CBSE Sample Paper -04 (solved)
SUMMATIVE ASSESSMENT –I
Class – XMathematics

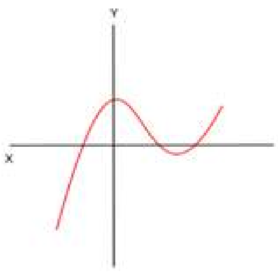
Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

SECTION – A

1. From the given graph, find the number of zeroes of the corresponding polynomial.



2. Prove that tan1°tan2°tan3°……tan89° = 1.

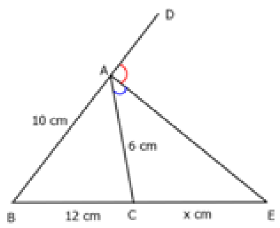
3. Find the value of x, if the mode of the following data is 25.

15, 20, 25, 18, 14, 15, 25, 15, 18, 16, 20, 25, 20, x, 18

4. Express sec67°+cosec58° in terms of trigonometric ratios of angles between 0° and 45°.

SECTION - B

5. In the given figure, AE is the bisector of the exterior ∠CAD meeting BC produced in E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, find CE.



6. If $p(x) = 2x^2 - 3x + 4$, find $p(3)$ and $p(-1)$.

7. The sum of two numbers is 100 and the difference between their squares is 256000. Find the numbers.

8. Taking A = 60° and B = 30°, verify that
   sin(A – B) = sinAcosB – cosAsinB

9. The hypotenuse of a right triangle is 6 m more than the twice of the shortest side. If the third side is 2 m less than the hypotenuse, find the sides of the triangle.

10. The following data gives the distribution of total household expenditure (in rupees) of manual workers in a city:

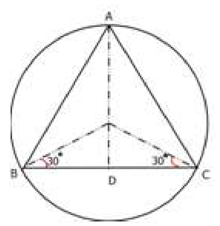
Expenditure (Rs)	Frequency	Expenditure (Rs)	Frequency
1000-1500	24	3000-3500	30
1500-2000	40	3500-4000	22
2000-2500	33	4000-4500	16
2500-3000	28	4500-5000	7

Find the average expenditure which is being done by the maximum number of manual workers.

SECTION – C

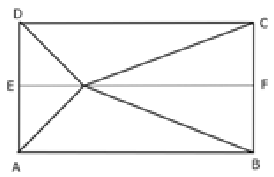
11. The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

12. An equilateral triangle is inscribed in a circle of radius 6 cm. Find its side.



13. If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, $0 < \theta < 90°$, find the values of cosθ and tanθ.

14. A point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D. Prove that $OB^2 + OD^2 = OC^2 + OA^2$



15. If 3 times the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as the remainder. Also, if 7 times the smaller number is divided by the larger one, we get 5 as quotient and 1 as the remainder. Find the numbers.

16. Draw the graph of the polynomial $f(x) = -4x^2 + 4x - 1$. Also, find the vertex of this parabola.

17. Prove that one of every three consecutive positive integers is divisible by 3.

18. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$.

19. If the median of the following frequency distribution is 46, find the missing frequencies.

Variable:

Less than20	Less than20	Less than20	Less than20	Less than20	Less than20	Less than20	Less than20	Less than20
0	4	16	30	46	66	82	92	100

20. Prove that if three or more parallel lines are intersected by two transversals, prove that the intercepts made by them on the transversals are proportional.

SECTION – D

21. If p is a prime number, then prove that $\sqrt{p}$ is irrational.

22. The mean of the following frequency table is 50. But the frequencies f_1 and f_2 in class 20-40 and 60-80 are missing. Find the missing frequencies.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

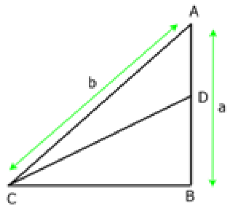
23. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

24. Prove that the areas of two similar triangles are in the ratio of the squares of the corresponding angle bisector segments.

25. Rama went to a stationary stall and purchased 2 pencils and 3 erasers for Rs 9. Her friend Sonal saw the new variety of pencils and erasers with Rama and she also bought 4 pencils and 6 erasers of the same kind for Rs 18. Represent this situation algebraically and graphically.

