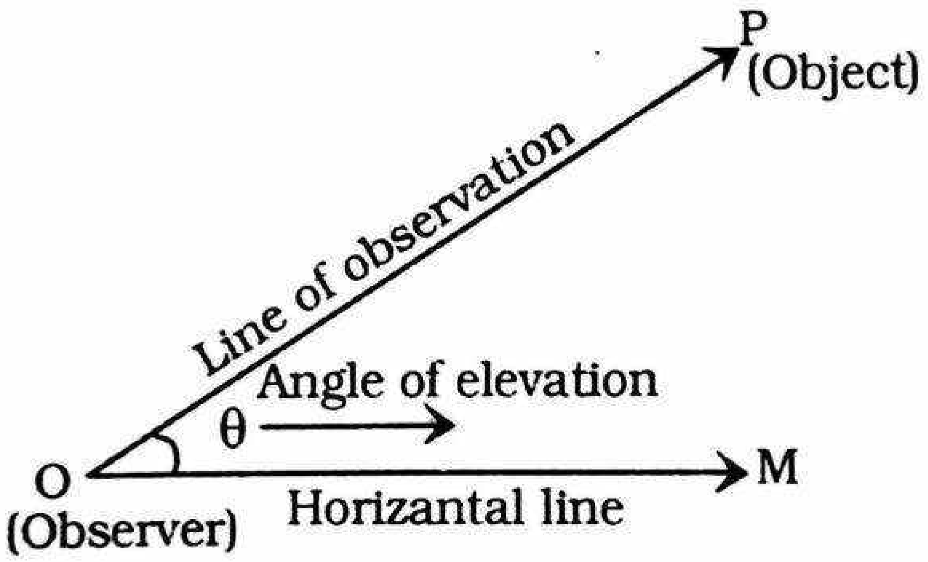
CHAPTER 19 HEIGHTS AND DISTANCES

INTRODUCTION:-

One of the important application of trigonomerty is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

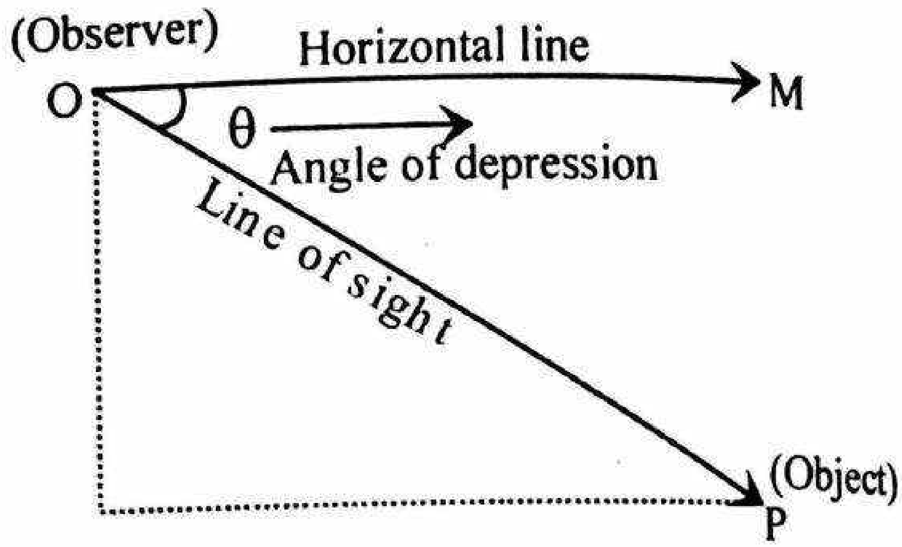
Angle of elevation:-

Let O and P be two point where P is at a higher level than O. Let O be at the position of the observer and P be the position of the object. Draw ahorizontal line OM through the point O. OP is called the line of observation or line of sight, Then $\angle POM = \theta$ is called of elevation of P as observed from O.



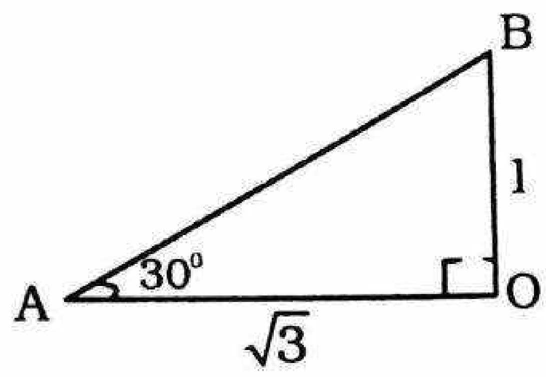
Angle of Depression :-

In the cbove figure, if P be at a loower level than O, then $\angle MOP = \theta$ is called the angle of depression.

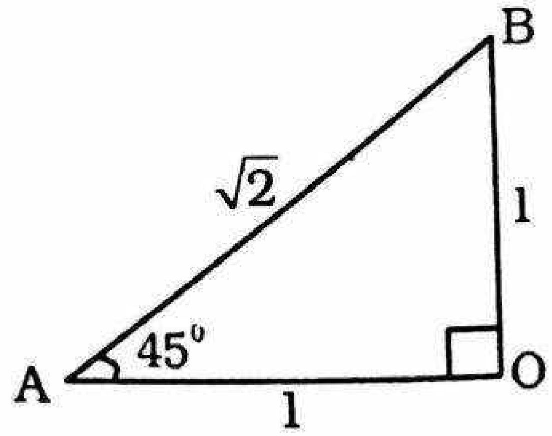


Note :-

In this chapter we solve all the questions with the help of ratio. Some important ratios are as following :-



(i)



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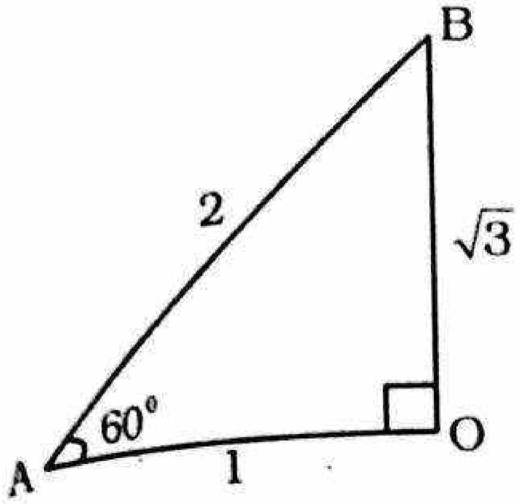

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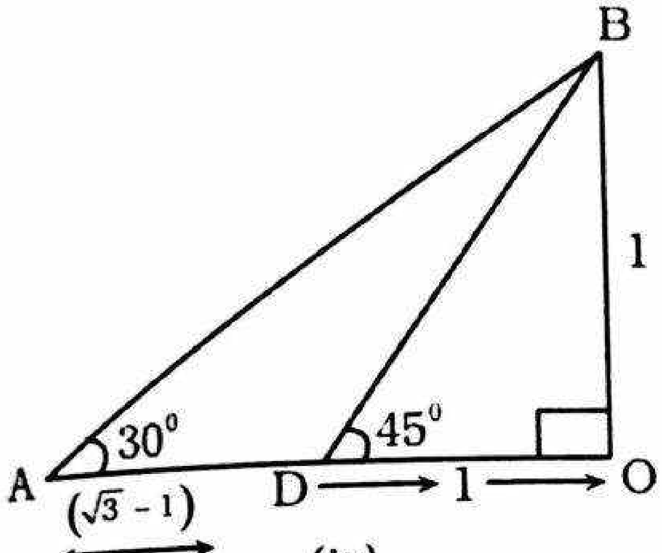

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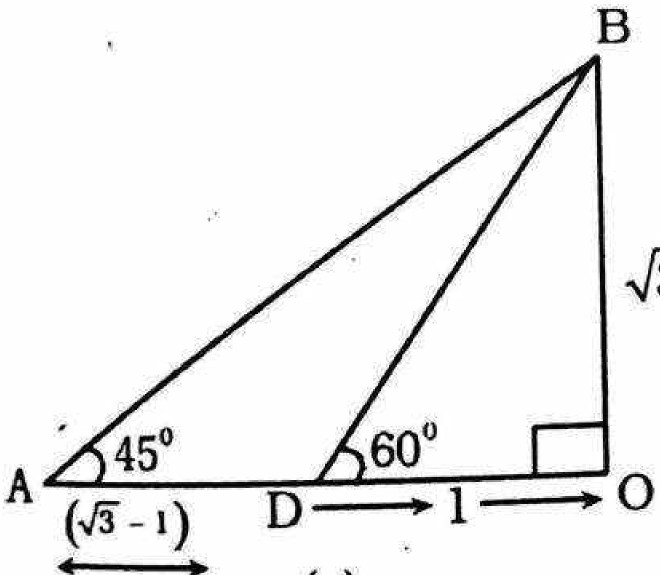

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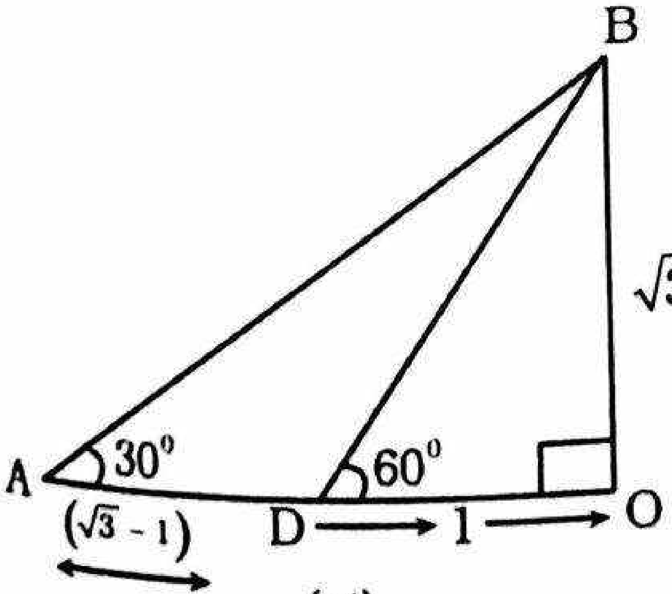

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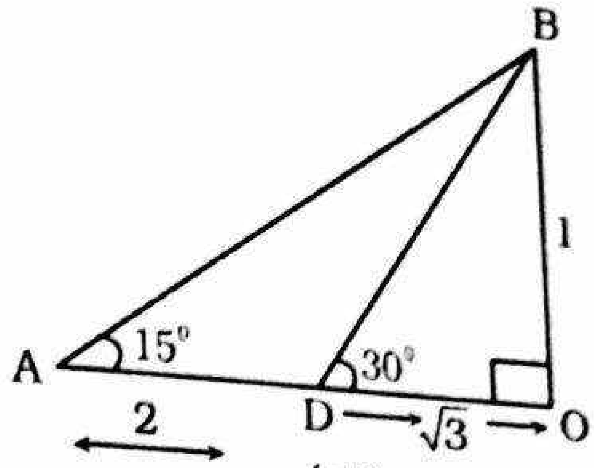


(vii)

Figure Angle Ratio (Base : Height : Hypotenuse)

(i) 30^0 $\sqrt{3}:1:2$

(ii) 45^0 $1:1:\sqrt{2}$

(iii) 60^0 $1:\sqrt{3}:2$

So remember all these ratios, and with the help of these ratios we can solve question very quickly.

Note :- you also can make other ratio for diffrent angle.

Note :- $\tan 15^0 = 2-\sqrt{3} = \frac{1}{2+\sqrt{3}}$

Useful Result :-

1 . If $\angle PAQ = 45^0, \angle BAM = 30^0$ and $\angle PBN = 60^0$ then

$$h = AB\left(\frac{\sqrt{3}+1}{2}\right)$$

LEVEL - I

1. A tower is $50\sqrt{3}$ meters high. Find the angle of elevation of its top from a point 50 meters away from its foot:-

(a) $\theta = 60^0$ (b) $\theta = 45^0$

(c) $\theta = 30^0$ (d) $\theta = 22\frac{1}{2}^0$

2. The angle of elevation of the top of a tower at a distance of 30 m from its foot is 45⁰. The height of the tower is:-

(a) 20 m (b) 30 m

(c) $15\sqrt{2}$ m (d) $\frac{15}{\sqrt{2}}$ m

3. If the ratio of the length of a pen to its shodow is $1:\sqrt{3}$, the angle of elevation of the source of light is :-

(a) 40^0 (b) 30^0

(c) 60^0 (d) 90^0

4. The angle of elevation of the top of a tower at a distance of 500 m from its foot is 30⁰. The height of the tower is :-

(a) $\frac{500(\sqrt{3}-1)}{3}$ m (b) 500 m

(c) $\frac{500\sqrt{3}}{3}$ m (d) $\frac{500(\sqrt{3}+1)}{3}$

5. If the angle of elevation of the top of a building from a point 50 m away from its base is 60⁰ , the height of the building is :-

(a) $100\sqrt{3}$ m (b) $50(\sqrt{3}-1)$ m

(c) $\frac{50}{\sqrt{3}}$ m (d) $50\sqrt{3}$

6. If the of the shadow of a pole is $\sqrt{3}$ times the height of the pole , the angle of elevation of sun is :-

(a) 60^0 (b) 30^0

(c) 45^0 (d) 90^0

7. If the angle of elevation of sun is θ and the length of the shadow of a pole of lenght P is S , then:-

(a) $P = S\cos\theta$ (b) $P = S\sin\theta$

(c) $P = \frac{s}{\cot\theta}$ (d) $P = S\cot\theta$

8. Determine the lenght of a ladder , if it is leaning against a vertical wall making an angle of inclination of 30⁰ with the ground and its foot is 15 m from the wall :-

(a) 15.77 m (b) 10 m

(c) $10\sqrt{3}$ m (d) 12.71 m

9. The foot of a ladder leaning against a wall of lenght 5 metre rest on a level ground $5\sqrt{3}$ metre from the base of the wall. The angle of inclination of the ladder with the ground is :-

(a) 60^0 (b) 50^0

(c) 40^0 (d) 30^0

10. A balloon is connected to a meteorological station by a cable of lenght 200 m , inclined at 60⁰ to the hoizontal. Find the height of the balloon from the ground. Assume that there is no slack in the cable :-

(a) 173.2 m (b) 17.35 m

(c) 123.2 m

(d) None of these

11. The angle of elevation of a moon when the lenght of the shadow of a pole is equal to its height is :-

(a) 30^0 (b) 45^0

(c) 60^0 (d) 90^0

12. If the length of shadow of a pole on a level ground is twice the length of that pole , the angle of elevation of the sun is :-

(a) 30^0 (b) 45^0

(c) 60^0

(d) None of these

13. The altitude of the sun at any istant is 60^0. The height of the vertical pole that will cast a shadow of 40 m is :-

(a) 20 m (b) $\frac{40}{\sqrt{3}}$ m

(c) $40\sqrt{3}$ m (d) $20\sqrt{3}$ m

14. When the sun is 30^0 above the horizontal , the lenght of shadow cast by a building 100 m high is :-

(a) $100\sqrt{3}$ m (b) $\frac{100}{\sqrt{3}}$ m

(c) 50 m (d) $50\sqrt{3}$ m

15. The banks of a river are parallel. A swimmer starts from a point on one of the banks and swims in a straight line inclined to the bank at 45^0 and reaches the opposite bank at a point 20 m from the point opposite to the starting point. The breadth of the river is :-

(a) 20 m (b) 28.28 m

(c) 14.14 m (d) 40 m

16. The length of a string between a kite and a point on the ground is 50 m. The string makes an angle of 60^0 with the level ground. If there is no slack in the string, the height of the kite is:-

(a) $50\sqrt{3}$ m (b) $25\sqrt{3}$ m

(c) 25 m (d) $25/\sqrt{3}$ m

17. The length of a shadow of a vertical tower is $\frac{1}{\sqrt{3}}$ times its height. The angle of elevation of the Sun is :

(a) 30° (b) 45°

(c) 60° (d) 90°

18. A ladder is resting against a wall at a height of 10m. If the ladder is inclined at an angle of 30° with the ground the distance of the ladder from the wall is:

(a) $\frac{10}{\sqrt{3}}$m (b) $\frac{20}{\sqrt{3}}$m

(c) $10\sqrt{3}$ m (d) $20\sqrt{3}$ m

19. One flies a kite with a thread 150 metre long. If the thread of the kite makes an angle of 60° with the horizontol line, then the height of the kite from the ground (assuming the thread to be in a straight line) is :

(a) 50 metre (b) $75\sqrt{3}$ m

(c) $25\sqrt{3}$ metre (d) 80 metre

LEVEL - II

1. The angle of elevation of the top of a tower at two points which are at a distance a and b from the foot in the same horizontal line and on the same side of the tower, are complementary. The height of the tower is :-

(a) ab (b) $\sqrt{ab}$

(c) $\sqrt{a/b}$ (d) $\sqrt{b/a}$

2. From the top of h meters high cliff, the angles of depression of the top and bottom of a tower are observed to be 30^0 and 60^0 respectively. The height of the tower is :-

(a) $2h/3$ (b) $h/3$

(c) $\frac{2h}{\sqrt{3}}$ (d) $h\sqrt{3}$

3. A man from the top a 50 m high tower, sees a car moving towards the tower at an angle of depression of 30^0. After some time , the angle of depression becomes 60^0. The distance (in m) travelled by the car during this time is :-

(a) $50\sqrt{3}$ (b) $\frac{50\sqrt{3}}{3}$

(c) $\frac{100\sqrt{3}}{3}$ (d) $100\sqrt{3}$

4. A pole is standing erect on the ground which is horizontal. The top of the pole is tied tight with a rope of lenght $\sqrt{12}$ m to a point on the ground. If the rope is making 30^0 angle with the horizontal, then the height of the pole is :-

(a) $2\sqrt{3}$ m (b) $3\sqrt{2}$ m

(c) 3 m (d) $\sqrt{3}$ m

5. Two observers are stationed due north of a tower at a distance of 20 m from each other. If the elevations of the tower observed by them are 30^0 and 45^0 respectively, then the height of the tower is :-

(a) 10 m (b) 16.32 m

(c) $10(\sqrt{3}+1)$ m (d) 30 m

6. The shadow of an electric pole standing on a ground is 40 m less when the angle of elevation changes from 30^0 to 45^0. The lenght of the pole is :-

(a) $20(\sqrt{3}+1)$ m

(b) $20(\sqrt{3}-1)$ m

(c) 20 m

(d) $20\sqrt{3}$ m

7. The angle of elevation of top of a tree on the bank of a river from its other bank is 60^0 and from a point 20 m further away from this is 30^0. The width of the river is :-

(a) $10\sqrt{3}$ m (b) 10 m

(c) 20 m (d) $20\sqrt{3}$ m

8. Find the decrease in the lenght of the shadow of a pole, when the angle of elevation becomes double. Give that at this moment, the shadow on the ground of a vertical pole of 16 m high is 64 m. :-

(a) 42 m (b) 40 m

(c) 30 m (d) 34 m

9. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite side of the bank is 60^0. When he moves 50 m away from the bank, the angle of elevation bacomes 30^0. The height of the tree and width of river respectively are:-

(a) $25\sqrt{3}$ m, 25 m

(b) $25\sqrt{3}$, $25\sqrt{3}$ m

(c) 25, $25\sqrt{3}$ m

(d) None of these

10. The angles of elevation of the top of a tower as observed from the bottom and top of a building of height 80 metre are 60^0 and 45^0 respectively. The distance of the base of the tower from the base of the building is :-

(a) $40(\sqrt{3}-1)$ m

(b) $40(3+\sqrt{3})$ m

(c) $40(3-\sqrt{3})$ m

(d) $40(\sqrt{3}+1)$ m

11. A man on the top of a rock rising on a sea-shore obeserves a boat coming towards it. If it takes 20 minute for the angle of depression to change from 30^0 to 60^0, how soon will the boat reach the shore ?

