

Chapter 4 Matrices and Determinants

Ex 4.5

Answer 1e.

We know that the number beneath the radical sign is the radicand of the expression.

The number 72 is beneath the radical sign. Therefore, in the given expression, 72 is called the radicand.

Answer 1gp.

Rewrite 27 as a product of two factors such that one factor is a perfect square.

$$\sqrt{27} = \sqrt{9 \cdot 3}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3}$$

Now, evaluate $\sqrt{9}$.

$$\sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$$

Therefore, the given expression simplifies to $3\sqrt{3}$.

Answer 1mr.

- a) For a falling object, the height h above the ground after t seconds can be modeled by the function $h = -16t^2 + h_0$ where h_0 is the object's initial height.

In this case, the initial height is 20 feet. Substitute 20 for h_0 to write a function that models the height of the pinecone as it falls.

$$h = -16t^2 + 20$$

- b) **STEP 1** We need to find some points to graph the function. For this, make table of values for the given function.

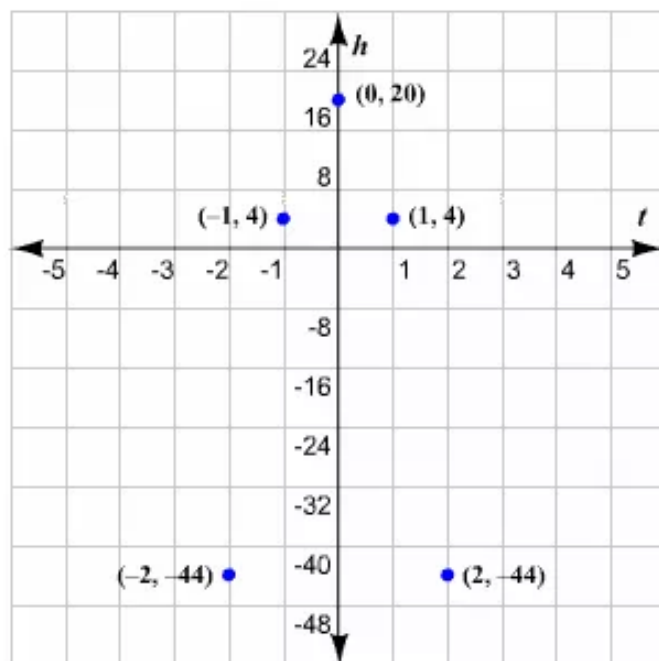
Substitute any value for x , say, -2 and evaluate y .

$$\begin{aligned} y &= -16(-2)^2 + 20 \\ &= -16(4) + 20 \\ &= 44 \end{aligned}$$

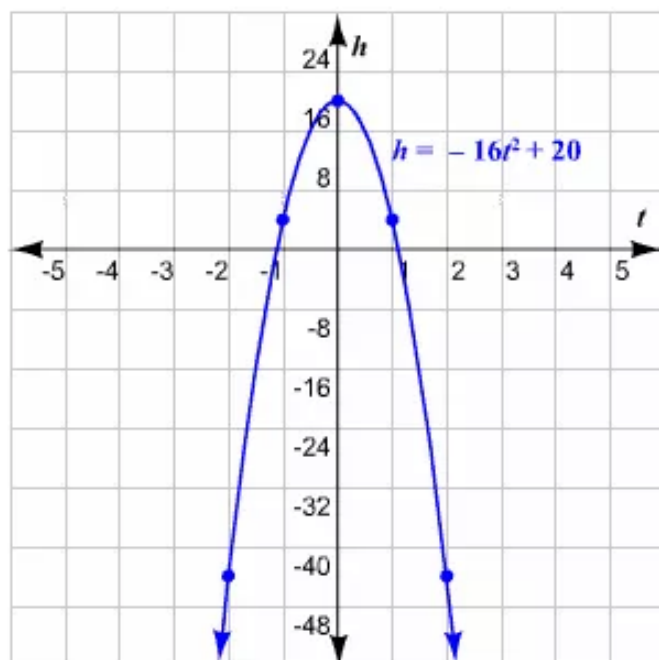
Choose some more x -values and find the corresponding y -values. Organize the results in a table.

x	-2	-1	0	1	2
y	44	4	20	4	44

STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



- c) We know that the height when the falling object hits the surface is 0.

Substitute 0 for h in the function that models the height of the pinecone as it falls.

$$0 = -16t^2 + 20$$

Subtract 20 from each side.

$$\begin{aligned}0 - 20 &= -16t^2 + 20 - 20 \\ -20 &= -16t^2\end{aligned}$$

Now, divide each side by -16 .

$$\begin{aligned}\frac{-20}{-16} &= \frac{-16t^2}{-16} \\ \frac{20}{16} &= t^2\end{aligned}$$

Take the square root on each side.

$$\begin{aligned}\sqrt{\frac{20}{16}} &= \sqrt{t^2} \\ \pm \frac{\sqrt{20}}{4} &= t\end{aligned}$$

Use a calculator to evaluate.

$$t \approx \pm 1.1$$

Since time cannot be negative, reject the negative value. Thus, the pinecone takes about 1.1 sec to hit the ground.

Answer 2e.

We are to explain the meaning of “rationalizing the denominator” of a quotient containing square roots”.

Rationalizing the denominator ; whenever we have a fraction whose denominator is an irrational number, we can multiply both numerator and denominator by an appropriate number (in most of case, a multiple of conjugate of the denominator) in order to eliminate the irrational number from the denominator.

This process is called rationalizing the denominator.

Answer 2gp.

Consider the expression,

$$\sqrt{98}$$

Simplify the following expression.

$$\sqrt{98}$$

The product property of square roots:

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \text{ Whenever } a > 0, b > 0$$

Now,

$$\begin{aligned}\sqrt{98} &= \sqrt{49 \cdot 2} \\ &= \sqrt{7^2 \cdot 2} \\ &= \sqrt{7^2} \cdot \sqrt{2} \quad \text{Apply the product property} \\ &= 7\sqrt{2}\end{aligned}$$

Therefore, $\sqrt{98} = 7\sqrt{2}$

Answer 2mr.

(a)

The function

$$y = -0.0035x(x - 143.9)$$

models the path of water shot by a water cannon where x is the horizontal distance (in feet) and y is the corresponding height (in feet).

The domain of a function is the set of x -values that make that function true and the range of a function is the set of y -values that make that function true.

We transform the function

$y = -0.0035x(x - 143.9)$ into vertex form as given below.

$$y = -0.0035x(x - 143.9) \quad [\text{The original function}]$$

$$y = -0.0035(x^2 - 143.9x) \quad [\text{Apply distributive property}]$$

$$y + (-0.0035)(?) = -0.0035(x^2 - 143.9x + ?) \quad [\text{Prepare to complete the square}]$$

$$y + (-0.0035)(5176.8025) = -0.0035(x^2 - 143.9x + 5176.8025) \quad \left[\begin{array}{l} \text{Add} \\ (-0.0035)(5176.8025) \\ \text{to each side} \end{array} \right]$$

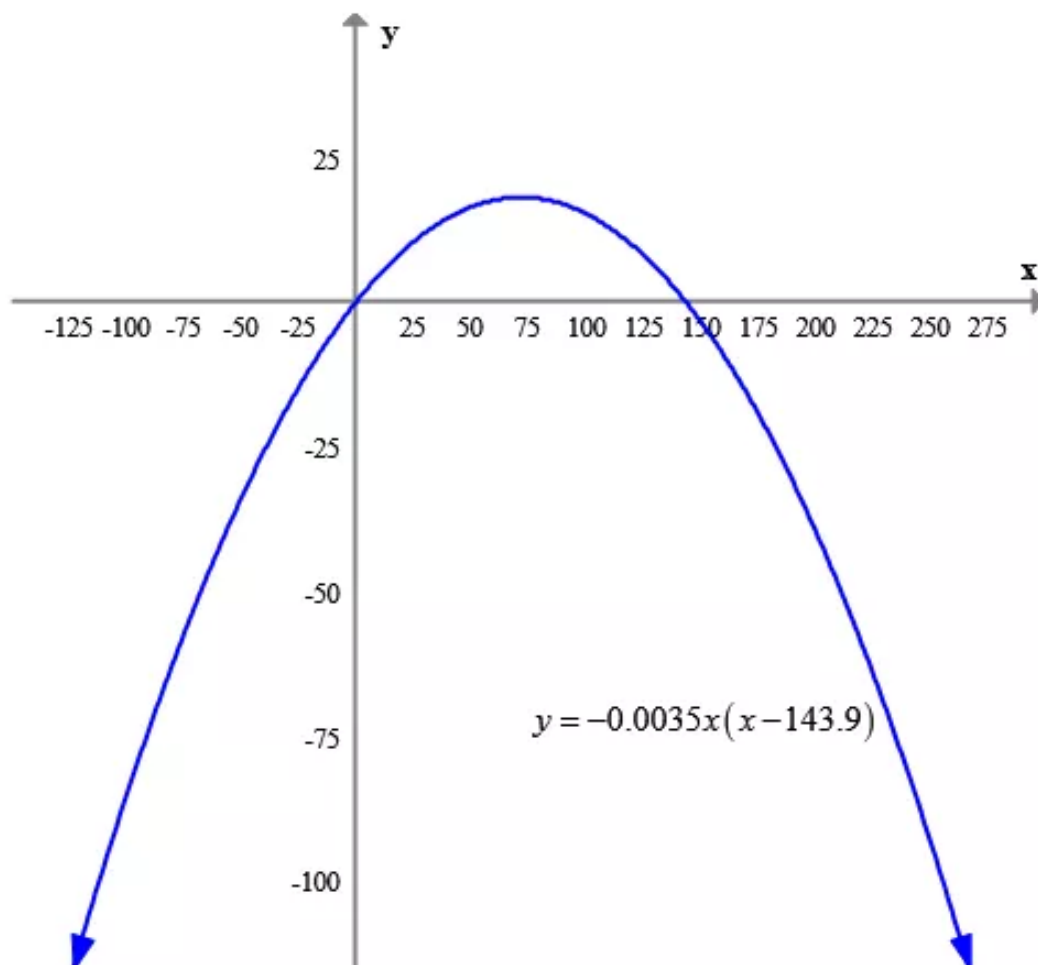
$$y - 18.1188 = -0.0035(x - 71.95)^2$$

$$y = -0.0035(x - 71.95)^2 + 18.1188 \quad [\text{Solve for } y]$$

So, the vertex is

$$(71.95, 18.1188)$$

So, the graph of the function $y = -0.0035x(x - 143.9)$ has as shown below.



Since, the graph continues down town the left and down toward the right, the domain contains all real numbers from negative infinity to positive infinity.

Hence, domain of the function $y = -0.0035x(x - 143.9)$ is

$$-\infty < x < \infty, \text{ or all real numbers} .$$

Since the maximum y-value of the function is 18.1188 and the graph continues to extend downward, the range contains all real numbers less than or equal to 18.1188.

Hence, range of the function $y = -0.0035x(x - 143.9)$ is

$$y \leq 18.1188 .$$

(b)

We first transform the function $y = -0.0035x(x - 143.9)$ into intercept form as given below.

$$y = -0.0035x(x - 143.9) \quad [\text{The original function}]$$

$$y = -0.0035(x - 0)(x - 143.9)$$

Now, comparing the equation $y = -0.0035(x - 0)(x - 143.9)$ with the standard intercept form $y = a(x - p)(x - q)$, we have

$$p = 0, q = 143.9.$$

Because $p = 0$ and $q = 143.9$, we know the x -intercepts are 0 and 143.9.