26. Prove that in any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median which bisects the third side.

27. In the given figure, AD = DB and ∠B is a right angle. Find $\sin^2\theta + \cos^2\theta$.



28. Prove $\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\cos A+\sin A$.

29. Compute the median from the following data:

Mid-value	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

30. Verify that 3, −1 and $-\frac{1}{3}$ are the zeros of the cubic polynomial $p(x)=3x^3-5x^2-11x-3$ and then verify the relationship between the zeros and its coefficients.

31. a. After every 6 months, price of petrol increases at the rate of Rs 4 per litre. Taking price of petrol in December 2010 as x and present price of petrol as y, form a linear equation showing the price of petrol in December 2014.

b. Due to continuous rise in the price of petrol, people are more interesting in CNG whose price is increasing at the rate of Rs 3 per litre in a year. Form a linear equation taking price of CNG in December 2010 as a and in December 2014 as b.

c. Which value is depicted by using CNG over petrol?

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SECTION – A

1. **Solution:**

Since the given graph intersects the x-axis at 3 points, the polynomial has 3 zeros.

2. **Solution:**

We have

$$
\begin{aligned}
\text{LHS} &= \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ \\
&= \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 44^\circ \tan 45^\circ \tan 46^\circ \ldots \tan 88^\circ \tan 89^\circ \\
&= (\tan 1^\circ . \tan 89^\circ)(\tan 2^\circ . \tan 88^\circ) \ \ldots (\tan 44^\circ \tan 46^\circ) \tan 45^\circ \\
&= \{\tan 1^\circ . \tan(90^\circ - 1^\circ)\}\{(\tan 2^\circ . \tan(90^\circ - 2^\circ)\} \ \ldots \{(\tan 44^\circ \tan(90^\circ - 44^\circ)\} \tan 45^\circ \\
&= \{\tan 1^\circ . \cot 1^\circ)(\tan 2^\circ . \cot 2^\circ) \ \ldots (\tan 44^\circ \cot 44^\circ) \tan 45^\circ
\end{aligned}
$$

$[\tan(90^\circ - \theta) = \cot\theta]$

= 1 = RHS $[\tan\theta\cot\theta = 1 \text{ and } \tan 45^\circ = 1]$

3. **Solution:**

We have cos60°cos30° + sin60°sin30°

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 2\left(\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{2}$$

4. **Solution:**

The frequency table of the given data is as given below:

Value (x_i)	14	15	16	18	20	25	x
Frequency (f_i)	1	3	1	3	3	3	1

It is given that the mode of the given data is 25. So, it must have the maximum frequency. That is possible only when x = 25.

Thus, x = 25.

SECTION – B

5. **Solution:**

Since AE is the bisector of the exterior ∠CAD.

$$\therefore \quad \frac{BE}{CE} = \frac{AB}{AC} \quad \Rightarrow \quad \frac{12+x}{x} = \frac{10}{6} \quad \Rightarrow \quad x = 18$$

6. **Solution:**

We have,

$p(x) = 2x^2 - 3x + 4$

$\Rightarrow$ $p(3) = 2 \times 3^2 - 3 \times 3 + 4$

$= 2 \times 9 - 9 + 4$

$= 18 - 9 + 4 = 22 - 9 = 13$

$p(-1) = 2 \times (-1)^2 - 3 \times (-1) + 4$

$= 2 + 3 + 4$

$= 9$

7. **Solution:**

Let the larger number be x and the smaller number be y.

Then, $x + y = 1000$ ...(i)

And, $x^2 - y^2 = 256000$ ...(ii)

On dividing (ii) by (i), we get

$$\frac{x^2 - y^2}{x + y} = \frac{256000}{1000}$$

$\Rightarrow$ $x - y = 256$ ...(iii)

Adding (i) and (iii), we get

$2x = 1256$

$\Rightarrow$ $x = 628$

Substituting $x = 628$ in (i), we get $y = 372$

Thus, the required numbers are 628 and 372.

8. **Solution:**

$A = 60°$ and $B = 30°$

$\Rightarrow$ $A - B = 60° - 30° = 30°$

$\therefore$ $\sin(A - B) = \sin 30° = \frac{1}{2}$

$\sin A \cos B - \cos A \sin B = \sin 60° \cos 30° - \cos 60° \sin 30°$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

∴ sin(A – B) = sinAcosB – cosAsinB

9. **Solution:**

Let the shortest side be $x$metres in length. Then,

Hypotenuse = $(2x + 6)$ m and third side = $(2x + 4)$ m

By Pythagoras theorem, we have

$$(2x+6)^2 = x^2 + (2x+4)^2$$

$\Rightarrow \quad 4x^2 + 24x + 36 = x^2 + 4x^2 + 16x + 16$

$\Rightarrow \quad x^2 - 8x - 20 = 0$

$\Rightarrow \quad (x-10)(x+2) = 0$

$\Rightarrow \quad x = 10$ or $x = -2$

$\Rightarrow \quad x = 10$

Thus, the sides of the triangles are 10 m, 26 m and 24 m.

