Class X Session 2023-24
Subject - Mathematics (Standard)
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

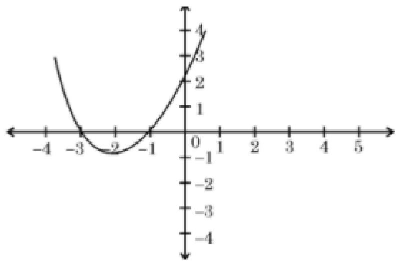
Section A

1. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then the product of two numbers is **[1]**

a) 205400 b) 203400

c) 194400 d) 198400

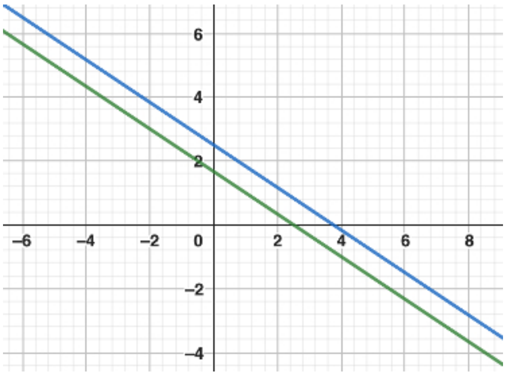
2. In the figure, the graph of the polynomial p(x) is given. The number of zeroes of the polynomial is: **[1]**



a) 2 b) 1

c) 0 d) 3

3. The pair of equations 2x + 3y = 5 and 4x + 6y = 15 has **[1]**



a) infinitely many solutions

b) exactly two solutions

c) no solution

d) a unique solution

4. If the sum of the roots of the equation $kx^2 + 2x + 3k = 0$ is equal to their product then the value of k is **[1]**

a) $\frac{1}{3}$

b) $\frac{-1}{3}$

c) $\frac{-2}{3}$

d) $\frac{2}{3}$

5. The common difference of the A.P whose $S_n = 3n^2 + 7n$ is **[1]**

a) 6

b) 2

c) 1

d) 5

6. The distance between the points $(\sin\theta, \cos\theta)$ and $(\cos\theta, -\sin\theta)$ is **[1]**

a) $\sqrt{2}$ units

b) 2 units

c) $2\sqrt{2}$ units

d) $\sqrt{\sin\theta + \cos\theta}$ units

7. If A(4, 9), B(2, 3) and C(6, 5) are the vertices of $\triangle$ABC, then the length of median through C is **[1]**

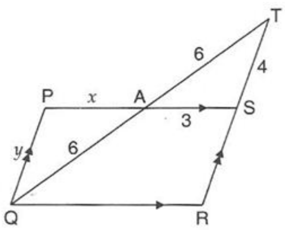
a) 10 units

b) 5 units

c) $\sqrt{10}$units

d) 25 units

8. In the given figure if PS||QR and PQ||SR and AT = AQ = 6, AS = 3, TS = 4, then **[1]**



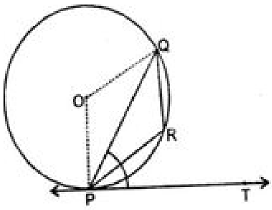
a) x = 2, y = 3.

b) x = 1, y = 2.

c) x = 3, y = 4.

d) x = 4, y = 5.

9. In the given figure, PQ is a chord of a circle and PT is the tangent at P such that $\angle QPT = 60^o$, Then $\angle PRQ$ is equal to: **[1]**



a) 150°

b) 120°

c) 140° | d) 110°

10. PQ is a tangent to a circle with centre O at the point P. If $\triangle$ OPQ is an isosceles triangle, then $\angle$OQP is equal to **[1]**

a) 60^o | b) 45^o
c) 90^o | d) 30^o

11. The value of cosec^4A - 2 cosec^2A + 1 is **[1]**

a) $\tan^4A$ | b) $\sec^4A$
c) cosec^4A | d) $\cot^4A$

12. ($\cos 0^o + \sin 30^o + \sin 45^o$) ($\sin 90^o + \cos 60^o - \cos 45^o$) = ? **[1]**

a) $\frac{5}{8}$ | b) $\frac{7}{4}$
c) $\frac{5}{6}$ | d) $\frac{3}{5}$

13. If the shadow of a boy 'x' metres high is 1.6m and the angle of elevation of the sun is 45°, then the value of 'x' is **[1]**

a) 0.8 m | b) 1.6 m
c) 3.2 m | d) 2 m

14. In a circle of radius 21 cm, an arc subtends an angle of 60^o at the centre. The area of the sector formed by the arc is: **[1]**

a) 231 cm^2 | b) 250 cm^2
c) 220 cm^2 | d) 200 cm^2

15. The area of a sector of angle θ^0 of a circle with radius R is **[1]**

a) $\frac{2\pi R\theta}{360}$ | b) $\frac{\pi R^2\theta}{180}$
c) $\frac{\pi R^2\theta}{360}$ | d) $\frac{2\pi R\theta}{180}$

16. The probability of guessing the correct answer to a certain test questions is $\frac{x}{12}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then x = **[1]**

a) 6 | b) 4
c) 2 | d) 3

17. A letter is chosen at random from the word ASSASSINATION. The probability that it is a vowel is **[1]**

a) $\frac{6}{13}$ | b) $\frac{7}{13}$
c) $\frac{6}{31}$ | d) $\frac{3}{13}$

18. In a data, if l = 60, h = 15, f_1 = 16, f_0 = 6, f_2 = 6, then the mode is **[1]**

a) 67.5 | b) 72
c) 60 | d) 62

19. **Assertion (A):** A sphere of radius 7 cm is mounted on the solid cone of radius 6 cm and height 8 cm. The volume of the combined solid is 1737.97 cm^3. **[1]**

Reason (R): Volume of sphere is $\frac{4}{3}\pi r^3$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Sum of natural number from 1 to 100 is 5050. **[1]**

Reason (R): Sum of n natural number is $\frac{n(n+1)}{2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

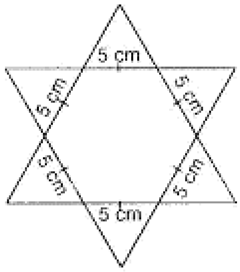
c) A is true but R is false.

d) A is false but R is true.

