CHAPTER-03

Shear Force & Bending Moment Diagrams (S.F.D. & B.M.D.)

Beam :- Beam is define as a structural Member which is subjected to transverse shear load during its functionality

⇒ Due to this transverse shear load beam are subjected to variable shear force and variable bending moment Hence to know type of variation and maximum value of shear force and bending Moment SFD & BMD are to be drawn.

⇒ SFD & BMD plays important role in design of beams, and design of shaft based in strength and rigidity criterion.

Representation of A Beam :-

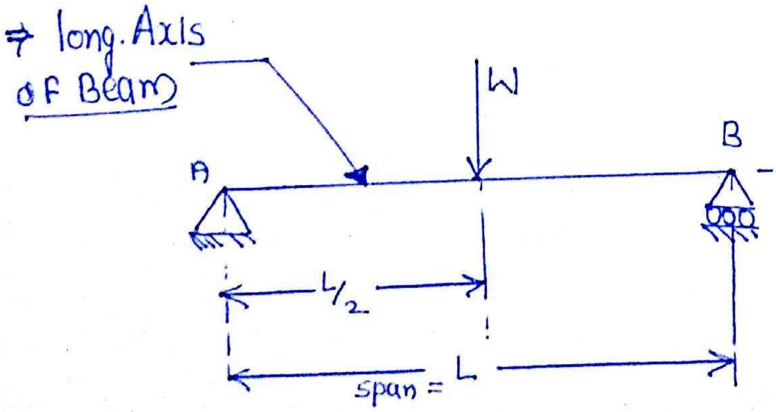


Fig:- Representation of Simply Support Beam.

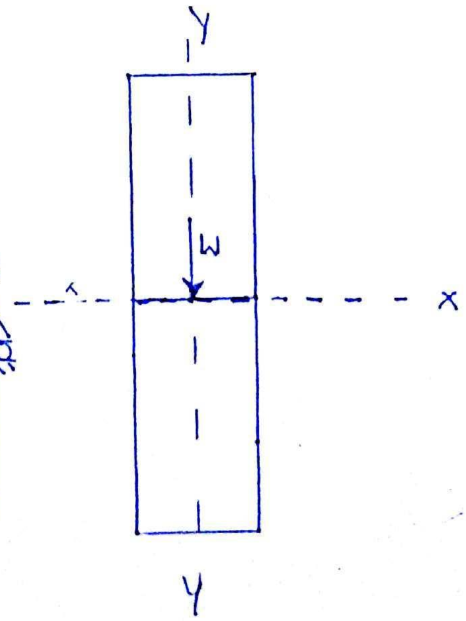


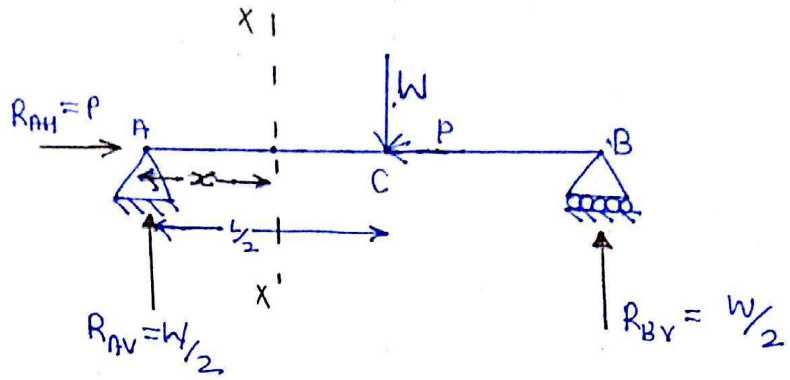
Fig:- X-S/C of Beam at C

(i) Its longitudnal axis

(ii) supports or support reaction

(iii) load acting on it

(iv) span



AC:- $[x = 0 \text{ to } L/2]$

$$(AL)_{X-X} = -P \text{ (Constant)}$$

$$(SF)_{X-X} = \frac{W}{2} \text{ (Const.)}$$

$$(BM_{X-X}) = \frac{Wx}{2} \text{ Variable (LHS)}$$

$$= -W\left(\frac{L}{2} - x\right) + \frac{W}{2}\left[\frac{L}{2} + \frac{L}{2} - x\right] \quad \text{(RHS)}$$

$$= -\frac{WL}{2} + Wx + \frac{WL}{2} - \frac{Wx}{2}$$

$$(BM_{x-x}) = \frac{Wx}{2}$$

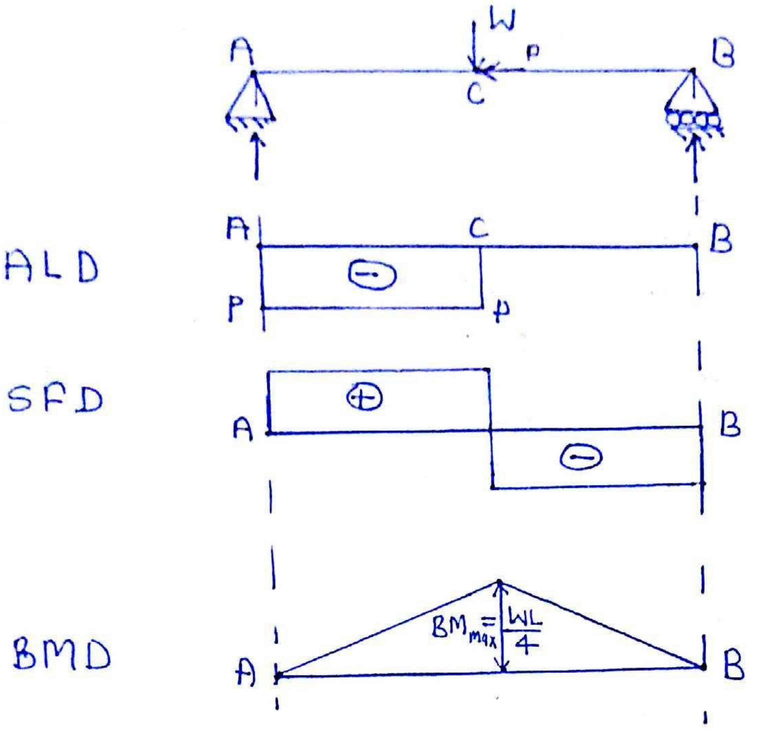
$$\Sigma M_X = 0 \Rightarrow \frac{M}{2}(x) + W\left(\frac{L}{2} - x\right) - \frac{W}{2}\left[\frac{L}{2} + \frac{L}{2} - x\right]$$

BC:- $x = [0 \text{ to } L/2]$

$$A_{x-x} = 0$$

$SF_{x-x} = \frac{-W}{2} = \text{Const}$ $\quad$ $BM = \frac{W}{2}x$ (variable)

ALD, SFD & BMD



$x\ [0\ to\ L/2]$

$\cancel{(BM)_{x-x} = \frac{W}{2}x - W(x-L)}$

$\cancel{(BM)_{x-x} = \frac{WL}{4} - \frac{Wx}{2}}$

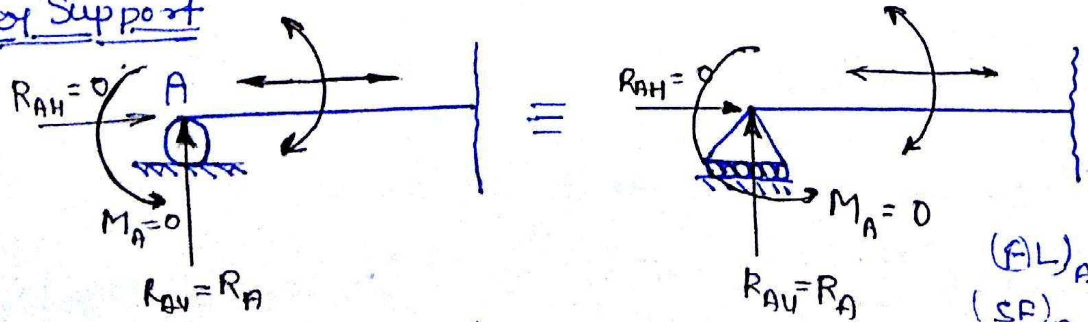
Types of (RIGID) support :-

1. Simple support
   - 1. Roller support
   - 2. Hinge support (Pin support)

2. Fixed support
   (OR) Built-in support
   (OR) clamped support

$$\left(\text{No. of Reactions At any support}\right) = \left(\text{No. of Restricted Motions by that Support}\right)$$

Roller Support



⇒ Fig:- Roller Support.

$(AL)_A = 0$

$(SF)_A = R_{AV}$

$(BM) = 0$

Hinge or Pin support

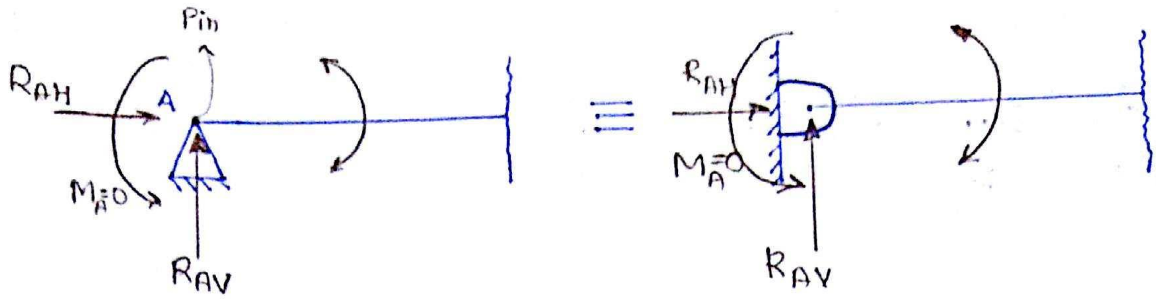
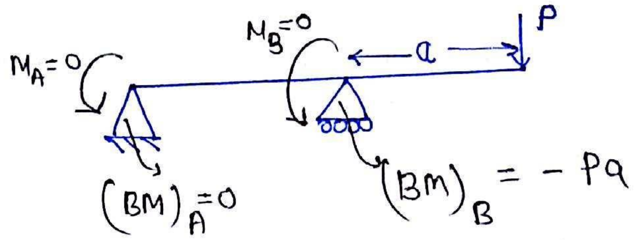


Fig:- Hinge or Pin support

Here $(AL)_A = -R_{AH}$ $\qquad (BM)_A = 0$

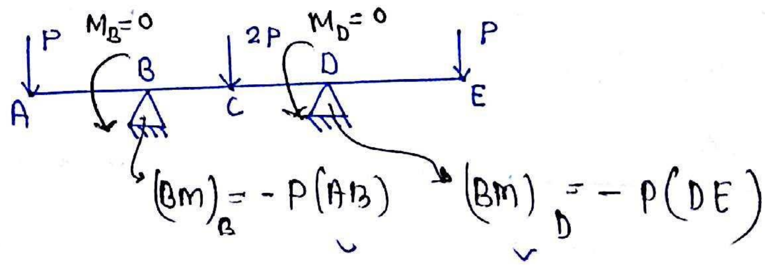
$(SF)_A = R_{AV}$

~~Hing~~ ⇒



V.V. Imp

* At simple support Always moment reaction zero and bending moment depand on where load provided.



