16

Probability

Short Answer Type Questions

Q. 1 **If the letters of the word 'ALGORITHM' are arranged at random in a row what is the probability the letters 'GOR' must remain together as a unit?**

Sol. Number of letters in the word 'ALGORITHM' = 9

If 'GOR' remain together, then considered it as 1 group.

$\therefore$ Number of letters = 6 + 1 = 7

Number of word, if 'GOR' remain together = 7!

Total number of words from the letters of the word 'ALGORITHM' = 9!

$\therefore$ Required probability $= \frac{7!}{9!} = \frac{1}{72}$

Q. 2 **Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks?**

Sol. Let the couple occupied adjacent desks consider those two as 1.

There are (4 + 1) *i.e.*, 5 persons to be assigned.

$\therefore$ Number of ways of assigning these five person = $5! \times 2!$

Total number of ways of assigning 6 persons = 6!

$\therefore$ Probability that the couple has adjacent desk $= \frac{5! \times 2!}{6!} = \frac{2}{6} = \frac{1}{3}$

Probability that the married couple will have non-adjacent desks $= 1 - \frac{1}{3} = \frac{2}{3}$

Q. 3 If an integer from 1 through 1000 is chosen at random, then find the probability that the integer is a multiple of 2 or a multiple of 9.

Sol. Multiple of 2 from 1 to 1000 are 2, 4, 6, 8, ..., 1000

Let n be the number of terms of above series.

$\therefore \quad n\text{th term} = 1000$

$\Rightarrow \quad 2 + (n-1)2 = 1000$

$\Rightarrow \quad 2 + 2n - 2 = 1000$

$\Rightarrow \quad 2n = 1000$

$\therefore \quad n = 500$

Since, the number of multiple of 2 are 500.

So, the multiple of 9 are 9, 18, 27, ..., 999

Let m be the number of term in above series.

$\therefore \quad m\text{th term} = 999$

$\Rightarrow \quad 9 + (m-1)\,9 = 999$

$\Rightarrow \quad 9 + 9m - 9 = 999$

$\Rightarrow \quad 9m = 999$

$\therefore \quad m = 111$

Since, the number of multiple of 9 are 111. So, the multiple of 2 and 9 both are 18, 36, ..., 990

Let p be the number of terms in above series.

$\therefore \quad p\text{th term} = 990$

$\Rightarrow \quad 18 + (p-1)18 = 990$

$\Rightarrow \quad 18 + 18p - 18 = 990$

$\Rightarrow \quad 18p = 990$

$\therefore \quad p = \frac{990}{18} = 55$

Since, the number of multiple of 2 and 9 are 55.

$\therefore$ Number of multiple of 2 or 9 = 500 + 111 − 55 = 556

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{556}{1000} = 0.556$

Q. 4 An experiment consists of rolling a die until a 2 appears.

(i) How many elements of the sample space correspond to the event that the 2 appears on the kth roll of the die?

(ii) How many elements of the sample space correspond to the event that the 2 appears not later than the kth roll of the die?

Sol. In a through of a die there is 6 sample points.

(i) If 2 appears on the kth roll of the die.

So, first $(k-1)$ roll have 5 outcomes each and kth roll results 2 *i.e.,* 1 outcome.

$\therefore$ Number of element of sample space correspond to the event that 2 appears on the kth roll of the die $= 5^{k-1}$

(ii) If we consider that 2 appears not later than kth roll of the die, then it is possible that 2 comes in first throw *i.e.,* 1 outcome.

If 2 does not appear in first throw, then outcomes will be 5 and 2 comes in second throw *i.e.,* 1 outcome, possible outcome $= 5 \times 1 = 5$

Similarly, if 2 does not appear in second throw and appears in third throw.

$\therefore$ Possible outcomes $= 5 \times 5 \times 1$

Given, series $= 1 + 5 + 5 \times 5 + 5 \times 5 \times 5 + \ldots + 5^{k-1}$

$$= 1 + 5 + 5^2 + 5^3 + \ldots + 5^{k-1}$$

$$= \frac{1(5^k - 1)}{5 - 1} = \frac{5^k - 1}{4}$$

Q. 5 A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die.

Sol. It is given that, 2 × Probability of even number = Probability of odd number

$$\Rightarrow \quad P(O) = 2P(E)$$

$$\Rightarrow \quad P(O) : P(E) = 2 : 1$$

$\therefore$ Probability of occurring odd number, $P(O) = \frac{2}{2+1} = \frac{2}{3}$

and probability of occurring 5each number,

$$P(E) = \frac{1}{2+1} = \frac{1}{3}$$

Now, G be the even that a number greater than 3 occur in a single roll of die.