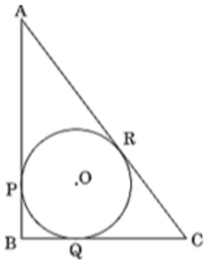
Section B

21. Find the HCF and LCM of 108, 120 and 252 using prime factorisation method. **[2]**

22. Complete the star by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. **[2]**



23. ABC is right triangle, right-angled at B, with BC = 6 cm and AB = 8 cm. A circle with centre O and radius r cm has been inscribed in $\triangle$ABC as shown in the figure. Find the value of r. **[2]**



24. If cosec$^2\theta$(1 + cosθ)(1 - cosθ) = λ, then find the value of λ. **[2]**

OR

Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$

25. Find the length of the arc of a circle of diameter 42 cm which subtends an angle of 60° at the centre. **[2]**

OR

Find the area of a sector of a circle with radius 6 cm, if the angle of the sector is 60^{o}.

Section C

26. Prove that 3 + $\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number. **[3]**

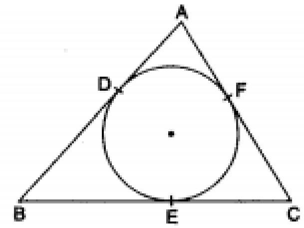
27. Find the zeroes of the polynomial 2s^2 + (1 + 2$\sqrt{2}$)s + $\sqrt{2}$ by factorisation method and verify the relationship between the zeroes and coefficient of the polynomial. **[3]**

28. A train covered a certain distance at a uniform speed. If the train had been 5 kmph faster, it would have taken 3 hours less than the scheduled time. And, if the train were slower by 4 kmph, it would have taken 3 hours more than the scheduled time. Find the length of the journey. **[3]**

OR

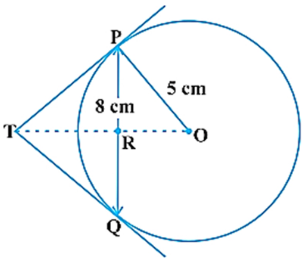
The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

29. In the given figure, a circle inscribed a in a triangle ABC, touches the sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, find the lengths of AD, BE and CF . **[3]**



OR

PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.



30. Prove that: $\frac{1}{\operatorname{cosec} A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec} A+\cot A}$. **[3]**

31. Compute the median for each of the following data: **[3]**

Marks	**No. of students**
Less than 10	0
Less than 30	10
Less than 50	25
Less than 70	43
Less than 90	65
Less than 110	87
Less than 130	96
Less than 150	100

Section D

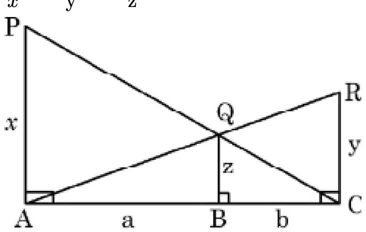
32. A plane left 30 minutes later than its scheduled time. In order to reach its destination 1500 km away in time it has to increase its speed by 250 km/h from its usual speed. Find its usual speed. **[5]**

OR

Solve for x

$\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$ where $a+b+x \neq 0$ and $a, b\ x \neq 0$

33. PA, QB and RC are each perpendicular to AC. If AP = x, QB = z, RC = y, AB = a and BC = b, then prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$. **[5]**



34. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and **[5]**

the level of the water is raised by 6.75 cm. Find the radius of the ball.

OR

A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is equal to the radius of the cone.

35. The following table shows the ages of the patients admitted in a hospital during a year: **[5]**

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

OR

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

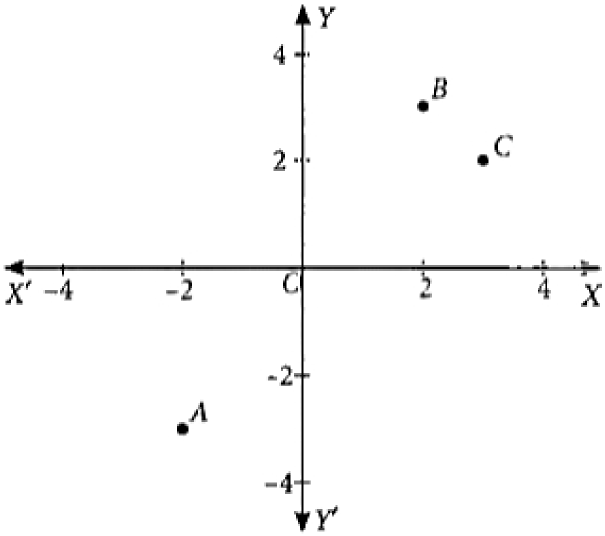
(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

37. **Read the text carefully and answer the questions:** **[4]**

There are two routes to travel from source A to destination B by bus. First bus reaches at B via point C and second bus reaches from A to B directly. The position of A, B and C are represented in the following graph: Based on the above information, answer the following questions.

Scale: x-axis : 1 unit = 1 km

y-axis: 1 unit = 1 km



(i) If the fare for the second bus is ₹15/km, then what will be the fare to reach to the destination by this bus?

(ii) What is the distance between A and B?

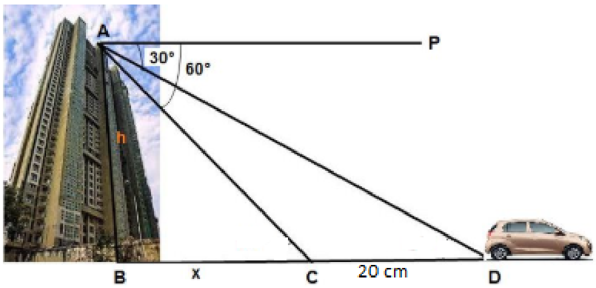
OR

If it is assumed that both buses have same speed, then by which bus do you want to travel from A to B?

