Chapter 5
Continuity and Differentiability

Exercise 5.3

Q. 1n Find dy /dx in the following:

$2x + 3y = \sin x$

Answer:

It is given that $2x + 3y = \sin x$

Differentiating both sides w.r.t. x, we get,

$= \frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx}(\sin x)$

$= 2 + 3\frac{dy}{dx} = \cos x$

$= 3\frac{dy}{dx} = \cos x - 2$

$= \frac{dy}{dx} = \frac{\cos x - 2}{3}$

Q. 2 Find dy/dx in the following:

$2x + 3y = \sin y$

Answer:

It is given that $2x + 3y = \sin y$

Differentiating both sides w.r.t. x, we get,

$= \frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx}(\sin y)$

$= 2 + 3\frac{dy}{dx} = \cos y\ \frac{dy}{dx}$

$= 2 = (\cos y - 3)\frac{dy}{dx}$

$= \frac{dy}{dx} = \frac{2}{(\cos y - 3)}$

Q. 3 Find dy/dx in the following:

$ax + by^2 = \cos y$

Answer:

It is given that $ax + by^2 = \cos y$

Differentiating both sides w.r.t. x, we get,

$\frac{d}{dx}(ax + by^2) = \frac{d}{dx}(\cos y)$

$= \frac{d}{dx}(ax) + \frac{d}{dx}(by^2) = \frac{d}{dx}(\cos y)$

$= a + b\frac{d}{dx}(y^2) = \frac{d}{dx}(\cos y)$

$= a + b \times 2y\frac{dy}{dx} = -\sin y\frac{dy}{dx}$

$= (2by + \sin y)\frac{dy}{dx} = -a$

$= \frac{dy}{dx} = \frac{-a}{(2by + siny)}$

Q. 4 Find dy/dx in the following:

$xy + y^2 = \tan x + y$

Answer:

It is given that $xy + y^2 = \tan x + y$

Differentiating both sides w.r.t. x, we get,

$\frac{d}{dx}(xy + y^2) = \frac{d}{dx}(\tan x + y)$

$= \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(\tan x) + \frac{dy}{dx}$

$= \left[y\frac{dy}{dx}(x) + x\frac{dy}{dx}\right] + 2y\frac{dy}{dx} = \sec^2 + \frac{dy}{dx}$

$= y.\, 1 + x\frac{dy}{dx} + 2y\,\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$

$= (x + 2y - 1)\,\frac{dy}{dx} = \sec 2 - y$

$= \frac{dy}{dx} = \frac{sec^2 x - y}{(x+2y-1)}$

Q. 5 Find dy/dx in the following:

$x^2 + xy + y^2 = 100$

Answer:

It is given that $x^2 + xy + y^2 = 100$

Differentiating both sides w.r.t. x, we get,

$\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}(100)$

$= \frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 0$

$= 2x + \left[y\frac{d}{dx}(x) + x\,\frac{dy}{dx}\right] + 2y\,\frac{dy}{dx} = 0$

$= 2x + y.1 + \text{x}\,\frac{dy}{dx} + 2y\,\frac{dy}{dx} = 0$

$= 2x + y + (x + 2y)\,\frac{dy}{dx} = 0$

$= \frac{dy}{dx} = -\frac{2x+y}{x+2y}$

Q. 6 Find dy/dx in the following:

$x^3 + x^2y + xy^2 + y^3 = 81$

Answer:

It is given that $x^3 + x^2y + xy^2 + y^3 = 81$

Differentiating both sides w.r.t. x, we get,

$\frac{d}{dx}(x^3 + x^2y + xy^2 + y^3) = \frac{d}{dx}(81)$

$= \frac{d}{dx}(x^3) + \frac{d}{dx}(x^2y) + \frac{d}{dx}(xy^2) + \frac{d}{dx}(y^3) = 0$

$= 3x^2 + \left[y\frac{d}{dx}(x^2) + x^2\frac{d}{dx}\right] + \left[y^2\frac{d}{dx}(x) + x\frac{d}{dx}(y^2)\right] + 3y^2\frac{dy}{dx} = 0$

$= 3x^2 + \left[y.2x + x^2\frac{dy}{dx}\right] + \left[y^2.1 + x.2y.\frac{dy}{dx}\right] + 3y^2\frac{dy}{dx} = 0$

$= (x^2 + 2xy + 3y^2)\frac{dy}{dx} + (3x^2 + 2xy + y^2) = 0$

$= \frac{dy}{dx} = \frac{-(3x^2 + 2xy + y^2)}{(x^2 + 2xy + 3y^2)}$

Q. 7

Find dy/dx in the following:

$\sin 2\,y + \cos xy = \pi$

Answer:

It is given that $\sin 2\,y + \cos xy = \pi$

Differentiating both sides w.r.t. x, we get,

$\frac{d}{dx}(\sin 2y + \cos xy) = \frac{d}{dx}(\pi)$

$= 2\sin y\cos y\frac{dy}{dx} - \sin xy\left[y\frac{d}{dx}(x) + x\frac{dy}{dx}\right]$

$= 2\sin y\cos y\frac{dy}{dx} - \sin xy\left[y.1 + x\frac{dy}{dx}\right] = 0$

$= 2\sin y\cos y\frac{dy}{dx} - y\sin xy - x\sin x\,y\frac{dy}{dx} = 0$

$= (2\sin y\cos y - x\sin xy)\frac{dy}{dx} = y\sin xy$

$= (\sin 2y - x\sin xy)\frac{dy}{dx} = y\sin xy$

$= \frac{dy}{dx} = \frac{y\,sinxy}{(sin2y - xsinxy)}$

Q. 8

Find *dy*/d*x* in the following:

sin2 *x* + cos2 *y* = 1

Answer:

It is given that sin2 *x* + cos2 *y* = 1

Differentiating both sides w.r.t. *x*, we get,

$\frac{d}{dx}$ (sin2*x* + cos2*y*) = $\frac{d}{dx}$ (1)

= $\frac{d}{dx}$ (sin2*x*) + $\frac{d}{dx}$ (cos2*x*) = 0

= 2 sin *x*. $\frac{d}{dx}$ (sin *x*) + 2 cos *y*. $\frac{d}{dx}$ (cos *y*) = 0

= 2 sin *x* cos *x* + 2 cos *y* (- sin *y*). $\frac{dy}{dx}$ = 0

= sin 2*x* – sin 2*y* $\frac{dy}{dx}$ = 0

= $\frac{dy}{dx} = \frac{sin2x}{sin2y}$

Q. 9

Find dy/dx in the following:

y = sin^{-1} $\left(\frac{2x}{1+x^2}\right)$

Answer:

Let *x* = tan A

then, A = tan^{-1}x

= $\frac{dA}{DX} = \frac{1}{1+x^2}$

y = sin^{-1} $\left(\frac{2\tan A}{1+tan^2 A}\right)$

also, we know $\left[\sin 2A = \frac{2\tan A}{1+tan^2 A}\right]$

And = y = $\sin^{-1}$ (sin 2A)

= y = 2A

= $\frac{dy}{dx} = 2\frac{dA}{dx}$ [by chain rule[

= $\frac{dy}{dx} = \frac{2}{1+x^2}$

Q. 10

Find dy/dx in the following:

y = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Answer:

It is given that:

y = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Assumption: Let x = tan θ, putting it in y, we get,

y = $\tan^{-1}\left(\frac{3\tan\theta - tan^3\theta}{1-3tan^2\theta}\right)$

we know by the formula that, tan 3x = $\frac{3\tan x - tan^3 x}{1-3tan^2 x}$

Putting this in y, we get, y = $\tan^{-1}$(tan3θ)

y = 3($\tan^{-1}$x)

Differentiating both sides, we get, $\frac{dy}{dx} = \frac{3}{1+x^2}$

Q. 11 Find dy/dx in the following:

y = $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Answer:

It is given that,

$y =$ $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \cos y = \frac{1-x^2}{1+x^2}$

$= \frac{1-tan^2\frac{y}{2}}{1+tan^2\frac{y}{2}} = \frac{1-x^2}{1+x^2}$

On comparing both sides, we get,

$\tan\frac{y}{2} = x$

Now, differentiating both sides, we get,

$\text{Sec}^2\left(\frac{y}{2}\right).\frac{d}{dx}\left(\frac{y}{2}\right) = \frac{d}{dx}(x)$

$= \sec^2\frac{y}{2} \times \frac{1}{2}\frac{dy}{dx} = 1$

$= \frac{dy}{dx} = \frac{2}{sec^2\frac{y}{2}}$

$= \frac{dy}{dx} = \frac{2}{1+tan^2\frac{y}{2}}$

$= \frac{dy}{dx} = \frac{2}{1+x^2}$

Q. 12 Find dy/dx in the following: $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

Answer:

It is given that $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \sin y = \frac{1-x^2}{1+x^2}$

$= (1 + x^2) \sin y = 1 - x^2$

$= (1 + \sin y)\, x^2 = 1 - \sin y$

$= x^2 = \frac{1-\sin y}{1+\sin y}$

Now, we can change the numerator and the denominator,

$1 = \sin 2\frac{y}{2} + \cos 2\frac{y}{2}$

We know that we can write, and

Sin $y = 2\sin\frac{y}{2}.\ \text{Cos}\frac{y}{2}$

Therefore, by applying the formula: $(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$, we get,

$= x^2 = \frac{\left(\cos\frac{y}{2}-\sin\frac{y}{2}\right)^2}{\left(\cos\frac{y}{2}+\sin\frac{y}{2}\right)^2}$