(a) 20 minute (b) 30 minute
(c) 10 minute (d) 15 minute

12. From the top of a 60 m high tower the angle of depression of the top and bottom of a building are observed to be 30^0 and 60^0 respectively. The height of the building is :-

(a) $60\sqrt{3}$ m (b) $40\sqrt{3}$ m
(c) 40 m (d) 20 m

13. From the top of a cliff 30 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower ?

(a) 50 m (c) 60 m
(c) $30\sqrt{3}$ m (d) 45 m

14. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 12 m. At a point on the plane, the angle of elevation of the bottom of the flagstaff is 30^0 and that of the top of the flagstaff is 45^0. Find the height of the tower ?

(a) $6(\sqrt{3}+1)$ m

(b) $12(\sqrt{3}+1)$ m

(c) $6(\sqrt{3}-1)$ m

(d) $12(\sqrt{3}-1)$ m

15. A vertial pole fixed to the ground is divided in the ratio 2:5 by a mark on it , the two parts subtend equal angle at a place on the ground, 14 m from the base of the pole. If the lower part be shorter than the upper one, the height of the pole is :-

(a) $\sqrt{21}$ m (b) $5\sqrt{21}$ m
(c) $6\sqrt{21}$ m (d) $7\sqrt{21}$ m

16. From the top of a pillar of height 80 m the angle of elevation and depression of the top and bottom of another pillar are 30^0 and 45^0 respectively . The height of second pillar (in metre) is :-

(a) $80\sqrt{3}$ m

(b) $\frac{80}{\sqrt{3}}(\sqrt{3}-1)$ m

(c) $\frac{80}{\sqrt{3}}(\sqrt{3}+1)$ m (d) $\frac{80}{\sqrt{3}}$ m

17. The angle of elevation of the top of a building and the top of the chimney on the roof of the building from a point on the ground are θ and 45^0 respectively. The height of building is h metre. Then, the height of the chimney (in metre) is :-

(a) $h \cot \theta + h$ (b) $h \cot \theta - h$
(c) $h \tan \theta - h$ (d) $h \tan \theta + h$

18. Two posts are k metres apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary, then the height of the shorter post (in metre) is :-

(a) $\frac{K}{2\sqrt{2}}$ (b) $\frac{K}{4}$

(c) $K\sqrt{2}$ (d) $\frac{K}{\sqrt{2}}$

19. Two poles of equal height are standing opposite to each other on either side of a road, which is 28 m wide. From a point between them on the road, the angles of elevation of the tops are 30^0 and 60^0. The height of each pole is :-

(a) $6\sqrt{3}$ m (b) $5\sqrt{3}$ m

(c) $4\sqrt{3}$ m (d) $7\sqrt{3}$ m

20. A, B, C are three collinear points on the ground such that B lies between A and C and AB=10 m. If the angles of elevation of the top of a vertical tower at C are respectively 30^0 and 60^0 as seen from A and B, then the height of the tower is :-

(a) $5\sqrt{3}$ m (b) 5 m

(c) $\frac{10\sqrt{3}}{3}$ m (d) $\frac{20\sqrt{3}}{3}$ m

21. A landmark on a river bank is observed from two points A and B on the opposite bank of the river. The lines of sight make equal angles of 45^0 with the bank of the river. If AB=1km , then the width of the river is :-

(a) 2 m (b) $\frac{3\sqrt{2}}{2}$ Km

(c) $\frac{1}{2}$ Km (d) $\frac{\sqrt{3}}{2}$ Km

22. When the angle of elevation of the sun increases from 30° to 60°, the shadow of a pole is diminished by 5 metres. Then the height of the pole is :

(a) $\frac{5\sqrt{3}}{2}$ m (b) $\frac{2\sqrt{3}}{5}$ m

(c) $\frac{2}{5\sqrt{3}}$ m (d) $\frac{4}{5\sqrt{3}}$ m

23. A man is climbing a ladder which is inclined to the wall at an angle of 30°. If he ascends at a rate of 2 m/s, then he apporaches the wall at the rate of:
(a) 1.5 m/s (b) 1 m/s
(c) 2 m/s (d) 2.5 m/s

24. P and Q are two points observed from the top of a building $10\sqrt{3}$ m high. If the angles of depression of the points are complementary and PQ = 20 m, then the distance of P from the building is :
(a) 25 m (b) 45 m
(c) 30 m (d) 40 m

25. From the top of cliff 90 metre high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. The height of the tower is :
(a) 45 m (b) 60 m
(c) 75 m (d) 30 m

26. The angles of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively 15° and 30°. If A and B are on the same side of the tower and AB = 48 metre, then the height of the tower is :

(a) $24\sqrt{3}$ metre (b) 24 metre

(c) $24\sqrt{2}$ metre (d) 96 metre

27. The angles of elevation of the top of a building from the top and bottom of a tree are x and y respectively. If the height of the tree is h metre, then, in metre, the height of the building is :

(a) $\frac{h \cot x}{\cot x + \cot y}$

(b) $\frac{h \cot y}{\cot x + \cot y}$

(c) $\frac{h \cot x}{\cot x - \cot y}$

(d) $\frac{h \cot y}{\cot x + \cot y}$

28. If the angle of elevation of the Sun changes from 30° to 45°, the length of the shadow of a pillar decreases by 20 metres. The height of the pillar is :

(a) $20(\sqrt{3}-1)$m

(b) $20(\sqrt{3}+1)$m

(c) $10(\sqrt{3}-1)$m

(d) $10(\sqrt{3}+1)$m

29. There are two vertical posts, one on each side of a road, just opposite to each other. One post is 108 metre high. From the top of this post, the angles of depression of the top and foot of the other post are 30° and 60° respectively. The height of the other post, in metre, is :

(a) 36 (b) 72
(c) 108 (d) 110

30. Two poles of equal heights are standing opposite to each other on either side of a road which is 100m wide. From a point between them on road, angles of elevation of their tops are 30° and 60°. The height of each pole in metre, is :

(a) $25\sqrt{3}$ (b) $20\sqrt{3}$

(c) $28\sqrt{3}$ (d) $30\sqrt{3}$

31. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60°. The height of the tower is :

(a) $\sqrt{3}$ m (b) $5\sqrt{3}$ m

(c) $10\sqrt{3}$ m (d) $20\sqrt{3}$ m

32. The angle of elevation of an aeroplane from a point on the ground is 60°. After 15 seconds flight, the elevation changes to 30°. If the aeroplane is flying at a height of $1500\sqrt{3}$ m, find the speed of the plane :

(a) 300 m/s (b) 200 m/s
(c) 100 m/s (d) 150 m/s

33. An aeroplane when flying at height of 5000 m from the ground passes vertically above another aeroplane at an instanct, when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. The vertical distar.ce between the aeroplanes at that instant is :

(a) $5000(\sqrt{3}-1)$

(b) $5000(3-\sqrt{3})$m

(c) $5000\left(1-\frac{1}{\sqrt{3}}\right)$m

(d) 4500 m

34. A man 6 ft tall casts a shadow 4 ft long at the same time when a flag pole casts a shadow 50 ft long. The height of the flag pole is :
(a) 80 ft (b) 75 ft
(c) 60 ft (d) 70 ft

35. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30°. The mand walks some distance towards the tower and then his angle of elevation of the top of the tower is 60°. If te height of the tower is 30 m, then the distance he moves is :
(a) 22 m (b) $22\sqrt{3}$ m
(c) 20 m (d) $20\sqrt{3}$ m

36. The distance between two vertical poles is 60 m. The height of one of the poles is double the height of the other. The angles of elevation of the top of the poles from the middle point of the line segment joining their feet are complementary to each other. The heights of the poles are :
(a) 10 m and 20 m
(b) 20 m and 40 m
(c) 20.9 m and 41.8 m
(d) $15\sqrt{2}$ m and $30\sqrt{2}$ m

37. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles or depression of the top and the foot of the other temple are 30° and 60° respectively. The length of the temple is :
(a) 18 m (b) 36 m
(c) $36\sqrt{3}$ m (d) $18\sqrt{3}$ m

38. An aeroplane when flying at a height of 3125 m from the ground passes vertically below another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 30° and 60° respectively. The distance between the two planes at that instant is :
(a) 6520 m (b) 6000 m
(c) 5000 m (d) 6250 m

LEVEL - III

1. From vertically situated aeroplane to the straight horizontal road, the angle of depression of two conseeutive km stones are α and β. If an aeroplane is in vertical plane in between two stones, then the height of the aeroplane from the road (in km) will be :-

(a) $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

(b) $\frac{\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

(c) $\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

(d) $\frac{\tan\alpha-\tan\beta}{\tan\alpha+\tan\beta}$

2. Each side of an equilateral triangle subtends an angle of 60^0 at the top of a tower h m high located at the centre of the tringle. If a is the length of each side of the trinagle, then :-

(a) $3a^2 = 2h^2$ (b) $2a^2 = 3h^2$

(c) $a^2 = 3h^2$ (d) $3a^2 = h^2$

3. A round balloon of radius r subtends an angle α at the eye of an observer while the angle of elevation of its centre is β. The height of the centre of the balloon is :-

(a) r sinb. cosec $\alpha/2$

(b) $r\cos\beta$. cosec $\alpha/2$

(c) r cosecα. $\sin\beta$

(d) $r^2\sin\beta/2 . \cos\alpha/2$

4. A tower on horizontal ground leans towards the north. at two points due south at distance a and b respectively from the foot, the angular elevations of the top of the tower are α and β. Find the inclination θ of the tower to the horizontal :-

(a) $\frac{b\cot\alpha+a\cot\beta}{a-b}$

(b) $\frac{b\sin\alpha+b\cos\beta}{b-a}$

(c) $\frac{b\cot\alpha-a\cot\beta}{b-a}$

(d) None of these

5. At the foot of the mountain the elevation of its summit is 45^0; after ascending 4km towards the mountain up a slope of 30^0 inclination, the elevation is found to be 60^0. Find the height of the mountain:-

(a) $2(\sqrt{3}+1)$km

(B) $4(\sqrt{3}+1)$km

(c) $2(\sqrt{3}-1)$km

(d) $4(\sqrt{3}+1)$km

6. A boy standing in the middle of a field, observes a flying bird in the north at an angle of elevation of 30° and after 2 minutes, he observes the same bird in the south at an angle of elevation of 60°. If the bird flies all along in a straight line at a height of $50\sqrt{3}$ m, then its speed in km/h is :

(a) 4.5 (b) 3

(c) 9 (d) 6

7. A pole broken by the storm of wind and its top struck the ground at a n angle of 30° and at a distance of 20 m from the foot of the pole, sthe height of the pole before it was broken was :

(a) $20\sqrt{3}$ m (b) $\frac{40\sqrt{3}}{3}$ m

(c) $60\sqrt{3}$ m (d) $\frac{100\sqrt{3}}{3}$ m

8. A tree is broken by the wind. If the top of the tree struck the ground at an angle of 30° and at a distance of 30 m from the root, then the height of the tree is :

(a) $25\sqrt{3}$ m (b) $30\sqrt{3}$ m

(c) $15\sqrt{3}$ m (d) $20\sqrt{3}$ m

9. At a point on a horizontal line through the base of a monument, the angle of elevation of the top of the monument is found to be such that its tangent is $\frac{1}{5}$. On walking 138 metres towards the monument the secant of the angle of elevation is found to be $\frac{\sqrt{193}}{12}$. The height of the monument (in metre) is :

(a) 35 (b) 49

(c) 42 (d) 56

10. A telegraph post is bent at a point above the ground due to storm. Its top just meets the ground at a distance of $8\sqrt{3}$ metres from its foot and makes an angle of 30°, then the height of post is :

(a) 16 metres (b) 23 metres

(c) 24 metres (d) 10 metres

11. The angle of elevation of a cloud from height h above the level of water in a lake is α and the angle of the depression of its image in the lake is β. Then, the height of the cloud above the surface of the lake is :

(a) $h \cot \beta$

(b) $h(\cot \alpha + \cot \beta)$

(c) $h \cot \alpha$

(d) $h\left(\frac{\cot \alpha + \cot \beta}{\cot \alpha - \cot \beta}\right)$

12. A vertical post 15 ft high is broken at a certain height and its upper part, not completely separated, meets the ground at an angle of 30°. Find the height at which the post is broken :

(a) 10 ft (b) 5 ft

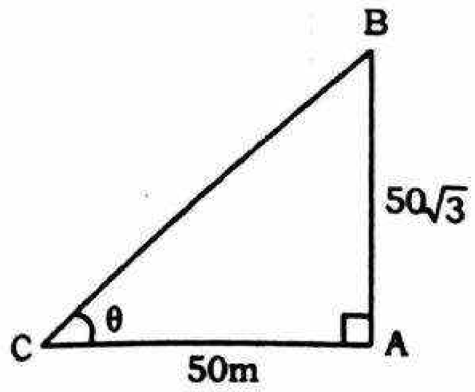
(c) $15\sqrt{3}(2-\sqrt{3})$ ft (d) $5\sqrt{3}$ ft

Solution I

1.(a) height of tower $(AB) = 50\sqrt{3}$ (given)

$$\therefore \tan\theta = \frac{AB}{AC} = \frac{50\sqrt{3}}{50} = \sqrt{3} = \tan 60^0$$

$$\Rightarrow \theta = 60^0$$



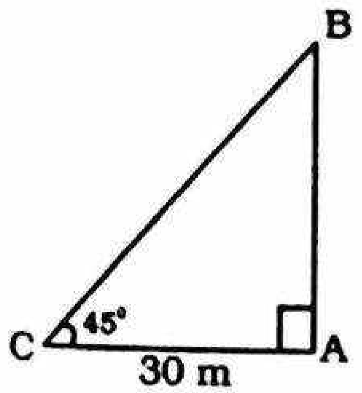
2.(b) in 45^0 Base : height $= 1:1$

$\therefore$ height = Base = 30 m

Alternativaly :-

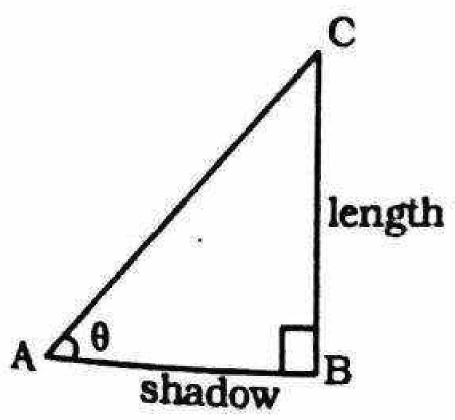
$$\text{In } \Delta ABC, \tan 45^0 = \frac{AB}{AC}$$

$$\Rightarrow 1 = \frac{AB}{30} \Rightarrow AB = 30 \text{ m}$$



3.(b) $\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{3}} = \tan 30^0$

$$\Rightarrow \theta = 30^0$$



4.(b) $\tan 30^0 = \frac{BC}{AB} = \frac{h}{500}$

$$\Rightarrow h = 500 \times \frac{1}{\sqrt{3}} = \frac{500\sqrt{3}}{3}$$

Alternatively,

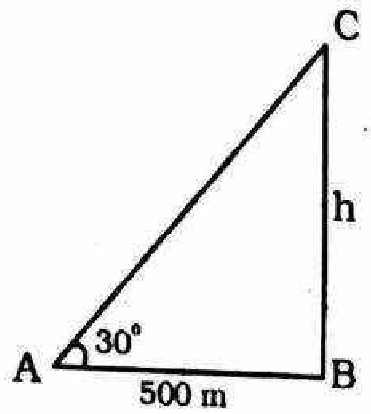
in 30° :- height : Base

1 : $\sqrt{3}$

but Base = 500(given)

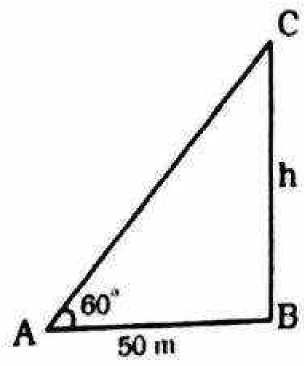
$\therefore \sqrt{3} \rightarrow 500$

$$\therefore 1 \rightarrow \frac{500}{\sqrt{3}} = \frac{500.\sqrt{3}}{3}$$



5.(d) $\tan 60^0 = \frac{h}{50} = \sqrt{3}$

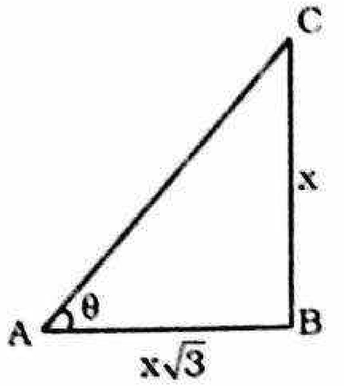
$$\Rightarrow h = 50\sqrt{3}$$



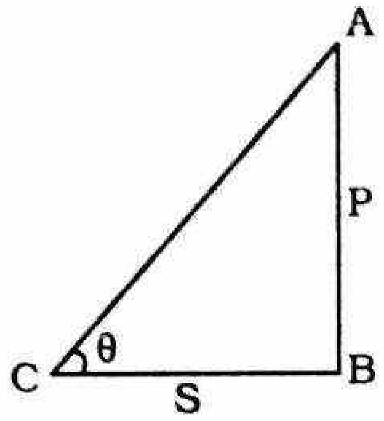
6. (B). let AB be the pole of x cm. Then , length of its shadow is $x\sqrt{3}$ in ΔABC,

$$\tan\theta = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^0$$



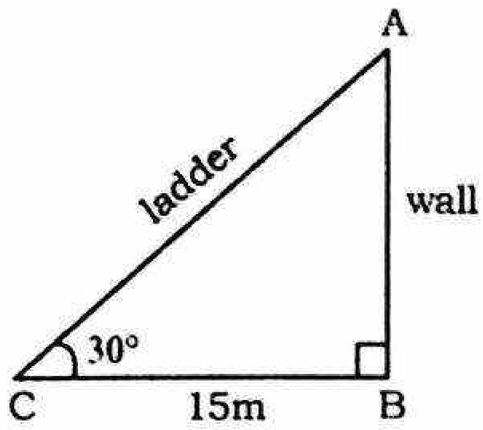
7.(c) $\tan\theta = \frac{P}{S} \Rightarrow P = \frac{S}{\cot\theta}$



8.(c) In right angled ΔABC,

$$\cos 30^0 = \frac{BC}{AC} = \frac{15}{AC}$$

$$\Rightarrow AC = \frac{15}{\cos 30^0} = \frac{15}{\sqrt{3}/2}$$



$$\Rightarrow AC = \frac{15 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$$

Hence, length of ladder is $10\sqrt{3}$ m

Alternatively :

$\cos 30° =$ BC : AC = $\sqrt{3} : 2$

$\sqrt{3} \rightarrow 15$

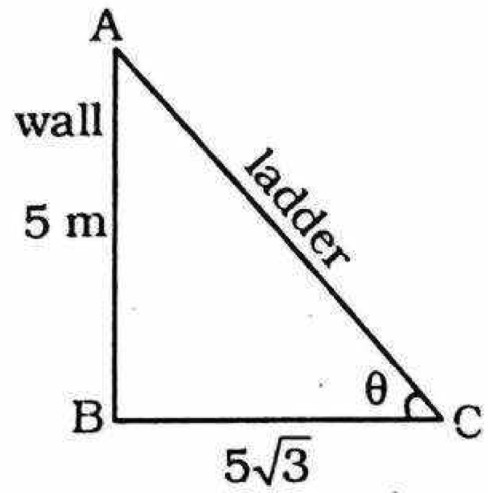
$1 \rightarrow \frac{15}{\sqrt{3}}$

$2 \rightarrow \frac{15}{\sqrt{3}} \times 2 = \frac{30}{\sqrt{3}} = 10\sqrt{3}$

9.(c) In right angled ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^0$$

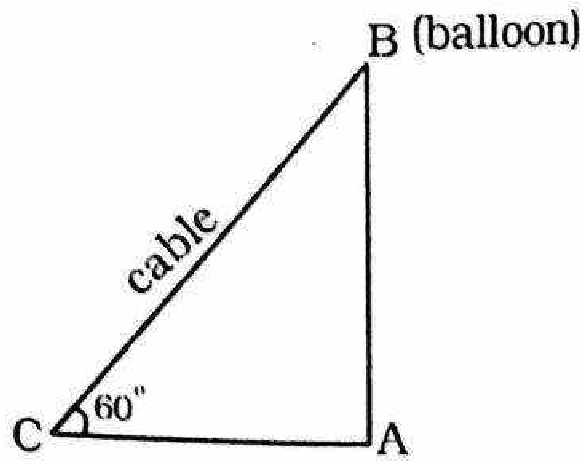
$$\Rightarrow \theta = 30^0$$



10.(a) Let B be the balloon and C be the meteorological station and CB be the bcable. Then, BC=200 m and $\angle ACB = 60^0$

then, $\sin 60^0 = \frac{AB}{BC} \Rightarrow \frac{AB}{200} = \frac{\sqrt{3}}{2}$