Fixed Support

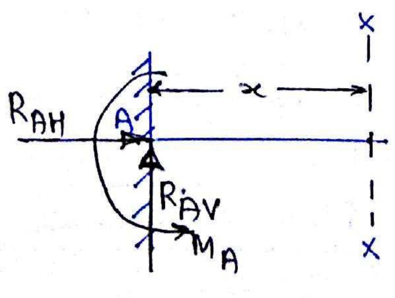


Fig - Fixed Support.

$(AL)_A = -R_{AH}$

$(SF)_A = R_{AV}$

$(BM)_A = -M_A$

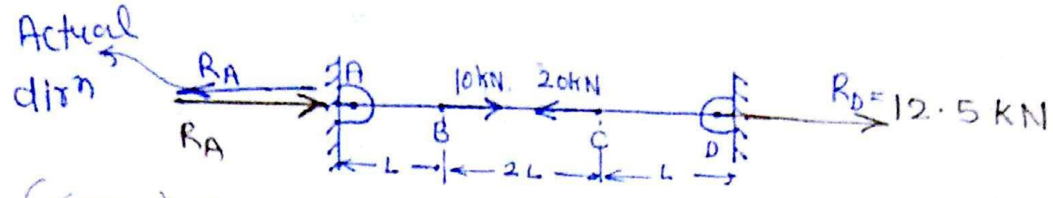
at X-S/C X-X

$(AL)_{x-x} = -R_{AH}$

$(SF)_{x-x} = R_{AV}$

$(BM)_{x-x} = -M_A + R_{AV}x$

Question Find max. tensile & Max. Comp. Axial load.



($\leftarrow$) ve

$$R_A = \frac{(-10)(3L) + (20 \times L)}{4L} = -2.5\,kN$$

$(AL)_{AB} = 2.5$ KN (T)

$(AL)_{BC} = 7.5$ KN (C) max Compressive

$(AL)_{CD} = 12.5$ KN (T) max tensile

Types of Beams

Beams

- Statically determinate Beams
  - Simple supported Beam (SSB)
  - Overhanging beam
  - Cantilever Beam
- Statically inderterminate Beams
  - Fixed beam
  - Propped Cantilever Beam
  - Continuous Beam

[Total No. of recns in a beam] = [No. of useful st. equm equns]

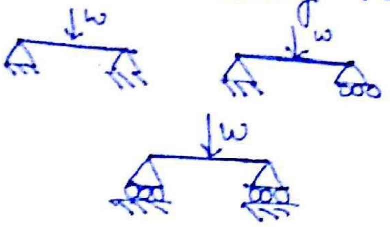
[Total no. of recns in a beam] > [No. of useful St. equm equns]

* SSP & overhanging can also be statically indeterminate beam when beam are supported by 2 hinge support in presence of axial load or temp. Variation

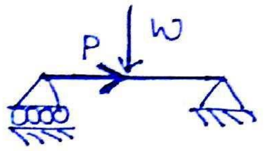
* Cantilever beams are always statically determinate beams.

Simply Supported beam (SSB)

* When only verticle load
  - two hinge ✓
  - onehinge, one roller ✓
  - two roller ✓

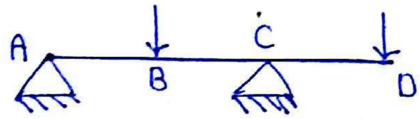


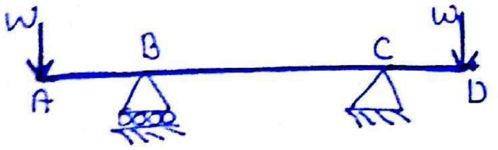
* When Verticle load as well as axial load also there
  - one hinge, one roller ✓ (other two become statically indeterminate beam)



✓ (Best Combination)

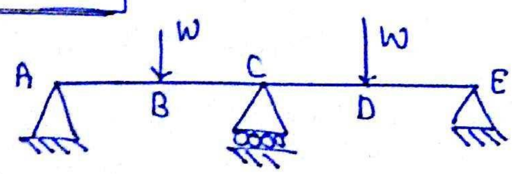
Overhanging Beam :-





∵ No axial load
⇒ statically determinate beam

Continuous beam



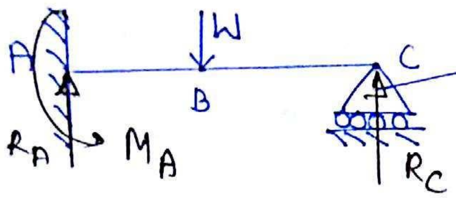
No. of rxn = 3
No. of useful st. equm eqn = 2.

⇒ Hence statically Indeterminate beam.

Cantilever Beam:-

Always Statically Determinate

Propped Cantilever Beam



Prop - it is a simple support which is provided near the free end.

No. of Rxn = 3

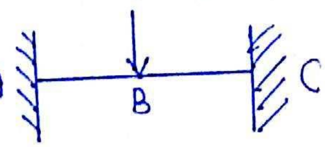
No. of useful eqⁿ = 2

⇒ Hence statically ~~interm~~ indeterminate beam.

* Compatibility equation obtain from deflection of beam.

$$\delta_c = \underbrace{(\delta_c)_D}_{\text{Downward}} + \underbrace{(\delta_c)_U}_{\text{upward}} = 0$$

Fixed Beam:-

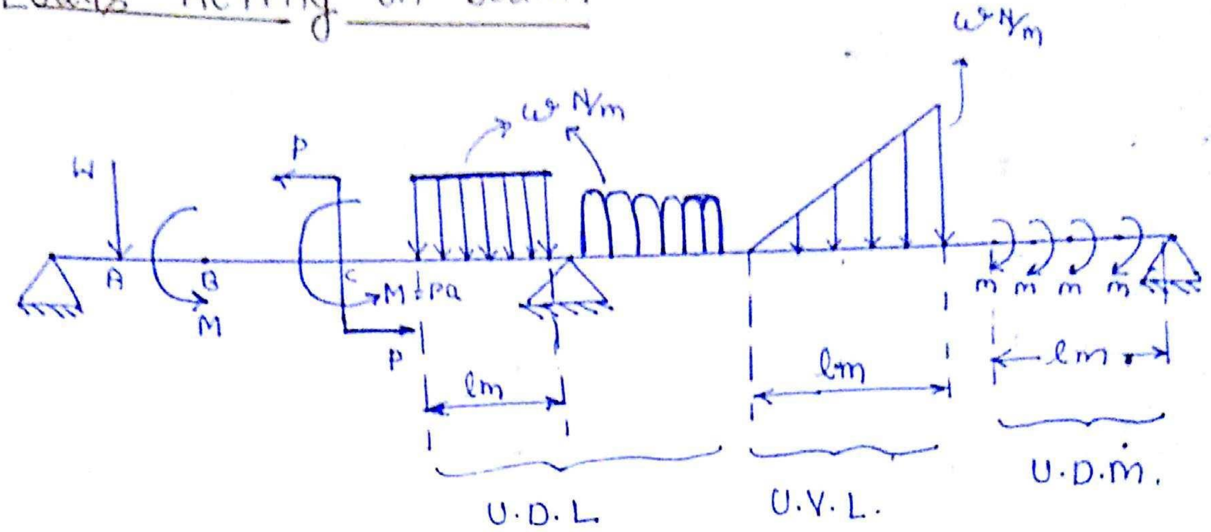


No. of rxn = 4

No. of useful st. equⁿ eqn = 2

Hence Interminate beam.

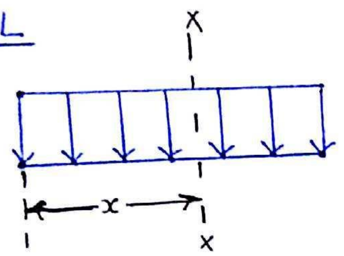
Loads Acting on Beam :-



w = load intensity (OR) load rating of a distributed load (N/m)

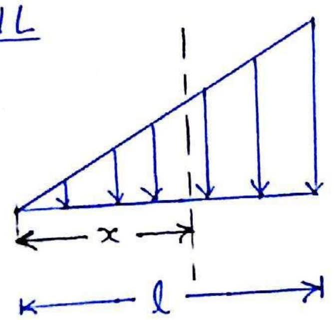
m = load intensity of a distributed moment