So, we conclude that the water cannon can shoot up to a distance of

$$\boxed{143.9 \text{ feet}}.$$

(c)

We first transform the function $y = -0.0035x(x - 143.9)$ into vertex form as given below.

$$y = -0.0035x(x - 143.9) \quad [\text{The original function}]$$

$$y = -0.0035(x^2 - 143.9x) \quad [\text{Apply distributive property}]$$

$$y + (-0.0035)(?) = -0.0035(x^2 - 143.9x + ?) \quad [\text{Prepare to complete the square}]$$

$$y + (-0.0035)(5176.8025) = -0.0035(x^2 - 143.9x + 5176.8025) \quad \left[\begin{array}{l} \text{Add} \\ (-0.0035)(5176.8025) \\ \text{to each side} \end{array} \right]$$

$$y - 18.1188 = -0.0035(x - 71.95)^2$$

$$y = -0.0035(x - 71.95)^2 + 18.1188 \quad [\text{Solve for } y]$$

So, the vertex is

$$(71.95, 18.1188)$$

The maximum height of the water is the y -coordinate of the vertex $(71.95, 18.1188)$ of the parabola with the given equation $y = -0.0035x(x - 143.9)$.

Hence, the maximum height of the water is

$$\boxed{18.1188 \text{ feet}}$$

Answer 3e.

Rewrite 28 as a product of two factors such that one factor is a perfect square.

$$\sqrt{28} = \sqrt{4 \cdot 7}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\sqrt{4 \cdot 7} = \sqrt{4} \cdot \sqrt{7}$$

Now, evaluate $\sqrt{4}$.

$$\sqrt{4} \cdot \sqrt{7} = 2\sqrt{7}$$

Therefore, the given expression simplifies to $2\sqrt{7}$.

Answer 3gp.

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\sqrt{10} \cdot \sqrt{15} = \sqrt{10 \cdot 15}$$

Simplify within the radical.

$$\sqrt{10 \cdot 15} = \sqrt{150}$$

Rewrite 150 as a product of two factors such that one factor is a perfect square.

$$\sqrt{150} = \sqrt{25 \cdot 6}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\sqrt{25 \cdot 6} = \sqrt{25} \cdot \sqrt{6}$$

Now, evaluate $\sqrt{25}$.

$$\sqrt{25} \cdot \sqrt{6} = 5\sqrt{6}$$

Therefore, the given expression simplifies to $5\sqrt{6}$.

Answer 3mr.

- a) The ratio of w to h is given as 4 : 3. Thus,

$$\frac{w}{h} = \frac{4}{3}.$$

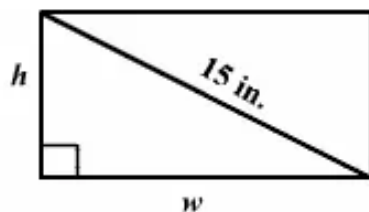
Find the cross products and solve for w .

$$w \cdot 3 = 4 \cdot h$$

$$\frac{w \cdot 3}{3} = \frac{4 \cdot h}{3}$$

$$w = \frac{4}{3}h$$

- b) We can draw a diagram that represents the given situation.



Apply the Pythagorean Theorem.

$$h^2 + w^2 = 15^2$$

Substitute for w obtained in part a.

$$h^2 + \left(\frac{4}{3}h\right)^2 = 225$$

$$h^2 + \frac{16}{9}h^2 = 225$$

Simplify to get an equation for h .

$$\frac{25}{9}h^2 = 225$$

$$\frac{25}{9}h^2 \cdot \frac{9}{25} = 225 \cdot \frac{9}{25}$$

$$h^2 = 81$$

- c) For solving the equation, take the square root on each side.

$$\begin{aligned}\sqrt{h^2} &= \pm\sqrt{81} \\ h &= \pm 9\end{aligned}$$

Since height cannot be negative, reject the solution $h = -9$. Thus, the possible solution is 9.

- d) The height of the laptop screen is 9 inches.

For finding the width, substitute 9 for h in the expression for w and evaluate.

$$\begin{aligned}w &= \frac{4}{3}(9) \\ &= 12\end{aligned}$$

The width of the screen is 12 inches.

Since the screen is in the shape of a rectangle, its area is the product of the width and the height.

$$\begin{aligned}\text{Area} &= 9(12) \\ &= 108\end{aligned}$$

Therefore, the area of the screen is 108 square inches.

Answer 4e.

We need to simplify the expression $\sqrt{192}$.

We proceed as follows.

$$\begin{aligned}\sqrt{192} \\ &= \sqrt{64 \cdot 3}\end{aligned}$$

By using the product property of square roots Which says $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ when ever $a > 0, b > 0$

We get,

$$\begin{aligned}\sqrt{192} \\ &= \sqrt{64} \cdot \sqrt{3} \\ &= \boxed{8\sqrt{3}}\end{aligned}$$

Answer 4gp.

Consider the expression,

$$\sqrt{8} \cdot \sqrt{28}$$

Simplify the following expression.

$$\sqrt{8} \cdot \sqrt{28}$$

The product property of square roots:

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \text{ Whenever } a > 0, b > 0.$$

Now,

$$\begin{aligned}\sqrt{8} \cdot \sqrt{28} &= \sqrt{8} \cdot \sqrt{4 \cdot 7} && \text{Since } 4 \cdot 7 = 28 \\ &= \sqrt{4 \cdot 2} \cdot \sqrt{4 \cdot 7} && \text{Since } 4 \cdot 2 = 8 \\ &= \sqrt{4} \cdot \sqrt{2} \cdot \sqrt{4} \cdot \sqrt{7} && \text{Apply the product property} \\ &= 4 \cdot \sqrt{2} \cdot \sqrt{7} && \text{Since } \sqrt{4} \cdot \sqrt{4} = 4 \\ &= 4\sqrt{14} && \text{Apply the product property}\end{aligned}$$

Therefore, $\boxed{\sqrt{8} \cdot \sqrt{28} = 4\sqrt{14}}$

Answer 4mr.

We have a wall mirror of dimensions 20 inches by 24 inches with a metal border of uniform width x inches with metal area 416 square inches.

Need to calculate the width.

$$\begin{aligned}\text{Now } (20+2x)(24+2x) - (20)(24) &= 416 \\ \Rightarrow 4x^2 + 88x - 416 &= 0 \\ \Rightarrow x^2 + 22x - 104 &= 0 \\ \Rightarrow x^2 - 4x + 26x - 104 &= 0 \\ \Rightarrow x(x-4) + 26(x-4) &= 0 \\ \Rightarrow (x-4)(x+26) &= 0 \\ \Rightarrow x-4 &= 0 && (\text{Since } x = -26 \text{ is not possible}) \\ \Rightarrow x &= 4\end{aligned}$$

If the double the width, then the metal area will be

$$\begin{aligned}&(20+2(8))(24+2(8)) - (20)(24) \\ &= 256 + 704 \\ &= 960 \text{ square inches}\end{aligned}$$

Which is more than double of previous area.

Hence doubling the width does not exactly require twice as much metal.

Answer 5e.

Rewrite 150 as a product of two factors such that one factor is a perfect square.

$$\sqrt{150} = \sqrt{25 \cdot 6}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\sqrt{25 \cdot 6} = \sqrt{25} \cdot \sqrt{6}$$

Now, evaluate $\sqrt{25}$.

$$\sqrt{25} \cdot \sqrt{6} = 5\sqrt{6}$$

Therefore, the given expression simplifies to $5\sqrt{6}$.

Answer 5gp.

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}}$$

Now, evaluate.

$$\frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$$

Therefore, the given expression simplifies to $\frac{3}{8}$.

Answer 5mr.

- a) After x price increases, the total number of slices sold is $80 - 5x$, and the price of each slice is $2 + 0.25x$.

We know that the shop's revenue is the product of the total sales and the price of each slice.

$$\begin{array}{ccccc} \text{Revenue} & = & \text{price} & \cdot & \text{sales} \\ \text{(dollars)} & & \text{(dollars)} & & \\ \Downarrow & & \Downarrow & & \Downarrow \\ R(x) & = & (2 + 0.25x) & \cdot & (80 - 5x) \end{array}$$

The function that gives the pizza shop's revenue is $R(x) = (2 + 0.25x)(80 - 5x)$.

- b) In order to find the value of x that maximizes R , we have to write the equation in intercept form. For this, first apply the FOIL method and simplify.

$$R(x) = 160 - 10x + 20x - 1.25x^2$$

$$R(x) = -1.25x^2 + 10x + 160$$

Take out the common factor -1.25 .

$$R(x) = -1.25(x^2 - 8x - 128)$$

Factor the expression within the parentheses. We need to find two factors m and n such that their product gives -128 and sum gives -8 . Two such factors are -16 and 8 .

$$\text{Thus, } R(x) = -1.25(x - 16)(x + 8).$$

Now, we have to find the x -coordinate of the vertex. For a quadratic function of the form $y = a(x - p)(x - q)$, the x -coordinate of the vertex of its graph is $\frac{p + q}{2}$.

In this case, p is 16 and q is -8 . The x -coordinate of the vertex is $\frac{-8 + 16}{2}$ or 4 .

Therefore, the value of x that maximizes R is 4 . This means that the revenue is maximum when the price per slice is $\$[2 + 0.25(4)]$ or $\$3$.

- c) After x price decreases, the total number of slices sold is $80 + 5x$, and the price of each slice is $2 - 0.25x$.

$$\begin{array}{ccccc} \text{Revenue} & = & \text{price} & \cdot & \text{sales} \\ \text{(dollars)} & & \text{(dollars)} & & \\ \Downarrow & & \Downarrow & & \Downarrow \\ R(x) & = & (2 - 0.25x) & \cdot & (80 + 5x) \end{array}$$

The function that gives the pizza shop's revenue is $R(x) = (2 - 0.25x)(80 + 5x)$.

Apply the FOIL method and simplify.

$$R(x) = 160 + 10x - 20x - 1.25x^2$$

$$R(x) = -1.25x^2 - 10x + 160$$

Take out the common factor -1.25 .

$$R(x) = -1.25(x^2 + 8x - 128)$$

Factor the expression within the parentheses. We need to find two factors m and n such that their product gives -128 and sum gives 8 . Two such factors are 16 and -8 .

$$\text{Thus, } R(x) = -1.25(x + 16)(x - 8).$$

Now, we have to find the x -coordinate of the vertex. $\frac{p+q}{2}$.

In this case, p is -16 and q is 8 . The x -coordinate of the vertex is $\frac{-16+8}{2}$ or -4 . Therefore, the value of x that maximizes R is -4 . We obtain a negative x -value because the major portion of the parabola lies on the negative x -axis.

Answer 6e.

We need to simplify the following expression

$$\sqrt{3} \cdot \sqrt{27}$$

By using the product property of square roots

Which says $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ when ever $a > 0, b > 0$

We get,

$$\sqrt{3} \cdot \sqrt{27}$$

$$= \sqrt{3 \cdot 27}$$

$$= \sqrt{81}$$

$$= \boxed{9}$$

Answer 6gp.