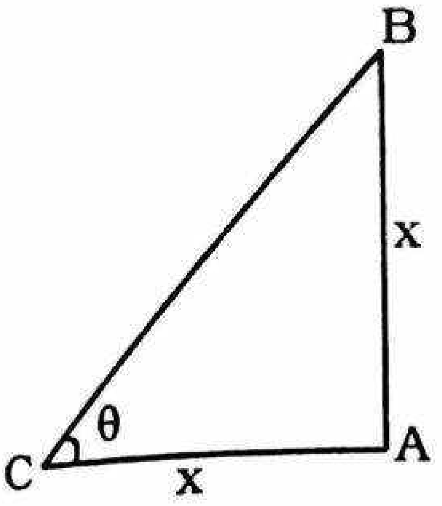
$\Rightarrow AB = 100\sqrt{3} = 173.2$ m



11.(b) Let $AB = x$, then AC=x

$\therefore \tan\theta = \frac{AB}{AC} = \frac{x}{x} = 1$

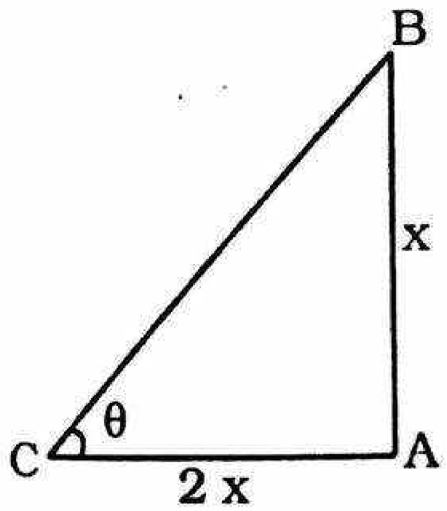
$\Rightarrow \theta = 45^0$



12.(d) Let AB=x , then AC=2x

$\therefore \tan\theta = \frac{AB}{AC} = \frac{x}{2x} = \frac{1}{2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$



13.(c) Let height of pole AB = h m

$\therefore \tan 60^0 = \frac{h}{40}$

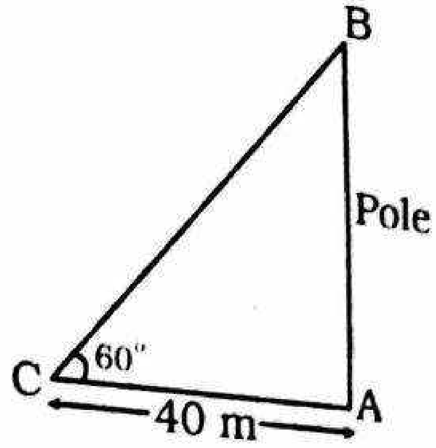
$\Rightarrow h = 40\sqrt{3}$ m

Alternatively :

In 60° = Base : height

$$\begin{array}{ccc} 1 & : & \sqrt{3} \\ \times 40) & & \times 40) \\ 40 & & 40\sqrt{3} \end{array}$$

$\therefore$ height = $40\sqrt{3}$

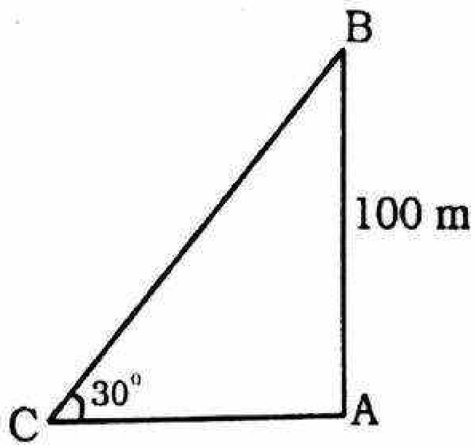


14.(a) Let AB be the building and AC be its shadow.

AB = 100 m and $\theta = 30^0$

$\therefore \frac{AB}{AC} = \tan 30^0 = \frac{1}{\sqrt{3}}$

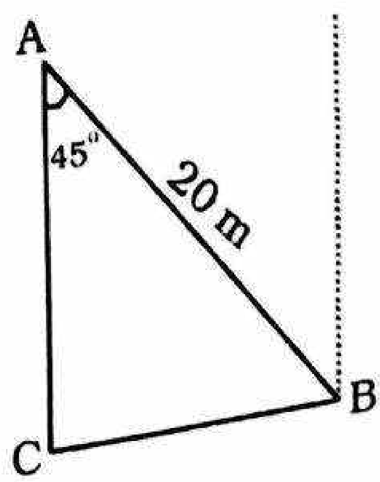
$\Rightarrow AC = AB\sqrt{3} = 100\sqrt{3}$ m



15.(c) Let A be the starting point and B, the end point of the swimmer. Then AB=20 m.& $\angle BAC = 45^0$

Now, $\sin 45^0 = \frac{BC}{AB}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{20} \Rightarrow BC = 10\sqrt{2} = 14.14$ m



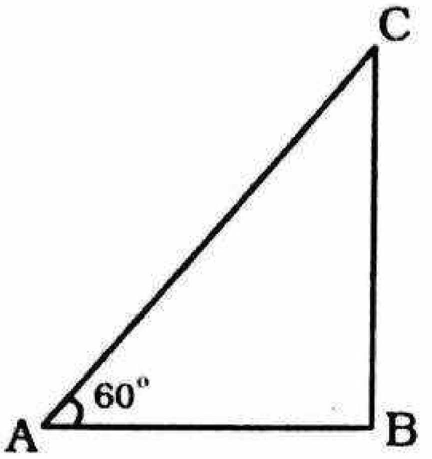
16.(b) Let C be the position of the kite and AC be the string.

$\therefore AC = 50$ m and $\angle BAC = 60^0$

$\therefore \frac{BC}{AC} = \sin 60^0 \Rightarrow \frac{BC}{50} = \frac{\sqrt{3}}{2}$

$\Rightarrow BC = 25\sqrt{3}$ m

Hence, Height of the kite = $25\sqrt{3}$ m



Alternatively : in 60° AC : BC

$= 2 : \sqrt{3}$

$\times 25) \quad \times 25)$

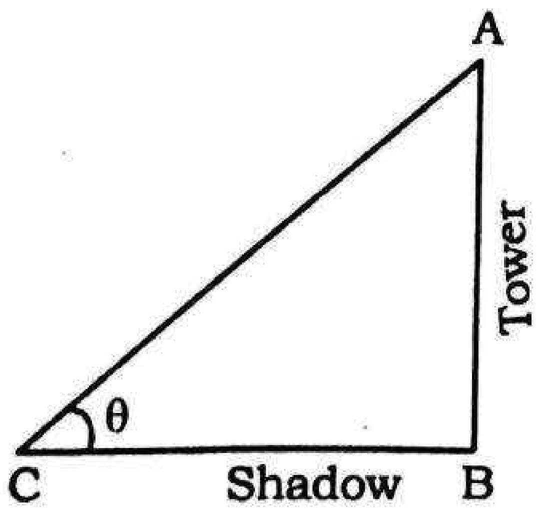
$50 \quad 25\sqrt{3}$

17.(c) Let height of tower (AB) = x

$\therefore$ shadow of tower = BC = $\frac{x}{\sqrt{3}}$

$\therefore \tan\theta = \frac{AB}{BC} = \frac{x}{\frac{x}{\sqrt{3}}} = \sqrt{3} = \tan 60°$

$\therefore \theta = 60°$

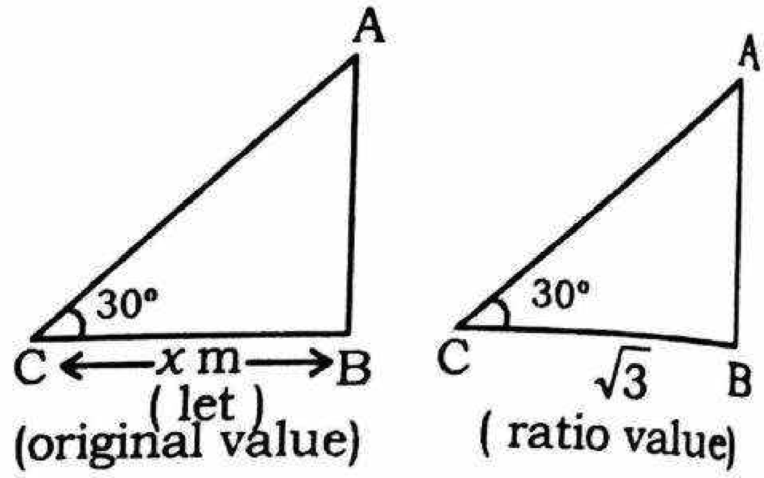


Alternatively :

Base : height = $\frac{x}{\sqrt{3}} : x = 1 : \sqrt{3}$

$\therefore \theta = 60°$

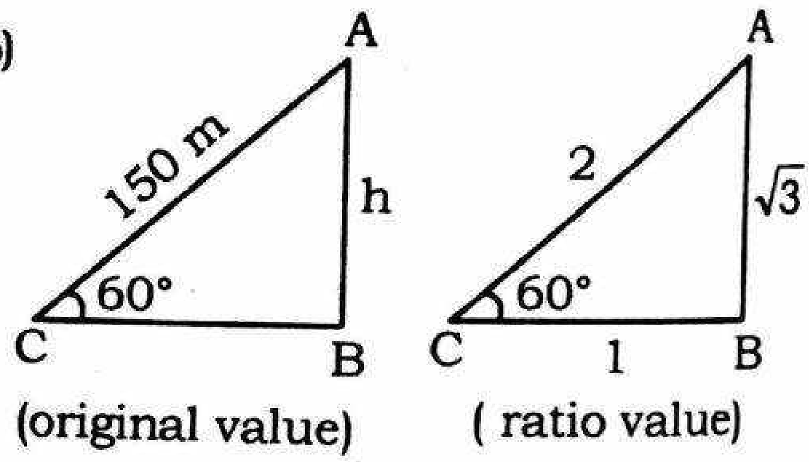
18.(c)



Ratio value		**Original value**
$AB \to 1$	$\to$	10 m
$\therefore BC \to \sqrt{3}$	$\to$	$10\sqrt{3}$ m

i.e. Required distance (BC ratio value = $\sqrt{3}$) = x = $10\sqrt{3}$ m

19.(b)



AC = length of thread, let height = h m

Ratio value		Original value
$AC \to 2$	$\longrightarrow$	150
$\therefore$ 1	$\longrightarrow$	$\frac{150}{2} = 75$
$\therefore$ $\sqrt{3}$	$\longrightarrow$	$75\sqrt{3}$

i.e. the height of the kite = AB = h $= 75\sqrt{3}$ m

Solution II

1. (B). Let PQ be the given tower of height h. If A, B be given points then suppose.

$\angle PAQ = \alpha$ and $\angle PBQ = \beta$

$\therefore \alpha + \beta = 90^0$

Now in ΔPAQ,

$\tan\alpha = \frac{h}{a}$..............(i)

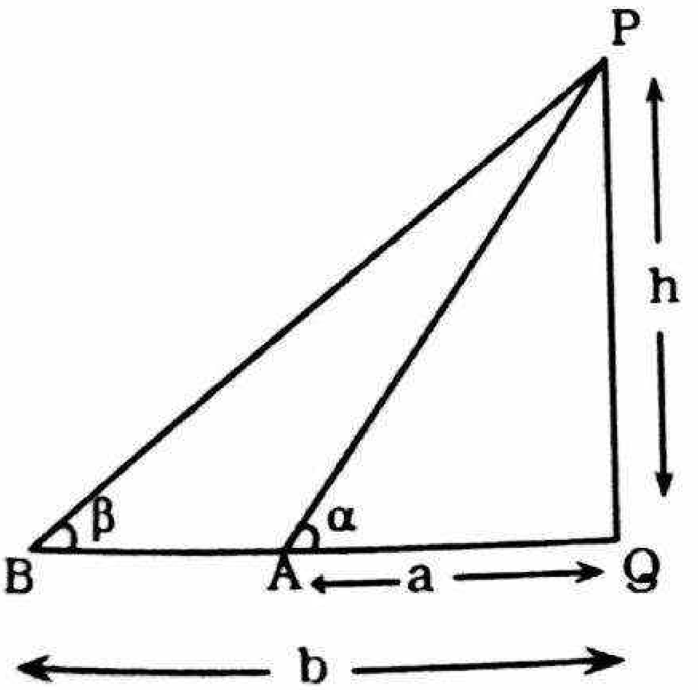
in ΔPBQ,

$\tan\beta = \frac{h}{b} \Rightarrow \tan(90^0 - \alpha) = \frac{h}{b}$

$\Rightarrow \cot\alpha = \frac{h}{b} \Rightarrow \tan\alpha = \frac{b}{h}$

$\Rightarrow \frac{h}{a} = \frac{b}{h}$ [from (i)- $\tan\alpha = \frac{h}{a}$]

$\Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$



2. (A). Let AB and PQ be given cliff and tower respectively.
If PQ=x
Now in ΔABQ and ΔACP,

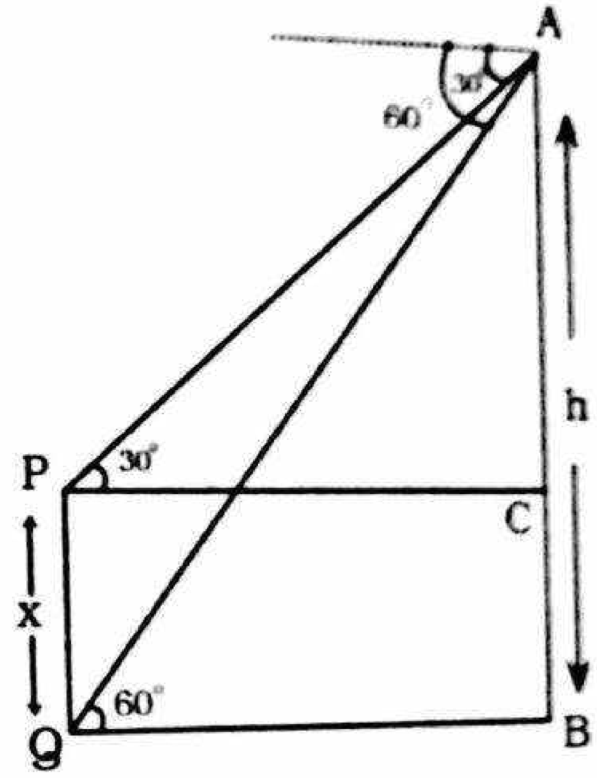
$\frac{h}{QB} = \tan 60^0 = \sqrt{3}$

$\Rightarrow QB = h/\sqrt{3}$

and $\frac{h-x}{PC} = \tan 30^0 = \frac{1}{\sqrt{3}}$ [PC = QB]

$\Rightarrow \frac{h-x}{QB} = \frac{1}{\sqrt{3}} \Rightarrow \frac{h-x}{\left(h/\sqrt{3}\right)} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{h-x}{h} = \frac{1}{3} \Rightarrow x = \frac{2h}{3}$



3. (C).

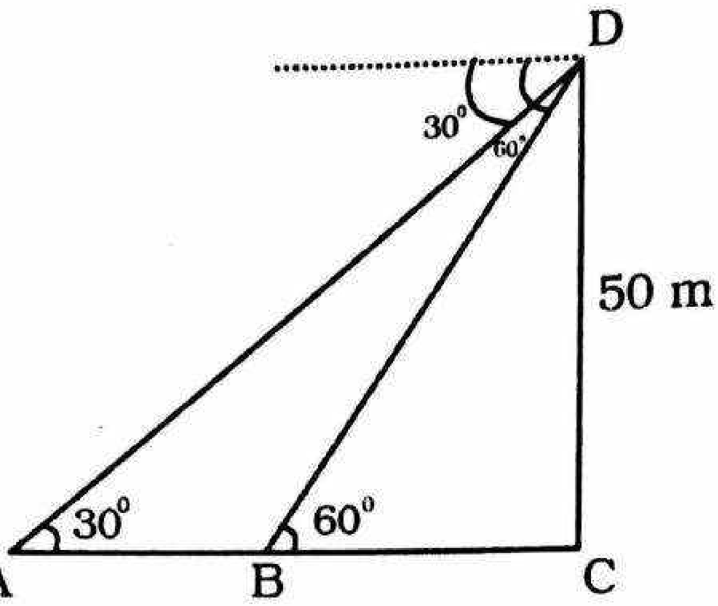
$BC = \frac{50}{\sqrt{3}}$, $AC = 50\sqrt{3}$

$AB = AC - BC$

$= 50\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$= \frac{100}{\sqrt{3}}$

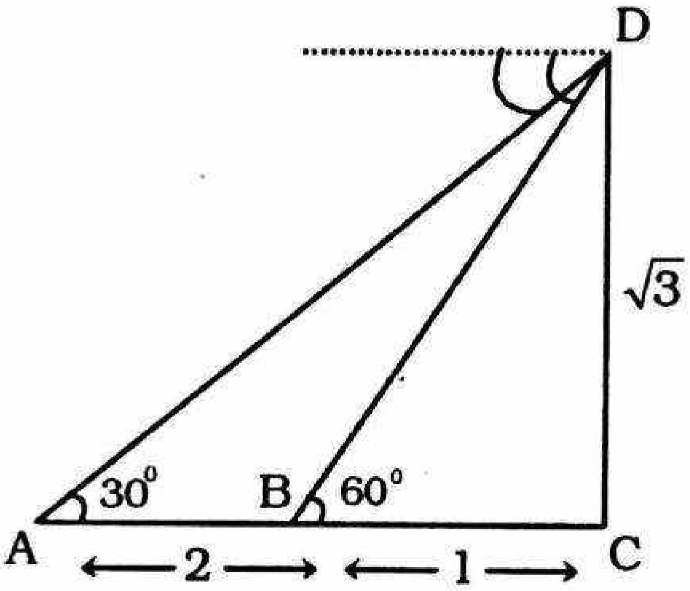
$= \frac{100\sqrt{3}}{3}$



Alternatively - (by ratio)

CD=50 m (given)

but CD = $\sqrt{3}$ (according to ratio)



Ratio value orignal value

CD $\sqrt{3} \longrightarrow 50$

$\therefore 1 \longrightarrow \frac{50}{\sqrt{3}}$

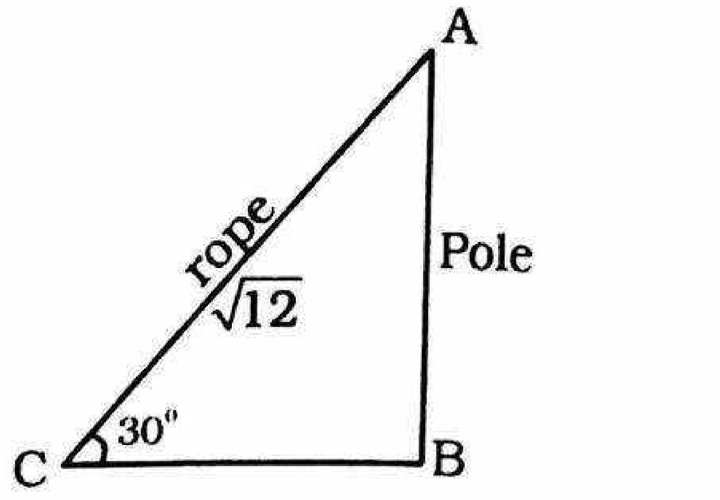
$\therefore 2 \longrightarrow \frac{100}{\sqrt{3}}$

