(a) 20 (b) 15 (c) 21 (d) 24

12. The mean of first five prime numbers is

(a) 5.0 (b) 4.5 (c) 5.6 (d) 6.5

WORKSHEET 5

Activity

Objective: To analyse language text by using frequency table **Method:** Take any text of about hundred words. Read the above lines and obtain the frequency table.

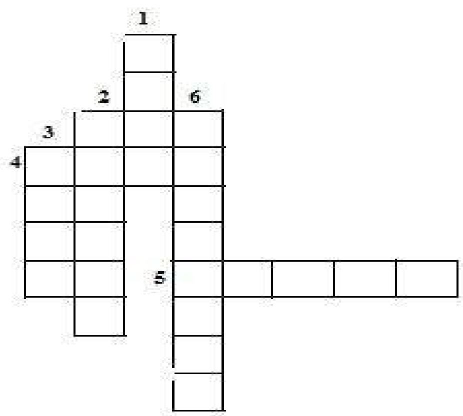
Letter	***Tally Marks***	***Frequency***
A		
B		
C		
D		
E		
.		
.		
.		
.		
.		
Z		

Now observe the table and answer the following:

1. The most frequency occurring letter is………………………..
2. The most commonly used vowel is……………………………..
3. The least frequency occurring letter is………………………..
4. The ratio of most to least frequency occurring letter is………………………..

WORKSHEET 6

Crossword Puzzle



Across

4 Average of observations

5 Upper Limit - Lower Limit

Down

1 Primary or Secondary

2 Middle most term

3 Most frequent observation

6 Gap

Solution:

Across: 4 MEAN 5 RANGE

Down: 1 DATA 2 MEDIAN 3 MODE 6 INTERVAL

WORKSHEET 7

Concept: Measures of central tendency-Mode of raw data

1. Sahil noted down the goals scored by a school team in a series of 10 matches. Calculate the mode score for the data 3, 2, 4, 2, 4, 1, 2, 2, 5, 1
2. In a mathematics test given to 15 students, the following marks (out of 100) are recorded: 51, 49, 49, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60

 Find the mode of this data.

3. Think of 5 situations from your surroundings where mode is used as a measure of central tendency.

____
____
____
____
____

MISCONCEPTIONS/COMMON ERRORS

1. For the data involving classes of varying widths, sometimes, students draw histogram as in the case of classes of uniform width without finding 'adjusted frequencies'. In such cases, "adjusted frequencies" have to be used in place of given frequencies.
2. Some students may draw a histogram of the following data by having an impression that the first column has continuous classes and 2nd column has frequencies.
3. Mean of a given raw data and mean calculated from the data converted in the form of a grouped frequency table may be different. But mean of a frequency distribution calculated using either assumed mean method or direct method or step deviation method will be the same.
4. Some students think that arithmetic average or mean is the only average. It is not so. Median and mode are also averages and there are some more averages also, which they will study in higher classes.
5. Some students think that relationship, 3 Median = Mode + 2 Mean, is always true. In fact, it is true only under some specific conditions.

PROBABILITY

INTRODUCTION

In our day-to-day life, we often come across statements such as:

1. It may rain today in the evening
2. Tomorrow, there is a cricket match between India and Pakistan. Probably, India will win the toss.
3. It is unlikely that bank make a mistake.
4. Chances are high that prices of petroleum products will go up next month.
5. If a drawing pin is dropped, it is likely not to land point down.

These are the statements made on certain direct past experiences or experiments. These statements involve the words like 'may', probably, unlikely, likely, chances etc which indicate an element of uncertainty. This uncertainty when measured numerically is called probability.

Since the above statements are made on the basis of past experiences or experiments, so we will call this probability as experimental (or empirical) probability.

Our life is full from probable situations with great opportunities. Some probable situations come true and some are not. Probability compels us to be more awakened, prepared, planned, systematic and organized to convert it into an opportunity. Our career planning is a form on probability statement and you planned your career path according to that. Our five year plans, GDP, Census etc. all are probable projections supported by previous happenings. So all our planning is based on some mathematical assumption and projections which are basically the functions of probability. For example which grade you will have in your SA-II, is also depends up upon your last performance in SA-I. It conveys that it is probable that your grade follow the last trend of SA-I. Our health profile conveys our probable health projections of future. This probability can be denoted empirically with mathematical techniques and methods.

Now let us have a probability statement: "It may rain tomorrow"

Probable happening regarding above said statement is as under:

1. Yes, it will rain tomorrow.
2. No, it will not rain tomorrow.
3. Can not say, may be, and may not be.

Consequence: for 1 & 2, we will plan our day according to probable happening. For 3 confused and no planning.

It means that this probable statement produce two sure consequences in yes and no. if we assign our favorable happening for example yes as 1 and non favorable no as 1*. So we will be able to express this probability statement mathematically as *P* (will rain tomorrow) = total favorable happening/total happening =1/2 or P (1) = 1/2 and

P (will not rain tomorrow) = 1/2 or P (1*) =1/2

Probability oscillates between 1 and 0

(Rain) —————- (no rain)

We can now conclude that there are some situations, phenomenon or event or question as above we discuss where it is not possible to give a definite answer. Each of our probable response has an element of uncertainty. In mathematics this element of uncertainty can be measured numerically and is studied under the topic "probability".

KEY CONCEPTS

Probability Concept Map

EXPERIMENT

***S* = SAMPLE SPACE (**$n(S)$ = TOTAL OUTCOMES IN SAMPLE SPACE

***A* = EVENT (SUBSET OF *S*) (**$n\ (A)$ = TOTAL OUTCOMES IN EVENT**)**

SIMPLE EVENTS & COMPLEMENTARY EVENTS

Event

An outcome or set of outcomes of an experiment or situation, e.g., rolling a 3 or higher is one possible event produced by a dice roll.

Experiment

In probability, any activity involving chance, such as a dice rolls.

Experimental Probability

A probability based on the statistical results of an experiment probability

Fair games

Games, where all players have the same chances of winning.

Independent events

The event in which the outcome of one event does not affect the probability of the subsequent event.

Outcome

One way an experiment or situation could turn out.

Probability

The likelihood of an event occurring.

Sample Space

All the possible outcomes of an experiment.

Theoretical probability

The ratio of the number of ways an event can happen to the total number of possible outcomes.

Sample and Event Spaces Sample Space A probabilistic experiment has the following characteristics: The set of all possible outcomes of the experiment can be described. The outcome of the experiment cannot be predicted with certainty prior to the performance of the experiment.

The set of all possible outcomes (or sample points) of the experiment is called the sample space and is denoted by S. For a given experiment it may be possible to define several sample spaces.

Example: For the experiment of tossing a coin three times, we could define S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}, each outcome being an ordered sequence of results; or (b) S = {0, 1, 2, 3}, each outcome being a possible value for the number of heads obtained.

Event Space

Events

A specified collection of outcomes in S is called an event: i.e., any subset of S (including S itself) is an event. When the experiment is performed, an event A occurs if the outcome is a member of A.

*Example**:***

In tossing a die once let the event A be the occurrence of an even number: i.e., A = {2, 4, 6}. If a 2 or 4 or 6 are obtained when the die is tossed, event A occurs.

The event S is called the certain event, since some member of S must occur. A single outcome is called an elementary event. If an event contains no outcomes, it is called the impossible or null event and is denoted by ∅.

