_1C_2}, \overline{C_2C_3}, \overline{C_3C_4}, \overline{C_4B}$. They are all equal. This is because intercepts on $\overrightarrow{AP}$ (or $\overrightarrow{BQ}$) are equal so they are equal on the transversal $\overline{AB}$.

DIVISION OF A LINE SEGMENT IN A GIVEN RATIO INTERNALLY

Ex.6 Divide a line segment $\overline{AB}$ = 6 cm the ratio 4 : 3 internally.

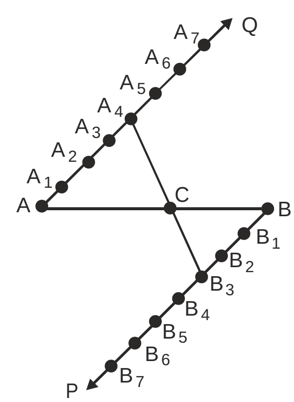
Sol. (i) Draw a line segment $\overline{AB}$ = 6 cm.

(ii) At A, draw ray $\overrightarrow{AP}$ and $\overrightarrow{BQ}$ parallel to each other and on opposite sides of $\overline{AB}$

(iii) Make 4 + 3 = 7 points on $\overrightarrow{AQ}$ and 7 on $\overrightarrow{BP}$ at equal distances namely

A_1, A_2, A_3, A_4, A_5 , A_6, A_7 on $\overrightarrow{AQ}$

and $B_1, B_2, B_3, B_4, B_5, B_6, B_7$ on BP respectively.

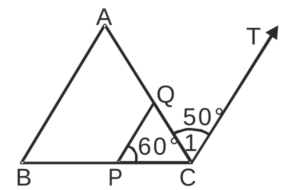


(iv) Join A_7 to B and B_7 to A.

(v) Join A_4 ot B_3 intersecting $\overline{AB}$ at C.

(vi) Thus, C divides AB internally in the ratio 4 : 3

Ex.7 In $\triangle ABC$, P is the mid-point of BC, Q is the mid-point of AC and $\overline{CT} \parallel \overline{AB}$. Find all angles of the triangle.



Sol. $\overline{PQ} \parallel \overline{AB}$ (line segment joining the midpoints of two sides of a triangle)

$\therefore \angle B = \angle P = 60°$ (Corresponding angles)

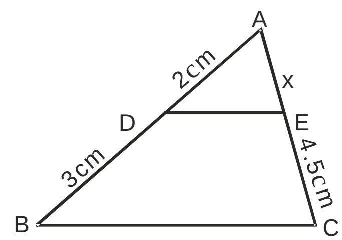
Also, $\angle A = \angle 1 = 50°$ ($\overline{AB} \parallel \overline{CT}$, AC is the transversal and angles from a pair of alternate interior angles)

$\therefore \angle C = 180° - (\angle A + \angle B)$ (Angle sum property of a triangle)

$\Rightarrow \angle C = 180° - (50° + 60°) = \angle C = 70°$

Thus, the angle of a ΔABC are $\angle A = 50°$, $\angle B = 60°$ and $\angle C = 70°$.

Ex.8 In $\triangle ABC$, $DE \parallel BC$. Find x.



Sol. Since $DE \parallel BC$,

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\therefore \quad \frac{2}{3} = \frac{x}{4.5}$$

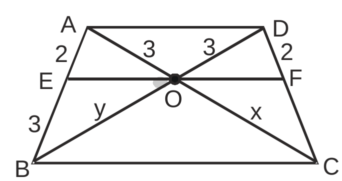
or $\quad x = \frac{2}{3} \times 4.5 = 2 \times 1.5$ cm

Thus, $\quad x = 3$ cm

Ex.9 In the figure, ABCD is a quadrilateral with O as point of intersection of diagonals, AB = DC and $AD \parallel BC$. Through O, EF is drawn parallel to the parallel sides of the quadrilateral. Find x & y and hence, show that AC = BD.

Sol. Since AB = DC and AE = DF = 2 cm, therefore, FC = FB = 3 cm.

