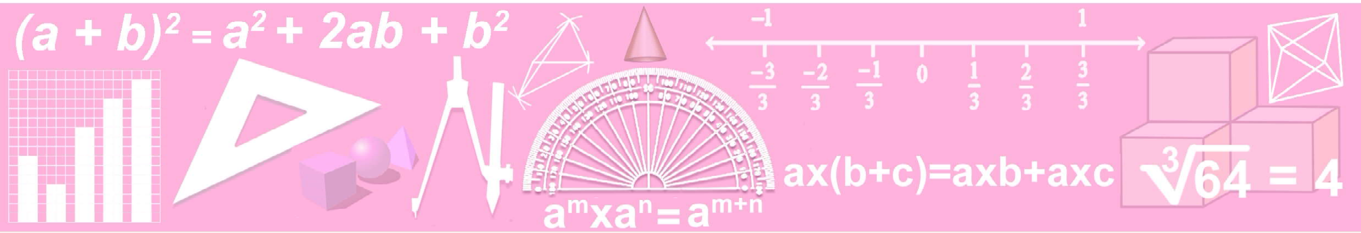

Chapter-12

Exponents and Powers

In the class VII, you have already learnt about exponent and powers.

Try to remember, the meaning of 2^5 !

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$
$$= 32$$

2^5 → Exponent

2^5 → Base

2^5 is Exponent form of 32

Here 2 is multiplied 5 times. Here 2 is called base. What is the power of 2? Can you tell? Here, power of 2 is raised to 5 i.e., it is 2^5. 5 is exponent of 2.

Now think of the meaning of 2^{-5}. Can you say that 2 is multiplied by (–5) times. Definitely not. Because, 2 multiplied by itself (–5) times is meaningless. Here exponent is a negative number.

2^{-5} → Exponent

2^{-5} → Base

In this lesson you will learn about negative exponents.

12.1 Power with negative exponents

You know that

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$2^3 = 2 \times 2 \times 2 = 8 = \frac{16}{2}$$

$$2^2 = 2 \times 2 = 4 = \frac{8}{2}$$

$$2^1 = 2 = \frac{4}{2}$$

$$2^0 = 1 = \frac{2}{2}$$

Could you understand the above pattern? Observe the value when the exponent is decreased by 1.

What is the value of 2^{-1} ?

So looking at the above pattern, you can see that $2^{-1} = \frac{1}{2} = \frac{1}{2^1}$

$$\text{Again,}\quad 2^{-2} = \frac{1}{2} \div 2 = \frac{1}{2 \times 2} = \frac{1}{2^2}$$

$$2^{-3} = \frac{1}{2^2} \div 2 = \frac{1}{2^2 \times 2} = \frac{1}{2^3}$$

$$2^{-4} = \frac{1}{2^3} \div 2 = \frac{1}{2^3 \times 2} = \frac{1}{2^4}$$

$$2^{-5} = \frac{1}{2^4} \div 2 = \frac{1}{2^4 \times 2} = \frac{1}{2^5} \text{ etc.}$$

$$\text{Similarly,}\quad 2^{-7} = \frac{1}{2^7}$$

$$3^{-19} = \frac{1}{3^{19}}$$

$$5^{-6} = \frac{1}{5^6}$$

$$10^{-1} = \frac{1}{10},\quad 10^{-2} = \frac{1}{10^2},\quad 10^{-3} = \frac{1}{10^3} \text{ etc}$$

Can you find the value of 10^{-5} in a similar pattern?
Now, let's see if both base and exponent are negative numbers.

$(-2)^{-5} = \frac{1}{(-2)^5}$, $(-3)^{-4} = \frac{1}{(-3)^4}$, $(-10)^{-7} = \frac{1}{(-10)^7}$ you get a definite value

Therefore, for any integer *a*, $a^{-m} = \frac{1}{a^m}$, where *m* is a positive integer.