Consider the following expression,

$$\sqrt{\frac{15}{4}}$$

Simplify the following expression,

$$\sqrt{\frac{15}{4}}$$

The quotient property of square roots:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ Whenever } a > 0, b > 0$$

Now,

$$\begin{aligned}\sqrt{\frac{15}{4}} &= \frac{\sqrt{15}}{\sqrt{4}} \\ &= \frac{\sqrt{15}}{2}\end{aligned}$$

Apply the quotient property of square roots

Since $\sqrt{4} = 2$

Therefore, $\boxed{\sqrt{\frac{15}{4}} = \frac{\sqrt{15}}{2}}$

Answer 6mr.

A rectangular vegetable garden measures 42 feet by 8 feet.

If the area has to be doubled by expanding the length and width by the same distance x feet, We need to find x

$$\text{Then, } (42+x)(8+x)=2(42)(8)$$

$$\Rightarrow x^2+50x-336=0$$

$$\Rightarrow x^2-6x+56x-336=0$$

$$\Rightarrow x(x-6)+56(x-6)=0$$

$$\Rightarrow (x-6)(x+56)=0$$

$$\Rightarrow x-6=0 \quad (\text{Since } x+56>0)$$

$$\Rightarrow x=6$$

Hence , we need to increment the length and width by 6 feet to double the rectangular garden.

Answer 7e.

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$4\sqrt{6} \cdot \sqrt{6} = 4\sqrt{6 \cdot 6}$$

Simplify within the radical.

$$4\sqrt{6 \cdot 6} = 4\sqrt{36}$$

Now, evaluate $\sqrt{36}$.

$$4\sqrt{36} = 4 \cdot 6$$

Multiply.

$$4 \cdot 6 = 24$$

Therefore, the given expression simplifies to 24.

Answer 7gp.

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$\sqrt{\frac{11}{25}} = \frac{\sqrt{11}}{\sqrt{25}}$$

Now, evaluate $\sqrt{25}$.

$$\frac{\sqrt{11}}{\sqrt{25}} = \frac{\sqrt{11}}{5}$$

Therefore, the given expression simplifies to $\frac{\sqrt{11}}{5}$.

Answer 7mr.

The vertex form of quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the vertex. Thus, the vertex form of a quadratic function having vertex at $(-3, 2)$ is $y = a[x - (-3)]^2 + 2$ where a can be any nonzero value.

Let $a = 1$. Substitute 1 for a , -3 for h , and 2 for k in the vertex form and simplify.

$$y = 1[x - (-3)]^2 + 2$$

$$y = (x + 3)^2 + 2$$

Apply the factoring pattern $a^2 + 2ab + b^2 = (a + b)^2$.

$$y = x^2 + 6x + 9 + 2$$

$$y = x^2 + 6x + 11$$

Thus, one of the quadratic functions for which the graph has a vertex of $(-3, 2)$ is $y = x^2 + 6x + 11$.

Similarly, choose any two different nonzero values for a and find the quadratic functions.

We get:

$$y = -x^2 - 6x - 7$$

$$y = 2x^2 + 12x + 20.$$

Answer 8e.

We need to simplify the following expression

$$5\sqrt{24} \cdot 3\sqrt{10}$$

$$= 15\sqrt{24} \cdot \sqrt{10}$$

By using the product property of square roots, which says $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ when ever $a > 0, b > 0$

We get,

$$\begin{aligned} & 5\sqrt{24} \cdot 3\sqrt{10} \\ &= 15\sqrt{24 \cdot 10} \\ &= 15\sqrt{240} \\ &= 15\sqrt{16 \cdot 15} \\ &= 15\sqrt{16} \cdot \sqrt{15} \\ &= 15(4\sqrt{15}) \\ &= \boxed{60\sqrt{15}} \end{aligned}$$

Answer 8gp.

Consider the following expression,

$$\sqrt{\frac{36}{49}}$$

Simplify the following expression.

$$\sqrt{\frac{36}{49}}$$

The quotient property of square roots:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ Whenever } a > 0, b > 0.$$

Now,

$$\begin{aligned} \sqrt{\frac{36}{49}} &= \frac{\sqrt{36}}{\sqrt{49}} \\ &= \frac{6}{7} \end{aligned}$$

Apply the quotient property of square roots

Since $\sqrt{36} = 6$ and $\sqrt{49} = 7$

Therefore, $\boxed{\sqrt{\frac{36}{49}} = \frac{6}{7}}$

Answer 9e.

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$\sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{\sqrt{16}}$$

Now, evaluate $\sqrt{16}$.

$$\frac{\sqrt{5}}{\sqrt{16}} = \frac{\sqrt{5}}{4}$$

Therefore, the given expression simplifies to $\frac{\sqrt{5}}{4}$.

Answer 9gp.

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$\sqrt{\frac{6}{5}} = \frac{\sqrt{6}}{\sqrt{5}}$$

We have to rationalize the denominator. For this, multiply the numerator and the denominator by $\sqrt{5}$.

$$\frac{\sqrt{6}}{\sqrt{5}} = \frac{\sqrt{6} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$$

Again, apply the product property.

$$\begin{aligned} \frac{\sqrt{6} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} &= \frac{\sqrt{6 \cdot 5}}{\sqrt{5 \cdot 5}} \\ &= \frac{\sqrt{30}}{\sqrt{25}} \end{aligned}$$

Evaluate $\sqrt{25}$.

$$\frac{\sqrt{30}}{\sqrt{25}} = \frac{\sqrt{30}}{5}$$

Therefore, the given expression simplifies to $\frac{\sqrt{30}}{5}$.

Answer 10e.

We need to simplify the following expression

$$\sqrt{\frac{35}{36}}$$

By using the quotient property of square roots ,which says $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ whenever

$$a > 0, b > 0$$

We get,

$$\begin{aligned}\sqrt{\frac{35}{36}} \\&= \frac{\sqrt{35}}{\sqrt{36}} \\&= \boxed{\frac{\sqrt{35}}{6}}\end{aligned}$$

Answer 10gp.

Consider the following expression,

$$\sqrt{\frac{9}{8}}$$

Simplify the following expression,

$$\sqrt{\frac{9}{8}}$$

The quotient property of square roots:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ Whenever } a > 0, b > 0.$$

The product property of square roots:

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \text{ Whenever } a > 0, b > 0$$

Now,

$$\begin{aligned}\sqrt{\frac{9}{8}} &= \frac{\sqrt{9}}{\sqrt{8}} && \text{Apply the quotient property of square roots} \\&= \frac{3}{\sqrt{8}} && \text{Since } \sqrt{9} = 3 \\&= \frac{3}{\sqrt{4 \cdot 2}} \\&= \frac{3}{\sqrt{4} \cdot \sqrt{2}} && \text{Apply the product property of square roots} \\&= \frac{3}{2\sqrt{2}} && \text{Since } \sqrt{4} = 2\end{aligned}$$

By multiplying both numerator and denominator by $\sqrt{2}$

$$\begin{aligned}&= \frac{3 \cdot \sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}} \\&= \frac{3\sqrt{2}}{4} && \text{Simplify}\end{aligned}$$

Answer 11e.

First, we have to rationalize the denominator. For this, multiply the numerator and the denominator by $\sqrt{3}$.

$$\frac{8}{\sqrt{3}} = \frac{8 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\begin{aligned}\frac{8 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} &= \frac{8\sqrt{3}}{\sqrt{3 \cdot 3}} \\ &= \frac{8\sqrt{3}}{\sqrt{9}}\end{aligned}$$

Now, evaluate $\sqrt{9}$.

$$\frac{8\sqrt{3}}{\sqrt{9}} = \frac{8\sqrt{3}}{3}$$

Therefore, the given expression simplifies to $\frac{8\sqrt{3}}{3}$.

Answer 11gp.

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$\sqrt{\frac{17}{12}} = \frac{\sqrt{17}}{\sqrt{12}}$$

We have to rationalize the denominator. For this, multiply the numerator and the denominator by $\sqrt{12}$.

$$\frac{\sqrt{17}}{\sqrt{12}} = \frac{\sqrt{17} \cdot \sqrt{12}}{\sqrt{12} \cdot \sqrt{12}}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\begin{aligned}\frac{\sqrt{17} \cdot \sqrt{12}}{\sqrt{12} \cdot \sqrt{12}} &= \frac{\sqrt{17 \cdot 12}}{\sqrt{12 \cdot 12}} \\ &= \frac{\sqrt{204}}{\sqrt{144}}\end{aligned}$$

Evaluate $\sqrt{144}$.

$$\frac{\sqrt{204}}{\sqrt{144}} = \frac{\sqrt{204}}{12}$$

Rewrite 204 as a product of two factors such that one factor is a perfect square.

$$\frac{\sqrt{204}}{12} = \frac{\sqrt{4 \cdot 51}}{12}$$

Again, apply the product property.

$$\begin{aligned}\frac{\sqrt{4 \cdot 51}}{12} &= \frac{\sqrt{4} \cdot \sqrt{51}}{12} \\ &= \frac{2\sqrt{51}}{12}\end{aligned}$$

Therefore, the given expression simplifies to $\frac{2\sqrt{51}}{12}$.

Answer 12e.

We need to simplify the following expression

$$\frac{7}{\sqrt{12}}$$

By using the product property of square roots, which says $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ whenever $a > 0, b > 0$

We get ,

$$\begin{aligned}\frac{7}{\sqrt{12}} \\ &= \frac{7}{\sqrt{4 \cdot 3}} \\ &= \frac{7}{\sqrt{4} \cdot \sqrt{3}} \\ &= \frac{7}{2\sqrt{3}}\end{aligned}$$

By multiplying both numerator and denominator by an appropriate number called rationalizing factor,

We get,

$$\begin{aligned}\frac{7}{\sqrt{12}} \\ &= \frac{7 \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} \\ &= \boxed{\frac{7\sqrt{3}}{6}}\end{aligned}$$

Answer 12gp.

Consider the following expression,

$$\sqrt{\frac{19}{21}}$$

Simplify the following expression,

$$\sqrt{\frac{19}{21}}$$

The quotient property of square roots:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{Whenever } a > 0, b > 0$$

The product property of square roots:

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \quad \text{Whenever } a > 0, b > 0$$

Now,

$$\sqrt{\frac{19}{21}} = \frac{\sqrt{19}}{\sqrt{21}} \quad \text{Apply the quotient property of square roots}$$

Multiplying both numerator and denominator by $\sqrt{21}$

$$= \frac{\sqrt{19} \cdot \sqrt{21}}{\sqrt{21} \cdot \sqrt{21}}$$

$$= \frac{\sqrt{19} \cdot \sqrt{21}}{21} \quad \text{Since } \sqrt{21} \cdot \sqrt{21} = 21$$

$$= \frac{\sqrt{19 \cdot 21}}{21} \quad \text{Apply the product property of square roots}$$

$$= \frac{\sqrt{399}}{21} \quad \text{Simplify}$$

Answer 13e.