10. **Solution:**

We observe that the class 1500-2000 has the maximum frequency 40. So, it is the modal class such that

$l = 1500$, $h = 500$, $f = 40$, $f_1 = 24$ and $f_2 = 33$

$$\therefore \quad \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h = 1500 + \frac{40-24}{80-24-33} \times 500$$

$$= 1500 + \frac{16}{23} \times 500 = 1847.826$$

SECTION – C

11. **Solution:**

Let the required numbers be x and y.

Then, $x + y = 16$

And, $\frac{1}{x} + \frac{1}{y} = \frac{1}{3} \quad \Rightarrow \quad \frac{x+y}{xy} = \frac{1}{3} \quad \Rightarrow \quad \frac{16}{xy} = \frac{1}{3} \quad \Rightarrow \quad xy = 48$

We can write

$$x - y = \sqrt{(x+y)^2 - 4xy}$$

$$= \sqrt{(16)^2 - 4 \times 48}$$

$= \sqrt{256-192} = \sqrt{64} = \pm 8$

$\therefore \quad x + y = 16 \qquad ...(i)$

$x - y = 8 \qquad ...(ii)$

Or, $\quad x + y = 16 \qquad ...(iii)$

$x - y = -8 \qquad ...(iv)$

On solving (i) and (ii), we get $x = 12$ and $y = 4$

On solving (iii) and (iv), we get $x = 4$ and $y = 12$

Thus, the required numbers are 12 and 4.

12. **Solution:**

Let ABC be an equilateral triangle inscribed in a circle of radius 6 cm. Let O be the centre of the circle. Then,

OA = OB = OC = 6 cm

Let OD be perpendicular from O on side BC. Then, D is the mid-point of BC and OB and OC are bisectors of $\angle$B and $\angle$C respectively.

$\therefore \quad \angle OBD = 30°$

In ΔOBD, right angled at D, we have

$\angle OBD = 30°$ and OB = 6 cm

$\therefore \quad \cos\angle OBD = \frac{BD}{OB}$

$\Rightarrow \quad \cos 30° = \frac{BD}{6}$

$\Rightarrow \quad BD = 6\cos 30°$

$\Rightarrow \quad BD = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

$\Rightarrow \quad BC = 2BD = 2 \times 3\sqrt{3} = 6\sqrt{3}$

Thus, the side of the equilateral triangle is $6\sqrt{3}$.

13. **Solution:**

We have,

$\sin\theta = \frac{a}{\sqrt{a^2 + b^2}}$

$\therefore \quad \cos\theta = \sqrt{1 - \sin^2\theta}$

$$= \sqrt{1-\frac{a^2}{a^2+b^2}}$$

$$= \sqrt{\frac{b^2}{a^2+b^2}}=\frac{b}{\sqrt{a^2+b^2}}$$

$\therefore$ $\tan\theta = \frac{\sin\theta}{\cos\theta}=\frac{\frac{a}{\sqrt{a^2+b^2}}}{\frac{b}{\sqrt{a^2+b^2}}}=\frac{a}{b}$

14. **Solution:**

Let ABCD be the given rectangle and let O be a point within it. Join OA, OB, OC and OD.

Through O, draw EOF || AB. Then, ABFE is a rectangle.

In right triangles OEA and OFC, we have

$OA^2 = OE^2 + AE^2$amd$OC^2 = OF^2 + CF^2$

$\Rightarrow$ $OA^2 + OC^2 = (OE^2 + AE^2) + (OF^2 + CF^2)$

$\Rightarrow$ $OA^2 + OC^2 = OE^2 + OF^2 + AE^2 + CF^2$ ...(i)

Now, in right triangles OFB and ODE, we have

$OB^2 = OF^2 + FB^2$ and $OD^2 = OE^2 + DE^2$

$\Rightarrow$ $OB^2 + OD^2 = (OF^2 + FB^2) + (OE^2 + DE^2)$

$\Rightarrow$ $OB^2 + OD^2 = OE^2 + OF^2 + DE^2 + BF^2$

$= OE^2 + OF^2 + CF^2 + AE^2$ [$\because$ DE = CF and AE = BF] ...(ii)