Examples:

Simple Event (& its complement)

A box contains 2 black balls and 2 red balls. A ball is selected at random, its colour recorded, and then it is replaced. A second ball is then selected at random, and its colour recorded.

Outcomes can be tabulated as shown below:

	R1	R2	B1	B2
B1	B1R1	B1R2	B1B1	B1B2
B2	B2R1	B2R2	B2B1	B2B2
R1	R1R1	R1R2	R1B1	R1B2
R2	R2R1	R2R2	R2B1	R2B2

The probability that both balls are black is 4/16 $= \frac{1}{4}$. The probability that both balls are not black = $1 - \frac{1}{4} = \frac{3}{4}$.

Random Experiments

(i) If you toss a coin, you know in advance that it will only land up in one of the two possible ways-either Head (H) or Tail (H).

However, we cannot say with certainly that it will land up head only or tail only.

(ii) If you throw a die, you know in advance, that it will show up only one number 1, 2, 3, 4, 5, or 6 (sometimes dots in place of number) But, we cannot say with certainty that it will show up a particular number.

In the above two experiments, one of tossing a coin and the other for throwing a die, we know in advanced the possible outcomes i.e., Head and Tail in tossing a coin and numbers 1, 2, 3, …6 in case of throwing a die, but we cannot say definitely the occurrence of any one particular outcome.

Such experiments are called random experiments.

Drawing a card from a well shuffled deck of 52 playing cards is also a random experiment because it can be any one of the 52 cards and we cannot say in advance what it will be.

Similarly, choosing a bulb from a lot of 100 bulbs without looking into the lot is a random experiment. This bulb may be good or may be defective.

If we drop a stone from a height on the ground, we can say in advance that it will fall down on the ground and there is no other outcome except it. It is not a random experiment.

Thus, a random experiment is one which has more than one outcome and it is not possible to predict in advance the occurrence of a particular outcome. Here, experiment means a random experiment.

Example: Which one of the following is a random experiment?

(i) Taking out a ball from a bag containing red and blue coloured balls of same size.

(ii) A bag contains 50 marbles of red colour.

Taking out a mark of red colour without looking into the bag.

Solution:

(i) It is a random experiment because that ball taken out can be red or blue and we cannot tell the colour of the ball in advance before the experiment.

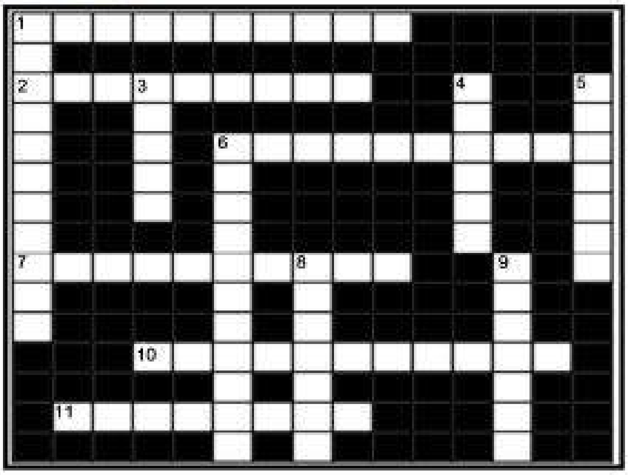
(ii) It is not a random experiment as it has only one outcome i.e., red ball.

LEARNING-TEACHING STATEGIES

WORKSHEET 1

Crossword Puzzle

Key concepts for Probability



Across

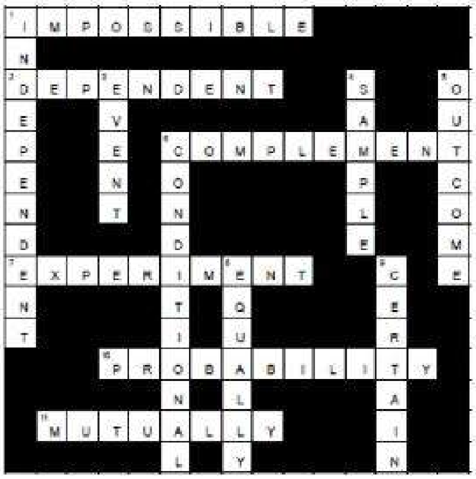
1. An event with a probability of 0.
2. Two events are _________ if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.
6. The _________ of an event is all outcomes different from the favourable outcome.
7. A situation involving chance or probability that leads to results called outcomes.
10. The chance that an event will occur expressed as the ratio of the number of favourable outcomes to the number of possible outcomes.
11. Events are _________ exclusive if they cannot occur at the same time.

Down

1. __________ events are events for which the outcome of the second event does not depend on the outcome of the first event.
3. A possible set of outcomes for an experiment.
4. A _______ space is the set of all possible outcomes for an experiment.
5. A result of an experiment.

6. _________ probability is the probability of an event occurring given that another event also occurs.
8. Outcomes are _______ likely if they have the same chance of occurring.
9. A _______ event has a probability of 1.

Solution:



ACTIVITY 1

SPIN AND WIN

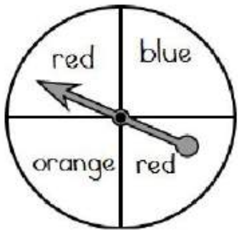
1. What is the probability of the spinner landing on a 3 _______________?
2. What is the probability of the spinner landing on a 1 _______________?
3. What is the probability of the spinner landing on a 2 _______________?
4. Are you more likely to spin an odd number or an even number? Explain.

____

____

ACTIVITY 2

SPIN AND WIN



1. What is the probability of the spinner landing on red?
 ____
2. What is the probability of the spinner landing on orange?
 ____
3. What is the probability of the spinner landing on a primary colour?
 ____
4. Rohan said, "You have a fifty-fifty chance of spinning red." Explain what he means.
 ____
 ____

ACTIVITY 3

The marbles pictured below are gray, white, and black. They are placed in a bag and one is drawn at random.

1. Which colour marble is least likely to be drawn from the bag?
2. What is the probability of drawing the black marble from the bag?
3. What is the probability of drawing a gray marble?
4. What is the probability of the drawing a white marble?
5. What is the probability of drawing a marble that is not white?

6. Would you be more likely to draw a marble that is not black or a marble that is not gray? Explain your answer.

Explanation: You'd be more likely to draw a marble that is not black. There is only one black marble and 6 that are not black. You have a 3 out of 7 chance of drawing a non-gray marble. You have 6 out of 7 chance of drawing a marble that is not black. ………………………………………..

7. If three more black marbles were added to the bag, what would be the probability of drawing a black marble?

WORKSHEET 2

Sample Questions

1. Which of the following cannot be empirical probability of an event?
 (a) 4/5 (b) 1 (c) 0 (c) 5/4
2. In a cricket match, a batsman hits a boundary 4 times out of 30 balls, he plays. Find the probability that he did not hit a boundary.
3. A die is thrown 400 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table.

Outcome:	1	2	3	4	5	6
Frequency:	72	65	70	71	63	59

 Find the probability of getting
 (i) A number less than 3.
 (ii) Getting an outcome 6.
 (iii) Getting a number more than 4.
4. Cards each marked with one of the numbers 4, 5, 6….20 are placed in a box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting an even prime number?
5. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting
 (i) a white ball or a green ball.
 (ii) Neither a green ball not a red ball.
6. One card is drawn from a well shuffled deck of 52 playing cards. Find the probability of getting
 (i) A non-face card

(i) A black king or a red queen.