(iii) What is the distance between A and C?

38. **Read the text carefully and answer the questions:** **[4]**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^{o}. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^{o}.



(i) Find the value of x.

(ii) Find the height of the building AB.

OR

Find the distance between top of the building and a car at position D?

(iii) Find the distance between top of the building and a car at position C?

Solution

Section A

1.

(c) 194400

Explanation: Let the HCF of the numbers be x and their LCM be y.

It is given that the sum of the HCF and LCM is 1260, therefore

x + y = 1260 ....(i)

And, LCM is 900 more than HCF.

y = x + 900 ..... (ii)

Substituting (ii) in (i), we get:

x + x + 900 =1260

$\Rightarrow$ 2x + 900 = 1260

$\Rightarrow$ 2x = 1260 - 900

$\Rightarrow$2x = 360

$\Rightarrow$ x = 180

Substituting x = 180 in (i), we get:

y = 180 + 900

$\Rightarrow$ y = 1080

We also know that the product the two numbers is equal to the product of their LCM and HCF

Thus, product of the numbers = 1080(180) = 194400

2. **(a)** 2

Explanation: 2

The number of zeroes is 2 as the graph does cut the x-axis 2 times.

3.

(c) no solution

Explanation: Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system has no solution.

4.

(c) $\frac{-2}{3}$

Explanation: Sum of roots = $\frac{-2}{k}$ and products of roots = $\frac{74}{k}$ = 3

$\therefore \frac{-2}{k} = 3 \Rightarrow k = \frac{-2}{3}$.

5. **(a)** 6

Explanation: Given: $S_n = 3n^2 + 7n$

Putting n = 1, 2, 3 we get

S = a = $3 \times (1)^2 + 7 \times 1 = 3 + 7 = 10$

$S_2 = 3 \times (2)^2 + 7 \times 2 = 12 + 14 = 26$

$S_3 = 3 \times (3)^3 + 7 \times 3 = 27 + 21 = 48$

Now, $a_2 = S_2 - S_1 = 26 - 10 = 16$

$\therefore$ Common difference (d) = a_2 - a = 16 - 10 = 6

6. **(a)** $\sqrt{2}$ units

Explanation: Distance between $(\sin\theta, \cos\theta)$ and $(\cos\theta, -\sin\theta)$

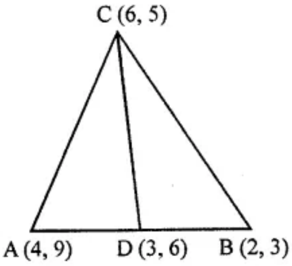
= $\sqrt{(\cos\theta - \sin\theta)^2 + (-\sin\theta - \cos\theta)^2}$

= $\sqrt{\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta + \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta}$

= $\sqrt{2\cos^2\theta + 2\sin^2\theta}$
= $\sqrt{2\left(\cos^2\theta + \sin^2\theta\right)}$
$\left[\because \cos^2\theta + \sin^2\theta = 1\right]$
= $\sqrt{2}$ units

7.

(c) $\sqrt{10}$units
Explanation: A(4, 9), B(2, 3) and C(6, 5) are the vertices of $\triangle ABC$
Let median CD has been drawn C(6, 5)



$\therefore$ D is mid point of AB
$D = \left(\frac{4+2}{2}, \frac{9+3}{2}\right)$
$\therefore$ D(3, 6)
$\therefore$ Length of CD
$= \sqrt{(6-3)^2 + (5-6)^2} = \sqrt{3^2 + (-1)^2}$
$= \sqrt{9+1} = \sqrt{10}$ units

8.

(c) x = 3, y = 4.
Explanation: In triangles APQ and ATS,
$\angle$PAQ = $\angle$TAS [Vertically opposite angles] $\angle$PQA = $\angle$ATS [Alternate angles]
$\therefore$ ΔAPQ$\sim$ ΔAST [AA similarity]
$\therefore \frac{\text{AQ}}{\text{AT}} = \frac{\text{AP}}{\text{AS}}$
$\Rightarrow \frac{6}{6} = \frac{x}{3}$
$\Rightarrow x = \frac{6\times3}{6}$ = 3
And $\frac{\text{AQ}}{\text{AT}} = \frac{\text{PQ}}{\text{ST}}$
$\Rightarrow \frac{6}{6} = \frac{y}{4}$
$\Rightarrow y = \frac{4\times6}{6}$ = 4
Therefore, x = 3, y = 4

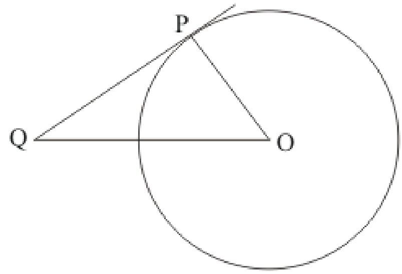
9.

(b) 120°
Explanation: Since OP is perpendicular to PT, then $\angle$OPT = 90°
$\Rightarrow\angle$OPQ + $\angle$QPT = 90°
$\Rightarrow\angle$OPQ + 60° = 90°
$\Rightarrow\angle$OPQ = 30°
$\therefore\angle$OPQ=$\angle$OQP = 30° [Angles opposite to radii]
$\therefore\angle$POQ = (180° - (30° + 30°) = 120° [Angle sum property of a triangle]
$\therefore$ Reflex$\angle$POQ = 360° - 120° = 240°
Now, since the degree measure of an arc of a circle is twice the angle subtended by it any point of the alternate segment of the circle with respect to the arc.
$\therefore$ Reflex$\angle$POQ = 2$\angle$PRQ
$\Rightarrow$240° = 2$\angle$PRQ
$\Rightarrow\angle$PRQ= 120°

10.

(b) 45^{o}

Explanation: Let us first put the given data in the form of a diagram.