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$\sqrt{\frac{18}{11}} = \frac{\sqrt{18}}{\sqrt{11}}$$

Rewrite 18 as a product of two factors such that one factor is a perfect square.

$$\frac{\sqrt{18}}{\sqrt{11}} = \frac{\sqrt{9 \cdot 2}}{\sqrt{11}}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\frac{\sqrt{9 \cdot 2}}{\sqrt{11}} = \frac{\sqrt{9} \cdot \sqrt{2}}{\sqrt{11}}$$

Now, evaluate $\sqrt{9}$.

$$\frac{\sqrt{9} \cdot \sqrt{2}}{\sqrt{11}} = \frac{3\sqrt{2}}{\sqrt{11}}$$

We have to rationalize the denominator. For this, multiply the numerator and the denominator by $\sqrt{11}$.

$$\frac{3\sqrt{2}}{\sqrt{11}} = \frac{3\sqrt{2} \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}}$$

Again, apply the product property.

$$\begin{aligned}\frac{3\sqrt{2} \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} &= \frac{3\sqrt{2 \cdot 11}}{\sqrt{11 \cdot 11}} \\ &= \frac{3\sqrt{22}}{\sqrt{121}}\end{aligned}$$

Evaluate $\sqrt{121}$.

$$\frac{3\sqrt{22}}{\sqrt{121}} = \frac{3\sqrt{22}}{11}$$

Therefore, the given expression simplifies to $\frac{3\sqrt{22}}{11}$.

Answer 13gp.

First, we have to rationalize the denominator. For this, multiply the numerator and the denominator by the conjugate of $7 - \sqrt{5}$, which is $7 + \sqrt{5}$.

$$\frac{-6}{7 - \sqrt{5}} = \frac{-6}{7 - \sqrt{5}} \cdot \frac{7 + \sqrt{5}}{7 + \sqrt{5}}$$

Simplify.

$$\frac{-6}{7 - \sqrt{5}} \cdot \frac{7 + \sqrt{5}}{7 + \sqrt{5}} = \frac{-42 - 6\sqrt{5}}{49 + 7\sqrt{5} - 7\sqrt{5} - 5}$$

Simplify the denominator.

$$\frac{-42 - 6\sqrt{5}}{49 + 7\sqrt{5} - 7\sqrt{5} - 5} = \frac{-42 - 6\sqrt{5}}{44}$$

Therefore, the given expression simplifies to $\frac{-42 - 6\sqrt{5}}{44}$.

Answer 14e.

We need to simplify the following expression

$$\sqrt{\frac{13}{28}}$$

By using the quotient property of square roots

Which says $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ whenever, $a > 0, b > 0$.

We get,

$$\sqrt{\frac{13}{28}}$$

$$= \frac{\sqrt{13}}{\sqrt{28}}$$

$$= \frac{\sqrt{13}}{\sqrt{4 \cdot 7}}$$

By using the product property of square roots ,which says $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ whenever $a > 0, b > 0$

We get ,

$$\sqrt{\frac{13}{28}}$$

$$= \frac{\sqrt{13}}{\sqrt{4} \cdot \sqrt{7}}$$

$$= \frac{\sqrt{13}}{2\sqrt{7}}$$

By multiplying both numerator and denominator by an appropriate number called the rationalizing factor,

We get,

$$\sqrt{\frac{13}{28}}$$

$$= \frac{\sqrt{13} \cdot \sqrt{7}}{2\sqrt{7} \cdot \sqrt{7}}$$

$$= \frac{\sqrt{91}}{14}$$

Answer 14gp.

Consider the following expression,

$$\frac{2}{4 + \sqrt{11}}$$

Simplify the following expression,

$$\frac{2}{4 + \sqrt{11}}$$

Multiply both numerator and denominator by $(4 - \sqrt{11})$

$$\begin{aligned}\frac{2}{4 + \sqrt{11}} &= \frac{2(4 - \sqrt{11})}{(4 + \sqrt{11})(4 - \sqrt{11})} \\ &= \frac{2(4 - \sqrt{11})}{4^2 - (\sqrt{11})^2} \quad \text{Apply } (a+b)(a-b) = a^2 - b^2\end{aligned}$$

On continuation,

$$\begin{aligned}&= \frac{8 - 2\sqrt{11}}{4^2 - (\sqrt{11})^2} \\ &= \frac{8 - 2\sqrt{11}}{4 - 11} \quad \text{Since } (\sqrt{11})^2 = 11 \\ &= \frac{8 - 2\sqrt{11}}{-7} \quad \text{Add like terms} \\ &= \frac{2\sqrt{11} - 8}{7} \quad \text{Simplify}\end{aligned}$$

Answer 15e.

First, we have to rationalize the denominator. For this, multiply the numerator and the denominator by the conjugate of $1 - \sqrt{3}$, which is $1 + \sqrt{3}$.

$$\frac{2}{1 - \sqrt{3}} = \frac{2}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

Simplify.

$$\frac{2}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{2 + 2\sqrt{3}}{1 + \sqrt{3} - \sqrt{3} - 3}$$

Simplify the denominator.

$$\frac{2 + 2\sqrt{3}}{1 + \sqrt{3} - \sqrt{3} - 3} = \frac{2 + 2\sqrt{3}}{-2}$$

Divide each term in the numerator by the denominator.

$$\frac{2 + 2\sqrt{3}}{-2} = -1 - \sqrt{3}$$

Therefore, the given expression simplifies to $-1 - \sqrt{3}$.

Answer 15gp.

First, we have to rationalize the denominator. For this, multiply the numerator and the denominator by the conjugate of $9 + \sqrt{7}$, which is $9 - \sqrt{7}$.

$$\frac{-1}{9 + \sqrt{7}} = \frac{-1}{9 + \sqrt{7}} \cdot \frac{9 - \sqrt{7}}{9 - \sqrt{7}}$$

Simplify.

$$\frac{-1}{9 + \sqrt{7}} \cdot \frac{9 - \sqrt{7}}{9 - \sqrt{7}} = \frac{-9 + \sqrt{7}}{81 - 9\sqrt{7} + 9\sqrt{7} - 7}$$

Simplify the denominator.

$$\frac{-9 + \sqrt{7}}{81 - 9\sqrt{7} + 9\sqrt{7} - 7} = \frac{-9 + \sqrt{7}}{74}$$

Therefore, the given expression simplifies to $\frac{-9 + \sqrt{7}}{74}$.

Answer 16e.

We need to simplify the following expression

$$\frac{1}{5 + \sqrt{6}}$$

By multiplying both numerator and denominator by an appropriate number called the rationalizing factor,

We get,

$$\begin{aligned} & \frac{1}{5 + \sqrt{6}} \\ &= \frac{1(5 - \sqrt{6})}{(5 + \sqrt{6})(5 - \sqrt{6})} \\ &= \frac{5 - \sqrt{6}}{5^2 - (\sqrt{6})^2} \\ &= \frac{5 - \sqrt{6}}{25 - 6} \\ &= \boxed{\frac{5 - \sqrt{6}}{19}} \end{aligned}$$

Answer 16gp.

Consider the following expression,

$$\frac{4}{8-\sqrt{3}}$$

Simplify the following expression,

$$\frac{4}{8-\sqrt{3}}$$

Now,

$$\frac{4}{8-\sqrt{3}}$$

Multiply both numerator and denominator by $8+\sqrt{3}$

$$\begin{aligned}\frac{4}{8-\sqrt{3}} &= \frac{4(8+\sqrt{3})}{(8-\sqrt{3})(8+\sqrt{3})} \\ &= \frac{32+4\sqrt{3}}{8^2-(\sqrt{3})^2} && \text{Apply } (a+b)(a-b)=a^2-b^2 \\ &= \frac{32+4\sqrt{3}}{64-3} && \text{Since } 8^2=64 \text{ and } (\sqrt{3})^2=3 \\ &= \frac{32+4\sqrt{3}}{61} && \text{Add like terms}\end{aligned}$$

Answer 17e.

First, we have to rationalize the denominator. For this, multiply the numerator and the denominator by the conjugate of $4+\sqrt{5}$, which is $4-\sqrt{5}$.

$$\frac{\sqrt{2}}{4+\sqrt{5}} = \frac{\sqrt{2}}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}}$$

Simplify.

$$\frac{\sqrt{2}}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{4\sqrt{2}-\sqrt{10}}{16-4\sqrt{5}+4\sqrt{5}-5}$$

Now, simplify the denominator.

$$\frac{4\sqrt{2}-\sqrt{10}}{16-4\sqrt{5}+4\sqrt{5}-5} = \frac{4\sqrt{2}-\sqrt{10}}{11}$$

Therefore, the given expression simplifies to $\frac{4\sqrt{2}-\sqrt{10}}{11}$.

Answer 17gp.

First, we have to isolate x^2 . For this, divide each side of the equation by 5.

$$\frac{5x^2}{5} = \frac{80}{5}$$

$$x^2 = 16$$

Take the square root on each side.

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

CHECK

Substitute the solutions for x in the original equation and evaluate.

$$\text{Let } x = 4$$

$$\text{Let } x = -4$$

$$5x^2 = 80$$

$$5x^2 = 80$$

$$5(4)^2 \stackrel{?}{=} 80$$

$$5(-4)^2 \stackrel{?}{=} 80$$

$$5(16) \stackrel{?}{=} 80$$

$$5(16) \stackrel{?}{=} 80$$

$$80 = 80 \quad \checkmark$$

$$80 = 80 \quad \checkmark$$

Therefore, the solutions are 4 and -4.

Answer 18e.

We need to simplify the following expression

$$\frac{3 + \sqrt{7}}{2 - \sqrt{10}}$$

By multiplying both numerator and denominator by an appropriate number called the rationalizing factor ,

We get,

$$\frac{3 + \sqrt{7}}{2 - \sqrt{10}}$$

$$= \frac{(3 + \sqrt{7})(2 + \sqrt{10})}{(2 - \sqrt{10})(2 + \sqrt{10})}$$

$$= \frac{6 + 2\sqrt{7} + 3\sqrt{10} + \sqrt{7} \cdot \sqrt{10}}{2^2 - (\sqrt{10})^2}$$

By using the product property of square roots .

Which says $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ when ever $a > 0, b > 0$

We get,

$$= \frac{6 + 2\sqrt{7} + 3\sqrt{10} + \sqrt{7 \cdot 10}}{4 - 10}$$

$$= \frac{6 + 2\sqrt{7} + 3\sqrt{10} + \sqrt{70}}{-6}$$

$$= \boxed{-1 - \frac{1}{3}\sqrt{7} - \frac{1}{2}\sqrt{10} - \frac{1}{6}\sqrt{70}}$$

Answer 18gp.

Consider the following equation,

$$z^2 - 7 = 29$$

Solve the following quadratic equation.

$$z^2 - 7 = 29$$

$$z^2 = 29 + 7 \quad \text{Add 7 on both sides}$$

$$z^2 = 36 \quad \text{Simplify}$$

$$z = \pm\sqrt{36} \quad \text{Take square root on both sides}$$

$$z = \pm 6 \quad \text{Since } \sqrt{36} = 6$$

Answer 19e.

Rewrite 108 as a product of two factors such that one factor is a perfect square.

$$\sqrt{108} = \sqrt{36 \cdot 3}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\sqrt{36 \cdot 3} = \sqrt{36} \cdot \sqrt{3}$$

Now, evaluate $\sqrt{36}$.

$$\sqrt{36} \cdot \sqrt{3} = 6\sqrt{3}$$

The given expression simplifies to $6\sqrt{3}$. Therefore, the correct answer is choice C.

Answer 19gp.