$\therefore AB(\text{ratio value}) = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$

4.(d) In right angled ΔABC,

$\sin 30^0 = \frac{AB}{AC} = \frac{AB}{\sqrt{12}}$

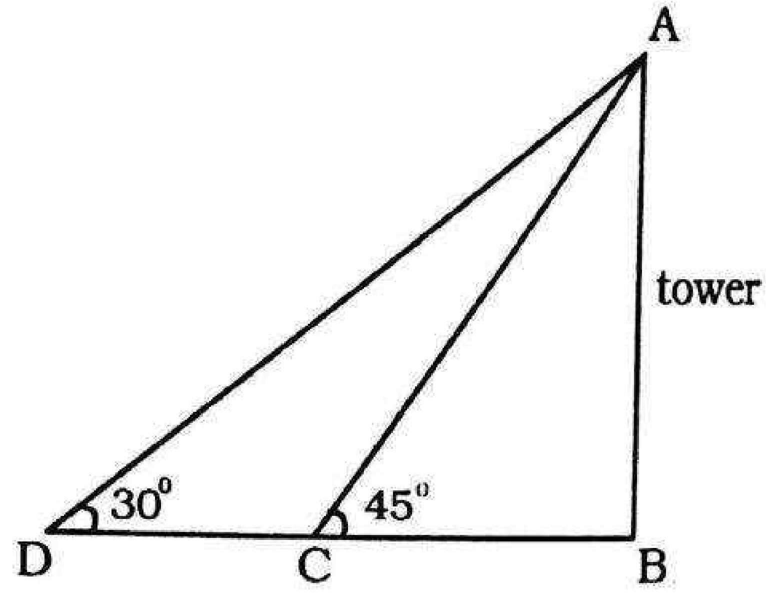
$\Rightarrow AB = \sqrt{12}\sin 30^0 = \sqrt{12} \times \frac{1}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$



Hence, height of the pole is $\sqrt{3}$ m

5.(c) Here, AB be the tower and C and D the two observers

Now, In right angled ΔABC,



$\tan 45^0 = \frac{AB}{BC} \Rightarrow BC = AB.....(i)$

In right angled ΔABD,

$\tan 30^0 = \frac{AB}{BD} \Rightarrow BD = AB\sqrt{3}$

$\Rightarrow BC + 20 = AB\sqrt{3}.........(ii)$

From (i) and (ii)

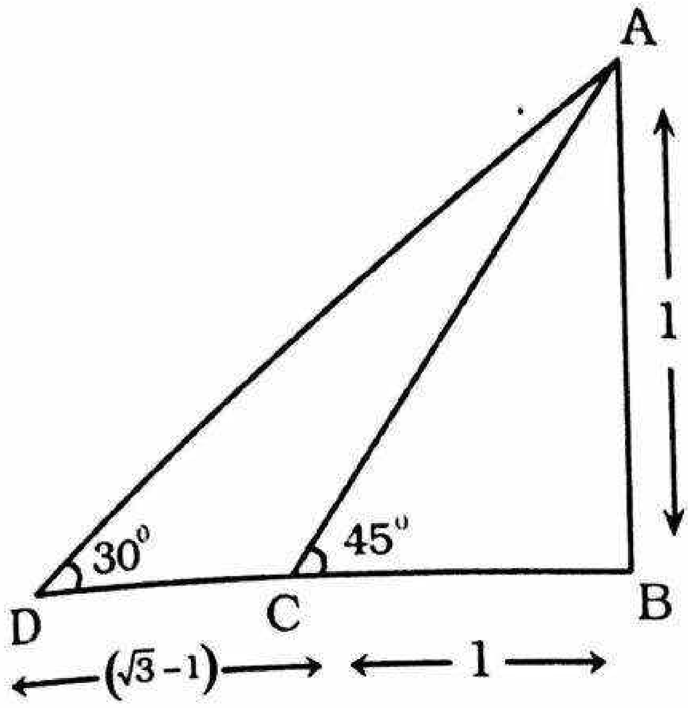
$AB + 20 = AB\sqrt{3}$

$\Rightarrow AB = \frac{20}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= 10(\sqrt{3}+1)$

Hence, height of the tower is $10(\sqrt{3}+1)$ m

Alternatively - (by ratio) :-



Ratio value original value

CD $(\sqrt{3}-1) \longrightarrow 20m$

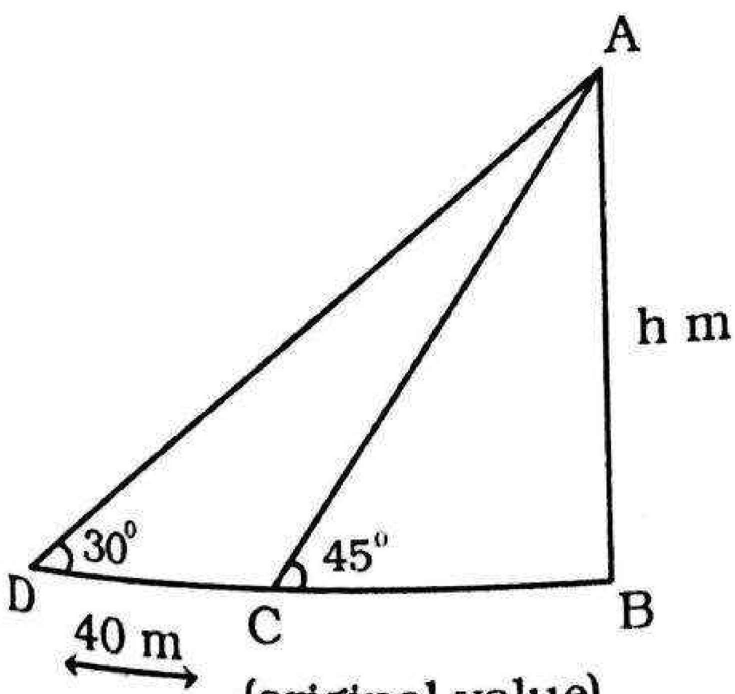
$\therefore 1 \longrightarrow \frac{20}{\sqrt{3-1}} m$

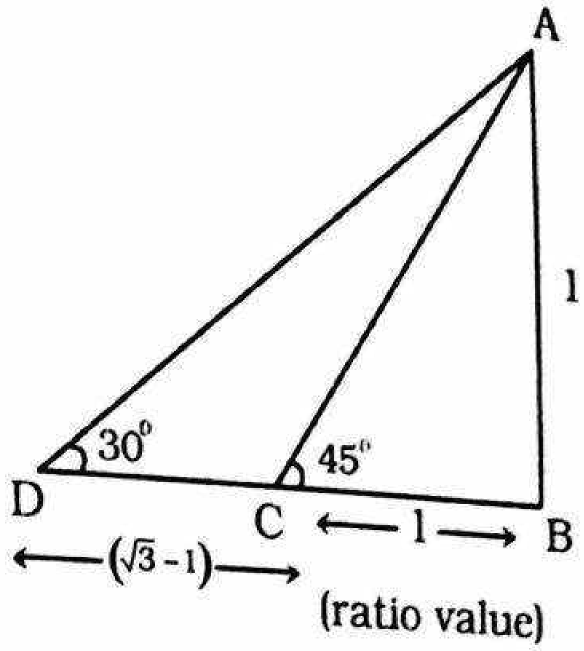
$\therefore$ Value of AB (height of tower) = (ratio value =1)

$= \frac{20}{\sqrt{3}-1} m = \frac{20}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= 10(\sqrt{3}+1) m$

6.(a) (By ratio)





Ratio value orignal value

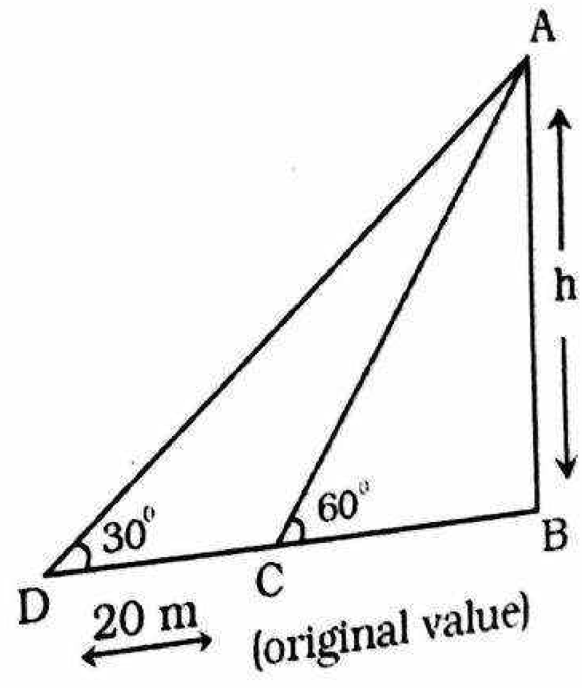
$\therefore (\sqrt{3}-1) \rightarrow 40$ m

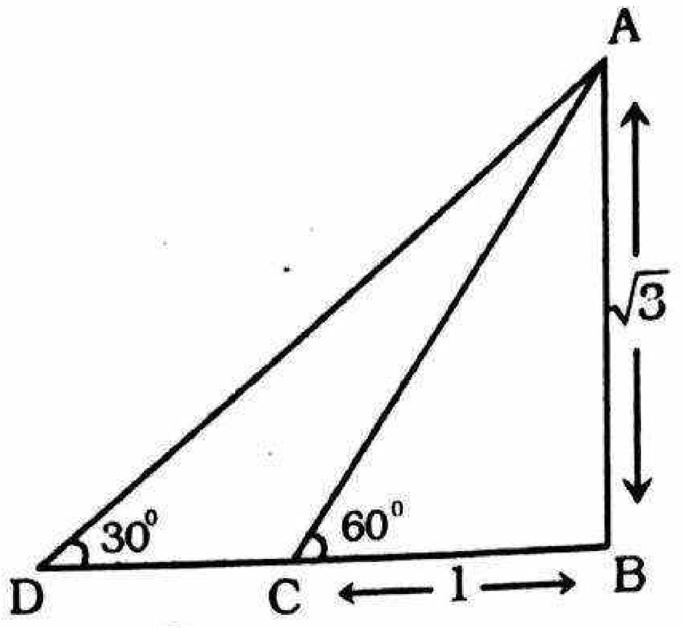
$\therefore 1 \rightarrow \frac{40}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 20(\sqrt{3}+1)$

$\therefore$ lenght of pole

AB(ratio value = 1) = $20(\sqrt{3}+1)\beta$

7.(b) (By ratio)
BC = width of river





(Ratio value)

Ratio value orignal value

2 ⟶ 20 m

∴ 1 ⟶ 10 m

Hence, width of the river=BC=10 m

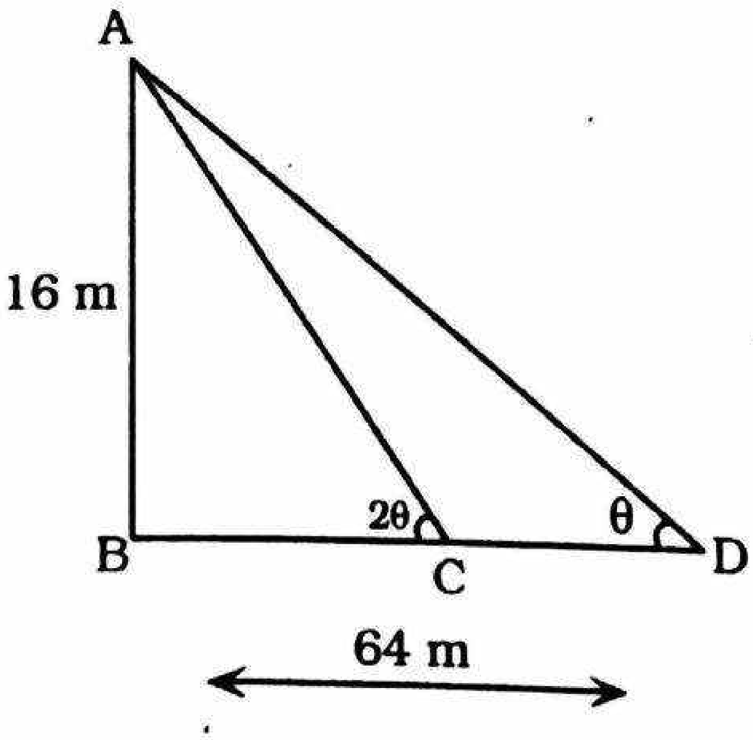
8.(d) Let AB be a pole of height 16 m. The angle of elevation at D is θ and the angle of elevation at C is 2θ.

In right angle ΔABD,

$$\tan\theta = \frac{AB}{BD} = \frac{16}{64} = \frac{1}{4}$$

In right angle ΔABC,

$$\tan 2\theta = \frac{AB}{BC}$$



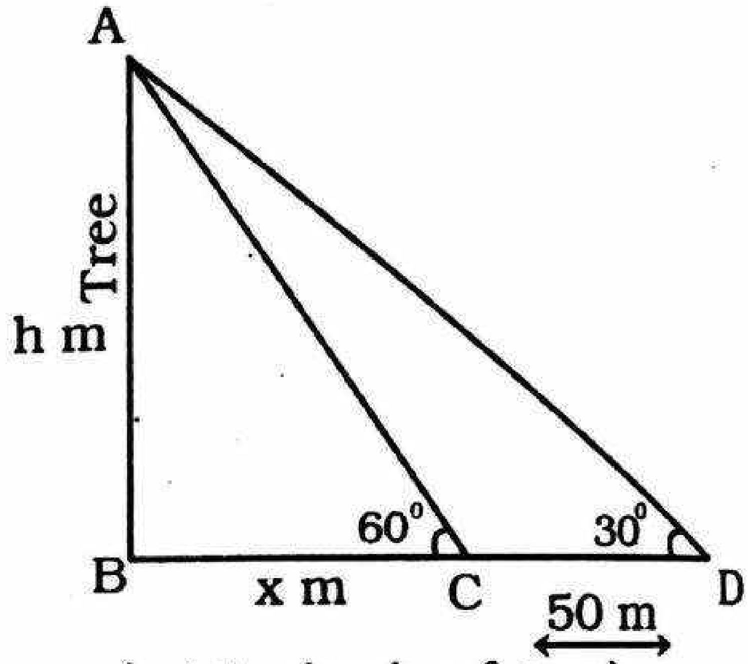
$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{16}{BC} \Rightarrow \frac{2\left(\frac{1}{4}\right)}{1-\left(\frac{1}{4}\right)^2} = \frac{16}{BC}$$

$\Rightarrow BC = 30$ m

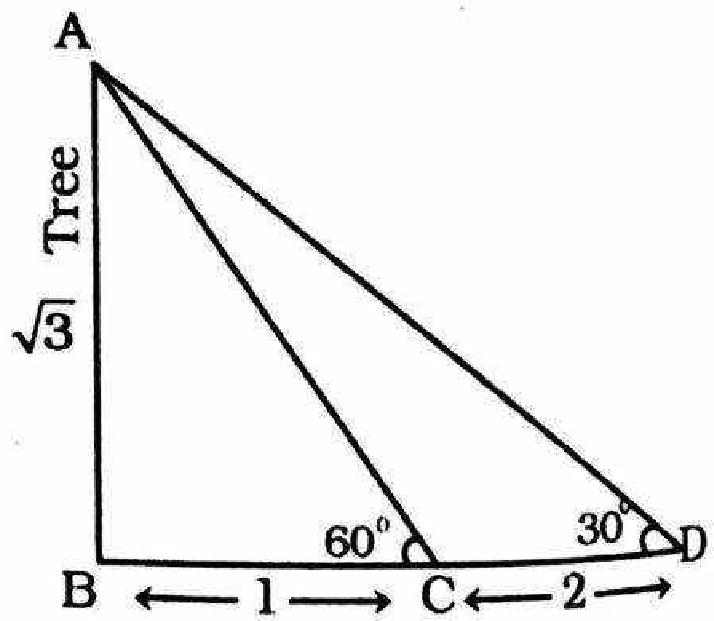
∴ Required decrease in the lenght of shadow

$= 64 - 30 = 34$

9.(a) (By ratio)



(original value figure)



(Ratio value figure)

Ratio value orignal value

2 ⟶ 50

∴ 1 ⟶ 25

and $\sqrt{3}$ ⟶

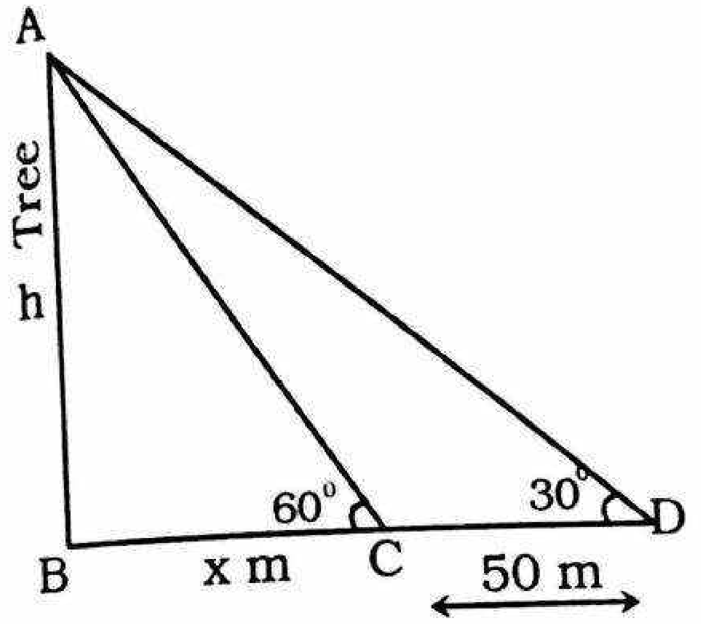
$25\sqrt{3}$

$\therefore$ height of the tree

$= h\left(\text{ratio value} = \sqrt{3}\right) = 25\sqrt{3}$ m

and width of the river = x (ratio value = 1) = 25m

Alternatively :-



In right angled ΔABC,

$\tan 60^0 = \frac{h}{x} \Rightarrow h = x\sqrt{3}$......(i)

In right angled ΔABD,

$\tan 30^0 = \frac{h}{x+50}$

$\Rightarrow h = \frac{x+50}{\sqrt{3}}$....(ii)

From (i) and (ii)

$\frac{x+50}{\sqrt{3}} = x\sqrt{3} \Rightarrow x + 50 = 3x$

$\Rightarrow 2x = 50 \Rightarrow x = 25$

Putting $x = 25$ in (i)

$h = 25\sqrt{3}$ m

Hence, height of the tree

$= h = 25\sqrt{3}$ m

and width of the tree $= x = 25$ m

10.(d) In ΔCDE,

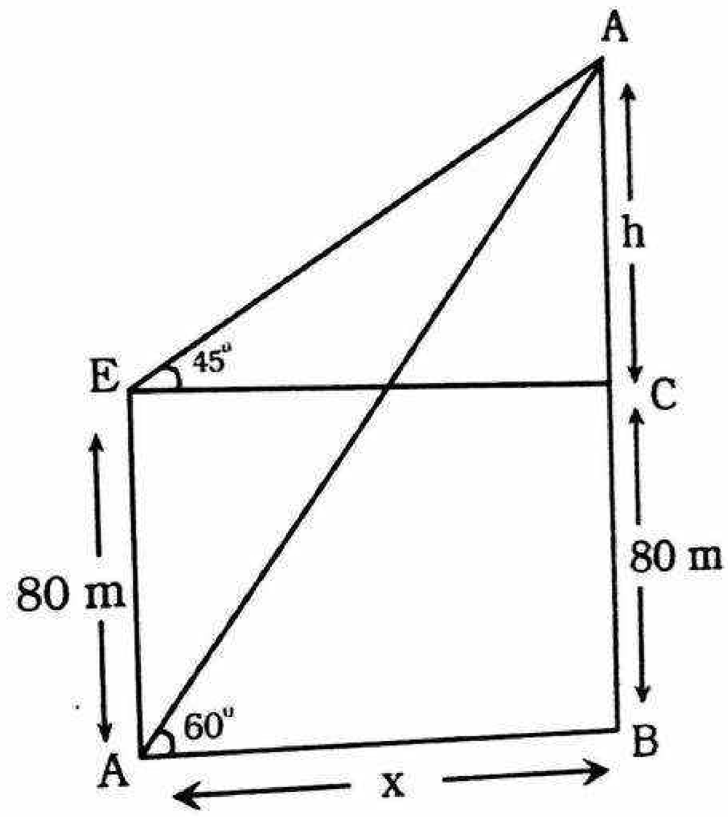
$\tan 45^0 = \frac{CD}{CE}$

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow x = h$..............(i)

