Linear Programming

Multiple Choice Questions

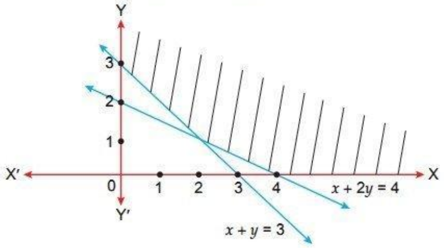
Choose and write the correct option in the following questions.

1. The number of feasible solutions of the linear programming problem given as Maximize $Z = 15x + 30y$ subject to constraints:

   $3x + y \leq 12, x + 2y \leq 10, x \geq 0, y \geq 0$ is [CBSE 2023 (65/3/2)]

   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) infinite

2. The feasible region of a linear programming problem is shown in the figure below :



   Which of the following are the possible constraints? [CBSE 2023 (65/3/2)]

   (*a*) $x + 2y \geq 4, x + y \leq 3, x \geq 0, y \geq 0$ (*b*) $x + 2y \leq 4, x + y \leq 3, x \geq 0, y \geq 0$

   (*c*) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$ (*d*) $x + 2y \geq 4, x + y \geq 3, x \leq 0, y \leq 0$

3. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true? [CBSE 2023 (65/2/1)]

   (*a*) $a = 9, b = 1$ (*b*) $a = 9, b = 2$ (*c*) $a = 3, b = 5$ (*d*) $a = 5, b = 3$

4. **The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then** **[*CBSE 2023 (65/1/1)*]**

(*a*) Z is maximum at (2, 72), minimum at (15, 20)

(*b*) Z is maximum at (15, 20), minimum at (40, 15)

(*c*) Z is maximum at. (40, 15), minimum at (15, 20)

(*d*) Z is maximum at (40, 15), minimum at (2, 72)

5. **The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2, x \geq 0, y \geq 0$ is** **[*CBSE 2023 (65/1/1)*]**

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

6. **If the corner points of the feasible region of an LPP are (0, 3), (3, 2) and (0, 5), then the minimum value of $Z = 11x + 7y$ is** **[*CBSE (Term-1) 2021-22 (65/1/4)*]**

(*a*) 21 (*b*) 33 (*c*) 14 (*d*) 35

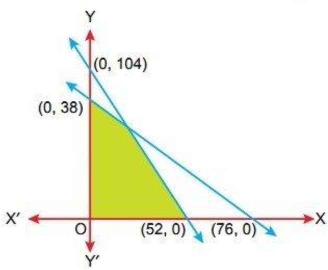
7. **The number of solutions of the system of inequations $x + 2y \leq 3, 3x + 4y \geq 12, x \geq 0, y \geq 1$ is** **[*CBSE (Term-1) 2021-22 (65/1/4)*]**

(*a*) 0 (*b*) 2 (*c*) finite (*d*) infinite

8. **The maximum value of $Z = 3x + 4y$ subject to the constraints $x \geq 0, y \geq 0$ and $x + y \leq 1$ is** **[*CBSE (Term-1) 2021-22 (65/1/4)*]**

(*a*) 7 (*b*) 4 (*c*) 3 (*d*) 10

9. **The feasible region of an LPP is given in the following figure:**



Then, the constraints of the LPP are $x \geq 0, y \geq 0$ and **[*CBSE (Term-1) 2021-22 (65/1/4)*]**

(*a*) $2x + y \leq 52$ and $x + 2y \leq 76$ (*b*) $2x + y \leq 104$ and $x + 2y \leq 76$

(*c*) $x + 2y \leq 104$ and $2x + y \leq 76$ (*d*) $x + 2y \leq 104$ and $2x + y \leq 38$

10. **For the following LPP**

Maximise $Z = 3x + 4y$

subject to constraints

$x - y \geq -1, x \leq 3$

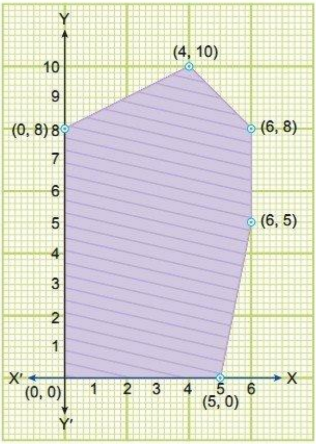
$x \geq 0, y \geq 0$

the maximum value is **[*CBSE (Term-1) 2021-22 (65/1/4)*]**

(*a*) 0 (*b*) 4 (*c*) 25 (*d*) 30

11. **The feasible region for an LPP is shown below:** [*NCERT Exemplar, CBSE 2020 (65/1/4)*]

Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at



(*a*) (0, 0) (*b*) (0, 8) (*c*) (5, 0) (*d*) (4, 10)

12. **In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points of which Z_{max} occurs is** [*CBSE 2020 (65/4/1)*]

(*a*) 0 (*b*) 2 (*c*) finite (*d*) infinite

13. **Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is**

(*a*) $p = 2q$ (*b*) $p = \frac{q}{2}$ (*c*) $p = 3q$ (*d*) $p = q$

14. **The optimal value of the objective function is attained at the points**

(*a*) given by intersection of inequation with y-axis only.

(*b*) given by intersection of inequation with x-axis only.

(*c*) given by corner points of the feasible region.

(*d*) none of these

15. **A Linear Programming Problem is as follows:**

Minimize: $Z = 2x + y$

Subject to the constraints

$$x \geq 3, x \leq 9, y \geq 0$$
$$x - y \geq 0, x + y \leq 14$$

The feasible region has [*CBSE (Term-1) 2021-22 (65/2/4)*]

(*a*) 5 corner points including (0, 0) and (9, 5) (*b*) 5 corner points including (7, 7) and (3, 3)

(*c*) 5 corner points including (14, 0) and (9, 0) (*d*) 5 corner points including (3, 6) and (9, 5)

16. **The corner points of the feasible region for a LPP are $P(0, 5)$, $Q(1, 5)$, $R(4, 2)$ and $S(12, 0)$. The minimum value of objective function $Z = 2x + 5y$ is at the point** [*CBSE (Term-1) 2021-22 (65/2/4)*]

(*a*) P (*b*) Q (*c*) R (*d*) S

17. **A LPP is as follows:**
Maximize/Minimise objective function $Z = 2x - y + 5$
Subject to constraints

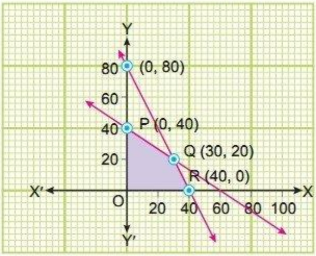
$$3x + 4y \le 60$$
$$x + 3y \le 30$$
$$x \ge 0, y \ge 0$$

If the corner points $A(0, 10)$, $B(12, 6)$, $C(20, 0)$ and $(0, 0)$, then which of the following is true.
[*CBSE (Term-1) 2021-22 (65/2/4)*]

(*a*) Maximum value of Z is 40.
(*b*) Minimum value of Z is –5.
(*c*) Difference of maximum and minimum value of Z is 35.
(*d*) At two corner points, value of Z are equal.

18. **The corner points of the feasible region determined by the set of constraints (linear inequalities) are $P(0, 5)$, $Q(3, 5)$, $R(5, 0)$ and $S(4, 1)$ and the objective function $Z = ax + 2by$ where $a, b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is** **[*CBSE (Term-1) 2021-22 (65/2/4)*]**
(*a*) $a - 5b = 0$ (*b*) $a - 3b = 0$ (*c*) $a - 2b = 0$ (*d*) $a - 8b = 0$

19. **For an LPP the objective function is $Z = 4x + 3y$ and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.** **[*CBSE (Term-1) 2021-22 (65/2/4)*]**



Which of the following statements is true?
(*a*) Maximum value of Z is at R.
(*b*) Maximum value of Z is at Q.
(*c*) Value of Z at R is less than the value at P.
(*d*) The value of Z at Q is less than the value at R.

20. **The maximum value of $Z = 4x + 3y$ subject to constraint $x + y \le 10, x, y \ge 0$ is**
(*a*) 36 (*b*) 40 (*c*) 20 (*d*) none of these

Answers

1. (*a*)	**2.** (*c*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)	**6.** (*a*)	**7.** (*a*)
8. (*b*)	**9.** (*b*)	**10.** (*c*)	**11.** (*b*)	**12.** (*d*)	**13.** (*b*)	**14.** (*c*)
15. (*b*)	**16.** (*c*)	**17.** (*b*)	**18.** (*d*)	**19.** (*b*)	**20.** (*b*)	

Solutions of Selected Multiple Choice Questions

1. We have LPP,

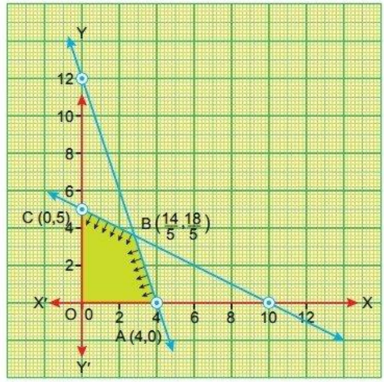
$$Z_{Max} = 15x + 30y$$

Subject to constraints

$$3x + y \le 12 \quad ...(i)$$
$$x + 2y \le 10 \quad ...(ii)$$
$$x, y \ge 0 \quad ...(iii)$$

On plotting (*i*), (*ii*) and (*iii*), we have required region (shaded) as *OABCO*.

Points of intersection of (*i*) and (*ii*), is $B\left(\frac{14}{5}, \frac{18}{5}\right)$.



Corner points	$Z = 15x + 10y$
O(0, 0)	0
A(4, 0)	60
$B\left(\frac{14}{5}, \frac{18}{5}\right)$	78 ⟵ Maximum
C(0, 5)	50

$Z_{Max} = 78$ when $x = \frac{14}{5}, y = \frac{18}{5}$.

$\therefore$ Its feasible solution is $\frac{14}{5}, \frac{18}{5}$.

$\therefore$ The number of feasible solution is 1

$\therefore$ Option (*a*) is correct.

2. As the region is away from origin from the line $x + y = 3$

$\therefore \quad x + y \geq 3$

Similarly, region is away from origin from the line $x + 2y = 4$

$\therefore \quad x + 2y \geq 4$

Also, $x \geq 0, y \geq 0$

$\therefore$ Option (*c*) is correct.

