CBSE Sample Paper-03 (Solved)
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
d) Use of calculator is not permitted.

Section A

1. The list of numbers $-12, -9, -6, -3, 0, 3, \ldots\ldots\ldots$ is:

   (a) not an AP (b) an AP with $d = 3$
   (c) an AP with $d = -3$ (d) an AP with $d = 0$

2. The height of a tree, if it casts a shadow 17 m long on the level of ground, when the angle of elevation of the Sun is 45°, is:

   (a) 17 m (b) 8 m (c) 10 m (d) 14 m

3. The probability that Apala and Meenu have same birthday (ignoring a leap year) is:

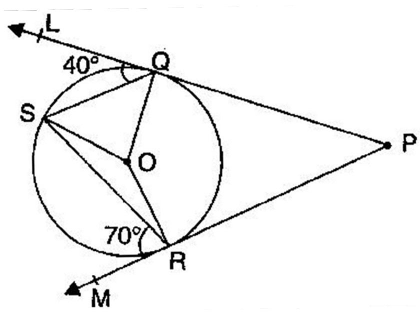
   (a) $\frac{364}{365}$ (b) $\frac{1}{73}$ (c) $\frac{1}{365}$ (d) $\frac{3}{73}$

4. If A$(5, y)$, B (1, 5), C (2, 1) and D (6, 2) are the vertices of a square, then the value of y is:

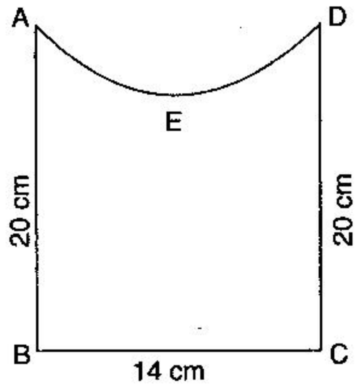
   (a) 1 (b) 3 (c) 2 (d) 6

Section B

5. Find the discriminant of the quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$
6. If 3 times the 3rd term of an AP is equal to 7 times its 7th term, then find its 10th term.
7. In figure, PQL and PRM are tangents to the circle with centre O at the points P and R respectively and S is a point on the circle such that $\angle$SQL = 40° and $\angle$SRM = 70°. Then find $\angle$QSR.



8. Find the perimeter of the figure, where AED is a semi-circle and ABCD is a rectangle.



9. How many spherical lead shots of radius 2 cm can be made out of a solid cube of lead whose edge measures 44 cm?
10. The rain water from a roof 22 m x 20 m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m of the vessel is just full, find the rainfall in cm.

Section C

11. Solve the quadratic equation: $4x^2+4\sqrt{3}x+3=0$
12. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th term is 44. Find the first three terms of the AP.
13. A circle touching the side BC of a ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that:

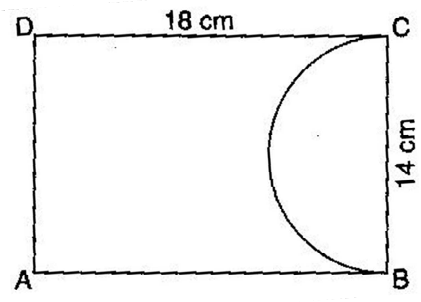
$$AQ = \frac{1}{2} \text{ (Perimeter of } \Delta ABC)$$

14. The length of a shadow of a tower standing on level plane is found to be 20 m longer when the Sun's altitude is 30°, than when it was 60°. Find the height of the tower.

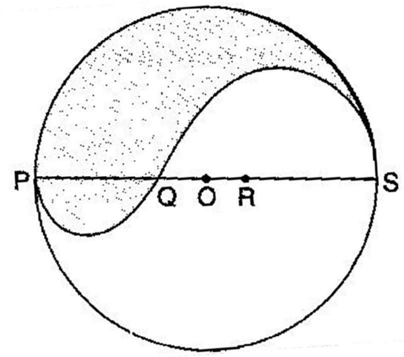
15. All the three face cards of spade are removed from a well shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting:
    (i) a black face card.
    (ii) a queen'
    (iii) a black card.
16. If the point $C(-1,2)$ divides the line segment AB in the ratio 3 : 4, where the coordinates of A are (2, 5), then find the coordinates of B.

17. Find the coordinates of the point equidistant from thee given points A (5, 1), $B(-3,-7)$ and C $(7,-1)$.

18. A paper is in the form of a rectangle ABCD with AB = 18 cm and BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining paper.



19. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in figure. Find the perimeter of the shaded region.



