Chapter - 11

Ray Optics

Introduction : Light is a form of energy. We can see objects when light fall on them. It was always a matter of curiosity to know about the production and nature of light and related phenomenon. From our common experience. We know that it travels in straight line and its speed is extreamly high.

The straight path traversed by light is called a ray and denoted by a straight line with an arrow. Group of rays is called a beam. Some of the optical phenomenon like reflection, refraction, dispersion etc. can easily be understood, using ray concept of light. In this chapter we will study reflection, refraction, dispersion and scattering using ray concept. In the later section we will study the working of optical instruments like microscope, telescope and human eye. The study involves the study of optical phenomenon related to daily life. The study is governed by the laws of geometry hence it is also called geometrical optics.

11.1 Reflection of Light

The light travelling in a medium, returns back at the boundary as shown in fig. 11.1. This phenomenon is called reflection. It obeys following laws.

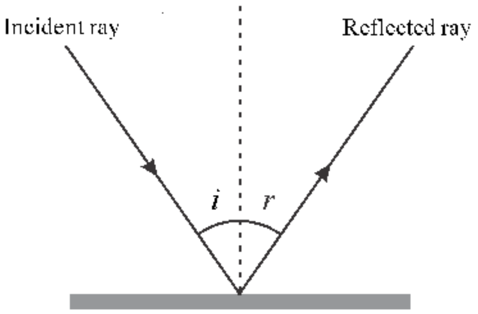


Fig. 11.1 : Reflection at a plane surface

(i) The angle of incidence *i* is equal to angle of reflection *r*.

$$\angle i = \angle r$$

(ii) The incident ray, reflected ray and the normal on reflecting surface lies in same plane called plane of incidence.

In reflection, there is no chnage in frequency and wavelength of light. If surface absorb light then intensity of reflected light decreases.

11.1.1 Formation of Image by a plane mirror

As shown in fig. 11.2 consider a point object at P, which is in front of a plane mirror AB. to construct the image we need two rays. The two rays PQ and PQ^1 get reflected by mirror and seems to be coming from P^1. Here P^1 is virtual image of the object. The image is virtual, erect and laterally inverted.

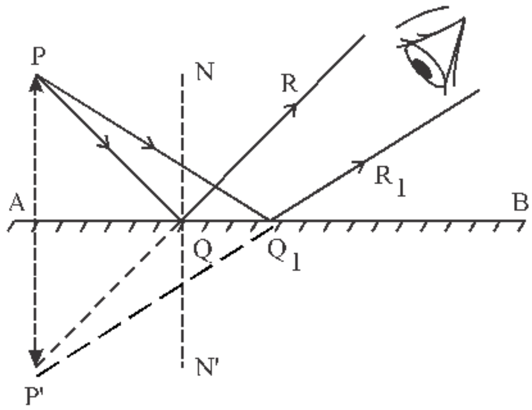


Fig. 11.2 : Image fromed by a plane mirror

11.2 Spherical Mirror

It is a part of a hollow sphere, whose one side is polished, normally these mirrors are of glass whose one side is silvered. If the reflecting surface is convex, the mirror is called convex mirror, and if the surface is concave, the mirror is called concave mirror. The figure 11.3 shows reflection of parallel rays by a concave mirror. Since the parallel rays get converged at a point these mirrors are also called convergent mirror. Similarly fig. 11.4 shows the reflection from a convex or divergent mirror.

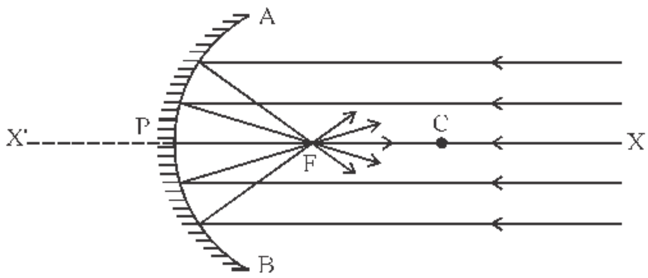


Fig. 11.3 : Concave mirror

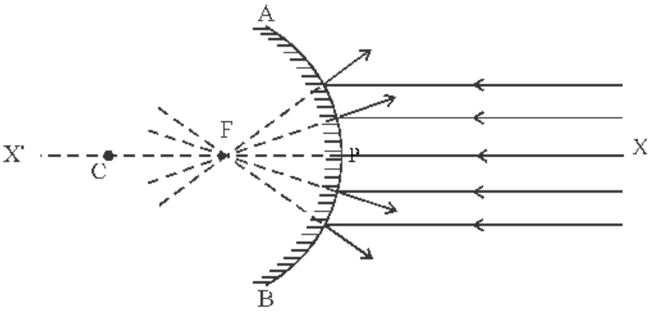


Fig. 11.4 : Convex mirror

11.2.1 The Terms and their Defenitions Related to Spherical Mirror

1. Aperture : The whole reflecting surface of a mirror is called its aperture.

2. Pole : The central point of the mirror is called pole. In fig. 11.3 and 11.4, the point P indicate pole.

3. Center of Curvature : The centre of the sphere, whose the mirror is a part, is called centre of curvature of the mirror. It is indicated as point C in fig 11.3 and 11.4.

4. Radius of Curvature : It is the distance between pole and centre of curvature of the mirror, indicated as CP in the figures.

5. Principle axis : The straight line passing through pole and centre of curvature is called principle axis. Shown as XX^1 in fig 11.3 and 11.4.

6. Principle focus : The incident rays parallel to principle axis, meet at a point on the principle axis after reflection from a concave mirror. This point is called principle focus of concave mirror. It is indicated as F in fig 11.3.

In case of a convex mirror, the incident rays parallel to principle axis are diversed after reflection from the mirror and seems to be coming from a point on principle axis. This point is called principle focus of a convex mirror and indicated as F in fig. 11.4.

7. Focal length: It is the distance between pole and the focus point. Its symbol is f.

11.2.2 Sign Convention

To obtain relation between different quantities like object distance, image distance, focal length, magnification etc. we should adopt a sign convention. Here we will follow cartesian sign convention. According to it -

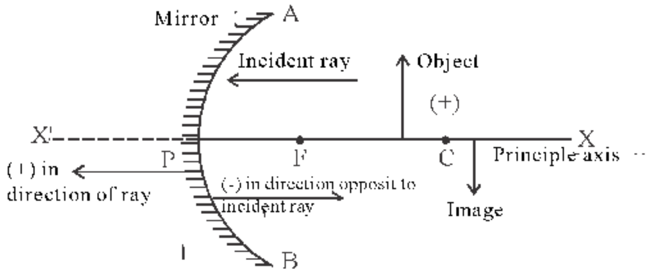


Fig. 11.5 Sign convention

1. All the distances are measured from pole.
2. The distance measured in the direction of incident rays is taken as positive and the distance measured in opposite direction of incident ray is taken as negative.
3. The height measured upwards with repect to X-axis and normal to principle axis is taken as positive. The height measured downwards is taken as negative.

According to above sign convention the object distance is taken as negative, but the image distance v can be positive or negative as per situation. Focal length of concave mirror is taken as negative while that of a convex mirror is positive.

11.2.3 Formation of images by Spherical Mirrors

To find the position of image we can take any two rays originating from object. After reflection from mirror these rays determines the position of image -

1. The incident ray which is parallel to principle axis pass through the focus (in concave mirror) or seems to be coming from focus (in convex mirror), after reflection from spherical mirror.
2. The incident ray passing through focus, becomes parallel to principle axis after reflection from spherical mirror.
3. The incident ray passing through center of curvature returns back on the same path after reflection from the spherical mirror.

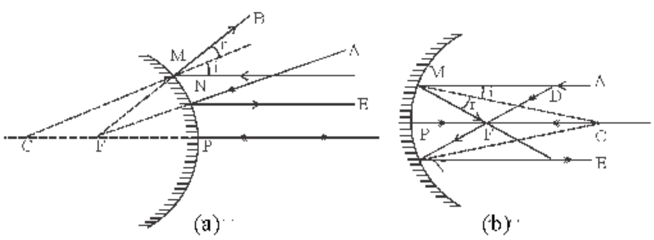
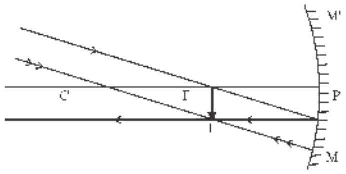
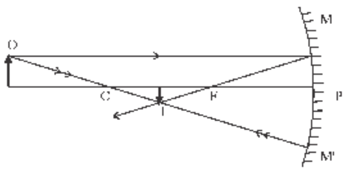
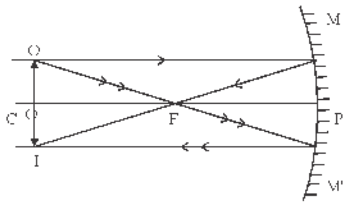
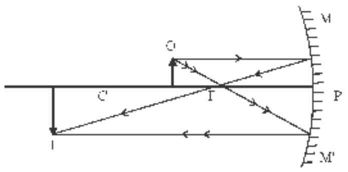
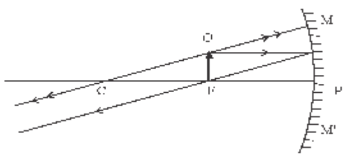
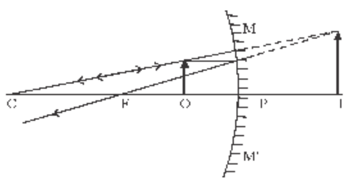


Fig. 11.6 : Rays used for the formation of images

If the two rays meet at a point after reflection a real image is formed. And if the rays do not meet but seems to be coming from that point, then a virtual image is formed. The real image is inverted while the virtual image is erect.

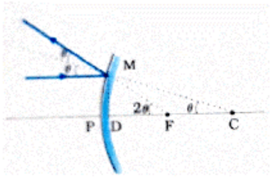
The different positions of images formed by concave and convex mirror are shown in table 11.1.

Table 11.1 (a) For concave mirror

Position of object	**Ray diagram**	**Form of image**
1. At infinity		Real, inverted, very small at focus
2. Between ∞ and C		Real, inverted, very small between F and C
3. At C		Real, inverted, and of same size, at C
4. Between F and C		Real, inverted, magnified between C and ∞
5. At focus		Real, inverted and very large at ∞
6. Between F and P		Virtual, erect, very large, behind the mirror

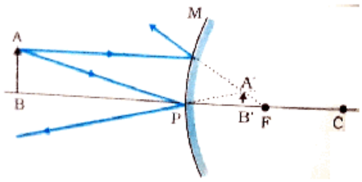
(b) For convex mirror

1. At ∞



virtual, erect,very small at focus

2. In front of mirror



Virtual,erect, small between pole and focus

11.3 Mirror Formula

11.3.1 Relation between R and F for a mirror

Consider a mirror of very small aparture (Fig. 11.7). AM is incident ray, MB is reflected ray and MC is perpendicular at M.

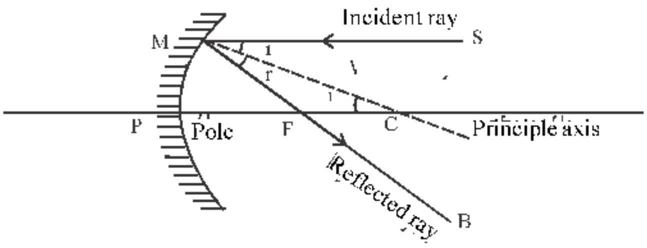


Fig. 11.7 : Reletion between R and f

From law of reflection $\angle AMC = \angle FMC$ or $\angle i = \angle r$ But $\angle AMC = \angle MCF$ (alternative angle).

Hence from $\angle MCF$, $\angle FMC = \angle MCF$ and MF= FC

If the point M is very near to P; MF = PF = f (focal length) but fron fig. 11.7.