7. A bag contains 5 red and 4 black balls. A ball is drawn at random from the bag. What is the probability of getting a black ball?
8. All cards of ace, jack and queen are removed from a deck of playing cards. One card is drawn at random from the remaining cards. Find the probability that the card drawn is
   (a) A face card
   (b) Not a face card

SUGGESTED PROJECTS

1. Collect some reports and data from different sources on "global warming" and based on them prepare a report on probable disastrous environmental events could happens in coming ten years from now.
2. Study the census report 2011 on Delhi and prepare detailed reports on probable trends in primary education.
3. Estimate the probable expenditure on white washing the school building and in how many days the work can be carried out.

WORKSHEET 3

Multiple Choice Questions:

1. A coin is tossed 1000 times and 560 times a "head" occurs. The probability of occurrence of a Head in this case is

(a) 0.5 (b) 0.56 (c) 0.44 (d) 0.056

2. What is the probability of getting tail in the above case?

(a) 0.056 (b) 0.044 (c) 0.50 (d) 0.44

3. What is the probability of getting a number less than in throwing a die?

(a) 0.50 (b) 0.54 (c) 0.46 (d) 0.52

4. What is the probability of getting a number greater than and 4 equal to in throwing a die?

(a) 1.0 (b) 0.25 (c) 0.75 (d) 0.50

5. In a bag, there are 100 bulbs out of which 30 are bad ones. A bulb is taken out of the bag at random. The probability of the selected bulb to be good is

(a) 0.50 (b) 0.70 (c) 0.30 (d) None of these

MISCONCEPTIONS/COMMON ERRORS

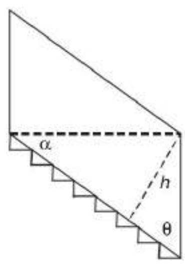
The concept of probability is assumed as reflected only as favorable and non favorable happenings. It is simply more than tossing a coin or throwing a dice or drawing a card or ball. This concept should be seen as an instrument of near to exact predictions, projections, planning ahead, probable profit or loss in business, pros and cons of a work. So it must be looked as a very unique tool of making our mind mathematised.

8

TRIGONOMETRY

INTRODUCTION

In general when we think of Trigonometry it appears to us that it does not have any use in daily life but look for the following example to find area of a wall moving with stairs or hilly area where we have to paint or to find area for that particular region for any other pur pose.



[Just measure the angle of a poi nt from the two corners of base and base length]

We can find height and area using Trigonometry when height is too large (i.e. not easy to measure).

In general we measures this area b y making it a rectangle in Mathematics we know the difference of a rectangle and parallelogram.

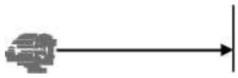
Beside this trigonometry is widely used in land surveying, architecture, astro nomy, cartography (science of drawing maps), civil e ngg., geo physics, medical imaging, optics etc.

KEY CONCEPTS

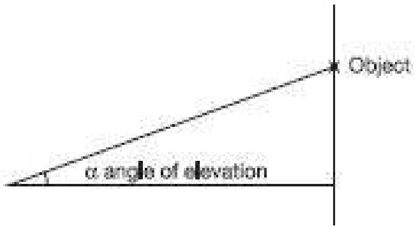
Angle of elevation and a ngle of depression.

In general when a person looks straight it is known as his eye-line

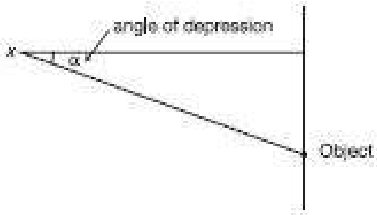
[redacted]



He makes an angle of ellevation with this eye-line when he watches a n object above this line, and



An angle of depression when he looks below this line.



- Observer i.e. position of eye is considered as a point.
- Diagrams showing object in the eyes of observer are stated or translated in a particular language.
  - (i) A man of height 1.5 m watches a bird flying at a height of 20 meter above earth. (height of man ignored or included)

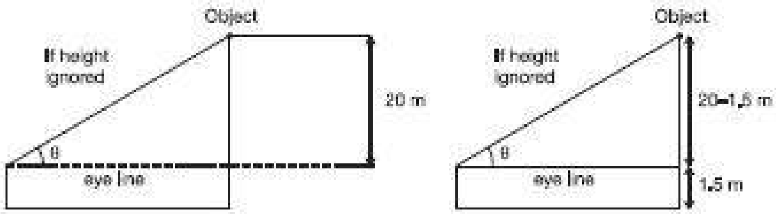


Fig. (i)

  - (ii) A man on the top of a building watches an object 50 m away from the base of the building

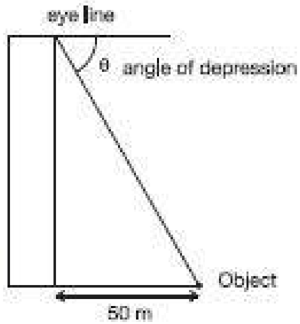


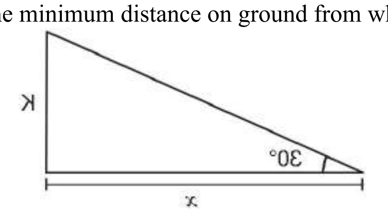
Fig. (ii)

LEARNING-TEACHING STRATEGIES

ACTIVITY I

(i) Wheel chair Ramps (To check how steep they are)

(ii) Suppose ideal angle for slope of a wheelchair Ramp is 30° and the height of the point up to which the slope is to be constructed is K then what would be the minimum distance on ground from where the slope needs to be started.



Measure 'x' if 'K' is known for a maximum angle 30°

Enrichment tasks

°

Draw a right angled triangle ABC with $\angle B$ = (a)

find Sin B = ? — Sin A = ? = (b)

Cos B = ? — Cos A = ? =

tan B = ? — Tan A = ?=

Cot B = ? Cot A = ?

Cosec B = ? Cosec A = ?

Sec A = ?

[Confirming the idea of Ratios]

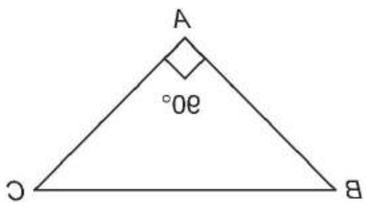
Discussion points (Group Discussion)

Angles and Ratios

Angles of elevation and .depression

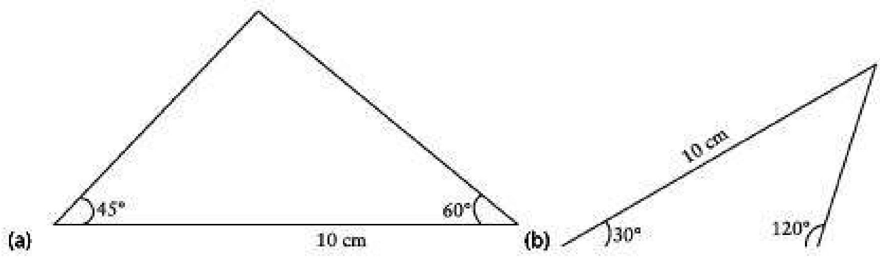
Correlating similarity with trigonometry as ratio of corresponding sides of similar figures are always .equal

All user of Pythagoras theorem if one acute angle can be determined with any one of the sides of a right angled .triangle



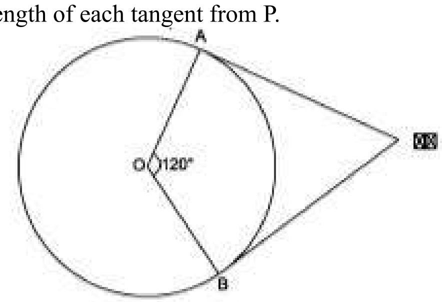
THOUGHT PROVOKING QUESTION

1. Area of triangle with two base angles and one side is given



(c) If semi vertical angle of a cone of base diameter 7cm is 30° find total surface area and volume of the cone.