So, the possible outcomes are 4, 5 and 6 out of which two are even and one odd.

$\therefore$ Required probability $= P(G) = 2 \times P(E) \times P(O)$

$$= 2 \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{9}$$

Q. 6 In a large metropolitan area, the probabilities are 0.87, 0.36, 0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets?

Sol. Let E_1 be the event that family own colour television set and E_2 be the event that family owns a black and white television set.

It is given that, $P(E_1) = 0.87$

$P(E_2) = 0.36$

and $P(E_1 \cap E_2) = 0.30$

We have to find probability that a family owns either anyone or both kind of sets *i.e.,* $P(E_1 \cup E_2)$.

Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$ [by addition theorem]

$$= 0.87 + 0.36 - 0.30$$

$$= 0.93$$

Q. 7 If A and B are mutually exclusive events, $P(A) = 0.35$ and $P(B) = 0.45$, then find

(i) $P(A')$ (ii) $P(B')$

(iii) $P(A \cup B)$ (iv) $P(A \cap B)$

(v) $P(A \cap B')$ (vi) $P(A' \cap B')$

Sol. Since, it is given that, A and B are mutually exclusive events.

$\therefore \quad P(A \cap B) = 0 \quad [\because A \cap B = \phi]$

and $\quad P(A) = 0.35,\ P(B) = 0.45$

(i) $P(A') = 1 - P(A) = 1 - 0.35 = 0.65$

(ii) $P(B') = 1 - P(B) = 1 - 0.45 = 0.55$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.35 + 0.45 - 0 = 0.80$

(iv) $P(A \cap B) = 0$

(v) $P(A \cap B') = P(A) - P(A \cap B) = 0.35 - 0 = 0.35$

(vi) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

Q. 8 A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple or very simple are respectively, 0.15, 0.20, 0.31, 0.26 and 0.08. Find the probabilities that a particular surgery will be rated

(i) complex or very complex.

(ii) neither very complex nor very simple.

(iii) routine or complex.

(iv) routine or simple.

Sol. Let E_1, E_2, E_3, E_4 and E_5 be the event that surgeries are rated as very complex, complex, routine, simple or very simple, respectively.

$\therefore \quad P(E_1) = 0.15, P(E_2) = 0.20, P(E_3) = 0.31, P(E_4) = 0.26, P(E_5) = 0.08$

(i) P (complex or very complex) $= P(E_1 \text{ or } E_2)$

$= P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 0.15 + 0.20 - 0$ [$P(E_1 \cap E_2) = 0$ because all events are independent]

$= 0.35$

(ii) P (neither very complex nor very simple), $(P(E_1' \cap E_5') = P(E_1 \cup E_5)'$

$= 1 - P(E_1 \cup E_5)$

$= 1 - [P(E_1) + P(E_5)]$

$= 1 - (0.15 + 0.08)$

$= 1 - 0.23$

$= 0.77$

(iii) P (routine or complex) $= P(E_3 \cup E_2) = P(E_3) + P(E_2)$

$= 0.31 + 0.20 = 0.51$

(iv) P (routine or simple) $= P(E_3 \cup E_4) = P(E_3) + P(E_4)$

$= 0.31 + 0.26 = 0.57$

Q. 9 Four candidates A, B, C and D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, then what are the probabilities that

(i) C will be selected?

(ii) A will not be selected?

Sol. It is given that A is twice as likely to be selected as D.

$$P(A) = 2P(B)$$

$$\Rightarrow \qquad \frac{P(A)}{2} = P(B)$$

while C is twice as likely to be selected as D.

$$P(C) = 2P(D) \Rightarrow P(B) = 2P(D)$$

$$\Rightarrow \qquad \frac{P(A)}{2} = 2P(D) \Rightarrow P(D) = \frac{P(A)}{4}$$

B and C are given about the same chance of being selected.

$$P(B) = P(C)$$

Now, sum of probability = 1

$$P(A) + P(B) + P(C) + P(D) = 1$$

$$P(A) + \frac{P(A)}{2} + \frac{P(A)}{2} + \frac{P(A)}{4} = 1$$

$$\Rightarrow \qquad \frac{4P(A) + 2P(A) + 2P(A) + P(A)}{4} = 1$$

$$\Rightarrow \qquad 9P(A) = 4 \Rightarrow P(A) = \frac{4}{9}$$

(i) P (C will be selected) $= P(C) = P(B) = \frac{P(A)}{2}$

$$= \frac{4}{9 \times 2} \qquad \left[\because P(A) = \frac{9}{4}\right]$$

$$= \frac{2}{9}$$

(ii) P (A will not be selected) $= P(A') = 1 - P(A) = 1 - \frac{4}{9} = \frac{5}{9}$

Q. 10 One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently the sample space consists of four elementary outcomes S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}. You are told that the chances of John's promotion is same as that of Gurpreet Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(i) Determine

P (John promoted), P (Rita promoted),

P (Aslam promoted), P (Gurpreet promoted).