Example 1 : Find the value of $(-3)^{-4}$

Solution : $(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{(-3)\times(-3)\times(-3)\times(-3)}$

$$= \frac{1}{81}$$

This means $(-3)^{-4} = \frac{1}{81}$

Example 2 : Evaluate

(i) 5^{-4} (ii) $(-5)^{-4}$ (iii) $(-5)^{-3}$ (iv) $(-4)^{-5}$

Solution : (i) $5^{-4} = \dfrac{1}{5^4}$

$= \dfrac{1}{5 \times 5 \times 5 \times 5}$

$= \dfrac{1}{625}$

(ii) $(-5)^{-4} = \dfrac{1}{(-5)^4}$

$= \dfrac{1}{(-5)\times(-5)\times(-5)\times(-5)}$

$= \dfrac{1}{625}$

(iii) $(-5)^{-3} = \dfrac{1}{(-5)^3}$

$= \dfrac{1}{(-5)\times(-5)\times(-5)}$

$= \dfrac{1}{-125}$

$= -\dfrac{1}{125}$

(iv) $(-4)^{-5}$

$= \dfrac{1}{(-4)\times(-4)\times(-4)\times(-4)\times(-4)}$

$= \dfrac{1}{-1024}$

$= -\dfrac{1}{1024}$

12.2 Expanded form of a number using exponents.

Example 3 : Write 57463 in expanded form.

Solution : $57463 = 50000 + 7000 + 400 + 60 + 3$

$= 5 \times 10^4 + 7 \times 10^3 + 4 \times 10^2 + 6 \times 10 + 3$

or

$= 5 \times 10^4 + 7 \times 10^3 + 4 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$

Example 4 : Write 57463.812 in expanded form

Solution : $57463.812 = 50000 + 7000+400+60+3 +0.8 +0.01 +0.002$

$= 50000 + 7000 + 400 + 60 + 3 + \dfrac{8}{10} + \dfrac{1}{100} + \dfrac{2}{1000}$

$= 5 \times 10^4+7\times10^3+4\times10^2+6\times10+3\times10^0+8\times10^{-1}+1\times10^{-2} + 2\times10^{-3}$

12.3 Laws of Indices

Laws of indices are

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$, $a \neq 0$

(iii) $(a^m)^n = a^{mn}$

(iv) $(a \times b)^m = a^m \times b^m$

(v) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, \quad b \neq 0$

(vi) $a^0 = 1, \quad a \neq 0$

Are the above laws correct for negative exponent?

Let us examine

(i) $(-3)^{-5} \times (-3)^{-6}$

Solution : $(-3)^{-5} \times (-3)^{-6}$

$= \dfrac{1}{(-3)^5} \times \dfrac{1}{(-3)^6}$

$= \dfrac{1}{(-3)^5 \times (-3)^6}$

$= \dfrac{1}{(-3)^{5+6}}$

$= (-3)^{-5+(-6)}$

(ii) $(-5)^3 \times (-5)^{-7}$

Solution : $(-5)^3 \times (-5)^{-7}$

$= (-5)^3 \times \dfrac{1}{(-5)^7}$

$= \dfrac{(-5)^3}{(-5)^7}$

$= (-5)^{3-7}$

$= (-5)^{3+(-7)}$

(iii) $(-4)^{-5} \times (-4)^6$

Solution : $(-4)^{-5} \times (-4)^6$

$= \dfrac{1}{(-4)^5} \times (-4)^6$

$= \dfrac{(-4)^6}{(-4)^5}$

$= (-4)^{6-5}$

$= (-4)^{-5+6}$

What have we learnt from the above three examples? Powers are added in case of multiplication when bases are the same. You may consider with different numbers. Are the properties same?

Therefore, for any nonzero integer a

$a^m \times a^n = a^{m+n}$ where m and n are integers.

In the same way you can verify the other laws of exponents where a and b are integers. Thus we see that all the above mentioned laws are correct for negative exponents..