We know that the radius of a circle will always be perpendicular to the tangent at the point of contact. Therefore, OP is perpendicular to QP. Therefore,

$\angle OQP = 90^o$

The side opposite to perpendicular is OQ.OQ will be the longest side of the triangle. So, in the isosceles right triangle ΔOPQ, OP = PQ

And the angles opposite to these two sides will also be equal. Therefore,

$\angle OQP = \angle POQ$

We know that sum of all angles of a triangle will always be equal to 180°. Therefore,

$\angle OQP + \angle POQ + \angle OPQ = 180^\circ$

$\angle OQP + \angle POQ + \angle OPQ = 180^\circ$

$90^\circ + 2\angle OQP = 180^\circ$

$2\angle OQP = 90^\circ$

$\angle OQP = 45^\circ$

11.

(d) $\cot^4 A$

Explanation: Given: $\text{cosec}^4 A - 2\ \text{cosec}^2 A + 1$

$= (\text{cosec}^2 A - 1)^2$

$= (\cot^2 A)^2$

$= \cot^4 A$

12.

(b) $\frac{7}{4}$

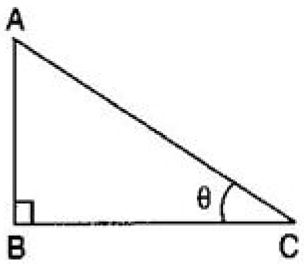
Explanation: $(\cos 0^o + \sin 30^o + \sin 45^o)(\sin 90^o + \cos 60^o - \cos 45^o) = ?$

$= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{9}{4} - \frac{1}{2}\right) = \frac{7}{4}$

13.

(b) 1.6 m

Explanation:



Given: Height of the boy = AB = x meters

And the length of the shadow of the boy = BC = 1.6 m

And angled of elevation $\theta = 45^\circ$

$\therefore \tan 45^\circ = \frac{\text{AB}}{\text{BC}} \Rightarrow 1 = \frac{x}{1.6}$

$\Rightarrow x = 1.6$ m

14. **(a)** 231 cm^2

Explanation: The angle subtended by the arc = 60^o

So, area of the sector = $(\frac{60^o}{360^o}) \times \pi r^2\ cm^2$

$= (\frac{441}{6}) \times (\frac{22}{7}) cm^2$

$= 231\ cm^2$

15.

(c) $\frac{\pi R^2\theta}{360}$

Explanation: $\frac{\pi R^2\theta}{360}$

16.

(b) 4

Explanation: Probability of guessing the correct answer

$= \frac{x}{12}$

and probability of not guessing the correct

answer $= \frac{2}{3}$

$\frac{x}{12} + \frac{2}{3} = 1 \because (A + \bar{A} = 1)$

$\Rightarrow \frac{x}{12} = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow x = \frac{12}{3} = 4$

$\therefore$ x = 4

17. **(a)** $\frac{6}{13}$

Explanation: Vowels present in the given word are A, A, I, A, I, O = 6

Number of possible outcomes = {A, A, I, A, I, O} = 6

Number of total outcomes = 13

Required Probability = $\frac{6}{13}$

18. **(a)** 67.5

Explanation: Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

= 60 + $\frac{16-6}{2\times16-6-6} \times 15$

= 60 + $\frac{10}{32-12} \times 15$

= 60 + $\frac{10}{20} \times 15$

= 60 + 7.5

= 67.5

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. By prime factorisation, we get

2	108
2	54
3	27
3	9
	3

2	120
2	60
2	30
3	15
	5

2	252
2	126
3	63
3	21
	7

108 = $\left(2^2 \times 3^3\right)$

120 = $\left(2^3 \times 3 \times 5\right)$

252 =$\left(2^2 \times 3^2 \times 7\right)$

HCF (108, 120, 252) = product of common terms with lowest power

$= \left(2^2 \times 3\right) = (4 \times 3)$

HCF = 12

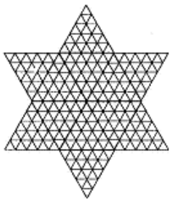
LCM (108, 120, 252) = product of prime factors with highest power

$= \left(2^3 \times 3^3 \times 5 \times 7\right)$

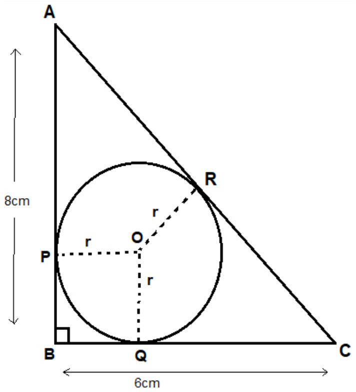
LCM = 7560

$\therefore$ HCF (108, 120, 252) = 12

and LCM (108, 120, 252) = 7560.

22. 

Number of triangles = $25 \times 12 = 300$

23. 

$\because$ AB = 8cm, BC = 6cm

$\therefore$ using Pythagoras theorem in $\triangle$ABC

$AC^2 = AB^2 + BC^2$

$AC^2 = (8)^2 + (6)^2$

$AC^2 = 64 + 36 = 100$

$AC = \sqrt{100} = 10$

Now, OP $\perp$ AB, OQ $\perp$ BC, OR $\perp$ AC (Radius and tangent are perpendicular to each other)

Also, BQ = BP, QC = RC, AP = AR (Tangents from an external point are equal)

Let BQ = x, QC = (6 - x), RC = (6 - x), AP = (8 - x), AR = (8 - x)

$\because$ AC = 10

$\Rightarrow$ AR + RC = 10

8 - x + 6 - x = 10

14 - 2x = 10

-2x = 10 - 14

-2x = -4

x = 2

Now, POQB is a square ($\because \angle PBQ = \angle BPO = 90^o$)

So, BQ = PO (Sides of square)

PO = r = 2cm

24. Given:

$cosec^2\theta(1+\cos\theta)(1-\cos\theta)=\lambda$

$\Rightarrow cosec^2\theta\{(1+\cos\theta)(1-\cos\theta)\}=\lambda$

$\Rightarrow \quad cosec^2\theta\left(1-\cos^2\theta\right)=\lambda$

$\Rightarrow \quad \operatorname{cosec}^2\theta\sin^2\theta=\lambda$

$\Rightarrow \quad \frac{1}{\sin^2\theta}\times\sin^2\theta=\lambda$

$\Rightarrow \quad 1=\lambda$

$\Rightarrow \quad \lambda=1$

Thus, the value of λ is 1.