(ii) If A = {John promoted or Gurpreet promoted}, find $P(A)$

Sol. Let E_1 = John promoted

E_2 = Rita promoted

E_3 = Aslam promoted

E_4 = Gurpreet promoted

Given, sample space, S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}

i.e., $S = \{E_1, E_2, E_3, E_4\}$

It is given that, chances of John's promotion is same as that of Gurpreet.

$$P(E_1) = P(E_4)$$

Rita's chances of promotion are twice as likely as John.

$$P(E_2) = 2P(E_1)$$

And Aslam's chances of promotion are four times that of John.

$$P(E_3) = 4P(E_1)$$

Now, $P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$

$\Rightarrow$ $P(E_1) + 2P(E_1) + 4P(E_1) + P(E_1) = 1$

$\Rightarrow$ $8P(E_1) = 1$

$\therefore$ $P(E_1) = \frac{1}{8}$

(i) P (John promoted) $= P(E_1) = \frac{1}{8}$

P (Rita promoted) $= P(E_2) = 2P(E_1) = 2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

P (Aslam promoted) $= P(E_3) = 4P(E_1) = 4 \times \frac{1}{8} = \frac{1}{2}$

P (Gurpreet promoted) $= P(E_4) = P(E_1) = \frac{1}{8}$

(ii) A = John promoted or Gurpreet promoted

$$A = E_1 \cup E_4$$

$$P(A) = P(E_1 \cup E_4) = P(E_1) + P(E_4) - P(E_1 \cap E_4)$$

$$= \frac{1}{8} + \frac{1}{8} - 0 \qquad [\because P(E_1 \cap E_4) = 0]$$

$$= \frac{2}{8} = \frac{1}{4}$$

Q. 11 The accompanying Venn diagram shows three events, A, B and C and also the probabilities of the various intersections [for instance, $P(A \cap B) = 0.7$]. Determine

(i) $P(A)$ (ii) $P(B \cap \bar{C})$

(iii) $P(A \cup B)$ (iv) $P(A \cap \bar{B})$

(v) $P(B \cap C)$ (vi) Probability of exactly one of the three occurs.

Sol. From the above Venn diagram,

(i) $P(A) = 0.13 + 0.07 = 0.20$

(ii) $P(B \cap \bar{C}) = P(B) - P(B \cap C) = 0.07 + 0.10 + 0.15 - 0.15 = 0.07 + 0.10 = 0.17$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

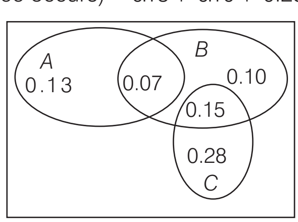
$= 0.13 + 0.07 + 0.07 + 0.10 + 0.15 - 0.07$

$= 0.13 + 0.07 + 0.10 + 0.15 = 0.45$

(iv) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.13 + 0.07 - 0.07 = 0.13$

(v) $P(B \cap C) = 0.15$

(vi) P (exactly one of the three occurs) $= 0.13 + 0.10 + 0.28 = 0.51$



Long Answer Type Questions

Q. 12 One urn contains two black balls (labelled B_1 and B_2) and one white ball. A second urn contains one black ball and two white balls (labelled W_1 and W_2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then, a second ball is chosen at random from the same urn without replacing the first ball.

(i) Write the sample space showing all possible outcomes.

(ii) What is the probability that two black balls are chosen?

(iii) What is the probability that two balls of opposite colour are chosen?

Sol. It is given that one of the two urn is chosen, then a ball is randomly chosen from the urn, then a second ball is chosen at random from the same urn without replacing the first ball.

(i) $\therefore$ Sample space $S = \{B_1B_2, B_1W, B_2B_1, B_2W, WB_1, WB_2, BW_1, BW_2, W_1B, W_1W_2, W_2B, W_2W_1\}$

$\therefore$ total sample point = 12

(ii) If two black ball are chosen.

So, the favourable events are $B_1\ B_2, B_2\ B_1$ *i.e.,* 2

$\therefore$ Required probability $= \dfrac{2}{12} = \dfrac{1}{6}$

(iii) If two balls of opposite colour are chosen.