First, divide each side of the equation by 3.

$$\frac{3(x-2)^2}{3} = \frac{40}{3}$$

$$(x-2)^2 = \frac{40}{3}$$

Take the square root on each side.

$$\sqrt{(x-2)^2} = \sqrt{\frac{40}{3}}$$

$$x-2 = \pm\sqrt{\frac{40}{3}}$$

Now, add 2 to each side.

$$x-2+2 = \pm\sqrt{\frac{40}{3}}+2$$

$$x = 2 \pm \sqrt{\frac{40}{3}}$$

CHECK

Substitute the solutions for x in the original equation and evaluate.

$$\text{Let } x = 2 + \sqrt{\frac{40}{3}}$$

$$3(x - 2)^2 = 40$$

$$3\left(2 + \sqrt{\frac{40}{3}} - 2\right)^2 \stackrel{?}{=} 40$$

$$3\left(\frac{40}{3}\right) \stackrel{?}{=} 40$$

$$40 = 40 \quad \checkmark$$

$$\text{Let } x = 2 - \sqrt{\frac{40}{3}}$$

$$3(x - 2)^2 = 40$$

$$3\left(2 - \sqrt{\frac{40}{3}} - 2\right)^2 \stackrel{?}{=} 40$$

$$3\left(\frac{40}{3}\right) \stackrel{?}{=} 40$$

$$40 = 40 \quad \checkmark$$

Therefore, the solutions are $2 + \sqrt{\frac{40}{3}}$ and $2 - \sqrt{\frac{40}{3}}$.

Answer 20e.

We need to describe and correct the error in simplifying the expression or solving the following equation.

$$\begin{aligned}\sqrt{96} &= \sqrt{4} \cdot \sqrt{24} \\ &= 2\sqrt{24}\end{aligned}$$

Now simplify the following expression

$$\begin{aligned}\sqrt{96} \\ &= \sqrt{16 \cdot 6}\end{aligned}$$

By using the product property of square roots

Which says $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ when ever , $a > 0, b > 0$

We get,

$$\begin{aligned}\sqrt{96} \\ &= \sqrt{16} \cdot \sqrt{6} \\ &= \boxed{4\sqrt{6}}\end{aligned}$$

Both the expressions yield the same result if simplified properly.

From what is given in the question,

$$\begin{aligned}\sqrt{96} &= \sqrt{4} \cdot \sqrt{24} \\ &= 2\sqrt{24} \\ &= \boxed{4\sqrt{6}}\end{aligned}$$

Answer 20gp.

Consider the statement,

A container which prevents egg from breaking is dropped form a height of 30 feet.

We need to calculate how long the container takes to hit the ground.

Consider the equation,

$$h = -16t^2 + h_0$$

Put $h = 0$ and $h_0 = 30$

$$0 = -16t^2 + 30$$

$$16t^2 = 30$$

$$t^2 = \frac{30}{16} \quad \text{Divide by 16 on both sides}$$

Now, take square root on both sides.

$$t = \pm \sqrt{\frac{30}{16}}$$

$$t = \sqrt{\frac{30}{16}}$$

$$t = \frac{\sqrt{30}}{4} \quad \text{Since } \sqrt{16} = 4$$

$$t \approx 1.37$$

Therefore, the container will fall for about 1.37 seconds before hitting the ground.

Answer 21e.

We know that a positive number has two square roots - a positive square root and a negative square root. Thus, the given equation has two solutions. The error is that only the positive square root of 81 is shown.

The square root of 81 is ± 9 .

$$x = \pm 9$$

Answer 22e.

To solve the following quadratic equation.

$$s^2 = 169$$

Write the original equation

$$\Rightarrow s = \pm \sqrt{169}$$

Take the square roots of each side

$$\Rightarrow \boxed{s = \pm 13}$$

Answer 23e.

Take the square root on each side.

$$\sqrt{a^2} = \sqrt{50}$$

$$a = \pm\sqrt{50}$$

Rewrite 50 as a product of two factors such that one factor is a perfect square.

$$\pm\sqrt{50} = \pm\sqrt{25 \cdot 2}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\pm\sqrt{25 \cdot 2} = \pm\sqrt{25} \cdot \sqrt{2}$$

Now, evaluate $\sqrt{25}$.

$$\pm\sqrt{25} \cdot \sqrt{2} = \pm 5\sqrt{2}$$

Thus, $a = \pm 5\sqrt{2}$.

CHECK

Substitute the solutions for a in the original equation and evaluate.

$$\text{Let } a = 5\sqrt{2} \qquad \text{Let } a = -5\sqrt{2}$$

$$a^2 = 50 \qquad a^2 = 50$$

$$(5\sqrt{2})^2 \stackrel{?}{=} 50 \qquad (-5\sqrt{2})^2 \stackrel{?}{=} 50$$

$$25(2) \stackrel{?}{=} 50 \qquad 25(2) \stackrel{?}{=} 50$$

$$50 = 50 \quad \checkmark \qquad 50 = 50 \quad \checkmark$$

Therefore, the solutions are $5\sqrt{2}$ and $-5\sqrt{2}$.

Answer 24e.

To solve the following quadratic equation.

$$x^2 = 84 \qquad \text{The original equation.}$$

$$x = \pm\sqrt{84} \qquad \text{Take square root each side.}$$

$$x = \pm\sqrt{4 \cdot 21}$$

(Product property of square roots: if $a > 0, b > 0$ then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$)

$$\Rightarrow x = \pm\sqrt{4} \cdot \sqrt{21} \qquad \text{Product property}$$

$$\Rightarrow x = \pm 2\sqrt{21}$$

Hence the roots of $x^2 = 84$ are $\boxed{2\sqrt{21}, -2\sqrt{21}}$

Answer 25e.

First, we have to isolate z^2 . For this, divide each side of the equation by 6.

$$\frac{6z^2}{6} = \frac{150}{6}$$

$$z^2 = 25$$

Take the square root on each side.

$$\sqrt{z^2} = \sqrt{25}$$

$$z = \pm 5$$

CHECK

Substitute the solutions for z in the original equation and evaluate.

Let $z = 5$

Let $z = -5$

$$6z^2 = 150$$

$$6z^2 = 150$$

$$6(5)^2 \stackrel{?}{=} 150$$

$$6(-5)^2 \stackrel{?}{=} 150$$

$$6(25) \stackrel{?}{=} 150$$

$$6(25) \stackrel{?}{=} 150$$

$$150 = 150 \quad \checkmark$$

$$150 = 150 \quad \checkmark$$

Therefore, the solutions are 5 and -5.

Answer 26e.

To solve the following quadratic equation

$$4p^2 = 448$$

The original equation.

$$p^2 = \frac{448}{4}$$

Divide each side by 4.

$$p^2 = 112$$

$$p = \pm\sqrt{112}$$

Take square root each side.

$$p = \pm\sqrt{16 \cdot 7}$$

$$p = \pm\sqrt{16} \cdot \sqrt{7}$$

Product property of square roots: if $a > 0, b > 0$

Then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

$$\boxed{p = \pm 4\sqrt{7}}$$

Answer 27e.

First, we have to isolate w^2 . For this, divide each side of the equation by -3.

$$\frac{-3w^2}{-3} = \frac{-213}{-3}$$

$$w^2 = 71$$

Take the square root on each side.

$$\sqrt{w^2} = \sqrt{71}$$

$$w = \pm\sqrt{71}$$

CHECK

Substitute the solutions for w in the original equation and evaluate.

Let $w = \sqrt{71}$	Let $w = -\sqrt{71}$
$-3w^2 = -213$	$-3w^2 = -213$
$-3(\sqrt{71})^2 \stackrel{?}{=} -213$	$-3(-\sqrt{71})^2 \stackrel{?}{=} -213$
$-3(71) \stackrel{?}{=} -213$	$-3(71) \stackrel{?}{=} -213$
$-213 = -213 \quad \checkmark$	$-213 = -213 \quad \checkmark$

Therefore, the solutions are $\sqrt{71}$ and $-\sqrt{71}$.

Answer 28e.

To solve the following quadratic equation

$$7r^2 - 10 = 25$$

$$7r^2 = 35$$

$$r^2 = 5$$

$$\boxed{r = \pm\sqrt{5}}$$

The original equation.

Add each 10 each sides.

Divide each side by 7.

Take square root each sides.

Answer 29e.

Multiply each side of the equation by 25 to clear the fraction.

$$25\left(\frac{x^2}{25} - 6\right) = 25(-2)$$

$$x^2 - 150 = -50$$

We have to isolate x^2 . For this, add 150 to each side.

$$x^2 - 150 + 150 = -50 + 150$$

$$x^2 = 100$$

Take the square root on each side.

$$\sqrt{x^2} = \sqrt{100}$$

$$x = \pm 10$$

CHECK

Substitute the solutions for x in the original equation and evaluate.

Let $x = 10$

Let $x = -10$

$$\frac{x^2}{25} - 6 = -2$$

$$\frac{x^2}{25} - 6 = -2$$

$$\frac{(10)^2}{25} - 6 \stackrel{?}{=} -2$$

$$\frac{(-10)^2}{25} - 6 \stackrel{?}{=} -2$$

$$\frac{100}{25} - 6 \stackrel{?}{=} -2$$

$$\frac{100}{25} - 6 \stackrel{?}{=} -2$$

$$4 - 6 \stackrel{?}{=} -2$$

$$4 - 6 \stackrel{?}{=} -2$$

$$-2 = -2 \quad \checkmark$$

$$-2 = -2 \quad \checkmark$$

Therefore, the solutions are 10 and -10.

Answer 30e.

To solve the following quadratic equation.

$$\frac{t^2}{20} + 8 = 15$$

$$\frac{t^2}{20} = 7$$

$$t^2 = 140$$

$$t = \pm\sqrt{140}$$

$$t = \pm\sqrt{4 \cdot 35}$$

$$t = \pm\sqrt{4} \cdot \sqrt{35}$$

$$\boxed{t = \pm 2\sqrt{35}}$$

The original equation.

Subtract each side with 8.

Both sides multiply with 20.

Take square root each side.

Product property of square roots: if

$$a > 0, b > 0 \quad \text{Then } \sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

Answer 31e.

First, divide each side of the equation by 4.

$$\frac{4(x-1)^2}{4} = \frac{8}{4}$$

$$(x-1)^2 = 2$$

Take the square root on each side.

$$\sqrt{(x-1)^2} = \sqrt{2}$$

$$x-1 = \pm\sqrt{2}$$

Now, add 1 to each side.

$$x-1+1 = \pm\sqrt{2}+1$$

$$x = 1 \pm \sqrt{2}$$

CHECK

Substitute the solutions for x in the original equation and evaluate.

$$\text{Let } x = 1 + \sqrt{2} \qquad \text{Let } x = 1 - \sqrt{2}$$

$$4(x-1)^2 = 8 \qquad 4(x-1)^2 = 8$$

$$4(1 + \sqrt{2} - 1)^2 \stackrel{?}{=} 8 \qquad 4(1 - \sqrt{2} - 1)^2 \stackrel{?}{=} 8$$

$$4(2) \stackrel{?}{=} 8 \qquad 4(2) \stackrel{?}{=} 8$$

$$8 = 8 \quad \checkmark$$

$$8 = 8 \quad \checkmark$$

Therefore, the solutions are $1 + \sqrt{2}$ and $1 - \sqrt{2}$.