3. Given objective function $Z = ax + by$

$\therefore \quad Z_{max.} = 42$ at (4, 6) $\Rightarrow$ $4a + 6b = 42$ ...(*i*)

and, $\quad Z_{min.} = 19$ at (3, 2) $\Rightarrow$ $3a + 2b = 19$ ...(*ii*)

On solving equation (*i*) and (*ii*), we have

$a = 3$ and $b = 5$

$\therefore$ Option (*c*) is correct.

4. Given corner points are (2, 72), (15, 20) and (40, 15) for the objective function

$Z = 18x + 9y$.

Corner Points	$Z = 18x + 9y$
(2, 72)	684
(15, 20)	450 ⟶ Minimum
(40, 15)	855 ⟶ Maximum

Thus, Z is maximum at (40, 15) and minimum at (15, 20).

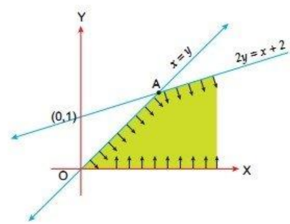
$\therefore$ Option (*c*) is correct.

5. Given constraints:

$x - y \geq 0$ ...(*i*)

$2y \leq x + 2 \Rightarrow x - 2y + 2 \geq 0$ ...(*ii*)

$x, y \geq 0$ ...(*iii*)



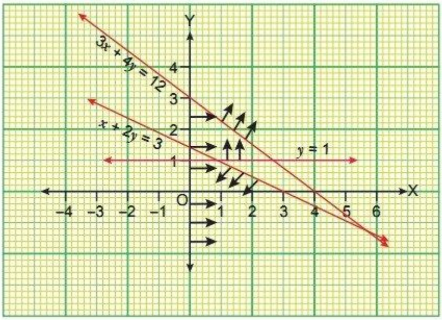
On plotting (*i*), (*ii*) and (*iii*), we have the figure.

There are only two corner points *O* and *A*.

$\therefore$ Option (*a*) is correct.

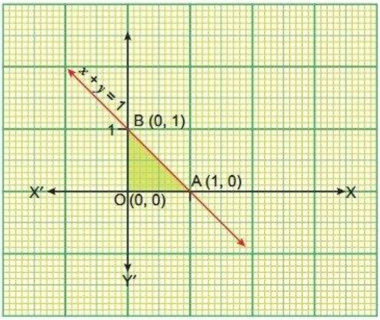
6. Let given corner points are $A(0, 3)$, $B(3, 2)$, $C(0, 5)$
and $Z = 11x + 7y$
At $A(0, 3)$, we have, $Z = 11 \times 0 + 7 \times 3 = 21$
At $B(3, 2)$, we have, $Z = 11 \times 3 + 7 \times 2 = 47$
At $C(0, 5)$, we have, $Z = 11x + 3y = 11 \times 0 + 7 \times 5 = 35$
$\Rightarrow Z_{min.} = 21$
$\therefore$ Option (*a*) is correct.

7. Given inequations, $x + 2y \leqslant 3$ ...(*i*)
$3x + 4y \geqslant 12$ ...(*ii*)
$x \geqslant 0$ ...(*iii*)
$y \geqslant 1$ ...(*iv*)



After plotting inequations (*i*) to (*iv*), we get there is no common region.
Hence, number of solutions of the system of inequations is 0.
$\therefore$ Option (*a*) is correct.

8. We have,
$$Z_{max} = 3x + 4y$$
Subject to $x, y \geqslant 0$ and $x + y \leqslant 1$



On plotting, we have

Corner points	$Z = 3x + 4y$
$O(0, 0)$	0
$A(1, 0)$	3
$B(0, 1)$	4 ← Maximum

$Z_{max.} = 4$
$\therefore$ Option (*b*) is correct.

9. Equation of line containing (0, 104) and (52, 0) is given by
$$\frac{y - 104}{x - 0} = \frac{0 - 104}{52 - 0} = \frac{-104}{52}$$
$$\Rightarrow \frac{y - 104}{x} = -\frac{104}{52} = -2 \quad \Rightarrow \quad y - 104 = -2x \quad \Rightarrow \quad 2x + y = 104$$
$\therefore$ Inequation satisfying region is $2x + y \leq 104$.

Also, equation of line containing points (0, 38) and (76, 0) is given by

$$\frac{y-38}{x-0} = \frac{0-38}{76-0} = -\frac{1}{2} \quad \Rightarrow \quad 2y - 76 = -x \quad \Rightarrow \quad x + 2y = 76$$

$\therefore$ Inequation satisfying region is $x + 2y \le 76$.

$\therefore$ Constraints are $2x + y \le 104$ and $x + 2y \le 76$.

$\therefore$ Option (b) is correct.

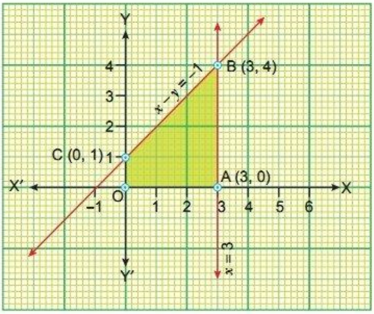
10. We have, $Z_{max.} = 3x + 4y$

Subject to constraints

$x - y \geqslant -1,\ x \le 3$

and $x, y \geqslant 0$

Corner points	$Z = 3x + 4y$
O (0, 0)	0
A(3, 0)	9
B(3, 4)	25 ← Maximum
C(0, 1)	4



$Z_{max.} = 25$

$\therefore$ Option (c) is correct.

11. Given objective function is $Z = 3x - 4y$.

On putting the corner points, we get

$Z_{min} = -32$ at (0, 8)

$\therefore$ Option (b) is correct.

13. At (3, 0), $Z_{min} = 3p + q \times 0 = 3p$

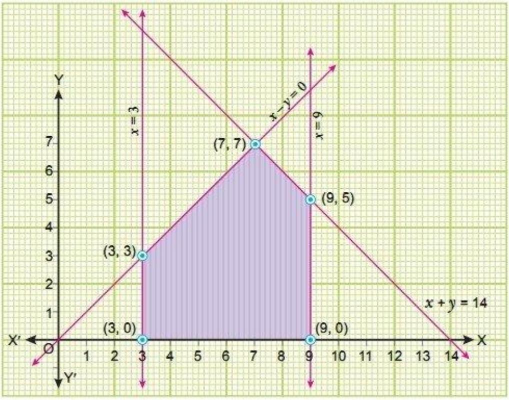
and, at (1, 1), $Z_{min} = p \times 1 + q + 1 = p + q$

$\therefore$ $3p = p + q$

$\Rightarrow$ $2p = q$ $\Rightarrow$ $p = \frac{q}{2}$

$\therefore$ Option (b) is correct.

15.



From above graph feasible region has 5 corner points including (7, 7) and (3, 3).

$\therefore$ Option (b) is correct.

16.

Corner Points	Objective Function $Z = 2x + 5y$
$P(0, 5)$	25
$Q(1, 5)$	27
$R(4, 2)$	18 ← Minimum
$S(12, 0)$	24

The minimum value of Z is 18 at $R(4, 2)$.

$\therefore$ Option (*c*) is correct.

17.

Corner Points	Objective Function $Z = 2x - y + 5$
$A(0, 10)$	–5 ← Minimum
$B(12, 6)$	23
$C(20, 0)$	25 ← Maximum
$O(0, 0)$	5

Minimum value of Z is –5 at $A(0, 10)$.

$\therefore$ Option (*b*) is correct.

18. We have objective function $Z = ax + 2by$ has maximum value at $Q(3, 5)$ and $S(4, 1)$.

$\therefore \quad Z(3, 5) = Z(4, 1)$

i.e., $3a + 10b = 4a + 2b$

$\Rightarrow \quad 0 = 4a + 2b - 3a - 10b$

$\Rightarrow \quad 0 = a - 8b$ *i.e.,* $a - 8b = 0$

$\therefore$ Option (*d*) is correct.

19. We are given objective function

$Z = 4x + 3y$ with corner points O, P, Q, R.

Corner Points	Objective Function $Z = 4x + 3y$
$O(0, 0)$	0
$P(0, 40)$	120
$Q(30, 20)$	180 ← Maximum
$R(40, 0)$	120

Maximum value of Z is 180 at $Q(30, 20)$.

$\therefore$ Option (*b*) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

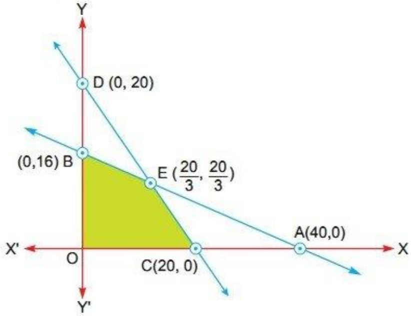
1. **Assertion (A) :** The maximum value of $Z = 5x + 3y$, satisfying the conditions $x \geq 0$, $y \geq 0$ and $5x + 2y \leq 10$, is 15.

Reason (R) : A feasible region may be bounded or unbounded.

2. **Assertion (A) :** The maximum value of $Z = x + 3y$. Such that $2x + y \leq 20$, $x + 2y \leq 20$, $x, y \geq 0$ is 30.

Reason (R) : The variables that enter into the problem are called decision variables.

3. **Assertion (A) :** Shaded region represented by $2x + 5y \geq 80$, $x + y \leq 20$, $x \geq 0, y \geq 0$ is



Reason (R) : A region or a set of points is said to be convex if the line joining any two of its points lies completely in the region.

Answers

1. (*b*) 2. (*b*) 3. (*d*)

Solutions of Assertion-Reason Questions

1. We have, corner points (0, 0), (2, 0), (0, 5).

$$\therefore \quad Z_{max.} = 5 \times 0 + 3 \times 5 = 15 \text{ at } (0, 5)$$

Clearly, both Assertion (*A*) and Reason (*R*) are true but Reason (*R*) is not the correct explanation of Assertion (*A*).

∴ Option (*b*) is correct.