So, the favourable events are $B_1\ W_1, B_2\ W_1, WB_1, WB_2, BW_1, BW_2, W_1B_1, W_2B$ *i.e.,* 8.

$\therefore$ Required probability $= \dfrac{8}{12} = \dfrac{2}{3}$

Q.13 A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that

(i) all the three balls are white.

(ii) all the three balls are red.

(iii) one ball is red and two balls are white.

Sol. $\because$ Number of red balls = 8

and number of white balls = 5

(i) P (all the three balls are white) $= \dfrac{^5C_3}{^{13}C_3} = \dfrac{\dfrac{5!}{3!2!}}{\dfrac{13!}{3!10!}} = \dfrac{5!}{3!2!} \times \dfrac{3!10!}{13!}$

$$= \frac{5 \times 4 \times 3 \times 2!}{2!} \times \frac{10!}{13 \times 12 \times 11 \times 10!} = \frac{5 \times 4 \times 3}{13 \times 12 \times 11}$$

$$= \frac{5}{13 \times 11} = \frac{5}{143}$$

$$= \frac{5 \times 4 \times 3}{13 \times 12 \times 11} = \frac{5}{13 \times 11} = \frac{5}{143}$$

(ii) P (all the three balls are red)

$$= \frac{{}^8C_3}{{}^{13}C_3} = \frac{\dfrac{8!}{3!5!}}{\dfrac{13!}{3!10!}} = \frac{8!}{3! \times 5!} \times \frac{3!10!}{13!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5!} \times \frac{10!}{13 \times 12 \times 11 \times 10!}$$

$$= \frac{8 \times 7 \times 6}{13 \times 12 \times 11} = \frac{28}{143}$$

(iii) P (one ball is red and two balls are white)

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3} = \frac{8 \times 10}{13 \times 6 \times 11} = \frac{40}{143}$$

Q. 14 If the letters of the word 'ASSASSINATION' are arranged at random. Find the probability that

(i) four S's come consecutively in the word.

(ii) two I's and two N's come together.

(iii) all A's are not coming together.

(iv) no two A's are coming together.

Sol. Total number of letters in the word 'ASSASSINATION' are 13.
Out of which 3A's, 4S's, 2 I's, 2 N's, 1 T's and 10.

(i) If four S's come consecutively in the word, then we considers these 4 S's as 1 group. Now, the number of laters is 10.

S	S	S	S	A	A	A	I	I	N	N	T	O
1				9								

Number of words when all S's are together $= \dfrac{10!}{3!2!2!}$

Total number of word using letters of the word 'ASSASSINATION'

$$= \frac{13!}{3!4!2!2!}$$

$\therefore$ Required probability $= \dfrac{\dfrac{10!}{3!2!2! \times 13!}}{3!4!2!2!}$

$$= \frac{10! \times 4!}{13!} = \frac{4!}{13 \times 12 \times 11} = \frac{24}{1716} = \frac{2}{143}$$

(ii) If 2 I's and 2 N's come together, then there as 10 alphabets.

Number of word when 2 I's and 2 N's are come together

$$= \frac{10!}{3!4!} \times \frac{4!}{2!2!}$$

$\therefore$ Required probability $= \dfrac{\dfrac{10!4!}{3!4!2!2!}}{\dfrac{13!}{3!4!2!2!}} = \dfrac{4!10!}{2!2!3!4!} \times \dfrac{3!4!2!2!}{13!}$

$$= \frac{4!10!}{13!} = \frac{4!}{13 \times 12 \times 11} = \frac{24}{13 \times 12 \times 11} = \frac{2}{143}$$

(iii) If all A's are coming together, then there are 11 alphabets.

Number of words when all A's come together

$$= \frac{11!}{4!2!2!}$$

Probability when all A's come together

$$= \frac{\dfrac{11!}{4!2!2!}}{\dfrac{13!}{4!3!2!2!}} = \frac{11!}{4!2!2!} \times \frac{4!3!2!2!}{13!} = \frac{11! \times 3!}{13!} = \frac{6}{13 \times 12} = \frac{1}{26}$$

Required probability when all A's does not come together

$$= 1 - \frac{1}{26} = \frac{25}{26}$$

(iv) If no two A's are together, then first we arrange the alphabets except A's.

	S		S		S		S		I		N		T		I		O		N	

All the alphabets except A's are arranged in $\dfrac{10!}{4!2!2!}$.

There are 11 vacant places between these alphabets.