$PF - FC - R$

$f \quad f - R$

$$f = \frac{R}{2} \qquad \ldots(11.4)$$

Hence the focal length is half of the radius of curvature for a spherical miror.

11.3.2 Mirror Equation

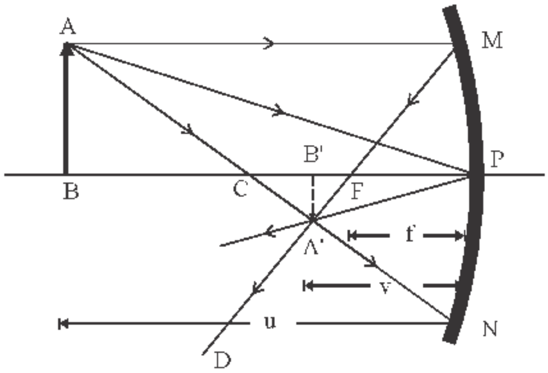


Fig. 11.8 : Ray diagram for image in concave mirror

In fig. 11.8 A^1B^1 is real image of the object . *u*, *v* and *f* are object distance,image distance and focal length respectively. To find relation between *u*, *v* and *f*, consider the similar tringle A^1B^1F and MPF (assuming MP as a straight line).

$$\frac{B'A'}{PM} = \frac{B'F}{FP} \text{ OR}$$

$$\frac{B'A'}{BA} = \frac{B'F}{FP} \quad (\because PM = BA) \qquad \ldots(11.5)$$

Again from Δ PAB and Δ PA^1B^1 (being similar) we get

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \qquad \ldots(11.6)$$

Comparing equation 11.5 and 11.6 we get

$$\frac{B'F}{FP}=\frac{B'P-FP}{FP}=\frac{B'P}{BP} \qquad ...(11.7)$$

using sign convention B'P = - *v*; FP = -*f* and $BP = -u$

$$\frac{-v+f}{-f}=\frac{-v}{-u} \text{ OR } \frac{v-f}{f}=\frac{v}{u}$$

or $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad ...(11.8)$$

This relation is called mirror equation, and is valid for both concave and convex mirrors.

Magnification :- Linear magnification *m* is defined as the ratio of image height h' and object height h.i.e

$m=\frac{h'}{h}$ As per sign convention, the distance above the principle axis is taken as position and vice versa.

From fig. 11.8 $\frac{B'A'}{BA}=\frac{-v}{-u}$

from similar triangle A'B'P and AB P we get

$$\frac{-h'}{h}>\frac{-v}{u}$$

(using sign convention) $$m=\frac{h'}{h}=-\frac{v}{u} \qquad ...(11.10)$$

Here - *ve* sign indicate that the image is inverted.

Using mirror equation (11.8) $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ and equation (11.10) we can obtain magnification in terms of *f* and *v*;

and f and u. *i.e* $$m=-\frac{v-f}{f} \qquad ...(11.10(a))$$

and $$m=-\frac{f}{u-f} \qquad ...(11.11a)$$

Example 11.1: Radius of curvature of a concave mirror is 50 cm. Find its focal length.

Solution : Given R = 50 cm

We know that $f=\frac{R}{2}$ So $f=\frac{R}{2}=\frac{50}{2}=25\text{ cm}$

$f=0.25\text{ m}$

Example 11.2 : An object is placed at a distance of 10 cm from a convex mirror of radius of curvature 15 cm. Find the nature, position and magnification of image.

Solution : Given $f=\frac{-15}{2}\text{ cm}$ and $u=-10\text{ cm}$

From mirror equation $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

hence $$\frac{1}{v}=-\frac{2}{15}-\left(\frac{1}{-10}\right)=\frac{-2}{15}+\frac{1}{10}=\frac{-4+3}{30}$$

$$\frac{1}{v}=-\frac{1}{30} \text{ hence } v=-30\text{ cm}$$

$$m=-\frac{v}{u}=-\left(\frac{-30}{-10}\right)=-\frac{30}{10}=-3$$

so the image will be at 30 cm in front of mirror, it is real and inverted and three times the size of object.

Example 11.3 : The image of an object placed in front of a concave mirror is obtained at 100 cm on the same side. If focal length of the mirror is 98 cm, find the object distance.

Soltuion : Given *f* = - 98 cm; *v* = - 100 cm

using mirror equation we get $\frac{1}{u}=\frac{1}{f}-\frac{1}{v}$

$$\frac{1}{u}=\frac{1}{-98}-\frac{1}{-100}=-\frac{1}{98}+\frac{1}{100}$$

$$\frac{1}{u}=\frac{-100+98}{9800}=\frac{-2}{9800}$$

$$u=\frac{9800}{2}=-4900\text{ cm}$$

Example 11.4 : Radius of curvature of a concave mirror in an amusement park is 4 m. A girl standing in front of this mirror sees herself 2.5 times taller. If the image is erect, find her distance from the mirror.

Solution : Given m = - 2.5 and R = - 4 cm

$$f = \frac{R}{2} = \frac{-4.0}{2} = -2.0\,\text{m}$$

$$m = \frac{h_2}{h_1} = -\frac{f}{u - f}$$

Substituting we get

$$2.5 = -\frac{(-2.0)}{u - (-2.0)} \Rightarrow u = -1.2\,\text{m}$$

Example 11.5 : An object is placed at a distance f in front of a convex mirror of focal length f. Find the image distance.

Solution : Taking u – -f and the mirror equation we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}\ ;\ -\frac{1}{f} + \frac{1}{v} = \frac{1}{f}\ ;\ \frac{1}{v} = \frac{2}{f}\ \ i.e.\ v = \frac{f}{2}$$

Note : The image of a real object by a convex mirror is between pole and focus. The image is aint, virtual and erect.

Example 11.6 : The image of an object placed at a distance 20 cm from a concave mirror is obtained at 40 cm. Find the focal length of the mirror.

Solution : Given u = - 20 cm; v = - 40 cm

from mirror equation we get

$$\frac{1}{f} = \frac{1}{(-20)} + \frac{1}{(-40)} = -\frac{1}{20} - \frac{1}{40} = \frac{-2-1}{40}$$

$$\therefore \qquad f = -\frac{40}{3} = -13.333\,\text{cm}$$

11.4 Refraction of light

Light travels in a straight line in a homogeneous medium. When light ray is incident at the interface of two media, a part of it is reflected and a part enters the other medium by bending from original direction. This phenomenon of bending of ray at the interface of two media is called refraction.

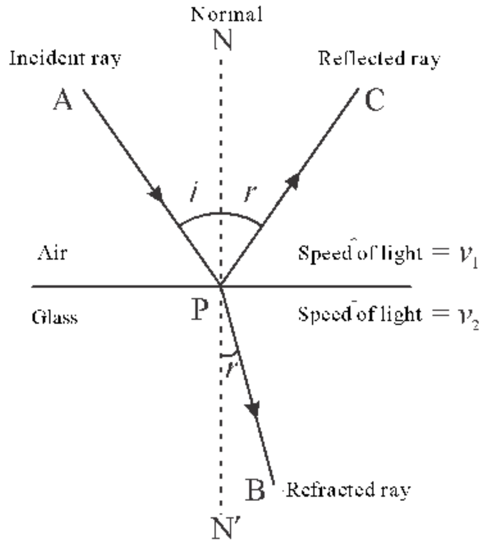


Fig 11.9 : Refraction of light

Following laws are obtained by simple experiments.

(i) Incident ray, the refracted ray and the normal on the interface at that point lie in the same plane.

(ii) The ratio of since of incident angle and rise of angle of refraction is a constant. This law is called Snell's law.

$$i.e.\ \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21} \qquad \ldots(11.11)$$

Here i and r are angle of incident and angle of refraction respectively. And n_1 and n_2 are refraction indexes of the first and second medium. Where as n_{21} is the refractive index of second medium with respect to the first medium.

If the light enters a medium from air or vaccum, then the refractive index is called the absolte refractive index of the medium. The Snell's law can be conveniently written as

$$n_1 \sin i = n_2 \sin r \qquad \ldots(11.12)$$

When $n_{21} > 1$; $r < i$, the refracted ray bends towards the normal, the second medium is denser than first medium.

If the ray enters a rarer medium from a denser medium then $r > i$ and ray bends away from the normal.

Speed of light C, is maximum in air/vaccum. The speed of light v is lesser in all other media.

Absolute refractive index of a medium is given by

$$n = \frac{\text{Speed of light in vaccum}}{\text{Speed of light in that medium}} = \frac{c}{v} \quad \ldots (11.13)$$

The relative refractive index of a medium is given by

$$n_{21} = \frac{\text{Speed of light in first medium}}{\text{Speed of light in second medium}} = \frac{v_1}{v_2}$$

Refractive index is a scalar and dimensionless quantity and it depends on the nature of the medium and the wavelength/frequency of the light. During refraction only wavelength of the light changes due to change in speed, but frequency remains unchanged.

11.4.1 Phenomena related to refraction of light

1. Twinkling of stars

Image of stars on retina are point images. Light from the stars travels a long distance in a turbulant atmosphere. Due to this the apparent position of the image on the ratina fluctuates and the amount of light entering the eye thickers. This is called twinkling of star.

Why don't planets twinkle? The planets are comparatively nearer and behave like an extended source. The light from a planets seems to be coming from large number of points. The change in positions of all the points, nullify the effect of twinking.

2. Sunrise and sunset

The sun is vissible to us a few minutes before the actual sunrise. It is also vissible for a few miniutes after the sunset. This is due to refraction of light by different layers of atmosphere having different refective indices.

When the Sun is still at horizon, the ray enters from denser layer to a rarer layer thus bending away from the normal. Due to this gradual bending by different layers. We see the Sun at at S[1] instead of S.

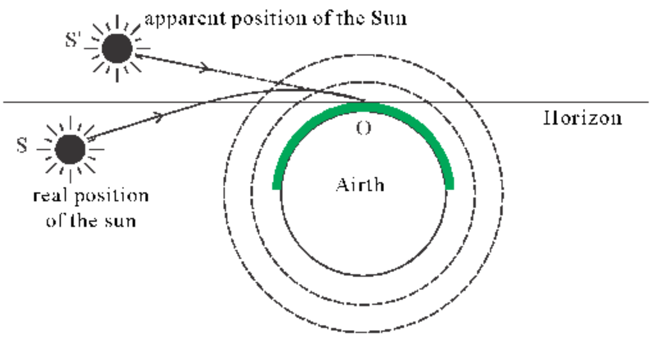


Fig. 11.10 The apparent position of the sun

3. Appearance of bottom of liquid as raised

The bottom of pot, filled with a liquid seems to be raised when viewed from above. This is due to refraction. As from fig. 11.11, the ray of light coming from the point P on the bottom, bends away from the normal at the liquid-air interface and seems to be coming from point P[1]. Hence the bottom seems to be raised.

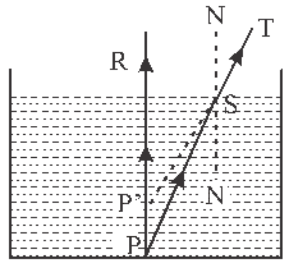


Fig. 11.11 : Virtual position of bottom of a liquid

4. Bending of a rod/spoon in a liquid

When a rod is partially dipped in a liquid, the rod appears to be bent. This is because the apparent positions of its different parts in liquid seems to be raised according to their depths. The rod ABE appears us as ABE[1] (Fig. 11.12).