OR

$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$

L.H.S. $=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$ $[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \tan\theta + \cot\theta + 1 = 1 + \tan\theta + \cot\theta = RHS$

Hence proved.

25. Diameter of a circle = 42 cm

$\Rightarrow$ Radius of a circle = r $= \frac{42}{2}$

= 21 cm

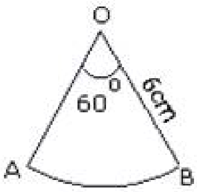
Central angle $= \theta = 60^\circ$

$\therefore$ Length of the arc $= \frac{2\pi r\theta}{360}$

$= \frac{2 \times \frac{22}{7} \times 21 \times 60^\circ}{360^\circ}$cm

= 22 cm

OR



We know that Area of sector $= \frac{\theta}{360}\pi r^2$

Here, $\theta = 60, r = 6$

$\therefore$ Required area $= \frac{60}{360} \times \frac{22}{7} \times (6)^2$

$= \frac{1}{6} \times \frac{22}{7} \times 36$

$= \frac{22 \times 6}{7}$

$= \frac{132}{7} = 18\frac{6}{7}cm^2$

Section C

26. Let's assume on the contrary that 3 + $\sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that 3 + $\sqrt{2}$ = $\frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{a}{b} - 3$

$\Rightarrow \sqrt{2} = \frac{(a-3b)}{b}$

$\Rightarrow \sqrt{2}$ is rational [$\because$ a and b are integers $\therefore$ $\frac{(a-3b)}{b}$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational.

So, our assumption is incorrect.

Hence, 3 + $\sqrt{2}$ is an irrational number.

27. $2s^2 + (1 + 2\sqrt{2})s + \sqrt{2}$

$= 2s^2 + s + 2\sqrt{2}s + \sqrt{2}$

$= s(2s+1) + \sqrt{2}(2s+1)$

$= (2s+1)(s + \sqrt{2})$

$\Rightarrow s = -\frac{1}{2}, -\sqrt{2}$ are zeroes of the polynomial.

Sum of zeroes = $-[\frac{1}{2} + \sqrt{2}] = -\frac{1+2\sqrt{2}}{2}$

Also, $\frac{-b}{a} = -\frac{1+2\sqrt{2}}{2}$

$\Rightarrow$Sum of zeroes = $\frac{-b}{a}$

Product of zeroes = $\frac{-1}{2} \times -\sqrt{2} = \frac{\sqrt{2}}{2}$

and $\frac{c}{a} = \frac{\sqrt{2}}{2}$

$\Rightarrow$Product of zeroes = $\frac{c}{a}$

28. Let us suppose that the original speed be x km/h and time taken be y hours.

We know that ,

Speed = distance / time

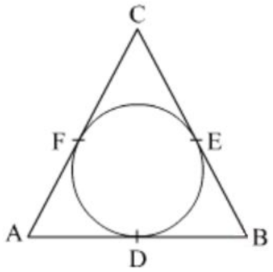
Distance = speed × time

Distance = xy km

Case I: if the train would have been 5 km/hr fast
i.e, Speed = (x + 5) km/h
it would have taken 3 hours less
i.e, time taken = (y - 3) hours
Distance covered = (x + 5)(y - 3) km
$\therefore$ (x + 5)(y - 3) = xy
$\Rightarrow$ xy + 5y - 3x - 15 = xy
$\Rightarrow$ 5y - 3x = 15.............(i)
Case II: if the train were slower by 4 km/hr
i.e, speed = (x - 4) km/h
It would have taken 3 hours more
i.e, time taken = (y + 3) hours
Distance covered = (x - 4)(y + 3) km
$\therefore$ (x - 4)(y + 3) = xy
$\Rightarrow$ xy - 4y + 3x - 12 = xy
$\Rightarrow$ 3x - 4y = 12............(ii)
Multiplying (i) by 4 and (ii) by 5, we get
20y - 12x = 60.........(iii)
-20y + 15x = 60.......(iv)
Adding (iii) and (iv), we get
20y - 12x + 15x - 20y = 60 + 60
3x = 120
or x = 40
Putting the value of x = 40 in equation (i), we get
5y - 3 $\times$ 40 = 15
$\Rightarrow$ 5y - 120 = 15
or, 5y = 135
$\Rightarrow$ y = 27
Hence, the length of the journey is (40$\times$27) km = 1080 km

OR

Let the ten's digit of required number be x and its unit digit be y respectively.
Then, As per given condition
The sum of digits of a two digit number is 15.
x + y = 15 ....(i)
Required number = 10x + y
Number formed on reversing the digits = 10y + x
So, as per given condition, the number obtained by reversing the order of digits of the given number exceeds the given number by 9.
$\therefore$ 10y + x -(10x + y) = 9
$\therefore$ 10y + x - 10x - y = 9
9y - 9x = 9
- x + y = 1.............(ii)
Adding (i) and (ii), we get
$2y = 16$
$\Rightarrow y = \frac{16}{2} = 8$
Putting y = 8 in (i), we get
x + 8 = 15
x = 15 - 8 = 7
Number = 10x + y
= 10 $\times$ 7 + 8
= 70 + 8
= 78
Hence the given two digit number is 78.

29. 

Tangents drawn from an external point to a circle are equal.

$\Rightarrow$ AD = AF, BD = BE, CE = CF.