2. We have, corner points be (0, 0), (10, 0), $\left(\frac{20}{3}, \frac{20}{3}\right)$, (0, 10).

$$\therefore \quad Z_{max.} = x + 3y = 0 + 3 \times 10 = 30 \text{ at } (0, 10)$$

Clearly, both Assertion (*A*) and Reason (*R*) are true but Reason (*R*) is not the correct explanation of Assertion (*A*).

∴ Option (*b*) is correct.

3. Clearly, Assertion (*A*) is false and Reason (*R*) is true.

∴ Option (*d*) is correct.

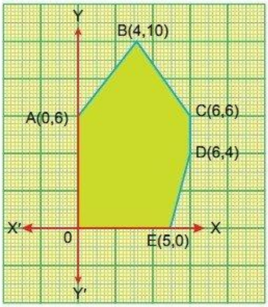
Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.**

Linear Programming Problem is a method of or finding the optimal values (maximum or minimum) of quantities subject to the constraints when relationship is expressed as a linear equations or linear inequations.

The corner points of a feasible region determined by the system of linear constraints are as shown below.



(*i*) Is this feasible region is bounded?

(*ii*) Write the number of corner points in the feasible region.

(*iii*) (*a*) If $Z = ax + by$ has maximum value at C (6, 6) and B (4, 10). Find the relationship between a & b.

OR

(*iii*) (*b*) If $Z = 2x - 5y$ then find the minimum value of this objective function.

Sol. (*i*) Yes the above feasible region is bounded.

(*ii*) Number of corner points = 6

(*iii*) (*a*) $Z = ax + by$

$Z(6, 6) = 6a + 6b$

Also $Z(4, 10) = 4a + 10b$

From question

$$6a + 6b = 4a + 10b \Rightarrow 2a = 4b \Rightarrow a = 2b$$

OR

(*iii*) (*b*)

Corner points	$Z = 2x - 5y$
O (0, 0)	0
A(0, 6)	– 30
B(4, 10)	– 42 ⟵ Minimum
C(6, 6)	– 18
D(6, 4)	– 8
E(5, 0)	10

Minimum value of Z is – 42 at the point B(4, 10).

2. **Read the following passage and answer the following questions.**

A dealer Ramprakash residing in a rural area opens a shop to start his business. He wishes to purchase a number of ceiling fans and table fans. A ceiling fan costs him ₹360 and table fan costs ₹240.

(*i*) If Ramprakash purchases x ceiling fans, y table fans. He has space in his store for at most 20 items, than write its constraints.

(*ii*) If he expects to sell ceiling fan at profit of ₹22 and table fan for a profit of ₹18, then express the profit Z (in terms of x and y).

(*iii*) (*a*) If he sells all the fans that he buys, then write the number x, y of both the type of fans in stock to get maximum profit.

OR

(*iii*) (*b*) What is the maximum profit of selling all the fans?

Sol. (*i*) From question

He has space in store for atmost 20 items.

$\therefore \quad x + y \leq 20$

(*ii*) Profit on ceiling fans = ₹ $22x$

Profit on table fans = ₹ $18y$

$\therefore \quad Z = 22x + 18y$

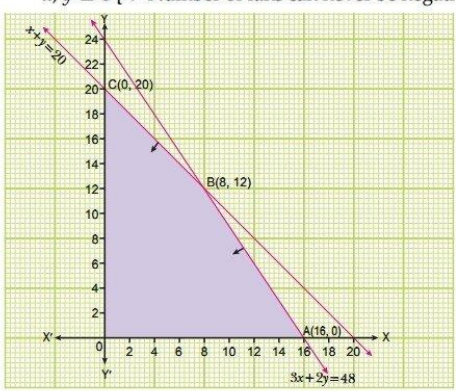
(*iii*) (*a*) We have

(Profit) $Z = 22x + 18y$, which is to be maximized under constraints

$$3x + 2y \leq 48$$

$$x + y \leq 20$$

$$x, y \geq 0 \; [\because \text{Number of fans can never be negative}]$$



Here, $OABC$ is a feasible region, which is bounded.

The co-ordinates of corner points are $O(0, 0)$, $A(16, 0)$, $B(8, 12)$ and $C(0, 20)$.

Now, we evaluate Z(profit) at each corner point.

Corner Point	$Z = 22x + 18y$
$O(0, 0)$	0
$A(16, 0)$	352
$B(8, 12)$	392 ← Maximum
$C(0, 20)$	360

Hence, maximum profit is for

x = No. of ceiling fans = 8

y = No. of table fans = 12

OR

Obviously, from table made above

(*iii*) (*b*) The maximum value of profit Z is ₹392.

3. Read the following passage and answer the following questions.

A share is referred to as a unit of ownership which represents an equal proportion of a company's capital. A share entities the shareholders to an equal claim on profit and loss of the company.

Dr. Ritam wants to invest at most ₹12,000 in two type of shares A and B. According to the rules, she has to invest at least ₹2000 in share A and at least ₹4000 in share B. If the rate of interest on share A is 8% per annum and on share B is 10% per annum.

(*i*) If Dr. Ritam invests ₹ x in share A, and invest ₹ y in share B. If the total interest recieved by Dr. Ritam from both type of shares is represented by Z. Formulate the LPP.

(*ii*) To maximise the interest on both types of share, find the invested amount on both shares A and B.

Sol. (*i*) Since, she has to invest at least ₹2000 in share A.

$\therefore \quad x \geqslant 2000$

Since, she has to invest atleast ₹4000 in share B.

$\therefore \quad y \geqslant 4000$

Interest on share A = $x \times \frac{8}{100} = ₹\frac{2x}{25}$

Interest on share B = $y \times \frac{10}{100} = ₹\frac{y}{10}$

$\therefore$ Her total interest = $Z = ₹\left(\frac{2x}{25} + \frac{y}{10}\right)$

Hence, LPP is given by

Maximise $Z = \frac{2x}{25} + \frac{y}{10}$ subject to

$x \geq 2000$

$y \geq 4000$

$x \geq 0,\ y \geq 0$

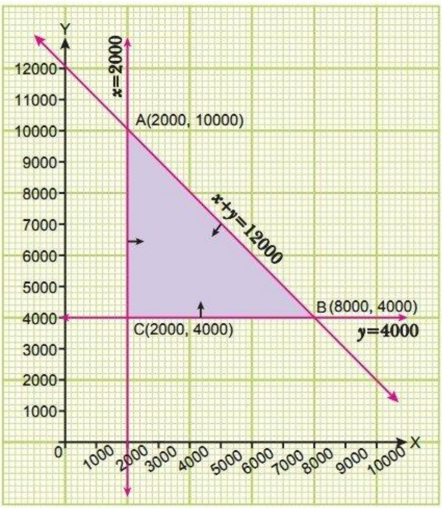
(*ii*) We have

$Z = \left(\frac{2x}{25} + \frac{y}{10}\right)$ which is to be maximised under constraints

$x \geqslant 2000$

$y \geqslant 4000$

and $x + y \leqslant 12000$



Here, *ABC* be bounded feasible region with corner points *A* (2000, 10000), *B* (8000, 4000), *C* (2000, 4000).

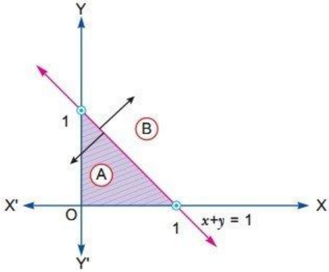
Now we evaluate *Z* at each corner points.

Corner Point	$Z = \left(\frac{2x}{25} + \frac{y}{10}\right)$
A (2000, 10000)	1160 ← Maximum
B (8000, 4000)	1040
C (2000, 4000)	560

i.e. for maximum interest $x = ₹2000$, $y = ₹10000$.

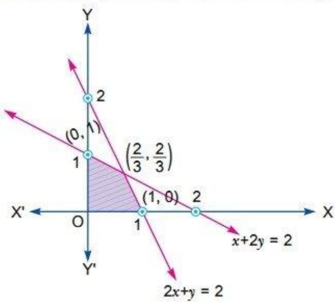
CONCEPTUAL QUESTIONS

1. In figure, which half plane (A) or (B) is the solution of $x + y > 1$? Justify your answer.



Sol. Half plane B because (0, 0) does not satisfy $x + y > 1$.

2. What is the maximum value of objective function $Z = 3x + y$ under given feasible region?



Corner Points	$Z = 3x + y$
(0, 0)	0
(1, 0)	3
$\left(\frac{2}{3}, \frac{2}{3}\right)$	$\frac{8}{3}$
(0, 1)	1

Sol. 3,

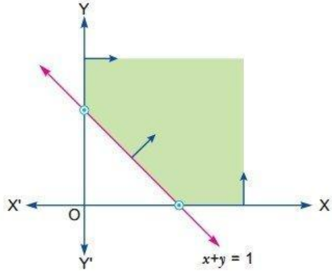
$\because$ $Z = 3x + y$ attains maximum value at (1, 0).

$\therefore$ $Z = 3 \times 1 + 0$

$= 3$

3. Is feasible region represented by $x + y \geq 1, x \geq 0, y \geq 0$ bounded? Justify your answer.

Sol. No, feasible region obtained is unbounded as shown in figure.



Short Answer Questions

1. Minimise $Z = 13x - 15y$ subject to the constraints $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$ and $y \geq 0$.

[*NCERT Exemplar*]

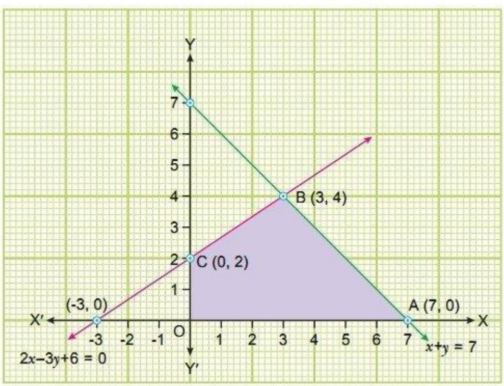
Sol. Minimise $Z = 13x - 15y$ ...(*i*)

Subject to the constraints

$$x + y \leq 7 \quad ...(ii)$$

$$2x - 3y + 6 \geq 0 \quad ...(iii)$$

$$x \geq 0, y \geq 0 \quad ...(iv)$$



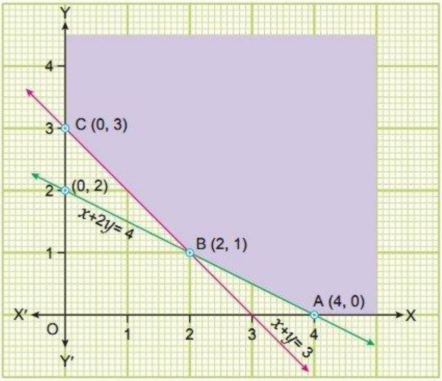
Feasible resion is shaded region shown as *OABC* is bounded and coordinates of its corner points are *O*(0, 0), *A*(7, 0), *B*(3, 4) and *C*(0, 2) respectively.