20. The radius of the base and the height of a solid right circular cylinder are in the ratio 2 : 3 and its volume is 1617 cm^3. Find the total surface area of the cylinder.

Section D

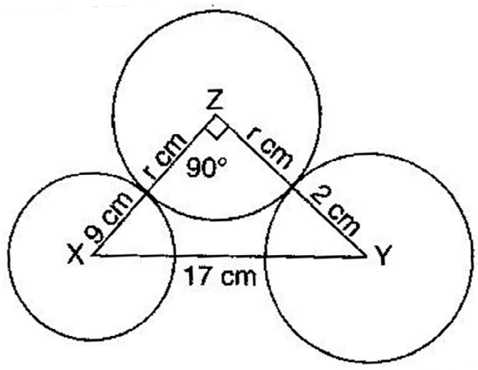
21. A motorboat whose speed is 9 km/h in still water goes 12 km downstream and comes back in a total time of 3 h.
    (i) Find the speed of the stream.
    (ii) Explain the situation when speed of the stream is more than the speed of the boat in still water.

[Value Based Question]

22. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

23. How many terms of the series 54, 51, 48, …….. be taken, so that their sum is 513? Explain the double answer.

24. X and Y are centres of circles of radius 9 cm and 2 cm and XY = 17 cm. Z is the centre of a circle of radius r cm, which touches the above circles externally. Given that $\angle XZY = 90^\circ$, write an equation in r and solve it for r.



25. Prove that the lengths of tangents drawn from an external point to a circle are equal.
Using the above result, prove the following:
If a circle touches all the four sides of a parallelogram, show that the parallelogram is a rhombus.
26. Construct a triangle ABC in which AB = 6.5 cm, $\angle B = 60^\circ$ and BC = 5.5 cm. Also construct a triangle AB'C' similar to ΔABC, whose each side is $\frac{3}{2}$ times the corresponding side of ΔABC.
27. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of the hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.
28. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is:
(i) a card of spades or an ace
(ii) a red king
(iii) neither a king nor a queen
(iv) either a king or a queen
29. Prove that the coordinates of the centroid of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and C (x_3, y_3) are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.
30. A right-angled triangle whose sides are 15 cm and 20 cm is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. $(\text{Use } \pi = 3.14)$
31. A bucket made up of a metal sheet in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the bucket, if the cost of the metal sheet used is Rs.15 per 100 cm^2. $(\text{Use } \pi = 3.14)$

CBSE Sample Paper-03 (Solved)
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

(Solutions)

SECTION-A

1. (b)
2. (a)
3. (c)
4. (d)
5. Here, $a = 3\sqrt{3}, b = 10, c = \sqrt{3}$

   Discriminant $= b^2 - 4ac$

   $= (10)^2 - 4(3\sqrt{3}).(\sqrt{3})$

   $= 100 - 36$

   $= 64$
6. According to question, $3a_3 = 7a_7$

   $\Rightarrow \quad 3(a+2d) = 7(a+6d)$

   $\Rightarrow \quad 3a + 6d = 7a + 42d$

   $\Rightarrow \quad 4a + 36d = 0$

   $\Rightarrow \quad 4(a+9d) = 0$

   $\Rightarrow \quad a + 9d = 0 \quad \Rightarrow \quad a_{10} = 0$
7. $\angle$OQS = $\angle$OQL – $\angle$SQL = $90° - 40° = 50°$

   $\Rightarrow \quad \angle$OQS = $50°$ ……….(i)

   $\angle$ORS = $\angle$ORM – $\angle$SQM = $90° - 70° = 20°$ ……….(ii)

   $\therefore \quad \angle$OSQ = $\angle$OQS = $50°$ ……….(iii) [From eq. (i)]

   $\because \quad$ OS = OR

   $\therefore \quad \angle$OSR = $\angle$ORS = $20°$ ………(iv) [From eq. (ii)]

   $\therefore \quad \angle$QSR = $\angle$ORS + $\angle$OSR

   $= 50° + 20° = 70°$
8. $r = \frac{14}{2}$ = 7 cm

   Length of arc = $\frac{\theta}{360°} 2\pi r$

   $\Rightarrow \quad \overset{\frown}{AED} = \frac{180°}{360°} \times 2 \times \frac{22}{7} \times 7$ = 22 cm