UDL



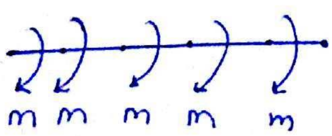
$$w_{x-x} = w$$

UVL

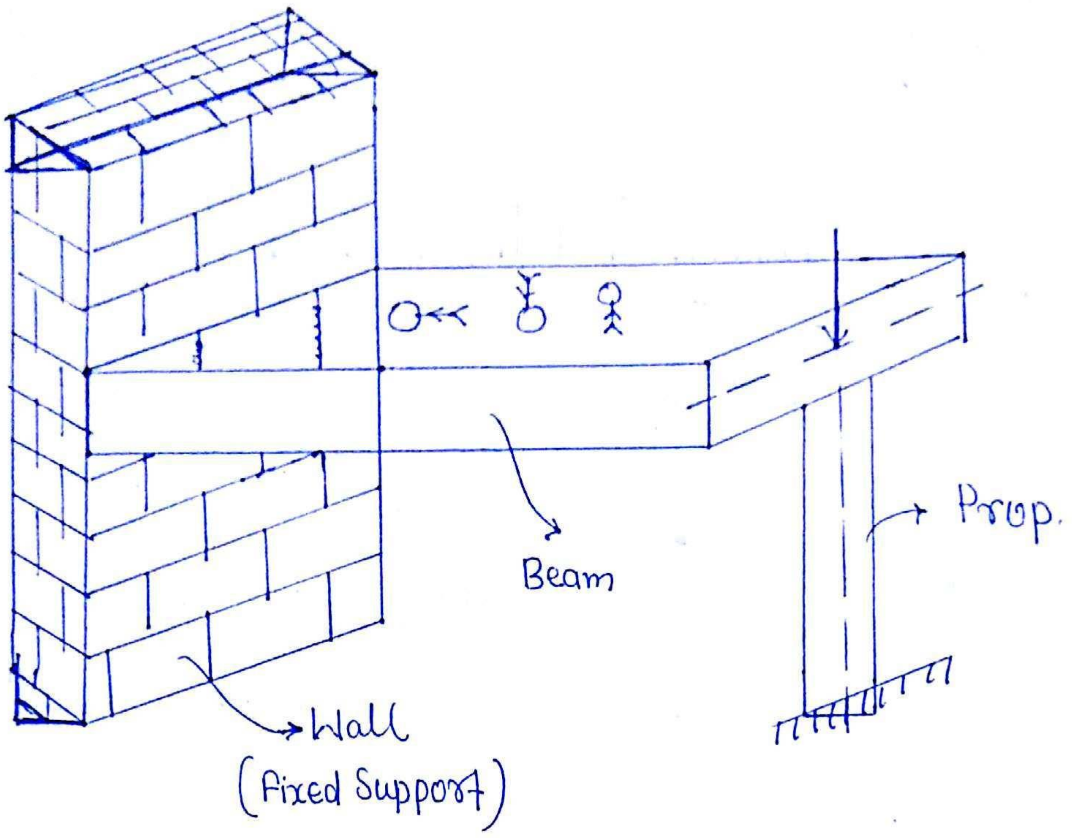


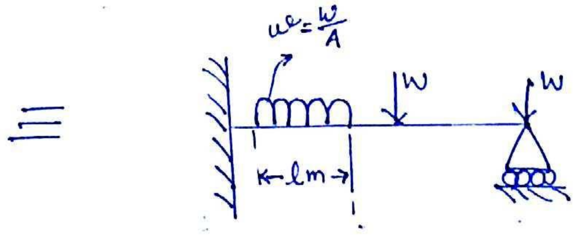
$$w_{x-x} = \frac{wx}{L} \text{ N/m} \left\{ \begin{array}{l} \text{varying from} \\ 0 \text{ to } w \end{array} \right.$$

UDM



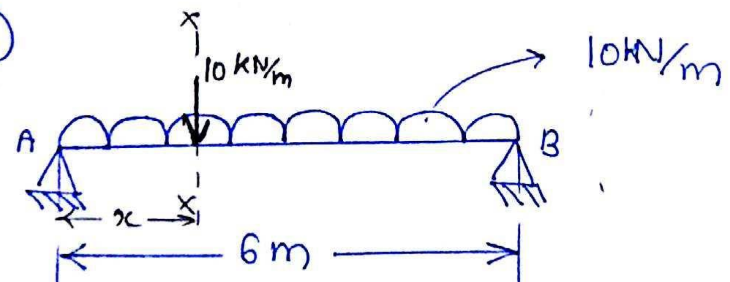
$$M_{x-x} = m \text{ N-m/m}$$


Beam
Prop.
Wall
(Fixed Support)

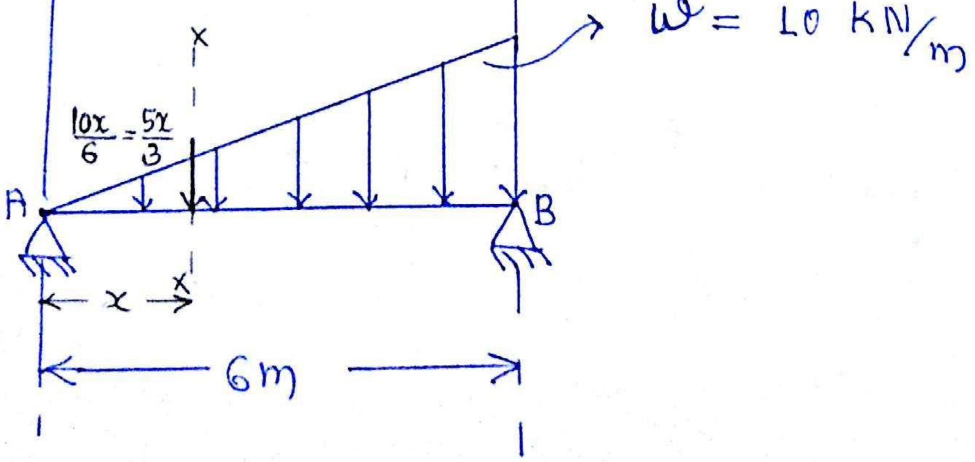

$\equiv$
$w = \frac{W}{A}$
W
W
$\leftarrow l\,m \rightarrow$

*

1


X
10 kN/m
10 kN/m
A
B
X
$\leftarrow x \rightarrow$
6 m

2


X
$w = 10$ kN/m
$\frac{10x}{6} = \frac{5x}{3}$
A
B
X
$\leftarrow x \rightarrow$
6 m

In presence of Distributed load. (D.L.)

① Total load of D.L. $= \int_a^b (w_{x-x})\, dx$

= Area of plane Fig of Corresponding distributed load.

② Total load of D.L. should be reprented through centroid of corresponding plane of Fig.

① U.D.L.

total load of U.D.L. $= \int_0^6 10\, dx = [10x]_0^6 = 60$ kN

total load of D.L. = Area of rectangle of U.D.L.

$= 6 \times 10 = 60$ kN

② U.V.L.

total load of UVL $= \int_0^6 \left(\frac{5x}{3}\right) dx = \frac{5}{3}\left[\frac{x^2}{2}\right]_0^6 = 30$ kN

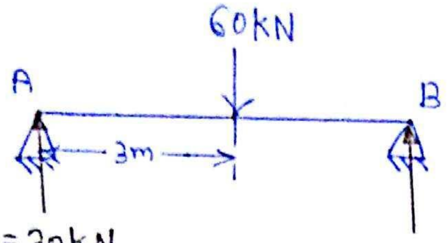
= Area of rectangle of U.V.L.

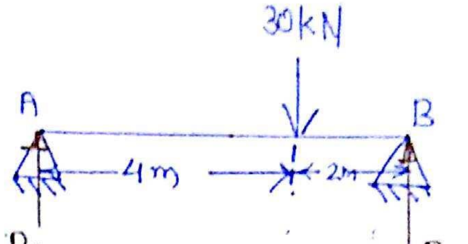
$= \frac{1}{2} \times 10 \times 6 = 30$ kN

⇒ e.g. if $W_{x-x} = \sin \frac{3\pi x}{L}$ to Calculate total load of UVL go through integration only

$= \int \sin \frac{3\pi x}{L}\, dx$

So ~~~~ Now Apply total load on Centroid





$R_A = 30kN$ $\quad$ $R_B = 30kN$

$R_A = \frac{60}{2} = 30kN$

R_A $\quad$ R_B

$R_A = \frac{30 \times 2}{6}$ $\quad$ $R_B = 20kN$

$R_A + R_B = 60$

$R_A = 10kN$

$R_A = \frac{60}{2} = 30kN$

$R_B = 30kN$

$\Sigma Y = 0 \rightarrow R_A - 30 + R_B = 0$

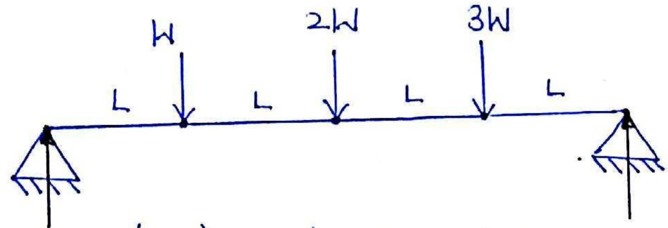
$\Rightarrow R_A + R_B = 30$

$\Sigma M_A = 0 \Rightarrow 30 \times 4 - R_B \times 6 = 0$

$R_B = 20kN (\uparrow)$

So $R_A = 10kN (\uparrow)$

eg

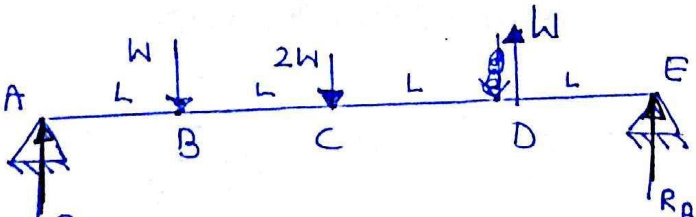


* Can Use Shortcut Method

$$R_A = \frac{W(3L) + 2W(2L) + 3W(L)}{4L} \qquad R_B = 3.5W$$

$R_A = 2.5W$

eg



* Can't Use Shortcut Method.

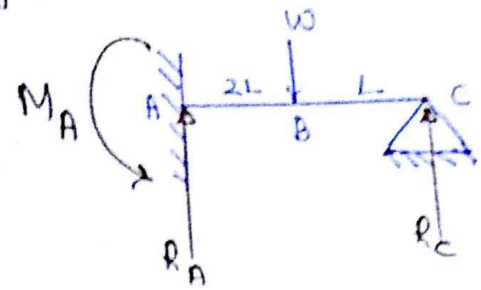
$\Sigma Y = 0 \Rightarrow -R_A + W + 2W - W - R_E = 0$

$R_A + R_E = 2W \quad -(1)$

$\Sigma M_E = 0 \Rightarrow W(L) - 2W(2L) - \cancel{W}(3L) + R_A(4L) = 0$

$R_A = 1.5W, \quad R_B = 0.5W$

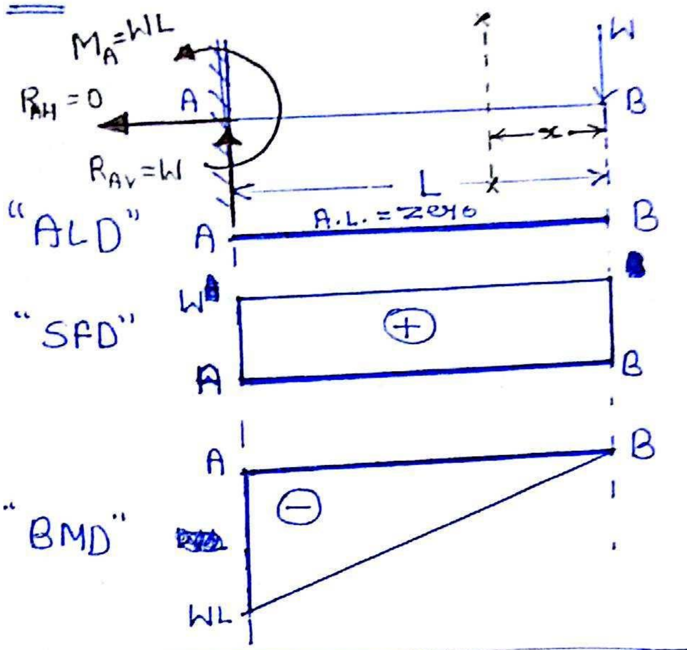
eq



Stattically Indeterminate

S.F.D. & B.M.D. (shear Force & Bending Moment diagram)

Ex.