Now observe the following multiplication :

$5^3 \times 5^{-3}$

$= 5^3 \times \dfrac{1}{5^3}$

$= \dfrac{5^3}{5^3}$

$= 1$

$7^6 \times 7^{-6}$

$= 7^6 \times \dfrac{1}{7^6}$

$= \dfrac{7^6}{7^6}$

$= 1$

$2^5 \times 2^{-5}$

$= 2^5 \times \dfrac{1}{2^5}$

$= \dfrac{2^5}{2^5}$

$= 1$

You have already learnt that if product of two numbers is 1, then one number is known as multiplicative inverse or *reciprocal* or *inverse* of the other.

That means, multiplicative inverse of 5^3 is 5^{-3}, and multiplicative inverse of 5^{-3} is 5^3.

Similarly, multiplicative inverse of 7^6 is 7^{-6} and multiplicative inverse of 7^{-6} is 7^6.

That means, multiplicative inverse of a^m is a^{-m} or $\frac{1}{a^m}$, where m is integer and $a \neq 0$.

Example 5 : Evaluate (using laws of indices)

(i) $(-4)^7 \times (-4)^{-5} \times (-4)^{-2}$ (ii) $(-3)^{-3} \div (-3)^{-2}$

(iii) $(-2)^{-2} \times 3^{-2}$ (iv) $(2^{-2})^{-3}$

Solution : (i) $(-4)^7 \times (-4)^{-5} \times (-4)^{-2}$

$= (-4)^{7+(-5)+(-2)}$ $[a^m \times a^n = a^{m+n}]$

$= (-4)^0$

$= 1$

(ii) $(-3)^{-3} \div (-3)^{-2}$

$= (-3)^{-3-(-2)}$ $[a^m \div a^n = a^{m-n}]$

$= (-3)^{-3+2}$

$= (-3)^{-1}$

$= \frac{1}{-3}$

$= -\frac{1}{3}$

(iii) $(-2)^{-2} \times 3^{-2}$

$= (-2 \times 3)^{-2}$ $[a^m \times b^m = (a \times b)^m]$

$= (-6)^{-2}$

$= \frac{1}{(-6)\times(-6)}$

$= \frac{1}{36}$

(iv) $(2^{-2})^{-3}$

$= 2^{(-2)\times(-3)}$ $[(a^m)^n = a^{mn}]$

$= 2^6$

$= 64$

Example 6 : Simplify and write the answer in exponential form.

(i) $3^{-7} \times 3^{-4} \times 3^2 \times 3^{-5}$ (ii) $(-7)^{-9} \div (-7)^{-2}$

(iii) $(-4)^{-5} \times (-3)^{-5} \times 2^{-5}$ (iv) $(3^{-5})^{-3}$

Solution : (i) $3^{-7} \times 3^{-4} \times 3^2 \times 3^{-5}$

$= 3^{-7+(-4)+2+(-5)}$

$= 3^{-16+2}$

$= 3^{-14}$

$= \frac{1}{3^{14}}$

(ii) $(-7)^{-9} \div (-7)^{-2}$

$= (-7)^{-9-(-2)}$

$= (-7)^{-9+2}$

$= (-7)^{-7}$

$= \frac{1}{(-7)^7}$

(iii) $(-4)^{-5} \times (-3)^{-5} \times 2^{-5}$

$= \{(-4)\times(-3)\times 2\}^{-5}$

$= (24)^{-5} \quad = \dfrac{1}{24^5}$

(iv) $(3^{-5})^{-3}$

$= 3^{(-5)\times(-3)}$

$= 3^{15}$

Example 7 : Find the value of $\left(\dfrac{2}{3}\right)^4$

Solution : $\left(\dfrac{2}{3}\right)^4 = \dfrac{2}{3}\times\dfrac{2}{3}\times\dfrac{2}{3}\times\dfrac{2}{3}$ [4 times $\dfrac{2}{3}$]

$= \dfrac{2\times2\times2\times2}{3\times3\times3\times3} \qquad \dfrac{[\text{4 times } 2]}{[\text{4 times } 3]}$