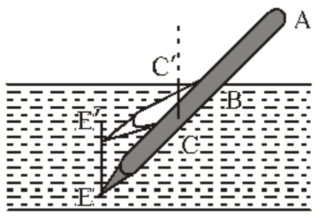


Fig. 11.12 : Bending of a rod in liquid

5. Refraction through a glass slab

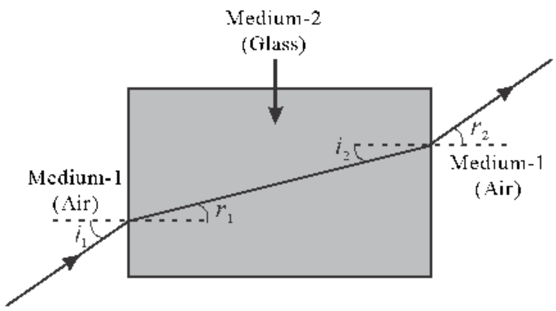
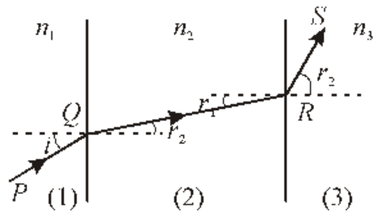


Fig. 11.13 : Refraction through a glass slab

As shown in the figure 11.13. The refraction of light occurs twice. First when the ray enters the slab and when it comes out of the slab. Here angle of incidence i_1 and angle of emergence r_2 are equal. The incident and emergent rays are parallel to each other, but there is a lateral shifting of emergent ray.

Example 11.7 : The given figure shows the path of a ray passing through three different media. The figure is based on scale. What do you conclude about the refractive indices of the three media.



Solution : As seen in the given figure, the refrated ray QR bends towards the normal, hence $n_2 > n_1$ and medium (2) is denser than medium (1).

Similarly the refracted ray RS is bending away from the normal hence $n_3 < n_2$ and the medium (3) is rarer than medium (2).

Also since $\angle r_2 > \angle i$ hence $n_2 > n_3$

Example 11.8 : Wavelength of a colour of light in air is 6000 Å, which changes to 4500 Å in water. Find the velocity of light in water.

Solution : We know that $\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

hence $\frac{v_\omega}{v_a} = \frac{\lambda_\omega}{\lambda_a}$

$$v_\omega = \left(\frac{4500}{6500}\right) 3\times10^8 = 2.25\times10^8 \text{ m/s}$$

Example 11.9 : The absolute refractive indexes of water and glass are 4/3 and 3/2 repsectively. Find refractive index of water when the ray enters from glass to water.

Solution : Given $n_\omega = \frac{4}{3}$ and $n_g = \frac{3}{2}$

hence the refractive nindex of water with respect to glass

$$n_{\omega g} = \frac{n_\omega}{n_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

11.5 Total Internal Reflection

We have learned in previous section, that when a ray enters a rarer medium from a denser medium it bends away from normal. Let us deal the phenomenon in detail.

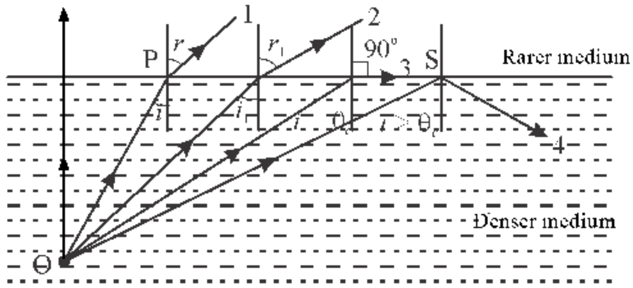


Fig. 11.14 Total internal reflection

A ray of light is incident on interface of denser and rarer medium. If the ray enters rarer medium from denser medium and $\angle i = 0$; the ray go undeviated. If we increase the incident angle of the refraction will increase. At a certain angle of incident, the angle of refraction r becomes 90^0, up to this stage both reflection and reflection simultaneously takes place. The angle of incident for which the corresponding angle of refraction becomes 90^0 is called critical angle and represented by θ_C.

If $i > \theta_C$ then $r > 90^0$. and the ray is totally reflected back (refraction ceases). This phenomon is called total internal reflection. for TIR two basic requirements are there - (1) The ray should enter a rarer medium from a denser medium.

(2) $\angle i > \theta_C$ *i.e.* angle of incidence should be greater than critical angle.

If the rarer medium is air, then from Snell's law

$$n_m \sin\theta_C = n_a \sin 90^\circ$$

$$n_m = \frac{1}{\sin\theta_C} \quad [\because n_a \approx 1] \qquad \ldots(11.14)$$

When n_m = refractive index of denser medium

The critical angle for some media are given in table 11.2 for air as rarer medium.

Substance	Refractive Index	Critical Angle
Water	1.33	48.75°
Crown glass	1.52	41.14°
Fliut glass	1.62	37.31°
Diamond	2.42	24.41°

11.5.1 Some Applications of Total Internal Reflection

1. Mirages

On a hot summer day, during driving on a coaltar raod, we experience an optical illusion in which we see that the road ahead of us is wet. Such type of experience is also there in desert during summer. This optical illusion is called mirage.

This phenomenon can be explained by assuming that the atmosphere near the earth surface consist of so many layer having different refractive indices, as we go up the refractive index go on increasing. The sun rays entering from uper layer have a refraction from denser to rarer layer hence bends away from the normal, a stage will come when the ray is totally reflected, and we feel the inverted image of tree or other object as if coming after reflection from a water surface.

2. Brilliance of diamond

Brilliance in diamond is due to total internal reflection. Reflective index of diamond is very large total approximately 2.42 hence the critical angle is small *i.e.* 24.41°. It is cut and polished in such a way that the light entering its different faces get totally internal reflected. Hence the brilliance.

3. Optical Fibre

Optical fibres are used to carry optical signals to a very long distance. It consists of central part core and outer coating, cladding. Refractive index of core is slightly greater than the refractive of cladding. An optical signal entering one end of the optical fibre at an angle $i > \theta c$ get multiple total internal reflection and ultimately comes out of the other end without much loss of energy.

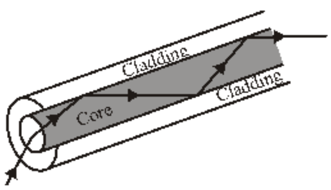


Fig.11.15 Optical Fibre

Example 11.10 : The refractive index of diamond is 2.42. Find critical angle for it.

Solution : Given n = 2.42

We know that $\sin C = \frac{1}{n}$

$$\sin C = \frac{1}{2.42} = 0.4132$$

$$\therefore \quad C = \sin^{-1}(0.4132)$$

From sine table we get θ_c = 24.4°

Example11.11 : The refractive indices of core and cladding of an optical fibre are 1.47 and 1.31 respectively. find the angle of incidence for which total internal reflection takes place in optical fibre.

Solution : For total internal reflection $\theta > \theta_c$

$$\theta > \theta_c = \sin^{-1}(n_2 / n_1)$$

$$\theta > \sin^{-1}(1.31/1.47) = \sin^{-1}(0.88)$$

$$\theta > 63^{\circ} \text{ (From sine table)}$$

11.6 Refraction at Spherical Surface

In this section we will study refraction at the spherical interface of two transparent homogeneons media. The law of refraction is applicable at every point of a spherical surface.

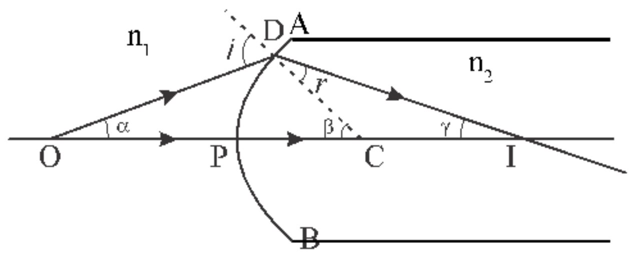


Fig. 11.16 Refraction at sperical surface

As in fig. 11.16 we consider a spherical surface AB of radius of curvature R, separating two media of refractive index n_1 and n_2. A point object is placed on the pricniple axis at O. The rays OD and OP meet at I after refraction and form an image. the point D is assumed very near to P. So that PD is straight line.

From fig. 11.16 $\tan \angle DOP = \tan \alpha = \frac{DP}{OP}$

$$\tan \angle DCP = \tan \beta = \frac{DP}{PC}$$

and $$\tan \angle DIP = \tan \gamma = \frac{DP}{PI}$$

In Δ DOC $\angle i$ is external angle hence

$$i = \angle DOP + \angle DCP = \alpha + \beta \quad \ldots (11.15)$$

For small angle (in radian) $\theta \approx \tan \theta$

$$i = \frac{DP}{OP} + \frac{DP}{PC} \quad \ldots (11.16)$$

Similarly in Δ DCI, is external angle hence $\beta = \gamma + r$

hence $$r = \angle DCP - \angle DIP \quad \ldots (11.17)$$

Or $$r = \frac{DP}{PC} - \frac{DP}{PI} \quad \ldots (11.18)$$

From Snell's law $n_i \sin i = n_2 \sin r$

$$n_i i = n_2 r \quad \text{(for small angle)} \quad \ldots (11.19)$$

$$n_1 \left(\frac{DP}{OP} + \frac{DP}{PC} \right) = n_2 \left(\frac{DP}{PC} - \frac{DP}{PI} \right)$$

$$\frac{n_1}{OP} + \frac{n_2}{PI} = \frac{(n_2 - n_1)}{PC} \quad \ldots (11.20)$$

Using cartesian coordinate sign convention and taking OP = - u; PI = v; and PC = + R

We get $$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_i}{R} \quad \ldots (11.21)$$

The above relation is valid for both convex and concave surfaces.

11.7 Lens

A transparent homogeneous medium enclosed by two curved surfaces is called lens. Out of the two surfaces, one can be a plane surface. The curved surface may be spherical, cylinderical or parabolic. We will discuss only spherical surface. The face in which light enters is called first face and the other is second face.

According to curved surfaces, lenses are of two types -

(1) Convex lens/ convergent lens. These lenses are thin on the outer rim and thick at the center.

(2) Concave lens/divergent lens. These lenses are thin at the center and thicker on the outer perifery.

11.7.1 Refraction Through Thin Lens

Consider a thin lens of refractive index n_2 placed in rarer medium of refractive index n_1. P_1 and P_2 are poles and R_1 and R_2 are radius of curvature of two surfaces of the lens.

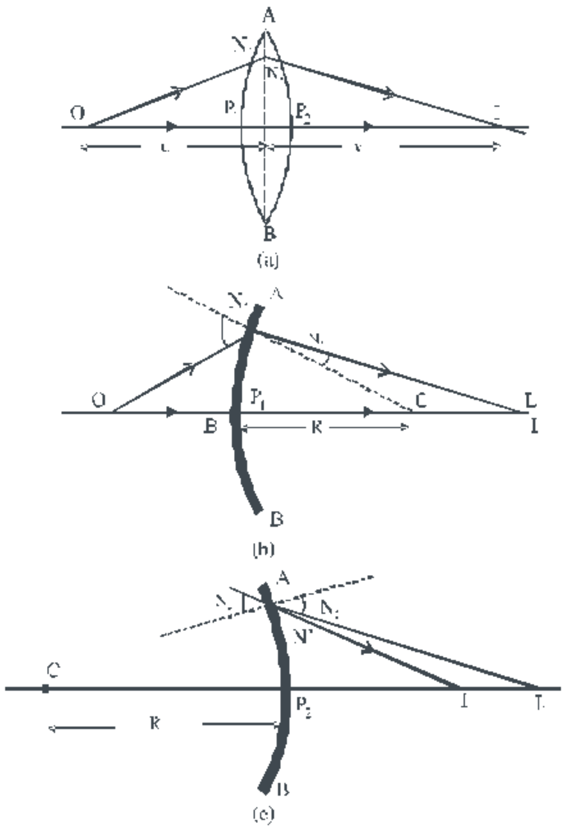


Fig. 11.17 : (a)Position of object and image
(b) Refraction through first surface and
(c) Refraction through second surface

In fig. 11.17 (a) the object is at O and image is at I. The formation of image in considered in two parts (i) action of the first surface and (ii) action of second surface.