BA :- $[x = 0 \text{ to } L]$

$(SF)_{x-x} = +W \text{ (Const.)}$

$(BM)_{x-x} = -Wx \text{ (Variable)}$

$x = 0 \longrightarrow (SF)_B = W$

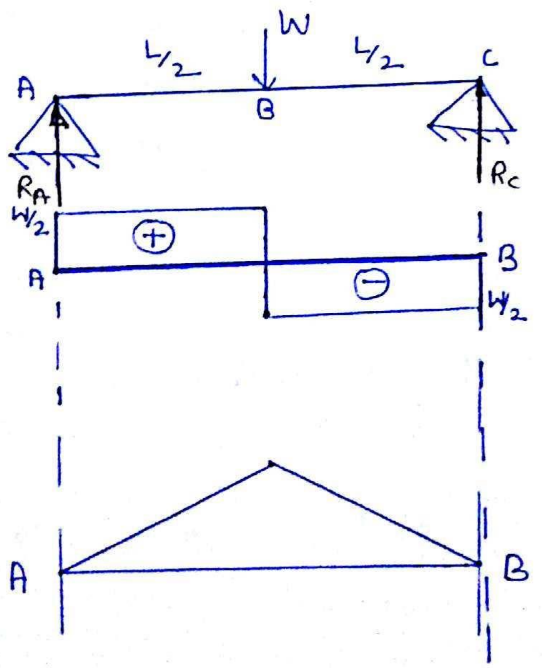
$(BM)_B = 0$

$x = L \longrightarrow (SF)_A = W$

$(BM)_A = -WL$

LHS	ACW	−Ve	RHS	ACW	+Ve
	CW	+W		CW	−Ve

Ex.



Max. Bending Moment $= (BM)_B$

$(BM)_B = \frac{W}{2}\left(\frac{L}{2}\right) = \frac{WL}{4}$

Max. BM = when S.F. Change Sign

$= \text{Max. of } \{(BM)_A, (BM)_B, (BM)_C\}$

Conclusion about) SFD & BMD

1. SFD is one degree higher than ALD and BMD is one degree higher than SFD.

2. If at any point a Concentrated point load or reaction act then ordinate of SFD will change by the magnitude of that load.

3. If at a point Concentrated moment (Couple) acts than ordinate of BMD will change by magnitude of that Couple

4. If shear force changes sign at a section then BM at that section is either maximum or minimum but inverse is not true

5. When shear force change it's ~~sign~~ magnitude, slope of BMD should change.

6. shear force at any X-S/c $(SF)_{x-x}$ = Slope of BMD at that X-S/c

$$(SF)_{x-x} = \left(\frac{dM}{dx}\right)_{x-x}$$

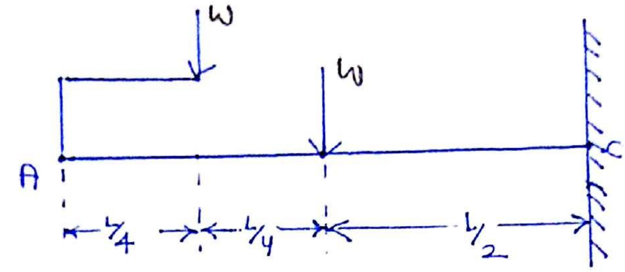
is not Valid in presence of Concentrated moment & U.D.M.

7. Point of Contraflexure is located by

(i) $(BM)_{x-x} =$ ______ $= 0$ (is must in Case of UDL & ULV)

(ii) Using similar triangle (is valid when B.M. Varies linearly)

Ques For the cantilever beam show in fig Draw. SFD & BMD and determine max. sagging bending moment & max. Hogging bending momem# location of Contraflexure from fixed end.



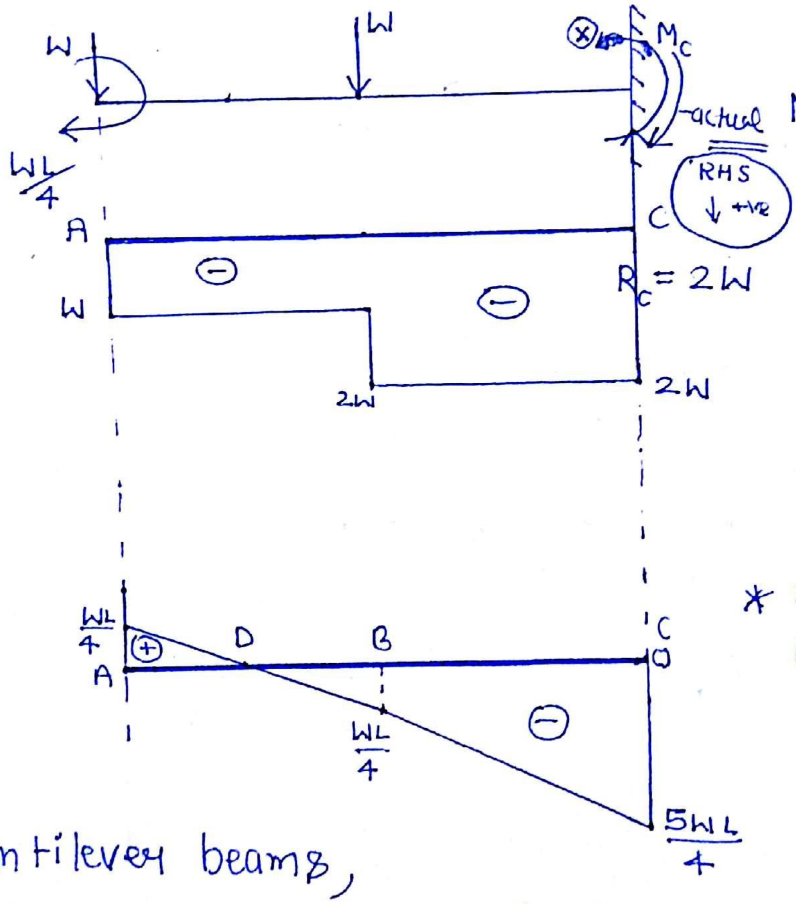
Solⁿ

LHS

($\uparrow$ = +ve)

SFD

BMD



$R_C = W + W = 2W$

$M_C + W\times\frac{L}{2} + W\times L - \frac{WL}{4} = 0$

$M_C = \frac{WL}{4} - \frac{3WL}{2}$

$M_C = -\frac{5WL}{4}$

or $\frac{5W}{4}$ (cw)

* Shear force change it mag. slope of BMD should change

In cantilever beams,

Vertical recⁿ at fixed end (R_V) = - Net Vertical load

Horizontal — " — (R_H) = - Net Horizontal load

Moment recⁿ — " — (M) = - Net Moment at Fixed end

Contraflexure:- where bending Moment change it sign.

shear force cal^n ($\uparrow$ = +ve , $\downarrow$ = -ve)

$(SF)_A = 0$

$(SF)_A = 0 - W = -W$

Cal^n start from zero and end with zero.

$(SF)_B = -W$

$(SF)_B = -W - W = -2W$

$(SF_a)_C = -2W$

$(SF)_C = -2W + 2W = 0$

Bending Moment cal^{ns} (($\curvearrowright$ = +ve, $\curvearrowleft$ = -ve)

$(BM)_A = 0$

$(BM)_A = 0 + \frac{WL}{4} = \frac{WL}{4}$

$(BM)_B = \frac{WL}{4} - \frac{WL}{2} = -\frac{WL}{4}$

First neglected concentrated load $(BM)_C = \frac{WL}{4} - WL - \frac{WL}{2} = -\frac{5WL}{4}$

Now Consider Concentrated load $(BM)_C = -\frac{5WL}{4} + \frac{5WL}{4} = 0$

⇒ Shear force from BMD

$$(SF)_{AB} = \left(\frac{dM}{dx}\right)_{AB} = \frac{M_B - M_A}{X_B - X_A} = \frac{\left(-WL/4\right) - \left(WL/4\right)}{L/2 - 0}$$

$$(SF)_{AB} = -W$$

$$(SF)_{BC} = \left(\frac{dM}{dx}\right)_{BC} = \frac{M_C - M_B}{X_C - X_B} = \frac{-5WL/4 - \left(-WL/4\right)}{L - L/2} = -2W$$

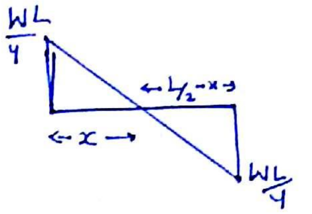
⇒ Shear at any x-s/c of beam (SF_{x-x}) = Slope of BMD at that x-s/c

i.e $\boxed{(SF)_{x-x} = \left(\frac{dM}{dx}\right)_{x-x}}$ — is not valid in presence of concentrated Moment & U.D.M.

location of Contraflexure from Fixed = $\frac{3L}{4}$
where bending moment change it sign.

$$\Rightarrow (BM)_{x-x} = \frac{WL}{4} - Wx = 0$$

$$x = L/4 \text{ (from free end)}$$

⇒ 

$$\frac{\frac{WL}{4}}{x} = \frac{WL/4}{L/2 - x}$$

$$x = L/4$$

AB $[x = 0 \text{ to } L/2]$

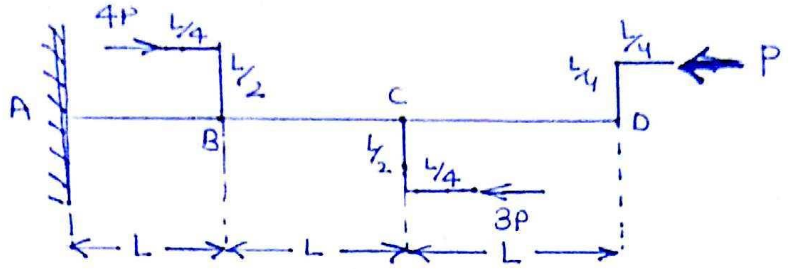
$$(SF)_{x-x} = -W = \frac{M_x - \frac{WL}{4}}{x - 0} \Rightarrow M_x = \frac{WL}{4} - Wx$$