From (i) and (ii), we get

$OA^2 + OC^2 = OB^2 + OD^2$

15. **Solution:**

Let the larger number be x and smaller one be y. We know that

Dividend = (Divisor × Quotient) + Remainder ...(i)

When $3x$ is divided by y, we get 4 as quotient and 3 as remainder. Therefore, by using (i), we get

$3x = 4y + 3$ $\Rightarrow$ $3x - 4y - 3 = 0$ ...(ii)

When $7y$ is divided by x, we get 5 as quotient and 1 as remainder. Therefore, by using (i), we get

$7y = 5x + 1$ $\Rightarrow$ $5x - 7y + 1 = 0$ ...(iii)

Solving equations (ii) and (iii), by cross-multiplication, we get

$$\frac{x}{-4-21}=\frac{-y}{3+15}=\frac{1}{-21+20}$$

$\Rightarrow \quad x = 25$ and $y = 18$

Thus, the required numbers are 25 and 18.

16. **Solution:**

Let $y = f(x)$ or $y = -4x^2 + 4x - 1$

The following table gives the values of y for various values of x.

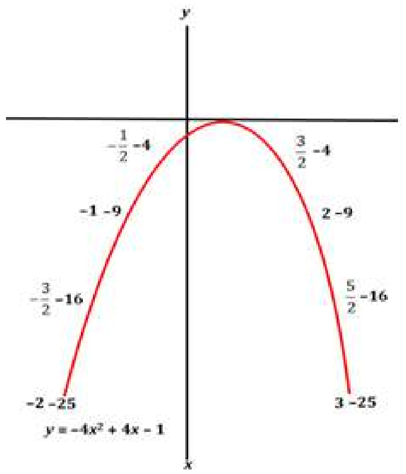
x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$y = -4x^2 + 4x - 1$	-25	-16	-9	-4	-1	0	-1	-4	-9	-16	-25

Thus, the following points lie on the graph of $y = -4x^2 + 4x - 1$:

$(-2, -25)$, $\left(-\frac{3}{2}, -16\right)$, $(-1, -9)$, $\left(-\frac{1}{2}, -4\right)$, $(0, -1)$, $\left(\frac{1}{2}, 0\right)$, $(1, -1)$, $\left(\frac{3}{2}, -4\right)$, $(2, -9)$, $\left(\frac{5}{2}, -16\right)$ and

$(3, -25)$.

Plot these points on a graph paper and draw a free hand smooth curve passing through these points.



The shape of the curve is shown in the figure. It is a parabola opening downward having its vertex at $\left(\frac{1}{2}, 0\right)$.

17. **Solution:**

Let n, $n + 1$ and $n + 2$ be three consecutive positive integers.

We know that n is of the form $3q$, $3q + 1$ or $3q + 2$.

So, we have the following cases:

Case I: When $n = 3q$

In this case, n is divisible by 3 but $n + 1$ and $n + 2$ are not divisible by 3.

Case II: When $n = 3q + 1$

In this case, $n + 2 = 3q + 1 + 2 = 3(q + 1)$, which is divisible by 3 but n and $n + 1$ are not divisible by 3.

Case III: When $n = 3q + 2$

In this case, $n + 1 = 3q + 1 + 2 = 3(q + 1)$,

which is divisible by 3 but n and $n + 2$ are not divisible by 3.

Thus, one of n, $n + 1$ and $n + 2$ is divisible by 3.

18. **Solution:**

We have,

LHS $= \left(m^2 + n^2\right)\cos^2\beta$

$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$ $\quad \left[\because m = \frac{\cos\alpha}{\cos\beta} \text{ and } n = \frac{\cos\alpha}{\sin\beta}\right]$

$= \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$= \cos^2\alpha\left(\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$= \frac{\cos^2\alpha}{\sin^2\beta} = \left(\frac{\cos\alpha}{\sin\beta}\right)^2 = n^2$ = RHS

19. **Solution:**

We are given the cumulative frequency distribution. So, we first construct a frequency table from the given cumulative frequency distribution and then we will make necessary computations to compute median.

Class intervals	Frequency (f)	Cumulative frequency (cf)
20-30	4	4
30-40	12	16
40-50	14	30
50-60	16	46
60-70	20	66
70-80	16	82
80-90	10	92
90-100	8	100
$N = \sum f_i = 100$		

Here, N = $\sum f_i$ = 100 $\Rightarrow$ $\frac{N}{2}$ = 50

We observe that the cumulative frequency just greater than $\frac{N}{2}$ = 50 is 66 and the corresponding class is 60-70.