As in fig. 11.17 (b) the object is at O and image is formed at I[1]. Applying equation 11.21 for this surface we get

$$-\frac{n_1}{OP_1} + \frac{n_2}{P_1 I_1} = \frac{n_2 - n_1}{R_1} \quad \ldots (11.22)$$

For the second surface the image at I[1] works as an object and final limage is formed at I.

Applying equation 11.21 for second surface we get

$$-\frac{n_2}{P_2 I_1} + \frac{n_1}{P_2 I} = \frac{n_1 - n_2}{R_2} \quad \ldots (11.23)$$

Adding equation 11.22 and equation 11.23, we get

$$-\frac{n_1}{OP_1}+\frac{n_1}{P_2I}=\left(n_2-n_1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\quad\ldots(11.24)$$

since $OP_1 = u$ and $P_2I = v$ we get

$$-\frac{1}{u}+\frac{1}{v}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\quad\ldots(11.25)$$

If the object is at ($u=-\infty,\ v=f$) the image will be formed at focus ($v=f$)

$$\text{We get }\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\quad\ldots(11.26)$$

The above relation is known as lens makers formula. Because for a particular value of refractive index, the required focal length is obtained by grinding the two surfaces. The above relation is also valid for concave lens.

A lens has two focii F and F^1 equidistance from the optical center on both sides. The focus on object side is called first focus and that of other side is called second focus.

As in case of a mirror, the magnification by lens is given by

$$m=\frac{\text{height of image}}{\text{height of object}}=\frac{h'}{h}=\frac{v}{u}\quad\ldots(11.28)$$

From sign convention we see that for virtual image m is positive, while for a real image m is negative.

11.7.2 Power of lens

The power of a lens is known by its ability to bend the incident ray by refraction. It is measured by the angle of deviation produced by lens.

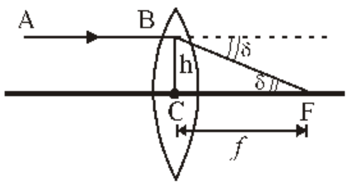


Fig. 11.18 : Power of lens

As in fig. 11.18 a ray parallel to principle axis is incident on lens at a point B which is at a distance h from the opticalcenter C. The ray will pass through F after refraction from lens. Hence for a small angle of deviation δ,

$$\delta=\tan\delta=\frac{h}{f}$$

$$\text{So}\quad P=\delta=\frac{h}{f}$$

The power is defined taking $h = 1\,m$. hence

$$h=1\quad P=\frac{1}{f_{(m)}}\quad\ldots(11.29)$$

The focal length is taken in metre. From eq. (11.29) it is evident that the power of a lens is inversly proportional to its focal length. If the focal length of a lens is 1m, its power is one diaptre *i.e.* 1D. Power is a dimentionless scalar quantity. The sign of power is according to sign of focal length. *i.e.* power of a convex lens is positive while the power of concave lens is negative.

11.7.3 Combination of thin lenses

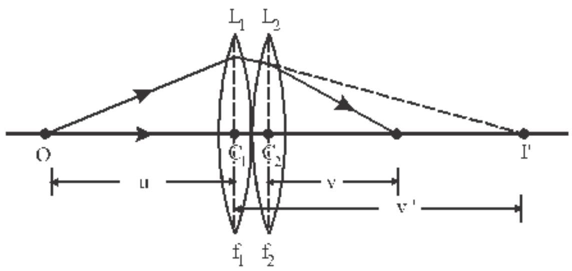


Fig. 11.19 : Combination of two lenses

The two lens kept in contact, behave like a single lens.

As in fig. 11.19 an object O is placed at a distance u from the combination of two lenses L_1 and L_2 of focal length f_1 and f_2. In the absence of lens L_2 the image formed by L_1 is at a distance V^1 from optical center C_1. Hence for the first lens

$$\frac{1}{v'}-\frac{1}{u}=\frac{1}{f_1}\quad\ldots(11.30)$$

Now the image I^1 behave as an object for the second lens since the lenses are assumed to be thin, C_1 and C_2 are very near to each other. So we, can take the object distance for second lens as V^1. Hence for the second lens we get

$$\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2}\quad\ldots(11.31)$$

Adding eq. 11.30 and eq. 11.31 we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2} \qquad \ldots(11.32)$$

The combination behave as a single lens for which the object distance is u and image distance is v. Hence f is focal length of the combination.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad \ldots(11.33)$$

So we get $$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \qquad \ldots(11.34)$$

for combination of many lenses of focal lengths $f_1, f_2, f_3 ...,$ etc. The resultant focal length and power will be -

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots \qquad \ldots(11.35)$$

and $P = P_1 + P_2 + P_3 +$ ...(11.36)

Here P is the net power of the combination. and the total magnification m is given by

$$m = m_1 \times m_2 \times m_3 \times \ldots \qquad \ldots(11.37)$$

where m_1, m_2, m_3 are magnification by individual lenses.

Such combinations are used in camera and optical microscopes (in eye pieces and objectives).

11.7.4 Image formation by lens

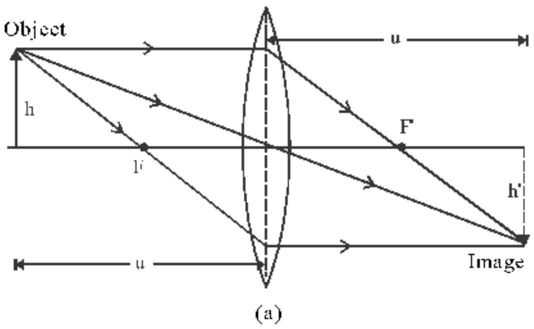


Fig. 11.20 (a) For convex lens

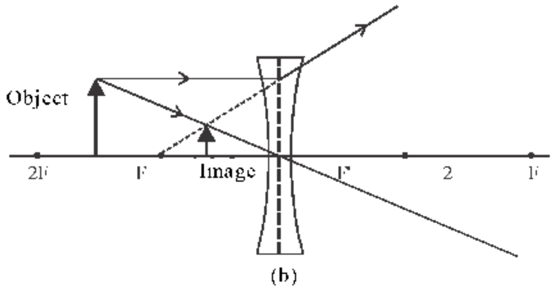


Fig. 11.20 (b) For concave lens

Formation of image by lens can be explained by tracing the path of at least two rays orriginating from the object. In case of a convex lens these two rays meet at a point after refraction and form a real image.

In case of a concave lens,these two rays from the object diverge after refraction, but seems to be coming from a point, where virtual image is formed. The path of the rays will be as follows-

(i) the incident ray parallel to principle axis will pass through the focus after refraction from convex lens.

But seems to be coming from focus in case of concave lens.

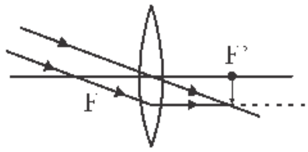
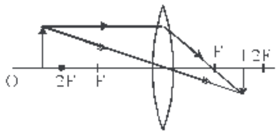
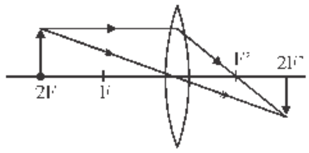
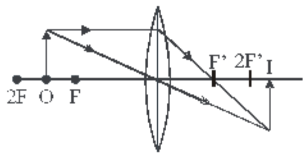
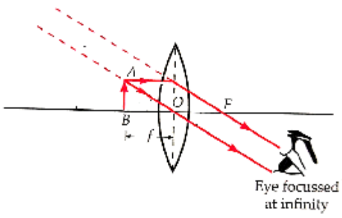
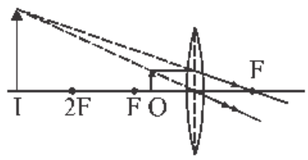
(ii) The incident ray passing through optical center will go undeviated in both types of lenses.

(iii) The incident ray passing through focus will go parallel to principle axis after refraction from convex lens.

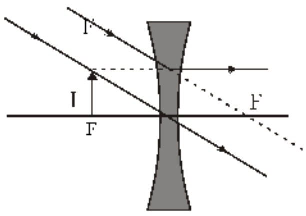
In case of concave lens,the incident ray targeted towards focus, will go parallel to principle axis after refraction.

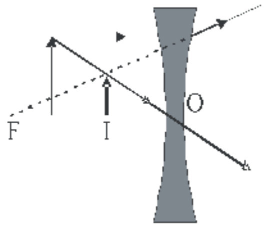
Table 11.2 For the positions of object and image

(a) For a converging or convex lens

S.No.	Position of object	Ray diagram	Position of image
1.	At ∞		Real, inverted, point image, at F ($m << -1$)
2.	Between ∞ and 2 F		Real, inverted, small ($m < -1$) between F and 2 F
3.	At 2 F		At 2 F, inverted, Real ($m = -1$)
4.	Between 2 F and F		Real,inverted, between 2 F and ∞
5.	At F		Real inverted,very large $m >> -1$ at ∞
6.	Between F and O		Virtual,erect,large $m > +1$ on the side of object between ∞ and object

(b) For divergent or concave lens

Position of object	Ray diagram	about image
1. At ∞		virtual, erect, small $m << 1$ at F (in the same side)

2. In front of lens  virtual, erect, small m < + 1 between F and O

Example 11. 12 : An object of height 6.0 cm is placed at 30.0 cm from a lens. the height of the image is 2.0 cm and is inverted. Find focal length of the lens.

Solution : $m = \frac{h_2}{h_1} = \frac{f}{u+f}$

Substituting $u = 30.0$ cm, $h_2 = 2.0$ cm and $h_1 = 6$ cm

$$\frac{(-2.0)}{(6.0)} = \frac{f}{(-30.0)+f}$$

$$f = \frac{60}{8.0} = 7.5\,\text{cm}$$

Example 11.13 : Radius of curvature of a convex lens is 20 cm and 30 cm. Refractive index of the material of lens is 1.5. If this lens is put in water (n = 1.33). Find its focal length.

Solution : Given $n_2 = 1.5$; $n_1 = 1.33$; $R_1 = 20$ cm and $R_2 = -30$ cm

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{20} + \frac{1}{30}\right) = \frac{1}{8}\left(\frac{5}{60}\right)$$

$$f = 96\,\text{cm}$$

Example 11.14 : What will be the object distance, when the magnification by a lens of focal length 10 cm is 2.

Solution : $f = +10\,\text{cm}$; $|m| = 2$

$$m = \frac{v}{m} = \frac{f}{u+f}$$

$$+2 = \frac{10}{u+10} \Rightarrow 2u + 20 = 10 \;;\; u = -5\,\text{cm}$$

$$\text{In } m - \text{-}\,2 \text{ then } -2 = \frac{10}{u+10} \Rightarrow -2u - 20 = 10$$

$$u = -15\,\text{cm}$$

Example 11.15 : A convex lens of focal length 5.0 cm is kept in contact with a concave lens of focal length 10.0 cm. What will be the focal length of this combination?

Solution : For combination of lenses

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

given $f_1 = +5\,\text{cm}$, $f_2 = -10\,\text{cm}$

$$\frac{1}{f} = \frac{1}{5} - \frac{1}{10} = +\frac{1}{10} \text{ hence } f = +10\,\text{cm}$$

Example 11.16 : Where should a candle of length 3 cm be placed from a lens of focal length 10 cm. So that its 6 cm long image is obtained on a screen placed at appropriate position?

Solution : h = 3 cm; h' = -6 cm; f= + 10;

$$m = I = \frac{h_2}{h_1} = \frac{v}{u};\quad \frac{-6}{3} = \frac{v}{u};\; v = -2u$$

using lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{-2u} - \frac{1}{u} = \frac{1}{10}$$

$$\frac{-1-2}{2u} = \frac{1}{10}$$

$$-\frac{3}{2u} = \frac{1}{10}$$

$$u = \frac{-3\times 10}{2} = -15\,\text{cm}$$

$$v = -2u = -2\times(-15) = 30\,\text{cm}$$

hence $u = -15\,\text{cm}$

Example 11.17 : Find the focal length of a double convex lens made up of glass whose radius of curvature and refractive index are 20 cm; 30 cm and 1.5 respectively.