$= x = \frac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}$

Dividing the numerator and denominator by cos (y/2), we get,

$= x\ \frac{1-\tan\frac{y}{2}}{1+\tan\frac{y}{2}}$

Now, we know that:

$\tan(A - B) = \frac{\tan A-\tan B}{1+\tan A.\tan B}$

$= x\tan\left(\frac{\pi}{4} - \frac{y}{2}\right)$

Now, differentiating both sides, we get,

$\frac{d}{dx}(x) = \frac{d}{dx}\left(\tan\left(\frac{\pi}{4} - \frac{y}{2}\right)\right)$

$= 1 = \sec^2\left(\frac{\pi}{2} - \frac{y}{2}\right) \times \frac{d}{dx}\left(\frac{\pi}{4} - \frac{y}{2}\right)$

$= 1 = \left[1 + tan^2\left(\frac{\pi}{4} - \frac{y}{2}\right)\right].\left(-\frac{1}{2}\frac{dy}{dx}\right)$

$= 1 = [1 + x^2].\left(-\frac{1}{2}\frac{dy}{dx}\right)$

$= \frac{dy}{dx} = \frac{-2}{1+x^2}$

Q. 13

Find dy/dx in the following:

$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

Answer:

It is given that $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

$= \cos y = \frac{2x}{1+x^2}$

Differentiating both sides w.r.t. x, we get,

$-\text{Sin } y\frac{dy}{dx} = \frac{(1+x^2).\frac{d}{dx}(2x) - 2x.\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$

$= \sqrt{1 - cos^2 y}\frac{dy}{dx} = \frac{(1+x^2)\times 2 - 2x.2x}{(1+x^2)^2}$

$= \sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}\frac{dy}{dx} = \left[\frac{(1-x^2)}{(1+x^2)}\right]$

$= \sqrt{\frac{(1-x^2)^2 - 4x^2}{(1+x^2)^2}}\frac{dy}{dx} = \frac{-2(1-x^2)}{(1+x^2)^2}$

$= \sqrt{\frac{(1-x^2)^2}{(1+x^2)^2}}\frac{dy}{dx} = \frac{-2(1-x^2)}{(1+x^2)^2}$

$= \frac{1-x^2}{1+x^2}\frac{dy}{dx} = \frac{-2(1-x^2)}{(1+x^2)^2}$

$= \frac{dy}{dx} = \frac{-2}{1+x^2}$

Q. 14 Find dy/dx in the following:

y = sin^-1^ $\left(2x\sqrt{1-x^2}\right), -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Answer:

It is given that y = $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

= $\sin y = 2x\sqrt{1-x^2}$

Differentiating both sides w.r.t. x, we get,

$\text{Cos } y\frac{dy}{dx} = 2\left[x\frac{d}{dx}\left(\sqrt{1-x^2}\right) + \sqrt{1-x^2}\frac{dy}{dx}\right]$

$= \sqrt{1-sin^2y}\frac{dy}{dx} = 2\left[\frac{x}{2}.\frac{-2x}{\sqrt{1-x^2}} + \sqrt{1+x^2}\right]$

$= \sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}\frac{dy}{dx} = 2\left[\frac{-x^2+1-x^2}{\sqrt{1-x^2}}\right]$

$= \sqrt{1-4x^2(1-x^2)}\frac{dy}{dx} = 2\left[\frac{1-2x^2}{\sqrt{1-x^2}}\right]$

$= (1 - 2x2)\frac{dy}{dx} = 2\left[\frac{1-2x^2}{\sqrt{1-x^2}}\right]$

$= \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

Q. 15 Find dy/dx in the following:

$y = \sec^{-1}\left(\frac{1}{2x^2+1}\right), 0 < x < \frac{1}{\sqrt{2}}$

Answer:

It is given that $y = \sec^{-1}\left(\frac{1}{2x^2+1}\right)$

$= \sec y = \frac{1}{2x^2+1}$

$= \cos y = 2x^2 + 1$

$= 2x^2 = 1 + \cos y$

$= 2x^2 = 2 \cos2 \frac{y}{2}$

$= x = \cos \frac{y}{2}$

Differentiating w.r.t. *x*, we get,

$\frac{d}{dx}(x) = \frac{d}{dx}\left(\cos\frac{y}{2}\right)$

$= 1 = -\sin\frac{y}{2} . \frac{d}{dx}\left(\frac{y}{2}\right)$

$= \frac{-1}{\sin\frac{y}{2}} = \frac{1}{2}\frac{dy}{dx}$

$= \frac{dy}{dx} = \frac{-2}{\sin\frac{y}{2}} = \frac{-2}{\sqrt{1-cos^2\frac{y}{2}}}$

$= \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$