Let AD = AF = a

BD = BE = b

CE = CF = c

AB = AD + DB = a + b = 8 .... (1)

BC = BE + EC = b + c = 10 .... (2)

AC = AF + FC = a + c = 12 .... (3)

Adding (1), (2) and (3), we get

2 (a + b + c) = 30

$\Rightarrow$ (a + b + c) = 15 .... (4)

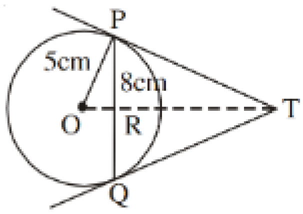
Subtracting (1) from (4), we get c = 7

Subtracting (2) from (4), we get a = 5

Subtracting (3) from (4), we get b = 3

Therefore, AD = a = 5 cm, BE = b = 3 cm, CF = c = 7 cm

OR



Let TR be x cm and TP be y cm

OT is perpendicular bisector of PQ

So PR = 4 cm (PR = $\frac{PQ}{2} = \frac{8}{2}$)

In $\triangle$OPR, $OP^2 = PR^2 + OR^2$

$5^2 = 4^2 + OR^2$

OR = $\sqrt{25-16}$

$\therefore$ OR = 3 cm

In $\triangle$PRT, $PR^2 + RT^2 = PT^2$

$y^2 = x^2 + 4^2$ .....(1)

In $\triangle$OPT, $OP^2 + PT^2 = OT^2$

$(x + 3)^2 = 5^2 + y^2$ (OT = OR + RT = 3 + x)

$\therefore$ $(x + 3)^2 = 5^2 + x^2 + 16$ [using (1)]

Solving, we get x = $\frac{16}{3}$ cm

From (1), $y^2 = \frac{256}{9} + 16 = \frac{400}{9}$

So, y = $\frac{20}{3}$cm = 6.667 cm

30. We have,

$\Rightarrow \frac{1}{\operatorname{cosec}A-\cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A+\cot A}$

$\Rightarrow \frac{1}{\operatorname{cosec}A-\cot A} + \frac{1}{\operatorname{cosec}A+\cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$

$\Rightarrow \frac{1}{\operatorname{cosec}A-\cot A} + \frac{1}{\operatorname{cosec}A+\cot A} = \frac{2}{\sin A}$

LHS = $\frac{1}{\operatorname{cosec}A-\cot A} + \frac{1}{\operatorname{cosec}A+\cot A}$

$\Rightarrow \frac{\operatorname{cosec}A+\cot A+\operatorname{cosec}A-\cot A}{(\operatorname{cosec}A-\cot A)(\operatorname{cosec}A+\cot A)}$

$\Rightarrow \frac{2\operatorname{cosec} A}{\operatorname{cosec}^2 A-\cot^2 A}$

$\Rightarrow \frac{\frac{2}{\sin A}}{1}=\frac{2}{\sin A}$ = RHS.

Hence Proved.

31.

Marks	No. of students	class interval	Frequency	Cumulative frequency
less than 10	0	0-10	0	0
less than 30	10	10-30	10	10
less than 50	25	30-50	15	25
less than 70	43	50-70	18	43(F)
less than 90	65	70-90	22(f)	65
less than 110	87	90-110	22	87
less than 130	96	110-130	9	96
less than 150	100	130-150	4	100
			N = 100	

We have

N = 100

$\therefore \frac{N}{2}=\frac{100}{2}$ = 50

The cumulative frequency just greater than $\frac{N}{2}$ is 65 then median class is 70 - 90 such that

l = 70, f = 22, F = 43, h = 90 - 70 =20

$\therefore$ Median = l + $\frac{\frac{N}{2}-F}{f}\times$ h

= 70 + $\frac{50-43}{22}\times$ 20

= 70 + $\frac{7\times 20}{22}$

= 70 + 6.36

= 76.36

Section D

32. Let the usual speed of the plane be x km/hr.

Then, time taken to cover 1500 km with the usual speed = $\frac{Distance}{Speed}$ = $\frac{1500}{x}hrs$

Increased speed be (x+250) km/hr

Time taken to cover 1500 km with the increased speed = $\frac{1500}{x+250}hrs$

According to the question

$\frac{1500}{x}-\frac{1500}{x+250}=30min=\frac{1}{2}$

$\implies \frac{1500x+1500\times 250-1500x}{x(x+250)}=\frac{1}{2}$

$\implies \frac{1500\times 250}{x^2+250x}=\frac{1}{2}$

$\implies x^2+250x-750000=0$

$\implies$ x^2+1000x - 750x -750000 = 0

$\implies$ x(x+1000) - 750 (x+1000) = 0

$\implies$ (x+1000) (x-750) = 0

Either x+1000 = 0 or x - 750 = 0

$\implies$ x= -1000, 750

But speed of the plane cannot be negative. So, x= 750.

Hence, the usual speed of the plane is 750 km/hr.

OR

$\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

$\Rightarrow \frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

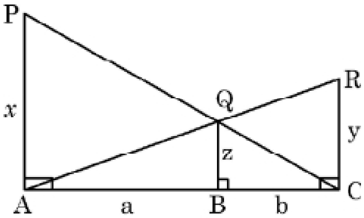
$\Rightarrow \quad \frac{-(a+b)}{x^2+(a+b)x}=\frac{b+a}{ab}$

$\Rightarrow \quad x^2+(a+b)x+ab=0$

$\Rightarrow \quad (x+a)(x+b)=0$

$\Rightarrow \quad x=-a, x=-b$

Hence ,x = - a, -b.

33. 

In the given figure we have $PA \perp AC$ and $QB \perp AC$

$\Rightarrow QB \| PA$

In $\triangle PAC$ and $\triangle QBC$, we have

$\angle QCB = \angle PCA$ (Common)

$\angle QBC = \angle PAC$ (both are 90^{o}).

So by AA similarity rule , $\triangle QBC \sim \triangle PAC$

$\therefore \frac{QB}{PA} = \frac{BC}{AC}$

$\Rightarrow \frac{z}{x} = \frac{b}{a+b}$ ...(i) [by the property of similar triangles]

In $\triangle RAC$, $QB \| RC$.

So, $\triangle QBA \sim \triangle RCA$.