Solution : Given $R_1 = +20\,\text{cm}$, $R_2 = -30\,\text{cm}$

$$n = 1.5$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-30}\right) = 0.5 \times \left(\frac{30+20}{600}\right)$$

$$\frac{1}{f} = 0.5 \times \frac{50}{600} = \frac{25}{600} = \frac{1}{24}$$

hence $f = +24\,\text{cm}$

Example 11.18 : A lens of refractive index 1.5 has focal length of 0.3 m. What will be its focal length in water?

Solution : Focal length of lens in air is

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{30} = 0.50 \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots (1)$$

When the lens is in water

$$\frac{1}{f'} = (n_{g\omega} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$n_{g\omega} = \frac{n_g}{n_\omega} = \frac{1.50}{1.33} = 1.1278$$

$$\frac{1}{f'} = (1.278 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f'} = 0.1278 \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots (2)$$

deviding equation (1) by eq. (2) we get

$$\frac{f'}{30} = \frac{0.50}{0.1278} = 3.912$$

$$f' = 3.912 \times 30 = 117.36\,\text{cm}$$

11.8 Prism

A homogeneous and transparent medium enclosed by two inclined plane surfaces is called prism. The angle betwee these two inclined planes is called prism angle, and the planes are called refracting planes. The section which is perpendicular to these planes is principle section of the prism.

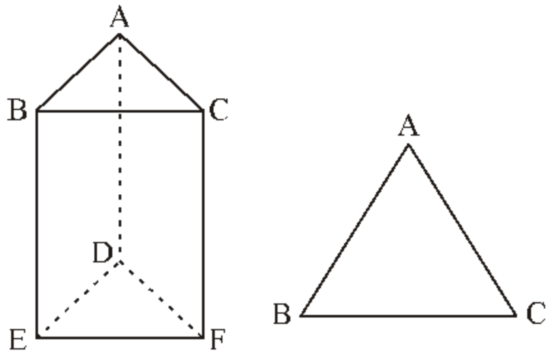


Fig. 11.21 Prism

ABC is the principle section and $\angle BAC$; $\angle ABC$ OR $\angle ACB$ is prism angle depending upon the refraction planes used. The prism can have any shape including tringular.

11.8.1 Refraction in prism

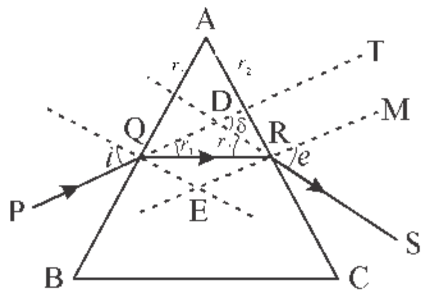


Fig. 11.22 Refraction by prism

In the above figure the planes AB and AC are used as refracting surfaces, hence $\angle A$ is taken as prism angle. The rays PQ, QR and RS are incident ray, refracted ray and emergent ray respectively. Similarly angle of incident e, angle of refraction r_1 and r_2 and angle of emergence e are marked in the figure. The deviation of the incident ray caused by prism is given as δ. The refractive index of the material of prism is n.

To find the relation between the deviation angle δ, prism angle A and refractive index n, we make use of few equations from the above diagram.

From quadriangle

$\sqcap$ AQERA; $\angle A + QER = 180^0$ ...(11.38 a)

But in Δ QER; $r_1 + r_2 + \angle QER = 180$ ...(11.38 b)

comparing eq (11.38a) and eq. (11.38 b) we get-

$A = r_1 + r_2$ ...(11.39)

similarly in Δ QDR; $\delta = \angle DQR + \angle DRQ$

or $\delta = (i - r_1) + (e - r_2)$

$= (i + e) - (r_1 + r_2)$

but from eq. 11.39; $r_1 + r_2 = A$

hence $\delta = i + e - A$ ...(11.40)

The above relation shows that the angle of deviation depends on angle of incidence i. This variation is shown by the following graph.

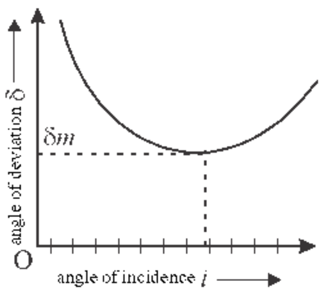


Fig. 11.23 Variation of δ with i for a triangular prism

From the above graph it is clear that the angle of deviation is minimum for a particular angle of incidence. In this condition of minimum deviation, $\delta = \delta_m$, the refracted ray QR is parallel to base of prism BC. Hence from geometry i = e and $r_1 = r_2 = \frac{A}{2}$

Hence when the prism is in the position of minimum deviation, it obey

$$\delta_m = 2i - A \ ; \quad i = \frac{A + \delta_m}{2} \text{ and } r = A/2$$

From Snell's law we get

$$n = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}} \quad \text{...(11.41)}$$

Practically we use this relation to find the refractive index of the material of the prism.

For small prism angle (expressed in radian) $(\sin\theta \approx \theta)$; the relation betwen A, n and δ_m can be expressed as

$$\delta_m = (\mu - 1)A \quad \text{...(11.42)}$$

It shows that for a thin prism, deviation is independent of angle of incident i.

Example 11.19 : A thin prism of refractive index $n = \sqrt{3}$ has prism angle $A = \delta_m$. Find prism angle.

Solution : Given $n = \sqrt{3}$ and $A = \delta_m$

$$n = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin \frac{A}{2}} = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin \frac{A}{2}}$$

$$= \frac{\sin A}{\sin \frac{A}{2}} = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin \frac{A}{2}}$$

$$\sqrt{3} = 2 \cos \frac{A}{2} \Rightarrow \cos \frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{A}{2} = 30^\circ \ ; \ A = 60^\circ$$

Example11.20 : A ray is incident on a prism of small prism angle A, and emerges perpendicular to other surface. Refractive index of the material of prism is n. Find angle of incidence.

Solution : Given $r_2 = 0$; but $r_1 + r_2 = A$, hence $r_1 = A$

From Snell's law $n = \frac{\sin i}{\sin r_1} = \frac{i}{r_1}$;

$i = n\, r_1$ (for small angle in radian) hence $i = nA$ radian

11.8.2 Dispersion of Light

Splitting of light into its constituant colours is called dispersion. We have learnt that the refractive index also depends on the wave length of light passing through it. Refractive index of a material is maximum for violet light and minimum for red light, *i.e.* $n_v > n_r$.

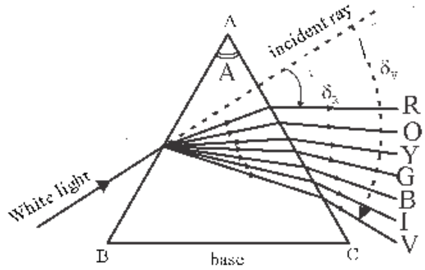


Fig. 11.24 : Dispersion by prism

White light consists of seven broad categories of colours known as VIBGYOR (for violet,indigo,blue, green,yellow, orange and red). The wavelengths of these categories is in increasing order and the refractive index of the material for these categories is in decreasing order.

From $\delta_m = A\,(n - 1)$ it is clear that different colours deviate differently according to n for that colour. $\delta_v = A\,(n_v - 1)$ is maximum while the deviation of red $\delta_R = A\,(n_r - 1)$ is minimum. The angle between these extreme ends is known as angle of dispersion. *i.e.*

$$\theta = \delta_v - \delta_R \qquad \ldots (11.43)$$

The pattern consisting of constituant colours in the given order is called spectrum.

The ratio of dispersion angle and the angle of deviation for yellow light is called dispessive power ω of the material of the prism.

$$w = \frac{q}{d_Y} = \frac{d_V - d_R}{d_Y} = \frac{n_v - n_r}{n_Y - 1} \qquad \ldots (11.44)$$

It is evident from above relation that dispersive power is independent of prism angle.

Example11.21: Refractive index of the material of prism for red and blue colours are 1.58 and 1.60 respectively. If the prism angle is 2^0. Find the deviation of two colours and dispersion angle.

Solution : Given $n_R = 1.58$, $n_B = 1.60$ and $A = 2^o$

Deviation of red is

$$\delta_R = (n_R - 1)A = (1.58 - 1) \times 2 = 1.16^o$$

Deviation of blue is

$$\delta_B = (n_B - 1)A = (1.60 - 1) \times 2^o = 1.20^o$$

Angle of disperson is

$$\theta = (\delta_B - \delta_R) = (1.20 - 1.16) = 0.04^o$$

Example 11.22 : The refractive index of crown glass for red and violet colours are 1.514 and 1.523 repectively. Find dispersion angle by the prism of prism angle 6^0.

Solution : Given

$$n_R = 1.514,\ n_v = 1.523 \text{ and } A = 6^o$$

$$\theta = (n_v - n_R)A$$

$$= (1.523 - 1.514) \times 6^o = 0.009 \times 6^o = 0.054^o$$

11.9 Scattering of Light

When a beam of light fall on the particles of atmosphere (gases and other suspended particles) it spreads in all directions. This phenomenon is called scattering of light. It is not simple reflection, basically the particles first absorbs the incident light and then re-emit in all directions.

Intensity of scattering depends on the incident wavelength and size of the particles. If the particle size is smaller than wavelength, the scattering is proportional to $1/\lambda^4$. This law is called Rayleigh's law. For the particles of size a >> λ *i.e.* rain drops, large dust or ice particles; this law is not true and nearly all colours are equally scattered.

11.9.1 Phenomenon Related to Scattering

(i) Blue appearance of sky : When sunlight fall on atmospheric particles,the smaller particles scatters the short wavelength of vissible light and out of these blue colour is most dominent. Hence sky appears blue to the observer on earth. Had there been no atmosphere the sky would have appeared black, and we could see stars during day time.

(ii) Red appearance of sun at sunrise and sunset : During sunset and sunrise the sun is at horizon. The sun rays have to travel a large distance to reach the observer, during this, most part of the shorter wavelength is scattered away and only the longer wavelength remains which reach the observer, hence the red appearance of the sun. This does not happen during day time because the sun is relatively nearer.

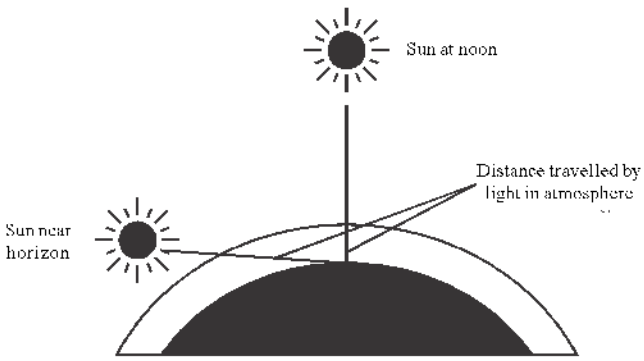


Fig. 11.25 Travelling large distance by sun rays at sunrise and sunset

(iii) Red colour (light) : Red light is used for danger signal,because this colour is least scattered by atmospheric particles and the signal is vissible in the most adverse atmospheric conditions.

(iv) White appearance of clouds : Clouds consist of water droplets,water vapours which has the particle size a >> λ; so all the wavelengths of sunlight are equally scattered. Hence the cloud appears white.

11.10 Rainbow

After stoping of rain, sometimes we see an arch shaped band of seven colours in the sky in the direction opposite to the sun. It is called rainbow. The center of the rainbow lies between the sun and the observer. When the order of the colours is from red to violet, it is called primary rainbow. Sometimes we see another rainbow just above the primary rainbow in which colours of the strips are in revers order. It is called secondary rainbow.