So, 60-70 is the median class.

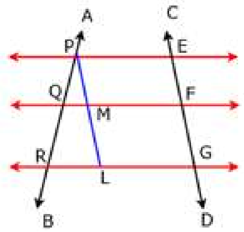
$\therefore$ l = 60, f = 20, F = 46 and h = 10

Now, median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$$= 60 + \frac{50-46}{20} \times 10 = 62$$

20. **Solution:**

Given: Three parallel lines l, m and n which are cut by the transversals AB and CD in P, Q, R and E, F, G, respectively.



To prove: $\frac{PQ}{QR} = \frac{EF}{FG}$

Construction: Draw PL || CD meeting the lines m and n in M and L, respectively.

Proof: Since PE || MF and PM || EF,

$\therefore$ PMFE is a parallelogram.

$\Rightarrow$ PM = EF ...(i)

Also, MF || LG and ML || FG,

$\therefore$ MLFG is a parallelogram.

$\Rightarrow$ ML = FG ...(ii)

Now, in ΔPRL, we have

QM || RL

$\Rightarrow$ $\frac{PQ}{QR} = \frac{PM}{ML}$ [By Thale's Theorem]

$\Rightarrow \quad \frac{PQ}{QR} = \frac{EF}{FG}$ [Using (i) and (ii)]

SECTION – D

21. **Solution:**

Let p be a prime number and if possible, let $\sqrt{p}$ be rational.

Let its simplest form be $\sqrt{p} = \frac{m}{n}$, where m and n are integers having no common factor other than 1, and $n \neq 0$.

Then $\sqrt{p} = \frac{m}{n}$

$\Rightarrow \quad p = \frac{m^2}{n^2}$ [On squaring both sides]

$\Rightarrow \quad pn^2 = m^2$ ...(i)

$\Rightarrow \quad p$ divides m^2 [$\because p$ divides pn^2]

$\Rightarrow \quad p$ divides m [$\because p$ is prime and p divides $m^2 \Rightarrow p$ divides m]

Let $m = pq$ for some integer q.

Putting $m = pq$ in (i), we get

$pn^2 = p^2q^2$

$\Rightarrow \quad n^2 = pq^2$

$\Rightarrow \quad p$ divides n^2 [$\because p$ divides pq^2]

$\Rightarrow \quad p$ divides n [$\because p$ is prime and p divides $n^2 \Rightarrow p$ divides n]

Thus, p is a common factor of m and n.

But this contradicts the fact that m and n have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{p}$ is rational.

Thus, $\sqrt{p}$ is irrational.

22. **Solution:**

Let the assumed mean be A = 50 and h = 20.

Calculation of mean

Class	Frequency f_i	Mid-values	$u_i = \frac{x_i - A}{h}$	f_iu_i

0-20	17	10	–2	–34
20-40	f_1	30	–1	$-f_1$
40-60	32	50	0	0
60-80	f_2	70	1	f_2
80-100	19	90	2	38
$N = \sum f_i = 68 + f_1 + f_2 \sum f_i u_i = 4 - f_1 + f_2$				

We have,

$N = \sum f_i = 120$ [Given]

$\Rightarrow \quad 68 + f_1 + f_2 = 120$

$\Rightarrow \quad f_1 + f_2 = 52$ ...(i)

Now,

Mean = 50

$$\Rightarrow \quad A + h\left\{\frac{1}{N}\sum f_i u_i\right\} = 50$$

$$\Rightarrow \quad 50 + 20 \times \left\{\frac{4 - f_1 + f_2}{120}\right\} = 50$$

$$\Rightarrow \quad 50 + \frac{4 - f_1 + f_2}{6} = 50$$

$$\Rightarrow \quad \frac{4 - f_1 + f_2}{6} = 0$$

$$\Rightarrow \quad 4 - f_1 + f_2 = 0$$

$$\Rightarrow \quad f_1 - f_2 = 4 \qquad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$f_1 = 28$ and $f_2 = 24$

23. **Solution:**

Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

Then,

Speed upstream = $(x - y)$ km/hr

Speed downstream = $(x + y)$ km/hr

Now,

Time taken to cover 32 km upstream = $\frac{32}{x-y}$ hrs

Time taken to cover 36 km downstream = $\frac{36}{x+y}$ hrs

But, total time of journey is 7 hours

$\therefore \quad \frac{32}{x-y}+\frac{36}{x+y}=7$ ...(i)

Time taken to cover 40 km upstream = $\frac{40}{x-y}$

Time taken to cover 48 km downstream = $\frac{48}{x+y}$

In this case, total time of journey is given to be 9 hours.