$= \dfrac{2^4}{3^4} = \dfrac{16}{81}$

Therefore, $\left(\dfrac{2}{3}\right)^4 = \dfrac{16}{81}$

Example 8 : Find the value of $\left(\dfrac{2}{3}\right)^{-4}$

Solution : $\left(\dfrac{2}{3}\right)^{-4} = \dfrac{1}{\left(\dfrac{2}{3}\right)^4} = \dfrac{1}{\dfrac{2^4}{3^4}} = \dfrac{3^4}{2^4} = \dfrac{81}{16}$

Therefore, $\left(\dfrac{2}{3}\right)^{-4} = \dfrac{3^4}{2^4} = \dfrac{81}{16}$

That means $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$ or $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

Example 9 : Find the value of $\left(\dfrac{2}{5}\right)^{-3}$

Solution : $\left(\dfrac{2}{5}\right)^{-3} = \left(\dfrac{5}{2}\right)^3 = \dfrac{5^3}{2^3} = \dfrac{125}{8}$

Example 10 : Convert $\left(\dfrac{3}{7}\right)^{-6}$ into positive exponent.

Solution : $\left(\dfrac{3}{7}\right)^{-6} = \left(\dfrac{7}{3}\right)^6$

Note : Previously in the laws of exponent we considered a and b as non zero integers and $m,\ n$ as any integers. Now, we examine if $a^m \times a^n = a^{m+n}$, hold for m, n integers and a, b are rational numbers.

Let us examine if the law is true in this case.

Example 11 : Find the value of $\left(\frac{2}{3}\right)^5 \times \left(\frac{2}{3}\right)^7$

Solution :

$$\left(\frac{2}{3}\right)^5 \times \left(\frac{2}{3}\right)^7 = \underbrace{\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)}_{5\text{ times}}\times\underbrace{\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)}_{7\text{ times}}$$

$$= \underbrace{\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)}_{12\text{ times}}$$

$$= \left(\frac{2}{3}\right)^{12}$$

That means, the law is applicable for rational numbers also.

Therefore, for any nonzero ratonal number a, $a^m \times a^n = a^{m+n}$ holds, where m and n are integers. You can also examine other laws of indices for any nonzero rational numbers a and b.

Example 12 : Simplify using laws of indices

(i) $\left(\frac{19}{15}\right)^8 \times \left(\frac{19}{15}\right)^{11} \times \left(\frac{19}{15}\right)^{-7}$ (ii) $\left(-\frac{2}{3}\right)^2 \times \left\{\left(-\frac{2}{3}\right)^7 \div \left(-\frac{2}{3}\right)^5\right\}$

(iii) $\left(-\frac{3}{5}\right)^{-4} \times \left(-\frac{5}{3}\right)^4 \times \left(-\frac{3}{5}\right)^{-3}$ (iv) $\left\{\left(\frac{2}{3}\right)^5\right\}^{-3}$ (v) $\left(\frac{3}{4}\right)^{-5} \div \left(\frac{4}{3}\right)^8$

Solution :

(i) $\left(\frac{19}{15}\right)^8 \times \left(\frac{19}{15}\right)^{11} \times \left(\frac{19}{15}\right)^{-7}$

$= \left(\frac{19}{15}\right)^{8+11+(-7)}$ $\quad [a^m \times a^n \times a^p = a^{m+n+p}]$

$= \left(\frac{19}{15}\right)^{12}$

(ii) $$\left(-\frac{2}{3}\right)^2 \times \left\{\left(-\frac{2}{3}\right)^7 \div \left(-\frac{2}{3}\right)^5\right\}$$

$$= \left(-\frac{2}{3}\right)^2 \times \left\{\left(-\frac{2}{3}\right)^{7-5}\right\} \qquad [a^m \div a^n = a^{m-n}]$$

$$= \left(-\frac{2}{3}\right)^2 \times \left(-\frac{2}{3}\right)^2$$

$$= \left(-\frac{2}{3}\right)^{2+2}$$

$$= \left(-\frac{2}{3}\right)^4 = \left(-1.\frac{2}{3}\right)^4 = (-1)^4\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^4$$