CB $[x = 0 \text{ to } L/2]$

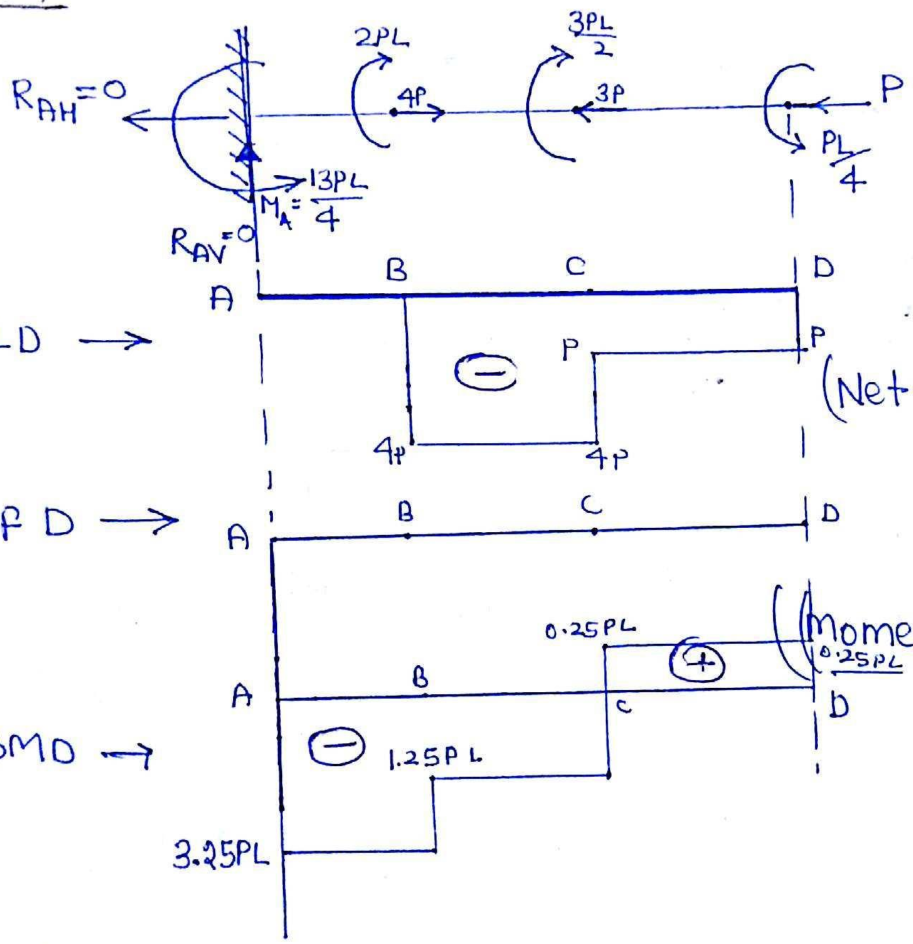
$$(SF)_{x-x} = +2W = \frac{M_x - \left(-\frac{5WL}{4}\right)}{x - 0} \Rightarrow M_x = -\frac{5WL}{4} + 2Wx$$

RHS ↓ +ve

Quest For the cantilever beam as showing in fig Draw SFD & BMD and determine max. sagging B.M. & Max. Hogging B.M. and no of point of Contra flexure.



Solution



$R_{AH} = P + 3P - 4P$

$R_{AH} = 0$

$R_{AV} = 0$

$(\text{Net Moment})_A = 2PL + \frac{3PL}{4} - \frac{PL}{4}$

$= \frac{13PL}{4}$ (↻)

(Moment reacn) $= \frac{13PL}{4}$ (↺)

$= 3.25PL$

$(B.M.)_A = M_A$

$-M_A + 2P + \frac{3PL}{2} - \frac{PL}{4} = 0$

$M_A = \frac{13PL}{4}$ (↺)

max. tensile load = zero

max. Comp = 4P

max. S.F. = 0

max sagging BM = $\frac{PL}{4}$

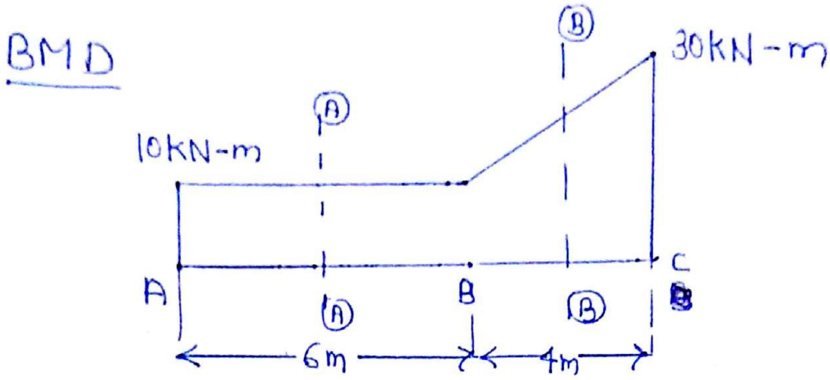
max. Hogging BM = $\frac{13}{4}PL$

AB → Pure bending (circular bending)

SF = AL = TM = 0

No. of P.O.C.F. = 1 [i.e. C]

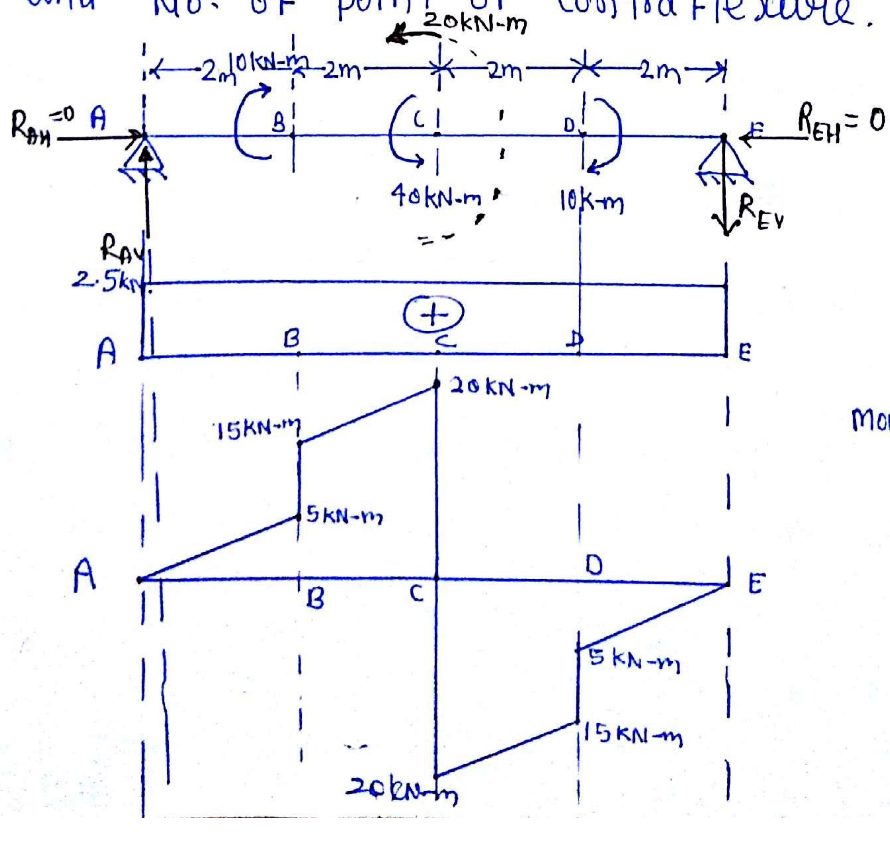
Ques BMD for a particular beam shown below determine shear force at x-S/c A-A & B-B



$$(SF)_{a-a} = \left(\frac{dM}{dx}\right)_{AB} = \frac{M_B - M_A}{X_B - X_A} = \frac{10-10}{6-0} = \text{zero}$$

$$(SF)_{b} = \left(\frac{dM}{dx}\right)_{BC} = \frac{M_C - M_B}{X_C - X_B} = \frac{30-10}{10-6} = 5$$

Ques For the S.S.B. draw SFD and BMD and determine max. shear force, max. hogging B.M., max. sagging B.M. and No. of point of contraflexure.



Sol^n

$R_{AV} = \frac{M_{net}}{L} = \frac{20}{8} = 2.5\text{ kN}(\uparrow)$

$R_{EV} = -2.5\text{ kN}$

or $2.5\text{ kN}(\downarrow)$

Moment about A = 0

$10 - 40 + 10 - R_{EV} \times 8 = 0$

$R_{EV} = -2.5\text{kN}$

$R_{AV} = 2.5\text{ kN}$

B.M. cal^n ($\curvearrowright$ = +ve, $\curvearrowleft$ = -ve)

$(BA)_A = 0$

$(BM)_B = 2.5(2) = 5$ KN-m

$(BM)_B = 5 + 10 = 15$ KN-m

$(BM)_C = 2.5 \times 4 + 10 = 20$ KN-m

$(BM)_C = 20 - 40 = -20$ KN-m

$(BM)_D = 2.5 \times (6) + 10 - 40 = -15$ KN-m

$(BM)_D = -15 + 10 = -5$ KN-m

$(BM)_E = 0$

Max. S.F. = 0

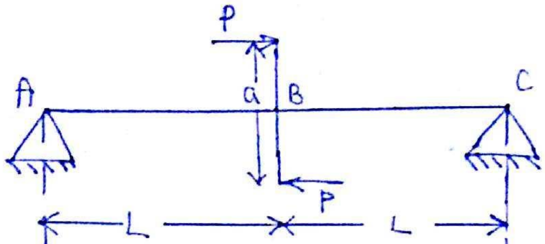
Max. Sagging B.M. = max. Hogging BM = 20 kN

No. of P.O.C.F. = 1 (At C)