When the cloud is exhausted of water vapours. The small water doplets do not grow further to fall down. They remain suspended because the mg force is balanced by bouyant force. When sunlight falls on these tiny droplets, the phenomenon of refraction, dispersion and total internal reflection takes place and rainbow is formed in the sky.

All droplets are not able to form rainbow. Only those droplets contribute to rainbow which lies between angle 40^0 to 42^0 from horizon (because of the conditions of TIR). Only one colour from one droplet reaches our eyes; So billions of droplets contribute to the whole rainbow. For secondary rainbow TIR occurs twice in a single droplet, that's why the order of colours is reversed and it appears faint.

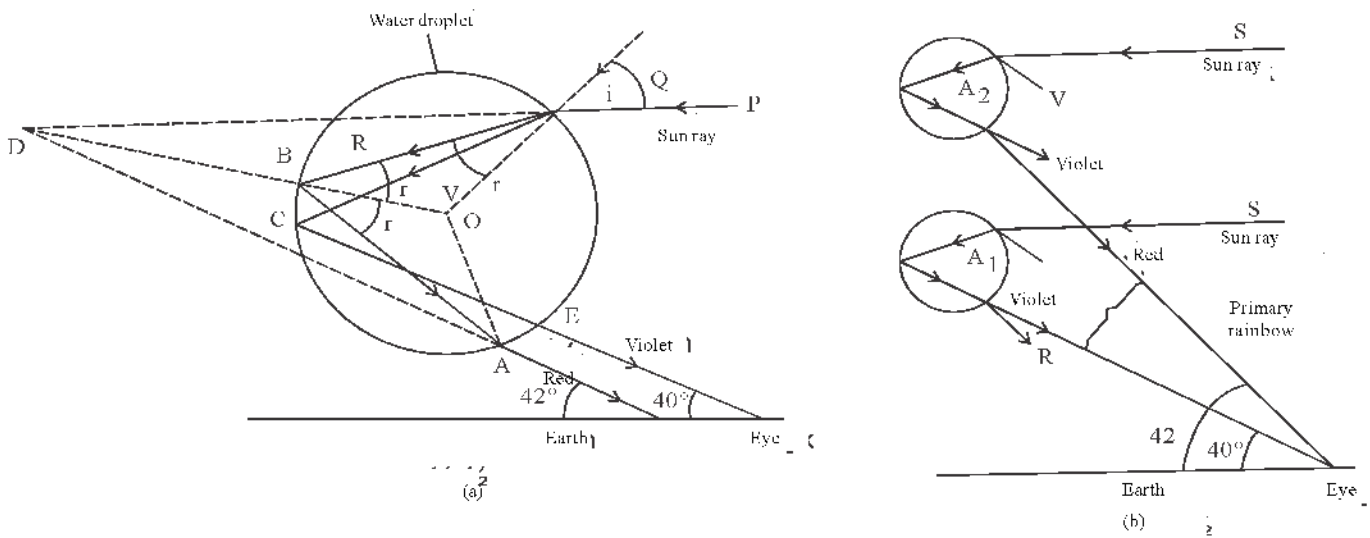


Fig. 11.26 : Formation of rainbow

11.11 Optical Instruments

11.11.1 Human Eye

The shape of our eyes is approximately spherical. As shown in the figure 11.27 (a),the front part of human eye is more sharply curved,which is covered by a transparent layer called cornea. Then an aparture called pupil controlls the amount of light entering our eyes as required. A transparent and fexible convex natural lens is held by cilliary muscles. A flued called aqueous humor is there between lens and cornea. After lens there is another fluid called vitreous humour. The refractive index of both these fluids is aproximately n = 1.336. At the end of the eye ball there is a screen called retina, which consist of photoreceptor cells called rods and cones. These photorecpter cells are connected to optical nerves, which caries the electrical singals to brain.

Cilliary muscles,which hold the lens can change the radius of curvature and focal length of the lens. They controlls the accomodation power of the lens. Lens obey the eq. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$; since the distance between lens and retina v is fixed, we have to adjust the focal length f of the lens, according to the object distance u. This is what the cilliary muscles are doing.

A healthy human eye can see the object placed between 25 cm and infinity from eye. This distance 25 cm is called near point and infinity is far point of heahlty, human eye. For children the near point may be 6-7 cm.

When we see at infinity (clouds and far distant object) cilliary muscles are relaxed,focal length is maximum. The muscles are heighly stressed when we see at near point.

Due to various reasons,such as againg, weaking of cilliary muscles and hardening of lens,various defects of vission (refractive errors) arises in eyes; the few are :

(i) Presbyopia : The effectivity of cilliary muscles and flexibility of lens reduces with age. In some people the near point which is 25 cm; reaches 200 cm at the age of 60 yrs. This error (defect) is called presbyopia, and can be corrected by using convex lens of appropriate focal length.

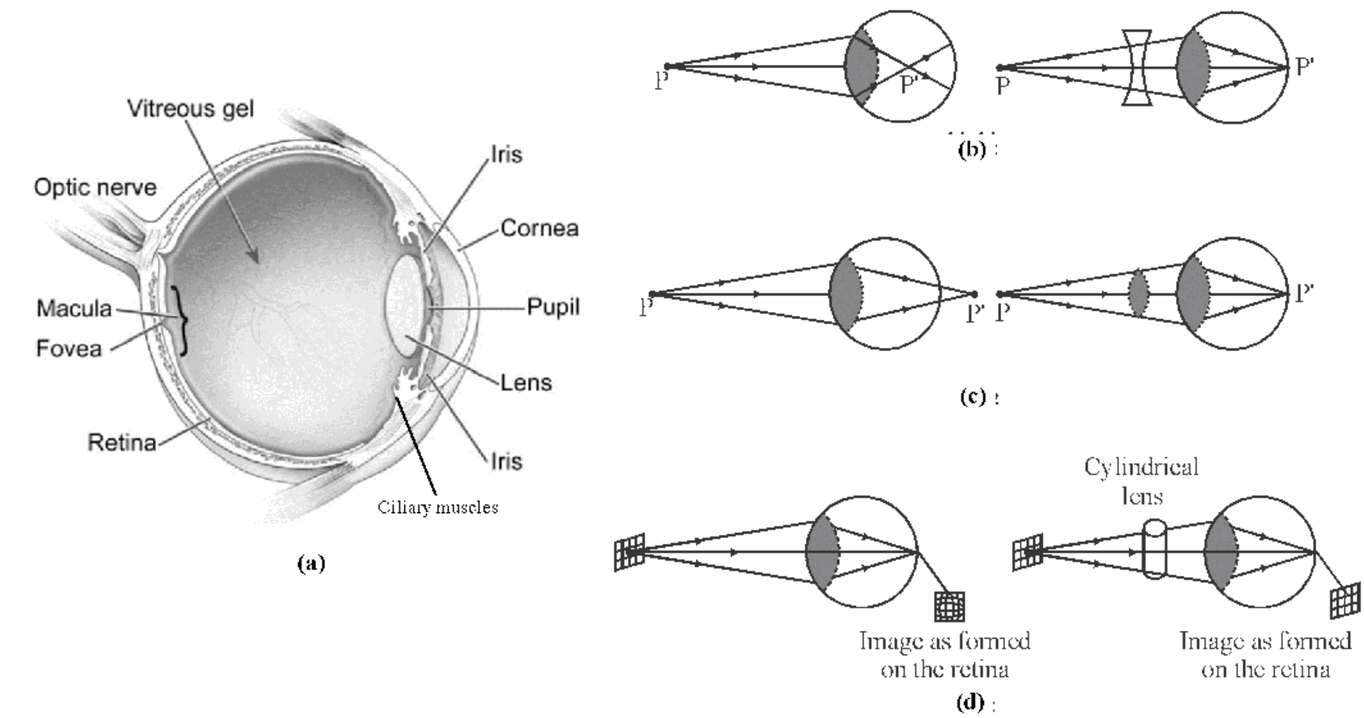


Fig. 11.27 (a) Construction of eye (b) Mopic eye and its correction (c) Hypermetropic eye and its corrtion (d) Astigmatic eye and its correction

(ii) Near sightedness or Myopia : The person having this defect can't see clearly,the object placed at infinity. The far point of such eye is no longer infinity. It is somewhere nearer. Shifting of far point towards eye shortens the range of vision, this defect is called myopia. The lens in eye becomes more convergent, and its focal length can't be increased beyond certain limit by cilliary muscles. The clear image of the object at infinity is formed before the retina, the image formed at retina is blurred. A divergent (concave)lens of suitable focal length is used for correction.

(iii) Far sightedness or Hypermetropia : The person with this defect can't see the near objects. The clear image is formed by such eye, beyond retina. A converging/convex lens of suitable focal length is used for correction.

(iv) Astigmatism : In such defect the radius of curvature of the eye lens is different in different planes; same vission is not obtained in all directions. As shown in diagram 11.27 (d); if the object is like a grid, the image may of horizontal/vertical lines or a distorted grid. A cylendrical lens of appropriate focal length and orientation is used for correction.

11.11.1.1 Apparant Size

The size of an object depends on the size of image formed on retina,which depends on the angle subtended by the object on eye. This angle is called visual angle.

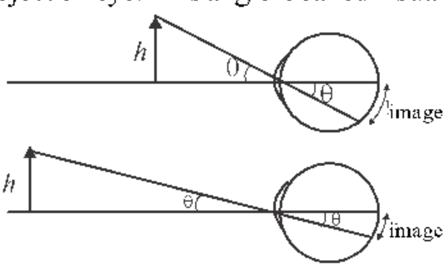


Fig. 11.28 Apparant size of an object

Size of object is h, but in fig. 11.28 (a) appear bigger than in fig. 11.28 (b) . In optical instruments this angle can be increased to get bigger and clear image.

11.11.2 Microscope

The visual angle formed by a very small object on eye is very small, so we can't see the object clearly. If an optical instrument can make its image bigger (magnified),the visual angle is increased and the object seems to be bigger and clearer. Such device is called micorscope.

11.11.2.1 Simple Microscope

It is also called magnifying lens or simply magnifier.It is actually a convex lens of small focal length. The object is placed between focus and the pole. The image formed is magnified, erect,virtual and in the same side between object and infinity.

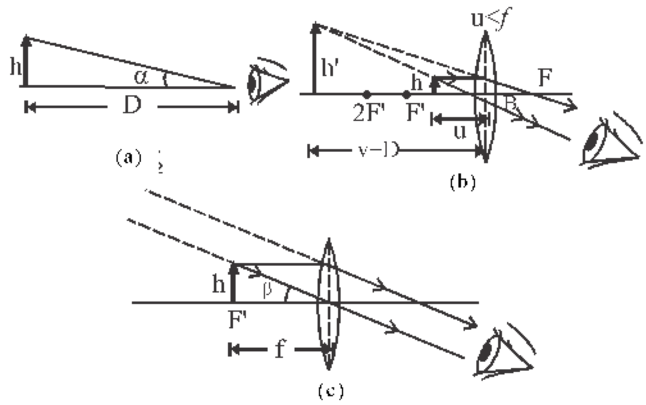


Fig. 11.29 Simple microscope

Magnification is given by

$$M = \frac{\text{Visual angle formed by image on eye}}{\text{Visual angle formed by object on eye}} = \frac{b}{a}$$

(When placed at same distance)

From fig. 1.29 (b) and (a)

$$\beta = \frac{h'}{v} = \frac{h}{u} \text{ and } \alpha = \frac{h}{D}$$

Because D = minimum distance for clear vission.

$$\text{So magnification } M = \frac{h/u}{h/D} = \frac{D}{u}$$

Normally there are two conditions in this case -

(i) If the image is at D (Near point of normal eye).

Fig. 11.29 (a) then v = - D and $u = -u$

$$\text{from lens formula } \frac{1}{v} - \frac{1}{u} = \frac{1}{f};$$

$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}; \quad -1 + \frac{D}{u} = \frac{D}{f}$$

$$\text{So } M = \frac{D}{u} = 1 + \frac{D}{f} \qquad \ldots(11.46)$$

In this case since the image is formed at D,the eye is in most stressed condition.