   $\therefore \quad$ Perimeter of given figure = 20 + 14 + 20 + 22= 76 cm

9. Let n spherical lead shots be made, then

According to question,

$n\times$ Volume of spherical lead shot = Volume of Solid Cube

$\Rightarrow \quad n.\frac{4}{3}\pi r^3 = (a)^3$

$\Rightarrow \quad n.\frac{4}{3}\times\frac{22}{7}\times(2)^3 = 44\times44\times44$

$\Rightarrow \quad n = \frac{44\times44\times44\times7\times3}{4\times22\times8}$ = 2541

10. Let the rainfall be x cm.

Volume of rain water = Volume of cylindrical vessel

$\Rightarrow \quad lbx = \pi r^2 h$

$\Rightarrow \quad 22\times20\times x = \frac{22}{7}\times1\times1\times3.5$

$\Rightarrow \quad x = \frac{22\times1\times3.5}{7\times22\times20}$ = 2.5 cm

11. $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow \quad (2x)^2 + 2(2x)(\sqrt{3}) + (\sqrt{3}) = 0$

$\Rightarrow \quad (2x+\sqrt{3})^2 = 0$

$\Rightarrow \quad (2x+\sqrt{3})(2x+\sqrt{3}) = 0$

$\Rightarrow \quad 2x = -\sqrt{3}, 2x = -\sqrt{3}$

$\Rightarrow \quad x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

12. According to question, $a_4 + a_8 = 24$

$\Rightarrow \quad (a+3d)+(a+7d) = 24$

$\Rightarrow \quad 2a+10d = 24$

$\Rightarrow \quad a+5d = 12$ ....(i)

Again, $a_6 + a_{10} = 44$

$\Rightarrow \quad (a+5d)+(a+9d) = 44$

$\Rightarrow \quad 2a+14d = 44$

$\Rightarrow \quad a+7d = 22$ ....(ii)

Solving eq. (i) and eq. (ii), we get,

$a = -13, \quad d = 5$

∴ First three terms are $-13, -8, -3$.

13. ∵ Tangent segments from an external point to a circle are equal in length.

∴ AQ = AR   BP = BQ   CP = CR

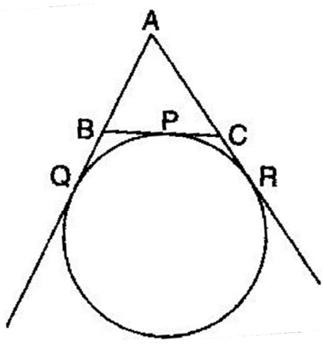
∵ Perimeter of ΔABC = AB + BC + AC

= AB + BP + CP + AC

= AB + BQ + CR + AC

= AQ + AR = AQ + AQ = 2AQ

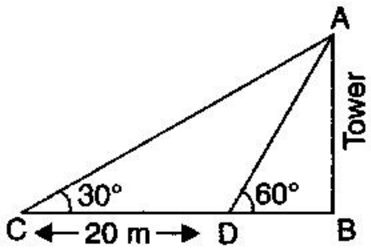
⇒ $AQ = \frac{1}{2}$ (Perimeter of ΔABC)



14. In right triangle ABD,

$$\tan 60^\circ = \frac{AB}{BD}$$

⇒ $\sqrt{3} = \frac{AB}{BD}$ ....(i)



In right triangle ABC,

$\tan 30^\circ = \frac{AB}{BC}$ ⇒ $\frac{1}{\sqrt{3}} = \frac{AB}{BD + 20}$

⇒ $\frac{1}{\sqrt{3}} = \frac{AB}{\frac{AB}{\sqrt{3}} + 20}$ [From eq. (i)]

⇒ AB = 17.32 m

15. There are 52 cards in a pack, out of which three cards are removed.

∴ Total number of possible outcome = 52 – 3 = 49

(i) Number of favourable outcomes = 3

Hence, Required probability = $\frac{3}{49}$

(ii) Number of favourable outcomes = 3

Hence required probability = $\frac{3}{49}$

(iii) Number of favourable outcomes = 23

Therefore there are 9 cards marked with the numbers which are perfect squares.