In ΔBAC, $EO \parallel BC$.



$$\therefore \quad \frac{AE}{EB} = \frac{AO}{OC}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{3}{x} \text{ or } x = \frac{9}{2} \text{ cm}$$

In ΔBDC, $OF \parallel BC$.

$$\therefore \quad \frac{DO}{OB} = \frac{DF}{FC}$$

$$\frac{3}{y} = \frac{2}{3}$$

$$\Rightarrow \quad y = \frac{9}{2}$$

Now, $AC = AO + OC = 3 + x = 3 + \frac{9}{2} = \frac{15}{2}$ cm and $BD = BO + OD = y + 3 = \frac{9}{2} + 3 = \frac{15}{2}$ cm

$\therefore \quad AC = BD$

EXERCISE - I UNSOLVED PROBLEMS

Q.1 Draw a line segment AB = 5 cm and divide it internally into 6 equal parts.

Q.2 Draw a line segment of length 6.4 cm and divide it into 4 equal parts. What is the length of each part ?

Q.3 Draw a line segment of length 5.5 cm. Divide it internally in the ratio 2 : 3. What is the length of each part ?

Q.4 Draw a line segment of length 6.3 cm and divide it internally in the ratio 3 : 4.

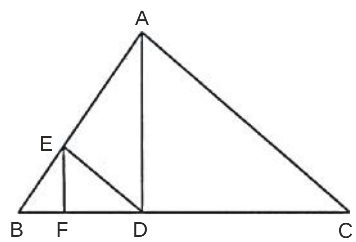
Q.5 Draw a line segment AB of length 7 cm and find a point P on it such that $\overline{AP}:\overline{PB}$ = 2: 3. Measure AP and PB.

Q.6 Draw a line segment of a given length. Divide it into four equal parts.

Q.7 Draw a line segment AB = 5.5 cm. Find a point P on it such that $\overline{AP} = \frac{2}{3}\overline{PB}$.

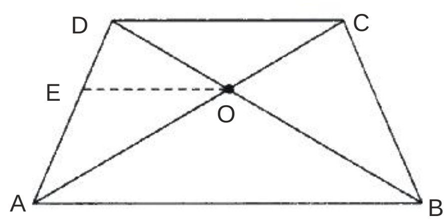
Q.8 Draw a line segment AB = 6 cm. Find a point Q on it such that $\overline{AQ} = \frac{2}{3}\overline{QB}$.

Q.9 In MBC, EF ∥ AD and ED ∥ AC. If BF = 2 cm, FD = 3 cm and BE = 4 cm, find BC.



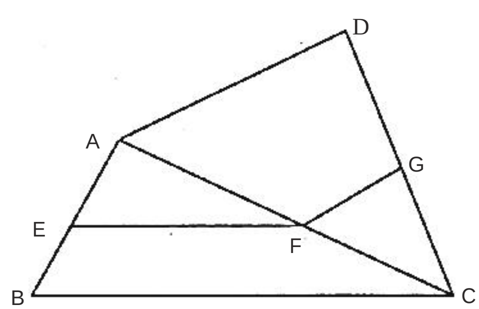
Q.10 In fig, ABCD is a quadrilateral with AB ∥ DC, and OE ∥ AB. If OA = 3 cm and OC = 2 cm, find

(i) AE : ED (ii) BO : OD (iii) If AE = 2.5 cm, find AD.

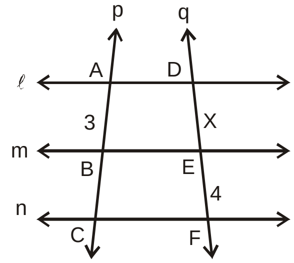


Q.11 ABC and DAC are two triangles having one side AC common and vertices Band D on opposite sides of AC. EF ∥ BC, FG ∥ AD. If AE = 4 cm, EB = 2.5 cm. and CG = 4 cm, find

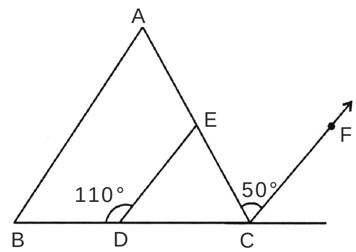
(i) AF : FC (ii) DG



Q.12 In the fig, $\ell \parallel m \parallel n$ and p and q are transversals. If AC = 9 cm, find x. Also find $\overline{DF}$.