(d) PA and PB are two tangents on a circle with centre O and radius 3cm. If $\angle AOB$= 120° find the distance of point P from the centre of the circle and the length of each tangent from P.



(e) What will be the two possible areas of a trapezium if one of the parallel sides is 12 cm in length and larger of the parallel side makes an angle of 60° with one non parallel side of length 5 cm. and an angle of 45° with other non parallel side with length 6 cm.

(f) A radar measures two angles α and β of an aeroplane coming towards it and flying at a height of 'h' meters above the radar in a span of 10 seconds. Give an expression to find the speed and distance of radar so that it can order the missile system to shoot the aeroplane in next 30 seconds.

ACTIVITY 2

Activity: To verify the value of different 'ratios' (Trigonometric) geometrically. (angles between 0° & 90°) **Material required:** Drawing sheet, Geometry Box **Knowledge:** Construction of right angled triangle.

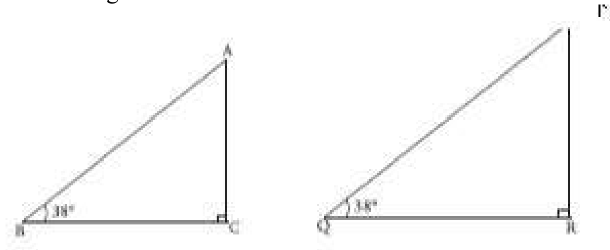
Steps of Activity

(i) consider the base angle for the ratio to be determined say 38°

(ii) Construct two right angled triangles with one base angle 38°

(iii) Measure the length of sides of triangles and determine the ratios to three nearest places of decimal for both triangles

(iv) Take the help of the teacher/trigonometric table to match the obtained values with the actual ratios.



Observations

ABC
PQR

AB = —— cm PQ = —— cm

BC = —— cm QR = —— cm

PR = —— cm

AC = —— cm

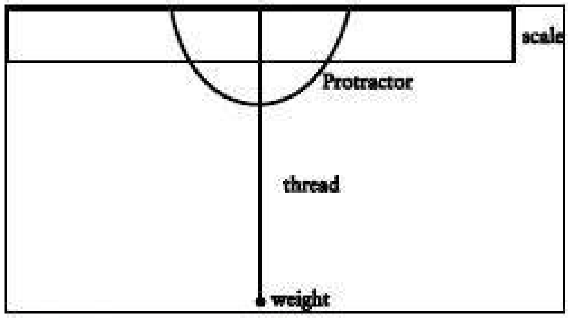
Ratio	***ABC***	***PQR***	***Avg. (May be taken)***	***Actual***
Sin 38°	$\frac{AC}{AB}$ =	$\frac{PR}{QP}$ =		
Sin 38°	$\frac{BC}{AB}$ =	$\frac{QR}{PQ}$ =		
Sin 38°	$\frac{AC}{BC}$ =	$\frac{PR}{QR}$ =		
Sin 38°	$\frac{AB}{AC}$ =	$\frac{PQ}{PR}$ =		
Sin 38°	$\frac{AB}{BC}$ =	$\frac{PQ}{QR}$ =		
Sin 38°	$\frac{BC}{AC}$ =	$\frac{QR}{PR}$ =		

Result: By comparing the ratios calculated as per construction with actual ratios we can verify the ratios and further it gives us an idea to use trigonometric ratios even if we have trigonometric table or not.

SUGGESTIVE PROJECTS/ACTIVITIES/ASSIGNMENT

Material—Clear plastic ruler, protractor, tape, cotton thread and small weight

Ask: students to make a clinometers using this item.



Give any activity to find height or distance using this clinometer like.

(i) height of building

(ii) height of tree

PROJECT WORK

(i) Finding height of school building by using clinometer.

(ii) Uses of trigonometry in industries relating size and angles of mechanical parts used in machinery, tools and equipment.

[This is helpful in automotive engineering allowing car companies to size each part correctly and to ensure their safe working]

[Helpful in garment industry regarding length of Fabric needed to craft a certain shape]

(iii) Musical theory and production

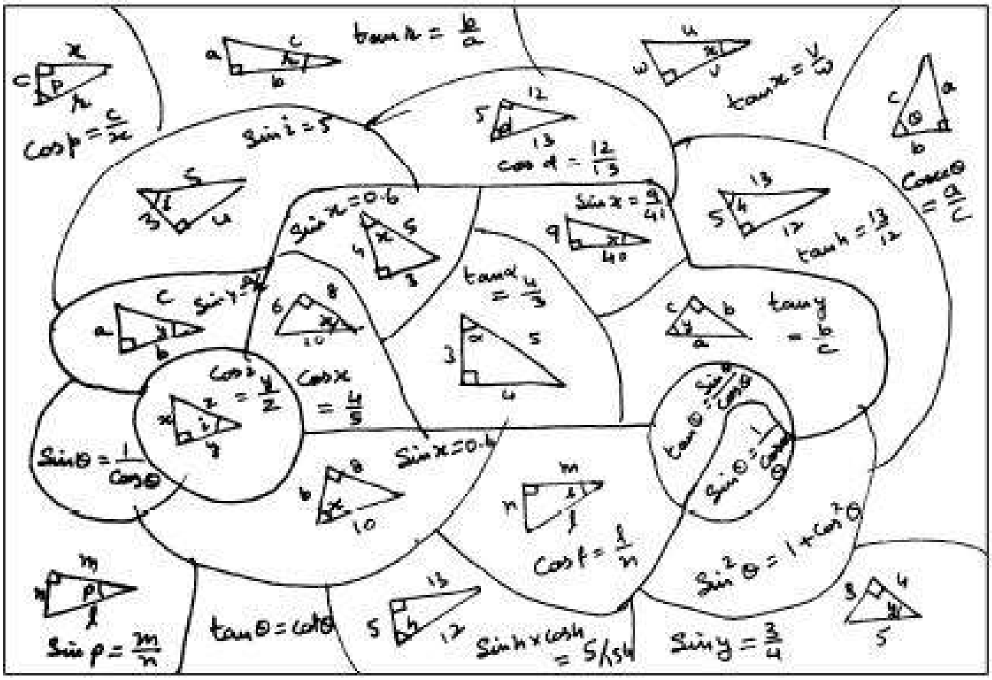
[A Sound waves travel in a repeating wave pattern, which can be represented graphically by sine and cosine functions]

[A graphical representation of music allows computers to create and understand sounds and also allows sound engineers to visualize sound waves so that they can adjust volume, pitch and other elements to create desired sound]

ACTIVITY 3

PUZZLE

Shade the region which display correct trigonometric ratios and find the hidden shape.