Hence, required probability = $\frac{23}{49}$

16. Let $B \rightarrow (x, y)$. Then

$$\frac{(3)(x)+(4)(2)}{3+4} = -1 \quad \Rightarrow \quad \frac{3x+8}{7} = -1$$

$$\Rightarrow \quad 3x+8 = -7 \quad \Rightarrow \quad x = -5$$

Also, $$\frac{(3)(y)+(4)(5)}{3+4} = 2 \quad \Rightarrow \quad \frac{3y+20}{7} = 2$$

$$\Rightarrow \quad 3y+20 = 14 \quad \Rightarrow \quad y = -2$$

17. Let the required point be $P(x, y)$.

Then PA = PB = PC

$\Rightarrow$ $PA^2 = PB^2 = PC^2$

$\Rightarrow$ $(x-5)^2 + (y-1)^2 = (x+3)^2 + (y+7)^2 = (x-7)^2 + (y+1)^2$

Taking, $(x-5)^2 + (y-1)^2 = (x+3)^2 + (y+7)^2$

$\Rightarrow$ $x + y = -2$ ....(i)

Taking, $(x+3)^2 + (y+7)^2 = (x-7)^2 + (y+1)^2$

$\Rightarrow$ $5x + 3y = -2$ ....(ii)

Solving eq. (i) and eq. (ii), we get

$x = 2, y = -4$

18. Area of remaining paper =

Area of rectangle – Area of semi-circle

= $l \times b - \frac{1}{2}\pi r^2$

= $18 \times 14 - \frac{1}{2} \times \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2}$

= 252 – 77

= 175 cm^2

19. PQ = QR = RS = $\frac{2 \times 6}{3}$ = 4 cm

$\therefore$ Perimeter of the shaded region = $\overset{\frown}{PQ} + \overset{\frown}{PS} + \overset{\frown}{QS}$

= $\pi \times 2 + \pi \times 6 + \pi \times 4$

= 12π cm

20. Let the radius and height be $2k$ cm and $3k$ cm respectively. Then,

Volume of cylinder = $\pi r^2 h$

$\Rightarrow$ 1617 = $\pi (2k)^2 (3k)$

$\Rightarrow \quad k = \frac{7}{2}$ cm

$\therefore \quad r =$ 7 cm and $h = \frac{21}{2}$ cm

$\therefore$ Total surface area of cylinder = $2\pi r\left(r+h\right)$

$= 2\times\frac{22}{7}\times 7\left(7+\frac{21}{2}\right)$

= 770 cm^2

21. (i) Let speed of the stream be x km/h

Speed of motorboat = 9 km/h

Speed of the motorboat to cover 12 km in downstream = $\left(9+x\right)$ km/h

Speed of the motorboat to cover 12 km in upstream = $\left(9-x\right)$ km/h

Time taken to cover 12 km in downstream = $\frac{12}{\left(9+x\right)}$ h

Time taken to cover 12 km in upstream = $\frac{12}{\left(9-x\right)}$ h

According to the question,

$$\frac{12}{9+x}+\frac{12}{9-x}=3$$

$\Rightarrow \quad 12\left[\frac{9-x+9+x}{81-x^2}\right]=3 \qquad \Rightarrow \quad 12\left[\frac{18}{81-x^2}\right]=3$

$\Rightarrow \quad 81-x^2=\frac{12\times 18}{3} \qquad \Rightarrow \quad -x^2=72-81$

$\Rightarrow \quad x^2=9 \qquad \Rightarrow \quad x=\pm 3$

Since the speed of stream cannot be negative.

$\therefore$ the speed of stream is 3 km/h.

(ii) Logically if speed of stream is more than the speed of boat in still water, then the boat will not sail.

22. Let the speed of the train be x km/h.

Time taken by the train with this speed for a journey of 360 km = $\frac{360}{x}$ h

Increased speed of the train = $\left(x+5\right)$ km/h

Time taken by train with increased speed for a journey of 360 km = $\frac{360}{x+5}$ h

According to the question,

$$\frac{360}{x}-\frac{360}{x+5}=1$$

$\Rightarrow \quad 360\left(\frac{x+5-x}{x^2+5x}\right)=1 \quad \Rightarrow \quad x^2+5x=1800$

$\Rightarrow \quad x^2+5x-1800=0 \quad \Rightarrow \quad x^2+45x-40x-1800=0$

$\Rightarrow \quad x(x+45)-40(x+45)=0 \quad \Rightarrow \quad (x+45)(x-40)=0 \quad \Rightarrow \quad x=-45, 40$

$x=-45$ is inadmissible as x is the speed which cannot be negative.

Hence the speed of the train is 40 km/h.