(ii) If the image is formed at infinity (far point of normal eye)

In this case $v = -\infty$, So $\frac{1}{-\infty} - \frac{1}{-u} = \frac{1}{f}$

here $u = f$,

$$M = \frac{D}{u} = \frac{D}{f} \qquad \ldots (11.47)$$

In the second case although the magnification is less by a factor 1 comparing the first case; but the eye of the observer remain relaxed or unstrained.

11.11.2.2 Compound Microscope

A practical simple microscope has small magnification (<10). To increase magnification we use two lenses. Such micorscope is called compound micorscope. Both lenses are coaxial. The lens which is towards the object is called objective lens or field lens, it is denoted by L_o. The other lens which is towards eye is called eye-piece or ocular,it is denoted by L_E. Their focal lengths are taken as f_0 and f_e respectively. Aparture and focal length of objective lens are smaller compared to that of eye-piece. Smaller apparture reduces sperical abration while small focal length increases magnification.

Formation of image (ray diagram) fig. 11.30 shows the ray diagram of a compound microscope.

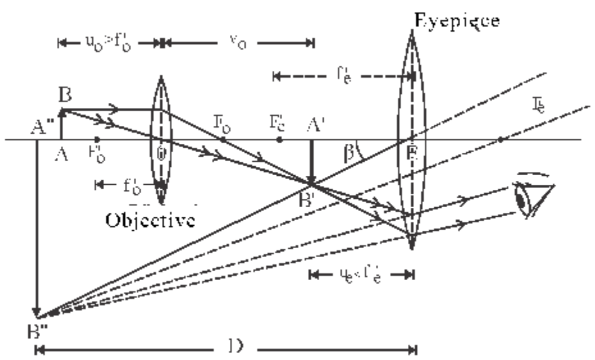


Fig.11.30 Compound microscope

An object AB is placed between F_0' and 2 F_0' of the objective lens. $u_0 > f'_0 (\approx f_0)$ fig. 11.30. Objective lens forms image A'B' on other side between 2F and infinity at a distance v_0. The image A'B' situated between eye-piece and its first focus, behaves like a virtual object, $[u_e < f'_e (\approx f_e)]$. The image formed by eye-piece A"B" is virtual,erect (comparing A'B') and very large. The final image A"B" of the object AB is very large, virtual and inverted (in comparison to AB). A"B" may be situated anywhere between D and infinity.

Magnifying power : Angular magnification of a compound microscope is given by -

$$M = \frac{\text{Visual angle formed by image on eye}}{\text{Visual angle formed by object on eye (when object is directly seen)}} = \frac{b}{a}$$

Because eye is very near to eye - piece, we take visual angle formed by A"B" on eye-piece as the visual angle formed on eye. Since object is small, α and β are very small we can approximate

$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \quad (\text{for small angle } \theta \approx \tan\theta)$$

$$= \frac{\frac{A'B'}{EA'}}{\frac{AB}{D}} = \frac{A'B'}{AB}\left(\frac{D}{EA'}\right)$$

(Max. visual angle formed by obejct on eye is $\alpha = \frac{AB}{D}$)

If the object distance and image distance for objective lens are u_0 and v_0; using sign convention we get

$$M = \frac{v_0}{-u_0}\left(\frac{-D}{-u_e}\right) = -\frac{v_0}{u_0}\left(\frac{D}{u_e}\right)$$

$$[\because \frac{A'B'}{AB} = \frac{+v_0}{-u_0}, \ EA' = -u_e \text{ and D is negative}] \qquad \ldots (11.48)$$

There are two situation in this case

(i) When the final image is formed at D (near point of normal eye).

$v_e = -D$, hence for eye-piece

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{or} \quad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

$$\frac{D}{u_e} = \frac{D}{f_e} + 1 \text{ and } u_e = \frac{f_e D}{f_e + D}$$

Substituting these values in equation 11.43 we get

$$M = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) \qquad \ldots(11.49)$$

(ii) When final image is formed at infinity

here $v_e = -\infty$, so for eye - piece

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{here } u_e = f_e \text{(maximum value)}$$

other wire $u_e < f_e$

$$\text{hence } M = -\frac{v_0}{u_0}\left(\frac{D}{f_e}\right) \qquad \ldots(11.50)$$

Again the magnification in second case less by a factor 1,but the eye of the observer remain relaxed or unstrained as compared to the first case.

11.11.3 Astronomical Telescope

The distant objects like aeroplane, planets, stars appears us very small, because they subtain small visual angle on our eyes. With help of proper lenses we obtain their image near our eyes, which produces large visual angle on eyes and the object appears enlarged.

11.11.3.1 Refracting Astonomical Telescope

Construction of such telescope is shown in fig. 11.31.

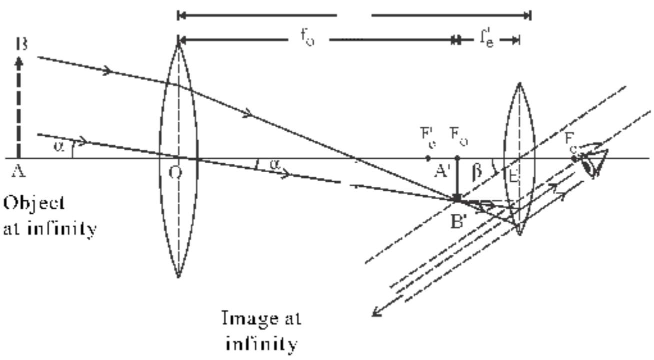


Fig. 11.31 Refracting astonomical telescope

If consists of two acromatic coaxial lenses whose principle axis coinside. A metallic pipe is fitted with a lens of large focal length and large apparture at one end. On the other end there is another small pipe which can move inside the larger pipe with help of rack and pinion arrangement. The smaller pipe is fitted with an eye-piece of smaller focal length.

The objective lens of focal length f_0 produces an inverted, real and bright image A'B' at its second focus,of the distant object, AB.The image A'B' acts as a real object for acromatic eye-piece of small focal length,which produces a magnified in image A"B" at infinity this image is inverted with respect to the object AB. The eye of the observer is relaxed (unstrained). For such telescope -

$$M = -\frac{f_0}{f_e} \text{ and } L = f_0 + f_e \text{ ; L = length of the tube}$$

11.11.3.2 Reflecting type of telescope

To obtain a bright image of a distant object,and to increase resolving power of refracting telescope, we have to use a lens of larger apparture. Obtaining such a lens is difficult. Moreover such lens creats so many problems, mechnical as well as optical such as spherical abration of the image.

To avoid all these problems reflecting telescope is used.Instead of lens a parabolic mirror of larger apparture is used intead of objective lens.

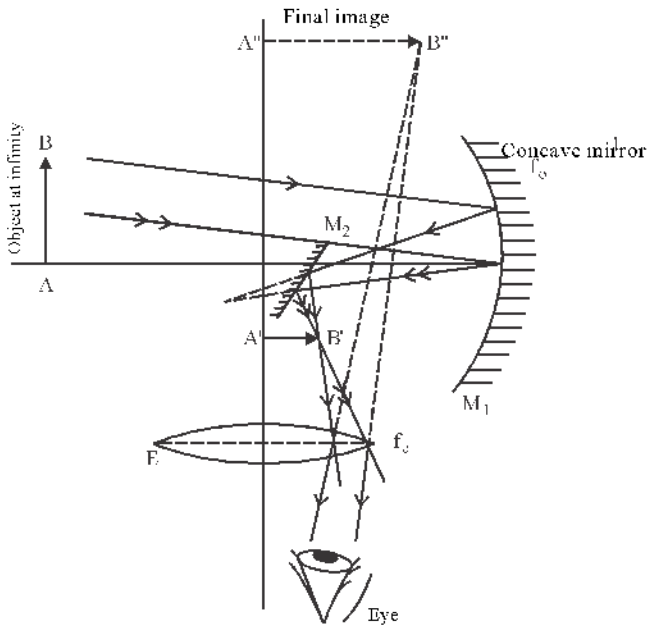


Fig.11.32 A reflecting telescope

As shown in fig 11.32 a parabolic mirror of large apparture is fitted at the end of a large tube. A plain mirror M_2 is fitted in the tube, inclined at 45^0. There is another smaller tube,which is fitted with an acromatic lens of smaller focal length, which works as eye-piece.

The incident parallel rays coming from distant object fall on parabolic (or spherical concave) mirror. The reflected rays fall on mirror M_2, and after reflection from M_2 these rays are recieved by the lens of eye-piece. We get a final magnified image A"B". Actually mirror M_2 form a real image A'B' of the object,this image act as a real object for eye-piece, which produces final virtual image A"B".

Example11.23 : Far point of a person is 5 m which statement is correct about the vission of the person?

(a) He suffers from hypermatropia,and needs a convex lens for correction.

(b) He suffers from hypermatropia, and needs a convex lens for correction.

(c) He suffers from myopia and needs a convex lens for correction.

(d) He suffers from far sightedness and needs a convex lens for correction.

Solution : Far point is infinity for a normal eye. For far point of the above person is 5 m, hence he suffers from myopia. For correction of myopia a concave lens of appropriate focal length is used.

Example 11.24 : An astronomical telescope is to be fabricated for a magnification of 50. If the length of the tube is 102 cm, find powers of objective and eye-piece.

Solution : Given $m = 50 = \frac{f_0}{f_e}$;

$$f_0 = 50 f_e \qquad \ldots (i)$$

also $f_0 + f_e = L = 102\,\text{cm} \qquad \ldots (ii)$

From these relations we get

f_o = 100 cm and f_e = 2 cm

Hence P_o = 1 D and P_e = 50 D

Example 11.25 : Magnification of a simple microscope is 11. The image is formed at the near point of clear vission. Find focal length of the lens.

Solution : Given M = 11; D = 25 cm

for a simple microscope when image is at D

$$M = 1 + \frac{D}{f} \text{ OR } 11 = 1 + \frac{25}{f} \text{ solving we get}$$

$$f = 2.5\,\text{cm}$$

Example 11.26 : Magnification of a telescope is 9. When it is set for parallel rays, the distance between objective and eye-piece is 20cm. Find the focal length of the two lenses.

Solution : As given,when the telescope is set for parallel rays, the final image is formed at infinity and magnification

$$M = -\frac{f_0}{f_e} = -9 \;;\; f_0 = -9 f_e \;;$$

since $L = f_0 + f_e = 20$ (given)

$$9 f_e + f_e = 20 \;;\; f_e = 2\,\text{cm} \;;$$

$L = f_0 + f_e = 20$ from we get

f_o = 18 cm

Important Points

1. In ray optics, the path of light is taken as straight line.

2. Laws of reflection

   (i) $\angle i = \angle r$

   (ii) The incident ray, reflected ray and the normal lie on the same incident plane.

3. Spherical mirror. Focus and focal length - for a concave mirror it is the point where incident rays parallel to principle axis meet after reflection. For convex mirror it is the point from where they seem to be originating after reflection. The distance between focus and pole is called focal length. Cartseian sign convetion - (1) All the distances are measured from pole. (2) The distance measured in the direction of incident rays is taken as positive and vise-versa. The distance upwards from principle axis is positive and vice-versa.

   Shperical mirror obey the following relation

   $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ and } f=\frac{R}{2}$$

4. Refraction - Bending of rays at the interface of two media when ray goes from one medium to another is called refraction.

   (i) Incident ray, refracted ray and the normal lie in the same plane.

   (ii) $\frac{\sin i}{\sin r}=\frac{n_2}{n_1}$ or $n_1 \sin i = n_2 \sin r$ (Snell's law)

5. Refractive index of a medium

   $$n=\frac{\text{speed of light in vaccum}}{\text{speed of light in that medium}}=\frac{c}{v}$$

6. Total internal reflection : (1) Angle of incidence $i > \theta_c$, the ray suffers total internal reflection. The critical angle $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ (2) They should go from the denser to rarer medium.