So, 3 A's can be place in 11 places in ${}^{11}C_3$ ways $= \dfrac{11!}{3!8!}$

$\therefore$ Total number of words when no two A's together

$$= \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$$

$$\text{Required probability} = \frac{11! \times 10!}{3!8!4!2!2!} \times \frac{4!3!2!2!}{13!} = \frac{10!}{8! \times 13 \times 12}$$

$$= \frac{10 \times 9}{13 \times 12} = \frac{90}{156} = \frac{15}{26}$$

Q. 15 If a card is drawn from a deck of 52 cards, then find the probability of getting a king or a heart or a red card.

Sol. $\because$ Number of possible event = 52

and favourable events = 4 king + 13 heart + 26 red – 13 – 2 = 28

$\therefore$ Required probability $= \dfrac{28}{52} = \dfrac{7}{13}$

Q.16 A sample space consists of 9 elementary outcomes $E_1, E_2, \ldots, E_9$ whose probabilities are

$$P(E_1) = P(E_2) = 0.08,\ P(E_3) = P(E_4) = P(E_5) = 0.1$$
$$P(E_6) = P(E_7) = 0.2,\ P(E_8) = P(E_9) = 0.07$$

Suppose $A = \{E_1, E_5, E_8\}$, $B = \{E_2, E_5, E_8, E_9\}$

(i) Calculate $P(A)$, $P(B)$ and $P(A \cap B)$.

(ii) Using the addition law of probability, calculate $P(A \cup B)$.

(iii) List the composition of the event $A \cup B$ and calculate $P(A \cup B)$ by adding the probabilities of the elementary outcomes.

(iv) Calculate $P(\overline{B})$ from $P(B)$, also calculate $P(\overline{B})$ directly from the elementary outcomes of $\overline{B}$.

Sol. Given,

$$S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9\}$$
$$A = \{E_1, E_5, E_8\},\ B = \{E_2, E_5, E_8\ , E_9\}$$
$$P(E_1) = P(E_2) = 0.08$$
$$P(E_3) = P(E_{4)} = P(E_5) = 0.1$$
$$P(E_6) = P(E_7) = 2,\ P(E_8) = P(E_9) = 0.07$$

(i) $P(A) = P(E_1) + P(E_5) + P(E_8)$
$= 0.08 + 0.1 + 0.07 = 0.25$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ... (i)

Now, $P(B) = P(E_2) + P(E_5) + P(E_8) + P(E_9)$
$= 0.08 + 0.1 + 0.07 + 0.07 = 0.32$

$A \cap B = \{E_5, E_8\}$

$P(A \cap B) = P(E_5) + P(E_8) = 0.1 + 0.7 = 0.17$

On substituting these values in Eq.(i), we get

$$P(A \cup B) = 0.25 + 0.32 - 0.17 = 0.40$$

(iii) $A \cup B = \{E_1, E_2, E_5, E_8, E_9\}$

$$P(A \cup B) = P(E_1) + P(E_2) + P(E_5) + P(E_8) + P(E_9)$$
$$= 0.08 + 0.08 + 0.1 + 0.07 + 0.07 = 0.40$$

(iv) $\because P(\overline{B}) = 1 - P(B) = 1 - 0.32 = 0.68$

and $\overline{B} = \{E_1, E_3, E_4, E_6, E_7\}$

$\therefore$ $P(\overline{B}) = P(E_1) + P(E_3) + P(E_4) + P(E_6) + P(E_7)$
$= 0.08 + 0.1 + 0.1 + 0.2 + 0.2 = 0.68$

Q. 17 Determine the probability p, for each of the following events.

(i) An odd number appears in a single toss of a fair die.

(ii) Atleast one head appears in two tosses of a fair coin.

(iii) A king, 9 of hearts or 3 of spades appears in drawing a single card from a well shuffled ordinary deck of 52 cards.

(iv) The sum of 6 appears in a single toss of a pair of fair dice.

Sol. **(i)** When a die is throw the possible outcomes are

$S = \{1, 2, 3, 4, 5, 6\}$ out of which 1, 3, 5 are odd,

$\therefore$ Required probability $= \frac{3}{6} = \frac{1}{2}$

(ii) When a fair coin is tossed two times the sample space is

$$S = \{HH, HT, TH, TT\}$$

In at least one head favourable enonts are HH, HT, TH

$\therefore$ Required probability $= \frac{3}{4}$

(iii) Total cards = 52

Favourable = 4 king + 2 of heart + 3 of spade = 4 + 1 + 1 = 6

$\therefore$ Required probability $= \frac{6}{52} = \frac{3}{26}$

(iv) When a pair of dice is rolled total sample parts are 36. Out of which (1, 5), (5, 1), (2, 4), (4, 2) and (3, 3).