Corner Points	$Z = 13x - 15y$
O (0, 0)	0
A (7, 0)	91
B (3, 4)	–21
C (0, 2)	–30 ← Minimum

Hence, the minimum value of Z is –30 at (0, 2).

2. The feasible region for a LPP is shown in the following figure. Evaluate $Z = 4x + y$ at each of the corner points of this region. Find the minimum value of Z, if it exists.

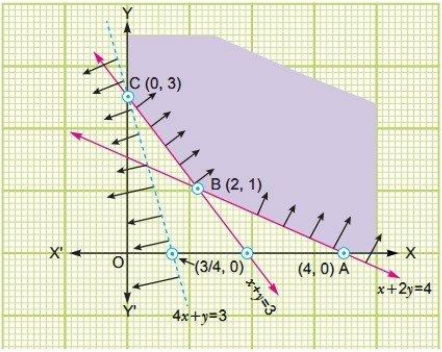
[*NCERT Exemplar*]



Sol. From the fig, it is clear that feasible region is unbounded with the corner points A (4, 0), B (2, 1) and C (0, 3). [$\because$ $x + 2y = 4$ and $x + y = 3 \Rightarrow y = 1$ and $x = 2$]

Also, we have $Z = 4x + y$

Corner Points	$Z = 4x + y$
A (4, 0)	16
B (2, 1)	9
C (0, 3)	3 ← Minimum



Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that, the region is unbounded, therefore 3 may or may not be the minimum value of Z.

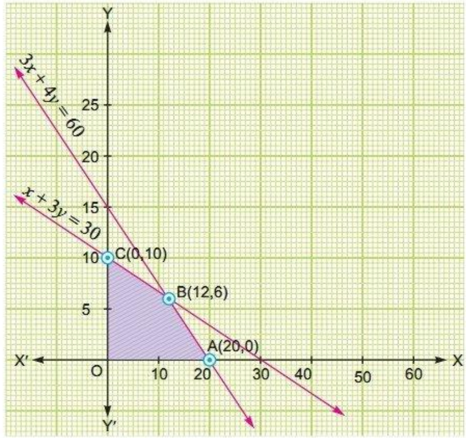
To decide this issue, we graph the inequality $4x + y < 3$ and check wether the resulting open half plane has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph, it is clear that there is no point common with feasible region and hence, Z has minimum value 3 at (0, 3).

3. Maximize: $Z = 80x + 120y$

Subject to the constraints:

$3x + 4y \leq 60, x + 3y \leq 30, x, y \geq 0$.



Sol. Objective function, $Z = 80x + 120y$ ...(*i*)

We have to maximize Z, subject to the constraints

$\Rightarrow \quad 3x + 4y \leq 60$ ...(*ii*)

$x + 3y \leq 30$ ...(*iii*)

$x \geq 0, y \geq 0$ ...(*iv*)

The graph of constraints are drawn and feasible region *OABC* is obtained, which is bounded having corner points *O* (0, 0), *A* (20, 0), *B* (12, 6) and *C* (0, 10).

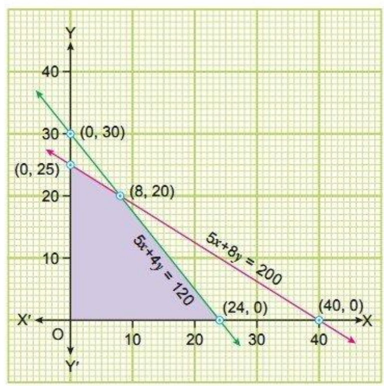
Now the value of objective function is obtained at corner points as

Corner points	$Z = 80x + 120y$
O (0, 0)	0
A (20, 0)	1600
B (12, 6)	1680 ← Maximum
C (0, 10)	1200

Z has maximum value 1680 at (12, 6).

4. Maximize: $Z = 100x + 120y$

Subject to: $5x + 8y \leq 200, 5x + 4y \leq 120, x, y \geq 0$.



Sol. Maximize $Z = 100x + 120y$ ...(*i*)

Subject to $5x + 8y \leq 200$ ...(*ii*)

$5x + 4y \leq 120$ ...(*iii*)

$x, y \geq 0$ ...(*iv*)

Plotting the constraints

Feasible region is shaded region with corner points (0, 0), (24, 0), (8, 20), and (0, 25).

Value of $Z = 100x + 120y$

At (0, 0), $Z = 0$

At (0, 25) $Z = 3000$

At (24, 0) $Z = 2400$

At (8, 20) $Z = 3200$ ← Maximum

$\therefore$ Maximum value of Z is ₹3200 at point (8, 20).

5. Solve the following linear programming problem graphically :

Maximum $Z = 3x + 9y$

Subject to the constraints $x + y \geq 10, x + 3y \leq 60, x \leq y, x \geq 0, y \geq 0$. [*CBSE 2023 (65/3/2)*]

Sol. We have,

$$Z_{\text{Max}} = 3x + 9y$$

Subject to the constraints

$x + y \geq 10$ ...(*i*)

$x + 3y \leq 60$ ...(ii)

$x \leq y \Rightarrow y - x \geq 0$ ...(iii)

$x, y \geq 0$ ...(iv)

From (*i*), we have equation $x + y = 10$, check at (0, 0) for $x + y \geq 10$, $0 \geq 10$, false.

x	0	10
y	10	0

$\therefore$ Region away from origin.

From (*ii*), we have equation $x + 3y = 60$, check at (0, 0) for $x + 3y \leq 60$, $0 \leq 60$, true.

x	0	60
y	20	0

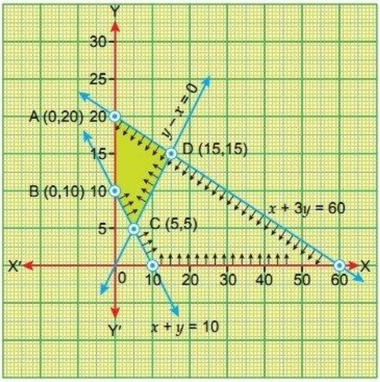
$\therefore$ Region towards from origin.

From (*iii*), we have equation $x = y$, check at (0, 0) for $x \leq y$, $0 \leq 0$, false.

x	10	2
y	0	2

$\therefore$ Region away from (10, 0).

On plotting (*i*), (*ii*) and (*iii*), we have required region (shaded) *ABCDA*.



Points of intersection:

On solving $x + y = 10$

$y - x = 0$, we get, $C(5, 5)$.

Also, on solving

$x + 3y = 60$

$y - x = 0$

$D(15, 15)$

Corner points	$Z = 3x + 9y$
$A(0, 20)$	180 ← Maximum
$B(0, 10)$	90
$C(5, 5)$	60
$D(15, 15)$	180 ← Maximum

$\therefore Z_{Max} = 180$ at infinitely many points lying on the line joining points (0, 20) and (15, 15).

6. Solve the following linear programming problem graphically:

Minimize: $Z = 5x + 10y$

subject to constraints : $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$. [*CBSE 2023 (65/2/1)*]

Sol. Objective function

$$Z = 5x + 10y$$

Subject to constraints

$x + 2y \leq 120$ ...(i)

$x + y \geq 60$ ...(ii)

$x - 2y \geq 0$ ...(iii)

$x, y \geq 0$ ...(iv)

From (i), equation is $x + 2y = 120$, check $x + 2y \leq 120$ at (0, 0)

x	0	120
y	60	0

$0 \leq 120$ true

$\therefore$ Region towards origin.

From (ii), equation is $x + y = 60$, check $x + y \geq 60$ at (0, 0)

x	0	60
y	60	0

$0 \geq 60$ false

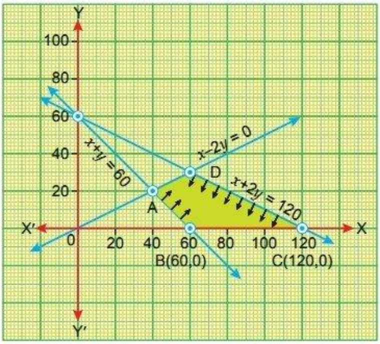
$\therefore$ Region away from origin.

From (iii), equation is $x - 2y = 0$, check at (60, 0), $x - 2y \geq 0$

x	0	2
y	0	1

$60 \geq 0$ true

$\therefore$ Region towards point B.



Points of intersection of $x + y = 60$ and $x - 2y = 0$ is A (40, 20)

and point of intersection of $x + 2y = 120$ and $x - 2y = 0$ is D(60,30).

$\therefore$

Corner Points	$Z = 5x + 10y$
A(40, 20)	400
B(60, 0)	300 ← Minimum
C(120, 0)	600
D(60, 30)	600

$\therefore$ $Z_{min.} = 300$ at (60, 0)

7. Solve graphically the following linear programming problem:

Maximise $Z = 6x + 3y$,

subject to the constraints

$$4x + y \geq 80,$$
$$3x + 2y \leq 150,$$
$$x + 5y \geq 115,$$
$$x \geq 0, y \geq 0.$$

[CBSE 2023 (65/1/1)]

Sol. $Z_{max.} = 6x + 3y$

Subject to the constraints

$$4x + y \geq 80 \quad ...(i)$$
$$3x + 2y \leq 150 \quad ...(ii)$$
$$x + 5y \geq 115 \quad ...(iii)$$
$$x, y \geq 0 \quad ...(iv)$$

Equation for (*i*), we have

$4x + y = 80$

x	0	20
y	80	0

Check at (0, 0) for $4x + y \geq 80$, $0 \geq 80$, false.

$\therefore$ Region away from origin.

Equation for (*ii*), we have

$3x + 2y = 150$

x	0	50
y	75	0

Check at (0, 0) for $3x + 2y \leq 150$, $0 \leq 150$, true.