In ΔABD,

$\tan 60^0 = \frac{BD}{AB}$



$\Rightarrow \sqrt{3} = \frac{80+h}{x}$

$\Rightarrow \sqrt{3}x = 80 + h$

$\Rightarrow \sqrt{3}x = 80 + x$ [From (i)]

$\Rightarrow \left(\sqrt{3} - 1\right)x = 80$

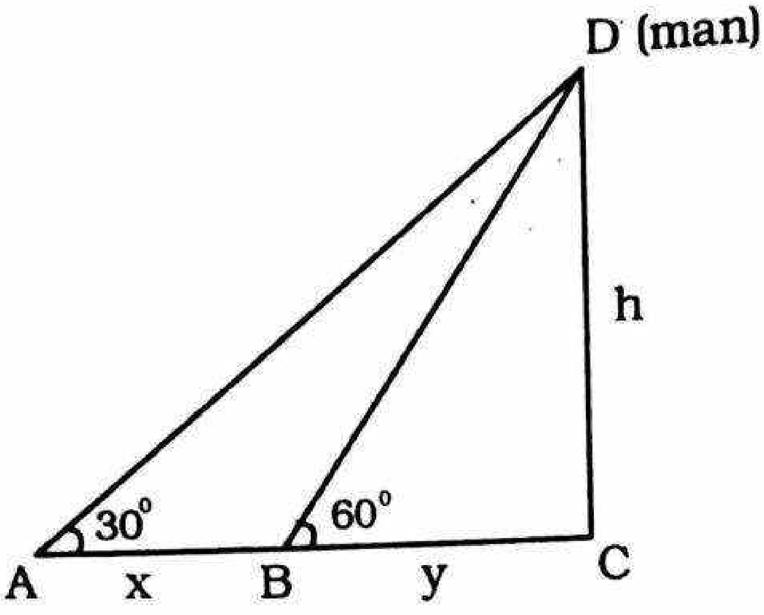
$\Rightarrow x = \frac{80}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow x = 40\left(\sqrt{3} + 1\right)$

$\therefore$ Distance between the bases

$= 40\left(\sqrt{3} + 1\right)$m

11.(c)



From figure, $\tan 60^0 = \frac{h}{y}$

$\Rightarrow h = y\sqrt{3}$............(i)

and $\tan 30^0 = \frac{h}{x+y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y\sqrt{3}}{x+y}$ [using (i)]

$\therefore 3y = x + y$

$\Rightarrow 2y = x \Rightarrow y = \frac{x}{2}$

$\because$ Time taken to cover a distance from A to B = 20 minute

$\therefore$ time taken to cover a unit distance

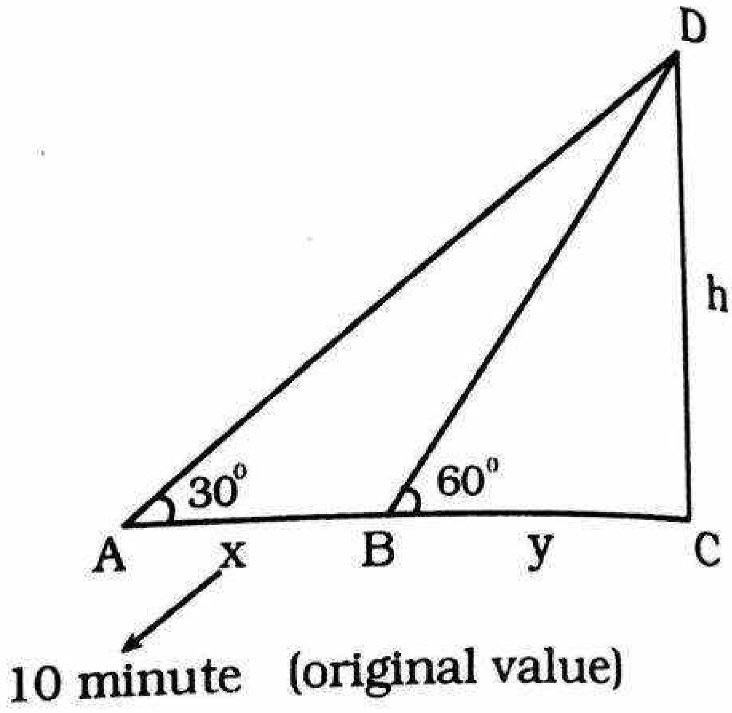
$= \frac{20}{x}$ minute

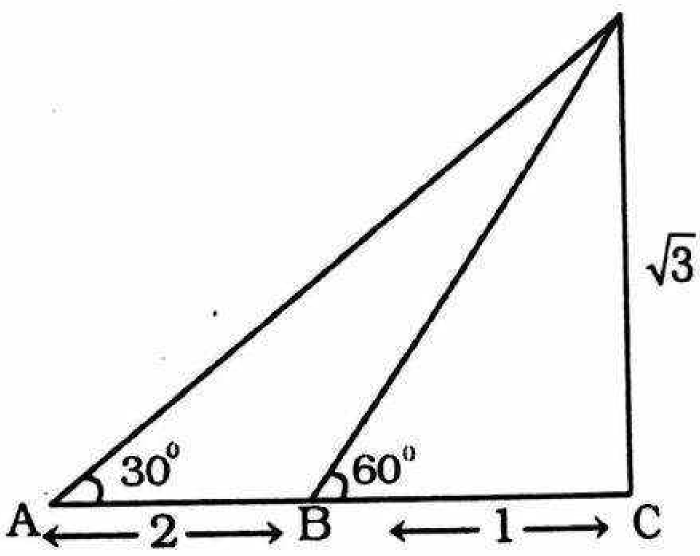
$\therefore$ For distance 'y' time taken

$= \frac{20}{x} \times y$

$= \frac{20}{x} \times \frac{x}{2} = 10$ minute

Alternatively. (By ratio)





(Real value)

Ratio value orignal value

2 ⟶ 20 minute

$\therefore$ 1 ⟶ 10 minute

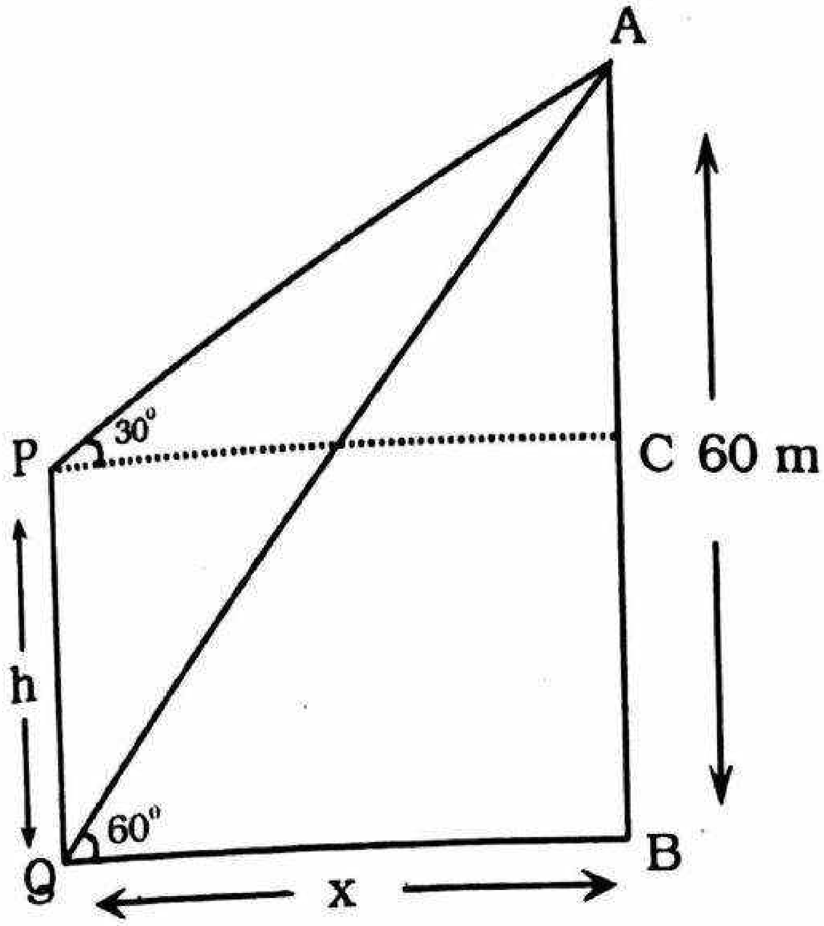
$\therefore$ For distance y (or ratio value=1) time taken =10minute

12.(c) Let AB be the tower and PQ the building.

From right angled ΔABQ,

$\tan 60^0 = \frac{AB}{BQ} \Rightarrow \sqrt{3} = \frac{60}{x}$

$\Rightarrow x = 20\sqrt{3}$



and From right angled ΔAPC,

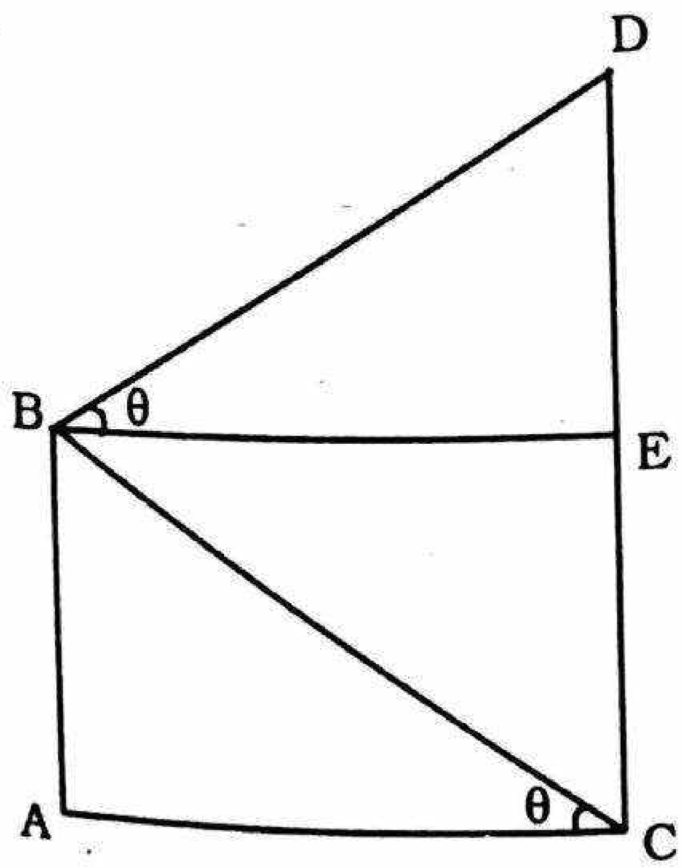
$$\tan 30^\circ = \frac{AC}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AC}{x}$$

$$\Rightarrow AC = \frac{x}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

$\therefore PQ = BC = AB - AC = 60 - 20 = 40$

$\therefore$ height of the building $= h = 40$ m

13.(b) Let AB be the cliff and CD be the tower. Then, AB=30 m. From B draw line $BE \perp CD$



Let $\angle EBD = \angle ACB = \theta$

Now in ΔBED, $\tan\theta = \frac{DE}{BE}$

and in ΔABC,

$$\tan\theta = \frac{AB}{AC}$$

$$\therefore \frac{DE}{BE} = \frac{AB}{AC} \Rightarrow DE = AB \quad [\because BE = AC]$$

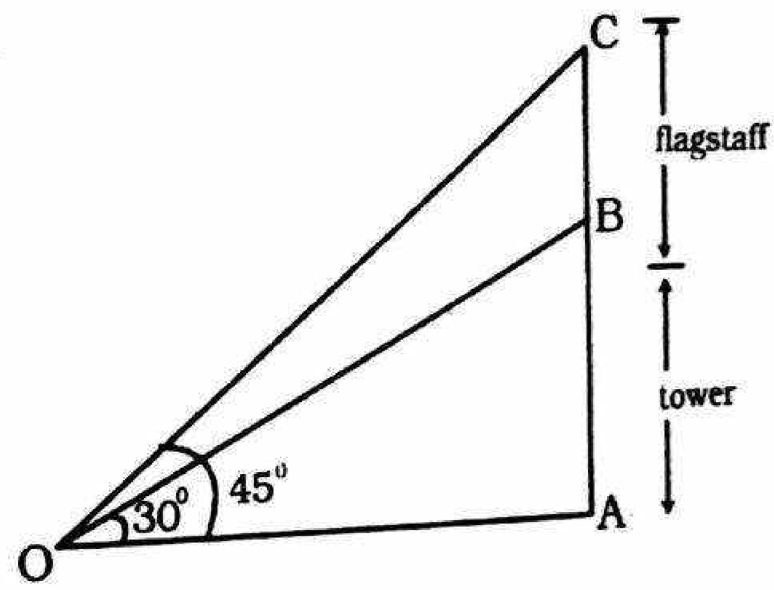
$\therefore CD = CE + DE = AB + AB = 2AB = 60\,m$

14.(a) In ΔOAB,

$$\tan 30^\circ = \frac{AB}{OA} \Rightarrow OA = AB\sqrt{3} \quad \text{.........(i)}$$

and In ΔOAC,

$$\tan 45^\circ = \frac{AC}{OA} \Rightarrow OA = AC$$



$\Rightarrow AB\sqrt{3} = AC$ [from (i)]

$\Rightarrow AB\sqrt{3} = AB + BC$

$\Rightarrow AB\left(\sqrt{3} - 1\right) = BC = 12$

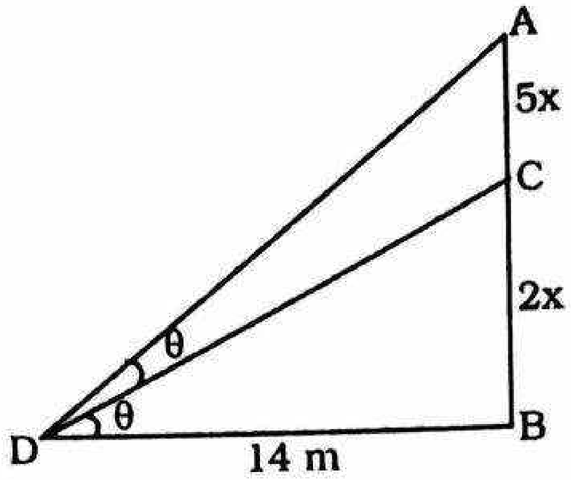
$$\Rightarrow AB = \frac{12}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} =$$

$$\frac{12\left(\sqrt{3} + 1\right)}{2} = 6\left(\sqrt{3} + 1\right) \text{ m}$$

15.(d) Let $BC = 2x$, then $CA = 5x$

$\therefore$ $AB = 7x$

According to question



$\angle ADC = \angle CDB = \theta$ and $BD = 14$ m

In ΔBDC, $\tan\theta = \frac{BC}{BD} = \frac{2x}{14} = \frac{x}{7}$

In ΔABD, $\tan 2\theta = \frac{AB}{BD} = \frac{7x}{14} = \frac{x}{2}$

$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{x}{2}$

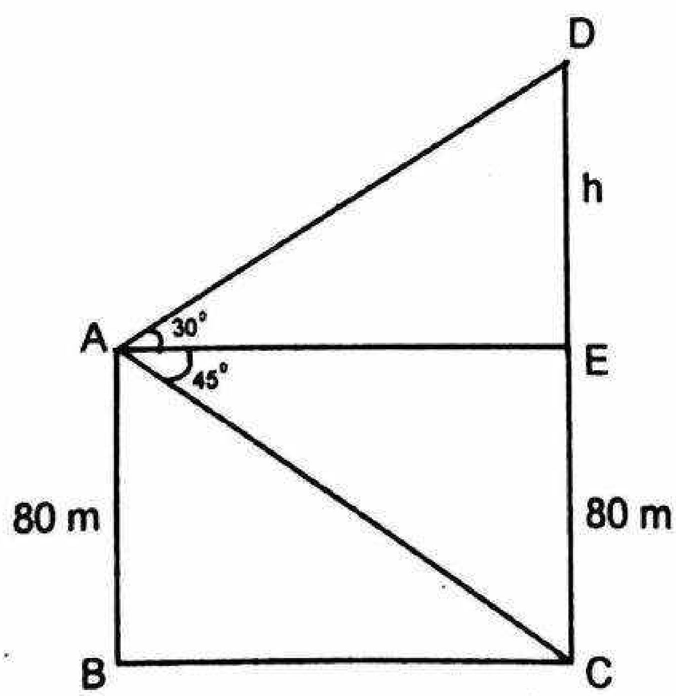
$\Rightarrow \frac{2\left(\frac{x}{7}\right)}{1-\left(\frac{x}{7}\right)^2} = \frac{x}{2}$

$\Rightarrow \frac{2x \times 7}{49 - x^2} = \frac{x}{2} \Rightarrow 49 - x^2 = 28$

$\Rightarrow x^2 = 21 \Rightarrow x = \sqrt{21}$

$\therefore$ height of the pole $= AB = 7x = 7\sqrt{21}$ m

16.(c) Let AB and CD are pillars.
Let DE=h



In ΔADE, $\tan 30^0 = \frac{h}{AE}$

$\Rightarrow AE = h\sqrt{3}$...........(i)

In ΔACE, $\tan 45^0 = \frac{80}{AE}$

$\Rightarrow AE = 80 \Rightarrow h\sqrt{3} = 80$ [From (i)]

$\Rightarrow h = \frac{80}{\sqrt{3}}$

$\therefore$ Required height

$= 80 + \frac{80}{\sqrt{3}} = \frac{80}{\sqrt{3}}\left(\sqrt{3}+1\right)$ m

Alternatively :