WORKSHEET 1

S.A.

Instructions

Q. 1. if x = a sec θ and y = a tan θ then the value of $x^2 - y^2$ is ………………

(a) a (b) a^2 (c) 1 (d) $\frac{1}{a}$

Q. 2. The value of sin 70° – cos 20° is ………………

(a) 0 (b) 1 (c) 70° (d) 20°

Q. 3. If 5 cos θ = 4; then the value of sin θ is ………..

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

Q. 4. If A and B are acute angles and sin A = cos B then the value of A + B is ………….

(a) 0 (b) 45° (c) 90° (d) 30°

Q. 5. If tan 3 θ = 1 then θ = ………….

(a) 1° (b) 15° (c) 9° (d) 7°

Q. 6. If tan (90° – θ). cot θ = 1 and θ is an acute angle then θ = ……………….

(a) 30° (b) 60° (c) 45° (d) 0

Q. 7. The value of (1 – sin θ)(1 + sin θ). $\sec^2\theta$ is……………….

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Q. 8. The angle θ between 0° and 90° at which the value of sinθ. and cosθ coincides is ……………….

(a) 90° (b) 45° (c) 30° (d) 0°

Q. 9. The angle θ between 0° and 90° at which the value of tanθ and cotθ coincides is ……………….

(a) 45° (b) 30° (c) 60° (d) 90°

Q. 10. $\tan^2 30°. \cot^2 60°$ = ……

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{6}$ (d) $\frac{4}{3}$

Q. 11. $\frac{\sin^2 45° + \cos^2 45°}{\tan^2 60°}$ = ...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 3 (d) 3

Q. 12. $\frac{\sin 27°}{\cos 63°}$ = ...

(a) 0 (b) 2 (c) 1 (d) 3

Q. 13. tan 48°. tan 23°. tan 42°. tan 67° = ………..

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 0

Q. 14. If sec θ + tan θ = x; then sec θ = ………..

(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2+1}{2x}$ (c) $\frac{x^2-1}{2x}$ (d) $\frac{x^2-1}{x}$

Q. 15. The value of $(\sin\theta + \cos\theta)^2 + (\cos\theta - \sin\theta)^2$ = ………..

(a) 1 (b) 2 (c) 3 (d) 4

Q. 16. The value of cosec 30° – $\sin^2 90°$ – $\sin^2 45°$ – $\sec^2 60°$ = ………..

(a) 2 (b) 1 (c) –2 (d) –1

Q. 17. If cotθ = $\frac{1}{3}$ then the value of $\frac{(1 - \cos^2\theta)}{(1 + \cos^2\theta)}$ = ………..

(a) $\frac{1}{5}$ (b) [redacted] $\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{1}{5}$

Q. 18. If cos x = cos60°. cos60°.cos30° + sin 60°.sin30° then the value of x is ………..

(a) 60° (b) 30° (c) 90° (d) 0°

Q. 19. If cotθ + $\frac{1}{\cot\theta}$ = 2 then the value of $\cot^2\theta + \frac{1}{\cot^2\theta}$

(a) 1 (b) 3 (c) 2 (d) 4

Q.20. $\frac{\tan i}{\cot 89°}$ = ………

(a) 1 (b) 0 (c) 2 (d) 3

Q. 1. If 16 cot A = 12, then find the value of

WORKSHEET 2

$\sin A + \cos A \quad \sin A - \cos A$

Q. 2. Find the value of θ if $\sqrt{3} \tan 2\theta - 3 = 0$

Q. 3. Find the value of θ, if $\sin 5\theta = \cos 4\theta$, Also 5θ and 4θ are acute angles.

Q. 4. Simplify

$\tan^2 60° + 4 \cos^2 45° + 3 (\sec^2 30° + \cos^2 90°)$.

Q. 5. If $\sec\alpha = \frac{5}{4}$; then evaluate $\frac{\tan\alpha}{(1 + \tan^2\alpha)}$

Q. 6. Prove that:

$\frac{\cos A}{(1 - \sin A)} + \frac{\cos A}{(1 + \sin A)} = 25 \sec A$

Q. 7. Prove that:

$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

Q. 8. Prove that:

$\sqrt{\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}} + \sqrt{\frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}} = 2 \sec A$

Q. 9. If $\tan (A - B) = \sqrt{3}$ and $\sin A = 1$ then find A and B.

Q. 10. If θ is an acute angle and $\sin\theta = \cos\theta$; then find the value of $3\tan^2\theta + 2\sin^2\theta - 1$

Q. 11. In a right angled triangle ABC, that right angled at B, if tan A = 1; then verify $2 \sin A.\cos A = 1$.

Q. 12. If $\frac{x}{a} \cos\theta + \frac{y}{b} \sin\theta = 1$ and $\frac{x}{a} \sin\theta - \frac{y}{b} \cos\theta = 1$ then prove that

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

WORKSHEET 3

Q. 1. Evaluate the following:

$\sin^2 25° + \sin^2 65° + \sqrt{3}(\tan 5° . \tan 15° \tan 30° . \tan 75° . \tan 85°)$

Q. 2. Prove that: $\tan^2\theta + \cot^2\theta + 2 = \operatorname{cosec}^2\theta . \sec^2\theta$

Q. 3. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$;

Show that $(m^2 + n^2)\cos^2\beta = n^2$

Q. 4. Prove that:

$(\sin\theta - \operatorname{cosec}\theta)(\cos\theta - \sec\theta) =$

1

$\tan\theta - \cot\theta$

Q. 5. If $\cot\theta = \frac{15}{8}$; Then evaluate:

$$(2 + 2\sin\theta)(1 - \sin\theta)\ (1 + \cos\theta)(2 - 2\sin\theta)$$

Q. 6. Prove that:

$$\frac{(\sec\theta + \tan\theta - 1)}{(\tan\theta - \sec\theta + 1)} = \frac{\cos\theta}{(1 - \sin\theta)}$$

Q. 7. If $x = r\sin A\cos C$

$y = r\sin A\sin C$

$z = r\cos A$

Prove that $r^2 = x^2 + y^2 + z^2$

Q. 8. Prove that:

$$\frac{1}{(\sec\theta - \tan\theta)} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta + \tan\theta}$$

Q. 9. Prove that:

$$\frac{\cos\theta}{(1 - \tan\theta)} - \frac{\sin^2\theta}{(\sin\theta - \cos\theta)} = \sin\theta + \cos\theta$$

Q. 10. If $x = a\sin\theta$; $y = b\tan\theta$;

Prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

ACTIVITY 4

Objective

To find the height of a building by using a clinometer Material Required

Clinometer

A measuring tape

White sheet of Paper

Pencil

Pre-requisite Knowledge

Understanding of angles of elevation and depression Knowledge about clinometer

Knowledge of trigonometric ratio

Procedure

(i) Place the clinometer at suitable place to measure the angle of elevation of the top of school building and look through the hollow pipe so that top of building is visible through the other end.

(i) Hold the clinometer steady and record the angle the string makes on the scale of the clinometer. This is the required angle of elevation (θ).