$\therefore$ Required probability $= \frac{5}{36}$

Objective Type Questions

Q.18 In a non-leap year, the probability of having 53 Tuesday or 53 Wednesday is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) None of these

Sol. ***(a)*** In a non-leap year' there are 365 days which have 52 weeks and 1 day. If this day is a Tuesday or Wednesday, then the year will have 53 Tuesday or 53 Wednesday.

$\therefore$ Required probability $= \frac{1}{7}$

Q. 19 Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive

(a) $\frac{186}{190}$ (b) $\frac{187}{190}$ (c) $\frac{188}{190}$ (d) $\frac{18}{{}^{20}C_3}$

Sol. ***(b)*** Since, the set of three consecutive numbers from 1 to 20 are 123, 234, 345, ..., 18, 19, 20 *i.e.,* 18.

$$P \text{ (numbers are consecutive)} = \frac{18}{{}^{20}C_3} = \frac{18}{1140} = \frac{3}{190}$$

$$P \text{ (three numbers are not consecutive)} = 1 - \frac{3}{190} = \frac{187}{190}$$

Q. 20 While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours.

(a) $\frac{29}{52}$ (b) $\frac{1}{2}$ (c) $\frac{26}{51}$ (d) $\frac{27}{51}$

Sol. ***(c)*** Since,in a back of 52 cards 26 are red colour and 26 are black colour.

$$\therefore P \text{ (both cards of opposite colour)} = \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$$

$$= 2 \times \frac{26}{52} \times \frac{26}{51} = \frac{26}{51}$$

Q. 21 If seven persons are to be seated in a row. Then, the probability that two particular persons sit next to each other is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{7}$ (d) $\frac{1}{2}$

Sol. ***(c)*** If two persons sit next to each other, then consider these two person as 1 group.

Now, we have to arrange 6 persons.

$\therefore$ Number of arrangement $= 2! \times 6!$

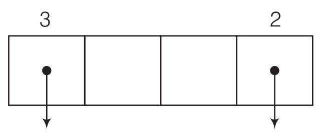
Total number of arrangement of 7 persons $= 7!$

$$\text{Required probability} = \frac{2!6!}{7!} = \frac{2}{7}$$

Q. 22 If without repetition of the numbers, four-digit numbers are formed with the numbers 0, 2, 3 and 5, then the probability of such a number divisible by 5 is

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{1}{30}$ (d) $\frac{5}{9}$

Sol. ***(d)*** We have, to form four-digit number using the digit 0, 2, 3 and 5 which are divisible by 5.



If 0 is fixed at units place $= 3 \times 2 \times 1 = 6$

If 5 is fixed at units place $= 2 \times 2 \times 1 = 4$

Total four-digit numbers divisible by 5 $= 6 + 4 = 10$

$\therefore$ Required probability $= \frac{10}{18} = \frac{5}{9}$

Q. 23 If A and B are mutually exclusive events, then

(a) $P(A) \le P(\bar{B})$ (b) $P(A) \ge P(\bar{B})$

(c) $P(A) < P(\bar{B})$ (d) None of these

Sol. ***(a)*** For mutually exclusive events,

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A \cup B) = P(A) + P(B)$$

$$\Rightarrow \quad P(A) + P(B) \le 1$$

$$\Rightarrow \quad P(A) + 1 - P(\bar{B}) \le 1 \qquad [\because P(B) = 1 - P(\bar{B})]$$

$$\therefore \quad P(A) \le P(\bar{B})$$

Q. 24 If $P(A \cup B) = P(A \cap B)$ for any two events A and B, then

(a) $P(A) = P(B)$ (b) $P(A) > P(B)$

(c) $P(A) < P(B)$ (d) None of these

Sol. ***(a)*** Given, $P(A \cup B) = P(A \cap B)$

$$P(A) + P(B) - P(A \cap B) = P(A \cap B)$$

$$\Rightarrow \quad [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$$

But $\quad P(A) - P(A \cap B) \ge 0$

and $\quad P(B) - P(A \cap B) \ge 0 \qquad [\because P(A \cap B) \le P(A) \text{ or } P(B)]$

$\Rightarrow \quad P(A) - P(A \cap B) = 0$

and $\quad P(B) - P(A \cap B) = 0$

[since, sum of two non-negative numbers can be zero only when these numbers aree zero]

$\Rightarrow \quad P(A) = P(A \cap B)$

and $\quad P(B) = P(A \cap B)$

$\therefore \quad P(A) = P(B)$

Q. 25 If 6 boys and 6 girls sit in a row at random, then the probability that all the girls sit together is

(a) $\frac{1}{432}$ (b) $\frac{12}{431}$

(c) $\frac{1}{132}$ (d) None of these

Sol. *(c)* If all the girls sit together, then considered it as 1 group.