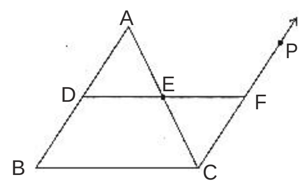


Q.13 In the fig, D and E are mid-points 'of sides $\overline{BC}$ and $\overline{AC}$ of ΔABC respectively. If $\overline{CF} \parallel \overline{AB}$, compute the angles of the triangle ABC.



Q.14 ABC is a triangle in which D and E are midpoints of sides AB and AC respectively. DE is extended to intersect $\overline{CP} \parallel \overline{AB}$ at F. Show that:

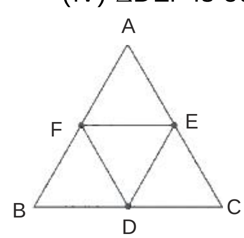
(i) $\angle$DAE = $\angle$FCE (ii) ΔADE $\cong$ ΔFCE (iii) $\overline{DE} \parallel \overline{BC}$ and $\overline{DE} = \frac{1}{2}\ \overline{BC}$



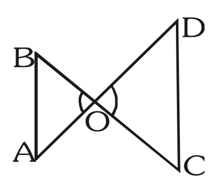
Q.15 Draw a ΔABC and join the mid-points of the three sides, D, E, F respectively. Show that:

(i) $\overline{FE} \parallel \overline{BC}$ (ii) $\overline{FD} \parallel \overline{AC}$

(iii) $\overline{DE} \parallel \overline{AB}$ (iv) ΔDEF is equi-angular to ΔABC.



Q.16 In the figure school that $\overline{AB} \parallel \overline{CD}$ given that $\angle$BOA = $\angle$BAO; $\angle$COD = $\angle$CDO.



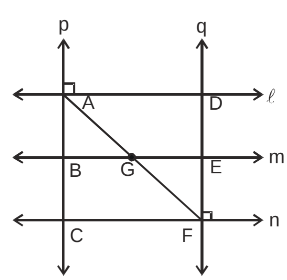
Q.17 In the fig no.1, $\ell \parallel m \parallel n$ and p $\perp$ ℓ, q $\perp$ n. Show that:

(i) $p \parallel q$

(ii) $\angle$ABE is a right angle.

(iii) $\angle$CFE is a right angle.

(iv) What about $\angle$ACF ? Is it also a right angle? Give reasons.



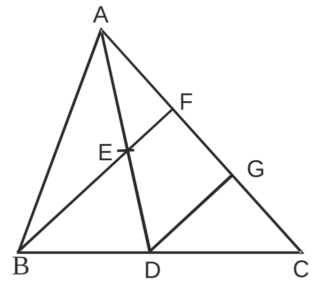
Q.18 In the fig. no.1 if AB = BC, show that:

(i) $\overline{AG} = \overline{GF}$ (ii) $\overline{DE} = \overline{EF}$

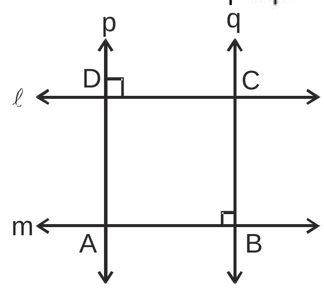
Q.19 In ΔABC of fig, AD is a median and E is the midpoint of AD. BE extended intersects AC at F. DG is drawn parallel to BE Show that:

(i) F is the mid-point of AG. (ii) G is the mid-point of FC

(iii) AF = FG = GC or F and G trisect AC



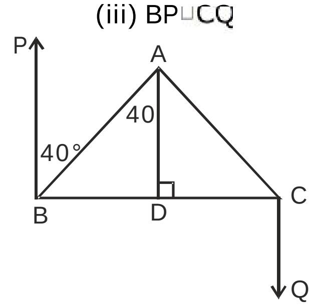
Q.20 In the fig no.2, $\ell \parallel m$ and p $\perp$ ℓ and p $\perp$ m. Show that p $\parallel$ q. Give reasons for your answer.



Q.21 What can you say about the quadrilateral ABCD given in fig no. 2. Is it a rectangle? Justify your answer.