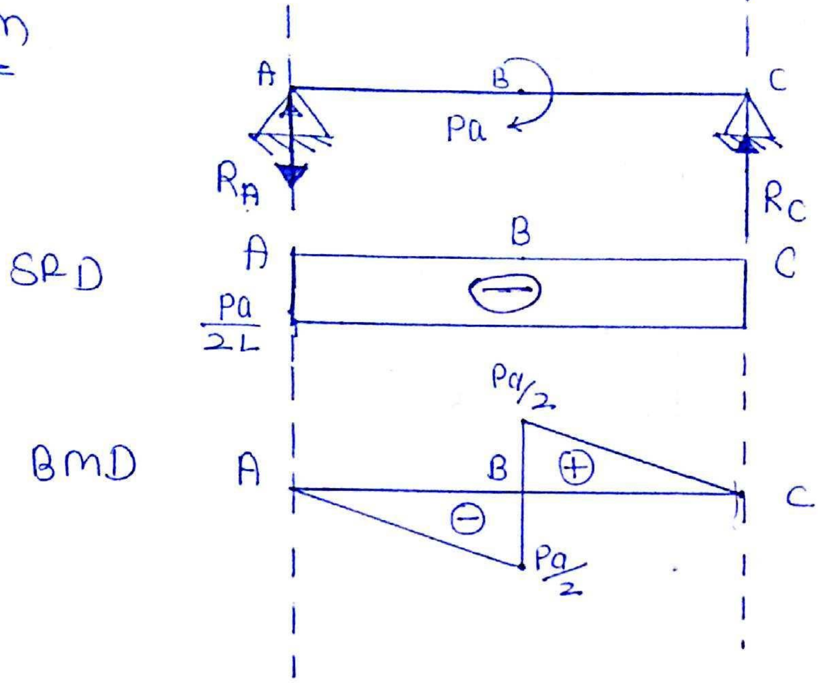
When S.S.B. is loaded with only concentrated moment

* vertical $reac^{ns}$ are equal & opposite.
* mag. of verticle rxn^s = $\dfrac{\text{Net moment on the beam}}{\text{length of the beam}}$
* S.F.D. is a rectange with a height is equal to vertical $reac^{n}s$

Ques



Sol^n



$$R_A = -\frac{Pa}{2L}$$

$$R_C = \frac{PL}{2L}$$

$$\left.\begin{aligned}(BM)_A &= 0\\ (BM)_C &= 0\end{aligned}\right\} \text{S.S.B.}$$

$$(BM)_B = -\frac{Pa}{2} + Pa = Pa/2$$

$$(BM)_B = Pa/2$$

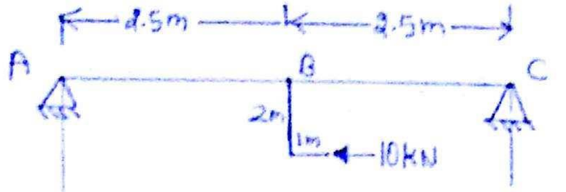
Question

Determine

(a) max. tensile & max. Comp.

(b) max. shear force

(c) max. Hogging & Sagging B.M.



Soln

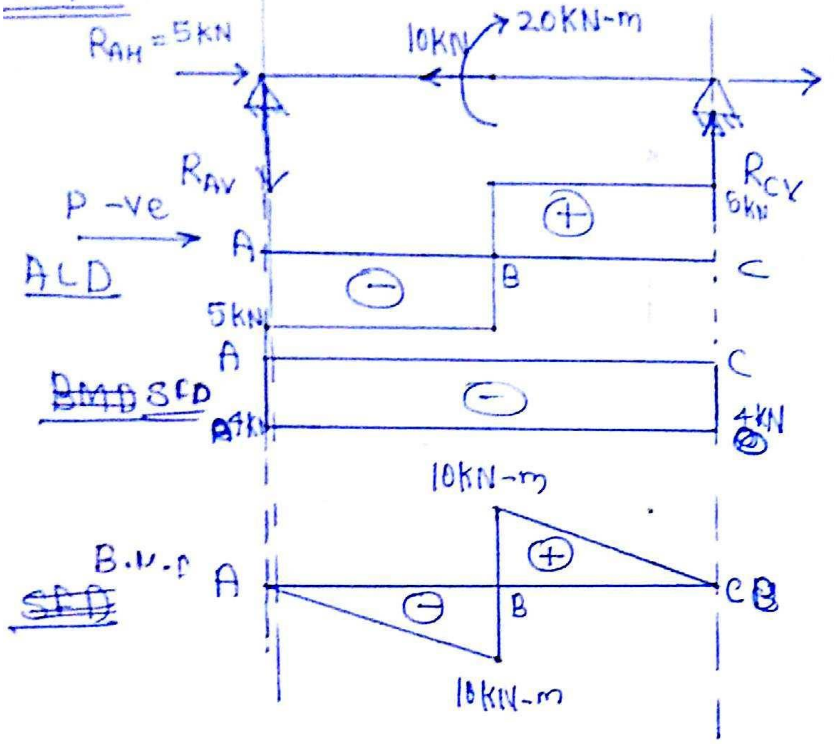
$R_{AH} + R_{CH} = 10KN$

$$R_{AH} = \frac{10 \times 2.5}{5}$$

$R_{AH} = 5KN$

$$R_{AV} = \frac{20}{5} = 4KN (\uparrow)$$

$R_{AV} = -4KN (\downarrow)$



$(AL)_{AB} = -5KN$ or $5KN$ (Comp)

$(AL)_{BC} = 5KN$ (tensile)

$(SF) = -4KN = $ Const.

$(BM)_A = (BM)_C = 0$

$(BM)_B = -4 \times 2.5 = -10$ KN-m

$(BM)_B = -10 + 20 = 10$ KN-m

max. tensile load = 5KN

max. Comp. load = 5KN

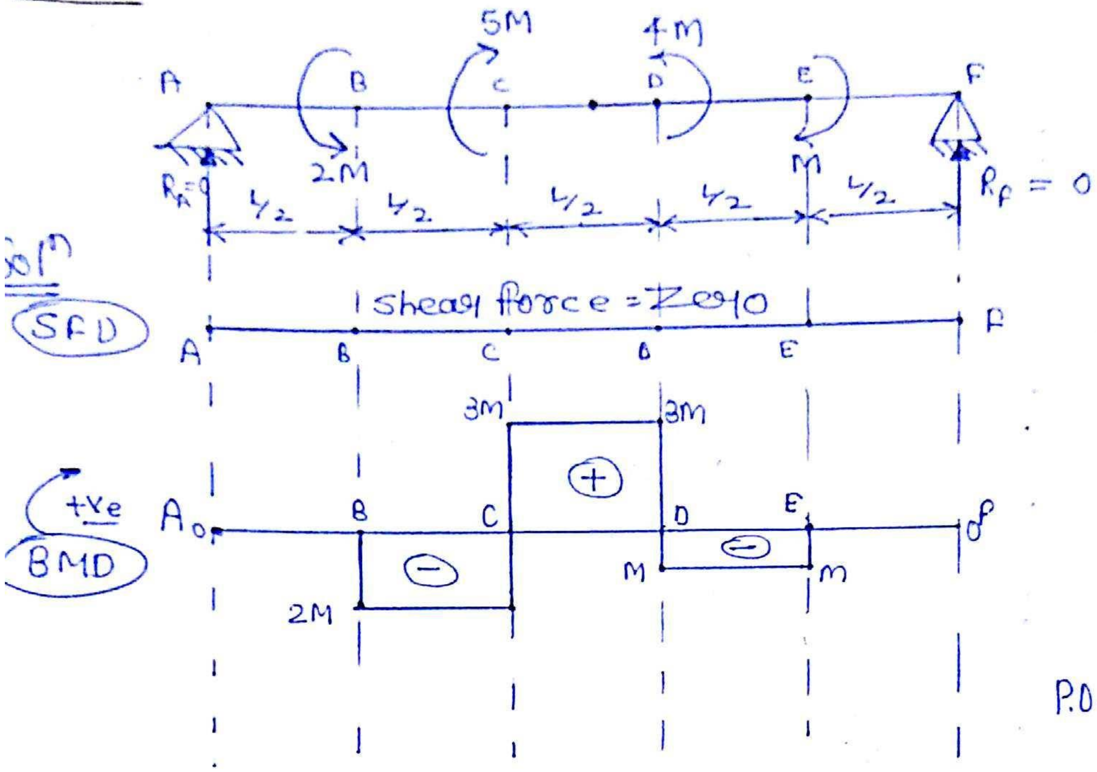
S.F. = 4KN

max. Sagging BM = 10 KN-m

" Hogging BM = 10 KN-m

Quest

Sol^n



Beam is Already in equilibrium so there is No verticle fxn

P.O.C.F. = C & D

BC & CD - Pure bending - Circular arcs (BM Const.)

AB & EF - State line

Possible ~~bendings~~

(i) Circular arc ⇒ S.F. = 0 & BM = Const.

(ii) St. line ⇒ S.F. = 0 & BM = 0

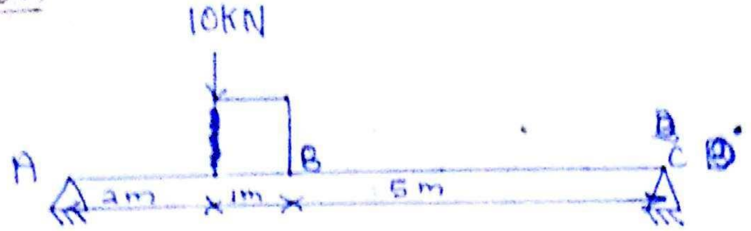
(iii) Parabolic ⇒ SF ≠ 0, BM ≠ 0

$$\frac{M}{I_{N.A.}} = \frac{E}{R}$$

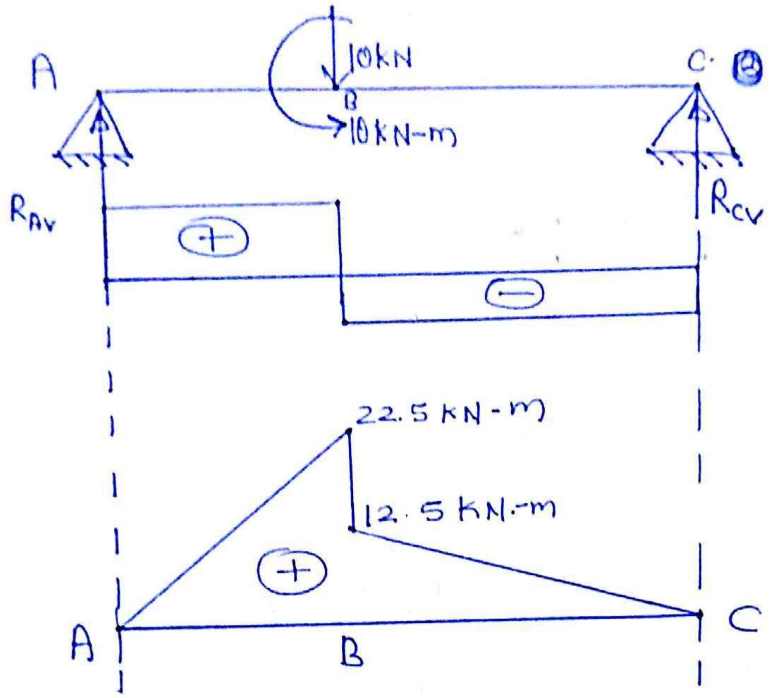
$$R = \frac{E I_{N.A.}}{M}$$ Radius of curvature