Answer 32e.

To solve the following quadratic equation

$$7(x-4)^2 - 18 = 10$$

The original equation.

$$7(x-4)^2 = 28$$

Add 18 each side

$$(x-4)^2 = 4$$

Divide each side by 7.

$$(x-4) = \pm\sqrt{4}$$

Take square root each side.

$$x-4 = \pm 2$$

$$x = 4 \pm 2$$

Add 4 each side

Hence, $\boxed{x=2,6}$

Answer 33e.

First, add 5 to each side of the equation.

$$2(x+2)^2 - 5 + 5 = 8 + 5$$

$$2(x+2)^2 = 13$$

Divide each side by 2.

$$\frac{2(x+2)^2}{2} = \frac{13}{2}$$

$$(x+2)^2 = \frac{13}{2}$$

Take the square root on each side.

$$\sqrt{(x+2)^2} = \sqrt{\frac{13}{2}}$$

$$x+2 = \pm\sqrt{\frac{13}{2}}$$

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$x + 2 = \pm \frac{\sqrt{13}}{\sqrt{2}}$$

Now, we have to rationalize the denominator. For this, multiply the numerator and the denominator by $\sqrt{2}$.

$$x + 2 = \pm \frac{\sqrt{13}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\begin{aligned} x + 2 &= \pm \frac{\sqrt{13 \cdot 2}}{\sqrt{2 \cdot 2}} \\ &= \pm \frac{\sqrt{26}}{2} \end{aligned}$$

Subtract 2 from each side.

$$\begin{aligned} x + 2 - 2 &= \pm \frac{\sqrt{26}}{2} - 2 \\ x &= -2 \pm \frac{\sqrt{26}}{2} \end{aligned}$$

CHECK

Substitute the solutions for x in the original equation and evaluate.

Let $x = -2 + \frac{\sqrt{26}}{2}$	Let $x = -2 - \frac{\sqrt{26}}{2}$
$2(x + 2)^2 - 5 = 8$	$2(x + 2)^2 - 5 = 8$
$2\left(-2 + \frac{\sqrt{26}}{2} + 2\right)^2 - 5 \stackrel{?}{=} 8$	$2\left(-2 - \frac{\sqrt{26}}{2} + 2\right)^2 - 5 \stackrel{?}{=} 8$
$2\left(\frac{26}{4}\right) - 5 \stackrel{?}{=} 8$	$2\left(\frac{26}{4}\right) - 5 \stackrel{?}{=} 8$
$13 - 5 \stackrel{?}{=} 8$	$13 - 5 \stackrel{?}{=} 8$
$8 = 8 \quad \checkmark$	$8 = 8 \quad \checkmark$

Therefore, the solutions are $-2 + \frac{\sqrt{26}}{2}$ and $-2 - \frac{\sqrt{26}}{2}$.

Answer 34e.

To solve the following quadratic equation

$$3(x+2)^2 + 4 = 13 \quad \text{The original equation}$$

$$3(x+2)^2 = 9 \quad \text{Subtract 4 each side.}$$

$$(x+2)^2 = 3 \quad \text{Divide both sides by 3.}$$

$$x+2 = \pm\sqrt{3} \quad \text{Take square root each side.}$$

$$x = -2 \pm \sqrt{3} \quad \text{Subtract 2 each side.}$$

Hence $x = -2 \pm \sqrt{3}$ are the solutions of $3(x+2)^2 + 4 = 13$

Therefore, Answer is (C).

Answer 35e.

One method to solve the given equation is by using the definition for factoring a difference of two squares.

Rewrite the given equation.

$$x^2 - 2^2 = 0$$

We know that $a^2 - b^2 = (a + b)(a - b)$.

Thus,

$$(x + 2)(x - 2) = 0.$$

Set each factor to zero and solve for x .

$$x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -2 \quad \quad \quad x = 2$$

Another method is taking the square root.

First, add 4 to each side.

$$x^2 - 4 + 4 = 0 + 4$$

$$x^2 = 4$$

Now, take the square root on each side.

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

Therefore, the solutions are 2 and -2.

Answer 36e.

(a)

To write an equation of the form $x^2 = s$ that has two real solutions.

$$x^2 = 4 \quad \text{Consider the equation}$$

$$x^2 = 2^2$$

$$x = \pm\sqrt{2^2} \quad \text{Take square root each side.}$$

$$x = \pm 2$$

These are two real solutions.

(b)

To write an equation of the form $x^2 = s$ that has exactly one real solution.

Consider $x^2 = 0$

$\Rightarrow x = 0$ is the only real solution of the given equation.

(c)

To write an equation of the form $x^2 = s$ that has no real solutions.

Consider $x^2 = -1$

$\Rightarrow x \notin \mathbb{R}$ (Because for any $x \in \mathbb{R} \Rightarrow x^2 \geq 0$)

Therefore the given equation has no real solution.

Answer 37e.

First, divide each side of the equation by a .

$$(x + b)^2 = \frac{c}{a}$$

Take the square root on each side.

$$\sqrt{(x + b)^2} = \sqrt{\frac{c}{a}}$$

$$x + b = \pm \sqrt{\frac{c}{a}}$$

Apply the quotient property.

For any numbers $a, b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

$$x + b = \pm \frac{\sqrt{c}}{\sqrt{a}}$$

Now, we have to rationalize the denominator. For this, multiply the numerator and the denominator by \sqrt{a} .

$$x + b = \pm \frac{\sqrt{c}}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}}$$

Apply the product property.

For any numbers $a, b > 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

$$\begin{aligned}x + b &= \pm \frac{\sqrt{c \cdot a}}{\sqrt{a \cdot a}} \\&= \pm \frac{\sqrt{ac}}{a}\end{aligned}$$

Subtract b from each side.

$$\begin{aligned}x + b - b &= \pm \frac{\sqrt{ac}}{a} - b \\x &= -b \pm \frac{\sqrt{ac}}{a}\end{aligned}$$

Therefore, the solutions are $-b \pm \frac{\sqrt{ac}}{a}$.

Answer 38e.

Consider a cliff diver dives off a cliff 40 feet above water.

To write an equation giving the diver is height h (in feet) above the water after t seconds

We also need to find how long the diver in the air is.

The required equation is $h = -16t^2 + h_0$, where t is time in seconds, h is height in feet

After t seconds and $h_0 = 40$ the initial height.

$$\Rightarrow h = -16t^2 + 40$$

We also need to find how long the diver is in the air.

Put $h = 0$

$$\Rightarrow -16t^2 + 40 = 0$$

$$\Rightarrow 16t^2 = 40$$

$$\Rightarrow t^2 = \frac{40}{16}$$

$$\Rightarrow t^2 = \frac{10}{4}$$

$$\Rightarrow t = \pm \sqrt{\frac{10}{4}}$$

$$\Rightarrow t = \sqrt{\frac{10}{4}} \quad (\text{Since } t \neq 0)$$

Quotient property of square roots: if $a > 0, b > 0$ then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$$\Rightarrow t = \frac{\sqrt{10}}{\sqrt{4}}$$

$$\Rightarrow t = \frac{\sqrt{10}}{2} \text{ Seconds.}$$

Answer 39e.

We know that the initial height is 150 and the height when the falling object hits the surface is 0.

Substitute 150 for h_0 , and 0 for h in the given equation.

$$0 = -\frac{g}{2}t^2 + 150 \quad (1)$$

In order to find the time the rock takes to hit the surface of the first planet, first substitute 32 for g in equation (1).

$$0 = -\frac{32}{2}t^2 + 150$$

Simplify.

$$0 = -16t^2 + 150$$

Subtract 150 from each side.

$$\begin{aligned} 0 - 150 &= -16t^2 + 150 - 150 \\ -150 &= -16t^2 \end{aligned}$$

Now, divide each side by -16 .

$$\begin{aligned} \frac{-150}{-16} &= \frac{-16t^2}{-16} \\ \frac{150}{16} &= t^2 \end{aligned}$$

Take the square root on each side.

$$\begin{aligned} \sqrt{\frac{150}{16}} &= \sqrt{t^2} \\ \pm\sqrt{\frac{150}{16}} &= t \end{aligned}$$

Use a calculator to evaluate.

$$t \approx \pm 3.1$$

Since time cannot be negative, reject the negative value. Thus, the rock takes about 3.1 sec to hit the surface of the first planet.

Similarly, we can find the time the rock takes to hit the surface of the remaining planets. The rock takes 5 sec to hit the surface of the second planet, about 2 sec to hit the surface of third, about 3.2 sec to hit the surface of fourth, and it takes about 12.2 sec to hit the surface of the fifth planet.

Answer 40e.

The equation $h = 0.019s^2$ gives the height h (in feet) of the largest ocean wave when the wind speed is s knots.

When $h = 5$

$$5 = 0.019s^2$$

Write original equation

$$s^2 = \frac{5}{0.019}$$

Divide both sides by 0.019

$$s = \pm \sqrt{\frac{5}{0.019}}$$

Take a square root each side of the equation

$$s = \sqrt{\frac{5}{0.019}}$$

Since s is not less than zero 0

When $h = 20$

$$20 = 0.019s^2$$

Write original equation

$$s^2 = \frac{20}{0.019}$$

Divide both sides by 0.019

$$s^2 = 4 \left(\frac{5}{0.019} \right)$$

$$s = \pm \sqrt{4 \left(\frac{5}{0.019} \right)}$$

Take a square root each side of the equation

$$s = \sqrt{4 \left(\frac{5}{0.019} \right)}$$

Since s is not less than zero 0

Product property of square roots: if $a > 0, b > 0$ then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

$$s = \sqrt{4} \cdot \sqrt{\frac{5}{0.019}}$$

$$s = 2 \sqrt{\frac{5}{0.019}}$$

By comparison, we clearly see that the wind speed required to generate a 20 foot wave is Twice the wind speed required to generate a 5 foot wave.

Answer 41e.

- a. First, we have to find the area of the square.
The area A of a square with side length s is $A = s^2$.

Substitute 10 for s and evaluate.

$$\begin{aligned} A &= 10^2 \\ &= 100 \end{aligned}$$

The area of the square is 100 ft².

It is given that the circle has the same area as that of the square. We know that the area of circle of radius r is πr^2 .

Equate the area of the square to that of the circle.

$$\pi r^2 = 100$$

An equation that can be used to find the radius r of the circular lot is $\pi r^2 = 100$.

- b.** We have the equation to find the radius r of the circular lot.

$$\pi r^2 = 100$$

Divide each side by π to isolate r^2 .

$$\begin{aligned}\frac{\pi r^2}{\pi} &= \frac{100}{\pi} \\ r^2 &= \frac{100}{\pi}\end{aligned}$$

Take the square root on each side.

$$\begin{aligned}\sqrt{r^2} &= \sqrt{\frac{100}{\pi}} \\ r &= \pm \sqrt{\frac{100}{\pi}}\end{aligned}$$

Since radius cannot be negative, discard the negative value.

$$r = \sqrt{\frac{100}{\pi}}$$

Use a calculator to evaluate.

$$r \approx 5.6$$

Therefore, the radius of circular lot is about 5.6 ft.