Q.22 In the fig, no.3 ABC is a triangle and AD is an altitude. Show that:

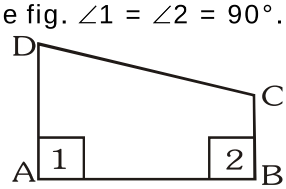
(i) BP $\parallel$ AD (ii) CQ $\parallel$ AD (iii) BP $\parallel$ CQ



Q.23 Draw a line segment AB of length 5 cm. At A and B construct lines perpendicular to AB. Also, draw the perpendicular bisector of AB. Are these three lines parallel to each other? Justify your answer.

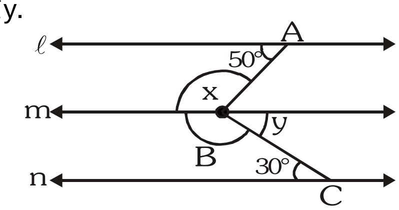
Q.24 $\angle$DAC = 30°, find the angles of ΔABC in the fig. no. 3

Q.25 In the quadrilateral ABCD shown in the fig. $\angle 1 = \angle 2 = 90°$. Is AD $\parallel$ BC? Justify your answer.



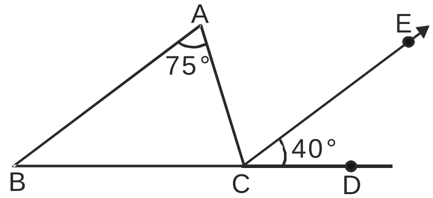
Q.25 Draw a line-segment $\overline{AB}$ = 6 cm. Mark two points P and Q on it. Draw lines perpendicular to AB through P and Q ($\overline{PR}$ and $\overline{QS}$). What can you say about $\overline{PR}$ and $\overline{QS}$? Are these parallel? Justify your answer.

Q.26 In the fig. $\ell \parallel m \parallel n$. Find $\angle$x and $\angle$y.

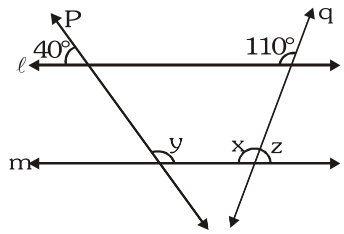


Q.27 ABCD is a quadrilateral in which all the four angles, $\angle A = \angle B = \angle C = \angle D = 90°$. Show that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$.

Q.28 In the fig, $\angle A = 75°$ and $\overline{CE} \parallel \overline{AB}$. If $\angle ECD = 40°$, find the other two angles of the triangle.



Q.29 In the fig, $\ell \parallel m$. Find $\angle x$, $\angle y$ and $\angle z$



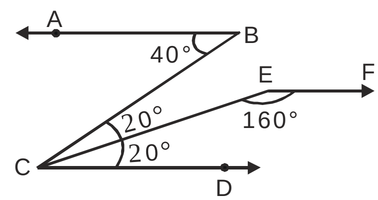
Q.30 In the fig, show that

(i) $\overline{AB} \parallel \overline{CD}$

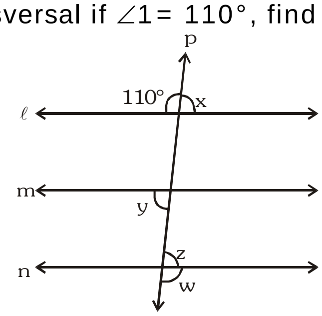
(ii) $\overline{CD} \parallel \overline{EF}$

(iii) $\overline{AB} \parallel \overline{EF}$

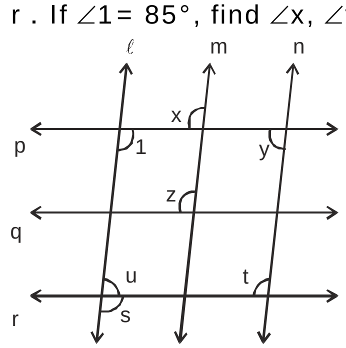
Justify your answer.



Q.31 In the figure $\ell \parallel m \parallel n$ and p is a transversal if $\angle 1 = 110°$, find angles x, y, z and w.



Q.32 In the figure, lines ℓ m n and p q r . If $\angle 1 = 85°$, find $\angle x$, $\angle y$, $\angle z$, $\angle t$, $\angle s$ and $\angle u$.



Q.33 A plot of land ABCD is divided into three as shown in figure if CD = 50 m and AD || BC || EF || GH, find the length of DF, FH and HC.