Question

Q. (Day 1)



Soln



$R_{AV} + R_{CV} = 10\ KN$

Net moment about A = 0

$10 + R_{CV} \times 8 - 10 \times 3 = 0$

$R_{CV} = \frac{20}{8} = 2.5\ KN$

$R_{AV} = 10 - 2.5 = 7.5\ KN$

Super possition Method

	R_A	R_C
due to 10kN	$\frac{10 \times 5}{8} = 6.25\ KN (\uparrow)$	$3.75\ KN (\uparrow)$
due to 10 KN	$\frac{10}{8} = 1.25\ KN (\uparrow)$	$1.25\ KN (\downarrow)$

$R_A = 7.5\ KN (\uparrow)$ $R_C = 2.5\ KN$

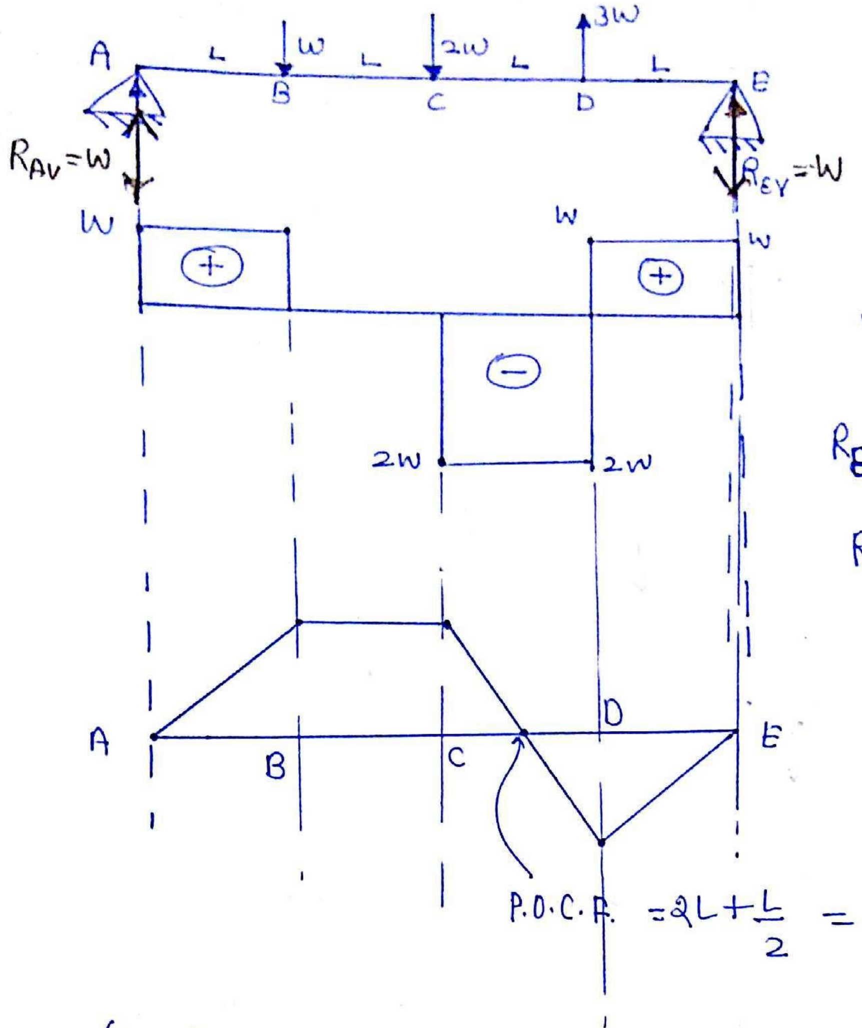
$(BM)_A = 0$ ($\uparrow$ +ve)

$(BM)_B = 7.5(3) = 22.5\ KN\text{-}m$

$(BM)_B = 22.5 - 10 = 12.5\ KN\text{-}m$

$(BM)_C = 0$

Question



$$(M)_A = 0$$

$$WL + 2W(2L) - 3W(3L) - R_{EV} \times 4L = 0$$

$$R_{EV} = \left(\frac{-9+4+1}{4}\right) W$$

$$R_{EV} = -\frac{4}{4} W = -1\, W$$

$$R_{EV} = -W$$

$$R_{AV} = W$$

P.O.C.F. $= 2L + \frac{L}{2} = 2.5\,L$ from (LHS)

$$(BM)_A = 0$$

$$(BM)_B = WL$$

$$(BM)_C = W(2L) - WL = WL$$

$$(BM)_D = W(3L) - W(2L) - 2WL = -WL$$

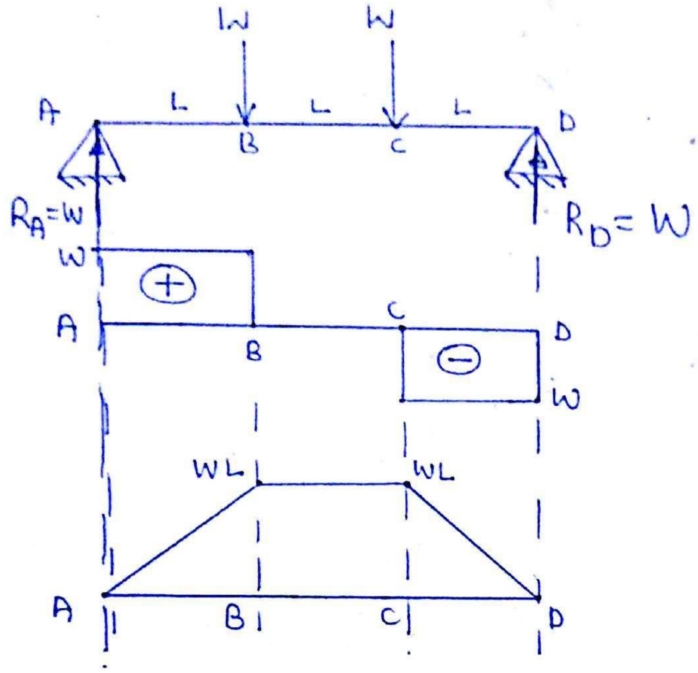
$$(BM)_E = 0$$

BC ⇒ S.F. = 0 , BM = Const. Circular arc.

Radius of curvature, R_{AB}, R_{CD} & R_{DE} varies Continuously

$$R_{BC} = \frac{E\, I_{N.A.}}{WL} = \text{Constant}$$

Question Draw the SFD & BMD and determine strain Energy of beam, max. S.F., Max. B.M.



$(BM)_A = 0$

$(BM)_B = WL$

$(BM)_C = W(2L) - W(L) = WL$

$(BM)_D = 0$

⇒ shear force zero so Bending Moment Constant.

Radius of Curvature

AB - Varies (Parabolic shape)

BC - Const. (circular arc)

CD - Varies (Parabolic shape)

$$U = U_{AB} + U_{BC} + U_{CD}$$

$$U = 2U_{AB} + U_{BC} \quad \because [U_{AB} = U_{CD}]$$

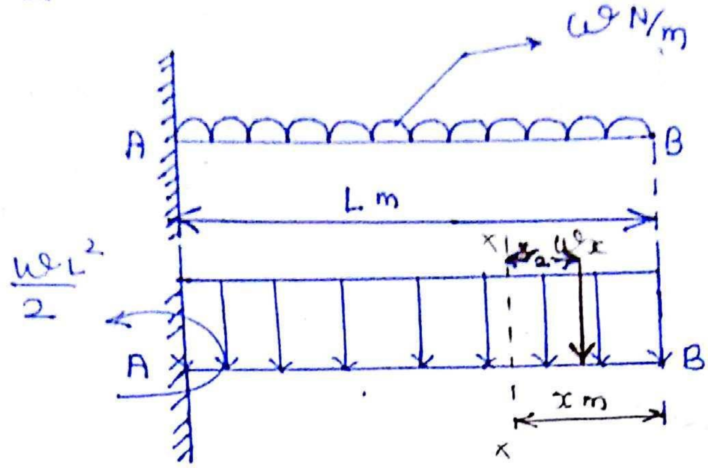
$$= 2\int_0^L \left(\frac{(Wx)^2 dx}{2EI}\right)_{AB} + \left(\frac{M^2 L}{2EI}\right)_{BC}$$

$$U = \frac{W^2 L^3}{3EI} + \frac{W^2 L^3}{2EI}$$

$$U = \frac{5}{6}\frac{W^2 L^3}{EI}$$

SFD & BMD Under Distribute load :-

UDL



BA :- $x = 0$ to L m

$w_{x-x} = -w$ N/m = Const. (Downward load intensity) —(1)

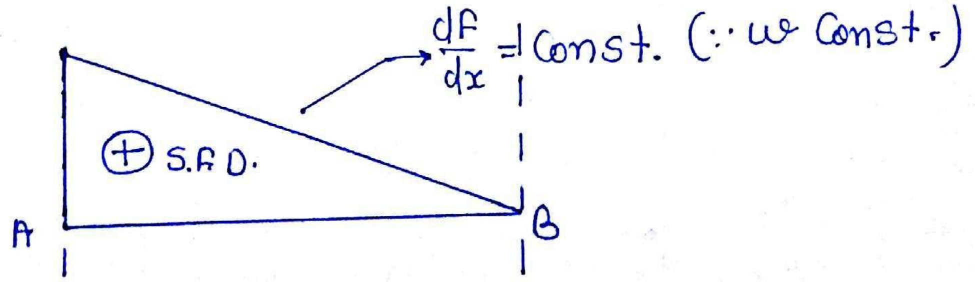
$(S.F.)_{x-x} = wx$ —(2)

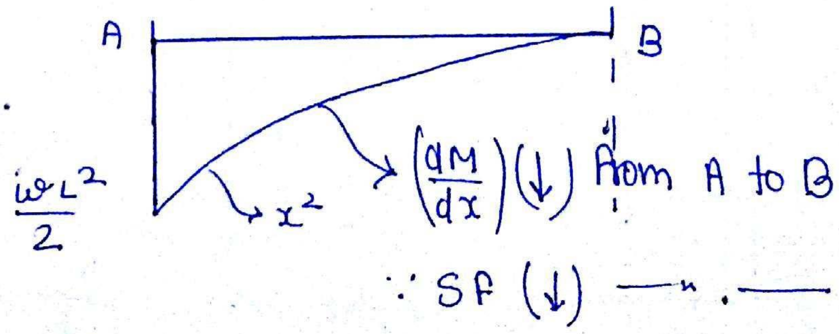
$(B.M.)_{x-x} = -wx\left(\frac{x}{2}\right) = -\frac{wx^2}{2}$ —(3)