$\therefore \quad \frac{40}{x-y}+\frac{48}{x+y}=9$ ...(ii)

Putting $\frac{1}{x-y}$ = u and $\frac{1}{x+y}$ = v in equations (i) and (ii), we get

$32u + 36v = 7 \quad \Rightarrow \quad 32u + 36v - 7 = 0$ ...(iii)

$40u + 48v = 9 \quad \Rightarrow \quad 40u + 48v - 9 = 0$ ...(iv)

Solving these equations by cross-multiplication, we get

$$\frac{u}{36\times-9-48\times-7}=\frac{-v}{32\times-9-40\times-7}=\frac{1}{32\times48-40\times36}$$

$$\Rightarrow \quad \frac{u}{-324+336}=\frac{-v}{-288+280}=\frac{1}{1536-1440}$$

$$\Rightarrow \quad \frac{u}{12}=\frac{v}{8}=\frac{1}{96}$$

$$\Rightarrow \quad u=\frac{12}{96}=\frac{1}{8} \text{ and } v=\frac{8}{96}=\frac{1}{8}$$

Now, $u = \frac{1}{8} \quad \Rightarrow \quad \frac{1}{x-y}=\frac{1}{8} \quad \Rightarrow \quad x-y = 8$ ...(v)

and, $v = \frac{1}{12} \quad \Rightarrow \quad \frac{1}{x+y}=\frac{1}{12} \quad \Rightarrow \quad x+y = 12$ ...(vi)

Solving equations (v) and (vi), we get x = 10 and y = 2

Thus, speed of the boat in still water = 10 km/hr

Speed of the stream = 2 km/hr

24. **Solution:**

Given: $\Delta ABC \sim \Delta DEF$ and AX and DY are bisector of $\angle A$ and $\angle D$ respectively.

To prove: $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AX^2}{DY^2}$

Proof: Since the ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

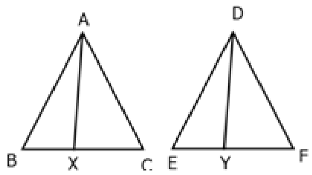
$\therefore \quad \frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AB^2}{DE^2}$ ...(i)

Now, $\Delta ABC \sim \Delta DEF$

$\Rightarrow \quad \angle A = \angle D$

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle D$

$\Rightarrow \quad \angle BAX = \angle EDY$



Thus, in triangles ABX and DEY, we have

$\angle BAX = \angle EDY$ and $\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

So, by AA-similarity criterion, we have

$\Delta ABX \sim \Delta DEY$

$\Rightarrow \quad \frac{AB}{DE} = \frac{AX}{DY}$

$\Rightarrow \quad \frac{AB^2}{DE^2} = \frac{AX^2}{DY^2}$ ...(ii)

From (i) and (ii), we get

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{AX^2}{DY^2}$$

25. **Solution:**

Let the cost of 1 pencil be Rsx and that of one eraser be Rsy.

It is given that Rama purchased 2 pencils and 3 erasers for Rs 9.

$\therefore \quad 2x + 3y = 9$

It is also given that Sonal purchased 4 pencils and 6 erasers for Rs 18.

$\therefore \quad 4x + 6y = 18$

Algebraic representation: The algebraic representation of the given situation is

$$2x + 3y = 9$$

$$4x + 6y = 18$$

Graphical representation: In order to obtain the graphical representation of the above pair of linear equations, we find two points on the line representing each equation. That is, we find two solutions of each equation. Let us find these solutions. We will try to find solutions having integral values.