(iii) Measure the height (x) of the centre of clinometer from the ground.

(iv) Measure the distance (y) of building from clinometer by using measuring tapes.

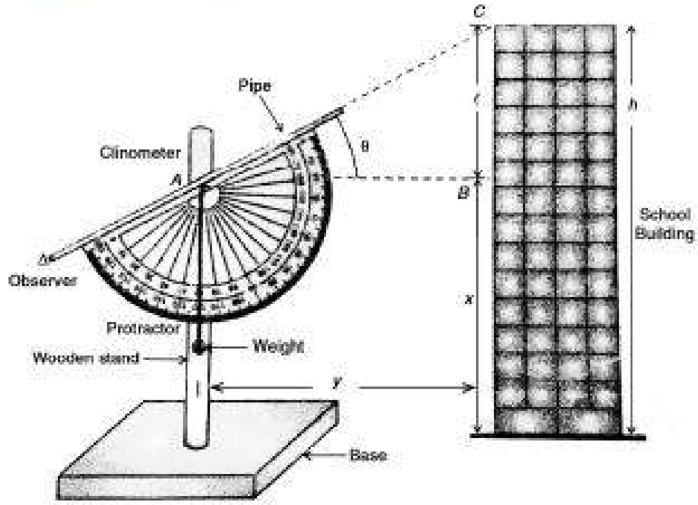
(v) Repeat this experiment by keeping clinometer at different position to measure the value of x, y and θ (angle of elevation) h = Height of building

x = Height of protractor from ground

θ = Distance of building from entre of protractor

Q = Angle measure through protractor (Angle of elevation)

h = Height of building $= x + b$



In right angle △ABC

$$\frac{BC}{AB} = \tan\theta$$

$$\frac{\ell}{y} = \tan\theta$$

$$\ell = y \tan\theta$$

$$h = x + y \tan\theta$$

S.No.	*Height of protractor from ground (x)*	*Distance of building from centre of protractor (y)*	*Angle measured through protractor (angle of elevation θ)*
1			
2			
3			
4			
5			

Calculate the height of building by using the formula given below: Height of building $= x + y \tan\theta$

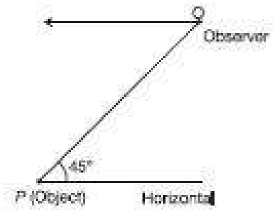
Result: Height of building =

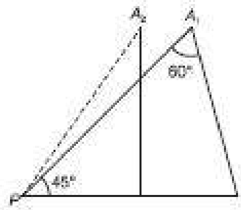
ACTIVITY 5

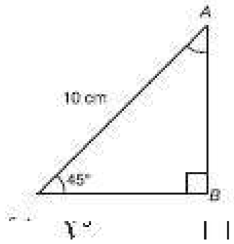
Divide the student of class in small groups of four or five children, ask them to find height of school building and height of T.V. Antenna mounted on roof top from ground level.

WORKSHEET 4

Very Short Answer Questions







1. In the adjoining fig the position of observer and object are marked. What is the angle of depression.

2. In the adjoining fig what is the angle of depression from the observing positionA1 and A2 of the object at P

3. From the adjoining fig; what is the length AB

√

4. If the length of shadow cast by a pole be 3 ________________B times the length of pole then what is the angle of elevation of the sun.

(Application of formula)

5. A river is 100 m wide. A tree of unknown height is on one bank. The angle of elevation of the top of the tree from the point exactly opposite to foot of the tree, on the other bank is 30°. What is the height of the tree.

(Application of formula)

WORKSHEET 5

Mark the correct alternative in each of the following:

1. If the angles of elevation of a tower from two points distant 'a' and b (a > b) from the foot and in the same straight line are 30° and 60°, then height of the tower is

(a) $\sqrt{a+b}$ (b) $\sqrt{\frac{a}{b}}$ (c) $\sqrt{ab}$ (d) $\sqrt{a-b}$ b

2. If the angle of elevation of a tower from a distance of 200 m from its foot is 60°, then the height of the tower is

(a) $200\sqrt{3}$ m (b) $\frac{200}{\sqrt{3}}$ m (c) $100\sqrt{3}$ m (d) $\frac{50}{\sqrt{3}}$ m

3. A circus artist is climbing a 50m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Angle made by the rope with the ground is 30° then height of pole is

(a) 10m (b) 20 m (c) 30 m (d) 40 m

4. Top of tower subtends an angle of 30° at a point on the same level at its foot. At a second point 20m along the first, the depression of the foot of the tower is 60°. The height of tower is

(a) $\frac{20}{3}m$ (b) 60 m (c) $20\sqrt{3}m$ (d) $\frac{20}{\sqrt{3}}m$

5. The tops of two poles of height 20m and 14m are connected by a wire. If the wire makes an angle 30° with horizontal, then the length of wire is

(a) 18 m (b) 12 m (c) 24m (d) 3m

6. A person is standing on the bank of a river, observes that the angle subtended by a tree on opposite bank is 60°. When he moves 20m from the bank he finds the angle to be 30°. Now the height of tree will be—

(a) 15m (b) $5\sqrt{3}m$ (c) $10\sqrt{3}m$ (d) $20\sqrt{3}m$

7. A tree is broken by the wind. The top struck the ground at an angle 30° and at a distance 30 m from the root then the height of tree is—

(a) $15\sqrt{3}$ m (b) $20\sqrt{3}$ m (c) $25\sqrt{3}$ m (d) $30\sqrt{3}$ m

8. The angles of depression of two ships from the top of a light house are 45° and 30° towards east. If the ships are 100m apart, the height of light house is—

(a) $50(\sqrt{3}+1)$ m (b) $100(\sqrt{3}+1)$ m (c) 50 m (d) 100m

9. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds, the angle of elevation changes to 30°. If the jet plane is flying at constant height of 3600 $\sqrt{3}$, then the speed of plane is—

$\sqrt{}$

(a) 864 km/hr (b) 3660 km/hr (c) 900 km/hr (d) 1000 km/hr

10. The angle of elevation of a ladder leaning against a wall is 60° and foot of ladder is 10m away from the wall then the height of ladder is—

(a) 10 m (b) 20 m (c) 30 m (d) 15 m

WORKSHEET 6

HOTS

1. A 1.2 m tall girl spots a fallen moving with a wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the fallen from the eyes of the girl at any instant is 60° after some time, the angle of elevation reduces to 30°. Find the distance travelled by the fallen during interval

2. A straight highway leads to the foot of tower. A man standing at the top of tower observes a car at angle of depression of 30°, which is approaching to the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by car to reach the foot of tower.

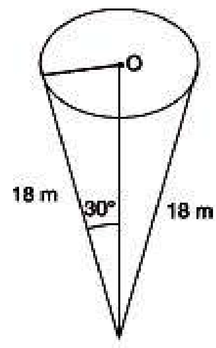
3. The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of cloud in the lake is 60°. Find the height of cloud.

4. From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite sides of street are 30°
and 45° respectively. Show that the height of the opposite house is 2326 m.

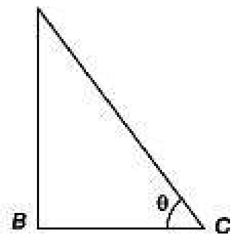
5. From the top of a building 60m high the angles of depression of the top and the bottom of tower are observed to be 30°and 60°. Find the height of tower.