(iii) $$\left(-\frac{3}{5}\right)^{-4} \times \left(-\frac{5}{3}\right)^4 \times \left(-\frac{3}{5}\right)^{-3}$$

$$= \left(-\frac{3}{5}\right)^{-4} \times \left(-\frac{3}{5}\right)^{-4} \times \left(-\frac{3}{5}\right)^{-3} \qquad \left[\because \left(-\frac{5}{3}\right)^4 = \left(-\frac{3}{5}\right)^{-4} \text{ Converting to the base same}\right]$$

$$= \left(-\frac{3}{5}\right)^{-4+(-4)+(-3)} = \left(-\frac{3}{5}\right)^{-11} = \left(-\frac{5}{3}\right)^{11}$$

(iv) $$\left\{\left(\frac{2}{3}\right)^5\right\}^{-3} = \left(\frac{2}{3}\right)^{5\times(-3)} = \left(\frac{2}{3}\right)^{-15} = \left(\frac{3}{2}\right)^{15}$$

(v) $$\left(\frac{3}{4}\right)^{-5} \div \left(\frac{4}{3}\right)^8 = \left(\frac{4}{3}\right)^5 \div \left(\frac{4}{3}\right)^8$$

$$= \left(\frac{4}{3}\right)^{5-8} = \left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^3$$

Example 13 : If $\left\{\left(\frac{2}{3}\right)^2\right\}^6 = \left(\frac{3}{2}\right)^{3k-3}$ find the value of k

Solution : Given that

$$\left\{\left(\frac{2}{3}\right)^2\right\}^6 = \left(\frac{3}{2}\right)^{3k-3}$$

or, $\left(\frac{2}{3}\right)^{2\times6} = \left(\frac{2}{3}\right)^{-(3k-3)}$ $[(a^m)^n = a^{mn}]$

or, $\left(\frac{2}{3}\right)^{12} = \left(\frac{2}{3}\right)^{3-3k}$

Therefore $12 = 3 - 3k$ [since bases are same, exponents are equal]

or, $3k = 3 - 12$

or, $3k = -9$

or, $k = -3$

$\therefore$ value of k is -3

Exercise 12.1

1. Evaluate :

   (i) 5^{-3} (ii) $(-4)^{-2}$ (iii) $(-4)^{-3}$

   (iv) $\left(-\frac{5}{7}\right)^{5}$ (v) $\left(-\frac{5}{7}\right)^{-5}$ (vi) $\left(-\frac{1}{3}\right)^{8}$

2. Express in exponential form :

   (i) $\frac{343}{125}$ (ii) $-\frac{1}{289}$ (iii) $-\frac{27}{343}$

   (iv) $-\frac{125}{216}$ (v) $-\frac{27}{16\times49}$ (vi) $\frac{128}{81}$

3. Simplify and express the result in power notation with positive exponent.

   (i) $(-2)^4 \times \left(\frac{3}{2}\right)^4$ (ii) $\left(-\frac{2}{3}\right)^4 \times \left(-\frac{3}{4}\right)^3$

   (iii) $5^{-7} \times \left(\frac{1}{5}\right)^3$ (iv) $3^{-5} \times (-2)^{-5} \times (-4)^{-5}$

4. Find the value of

   (i) $\left(\frac{1}{3}\right)^4 \times \left(-\frac{3}{5}\right)^3 \times \left(-\frac{7}{9}\right)^2$ (ii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{5}\right)^{-2}$

   (iii) $\left(\frac{1}{3}\right)^0 + \left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{3}\right)^{-3}$ (iv) $(5^{-1} + 3^{-1} + 7^{-2})^0$