23. $a=54, d=-3, S_n=513$

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$\Rightarrow \quad 513 = \frac{n}{2}\left[108+(n-1)(-3)\right]$

$\Rightarrow \quad n^2-37n+342=0$

$\Rightarrow \quad (n-18)(n-19)=0$

$\Rightarrow \quad n$ = 18 or 19

Explanation

$a_{19}=a+18d=54+8(-3)=0$

24. $\because \quad \angle XZY = 90^\circ$

$\therefore \quad (9+r)^2+(2+r)^2=(17)^2$ [By Pythagoras theorem]

$\Rightarrow \quad r^2+11r-102=0$

$\Rightarrow \quad r^2+17r-6r-102=0$

$\Rightarrow \quad r(r+17)-6(r+17)=0$

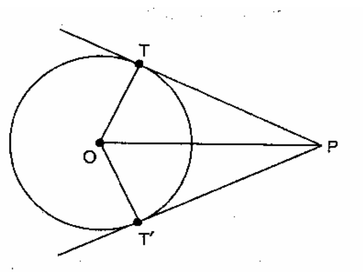
$\Rightarrow \quad (r+17)(r-6)=0$

$\Rightarrow \quad r=-17, 6$

Radius cannot be negative.

Hence r = 6 cm

25. **First part**: Given : A circle with centre O and a point P outside the circle. PT and PT' are tangents from P to the circle.



To Prove : We need to prove that PT = PT'

Construction : Joined OP, OT and OT'

Proof : $\because$ OT is a radius and PT is a tangent.

$\therefore \quad \angle OTP = 90^\circ$

Similarly, $\angle OT'P = 90^\circ$

Now in right triangles OTP and OT'P,

OT = OT' ……. (Radii of the same circle)

and OP = OP ……..(Common)

$\therefore \quad \Delta OTP \cong \Delta OT'P$ ……..(RHS congruency)

Hence, PT = PT’

Second part: Using the above, we get,

AP = AS, BP = BQ, CR = CQ, DR = DS

On adding, we get,

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD + AD + BC

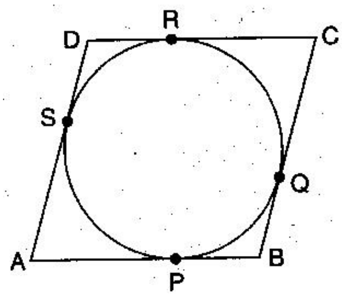
$\Rightarrow$ 2AB = 2BC [$\because$ ABCD is a parallelogram]

$\Rightarrow$ AB = BC

$\Rightarrow$ AB = BC = CD = DA

[$\because$ ABCD is a parallelogram]

$\Rightarrow$ ABCD is a rhombus.



26. **Steps of construction**:

(a) Draw a right angled triangle ABC with given measurements.

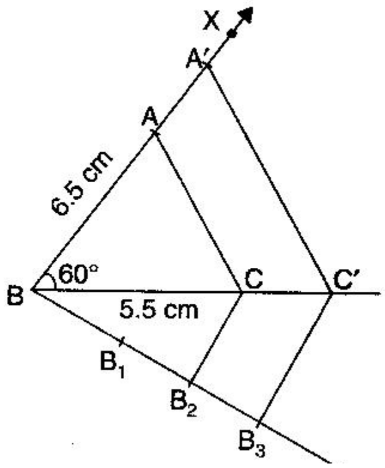
(b) Draw any ray BY making an acute angle with BC on the side opposite to the vertex A.

(c) Locate 3 points B_1, B_2 and B_3 on BY so that $BB_1 = B_1B_2 = B_2B_3$.

(d) Join B_2 to C and draw a line through B_3 parallel to B_2C, intersecting the extended line segment BC at C’.

(e) Draw a line through C’ parallel to CA intersecting the extended line segment BA at A’.

The A’BC’ is the required triangle.



27. In right triangle AED,

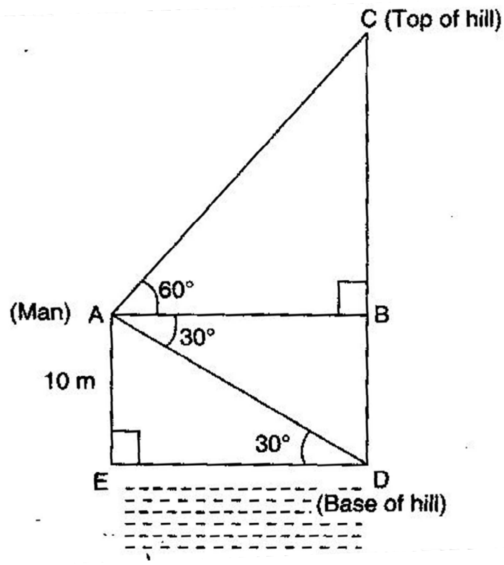
$$\tan 30^\circ = \frac{AE}{ED}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{ED}$$