6. For a polio eradication campaign, large spherical balloon is tied up with the two ropes each of length 18 m as shown is fig.

(a) Find the diameter of balloon.
(b) What value is depicted in the balloon.



7. In a given right triangle, base and perpendicular are represented by hardwork and success and are in the ratio of 1 : 1. A



(a) Find *ACB*
(b) What values are depicted from the problem?

8. A student was on the top of vertical tower 100 m high observes a moving car and angle of depression is 30°. Student immediately inform to the fire station on mobile phone.
   (a) Find the distance of car from foot of tower.
   (b) What value is shown by the student?

MISCONCEPTIONS/COMMON ERRORS

	Misconceptions	How to deal
1.	Sinθ, cosθ, tanθ etc are the ratios while some students consider them as angles.	A thorough practice by using different triangles with right angles
2.	Trigonometry only involves triangles and nothing else.	
3.	Understanding word problems.	Situations maybe created on blackboard.

9

MENTAL MATHEMATICS

INTRODUCTION

Traditionally, much of the focus of school mathematics has been on teaching algorithms for arithmetic calculation. However, mental computation and estimation are just as important for everyday life as well as to enhance the learning of mathematics. In everyday life it is very common that an approximate answer to an arithmetic problem is needed, rather than an exact one. This is especially so if the answer can be found quickly, without resorting to tools such as pencil and paper or a calculator. It is important, then, that children larn to apply efficient mental computation an estimation strategies.

Facts about mental computation

Many people work out strategies for mental computation from their good understanding of place value, their number sense and their understanding of the meaning of the arithmetic operation and its properties. Many children can compute mentally before they learn the relevant formal written algorithms at school. People good at mental computation use a wide variety of methods, for example, making use of the distributive law, factors, halving and doubling.

The methods used for mental computation are often quite different from the paper-and-pencil algorithms taught at school. In general, less competent students use less efficient strategies (such as counting on by ones rather than by tens) and they use them for longer. Focused teacher is needed to help them move on.

Characteristics of mental methods

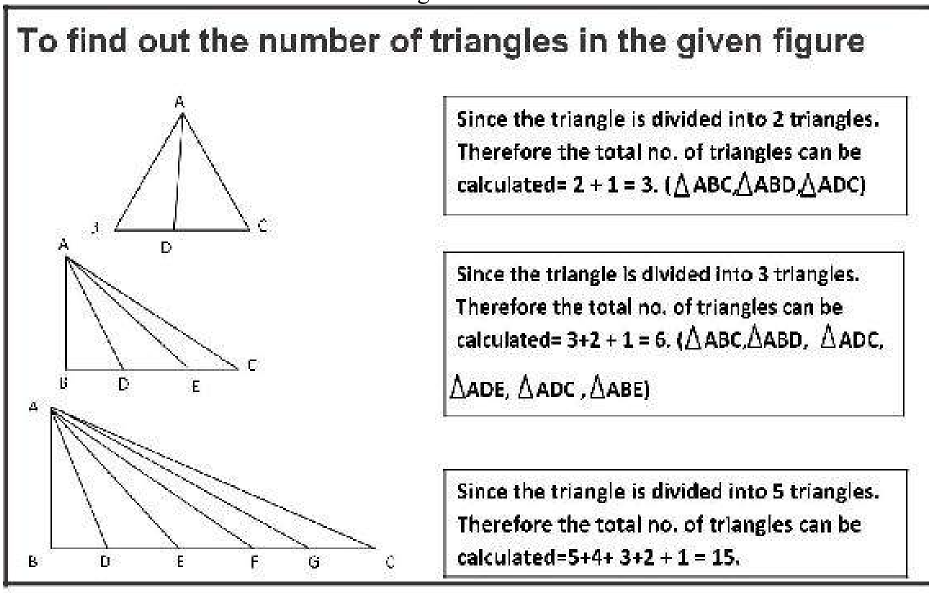
Mental methods are often varied to take advantage of known properties of the actual numbers in the problem. For example, mental methods use facts such as 8 is close to 10, 25 is one quarter of 100 or 6 and 4 add to 10. Favourite number combinations are often used as a basis of computation.

Mental methods are often based on using round numbers (e.g. 600, 1400, 30). In contrast, some formal written algorithms are hard to carry out with round numbers (think about 1000-657 done by a formal subtraction algorithm). Children make many mistakes dealing with zero informal written algorithms.

Mental computation is often step-by-step, rather than dealing with all the relationships in the problem simultaneously. For many people, the types of numbers that can be dealt with by mental computation are limited. For example, many people can calculate with ½ but not with other fractions.

Teachers should learn about the strategies that children use and learn how to describe mental strategies to children. Class discussion is important for sharing mental methods among students. Even the weaker students have interesting methods. Some strategies can be taught through class discussion, explanation and practice. Be wary of including rules to learn by rote (e.g. adding zeros) since they are almost invariably misused by all but the most competent.

Some suggestive activities based on mental calculators are given below:



To find the difference of 19th and 6th terms of arithmetic progression (A.P.) 1,3, 5,..........

Solution: d is the difference

nineteenth term

sixth term

d= 3 – 1= 2

– = (19 – 6) d= 13X 2 =26

A_{19}

A_{6}

A_{19} A_{6}

Multiplication of two digit numbers by 11

Add the two digits and put them in the middle of the number

Example, 43 x 11

4 | 4+3 | 3

4	7	3

therefore, 43X11 = 473

Example, 84 x 11

8 | 8+4= 12 | 4

(+ 1 carried to 8)

9	2	4

therefore, 84X11 = 924

To find out the square of the number with unit place as 5

Multiply the first digit by (itself + 1), and put 25 on the end.

15^2 = 225

1 x 2 = 2 | 5 x 5 = 25

25^2 = 625

2 x 3 = 6 | 5 x 5 = 25

In any three consecutive whole numbers, product of extreme is one less than square of middle number

e.g. 29, 30, 31

$29 \times 31 = 30^2 - 1$

$= 900 - 1$

$= 899$

39, 40 ,41

$39 \times 41 = 40^2 - 1$

$= 1600 - 1 = 1599$

To find the sum of the consecutive odd numbers beginning from 1

Solution : Square the number of the terms

$1 + 3 = 2^2 = 4$

(number of the terms are 2)

$1 + 3 + 5 = 3^2 = 9$

(number of the terms are 3)

$1 + 3 + 5 + 7 = 4^2 = 16$

(number of the terms are 4)

$1 + 3 + 5 + 7 + 9 = 5^2 = 25$

(number of the terms are 5)

$1 + 3 + 5 + 7 + 9 + 11 + 13 = 7^2 = 49$

(number of the terms are 7)

To find the difference of square of two numbers

When the difference between the numbers is one add the numbers

$29^2 - 28^2 = (29 + 28) = 57$

When the difference between the numbers is two, add the numbers and multiply with two

$37^2 - 35^2 = (37 + 35) \times 2 = 72 \times 2 = 144$

When the difference between the numbers is three, add the numbers and multiply with three

$78^2 - 75^2 = (78 + 75) \times 3 = 153 \times 3 = 459$

To find out the squares of numbers Less than or nearing 100 (Base 100)