In 45°; Base : height = 1 : 1

$\therefore$ AE(Base) = 80m $\therefore$ height (EC) = 80 m

In 30°; Base : height

$= \sqrt{3} : 1$

$\times\frac{80}{\sqrt{3}}$ $\times\frac{80}{\sqrt{3}}$

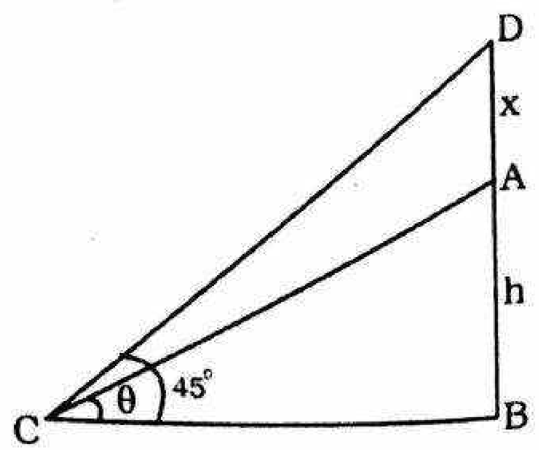
80 $\frac{80}{\sqrt{3}}$ m

$h = \frac{80}{\sqrt{3}}$ m

$\therefore$ Required height =

$CD = 80 + \frac{80}{\sqrt{3}} = \frac{80}{\sqrt{3}}\left(\sqrt{3}+1\right)$

17.(b) AB = Building = h metre
AD = Chimny = x metre



In ΔBCD,

$$\tan 45^0 = \frac{h+x}{BC} \Rightarrow BC = h + x..........(i)$$

In ΔABC,

$$\tan\theta = \frac{h}{BC} \Rightarrow BC = h\cot\theta............(i)$$

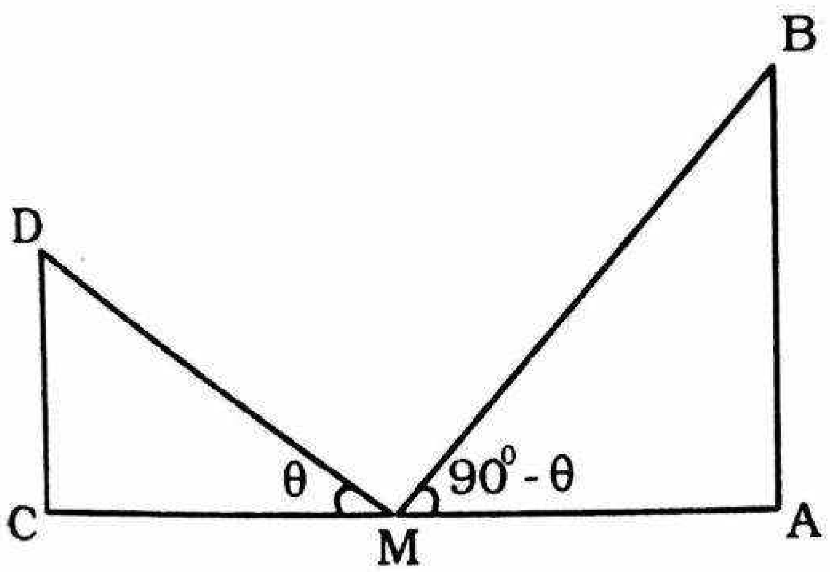
From (i) and (ii)

$$h + x = h\cot\theta$$

$$\Rightarrow x = (h\cot\theta - h)\ m$$

18.(a) Let AB and CD be the two posts such that AB=2CD. Let M be the mid-point of CA. Let $\angle CMD = \theta$ and $\angle AMB = 90^0 - \theta$

Let CD=h, then AB = 2h



Now,

$$\tan(90^0 - \theta) = \frac{AB}{AM} \Rightarrow \cot\theta = \frac{2h}{(K/2)}$$

$$\Rightarrow \cot\theta = \frac{4h}{K}...............(i)$$

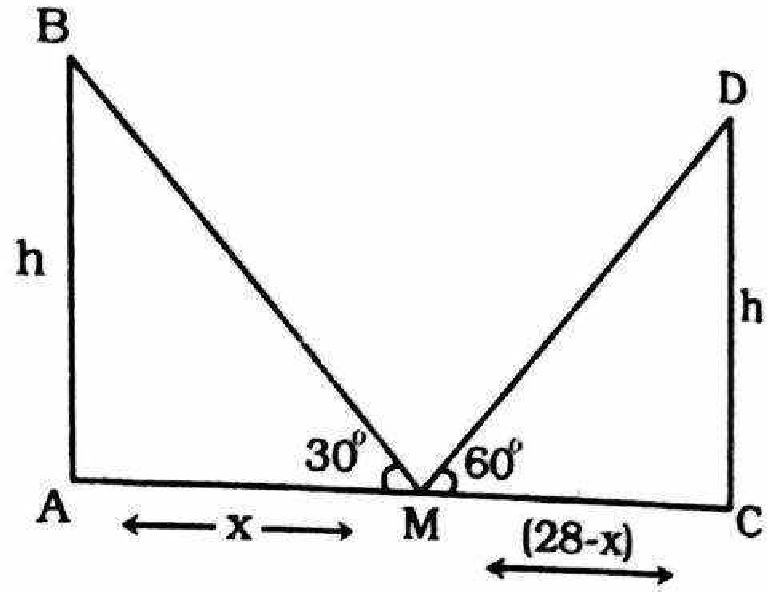
$$\frac{CD}{CM} = \tan\theta \Rightarrow \tan\theta = \frac{h}{K/2} = \frac{2h}{K}..........(ii)$$

Multiplying (i) and (ii), we get :

$$\frac{4h}{K} \times \frac{2h}{K} = 1$$

$$\Rightarrow h^2 = \frac{K^2}{8} \Rightarrow h = \frac{K}{2\sqrt{2}}$$

19.(a) Let AB and CD be the pole and AC be the road.
Let AE = x, then EC = 28-x and AB =CD = h. Then



$$\tan 30^0 = \frac{h}{x} \Rightarrow x = \sqrt{3}h..........(i)$$

$$\tan 60^0 = \frac{h}{28-x} \Rightarrow 28 - x = \frac{h}{\sqrt{3}}$$

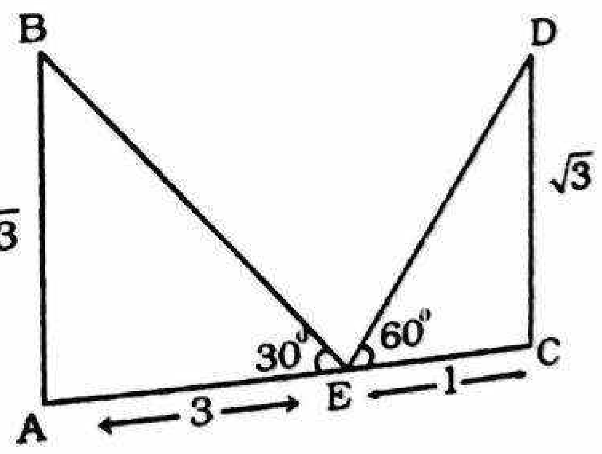
$$\Rightarrow 28 - \sqrt{3}h = \frac{h}{\sqrt{3}}$$

[From (i)]

$$\Rightarrow \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)h = 28 \Rightarrow \frac{4}{\sqrt{3}}h = 28$$

$$\Rightarrow h = 7\sqrt{3}$$

Alternatively - (By ratio)



let AB = CD = $\sqrt{3}$

then , EC=1 and AE=3

$\therefore$ AC (ratio value) =3+1=4

$\therefore 4 \longrightarrow 28$

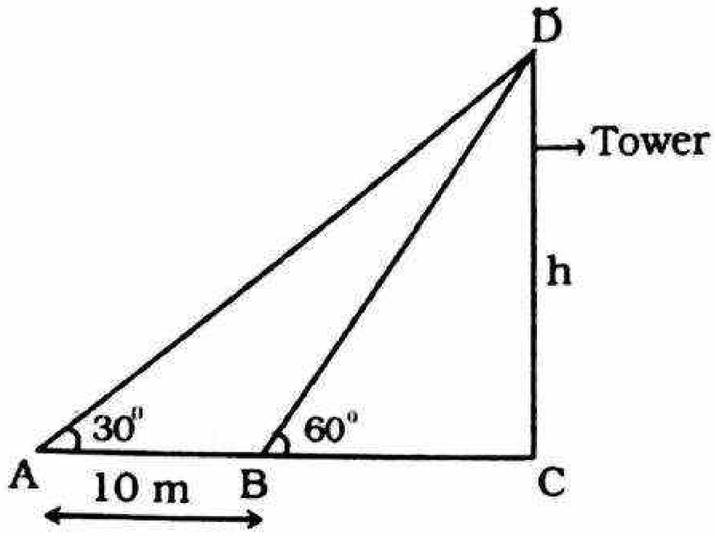
$\Rightarrow 1 \longrightarrow 7$

$\therefore \sqrt{3} \longrightarrow 7\sqrt{3}$

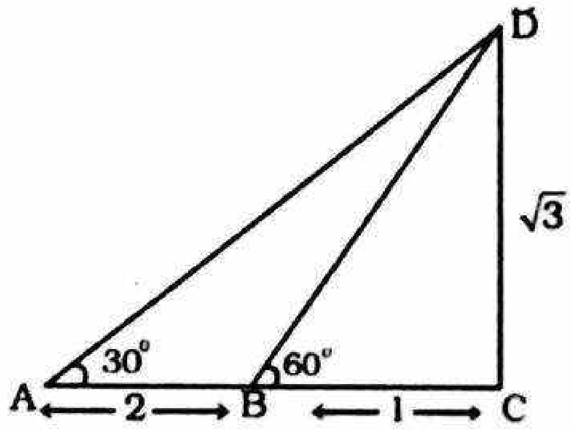
i.e. height = h(ratio value = $\sqrt{3}$)

$= 7\sqrt{3}$ m

20.(a) (By ratio)



(orignal value)



(ratio value)

i.e. $2 \longrightarrow 10$

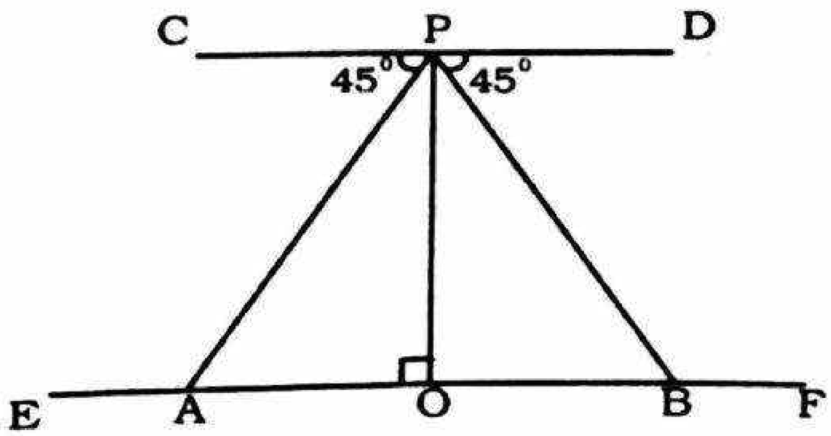
$\therefore 1 \longrightarrow 5$

$\therefore \sqrt{3} \longrightarrow 5\sqrt{3}$

hence, height of tower (CD)= h = (ratio value = $\sqrt{3}$) = $5\sqrt{3}$ m

21.(c) Let CD and EF be the two banks of a river and P be the landmark.

From P draw $PO \perp AB$.



As given $\angle DPB = \angle CPA = 45^0$

$\therefore \angle A = \angle B = 45^0$

$\therefore$ So PA = PB

Thus ΔABP is an isosceles triangle and $PO \perp AB$ So, O is the mid-point of AB.

$\therefore$ AO = OB = 0.5 km

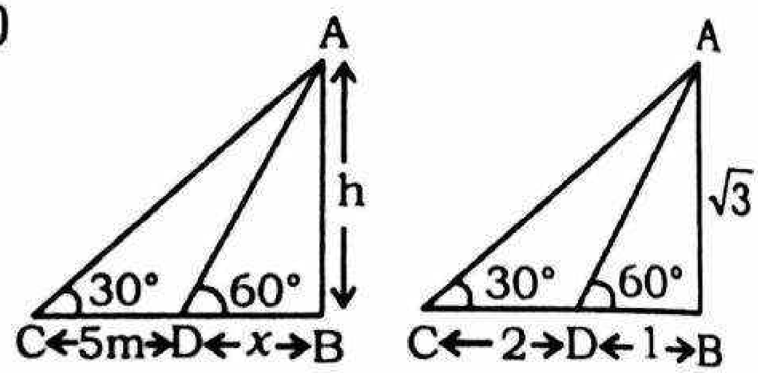
In ΔOBP, $\angle OBP = \angle BPO = 45^0$

$\therefore$ PO = BO =0.5 km

Hence, the width of the river

$= 0.5km = 500m = \frac{1}{2}km$

22.(a)



(original value) (ratio value)

AB = Pole = h metre (let)

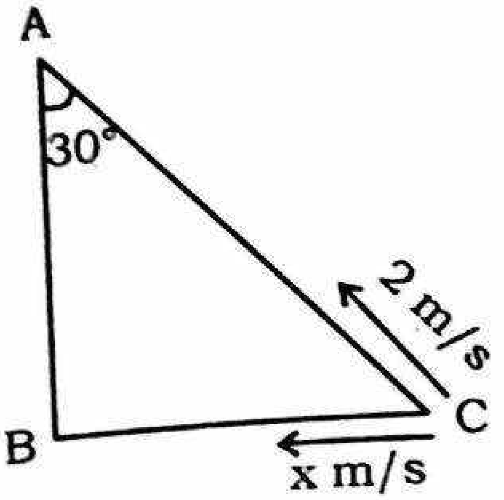
let BD = x metre

Ratio value		Original value
CD → 2	⟶	5
∴ 1	⟶	$\frac{5}{2}$
∴ $\sqrt{3}$	⟶	$\frac{5}{2}\sqrt{3}$

i.e. the height of the pole = AB

$= h = \frac{5\sqrt{3}}{2}$ metre

23.(b)



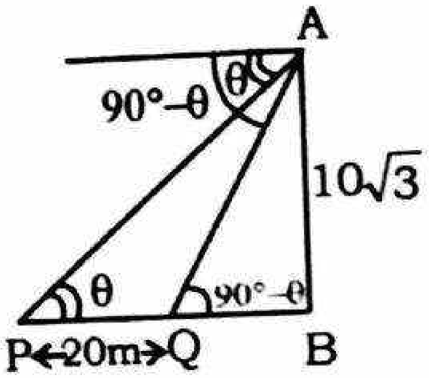
$\angle C = 90° - 30° = 60°$

let he approaches the wall at the rate of x m/s.

$\therefore$ components of 2 m/s in horizontal direction $x = 2\cos 60° = 2 \times \frac{1}{2}$

$= 1$ m/s

24.(c)



AB = Building = $10\sqrt{3}$ m

PQ = 20 m

let BQ = x m

As we know that the two points at a distance of a and b from the base of a building respectively, makes angles at the top of the Building which are complementary, then height of building,

$h = \sqrt{ab}$

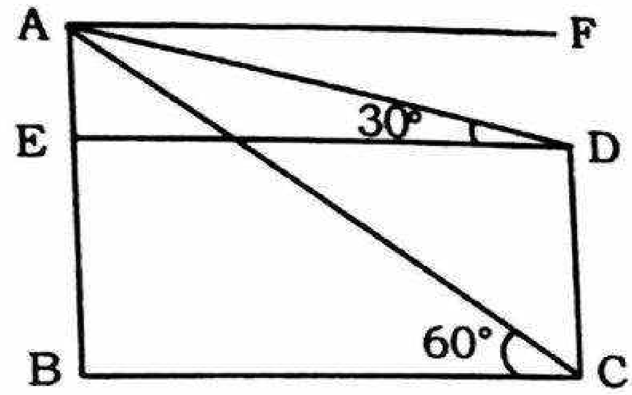
$\therefore 10\sqrt{3} = \sqrt{x(x+20)}$

or $300 = x(x+20)$

On solving we get $x = 10$ m.

$\therefore$ BP = 10 + 20 = 30 m

25.(b)



AB = cliff = 90 metre

$\angle ADE = 30°$

$\angle ACB = 60°$

CD = Tower = h metre

BC = x metre

From Δ ABC,

$$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{90}{x}$$

$$\Rightarrow x = \frac{90}{\sqrt{3}} = 30\sqrt{3} \text{ metre}$$

From Δ ADE,

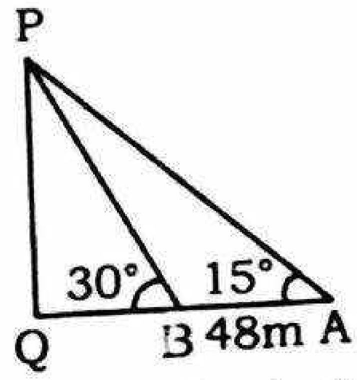
$$\tan 30° = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{90-h}{30\sqrt{3}}$$

$$\left[\because ED = x = 30\sqrt{3}\right]$$

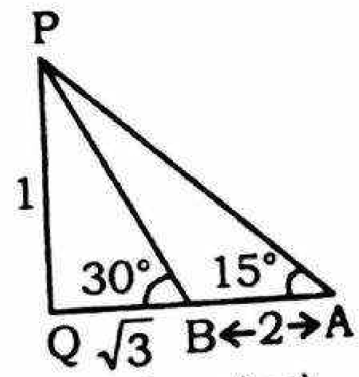
$\therefore 90 - h = 30$

$\Rightarrow h = 90 - 30 = 60$metre

26.(b)



(original value)



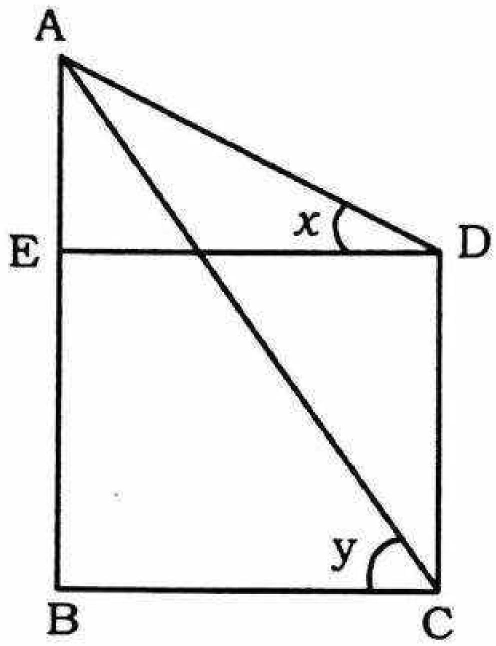
(ratio value)

PQ = Tower = h metre

Ratio value		Original value
AB → 2	⟶	48
∴ 1	⟶	$\frac{48}{2}$ = 24 metre

i.e. height of the building = PQ
(ratio value = 1) = h = 24 metre.

27.(c)



CD = tree = h metre
AB = building = a metre
BC = ED = b metre
∴ From Δ AED,

$$\tan x = \frac{AE}{ED} \Rightarrow \tan x = \frac{a-h}{b}$$

$\Rightarrow$ b = (a – h)cot x

$\Rightarrow$ b = a cot y

From equations (i) and (ii)

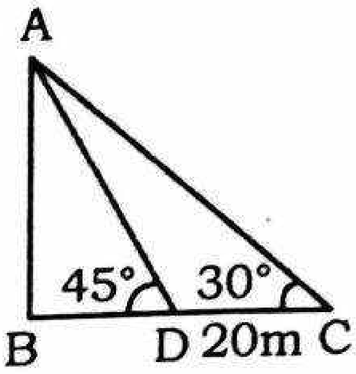
(a – h)cot x = a cot y

$\Rightarrow$ a cot x – h cot x = a cot y

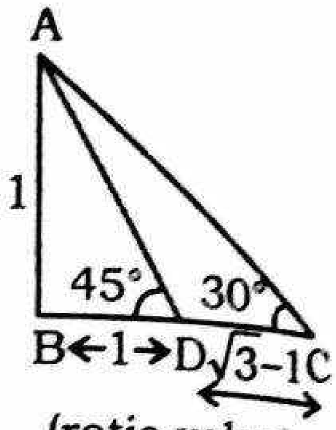
$\Rightarrow$ h cot x = a (cot x – cot y)

$$\Rightarrow a = \frac{h \cot x}{\cot x - \cot y}$$

28.(d)



(original value)



(ratio value)

Let AB be a pillar of height h metre.