$\Rightarrow$ ED = $10\sqrt{3}$ = 10 x 1.73 = 17.3 m

In right triangle ABC,

$$\tan 60^\circ = \frac{CB}{AB}$$



$\Rightarrow \quad \sqrt{3} = \frac{CB}{ED}$

$\Rightarrow \quad \sqrt{3} = \frac{CB}{10\sqrt{3}}$

$\Rightarrow \quad CB = 30$ m

$\therefore \quad CD = CB + BD = CB + AE = 30 + 10 = 40$ m

28. Total number of cards in the deck = 52

$\therefore$ Number of all possible outcomes = 52

(i) Number of cards of spades or ace = 13 + 4 – 1 = 16

$\therefore$ Required probability = $\frac{16}{52} = \frac{4}{13}$

(ii) Number of a red king = 2

$\therefore$ Required probability = $\frac{2}{52} = \frac{1}{26}$

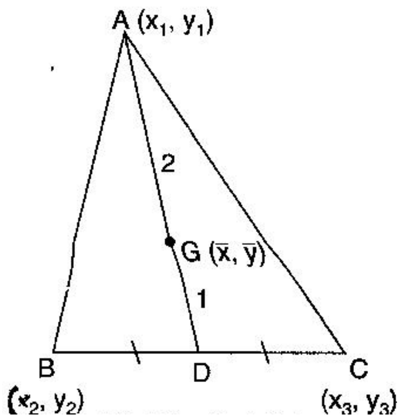
(iii) Number of neither a king nor a queen = 52 – (4 + 4) = 44

$\therefore$ Required probability = $\frac{44}{52} = \frac{11}{13}$

(iv) Number of either a king or a queen = (4 + 4) = 8

$\therefore$ Required probability = $\frac{8}{52} = \frac{2}{13}$

29. $\because$ D is the mid-point of BC.



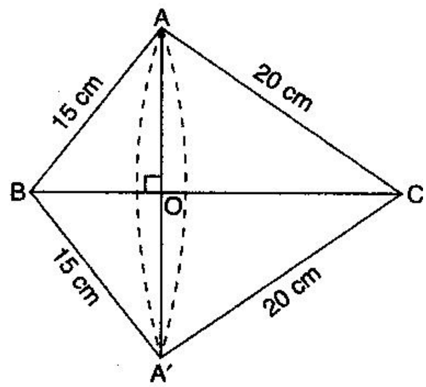
$\therefore \quad D \rightarrow \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

G divides AD internally in the ratio 2 : 1.

$\therefore \quad \bar{x} = \frac{2\left(\frac{x_2 + x_3}{2}\right) + 1(x_1)}{2+1} = \frac{x_1 + x_2 + x_3}{3}$

And $\quad \bar{y} = \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1(y_1)}{2+1} = \frac{y_1 + y_2 + y_3}{3}$

30. BC = $\sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625} = 25$ cm



Δ AOB ~ Δ CAB [AA similarity criterian]

$\therefore \quad \frac{AO}{20} = \frac{15}{25}$

$\Rightarrow \quad AO = 12$ cm

And $\quad \frac{BO}{15} = \frac{15}{25}$

$\Rightarrow \quad BO = 9$ cm

$\Rightarrow$ CO = 25 – 9 = 16 cm

Now, Volume of the double cone = $\frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r_1^2 h_1$

= $\frac{1}{3}\times 3.14\times(12)^2\times 9+\frac{1}{3}\times 3.14\times(12)^2\times 16$

= $\frac{1}{3}\times 3.14\times 144(9+16)$

= $3.14\times 48\times 25$ = 3768 cm^3

Surface area of the double cone = $\pi rl + \pi r_1 l_1$

= 3.14 x 12 x 15 + 3.14 x 12 x 20

= 3.14 x 12 (15 + 20)

= 3.14 x 12 x 35 = 1318.8 cm^2

31. h = 16 cm, r_1 = 20 cm, r_2 = 8 cm

$l=\sqrt{h^2+(r_1-r_2)^2}$ = $\sqrt{(16)^2+(20-8)^2}$ = $\sqrt{256+144}$ = 20 cm

Total surface area of bucket = $\pi(r_1+r_2)l+\pi r_2^2$

= $3.14(20+8)20+3.14(8)^2$

= 3.14 (28 x 20 + 64)

= 3.14 (560 + 64)

= 3.14 x 624

= 1959.36 cm^2

$\therefore$ Cost of bucket = $\frac{15}{100}\times 1959.36$

= Rs. 293.90 (approx.)