   (v) $(5^{-1} \times 2^{-1}) \times 6^{-1}$ (vi) $(3^{-2})^{-3}$

5. Write the multiplicative inverse of

(i) 3^4 (ii) $\left(\frac{2}{3}\right)^6$ (iii) $\left(-\frac{4}{9}\right)^{50}$

(iv) $\left(\frac{3}{4}\right)^{-5}$ (v) $\left(-\frac{2}{3}\right)^{-7}$ (vi) $\left(\frac{3}{8}\right)^{-4}$

6. Simplify using laws of indices

(i) $\left(-\frac{4}{5}\right)^3 \times \left(-\frac{4}{5}\right)^2 \times \left(-\frac{4}{5}\right)$ (ii) $\left(\frac{5}{3}\right)^0 \times \left(\frac{5}{3}\right)^{-3} \times \left(\frac{5}{3}\right)^{-2}$

(iii) $\left\{\left(-\frac{5}{3}\right)^{15} \times \left(-\frac{5}{3}\right)^{-8}\right\} \div \left(-\frac{5}{3}\right)^6$ (iv) $\left(-\frac{3}{2}\right)^{-5} \times \left(-\frac{3}{2}\right)^{-7} \times \left(-\frac{2}{3}\right)^8 \times \left(-\frac{2}{3}\right)^4$

(v) $(3^{-4})^{-2} \times (3^{-5})^2 \div (3^{-2})^{-3}$

7. (i) If $\left(\frac{5}{7}\right)^{-7} \times \left(\frac{7}{5}\right)^{-9} = \left(\frac{5}{7}\right)^{2m}$, find the value of m.

(ii) If $\left(\frac{9}{49}\right)^{-5} \times \left(\frac{9}{49}\right)^{7} = \left(\frac{9}{49}\right)^{-6k}$, find the value of k.

(iii) If $(1.4)^8 \times (1.4)^5 = (1.4)^3 \times (1.4)^k$, Find the value of k

(iv) Find the value of m, such that $5^m \div 5^{-3} = 5^5$

8. Simplify

(i) $\dfrac{125 \times 3^{-4} \times 2^2}{5^{-4} \times 100 \times 3^{-7}}$ (ii) $\dfrac{3^{2k} \times 27 \times 9^{-3}}{81^{-2k} \times 3^{-4} \times 3^5}$

(iii) $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$ (iv) $\dfrac{25 \times l^{-4}}{5^{-3} \times 10 \times l^{-8}}$ $(l \neq 0)($

v) $\dfrac{2^{m+2} \times 3^{2m-n} \times 6^n}{6^m \times 2^n \times 4 \times 3^m}$

9. Express in expanded form :

(i) 15737.348

(ii) 35792.39

12.4 Expressing large numbers in the standard form

You have already learnt, how to express a number in exponential form where 10^0, 10^1, 10^2 are used for unit's place, ten's place, hundred's place etc., and 10 as base.

Observe the following numbers

$$58 = 5.8 \times 10 = 5.8 \times 10^1$$
$$580 = 5.8 \times 100 = 5.8 \times 10^2$$
$$5800 = 5.8 \times 1000 = 5.8 \times 10^3$$
$$58000 = 5.8 \times 10000 = 5.8 \times 10^4$$
$$580000 = 5.8 \times 100000 = 5.8 \times 10^5$$

This form of a number is known as *standard form* or *scientific notation*.

We can express 5800 in different ways.

$$5800 = 58 \times 100 = 58 \times 10^2$$
$$5800 = 580 \times 10 = 580 \times 10^1$$
$$5800 = 0.5800 \times 10000 = 0.5800 \times 10^4$$

They are not standard form of 5800. But, 5.8×10^3 is called standard form of 5800.

We express this number in standard form as follows

$$5\underset{3\,2\,1}{800} = 5.8 \times 10^3$$

Since decimal point is moved 3 places to the left, therefore power of 10 is 3

or,

Total number of digits is 4, therefore power of 10 is 4–1 = 3

Number between 1 and 10 (including 1)

That means, the standard form of a large number is $K \times 10^n$

where $1 \leq K < 10$ and n are whole numbers. The value of n will mean how many places the decimal point moves to the left from its initial point, just right to the first digit.

or, value of n = number of digits left to the decimal–1.