We have,

$$2x + 3y = 9$$

Putting $x = -3$, we get

$$-6 + 3y = 9 \quad \Rightarrow \quad 3y = 15 \quad \Rightarrow \quad y = 5$$

Putting $x = 0$, we get

$$0 + 3y = 9 \quad \Rightarrow \quad y = 3$$

Thus, two solutions of $2x + 3y = 9$ are:

x	−3	0
y	5	3

We have,

$$4x + 6y = 18$$

Putting $x = 3$, we get

$$12 + 6y = 18 \quad \Rightarrow \quad 6y = 6 \quad \Rightarrow \quad y = 1$$

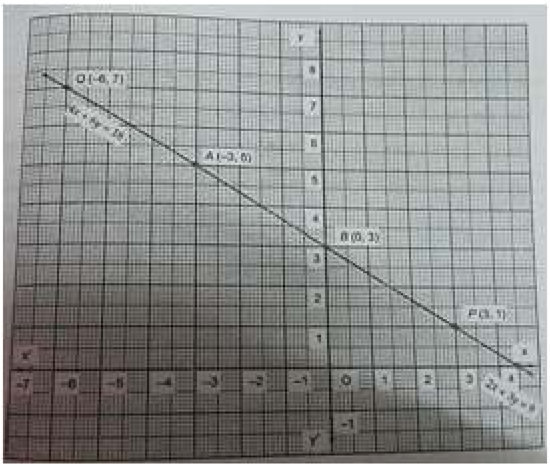
Putting $x = -6$, we get

$$-24 + 6y = 18 \Rightarrow \quad 6y = 42 \quad \Rightarrow \quad y = 7$$

Thus, two solutions of $4x + 6y = 18$ are:

x	3	−6
y	1	7

Now, we plot the points A(−3, 5) and B(0, 3) and draw the line passing through these points to obtain the graph of the line $2x + 3y = 9$. Points P(3, 1) and Q(−6, 7) are plotted on the graph paper and we join them to obtain the graph of the line $4x + 6y = 18$. We find that both the lines AB and PQ coincide.



26. **Solution:**

Given: A ΔABC in which AD is a median.

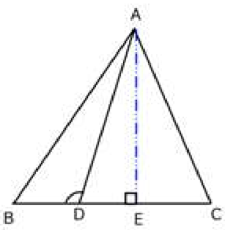
To prove: $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$ or $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Construction: Draw $AE \perp BC$

Proof: Since $\angle AED = 90°$, therefore, in ΔADE, we have

$\angle ADE < 90° \quad \Rightarrow \quad \angle ADB > 90°$

Thus, ΔADB is an obtuse-angled triangle and ΔADC is an acute-angled triangle.



ΔADB is an obtuse-angled triangle at D and $AE \perp BD$ produced. Therefore, we have

$AB^2 = AD^2 + BD^2 + 2BD \times DE$ ...(i)

ΔACD is an acute-angled triangle at D and $AE \perp CD$. Therefore, we have

$AC^2 = AD^2 + DC^2 - 2DC \times DE$

$\Rightarrow \quad AC^2 = AD^2 + BD^2 - 2BD \times DE$ [$\because$ CD = BD] ...(ii)

Adding equations (i) and (ii), we get

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow \quad AB^2 + AC^2 = 2\left\{AD^2 + \left(\frac{BC}{2}\right)^2\right\}$

$= 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

$= 2AD^2 + 2BD^2$

$\Rightarrow \quad AB^2 + AC^2 = 2(AD^2 + BD^2)$

27. **Solution:**

We have,

$AB = a$

$\Rightarrow \quad AD + DB = a$

$\Rightarrow \quad AD + AD = a$

$\Rightarrow \quad 2AD = a \qquad \Rightarrow \qquad AD = \frac{a}{2}$

Thus, $AD = DB = \frac{a}{2}$

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad b^2 = a^2 + BC^2$

$\Rightarrow \quad BC^2 = b^2 - a^2$

$\Rightarrow \quad BC^2 = \sqrt{b^2 - a^2}$

Thus, in ΔBCD, we have

Base = BC = $\sqrt{b^2 - a^2}$ and perpendicular = BD = $\frac{a}{2}$

Applying Pythagoras theorem in ΔBCD, we have

$BC^2 + BD^2 = CD^2$

$\Rightarrow \quad \left(\sqrt{b^2 - a^2}\right)^2 + \left(\frac{a}{2}\right)^2 = CD^2$

$\Rightarrow \quad CD^2 = b^2 - a^2 + \frac{a^2}{4}$

$\Rightarrow \quad CD^2 = \frac{4b^2 - 4a^2 + a^2}{4}$

$\Rightarrow \quad CD^2 = \frac{4b^2 - 3a^2}{4}$

$\Rightarrow \quad CD = \frac{\sqrt{4b^2 - 3a^2}}{2}$

Now, $\sin\theta = \frac{BD}{CD} = \frac{\frac{a}{2}}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \frac{a}{\sqrt{4b^2 - 3a^2}}$