- c. In order to find the radius of a circle which has the same area as that of a square, equate the area of a circle with radius r to that of a square with side length s .

$$\pi r^2 = s^2$$

Divide each side by π to isolate r^2 .

$$\frac{\pi r^2}{\pi} = \frac{s^2}{\pi}$$

$$r^2 = \frac{s^2}{\pi}$$

Take the square root on each side.

$$\sqrt{r^2} = \sqrt{\frac{s^2}{\pi}}$$

$$r = \pm \sqrt{\frac{s^2}{\pi}}$$

Since radius cannot be negative, discard the negative value.

$$r = \sqrt{\frac{s^2}{\pi}}$$

The radius of a circle with the same area as that of a square is $\sqrt{\frac{s^2}{\pi}}$.

Answer 42e.

Consider the air resistance R (in pounds) on a racing cyclist is given by the equation $R = 0.00829s^2$ where s is the speed of the bicycle (in miles per hour).

(a)

Need to find the speed of a racing cyclist when the air resistance is 5 pounds.

That is $R = 5$

$$5 = 0.00829s^2 \quad \text{Substitute } R = 5$$

$$s^2 = \frac{5}{0.00829} \quad \text{Divide by 0.00829 on each side}$$

$$s = \pm \sqrt{\frac{5}{0.00829}} \quad \text{Apply square root on each side}$$

$$s = \sqrt{\frac{5}{0.00829}} \quad \text{Since } s \neq 0$$

$$= \sqrt{603.136} \quad \text{By using calculator}$$

$$s \approx 24.56$$

Therefore,

The speed of racing cyclist is $s \approx 24.56$ mph

(b)

If the speed of the cyclist doubles, then

$$\begin{aligned} R &= 0.00829(2s)^2 \\ &= 0.00829(4s^2) \\ &= 4[0.00829s^2] \end{aligned}$$

The air resistance will be four times as that of initial speed.

Answer 43e.

We know that when the pool is drained, the height of water is 0.

Substitute 0 for h in the equation.

$$0 = \left(\sqrt{h_0} - \frac{2\pi d^2 \sqrt{3}}{lw} t \right)^2$$

Take the positive square root on each side.

$$\begin{aligned} \sqrt{0} &= \sqrt{\left(\sqrt{h_0} - \frac{2\pi d^2 \sqrt{3}}{lw} t \right)^2} \\ 0 &= \sqrt{h_0} - \frac{2\pi d^2 \sqrt{3}}{lw} t \end{aligned}$$

Now, subtract $\sqrt{h_0}$ from each side.

$$\begin{aligned} 0 - \sqrt{h_0} &= \sqrt{h_0} - \frac{2\pi d^2 \sqrt{3}}{lw} t - \sqrt{h_0} \\ -\sqrt{h_0} &= -\frac{2\pi d^2 \sqrt{3}}{lw} t \end{aligned}$$

Multiply each side by $-\frac{lw}{2\pi d^2 \sqrt{3}}$ to isolate t .

$$\begin{aligned} \left(-\sqrt{h_0} \right) \left(-\frac{lw}{2\pi d^2 \sqrt{3}} \right) &= \left(-\frac{2\pi d^2 \sqrt{3}}{lw} t \right) \left(-\frac{lw}{2\pi d^2 \sqrt{3}} \right) \\ \frac{\sqrt{h_0} lw}{2\pi d^2 \sqrt{3}} &= t \end{aligned}$$

Therefore, the time required to drain the pool when it is completely filled is

$$\frac{lw}{2\pi d^2 \sqrt{3}} \sqrt{h_0}.$$

Answer 44e.

Consider the power $(-5)^2$.

Need to evaluate the given power.

$$\begin{aligned}(-5)^2 &= ((-1)(5))^2 \\&= (-1)^2 (5)^2 && \text{Since } (ab)^2 = a^2 b^2 \\&= (1)(25) \\&= 25\end{aligned}$$

Therefore $(-5)^2 = \boxed{25}$.

Answer 45e.

We know that the exponent represents the number of times the base is used as a factor.

In the given expression, the base -4 is used as a factor 2 times.

$$(-4)^2 = (-4) \cdot (-4)$$

Multiply.

$$(-4) \cdot (-4) = 16$$

Therefore, $(-4)^2 = 16$.

Answer 46e.

Consider the power $(-8)^2$.

Need to evaluate the power.

$$\begin{aligned}(-8)^2 &= ((-1)(8))^2 \\&= (-1)^2 (8)^2 && \text{Since } (ab)^2 = a^2 b^2 \\&= (1)(64) \\&= 64\end{aligned}$$

Therefore $(-8)^2 = \boxed{64}$.

Answer 47e.

We know that the exponent represents the number of times the base is used as a factor.

In the given expression, the base -13 is used as a factor 2 times.

$$(-13)^2 = (-13) \cdot (-13)$$

Multiply.

$$(-13) \cdot (-13) = 169$$

Therefore, $(-13)^2 = 169$.

Answer 48e.

Consider the power -3^2 .

Need to evaluate the power.

$$\begin{aligned}-3^2 &= (-1)(3^2) \\ &= (-1)(9) \\ &= -9\end{aligned}$$

Therefore $-3^2 = \boxed{-9}$.

Answer 49e.

We know that the exponent represents the number of times the base is used as a factor.

In the given expression, the base 11 is used as a factor 2 times.

$$-11^2 = -(11 \cdot 11)$$

Multiply.

$$-(11 \cdot 11) = -121$$

Therefore, $-11^2 = -121$.

Answer 50e.

Consider the power -15^2

Need to evaluate the power.

$$\begin{aligned}-15^2 &= (-1)(15^2) \\ &= (-1)(225) \\ &= -225\end{aligned}$$

Therefore $-15^2 = \boxed{-225}$.

Answer 51e.

We know that the exponent represents the number of times the base is used as a factor.

In the given expression, the base 7 is used as a factor 2 times.

$$-7^2 = -(7 \cdot 7)$$

Multiply.

$$-(7 \cdot 7) = -49$$

Therefore, $-7^2 = -49$.

Answer 52e.

Consider the equation $x - 8 = 2$

Need to solve the equation.

$$x - 8 = 2$$

$$(x - 8) + 8 = 2 + 8 \quad \text{Add 8 on each side}$$

$$x + (-8 + 8) = 10$$

$$x + 0 = 10$$

$$x = 10$$

Thus the solution is $x = 10$.

Check:

Now need to check the solution for $x = 10$.

$$x - 8 = 2$$

$$10 - 8 = 2 \quad \text{Substitute 10 for } x$$

$$2 = 2$$

This is TRUE.

Therefore the solution set is $\{10\}$.

Answer 53e.

Apply the subtraction property of equality.

If $a = b$, then $a - c = b - c$.

Subtract 4 from each side.

$$3x + 4 - 4 = 13 - 4$$

$$3x = 9$$

Apply the division property of equality.

If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

Divide each side by 3.

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

CHECK

Substitute the solution for x in the original equation and evaluate.

$$3x + 4 = 13$$

$$3(3) + 4 \stackrel{?}{=} 13$$

$$9 + 4 \stackrel{?}{=} 13$$

$$13 = 13 \quad \checkmark$$

Therefore, the solution is 3.

Answer 54e.

Consider the equation $2x - 1 = 6x + 3$.

Need to solve the equation.

$$2x - 1 = 6x + 3$$

$$2x - 6x = 3 + 1 \quad \text{Collect like terms}$$

$$(2 - 6)x = 4 \quad \text{Combine like terms}$$

$$-4x = 4$$

$$x = \frac{4}{-4} \quad \text{Divide by -4 on each side}$$

$$x = -1$$

Thus the solution is $x = -1$.

Check:

Need to check the solution for $x = -1$.

$$2x - 1 = 6x + 3$$

$$2(-1) - 1 = 6(-1) + 3 \quad \text{Substitute -1 for } x$$

$$-2 - 1 = -6 + 3$$

$$-3 = -3$$

This is TRUE.

Therefore the solution set is $\boxed{\{-1\}}$.

Answer 55e.

Subtract 9 from each side to solve the inequality.

$$x + 9 - 9 > 5 - 9$$

$$x > -4$$

Therefore, the solution of the inequality is $x > -4$.

Answer 56e.

Consider the inequality $-7x - 15 \geq 6$.

Need to solve the inequality.

$$-7x - 15 \geq 6$$

$$-7x \geq 15 + 6 \quad \text{Add 15 on each side}$$

$$-7x \geq 21$$

$$\frac{-7x}{-7} \leq \frac{21}{-7} \quad \text{Divide by -7 so the sign will be reverse}$$

$$x \leq -3$$

Thus $x \in (-\infty, -3]$

Therefore,

The solution set of the inequality $-7x - 15 \geq 6$ is $\boxed{(-\infty, -3]}$.

Answer 57e.

In order to solve the inequality, first add $10x$ to each expression.

$$3 - 6x + 10x \leq 23 - 10x + 10x$$

$$3 + 4x \leq 23$$

Now, subtract 3 from each expression.

$$3 + 4x - 3 \leq 23 - 3$$

$$4x \leq 20$$

Divide each expression by 4.

$$\frac{4x}{4} \leq \frac{20}{4}$$

$$x \leq 5$$

Therefore, the solutions are all real numbers less than or equal to 5.

Answer 58e.

Consider the equation $|x+12|=5$.

Need to solve the equation.

$$|x+12|=5$$

$$x+12=\pm 5$$

$$x=-12\pm 5 \quad \text{Subtract 12 from each side}$$

$$x=-12+5 \text{ or } x=-12-5$$

$$x=-7 \quad \text{or } x=-17$$

$$x=-17, -7$$

Thus the roots of the equation are $\boxed{x=-17, -7}$.

Answer 59e.

In order to solve an absolute value equation $|ax+b|=c$ where $c > 0$, we have to rewrite as two equations: $ax+b=c$ or $ax+b=-c$. Thus,

$$-2 + 3x = 10 \quad (1)$$

or

$$-2 + 3x = -10 \quad (2).$$

Now, solve each equation.

Add 2 to each side of equation (1).

$$-2 + 3x + 2 = 10 + 2$$

$$3x = 12$$

Divide each side by 3.

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

Add 2 to each side of equation (2).

$$-2 + 3x + 2 = -10 + 2$$

$$3x = -8$$

Divide each side by 3.

$$\frac{3x}{3} = -\frac{8}{3}$$

$$x = -\frac{8}{3}$$

CHECK

Substitute the solutions in the original equation and evaluate.

$$\text{Let } x = 4$$

$$|-2 + 3x| = 10$$

$$|-2 + 3(4)| \stackrel{?}{=} 10$$

$$|-2 + 12| \stackrel{?}{=} 10$$

$$|10| \stackrel{?}{=} 10$$

$$10 = 10 \quad \checkmark$$

$$\text{Let } x = -\frac{8}{3}$$

$$|-2 + 3x| = 10$$

$$\left| -2 + 3\left(-\frac{8}{3}\right) \right| \stackrel{?}{=} 10$$

$$|-2 - 8| \stackrel{?}{=} 10$$

$$|-10| \stackrel{?}{=} 10$$

$$10 = 10 \quad \checkmark$$

Therefore, the solutions are 4 and $-\frac{8}{3}$.

Answer 60e.