In case of 5 8 0 0 the decimal point is moved three points left, therefore value of n is 3.

or, $4 - 1 = 3$

Similarly, standard, form of 47300000 is 4.73×10^7.

You know that the sun is at a distance of 149600,000,000 metres from us. This number is difficult to read. Let us express it in standard form. The decimal point must be moved to the right of first digit 1. Here, there are 12 digits. Therefore, 12–1=11

$$1\,\underset{11\,10\,9\,8\,7\,6\,5\,4\,3\,2\,1}{49600000000}$$

You have seen that after moving 11 places towards left we get one.

Therefore, $149600{,}000{,}000 = 1.496 \times 10^{11}$

That means, we can say that sun is at a distance of 1.49×10^{11} metres away from us.

We observe that the large numbers are difficult to read and write. Therefore, it is easy to understand as well as to read and write if we express it in standard form. You can study in books of higher classes that mass of the earth is 5976000000000000000000000 kg. How will you read this number? Think how to express it in standard form. The first digit of the number is 5, count how many places the decimal point has to be moved from right to left, that means, 24 places are to be moved upto 5 excluding the digit 5. Therefore the standard form is 5.976×10^{24}. That means, the mass of earth is 5.976×10^{24} kg.

Example 14 : Express the following in standard form.

(i) 4000,000,000 (ii) 5376.3×10^5

(iii) 0.278×10^5 (iv) 0.00327×10^7

Solution : (i) $4000,000,000 = 4 \times 10^9$

(ii) $5376.3 \times 10^5 = 5.3763 \times 10^3 \times 10^5$
$= 5.3763 \times 10^8$

(iii) $0.278 \times 10^5 = \frac{2.78}{10} \times 10^5$
$= 2.78 \times 10^{5-1}$
$= 2.78 \times 10^4$

(iv) $0.00327 \times 10^7 = \frac{3.27}{1000} \times 10^7$
$= \frac{3.27}{10^3} \times 10^7$
$= 3.27 \times 10^{7-3}$
$= 3.27 \times 10^4$

12.5 Use of the exponents to express small numbers in standard form :

Like large numbers, we can express small numbers also in standard form.

Observe the following

$$0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$$

$$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

$$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$$

$$0.0001 = \frac{1}{10000} = \frac{1}{10^4} = 10^{-4}$$

$$0.00001 = \frac{1}{100000} = \frac{1}{10^5} = 10^{-5} \quad \text{etc.}$$

You have seen that–

$$0.00001 = 10^{-5} \text{ or } 1 \times 10^{-5}$$

Similarly, think what is the standard form of 0.00004.

$$0.00004 = \frac{4}{100000}$$

$$= \frac{4}{10^5}$$

$$= 4 \times 10^{-5}$$

$\therefore$ Standard form of 0.00004 is 4×10^{-5}

You can think that the decimal point comes after moving 5 places from left to right and across the digit 4. Therefore, power of 10 is –5.

0 .|0|0|0|0|4|
1 2 3 4 5

$$\therefore \quad 0.00004 = 4 \times 10^{-5}$$

Similarly, what is the standard form of 0.000000123? Here decimal point should be shifted to just right to digit 1, the first significant digit after decimal point. How many places to be shifted?

0 .|0|0|0|0|0|0|1|2 3
1 2 3 4 5 6 7

Definitely 7 places are to be shifted. Then $0.000000123 = 1.23 \times 10^{-7}$

Therefore, you note that *the standard form of a number (may be small or large) is of the form* $K \times 10^n$ *where* $1 \leq K < 10$ *and n is integer.*

Example 15 : Express the following in standard form.