$\therefore$ Arrangement of $6+1=7$ person in a row is $7!$ and the girls interchanges their seets in 6! ways.

$$\therefore \quad \text{Required probability} = \frac{6!7!}{12!} = \frac{1}{132}$$

Q. 26 If a single letter is selected at random from the word 'PROBABILITY', then the probability that it is a vowel is

(a) $\frac{1}{3}$ (b) $\frac{4}{11}$ (c) $\frac{2}{11}$ (d) $\frac{3}{11}$

Sol. *(b)* Total number of alphabet in the word probability $= 11$

$$\text{Number of vowels} = 4$$
$$P(\text{letter is vowel}) = \frac{4}{11}$$

Q. 27 If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

(a) > 0.5 (b) 0.5 (c) ≤ 0.5 (d) 0

Sol. *(c)* Given, $P(A \text{ fail}) = 0.2$

and $P(B \text{ fail}) = 0.3$

$$\therefore \quad P(\text{either } A \text{ or } B \text{ fail}) \leq P(A \text{ fail}) + P(B \text{ fail})$$
$$\leq 0.2 + 0.3$$
$$\leq 0.5$$

Q. 28 The probability that atleast one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is equal to

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.6

Sol. *(c)* Given, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

$$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow \quad 0.6 = P(A) + P(B) - 0.2$$
$$\Rightarrow \quad P(A) + P(B) = 0.8$$
$$\therefore \quad P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$$
$$= 2 - [P(A) + P(B)]$$
$$= 2 - 0.8 = 1.2$$

Q. 29 If M and N are any two events, the probability that atleast one of them occurs is

(a) $P(M) + P(N) - 2P(M \cap N)$ (b) $P(M) + P(N) - P(M \cap N)$
(c) $P(M) + P(N) + P(M \cap N)$ (d) $P(M) + P(N) + 2P(M \cap N)$

Sol. ***(b)*** If M and N are any two events.

$\therefore$ $P(M \cup N) = P(M) + P(N) - P(M \cap N)$

True/False

Q. 30 The probability that a person visiting a zoo will see the giraffee is 0.72, the probability that he will see the bears is 0.84 and the probability that he will see both is 0.52.

Sol. ***False***

P (to see girafee) = 0.72
P (to see bear) = 0.84
P (to see giraffee and bear) = 0.52
P (to see giraffee or bear) = P (giraffee) + P (bear) – P (giraffee and bear)
= 0.72 + 0.84 – 0.52
= 1.04

which is not possible. Hence statement is false.

Q. 31 The probability that a student will pass his examination is 0.73, the probability of the student getting a compartment is 0.13 and the probability that the student will either pass or get compartment is 0.96.

Sol. ***False***

Let A = Student will pass examination
B = Student will getting compartment
$P(A) = 0.73$ and $P(A \text{ or } B) = 0.96$ and $P(B) = 0.13$

$\therefore$ $P(A \text{ or } B) = P(A) + P(B) = 0.73 + 0.13 = 0.86$

But $P(A \text{ or } B) = 0.96$

Hence, it is **false** statement.

Q. 32 The probabilities that a typist will make 0, 1, 2, 3, 4 and 5 or more mistakes in typing a report are respectively, 0.12, 0.25, 0.36, 0.14, 0.08 and 0.11.

Sol. ***False***

Sum of these probabilities must be equal to 1.

$P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$
$= 0.12 + 0.25 + 0.36 + 0.14 + 0.08 + 0.11 = 1.06$

which is greater than 1,
So, it is **false** statement.

Q. 33 If A and B are two candidates seeking admission in an engineering college. The probability that A is selected is 0.5 and the probability that both A and B are selected is at most 0.3. Is it possible that the probability of B getting selected is 0.7?

Sol. ***False***

Here, $P(A) = 0.5, P(A \cap B) \le 0.3$

Now, $P(A) \times P(B) \le 0.3$

$\Rightarrow$ $0.5 \times P(B) \le 0.3$

$\Rightarrow$ $P(B) \le 0.6$

Hence, it is **false** statement.

Q. 34 The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

Sol. ***True***

$$P(A \cap B) \le P(A)$$

Hence, it is **true** statement.