$\therefore$ Region is towards the origin.

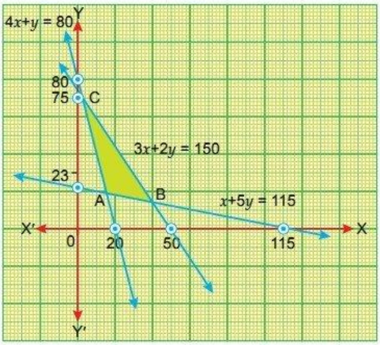
Equation for (*iii*), we have

$x + 5y = 115$

x	0	115
y	23	0

Check at (0, 0) for $x + 5y \geq 115$, false.

$\therefore$ Region away from origin.



On solving equation $4x + y = 80$ and $x + 5y = 115$, we get corner point $A(15, 20)$.

On solving equation $3x + 2y = 150$ and $x + 5y = 115$, we get corner point $B(40, 15)$.

On solving equation $4x + y = 80$ and $3x + 2y = 150$, we get corner point $C(2, 72)$.

Now, we have

Corner Points	$Z = 6x + 3y$
A (15, 20)	150
B (40, 15)	285 ← Maximum
C (2, 72)	228

$\therefore Z_{max.} = 285$ at $B(40, 15)$

8. Maximize: $Z = 300x + 190y$

Subject to constraints: $x + y \leq 24, x + \frac{1}{2}y \leq 16, x, y \geq 0.$

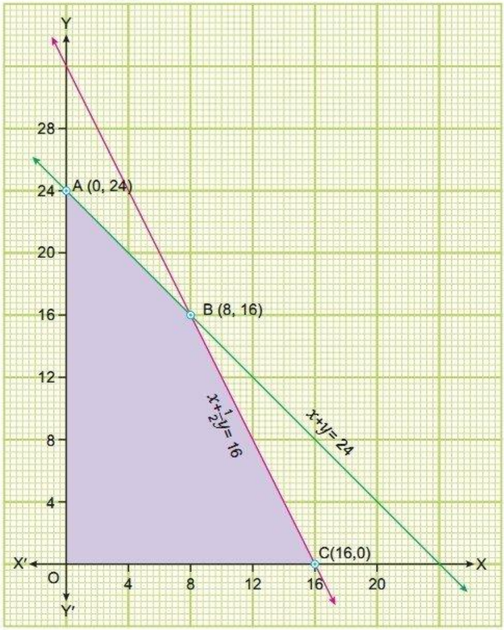
Sol. LPP is

Maximize $\quad Z = 300x + 190y \quad$ ...(*i*)

Subject to constraints $\quad x + y \leq 24 \quad$ ...(*ii*)

$$x + \frac{1}{2}y \leq 16 \qquad ...(iii)$$

$$x \geq 0, y \geq 0 \qquad ...(iv)$$



Feasible region is shaded region with corner points $O(0, 0)$, $A(0, 24)$, $B(8, 16)$, $C(16, 0)$.

Corner Points	$Z = 300x + 190y$
$O(0, 0)$	0
$A(0, 24)$	4560
$B(8, 16)$	5440 ← Maximum
$C(16, 0)$	4800

Z is maximum at (8, 16) and maximum value is 5440.

9. Maximize: $Z = 5x + 8y$

Subject to the constraints:

$$x + y = 5, x \leq 4, y \geq 2, x, y \geq 0.$$

Sol. Here, $Z = 5x + 8y$ which is objective function and is to be maximised subjected to following constraints.

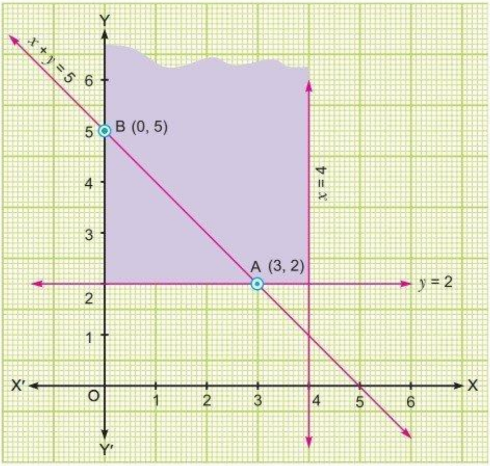
$$x + y = 5 \qquad \dots (i)$$

$$x \leq 4 \qquad \dots (ii)$$

$$y \geq 2 \qquad \dots (iii)$$

$$x \geq 0, y \geq 0 \qquad \dots (iv)$$

In this case, constraint (*i*) is a line passing through the feasible region determined by constraints (*ii*), (*iii*) and (*iv*).



Therefore, maximum or minimum value of objective function 'Z' exist on end points of line (constraint) (*i*) in feasible region *i.e.*, at *A* or *B*.

At A (3, 2), $Z = 5 \times 3 + 8 \times 2 = 15 + 16 = 31$

At B (0, 5), $Z = 5 \times 0 + 8 \times 5 = 0 + 40 = 40$ ⟵ Maximum

Maximum value of Z is 40 at (0, 5).

10. Maximize: $Z = 20x + 10y$

Subject to constraints:

$$1.5x + 3y \leq 42,\ 3x + y \leq 24,\ x, y \geq 0.$$

Sol. $Z = 20x + 10y$ ...(*i*)

We have to maximise Z subject to the constraints:

$1.5x + 3y \leq 42$ ...(*ii*)

$3x + y \leq 24$ ...(*iii*)

$x, y \geq 0$ ...(*iv*)

Graph of $x = 0$ and $y = 0$ is the y-axis and x-axis respectively.

∴ Graph of $x \geq 0, y \geq 0$ is the Ist quadrant.

Graph of $1.5x + 3y = 42$

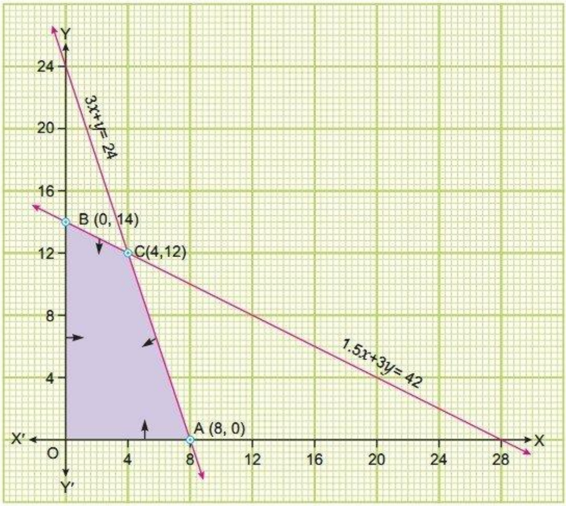
x	0	28
y	14	0

∴ Graph for $1.5x + 3y \leq 42$ is the part of Ist quadrant which contains the origin.

Graph of $3x + y = 24$

x	0	8
y	24	0

∴ Graph of $3x + y \leq 24$ is the part of Ist quadrant in which origin lie.



Hence, shaded area $OACB$ is the feasible region.

For coordinate of C, equation $1.5x + 3y = 42$ and $3x + y = 24$ are solved as

$$1.5x + 3y = 42 \quad \ldots(v)$$

$$3x + y = 24 \quad \ldots(vi)$$

$$2 \times (v) - (vi) \Rightarrow \begin{array}{r} 3x + 6y = 84 \\ \underline{{}_-3x \pm y = {}_-24} \\ 5y = 60 \end{array}$$

$\Rightarrow \quad y = 12 \quad \Rightarrow \quad x = 4 \qquad$ (Substituting $y = 12$ in (vi))

Now, value of objective function Z at each corner of feasible region is

Corner Points	$Z = 20x + 10y$	
O (0, 0)	0	
A (8, 0)	$20 \times 8 + 10 \times 0 = 160$	
B (0, 14)	$20 \times 0 + 10 \times 14 = 140$	
C (4, 12)	$20 \times 4 + 10 \times 12 = 200$	← Maximum

Maximum value of Z is 200 at (4, 12).

11. Maximize: $Z = 22x + 18y$

Subject to constraints: $3x + 2y \leq 48$, $x + y \leq 20$, $x, y \geq 0$.

Sol. We are given LPP

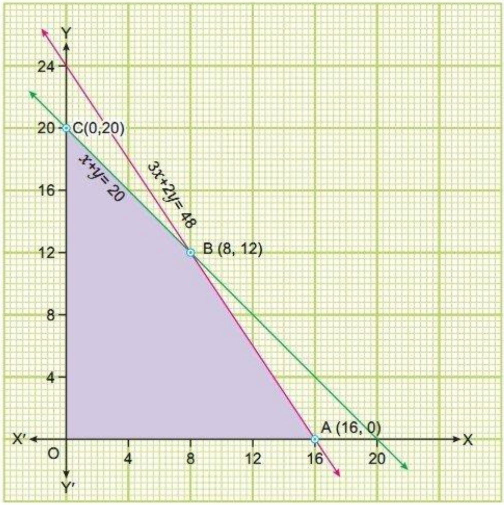
Maximise $Z = 22x + 18y \quad \ldots(i)$

Subject to the constraints:

$$3x + 2y \leq 48 \quad \ldots(ii)$$

$$x + y \leq 20 \quad \ldots(iii)$$

$$x, y \geq 0 \quad \ldots(iv)$$



The region satisfying inequalities (*ii*) to (*iv*) is shown (shaded) in the figure.

Feasible reagion is shaded region with corner points $O(0, 0)$, $A(16, 0)$, $B(8, 12)$, $C(0, 20)$.

Let us evaluate, $Z = 22x + 18y$ at each corner point.

Corner Points	$Z = 22x + 18y$
O (0, 0)	0
A (16, 0)	352
B (8, 12)	392 ← Maximum
C (0, 20)	360

Thus, maximum value of Z is 392 at B (8, 12).

12. Minimize: $Z = 10x + 4y$

Subject to constraints:

$$4x + y \geq 80,\ 2x + y \geq 60,\ x, y \geq 0.$$

Sol. $Z = 10x + 4y$ ...(*i*)

is objective function, which we have to minimize.