Ratio value Original value

CD → $\sqrt{3} - 1$ ⟶ 20

$$\therefore \quad 1 \rightarrow \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

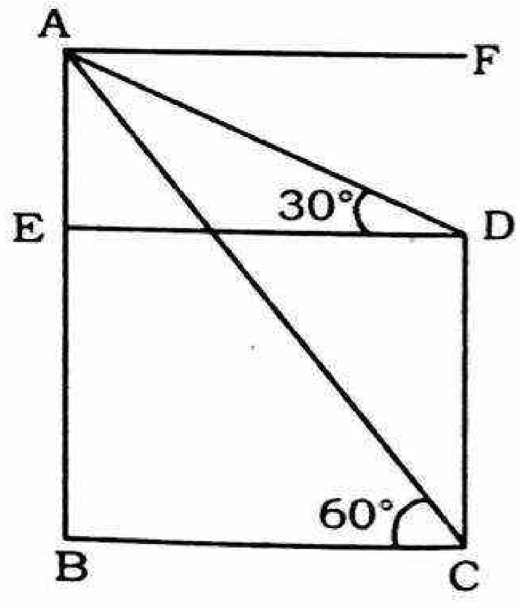
$$= 10\left(\sqrt{3}+1\right)\text{metre}$$

29.(b) AB = 108 m

CD = x metre

From Δ ABC,

$$\tan 60° = \frac{AB}{BC}$$



$$\Rightarrow \sqrt{3} = \frac{108}{BC}$$

$$\Rightarrow BC = \frac{108}{\sqrt{3}} = 36\sqrt{3}\text{m}$$

From Δ AED,

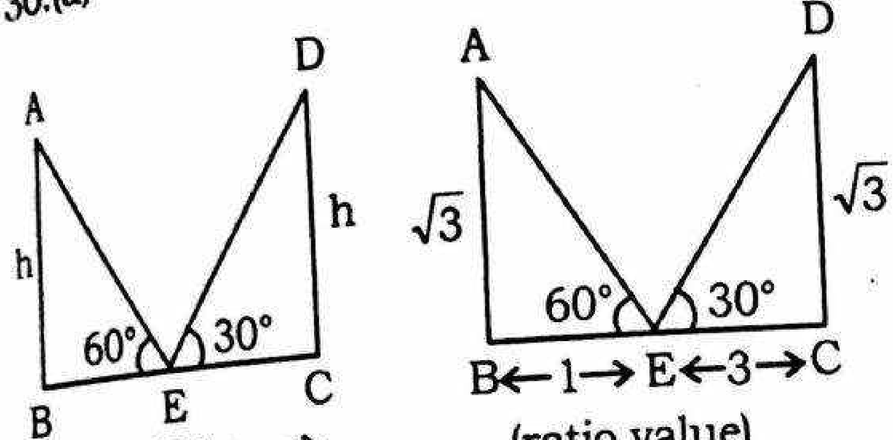
$$\tan 30^\circ = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{108 - x}{36\sqrt{3}}$$

$$\left[\because ED = BC = 36\sqrt{3}\right]$$

$\Rightarrow 108 - x = 36$

$\Rightarrow x = 108 - 36 = 72\text{m}$

30.(a)



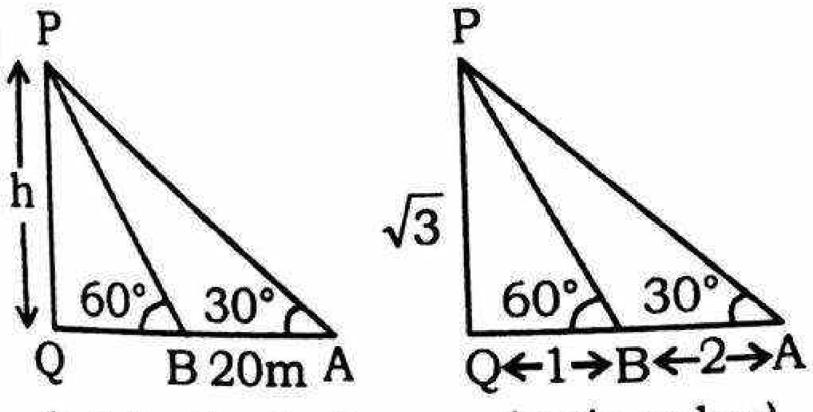
(original value)

(ratio value) [as heights are same]

Ratio value		Original value
BC → 4	⟶	100
∴ 1	⟶	25
∴ $\sqrt{3}$	⟶	$25\sqrt{3}$

∴ height of each pole = h (ratio value = $\sqrt{3}$) = $25\sqrt{3}$

31.(c)



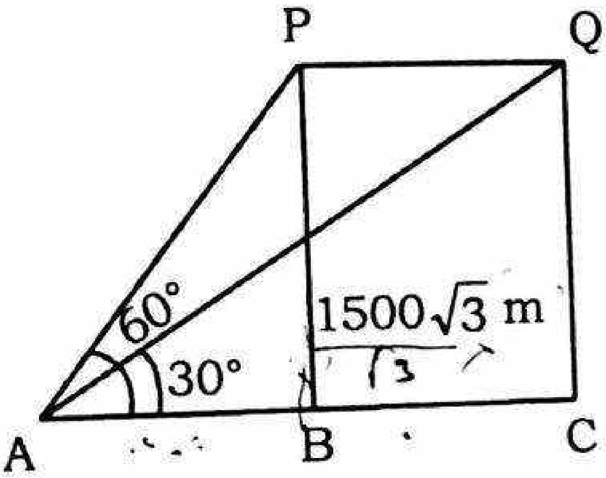
(original value) (ratio value)

PQ = Tower = h metre (let)

Ratio value		Original value
AB → 2	⟶	20
∴ 1	⟶	10
∴ $\sqrt{3}$	⟶	$10\sqrt{3}$

i.e. height of the tower = h(ratio value = $\sqrt{3}$) = $10\sqrt{3}$ metre.

32.(b)



P & Q are the positions of the plane.

$\angle PAB = 60^\circ$; $\angle QAB = 30^\circ$

$PB = 1500\sqrt{3}$ m

In Δ ABP

$$\tan 60^\circ = \frac{BP}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{AB}$$

$\Rightarrow$ AB = 1500 metre

In Δ ACQ

$$\tan 30^\circ = \frac{CQ}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AC}$$

$= AC = 1500 \times 3$
$= 4500$ metre

$PQ = BC = AC - AB$
$= 4500 - 1500$
$= 3000$ metre

$\therefore$ Speed of plane $= \frac{3000}{15}$

$= 200$ metre/second

33.(c) $\angle ACB = 60°$

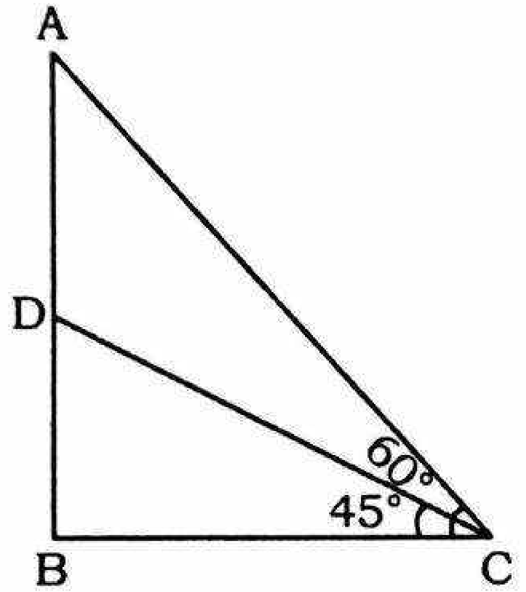
$\angle DCB = 45°$

AB = 5000 metre

AD = x metre

$\therefore$ From Δ ABC,

$$\tan 60° = \frac{AB}{BC}$$



$$\Rightarrow \sqrt{3} = \frac{5000}{BC}$$

$$\Rightarrow BC = \frac{5000}{\sqrt{3}} \text{ metre}$$

From Δ DBC,

$$\tan 45° = \frac{DB}{BC} \quad \Rightarrow DB = BC = \frac{5000}{\sqrt{3}}$$

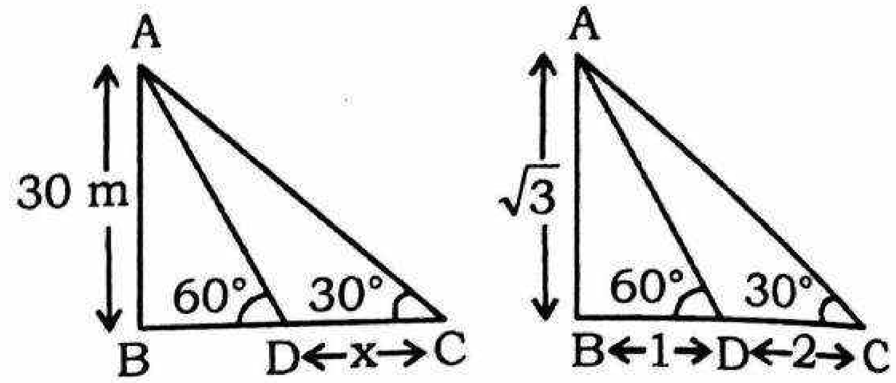
$$\therefore AD = AB - BD = 5000 - \frac{5000}{\sqrt{3}}$$

$$= 5000\left(1 - \frac{1}{\sqrt{3}}\right) = 5000\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \text{ metre}$$

34.(b) $\frac{6}{4} = \frac{h}{50}$

$$\Rightarrow h = \frac{50 \times 6}{4} = 75 \text{ feet}$$

35.(d)



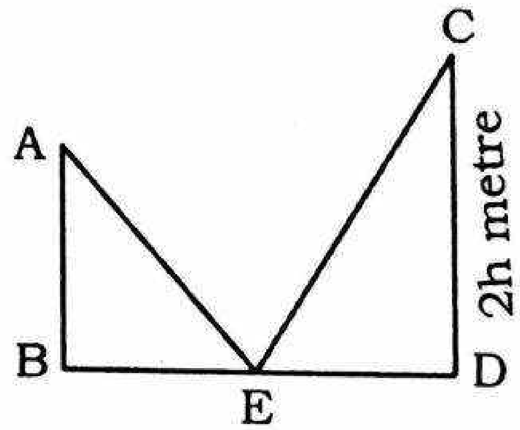
(original value) (ratio value)

AB = tower = 30 metre, let CD = x metre

Ratio value	**Original value**
AB → $\sqrt{3}$ ⟶	30
$\therefore$ 1 ⟶	$\frac{30}{\sqrt{3}} = 10\sqrt{3}$
$\therefore$ 2 ⟶	$20\sqrt{3}$ metre

$\therefore$ Required distance moved = x (ratio value = 2) = $20\sqrt{3}$ metre.

36.(d)



BE = DE = 30 metre

$\angle AEB = \theta$

$\therefore \angle CED = 90° - \theta$

From Δ ABE,

$$\tan \theta = \frac{AB}{BE}$$

$\Rightarrow \tan\theta = \frac{h}{30}$

$\Rightarrow h = 30\tan\theta$ ....(i)

From Δ CDE,

$\tan(90^\circ - \theta) = \frac{2h}{30}$

$\Rightarrow \cot\theta = \frac{h}{15} \Rightarrow 15\cot\theta$ .....(ii)

by multiplying both equations,

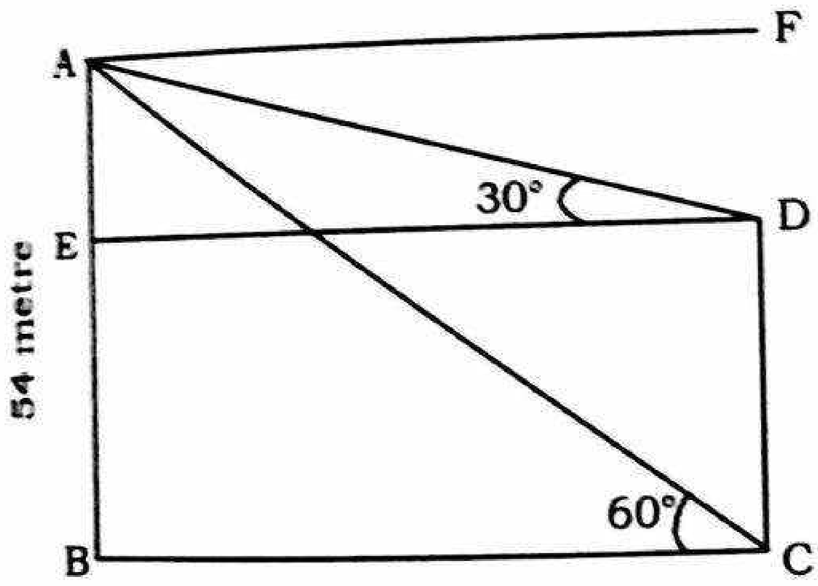
$h^2 = 30 \times 15$

$[\because \tan\theta.\cot\theta = 1]$

$\Rightarrow h = 15\sqrt{2}$ metre = AB

$\Rightarrow 2h = 30\sqrt{2}$ metre = CD

37.(b)



AB = temple = 54 metre

CD = temple = h metre

BC = width of river = x metre

From Δ ABC,

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow BC = \frac{54}{\sqrt{3}} = 18\sqrt{3}$ metre

From Δ ADE,

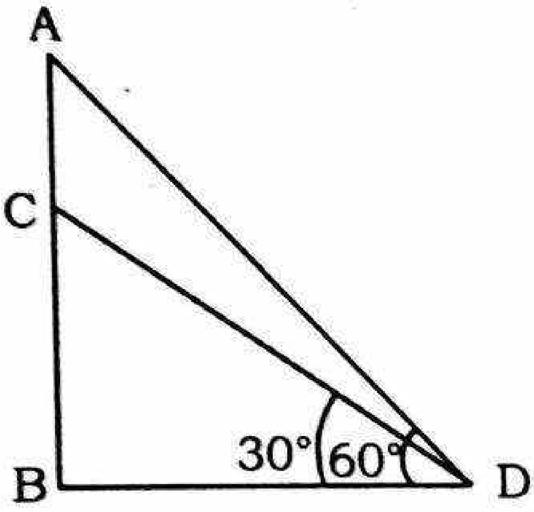
$\tan 30^\circ = \frac{AE}{DE}$ $\left[\because DE = BC = 18\sqrt{3}\right]$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{54-h}{18\sqrt{3}}$

$\Rightarrow 54 - h = 18$

$\Rightarrow h = 54 - 18 = 36$ metre

38.(d)



A and C $\Rightarrow$ position of planes

BC = 3125 m

AC = x metre

In Δ ABD,

$\tan 60^\circ = \frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{3125 + x}{BD}$

$\Rightarrow BD = \frac{3125 + x}{\sqrt{3}}$

In Δ BCD,

$\tan 30^\circ = \frac{BC}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3125}{\frac{3125 + x}{\sqrt{3}}}$

$\Rightarrow 3(3125) = 3125 + x$

$\Rightarrow x = 9375 - 3125$

$x = 6250$ metre

Solution - III

1.(a) Let P be the position of the aeroplane and A, B, given km stones on the road. Then as given

$\angle PAB = \alpha$ and $\angle PBA = \beta$

If height of the aeroplane from the road be PQ=h then,

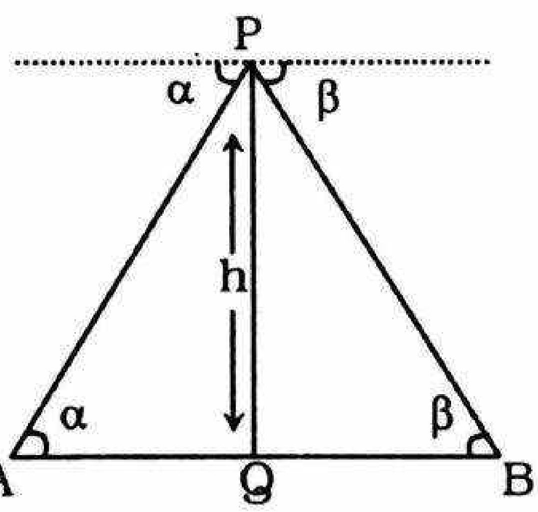
$\frac{h}{AQ} = \tan\alpha$ and

$\frac{h}{BQ} = \tan\beta$

$$\Rightarrow AQ + BQ = h\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$$

$$\Rightarrow 1 = h\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$$

$$\Rightarrow h = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$$



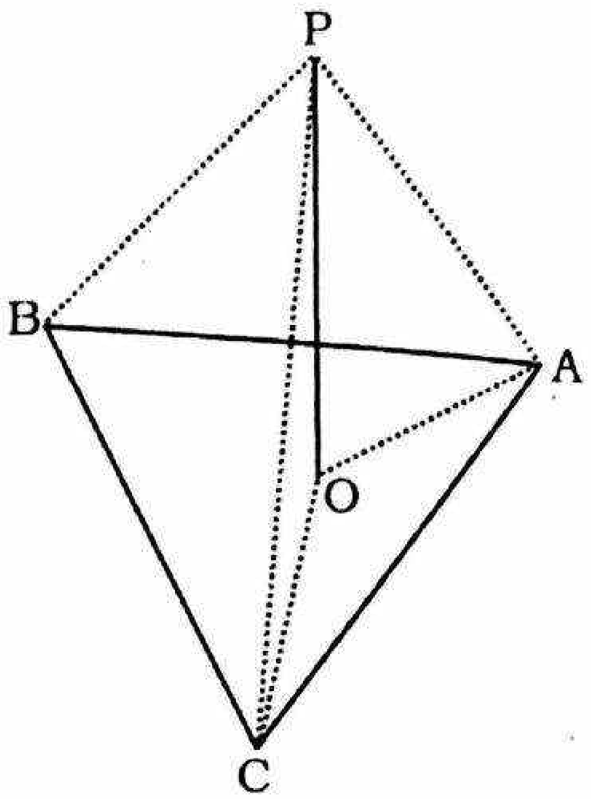
2.(b) Let O be the centre of the equilateral ΔABC and OP the tower of height h. Then each of the ΔPAB, ΔPBC and ΔPCA equilateral. Thus, PA = PB = PC = a. Therefore from right-angle triangle POA, we have

$$PA^2 = PO^2 + OA^2$$

$$\Rightarrow a^2 = h^2 + \left(\frac{a}{2}\sec 30^0\right)^2$$

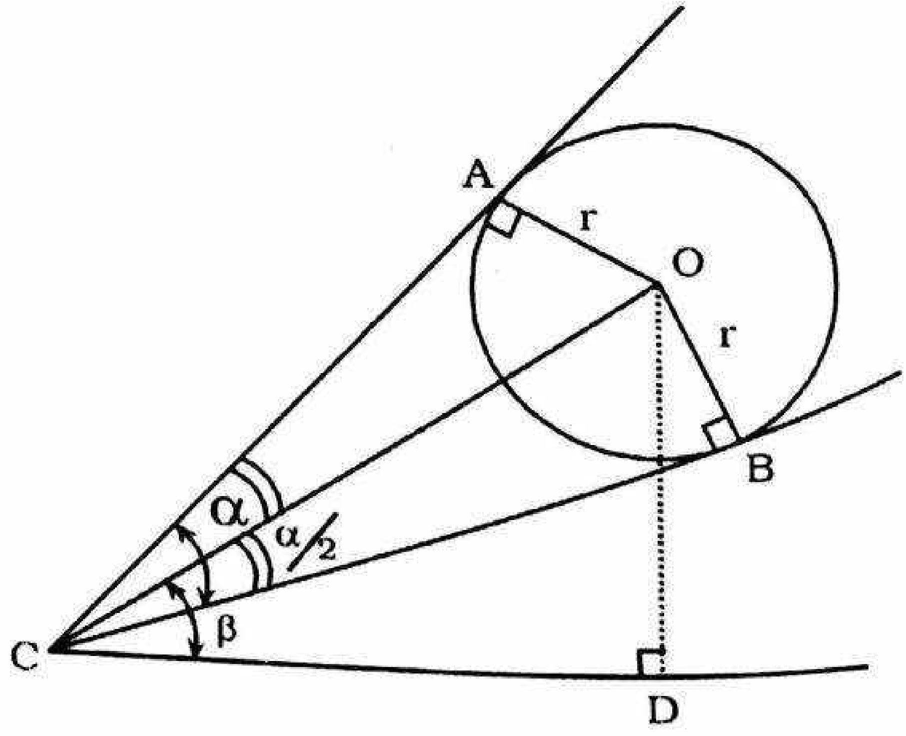
$$= h^2 + \frac{a^2}{4}\frac{4}{3} = h^2 + \frac{a^2}{3}$$

$$\Rightarrow \frac{2}{3}a^2 = h^2 \Rightarrow 2a^2 = 3h^2$$



3.(a) Let O be the centre of balloon of radius r. The observer's eye is at C, s.t. $\angle ACB = \alpha$ and $\angle OCD = \beta$ clearly, CA and CB are tangents to the circle. so $\angle ACO = \angle BCO = \alpha/2$