Q. 35 The probability of an occurrence of event A is 0.7 and that of the occurrence of event B is 0.3 and the probability of occurence of both is 0.4.

Sol. ***False***

Here, $P(A) = 0.7$

and $P(B) = 0.3$

$\therefore$ $P(A \cap B) = P(A) \times P(B)$

$= 0.7 \times 0.3 = 0.21$

Hence, it is **false** statement.

Q. 36 The sum of probabilities of two students getting distinction in their final examinations is 1.2.

Sol. ***True***

Since, these two events not related to the same sample space.

So, sum of probabilities of two students getting distinction in their final examination may be 1.2.

Hence, it is **true** statement.

Fillers

Q. 37 The probability that the home team will win an upcoming football game is 0.77, the probability that it will tie the game is 0.08 and the probability that it will lose the game is ... .

Sol. P (lossing) $= 1 - (0.77 + 0.08) = 0.15$

Q. 38 If e_1, e_2, e_3 and e_4 are the four elementary outcomes in a sample space and $P(e_1) = 0.1$, $P(e_2) = 0.5$ and $P(e_3) = 0.1$, then the probability of e_4 is .... .

Sol. $\because$ $P(e_1) + P(e_2) + P(e_3) + P(e_4) = 1$

$\Rightarrow$ $0.1 + 0.5 + 0.1 + P(e_4) = 1$

$\Rightarrow$ $0.7 + P(e_4) = 1$

$\therefore$ $P(e_4) = 0.3$

Q. 39 If $S = \{1, 2, 3, 4, 5, 6\}$ and $E = \{1, 3, 5\}$, then $\overline{E}$ is .... .

Sol. Here, $S = \{1, 2, 3, 4, 5, 6\}$

and $E = \{1, 3, 5\}$

$\therefore$ $\overline{E} = S - E = \{2, 4, 6\}$

Q. 40 If A and B are two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, then the value of $P(A \cap \overline{B})$ is ... .

Sol. $P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.3 - 0.1 = 0.2$

Q. 41 The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is ....

Sol. $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B)]$ [since, A and B are mutually exclusive]

$= 1 - (0.5 + 0.3) = 1 - 0.8 = 0.2$

Matching The Columns

Q. 42 Match the following.

	Column I		Column II
(i)	0.95	(a)	An incorrect assignment
(ii)	0.02	(b)	No chance of happening
(iii)	– 0.3	(c)	As much chance of happening as not
(iv)	0.5	(d)	Very likely to happen
(v)	0	(e)	Very little chance of happening

Sol. **(i)** 0.95 is very likely to happen, so it is close to 1.

(ii) 0.02 very little chance of happening because probability is very low.

(iii) – 0.3 an incorrect assignment because probability of any events lie between 0 and 1.

(iv) 0.5, as much chance of happening as not because sum of chances of happening and not happening is zero.

(v) 0, no chance of happening.

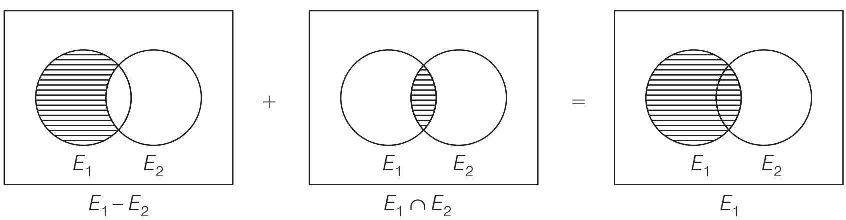
Q. 43 Match the following.

	Column I		Column II
(i)	If E_1 and E_2 are the two mutually exclusive events	(a)	$E_1 \cap E_2 = E_1$
(ii)	If E_1 and E_2 are the mutually exclusive and exhaustive events	(b)	$(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$
(iii)	If E_1 and E_2 have common outcomes, then	(c)	$E_1 \cap E_2 = \phi, E_1 \cup E_2 = S$
(iv)	If E_1 and E_2 are two events such that $E_1 \subset E_2$	(d)	$E_1 \cap E_2 = \phi$

Sol. **(i)** If E_1 and E_2 are two mutually exclusive event, then $E_1 \cap E_2 = \phi$.

(ii) If E_1 and E_2 are mutually exclusive and exhaustive events, then $E_1 \cap E_2 = \phi$ and $E_1 \cup E_2 = S$.

(iii) If E_1 and E_2 have common outcomes, then $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$



(iv) If E_1 and E_2 are two events such that $E_1 \subset E_2 \Rightarrow E_1 \cap E_2 = E_1$