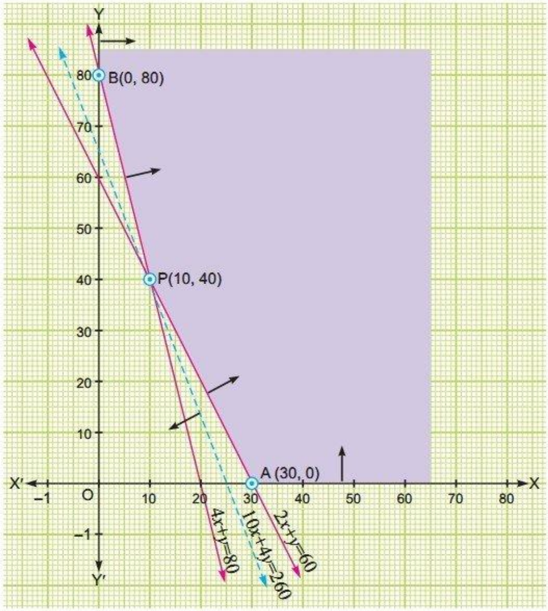
Here, constraints are:

$4x + y \geq 80$ ...(*ii*)

$2x + y \geq 60$ ...(*iii*)

Also, $x, y \geq 0$ ...(*iv*)

On plotting graph of above constraints or inequalities (*ii*), (*iii*) and (*iv*), we get shaded region having corner point A, P, B as feasible region.



For coordinate of P.

Point of intersection of

$2x + y = 60$ ...(v)

and $4x + y = 80$ ...(vi)

$(v) - (vi)$

$\Rightarrow$ $2x + y - 4x - y = 60 - 80$

$\Rightarrow$ $-2x = -20$

$\Rightarrow$ $x = 10$

$\Rightarrow$ $y = 40$

Coordinate of $P \equiv (10, 40)$

Now the value of Z is evaluated at corner point in the following table

Corner Points	$Z = 10x + 4y$	
A (30, 0)	300	
P (10, 40)	260	← Minimum
B (0, 80)	320	

Since, feasible region is unbounded. Therefore we have to draw the graph of the inequality.

$10x + 4y < 260$ ...(vii)

Since, the graph of inequality (vii) does not have any point common.

So, the minimum value of Z is 260 at (10, 40).

13. Maximize: $Z = 10500x + 9000y$

Subject to constraints: $x + y \leq 50, 2x + y \leq 80, x, y \geq 0$.

Sol. Given

$$Z = 10500x + 9000y \quad \ldots(i)$$

We have to maximize Z subject to the constraints:

$$x + y \leq 50 \quad \ldots(ii)$$

$$2x + y \leq 80 \quad \ldots(iii)$$

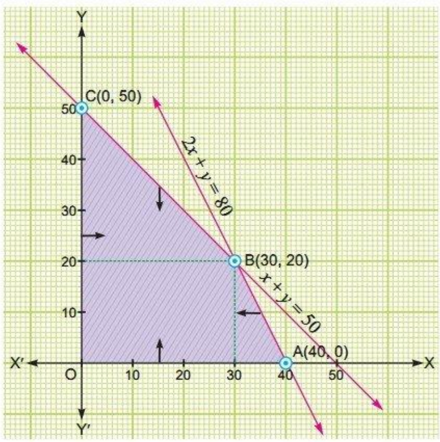
$$x \geq 0, y \geq 0 \quad \ldots(iv)$$

Table for $x + y = 50$

x	0	50
y	50	0

Table for $2x + y = 80$

x	0	40
y	80	0



The graph of system of inequalities (*ii*) to (*iv*) are drawn, which gives feasible region *OABC* with corner points *O* (0, 0), *A* (40, 0), *B* (30, 20) and *C* (0, 50).

Feasible region is bounded.

Now,

Corner points	$Z = 10500x + 9000y$
O (0, 0)	0
A (40, 0)	420000
B (30, 20)	495000 ← Maximum
C (0, 50)	450000

Maximum value of Z is 495000 at (30, 20).

Long Answer Questions

1. **Solve the following Linear Programming Problem graphically:**

Maximize : $P = 70x + 40y$

subject to : $3x + 2y \leq 9$

$3x + y \leq 9$

$x \geq 0, y \geq 0$ **[CBSE 2023 (65/5/1)]**

Sol. We are given LPP

Max $P = 70x + 40y$ subject to

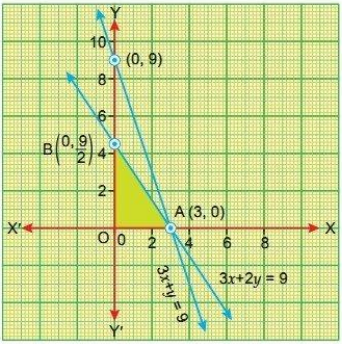
$3x + 2y \leq 9$ ...(*i*)

$3x + y \leq 9$ ...(*ii*)

$x \geq 0$ ...(*iii*)

$y \geq 0$ ...(*iv*)

On plotting the Constraints (*i*), (*ii*), (*iii*) and (*iv*) we have



We have feasible region is shaded region with corner points $O(0, 0)$, $A(3, 0)$, $B\left(0, \frac{9}{2}\right)$.

Corner Points	$P = 70x + 40y$
$O(0, 0)$	0
$A(3, 0)$	210 ← Maximum
$B\left(0, \frac{9}{2}\right)$	180

Maximum value of P is 210 at the point $A(3, 0)$.

2. **Maximize: $Z = 100x + 120y$**

Subject to the constraints: $2x + 3y \leq 30, 3x + y \leq 17, x, y \geq 0$.

Sol. Here, $Z = 100x + 120y$ ...(*i*)

Subjects to constraints:

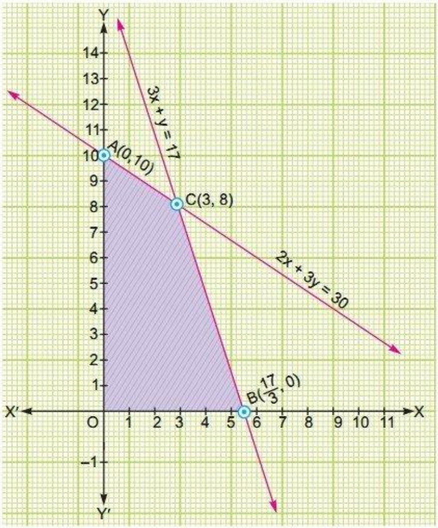
Also $2x + 3y \leq 30$ ...(*ii*)

$$3x + y \leq 17 \qquad ...(iii)$$
$$x,\ y \geq 0 \qquad ...(iv)$$

On plotting graph of above inequalities (*ii*), (*iii*) and (*iv*). We get shaded region as feasible region having corner points *A*, *O*, *B* and *C*.

For coordinate of 'C'

Two equations (*ii*) and (*iii*) are solved and we get coordinate of *C* = (3, 8).



Now, the value of *Z* is evaluated at corner points as:

Corner points	$Z = 100x + 120y$
$O\ (0, 0)$	0
$A\ (0, 10)$	1200
$B\left(\frac{17}{3}, 0\right)$	$\frac{1700}{3}$
$C\ (3, 8)$	1260 ← Maximum

Maximum value of *Z* is 1260 at (3, 8).

3. Maximize: $Z = 60x + 40y$

Subject to the constraints:

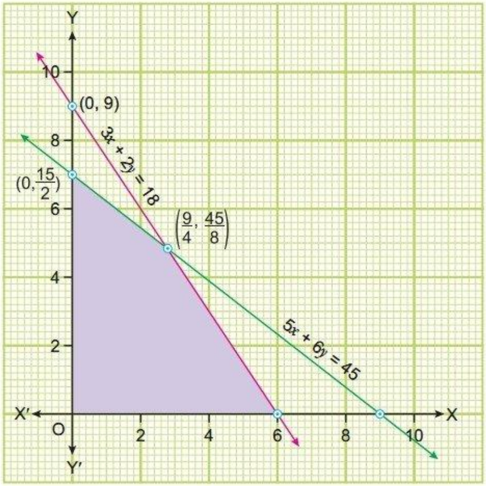
$$5x + 6y \leq 45,\ 3x + 2y \leq 18,\ x,\ y \geq 0.$$

Sol. Objective function is to maximize $Z = 60x + 40y$

$$5x + 6y \leq 45 \qquad ...(i)$$
$$3x + 2y \leq 18 \qquad ...(ii)$$
$$x, y \geqslant 0 \qquad ...(iii)$$

We plot the graph of inequations shaded region is the feasible solution (*i*) (*ii*) and (*iii*).

The shaded region in the figure represents the feasible region which is bounded. Let us now evaluate Z at each corner point.



Z at (0, 0) is $60 \times 0 + 40 \times 0 = 0$

Z at $\left(0, \frac{15}{2}\right)$ is $60 \times 0 + 40 \times \frac{15}{2} = 300$

Z at $\left(\frac{9}{4}, \frac{45}{8}\right)$ is $60 \times \frac{9}{4} + 40 \times \frac{45}{8} = 135 + 225 = 360$ ⟵ Maximum

Z at (6, 0) is $60 \times 6 + 0 = 360$ ⟵ Maximum

Maximum value of Z is 360 obtained at any point on the line segment joining $\left(\frac{9}{4}, \frac{45}{8}\right)$ and (6, 0).

4. Maximize: $Z = 15x + 10y$
Subject to the constraints: $2x + y \le 40$, $2x + 3y \le 80$, $x, y \ge 0$.

Sol. We have LPP

$$Z_{Max.} = 15x + 10y$$

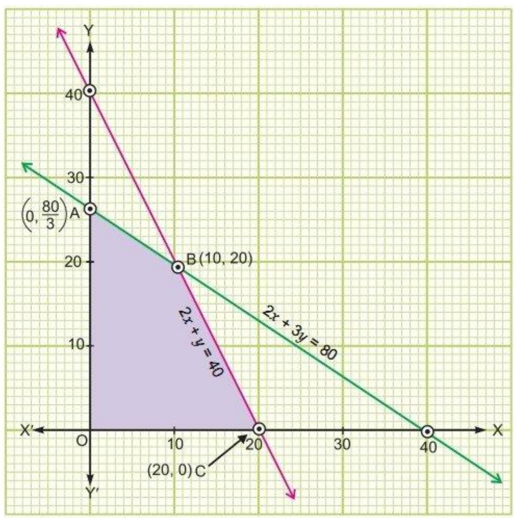
Subject to constraints

$$2x + y \le 40 \quad ...(i)$$

$$2x + 3y \le 80 \quad ...(ii)$$

$$x, \ y \ge 0 \quad ...(iii)$$

The feasible region determined by the system of constraints is $OABC$. The corner points are, $O(0, 0)$, $A\left(0, \frac{80}{3}\right)$, $B(10, 20)$, $C(20, 0)$.