In right angled ΔOBC,



$$\sin\frac{\alpha}{2} = \frac{OB}{OC} \Rightarrow OC = \frac{OB}{\sin\alpha/2} = r\,\text{cosec}\,\alpha/2$$

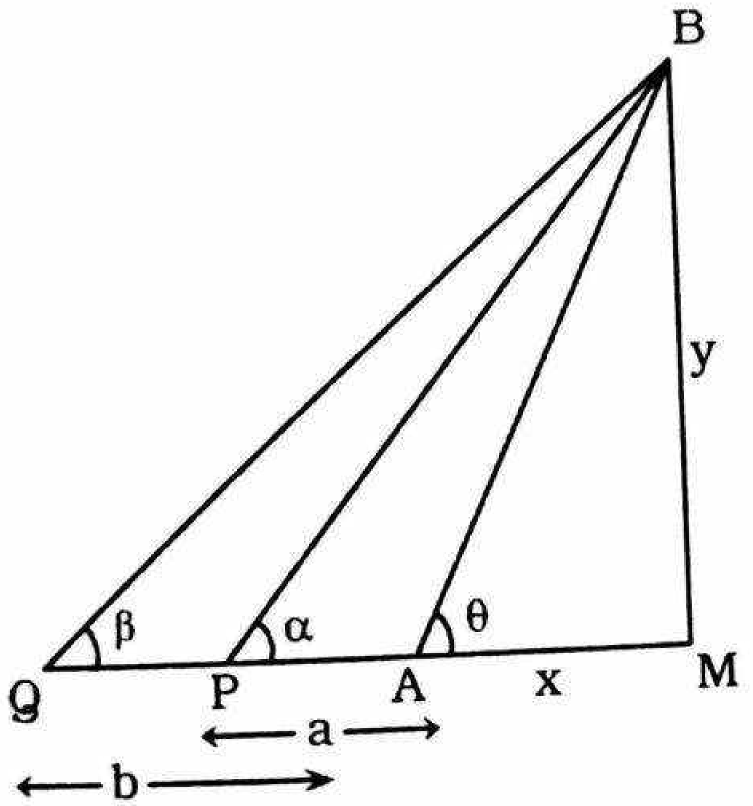
In right angled ΔOCD,

$$\sin\beta = \frac{OD}{OC} \Rightarrow OD = OC\sin\beta = r\,\text{cosec}\,\alpha/2 . \sin\beta$$

$\therefore$ Height of the centre of the balloon is

$r\sin\beta . \text{cosec}\,\alpha/2$

4.(c) Let AB be the tower and its inclination from the horizontal is θ.
Let AM = x and BM = y.
In ΔBMP,



$$\cot\alpha = \frac{PM}{BM} = \frac{a+x}{y}$$

$$\Rightarrow a + x = y\cot\alpha ....(i)$$

In ΔBQM,

$$\cot\beta = \frac{QM}{BM} = \frac{b+x}{y}$$

$$\Rightarrow b + x = y\cot\beta ....(ii)$$

Subtracting (ii) from (i)

$$a - b = y(\cot\alpha - \cot\beta)$$

$$\Rightarrow y = \frac{a-b}{\cot\alpha - \cot\beta}$$

Now from (i)-

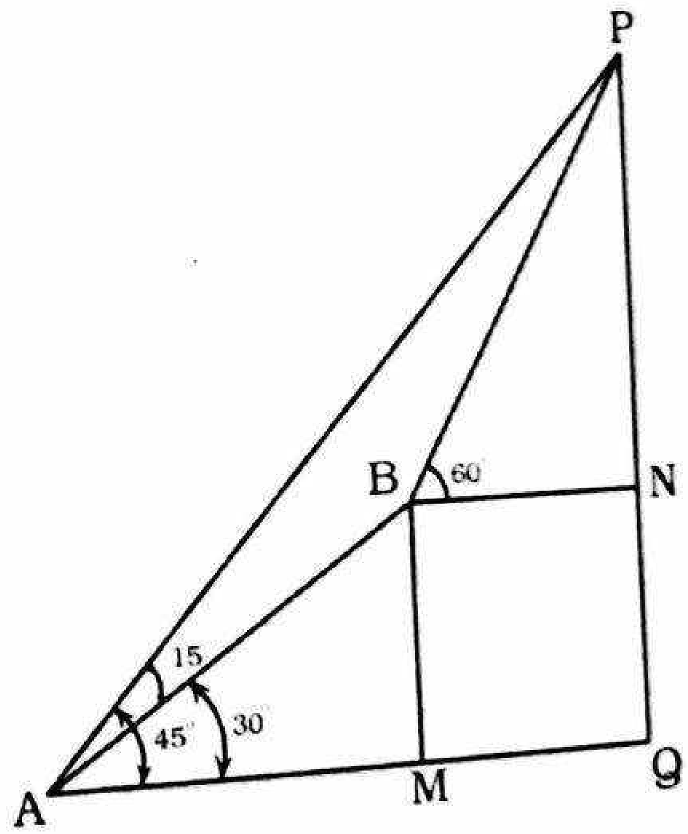
$$x = y\cot\alpha - a = \frac{(a-b)\cot\alpha}{\cot\alpha - \cot\beta} - a$$

$$= \frac{a\cot\beta - b\cot\alpha}{\cot\alpha - \cot\beta}$$

In ΔABM, $\cot\theta = \frac{x}{y}$

$$\Rightarrow \cot\theta = \frac{\frac{a\cot\beta - b\cot\alpha}{\cot\alpha - \cot\beta}}{\frac{a-b}{\cot\alpha - \cot\beta}} = \frac{b\cot\alpha - a\cot\beta}{b-a}$$

5.(a) Suppose P be the summit of the mountain and Q be the foot.



Here, $BN \perp PQ$ and $BM \perp AQ$.

$AB = 4\text{km}$, $\angle MAB = 30^0$.

$\angle MAP = 45^0$

$\angle NBP = 60^0 \quad \therefore \angle BAP = 15^0$

and $\angle APQ = 45^0$

and $\angle BPN = 30^0$

$\therefore \angle APB = 15^0$

$\therefore \Delta ABP$ is isosceles and

$AB = BP = 4km$

In ΔPBN,

$PN = BP \sin 60^0$

In ΔABM,

$BM = AB \sin 30^0$

$\therefore PQ = PN + NQ = PN + BM$

$= BP \sin 60^0 + AB \sin 30^0$

$$= 4\frac{\sqrt{3}}{2} + 4\frac{1}{2} = 4\left(\frac{\sqrt{3}+1}{2}\right)$$

$= 2\left(\sqrt{3}+1\right)$ km

$\therefore$ Height of the mountain is

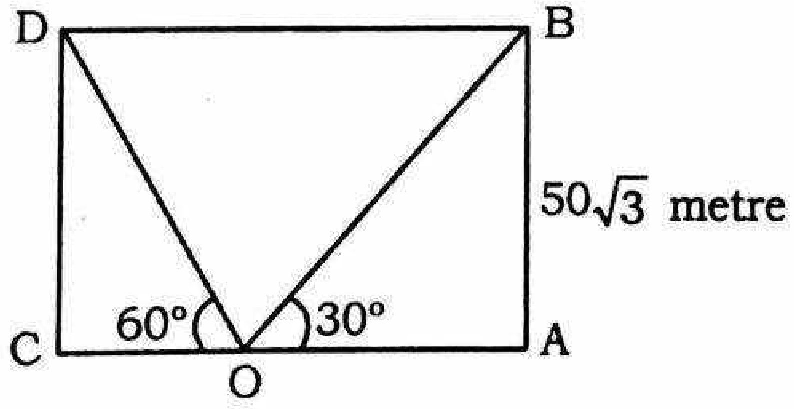
$= 2\left(\sqrt{3}+1\right)$ km

Short-cut :-

$$PQ = AB\left(\frac{\sqrt{3}+1}{2}\right) = 4\left(\frac{\sqrt{3}+1}{2}\right)$$

$$= 2\left(\sqrt{3}+1\right) km$$

6.(d)



$AB = CD = 50\sqrt{3}$ metre

From Δ OAB,

$$\tan 30° = \frac{AB}{OA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{OA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{OA}$$

$\Rightarrow OA = 50\sqrt{3} \times \sqrt{3} = 150$ metre

From Δ OCD,

$$\tan 60° = \frac{CD}{OC}$$

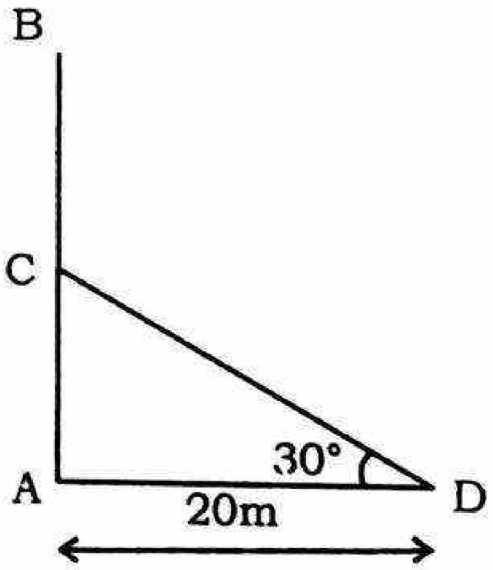
$$\sqrt{3} = \frac{50\sqrt{3}}{OC} \Rightarrow OC = 50 \text{ metre}$$

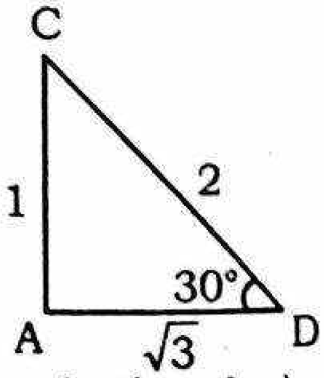
$\therefore BD = AC = 150 + 50 = 200$ metre

$$\therefore \text{Speed of bird} = \frac{200}{2} = 100\text{m/minute}$$

$$= \frac{100}{1000} \times 60 \text{ kmph} = 6 \text{ kmph}$$

7.(a)





(ratio value)

AB = Pole

BC = CD = broken part of pole

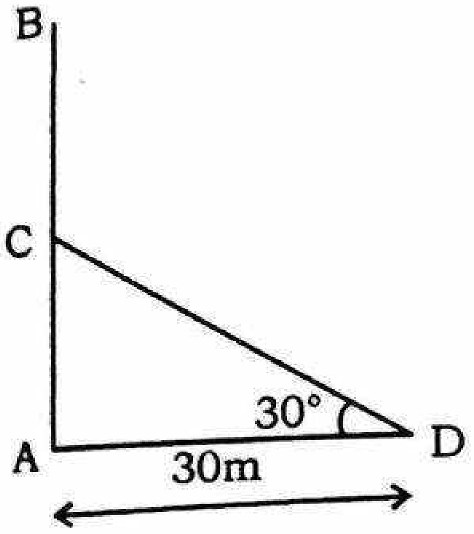
AD = 20 m

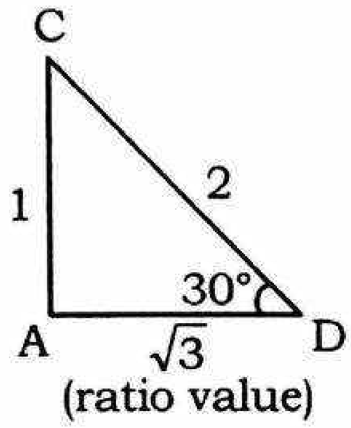
Ratio value	**Original value**
$AD \rightarrow \sqrt{3}$ ⟶	20
$\therefore AC \rightarrow 1$ ⟶	$\frac{20}{\sqrt{3}}$
$\therefore CD \rightarrow 2$ ⟶	$\frac{40}{\sqrt{3}}$

$\therefore AC + CD = \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$ metre

i.e. height of the pole = AB = AC + CD = $20\sqrt{3}$ metre.

8.(b)



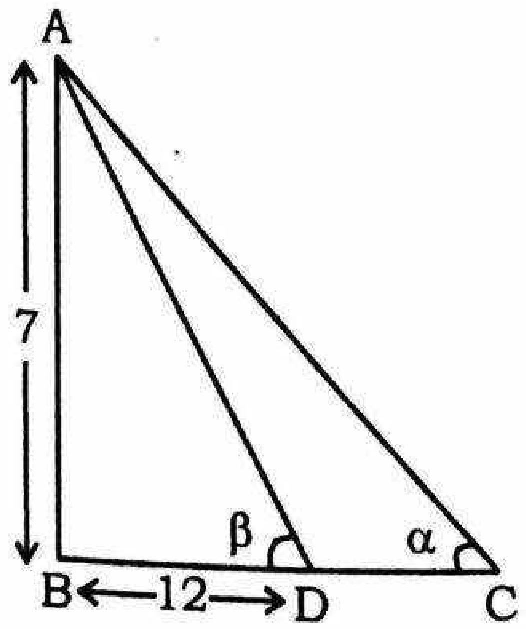


AB = tree
BC = CD = broken part of tree
AD = 30 m

Ratio value		Original value
AD → $\sqrt{3}$	⟶	30
∴ AC → 1	⟶	$\frac{20}{\sqrt{3}} = 10\sqrt{3}$
∴ CD → 2	⟶	$20\sqrt{3}$

Height of the tree = AB = AC + BC = AC + CD = $10\sqrt{3} + 20\sqrt{3} = 30\sqrt{3}$ metre.

9.(c)



AB = monument = h metre
DC = 138 metre

Given that, $\tan\alpha = \frac{1}{5}$

and $\sec\beta = \frac{\sqrt{193}}{12} = \frac{AD}{BD}$

$\therefore AB = \sqrt{AD^2 - BD^2} = \sqrt{193 - 144}$

or $AB = \sqrt{49} = 7$

Also, $\tan\alpha = \frac{1}{5} = \frac{AB}{BC}$

but $AB = 7$ [according to $\sec\beta$]

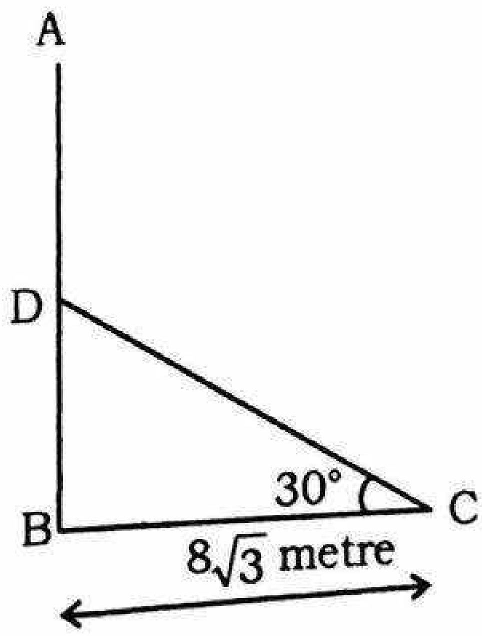
$\therefore BC = 5 \times 7 = 35$ (by ratio)

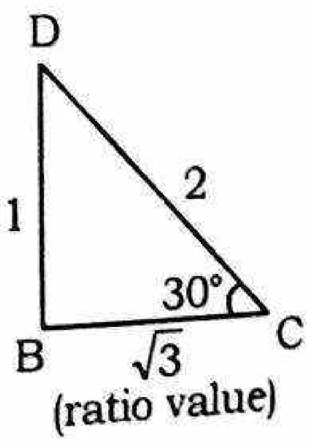
$\therefore DC = BC - BD = 35 - 12 = 27$

Ratio value		Original value
DC → 27	⟶	138
∴ 1	⟶	$\frac{138}{27} = 6$
∴ 7	⟶	42 metres

∴ height of the monument = h(ratio value = 7) = 42 metre.

10.(c)





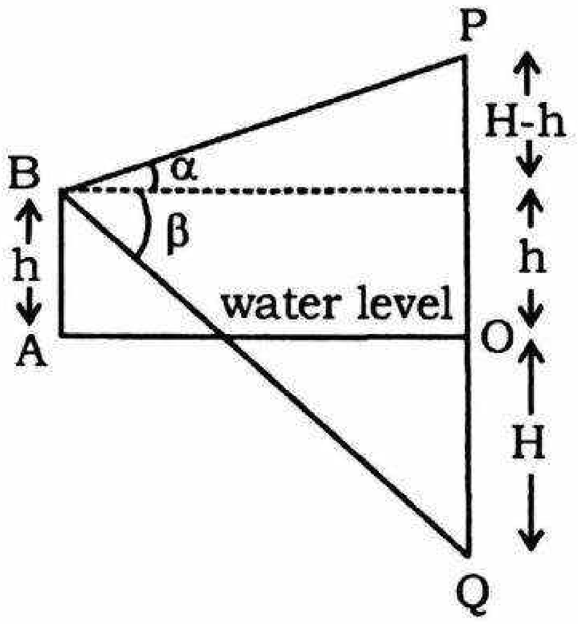
AB = telegraph post
AD = CD = broken part of the post

$BC = 8\sqrt{3}$ metre

Ratio value	**Original value**
$BC \to \sqrt{3}$ ⟶	$8\sqrt{3}$
$\therefore BD \to 1$ ⟶	$\frac{8\sqrt{3}}{\sqrt{3}}$
and $CD \to 2$ ⟶	16

∴ height of the telegraph post = AB
= BD + AD
= BD + CD
= 8 + 16
= 24 metre.

11.(d) Let P be the cloud at height H above the level of the water in the lake Q its image in the water.



$\therefore OQ = OP = H$

B is at a point at a height AB = h, above the water, Angle of elevation of P and depression of Q from B are α and β respectively.

In Δ PBM,

$$\tan\alpha = \frac{H-h}{BM}$$

$$\therefore BM = (H-h)\cot\alpha \quad \text{.........(i)}$$

In Δ QMB,

$$\tan\beta = \frac{QM}{BM}$$

$$\therefore BM = (H+h)\cot\beta \quad \text{..........(ii)}$$

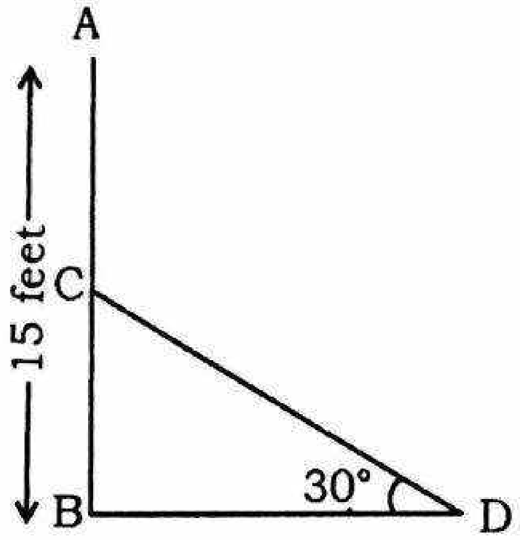
From equations (i) and (ii),

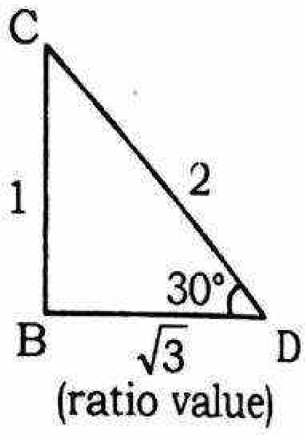
$$(H-h)\cot\alpha = (H+h)\cot\beta$$

$$\Rightarrow H(\cot\alpha - \cot\beta) = h(\cot\alpha + \cot\beta)$$

$$\therefore H = \frac{h(\cot\alpha + \cot\beta)}{\cot\alpha - \cot\beta}$$

12.(b)





AB = Post = 15 feet
AC = CD = Broken part of the post
∵ AC = CD
∴ AB = BC + AC = BC + CD
∴ ratio value of AB = Ratio value of (BC + CD)
= 1 + 2 = 3

Ratio value	**Original value**
$AB \to 3$ ⟶	15
$\therefore BC \to 1$ ⟶	$\frac{15}{3} = 5$ feet

i.e. the required height = BC (ratio value =1) = 5 feet.