$\therefore \frac{QB}{RC} = \frac{AB}{AC}$

$\Rightarrow \frac{z}{y} = \frac{a}{a+b}$ ...(ii) [by the property of similar triangles]

Form (i) and (ii), we obtain

$\frac{z}{x} + \frac{z}{y} = \left(\frac{b}{a+b} + \frac{a}{a+b}\right) = 1$

$\Rightarrow \frac{z}{x} + \frac{z}{y} = 1$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

or $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.

Hence proved.

34. According to question it is given that

Radius of cylindrical tub = 12 cm

Depth of cylindrical tub = 20 cm

Let us suppose that (r) be the radius of spherical ball

Again it is given that level of water is raised by 6.75 cm

Now, according to the question,

Volume of spherical ball = Volume of water rise in cylindrical tub

$\Rightarrow \frac{4}{3}\pi r^3 = \pi\ (12)^2 \times 6.75$

$\Rightarrow \frac{4}{3} r^2$ = 12 $\times$ 12 $\times$ 6. 75

$\Rightarrow$ r^3 = $\frac{12 \times 12 \times 6.75 \times 3}{4}$

$\Rightarrow$ r^3 = = 729

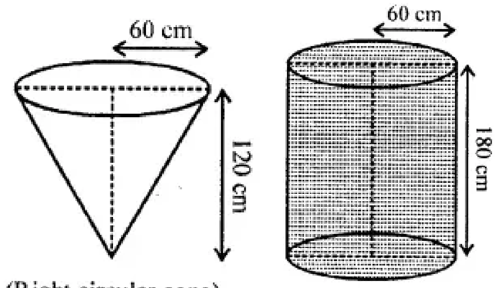
$\Rightarrow r = \sqrt[3]{729}$ = 9cm

Therefore, Radius of the ball = 9 cm

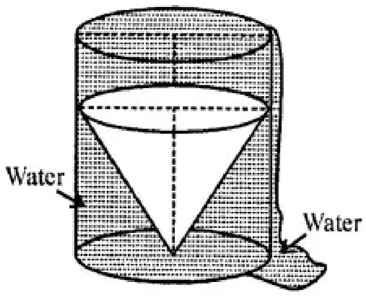
OR

i. Whenever we placed a solid right circular cone in a right circular cylinder ,cylinder with full of water, then volume of a solid right circular cone is equal to the volume of water filled from the cylinder.
ii. Total volume of water in a cylinder is equal to the volume of the cylinder.

iii. Volume of water left in the cylinder is = Volume of the right circular cylinder - Volume of a right circular cone.



(Right circular cone) (Right circular cylinder)



(Cylinder contained a cone)

Now, given that
Height of a right circular cone = 120cm
Radius of a right circular cone = 60cm
∴ The volume of a right circular cone = $\left(\frac{1}{3}\right)\pi r^2 \times h$
$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 60 \times 60 \times 120$
$= \left(\frac{22}{7}\right) \times 20 \times 60 \times 120$
= 14000 π cm^3
∴ Volume of a right circular cone = Volume of water spilled from the cylinder =144000π cm^3 [from point (i)]
Given that, the height of a right circular cylinder =180cm
and radius of a right circular cylinder = Radius of a right circular cone = 60 cm
∴ Volume of a right circular cylinder $= \pi \mathrm{r}^2 \times \mathrm{h}$
$= \pi \times 60 \times 60 \times 180 = 648000\pi$ cm^3 So, volume of a right circular cylinder = Total volume of water in a cylinder = 648000 π cm^3 [from point (ii)]
From point (iii),
Volume of water left in the cylinder = Total volume of water in a cylinder - Volume of water failed from the cylinder when solid cone is placed in it
$= 648000\pi - 144000\pi$
$= 504000\pi = 504000 \times \left(\frac{22}{7}\right) = 1584000$ cm^3
$= \left(\frac{1584000}{(10)6}\right) m^3 = 1.584 m^3$
Hence, the required volume of water left in the cylinder is 1.584 m^3

35. <u>Mode:</u>
Here, the maximum frequency is 23 and the class corresponding to this frequency is 35 - 45.
So, the modal class is 35 - 45.
Now, size (h) = 10
lower limit it (l) of modal class = 35
frequency (f_1) of the modal class = 23
frequency (f_0) of class previous the modal class = 21
frequency (f_2) of class succeeding the modal class = 14
∴ Mode = l + $\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times$ h = 35 + $\frac{23-21}{2\times23-21-14} \times 10$
= 35 + $\frac{2}{11} \times 10$ = 35 + $\frac{20}{11}$
= 35 + 1.8 (approx.)

= 36.8 years (approx.)

Mean:-

Take a = 40, h = 10.

Age (in years)	**Number of patients (f_i)**	**Class marks (x_i)**	**$d_i = x_i - 40$**	$u_i = \frac{x_i - 40}{10}$	**f_iu_i**
5-15	6	10	–30	–3	–18
15-25	11	20	–20	–2	–22
25-35	21	30	–10	–1	–21
35-45	23	40	0	0	0
45-55	14	50	10	1	14
55-65	5	60	20	2	10
Total	$\sum f_i = 80$				$\sum f_iu_i = -37$

Using the step deviation method,

$\overline{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times$ h = 40 + $\left(\frac{-37}{80}\right) \times 10$

= 40 - $\frac{37}{8}$ = 40 - 4.63

= 35.37 years

Interpretation:- Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.