Corner points	$Z = 15x + 10y$
$O(0, 0)$	0
$A\left(0, \frac{80}{3}\right)$	$\frac{800}{3}$
$B(10, 20)$	350 ← Maximum
$C(20, 0)$	300

The maximum value of $Z = 350$ which is attained at $B(10, 20)$.

5. **Maximize: $Z = 1000x + 500y$**

Subject to the constraints: $3x + 5y \le 225$, $2x + y \le 80$, $x, y \ge 0$.

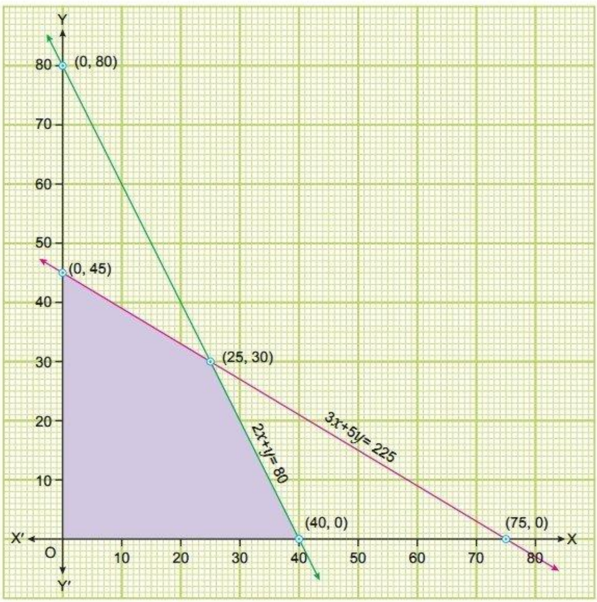
Sol. LPP is

Maximise $Z = 1000x + 500y$

Subject to, $3x + 5y \le 225$; $2x + y \le 80$; $x \ge 0, y \ge 0$

From the shaded feasible region, it is clear that coordinates of corner points are (0, 0), (40, 0), (25, 30) and (0, 45).

On Solving $3x + 5y = 225$ and $2x + y = 80$, we get $x = 25, y = 30$



Corner Points	$Z = 1000x + 500y$
(0, 0)	0
(40, 0)	40000 ← Maximum
(25, 30)	25000 + 15000 = 40000 ← Maximum
(0, 45)	22500

Maximum value of Z is 40000 obtained at any point on the line segment joining (40, 0) and (25, 30).

6. Solve the following LPP graphically:

Minimise $Z = 5x + 7y$

Subject to the constraints

$$2x + y \geq 8$$
$$x + 2y \geq 10$$
$$x, y \geq 0$$

[CBSE (F) 2020, (65/3/1)]

Sol. Given constraints are

$$2x + y \geq 8$$
$$x + 2y \geq 10$$

and $x, y \geq 0$

For the graph of $2x + y \geq 8$, we draw the graph of $2x + y = 8$

x	0	4
y	8	0

Now, checking for (0, 0) we have $2\times 0 + 0 \geq 8 \quad \Rightarrow \quad 0 \geq 8$

$\therefore$ Origin (0, 0) does not satisfy $2x + y \geq 8$.

$\therefore$ Region lies away from origin.

For the graph of $x + 2y \geq 10$, we draw the graph of $x + 2y = 10$.

x	0	10
y	5	0

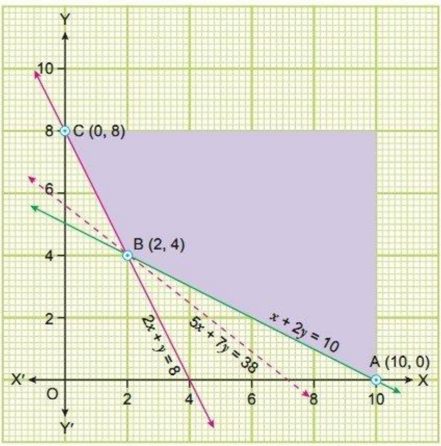
Now, checking for origin (0, 0), we have

$$0 + 2\times 0 \geq 10 \quad \Rightarrow \quad 0 \geq 10$$

$\therefore$ Origin (0, 0) does not satisfy $x + 2y \geq 10$.

$\therefore$ Region lies away from origin.

Now $x, y \geq 0$, it means region will lie in first quadrant.



On plotting graph of given inequalities (or constraints)

We get the region (shaded) with corner points

A (10, 0), B(2, 4) and C(0, 8).

Now, the value of Z is evaluated at corner points in the following table.

Corner Points	$Z = 5x + 7y$
A (10, 0)	50
B (2, 4)	38 ← Minimum
C (0, 8)	56

Since, feasible region is unbounded. Therefore, we have to draw the graph of the inequality.

$$5x + 7y < 38$$

Since, the graph of this inequality does not have any point common.

So, the minimum value of Z is 38 at (2, 4).

Hence, $Z_{min} = 38$ at (2, 4).

7. Maximise $Z = 8x + 9y$ subject to the constraints given below :

$2x + 3y \le 6;\ 3x - 2y \le 6;\ y \le 1;\ x, y \ge 0$ **[CBSE (F) 2015]**

Sol. Objective function is $Z = 8x + 9y$. Given constraints are

$2x + 3y \le 6$

$3x - 2y \le 6$

$y \le 1$

$x, y \ge 0$

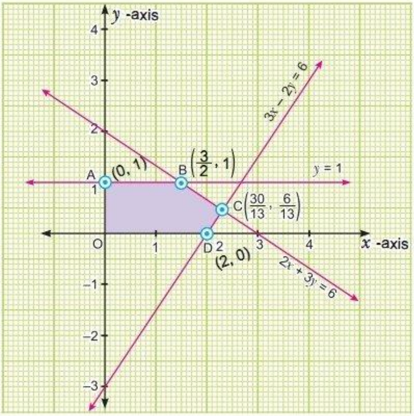
For graph of $2x + 3y \le 6$

We draw the graph of $2x + 3y = 6$

x	0	3
y	2	0

$2 \times 0 + 3 \times 0 \le 6 \Rightarrow$ (0,0) satisfy the constraints.

Hence, feasible region lie towards origin side of line.



For graph of $3x - 2y \le 6$

We draw the graph of line $3x - 2y = 6$.

x	0	2
y	–3	0

$3 \times 0 - 2 \times 0 \le 6$

$\Rightarrow$ Origin (0, 0) satisfy $3x - 2y \le 6$.

Hence, feasible region lie towards origin side of line.

For graph of $y \le 1$

We draw the graph of line $y = 1$, which is parallel to x-axis and meet y-axis at 1.

$0 \le 1 \Rightarrow$ feasible region lie towards origin side of $y = 1$.

Also, $x \ge 0, y \ge 0$ says feasible region is in Ist quadrant.

Therefore, $OABCDO$ is the required feasible region, having corner point $O(0, 0)$, $A(0, 1)$ $B\left(\frac{3}{2}, 1\right), C\left(\frac{30}{13}, \frac{6}{13}\right), D(2, 0)$.

Here, feasible region is bounded. Now the value of objective function $Z = 8x + 9y$ is obtained as.

Corner Points	$Z = 8x + 9y$	
$O(0, 0)$	0	
$A(0, 1)$	9	
$B\left(\frac{3}{2}, 1\right)$	21	
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	22.6	← Maximum
$D(2, 0)$	16	

Z is maximum when $x = \frac{30}{13}$ and $y = \frac{6}{13}$.

8. Minimize and maximize $Z = 5x + 2y$ subject to the following constraints:

$x - 2y \le 2$, $3x + 2y \le 12$, $-3x + 2y \le 3$, $x \ge 0, y \ge 0$ **[*CBSE Panchkula 2015*]**

Sol. Here, objective function is

$$Z = 5x + 2y \quad ...(i)$$

Subject to the constraints :

$$x - 2y \le 2 \quad ...(ii)$$

$$3x + 2y \le 12 \quad ...(iii)$$

$$-3x + 2y \le 3 \quad ...(iv)$$

$$x \ge 0, y \ge 0 \quad ...(v)$$

Graph for $x - 2y \le 2$

We draw graph of $x - 2y = 2$ as

x	0	2
y	–1	0

$0 - 2 \times 0 \le 2$ [By putting $x = y = 0$ in the equation]

i.e., (0, 0) satisfy (*ii*) $\Rightarrow$ feasible region lie origin side of line $x - 2y = 2$.

Graph for $3x + 2y \le 12$

We draw the graph of $3x + 2y = 12$.

x	0	4
y	6	0

$3 \times 0 + 2 \times 0 \le 12$ [By putting $x = y = 0$ in the given equation]

i.e., (0, 0) satisfy (*iii*) $\Rightarrow$ feasible region lie origin side of line $3x + 2y = 12$.

Graph for $-3x + 2y \le 3$

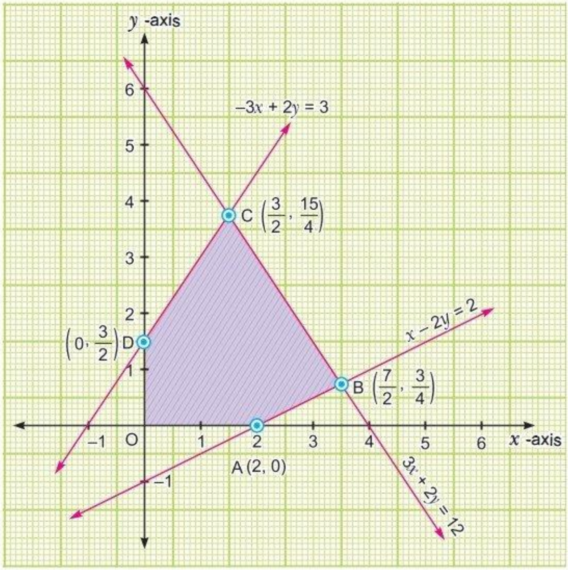
We draw the graph of $-3x + 2y \le 3$

x	–1	0
y	0	1.5

$-3 \times 0 + 2 \times 0 \le 3$ [By putting $x = y = 0$]

i.e., (0, 0) satisfy (*iv*) $\Rightarrow$ feasible region lie origin side of line $-3x + 2y = 3$.