Consider the inequality

$$\left| \frac{1}{2}x + 9 \right| \geq 4$$

Need to solve the inequality.

$$\frac{1}{2}x + 9 \geq 4 \quad \text{Or} \quad \frac{1}{2}x + 9 \leq -4$$

$$\frac{1}{2}x \geq -5 \quad \text{Or} \quad \frac{1}{2}x \leq -13$$

$$x \geq -10 \quad \text{Or} \quad x \leq -26$$

$$x \in [-10, \infty) \quad \text{Or} \quad x \in (-\infty, -26]$$

$$x \in (-\infty, -26] \cup [-10, \infty)$$

Therefore,

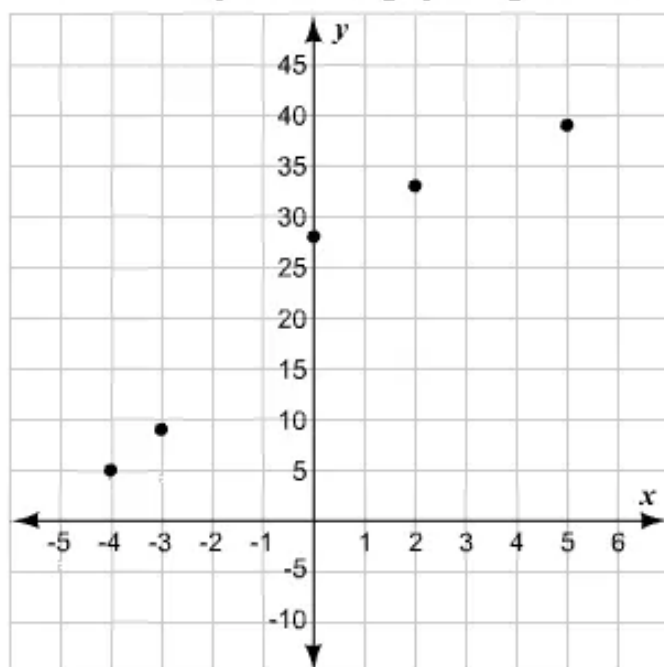
The solution set of the inequality $\left| \frac{1}{2}x + 9 \right| \geq 4$ is $\boxed{(-\infty, -26] \cup [-10, \infty)}$.

Subtract 9 from each side
multiply by 2 on each side

Answer 61e.

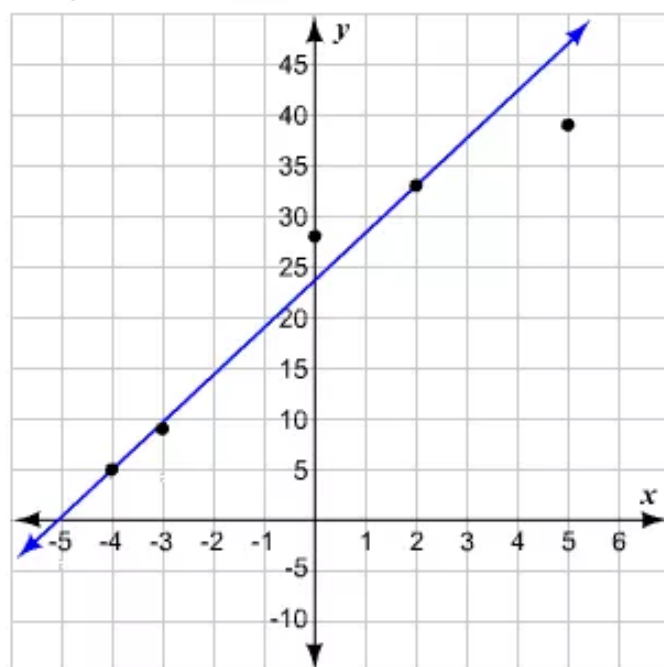
- (a) The given data can be represented in the ordered pair form as $(-4, 5)$, $(-3, 9)$, $(0, 28)$, $(2, 33)$, and $(5, 39)$.

Plot the above points on a graph to get the scatter plot.



- (b) The line that follows the trends given by the data points most closely is called a best-fitting line.

First, sketch the line that best fits the data.



Now, choose two data points that appear to lie on the line. Let the points be $(-4, 5)$ and $(2, 33)$.

Find the slope using these points.

$$\begin{aligned} m &= \frac{33 - 5}{2 - (-4)} \\ &= \frac{28}{6} \\ &\approx 5 \end{aligned}$$

The point slope form of an equation is $y - y_1 = m(x - x_1)$. Choose $(2, 33)$ as the point (x_1, y_1) .

Substitute 5 for m , 2 for x_1 , and 33 for y_1 in the above equation.

$$y - 33 = 5(x - 2)$$

Simplify.

$$y - 33 = 5x - 10$$

Add 33 to both sides of the equation.

$$\begin{aligned} y - 33 + 33 &= 5x - 10 + 33 \\ y &= 5x + 23 \end{aligned}$$

Thus, an approximation of the best fitting line is $y = 5x + 23$.

(c) Substitute 20 for x in the equation $y = 5x + 23$ and simplify.

$$\begin{aligned} y &= 5(20) + 23 \\ &= 100 + 23 \\ &= 123 \end{aligned}$$

Therefore, the value of y is 123 when x is 20.

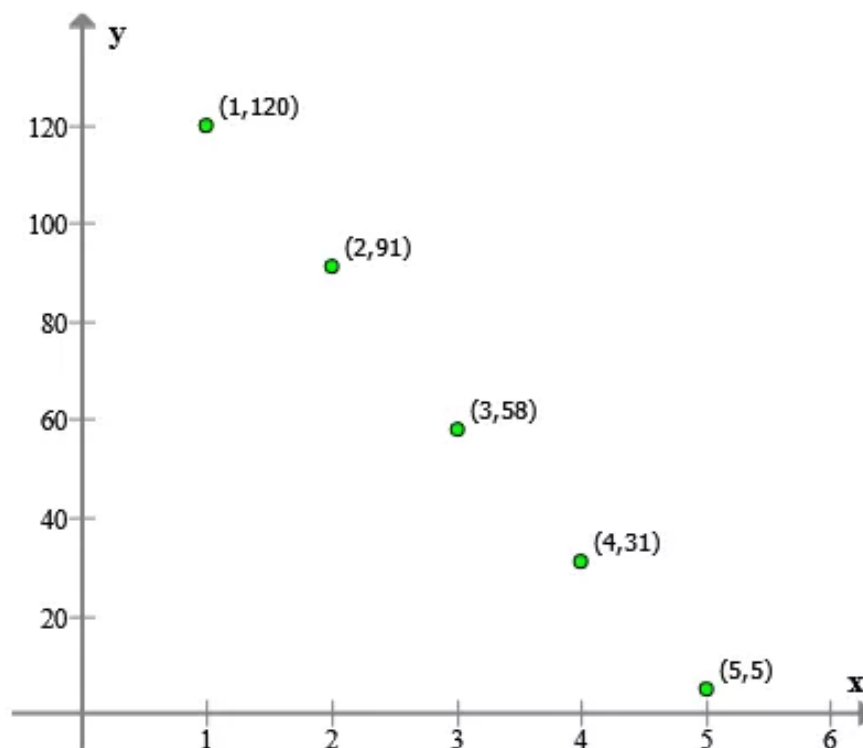
Answer 62e.

We consider the table given below.

x	1	2	3	4	5
y	120	91	58	31	5

(a)

We can draw a scatter plot of the data given above in the table by plotting the points $(1,120), (2,91), (3,58), (4,31), (5,5)$ where the values of x along the horizontal axis and those of y along the vertical axis as shown below.



(b)

We can approximate the best-fitting line as given below.

Now, we have to find an equation of a best-fitting line that comes close to the data points. For the equation of the form $y = mx + b$ of a best-fitting line, we first use the points $(1,120)$ and $(4,31)$ to find the slope, m .

The slope, m , of a line that containing two points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So, the slope, m of the best-fitting line is

$$\begin{aligned} m &= \frac{31 - 120}{4 - 1} && [\text{Substitute } 31, 120, 4, 1 \text{ for } y_2, y_1, x_2, x_1 \text{ respectively}] \\ &= \frac{-89}{3} \\ &\approx -29.67 \end{aligned}$$

We substitute -29.67 for m in the equation $y = mx + b$:

$$y = -29.67x + b$$

We can find the constant b by substituting the coordinates of the point $(1,120)$ into the equation $y = -29.67x + b$ and then solving for b :

$$120 = -29.67(1) + b \quad [\text{Substituting 1 for } x \text{ and 120 for } y]$$

$$120 = -29.67 + b$$

$$149.67 = b \quad [\text{Add 29.67 from both sides}]$$

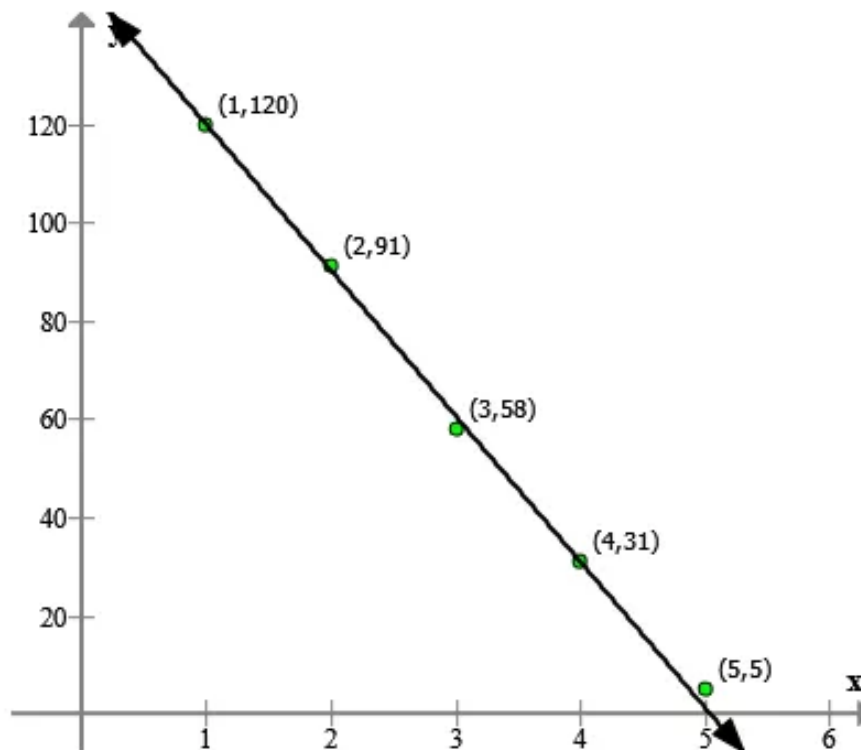
We substitute 149.67 for b in the equation $y = -29.67x + b$:

$$y = -29.67x + 149.67$$

Hence, the equation of the best-fitting line is

$$y = -29.67x + 149.67$$

The graph of the best-fitting line has as shown below.



Hence, we can that the graph of the best-fitting line that contains $(1,120)$ and $(4,31)$ comes close to the all data points $(1,120), (2,91), (3,58), (4,31), (5,5)$.

(c)

We estimate y when $x = 20$ by plotting 20 for x in the equation $y = -29.67x + 149.67$ of the best-fitting line as given below.

$$y = -29.67(20) + 149.67 \quad [\text{Substitute 20 for } x]$$

$$= -593.4 + 149.67$$

$$= \boxed{-443.73}$$