Section E

36. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, a_3 = 600 and

a_3 = 600

$\Rightarrow$ 600 = a + 2d

$\Rightarrow$ a = 600 - 2d ...(i)

$\Rightarrow$ a_7 = 700

$\Rightarrow$ a_7 = 700

$\Rightarrow$ 700 = a + 6d

$\Rightarrow$ a = 700 - 6d ...(ii)

From (i) and (ii)

600 - 2d = 700 - 6d

$\Rightarrow$ 4d = 100

$\Rightarrow$ d = 25

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

a_n = 1000

$\Rightarrow$ 1000 = a + (n - 1)d

$\Rightarrow$ 1000 = 550 + 25n - 25

$\Rightarrow$ 1000 - 550 + 25 = 25n

$\Rightarrow$ 475 = 25n

$\Rightarrow$ n = $\frac{475}{25}$ = 19

OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$

$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$

$\Rightarrow$ S_7 = 4375

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

The production in the 10th term is given by a_{10}. Therefore, production in the 10th year = a_{10} = a + 9d = 550 + 9 $\times$ 25 = 775. So, production in 10th year is of 775 TV sets.

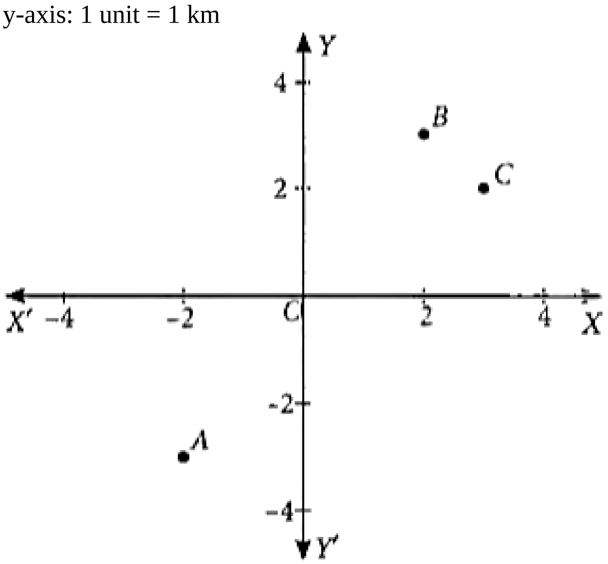
37. **Read the text carefully and answer the questions:**

There are two routes to travel from source A to destination B by bus. First bus reaches at B via point C and second bus reaches from A to B directly. The position of A, B and C are represented in the following graph:

Based on the above information, answer the following questions.

Scale: x-axis : 1 unit = 1 km

y-axis: 1 unit = 1 km



(i) Distance travelled by second bus = 7.2 km

$\therefore$ Total fare = 7.2 $\times$ 15 = ₹108

(ii) Required distance = $\sqrt{(2+2)^2 + (3+3)^2}$

$= \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = 2\sqrt{13}$ km $\approx$ 7.2 km

OR

Distance between B and C

= $\sqrt{(3-2)^2 + (2-3)^2}$ = $\sqrt{1+1} = \sqrt{2}$ km

Thus, distance travelled by first bus to reach to B

= AC + CB = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$ km $\approx$ 8.48 km

and distance travelled by second bus to reach to B

= AB = $2\sqrt{13}$ km = 7.2 km

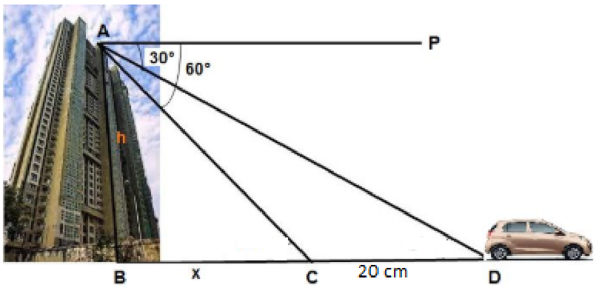
∴ Distance of first bus is greater than distance of the cond bus, therefore second bus should be chosen.

(iii)Required distance = $\sqrt{(3+2)^2+(2+3)^2}$

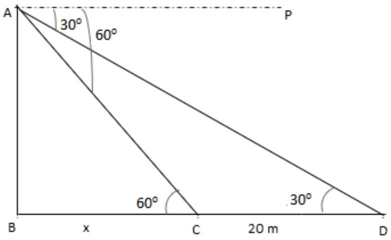
= $\sqrt{5^2+5^2} = 5\sqrt{2}$ km

38. **Read the text carefully and answer the questions:**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^o. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^o.



(i) The above figure can be redrawn as shown below:



From the figure,

let AB = h and BC = x

In △ABC,

tan 60 = $\frac{AB}{BC} = \frac{h}{x}$

$\sqrt{3}$ = $\frac{h}{x}$

h = $\sqrt{3}$x ...(i)

In △ABD,

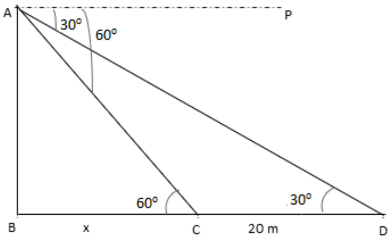
tan 30 = $\frac{AB}{BD} = \frac{h}{x+20}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x+20}$ [using (i)]

x + 20 = 3x

x = 10 m

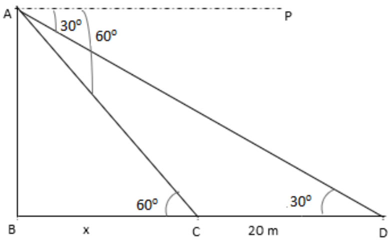
(ii) The above figure can be redrawn as shown below:



Height of the building, h = $\sqrt{3}$x = $10\sqrt{3}$ = 17.32 m

OR

The above figure can be redrawn as shown below:



Distance from top of the building to point D.

In $\triangle$ABD

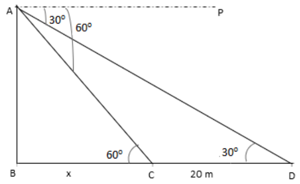
sin 30^o = $\frac{AB}{AD}$

$\Rightarrow AD = \frac{AB}{\sin 30^0}$

$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}m$

(iii)The above figure can be redrawn as shown below:



Distance from top of the building to point C is

In $\triangle$ABC

sin 60^o = $\frac{AB}{AC}$

$\Rightarrow AC = \frac{AB}{\sin 60^0}$

$\Rightarrow AC = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AD = 20 m