$x \geq 0, y \geq 0 \Rightarrow$ feasible region is in Ist quadrant.



Now, we get shaded region having corner points *O*, *A*, *B*, *C* and *D* as feasible region.

The co-ordinates of *O*, *A*, *B*, *C* and *D* are $O(0, 0)$, $A(2, 0)$, $B\left(\frac{7}{2}, \frac{3}{4}\right)$, $C\left(\frac{3}{2}, \frac{15}{4}\right)$ and $D\left(0, \frac{3}{2}\right)$ respectively. Now, we evaluate Z at the corner points.

Corner Points	$Z = 5x + 2y$	
$O(0, 0)$	0	← Minimum
$A(2, 0)$	10	
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	19	← Maximum
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	15	
$D\left(0, \frac{3}{2}\right)$	3	

Hence, Z is minimum at $x = 0, y = 0$ and minimum value = 0

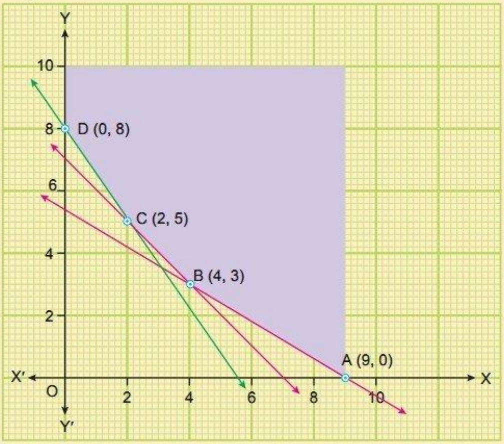
also Z is maximum at $x = \frac{7}{2}, y = \frac{3}{4}$ and maximum value = 19.

Questions for Practice

■ Objective Type Questions

1. **Choose and write the correct option in each of the following questions.**

(*i*) Feasible region (shaded) for a LPP is shown in the given figure. Minimum of $Z = 4x + 3y$ occurs at the point.



(*a*) (0, 8) (*b*) (2, 5) (*c*) (4, 3) (*d*) (9, 0)

(*ii*) The solution set of the inequation $3x + 2y > 3$ is

(*a*) half plane not containing the origin (*b*) half plane containing the origin

(*c*) the point being on the line $3x + 2y = 3$ (*d*) None of these

(*iii*) If the constraints in a linear programming problem are changed

(*a*) solution is not defined (*b*) the objective function has to be modified

(*c*) the problems is to be re-evaluated (*d*) none of these

(*iv*) Which of the following statement is correct?

(*a*) Every LPP admits an optimal solution.

(*b*) Every LPP admits unique optimal solution.

(*c*) If a LPP gives two optimal solutions it has infinite number of solutions.

(*d*) None of these

(*v*) The maximum value of $p = x + 3y$ such that $2x + y \leq 20$, $x + 2y \leq 20$, $x \geq 0$, $y \geq 0$ is

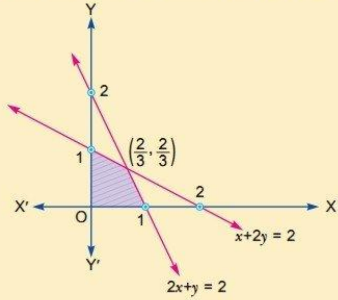
(*a*) 10 (*b*) 30 (*c*) 60 (*d*) $\frac{80}{3}$

■ Conceptual Questions

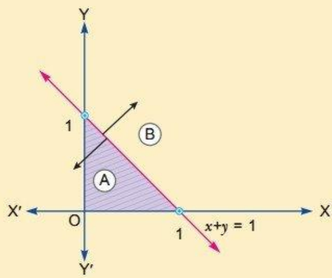
2. Determine the maximum value of $Z = 3x + 4y$,
   Subject to the constraints: $x + y \leq 1, x \geq 0, y \geq 0$.
3. If a linear programming problem is $Z_{max} = 3x + 2y$,
   Subject to the constraints: $x + y \leq 2$, find Z_{max}.
4. Maximise the function $Z = 11x + 7y$, subject to the constraints: $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.

Short Answer Questions

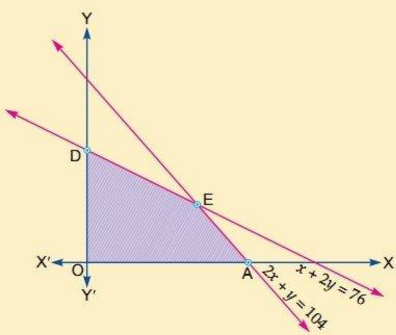
5. Solve the linear inequation $-3x + 2y \geq 6$ graphically.
6. What is the maximum value of objective function $Z = 3x + y$ under given feasible region?



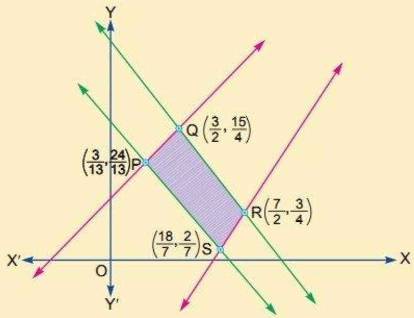
7. In figure, which half plane (A) or (B) is the solution of $x + y > 1$? Justify your answer.



8. Maximize $Z = 6x + 16y$ subject to constraints $x + y \geq 2, x, y \geq 0$.
9. Find the maximum value of $Z = 3x + 2y$ where corner points of feasible region are (0, 0), (0, 8), (2, 7), (5, 4) and (6, 0).
10. Let $Z = ax + by$ has optimal value at two points (2, 3) and (5, 7), then derive the relationship between a and b.
11. Determine the maximum value of $Z = 3x + 4y$ of the feasible region (shaded) for a LPP is shown in figure.



12. In figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of $Z = x + 2y$.



■ **Long Answer Questions**

13. Solve the following linear programming problem graphically:

Minimise $Z = x - 5y + 20$

Subject to constraints: $x - y \geq 0, -x + 2y \geq 2;$

$x \geq 3, y \leq 4, x, y \geq 0$

14. Solve the following LPP:

Maximise $Z = 5x_1 + 7x_2,$

Subject to constraints: $x_1 + x_2 \leq 4,$

$3x_1 + 8x_2 \leq 24,$

$10x_1 + 7x_2 \leq 35,$

$x_1, x_2 \geq 0.$

15. Maximise $Z = x + y$ subject to $x + 4y \leq 8, 2x + 3y \leq 12, 3x + y \leq 9, x \geq 0, y \geq 0.$

16. Solve the following LPP graphically:

Maximise $Z = 1000x + 600y$

Subject to the constraints

$x + y \leq 200$

$x \geq 20$

$y - 4x \geq 0$

$x, y \geq 0$ **[*CBSE (F)* 2017]**

17. Solve the following LPP graphically:

Maximise $Z = 4x + y$

Subject to following constraints $x + y \leq 50$,

$3x + y \leq 90$, $x \geq 10$, $x, y \geq 0$ **[*CBSE Delhi* 2017]**

18. Solve the following linear programming problem graphically:

Maximise $Z = 7x + 10y$

Subject to constraints

$4x + 6y \leq 240$

$6x + 3y \leq 240$

$x \geq 10$

$x \geq 0, y \geq 0$ **[*CBSE (AI)* 2017]**

19. Find graphically, the maximum value of $Z = 2x + 5y$, subject to constraints given below:

$$2x + 4y \le 8,\ 3x + y \le 6,\ x + y \le 4,\ x \ge 0, y \ge 0$$ [*CBSE Delhi* 2015]

20. Solve the following linear programming problem graphically.

Minimise $Z = 3x + 5y$

Subject to the constraints:

$$x + 2y \ge 10;\ x + y \ge 6;\ 3x + y \ge 8;\ x, y \ge 0$$ [*CBSE Ajmer* 2015]

21. Maximize $Z = x + y$

Subject to the constraints

$$x \le \frac{4}{5}, 3x + y \le 5, x + 2y \le 6, x, y \ge 0$$

22. Maximize $Z = 45000x + 5000y$

Subject to constraints

$$x + y \le 250, 5x + 8y \le 1400, x, y \ge 0$$

23. Maximize $Z = 0.7x + y$

Subject to constraints

$$2x + 3y \le 120,\ 2x + y \le 80,\ x, y \ge 0.$$

24. Maximize $Z = 1000x + 600y$

Subject to constraints

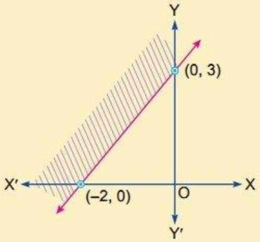
$$x + y \le 200,\ x \ge 20,\ y - 4x \ge 0,\ x, y \ge 0$$

Answers

1. (*i*) (*b*) (*ii*) (*a*) (*iii*) (*c*) (*iv*) (*c*) (*v*) (*b*)

2. 4 3. 6 4. 47

5.



6. 3

7. Half plane B because (0, 0) does not satisfy $x + y > 1$.

8. 12 9. 23 10. $3a + 4b = 0$ 11. 196 12. Maximum= 9, minimum = $3\frac{1}{7}$

13. At (4, 4), $Z_{min} = 4$ 14. $x_1 = \frac{8}{5}, x_2 = \frac{12}{5}; Z_{max} = \frac{124}{5}$ 15. $\frac{43}{11}$ at $\left(\frac{28}{11}, \frac{15}{11}\right)$

16. $Z_{max} = 136000$ at $x = 40$ and $y = 160$ 17. $Z_{max} = 120$ when $x = 30, y = 0$

18. $Z_{max} = 410$ for $x = 30, y = 20$ 19. Maximum value of Z is 10 at $x = 0, y = 2$.

20. Minimum value of Z is 26 at (2, 4). 21. 3.4 at $\left(\frac{4}{5}, \frac{13}{5}\right)$

22. 11250000 at (250, 0) 23. 41 at (30, 20) 24. 1, 36,000 at (40, 160)