7. Refraction through prism - A prism deviates a ray of light passing through it. The angle of deviation remain same even if $\angle i$ and $\angle e$ are exchanged. For $\delta = \delta_m$; $i = e$, $r_1 = r_2 = A/2$ and

   $$n=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)}$$

   for small (in radian) $\delta_m = (n-1)A$

8. Refraction from a spherical surface. Shperical interface of the two media obey the relation

   $$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

9. For thin lens : There are two focii, the second focus is that where the incident rays mparallel to prinicple axiseet after refraction. The rays originating from first focus go parallel after refraction. For a thin lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \text{ or } v=\frac{uf}{u+f}$$

and $$\frac{1}{f}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

The linear magnification by a lens is given by -

$$m=\frac{\text{height of image}}{\text{height of object}}=\frac{h'}{h}=\frac{v}{u}=\frac{f}{u+f}$$

10. Power of lens- It is ability to deviate the incident ray. It is given by $P=\frac{1}{f}$; (f in metre) and its unit is (dioptre).

The resultant focal length of combination of lenses is given by $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+...+\frac{1}{f_n}$ and resultant power $P=P_1+P_2+P_3+...P_n$ and the net magnification $m=m_1\times m_2\times m_3\times...\times m_n$

11. Dispersion by a prism - dividing of light into its consistuant colours is called dispersion. White light breaks up into seven colours (VIBGYOR). The angle between two extream colours is called angle of dispersion dispersion $\theta=\phi_v-\delta_R=\left(\mu_v-\mu_R\right)A$

$$\omega=\frac{\theta}{\delta}=\frac{\delta_v-\delta_R}{\delta_y}=\frac{n_v-n_R}{n_y-1}$$

$$\omega=\frac{\left(n_v-n_R\right)}{\left(n-1\right)}$$

12. Microscope : It is a device which gives magnified image of small object.

(i) Simple microscope (ii) Compound microscope

(1) For simple microscope (i) $M=1+\frac{D}{f}$ (when image is formed at D)

(ii) $M=\frac{D}{f}$ (when image is formed at infinity)

(2) For compound microscope

(i) $M=-\frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right)$ (when image is at D)

(ii) $M=-\frac{v_0}{u_0}\times\frac{D}{f_e}$ (when image is at infinity)

13. Telescope - device use to see distant object resolved.

Magnification $M = -\frac{f_0}{f_e}$ when image is at infinity.

Questions for Practice

Multiple Choice Questions -

1. Only paraxial rays are considered for the formation of image in spherical mirrors because -

   (a) It is geometrically easy to use

   (b) Most of the intensity lies in them

   (c) They form point image of a point object

   (d) They show minimum dispersion

2. An object is placed at 30 cm from the concave mirror of focal length 20 cm, the nature of image and magnification will be -

   (a) Real and -2 (b) Virtual and - 2

   (c) Real and + 2 (d) Virtual and + 2

3. The reflective index for infrared rays is -

   (a) Equal to that for UV rays

   (b) Equal to that for red light

   (c) Less than that for UV rays

   (d) More than that for UV rays

4. Total internal reflection will occur if -

   (a) They ray enters denser medium from a rarer medium and $i > i_c$

   (b) The ray enters rarer medium from a denser medium and $i > i_c$

   (c) The refractive index of two media are nearly same

   (d) The refractive index of two media are different but $i > i_c$

5. An object is placed at a distance of 20 cm from a concave lens, the image formed is small which statement is certainly correct?

   (a) Image is inverted

   (b) Image may be real

   (c) Image distance is > 20 cm

   (d) Focal length of the lens may be < 20 cm

6. A convex lens of + 6 D is kept in contact with a concave lens of power - 4 D. The focal length and nature of combination lens will be -

   (a) 25 cm, concave (b) 50 cm, convex

   (c) 20 cm, concave (d) 100 cm convex

7. A ray passes through an equilateral glass prism such that $i = e$, which is 3/4 of the prism angle. Then deviation will be -

   (a) 45^0 (b) 70^0

   (c) 39^0 (d) 30^0

8. The image formed by a compound microscope will be -

   (a) Virtual and magnified (b) Virtual and small

   (c) Real point image (d) Real and magnified

9. A double convex lens of refractive index 1.47 is immerged in a liquid. It behave like a plane sheet. It means the refractive index of the liquid is -

   (a) Greater than refractive of glass

   (b) Less than the refractive index of glass

   (c) Equal to the refractive index of glass

   (d) Less than 1

10. The angle of minimum deviation for a prism would be equal to prism angle if the refractive of the medium of prism is -

    (a) Between $\sqrt{2}$ and 2 (b) Less than 1

    (c) More than 2 (d) Between $\sqrt{2}$ and 1

11. A ray of light falls normal to a plane mirror, the reflection angle will be -

    (a) 90^0 (b) 180^0

    (c) 0^0 (d) 45^0

12. An object is placed at 20 cm from a concave mirror of focal length 20 cm, its image will be formed at -

(a) $2f$ (b) f

(c) 0 (d) Infinity

13. As observed from earth, stars seems to be twinkling, the reason is -

(a) It is true that stars do not emit light continuously

(b) Absorption of frequency by the atmosphere of star itself

(c) Absorption of frequency by earth atmosphere

(d) Variation in the refractive index of earth atmosphere

14. If the yellow light is reflected by a prism at angle of minimum deviation then -

(a) $\angle i = \angle e$ (b) $i + e = 90^0$

(c) $i < e$ (d) $i > e$

15. The minimum and maximum distance for clear vission for normal eye is -

(a) 25 cm and 100 cm

(b) 25 cm and infinity

(c) 100 cm and infinity

(d) Zero and from zero to infinity

16. The length of a normal astronomical telescope is -

(a) Equal to the difference in focal length of the two lenses

(b) Half of the sum of the focal lengths

(c) Equal to sum of focal lengths

(d) Equal to the product of focal lengths

17. A virtual and magnified image can be obtained by -

(a) A convex mirror (b) Concave mirror

(c) Plane mirror (d) Concave lens

18. Final image obtained from a compound microscope is -

(a) Real and erect (b) Virtual and inverted

(c) Virtual and erect (d) Real and inverted

19. The objective used in reflecting telescope is -

(a) Convex lens (b) Convex mirror

(c) Prism (d) Concave mirror

20. The power of objective and eye-piece is 5 and 20 diaptor respectively and the image is formed at infinity. Magnifiying power of the telescope will be-

(a) 4 (b) 2

(c) 100 (d) 0.25

21. Power of a convex lens is -

(a) Negative (b) Positive

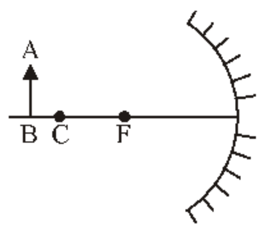
(c) Zero (d) Imaginary

Very Short Answer Questions -

1. Wha tis the focal length of a plane mirror?
2. Which lens has a magnification less than 1?
3. What is cause of refraction?
4. What is the cause of mirage seen in desert in summer?
5. For equal angle of incidence, refraction angle for these media. A, B and C are 15^0, 25^0 and 35^0 respectively. In which medium speed of light is minimum?
6. Write the working principle of an opitcal fibre.
7. What is relation between i and e when prism is in the position of minimum deviation?
8. Two lenses convex and concave of equal focal length are coaxially in contact. What will be the resultant focal length?
9. Why sun appears redish at sunrise and sunset?
10. What is the cause of rainbow?
11. What is myopia? Which lens is used for its correction?
12. On what factor, the intensity of scattered light depends?
13. Which type lens is used in a simple mirocscope?
14. How can you differentiate between a compound microscope and a telescope just by looking at it?

Short Answer Type Questions -

1. An object AB is placed is front of a concave mirror as shown in the given diagram -



(i) Draw ray diagram to form image.

(ii) If half of the apparture of mirror is blankend, how the position and intensity of image is effected.

2. Write uses of spherical mirrors.

3. Establish relation between focal length and radius of curvature for a spherical mirror.

4. (i) Why the sun appears redish at sun rise and sunset?

(ii) For which colour, the refractive index of a medium is minimum and maximum?

5. (i) What is the relation between critical angle and refractive index of a medium?

(ii) Do the critical angle depends on the colour of light? Explain.

6. On what factors the focal length of a lens depends?

7. How we can increase the magnifying power of a compound micorscope?

8. What do you understand by scattering of light? Write down its uses in daily life?

9. Define power of a lens and write down its unit. For two coaxially contacted lenses, derive the redeation -

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Esaay Type Questions -

1. Define a spherical mirror. Find relation between object distance, image distance and focal length for it.

2. Explain formation of images by convex and concave lenses for different object positive. Show the size, position and nature of image by ray diagrams.

3. What types of lenses are there? Establish relation between object distance, image distance and focal length of a lens.

4. Find relation between u, v and R for a convex refracting surface, when light enters a denser medium from rarer medium.

5. Draw a labled ray diagram of a compound micorscope showing image formation at near point of normal eye.

6. Draw a ray diagram for a monochromatic ray refracting from a glass prism. Write the expression for refractive index of glass in terms of prism angle and angle of minimum deviation.

7. Find relation between u, v and f for a lens assuming it to be surrounding by two spherical surfaces.

8. How many types of telescope are there? Obtain an expression for magnifying power of a refracting telescope, explaining its construction and working.

Answer MCQ

1. (c) 2. (a) 3. (b) 4. (b) 5. (d) 6. (b) 7. (d)

8. (d) 9. (c) 10. (a) 11. (c) 12. (d) 13. (d)

14. (A) 15. (b) 16. (c) 17. (b) 18. (b)

19. (d) 20. (a) 21. (b)

Very short answer type questions -

(1) Infinity (2) Concave

(3) Speed of light is different in different media

(4) Total internal reflection

(7) $\angle i = \angle e$

(9) Scattering (10) Dispersion

(12) Wavelength

(13) Convex of small focal length

(14) In compound micorscope apparture of the objective is smaller then eye-piece where in telescope apparture of objective is larger than its eye-piece.

Numberical Questions

1. An object is placed at a distance of 24 cm from a concave mirror of focal length 36 cm. Find image distance.

(Ans : 72 cm, towards object)

2. Find the speed of ligth in a medium of refrative index 1.33 speed of light in air is $c = 3 \times 10^8$ m/s.

(Ans : 2.25×10^8 m/s)

3. Radius of curvature of the two surfaces of convex lens of focal length 20 cm are 18 cm and 24 cm respectively. Find refractive index of the material of the lens.

(Ans : 1.514)

4. A ray of light is incident at an angle 50^0 on a glass slab. If the angle of refraction is 30^0. Find refractive index of glass.

(Ans : 1.532)

5. An object is placed at distance of 0.06 m from convex lens of focal length 0.10 m. Find the position of image.

(Ans : 15 m)

6. Find angular dispersion due to glass prism of refrating angle 6^0. Refractive index for red and violet light is given as 1.514 and 1.523 respectively.

(Ans : 0.054^0)

7. Find the resultant power of the combination of two lenses of power + 5 D and - 7 D will the combination be converging or diverging.

(Ans : - 2 D, diverging)

8. Focal lengths of objective and eye-piece of a compound microscope are 0.95 cm and 5 cm, and they are 20 cm appart from each other. Find magnification of micorscope, if the image is formed at 25 cm away eye-piece.

(Ans : 94)

9. A thin convex lens of glass (n_g = 1.5) has power of + 5.0 D. When this lens is placed in a liquid of refractive index n_e, its behave like a concave lens of focal length 100 cm. Find value of n_l.

(Ans : n_e = 5/3)

10. Find the angle of minimum deviation for a prism of refracting angle A and refractive index ($\cot A/2$).

(Ans : $180^0 + 2A$)