And, $\cos\theta = \dfrac{BC}{CD} = \dfrac{\sqrt{b^2-a^2}}{\dfrac{\sqrt{4b^2-3a^2}}{2}} = \dfrac{2\sqrt{b^2-a^2}}{\sqrt{4b^2-3a^2}}$

Thus, $\sin^2\theta + \cos^2\theta = \left(\dfrac{a}{\sqrt{4b^2-3a^2}}\right)^2 + \left(\dfrac{2\sqrt{b^2-a^2}}{\sqrt{4b^2-3a^2}}\right)^2$

$$= \frac{a^2}{4b^2-3a^2} + \frac{4\left(b^2-a^2\right)}{4b^2-3a^2}$$

$$= \frac{a^2+4b^2-4a^2}{4b^2-3a^2}$$

$$= \frac{4b^2-3a^2}{4b^2-3a^2} = 1$$

28. **Solution:**

We have,

LHS $= \dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A}$

$$= \frac{\cos A}{1-\dfrac{\sin A}{\cos A}} + \frac{\sin A}{1-\dfrac{\cos A}{\sin A}}$$

$$= \frac{\cos A}{\dfrac{\cos A-\sin A}{\cos A}} + \frac{\sin A}{\dfrac{\sin A-\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} + \frac{\sin^2 A}{\sin A-\cos A}$$

$$= \frac{\cos^2 A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A}$$

$$= \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$

$$= \frac{\left(\cos A-\sin A\right)\left(\cos A+\sin A\right)}{\cos A-\sin A}$$

= cosA + sinA

= RHS

29. **Solution:**

Here, we are given the mid-values. So, should first find the upper and lower limits of the various classes. The difference between two consecutive values is h = 125 – 115 = 10.

∴ Lower limit of a class = Mid-value − $\frac{h}{2}$

Upper limit of a class = Mid-value + $\frac{h}{2}$

Calculation of Median

Mid-value	Class groups	Frequency	Cumulative frequency
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151
155	150-160	116	267
165	160-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390
			$N = \sum f_i = 390$

We have,

N = 390 ∴ $\frac{N}{2} = \frac{390}{2} = 195$

The cumulative frequency just greater than $\frac{N}{2}$, i.e., 195 is 267 and the corresponding class is 150-160. So, 150-160 is the median class.

Now,

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 150 + \frac{195 - 151}{116} \times 10 = 153.80$$

30. **Solution:**

We have,

$$p(x) = 3x^3 - 5x^2 - 11x - 3$$

$$\Rightarrow \quad p(3) = 3\times 3^3 - 5\times 3^2 - 11\times 3 - 3$$

$$= 81 - 45 - 33 - 3$$

$= 0$

$p(-1) = 3\times(-1)^3 - 5\times(-1)^2 - 11\times(-1) - 3$

$= -3 - 5 + 11 - 3$

$= 0$

$p\left(-\frac{1}{3}\right) = 3\times\left(-\frac{1}{3}\right)^3 - 5\times\left(-\frac{1}{3}\right)^2 - 11\times\left(-\frac{1}{3}\right) - 3$

$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3$

$= 0$

So, 3, −1 and $-\frac{1}{3}$ are zeros of polynomial $p(x)$.

Let $\alpha = 3$, $\beta = -1$ and $\gamma = -\frac{1}{3}$. Then,

$$\alpha + \beta + \gamma = 3 - 1 - \frac{1}{3} = \frac{5}{3} = -\left(\frac{-5}{3}\right) = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3\times(-1) + (-1)\times\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)\times 3$$

$$= -3 + \frac{1}{3} - 1 = \frac{-11}{3} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = 3\times(-1)\times\left(-\frac{1}{3}\right) = 1 = -\left(\frac{-3}{3}\right) = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

31. **Solution:**

a. Price of petrol in December 2010 = x

Price of petrol in December 2014 = y

Price of petrol increased in 1 year = 4 × 2 = Rs 8

Price of petrol increased in 4 years (December 2010- December 2014) = 8 × 4 = Rs 32

Equation representing the price of petrol in December 2014 = y = x + 32

b. Price of CNG in December 2010 = a

Price of CNG in December 2014 =b

Price of CNG increased in 1 year = Rs 3

Price of CNG increased in 4 years (December 2010- December 2014) = 3 × 4 = Rs 12

Equation representing the price of CNG in December 2014 = b = a + 12

c. The value depicted by using CNG over petrol is environmental protection.