(i) 0.0000000007 (ii) 0.0000001234

(iii) 0.00000008034 (iv) $\frac{1}{100000000}$

Solution : (i) $0.0000000007 = 7 \times 10^{-10}$

(ii) $0.0000001234 = 1.234 \times 10^{-7}$

(iii) $0.00000008034 = 8.034 \times 10^{-8}$

(iv) $\frac{1}{100000000} = \frac{1}{10^8} = 1 \times 10^{-8}$

12.6 Addition and subtraction of large numbers

Example 16 : The distance between the Earth and the Sun is 3.844×10^8 metres during solar eclipse. The distance between the Moon and the Sun is 1.492×10^{11} metres. Find the distance between the Earth and the Sun.

Solution : During solar eclipse, the centres of the sun, the moon and the earth lie on a straight line. In this case, we should convert the numbers from standard form to a number of same exponent.

Distance between the sun and the earth = Distance between the earth and the moon + Distance between the moon and the sun.

$\therefore$ Distance between the earth and the sun $= 3.844 \times 10^8 + 1.492 \times 10^{11}$ m

$= 3.844 \times 10^8 + 1.492 \times 10^3 \times 10^8$ m

$= 3.844 \times 10^8 + 1492 \times 10^8$ m

$= (3.844 + 1492) \times 10^8$ m

$= 1495.844 \times 10^8$ m

$= 1.495844 \times 10^3 \times 10^8$ m

$= 1.495844 \times 10^{11}$ m

Example 17 : Masses of the earth and the moon are 5.972×10^{24} kg and 7.352×10^{22} kg respectively. Which has more mass and by how much?

Solution : Here $5.972 \times 10^{24} > 7.352 \times 10^{22}$

Now, $5.972 \times 10^{24} - 7.352 \times 10^{22}$

$= 5.972 \times 10^2 \times 10^{22} - 7.352 \times 10^{22}$ [converting each number with the same exponent]

$= 597.2 \times 10^{22} - 7.352 \times 10^{22}$

$= (597.2 - 7.352) \times 10^{22}$

$= 589.848 \times 10^{22}$

$= 5.89848 \times 10^2 \times 10^{22}$

$= 5.89848 \times 10^{24}$

Therefore, mass of the earth is more than that of the moon by 5.89848×10^{24} kg.

Exercise 12.2

1. Express the following in standard form :
   (i) 35700000 (ii) 705030000 (iii) 37800.35
   (iv) 5362.8×10^6 (v) 4003.2×10^5
2. Express the following in standard form :
   (i) 0.0000000382 (ii) 0.00000009057 (iii) 0.00000756
   (iv) 0.00023×10^{-2} (v) 0.000314×10^{-3}
3. Express the following in usual form :
   (i) 7.02×10^5 (ii) 3.972×10^7 (iii) 1.001×10^8
   (iv) 3×10^{-8} (v) 2.1×10^{-6} (vi) 3.09×10^{-5}

4. Write the statement in standard form :
   (i) Speed of light is 300000 km per second.
   (ii) Distance between the sun and saturn is 1,433,500,000,000 m.
   (iii) 18 gm water contains 602,300,000,000,000,000,000,000 molecules.
   (iv) Diameter of a molecule of an object is 0.000000015 cm.
   (v) Size of a virus is 0.0000005 m.
   (vi) Diameter of a wire is 0.0000032 m.
   (vii) 1 micron = $\frac{1}{1000000}$ m

5. Express the following in standard form and arrange in ascending order.
   925×10^4; 94.2×10^5, 875×10^5, 87.5×10^4

6. Add :
   (i) $3.04 \times 10^{11} + 5.02 \times 10^{10}$
   (ii) $6.03 \times 10^7 + 6.03 \times 10^8$

7. Subtract :
   (i) $6.47 \times 10^8 - 3.15 \times 10^6$
   (ii) $3.76 \times 10^7 - 3.76 \times 10^5$

What we have learnt

1. If m and n are integers
   (a) $a^m \times a^n = a^{m+n}$
   (b) $a^m \div a^n = a^{m-n}$
   (c) $(a^m)^n = a^{mn}$
   (d) $a^m \times b^m = (ab)^m$
   (e) $a^0 = 1$
   (f) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

2. Using exponents, very large and very small numbers can be expressed in standard form.

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