Example $(94)^2$

Difference from the Base = 6

Square it

Subtract 6 from 94

88 36

$(94)^2 = 8836$

$(98)^2$

Difference from the Base = 2

Square it

Subtract 2 from 98

$(98)^2 = 9604$

96 04

Multiplication of two 2- digit numbers(nearing 100)

Example 97 X 94

Base for both the numbers : 100

Complement of 97 is 03

Complement of 94 is 06

97 → 3
94 → 6) X

91 18

For right hand part of the answer, multiply 3 and 6

For left hand part of answer, cross subtract, (97 – 6) or (94 – 3) or 100 - 9

So, 97 X 94 = 9118

How to add mentally-

58 - 55 + 67 - 62 + 98 - 92 + 5

Make the pairs and then do the calculations

= 3 + 5 + 6 + 5

= 19

To add the decimal numbers -

17.13 + 16.27 + 34.53

Add whole numbers together and then add the decimals

= 17 + 16 + 34 + 0.13 + 0.27 + 0.53

= 17 + 50 + 0.93

= 67.93

Multiplication of two numbers when the digit at 10's place is common and sum of the digit at unit place is equal to 10

26 X 24

multiply common digit at 10's place with next consecutive number

= 2 X 3

multiply the digits in unit place

6 x 4

6 24

---

38 X 32

= 3 X 4

8 X 2

12 16

Multiplication by 5

If multiplying even number, halve the number and place '0' after the number

48 X 5 = 240

(Half of 48 = 24 and place '0')

86 X 5 = 430

(Half of 86 = 43 and place '0')

If multiplying odd number, subtract one from the number you are multiplying, then halve that number and place '5' after the resulting number.

43 X 5 = 215

(43-1 =42, Half of 42 = 21 and place '5')

Multiplication by 25

Divide the number by 4 and place '00' after the number

288 X 25 = 288/4 X 100 = 7200

(one fourth of 288 = 72 and place '00')

64 X 5 = 1600

(one fourth of 64 = 16 and place '00')

To find the square of the number where the digit at ten's place is '5'

Square '5' and add the number at unit place. Place it before the square of number at unit place.

$53^2 = 5^2 + 3 \quad 09$

= 28 09

$56^2 = 5^2 + 6 \quad 36$

=31 36

Sum of consecutive numbers

Number of terms multiplied with next number

1 + 2	= 2	X	3	= 6 /2 = 3
(two numbers)		(next number)		
1 + 2 +3	= 3	X	4	= 12 /2 = 6
1 + 2 +3 + 4	= 4	X	5	= 20/2 = 10
1 + 2 +3 + 4 + 5	= 5	X	6	= 30/2 = 15
1 + 2 +3 + 4 + 5 + 6	= 6	X	7	= 42/2 = 21
1 + 2 +3 + 4 + 5 + 6 + 7	= 7	X	8	= 56/2 = 28

Sum of consecutive even numbers

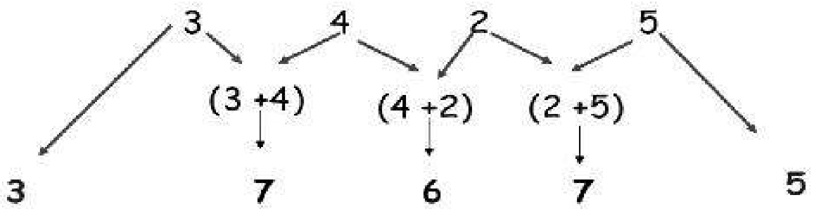
Number of terms multiplied with next number

2 + 4	= 2	X	3	= 6
(two numbers)		(next number)		
2 + 4 + 6	= 3	X	4	= 12
2 + 4+ 6 + 8	= 4	X	5	= 20
2 + 4 + 6 + 8 + 10	= 5	X	6	= 30
2 + 4 +6+ 8 + 10 + 12	= 6	X	7	= 42
2 + 4 +6 + 8 + 10 + 12 + 14	= 7	X	8	= 56

To multiply with 11

Starting from the left of the number to be multiplied by 11, write down the first digit. Next, add the first digit to the subsequent digit. Write that number down. Follow this pattern of adding the subsequent digit to the previous digit until you arrive at the last digit. Finally, write down the last digit.(note: if the numbers, when added together, sum to more than 9, then it has to carried over).

3425 X 11 =

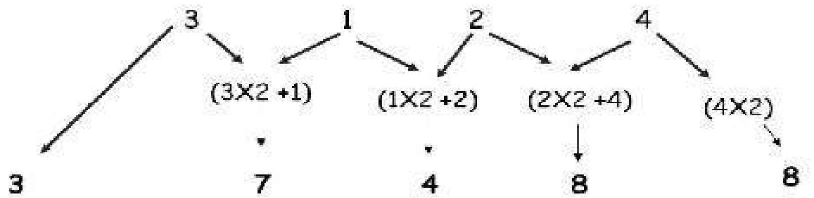


= 37675

To multiply with 12

Double the first digit on the far right and write it down
For each next digit, double it, add its neighbour to the right, and write the result down to the left of the last digit found
Write down the digit on the far left to the left of the last digit found

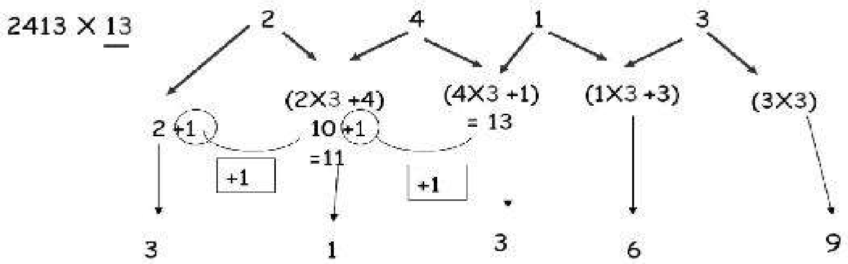
3124 X 1 2



= 37488

To multiply with 13

Starting from the left of the number to be multiplied by 13, multiply first digit by 3. Next, multiply the subsequent number with 3 and add the first digit . Write that number down. Follow this pattern of multiplication by 3 and adding the subsequent digit to the previous digit until you arrive at the last digit. Finally, write down the last digit.(note: if the numbers, when added together, sum to more than 9, then it has to carried over).



= 31369

To convert given percent into actual value

Percentage	Actual Value
100%	1 X Total value
200%	2 X Total value
50%	½ X Total value
25%	¼ X Total value
12 ½%	1/8 X Total value
10%	1/10 X Total value

Example, **What is 25% of 800 ?**

Total value = 800 So, actual value = ¼ X Total value

= ¼ X 800 = 200

Square of numbers having any numbers of digits when all the digits are same

Examples,

$(222)^2$ Here, no. of digits = 3

Repeated digit = 2

so, $(222)^2$ $= 1\,2\,3\,2\,1 \times 2^2$

$= 12321 \times 4$

$= 49284$

Square of numbers having any numbers of digits when all the digits are same

Examples,

$(3333)^2$ Here, no. of digits = 4

Repeated digit = 3

so, $(333)^2$ $= 1\,2\,3\,4\,3\,2\,1 \times 3^2$

$= 1234321 \times 9$

$= 11108889$

$(555^2) = (12321 \times 5^2)$

$= 308025$

Teachers may use these activities for making calculations easy and interesting so that the students enjoy mathematics.