$x = 0 \Rightarrow (SF)_B = 0$; $(BM)_B = 0$

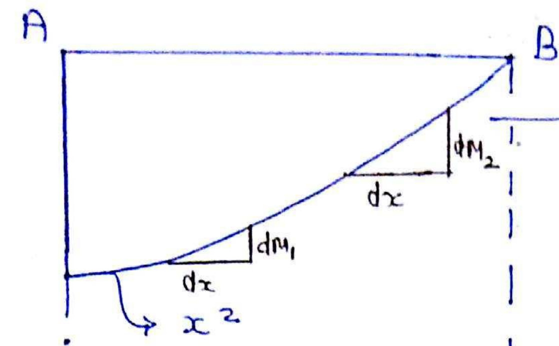
$x = L \Rightarrow (SF)_A = wL$; $(BM)_A = -\frac{wL^2}{2}$

SFD



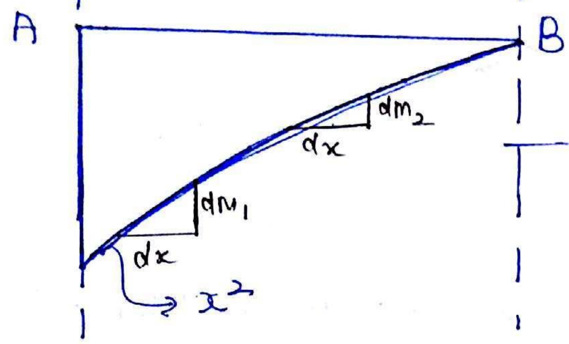


Curve of Parabola :-



Parabola with a increasing slope $\left[\text{i.e. } \frac{dM}{dx} \uparrow\right]$ from A to B

$[\because dM_2 > dM_1]$



Parabola with a decreasing slop $\left[\text{i.e } \frac{dM}{dx} \downarrow\right]$ from A to B

$[\because dM_2 < dM_1]$

* If shear force magnitude increase (↑) from L to R slope of BMD $\left[\text{i.e. } \frac{dM}{dx}(\uparrow)\right]$ increase from Left to Right
(table 2)

* If 'w' of a distrubuted load increase (↑) from Left to right slope of S.F.D. $\left[\text{i.e. } \frac{dF}{dx}(\uparrow)\right]$ Increase from left to right. and vice versa $\left[\because -w_{x-x} = \frac{dF_{x-x}}{dx}\right]$
table①

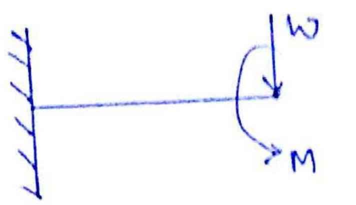
Table ①

w	$\frac{dF}{dx}$
Const.	Const.
↑	↑
↓	↓

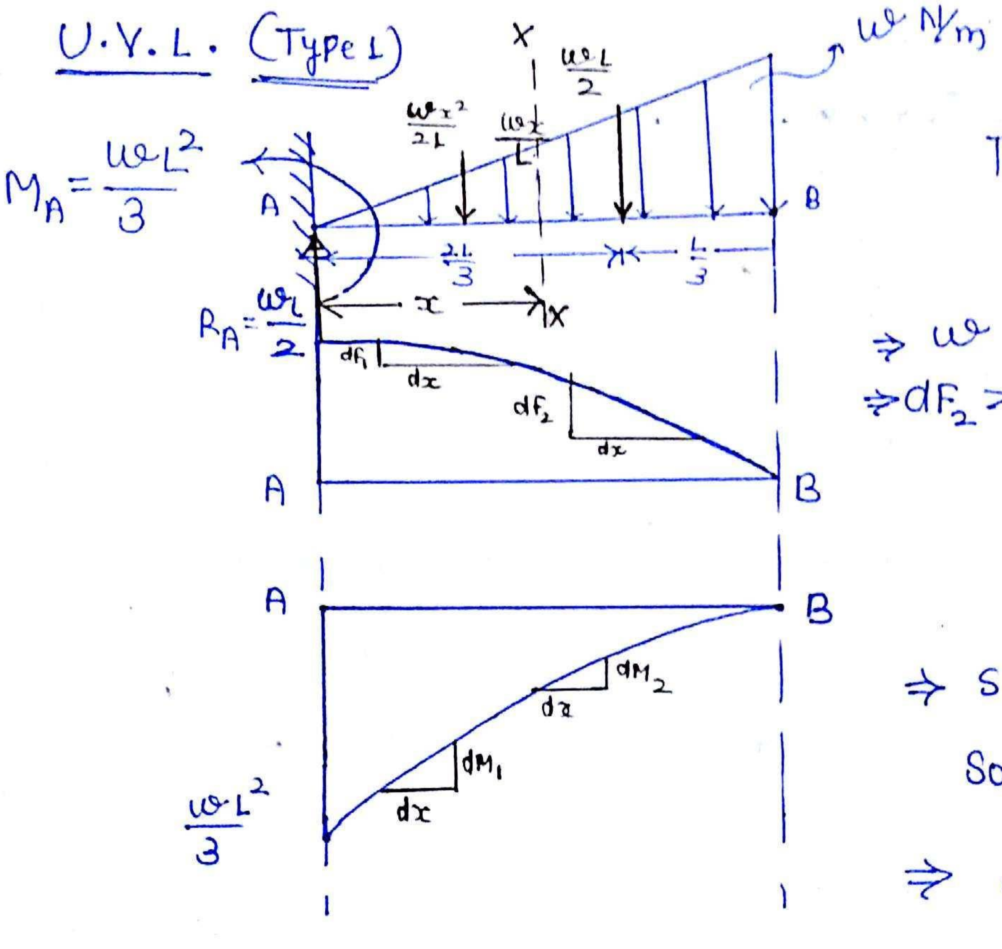
Table ②

S.F.	dM/dx
Const.	Const.
↑	↑
↓	↓

* If there is no external moment, BM of free end will be zero. if there is BM then non-zero



U.V.L. (Type 1)



Total load $= \frac{1}{2} \times L \times w$

$= \frac{wL}{2}$

$\Rightarrow$ w increasing L to R

$\Rightarrow dF_2 > dF_1$ (Increasing slope)

$\Rightarrow$ S.F. mag. $\downarrow$ from L to R

So $\left(\frac{dM}{dx}\right)$ also $\downarrow$

$\Rightarrow dM_2 < dM_1$

x [$\because$ 0 to L] from fixed end

first we have to calculate unknows $R_A = \frac{wL}{2}$

$M_A = \frac{wL^3}{3}$ (ACW)

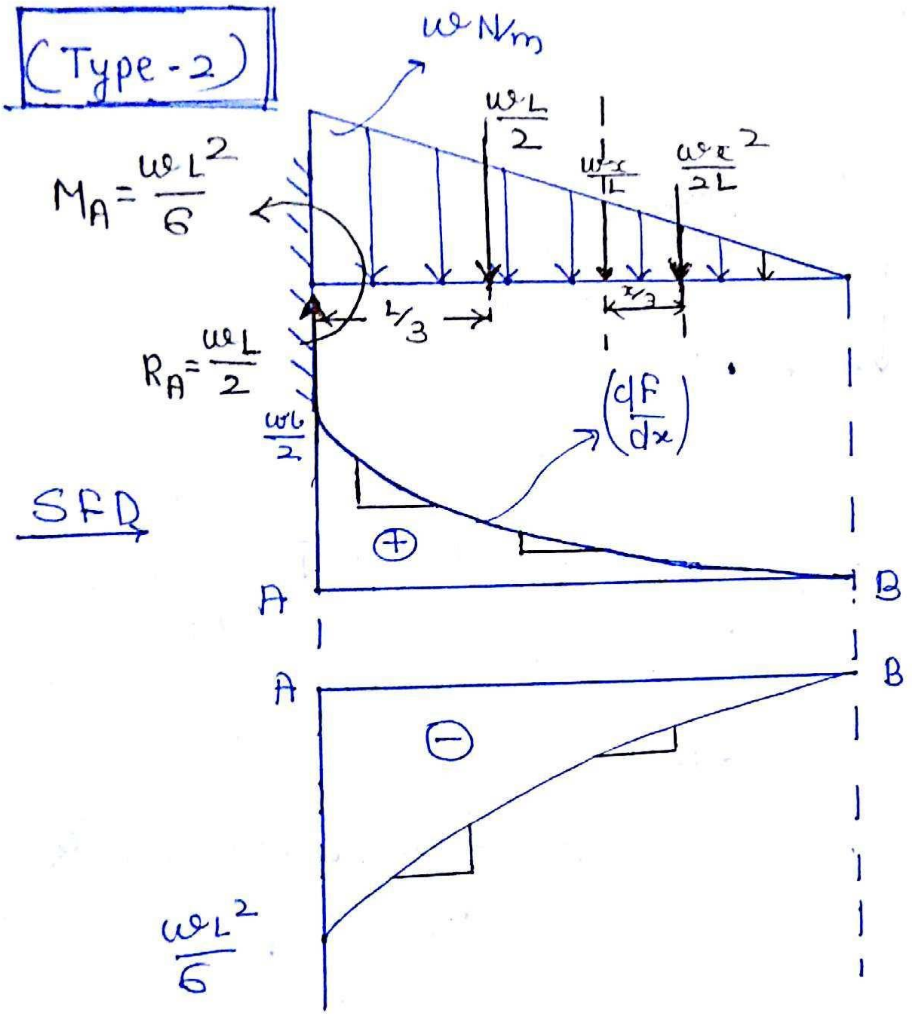
$$w_{x-x} = -\frac{wx}{L} \quad -①$$

$$(SF)_{x-x} = \frac{wL}{2} - \frac{wx^2}{2L} \quad -②$$

$$(BM)_{x-x} = -\frac{wL^2}{3} + \frac{wL}{2}(x) - \frac{wx^2}{2L}\left(\frac{x}{3}\right)$$

$$(BM)_{x-x} = -\frac{wL^2}{3} + \frac{wLx}{2} - \frac{wx^3}{6L} \quad -(3)$$

$\boxed{\text{(Type - 2)}}$



[w (↓) from L to B]
[So $\frac{dF}{dx}$ (↓) from L to B]

from A to B
$\frac{dM}{dx}$ (↓)
[∵ S.F. = (↓)]

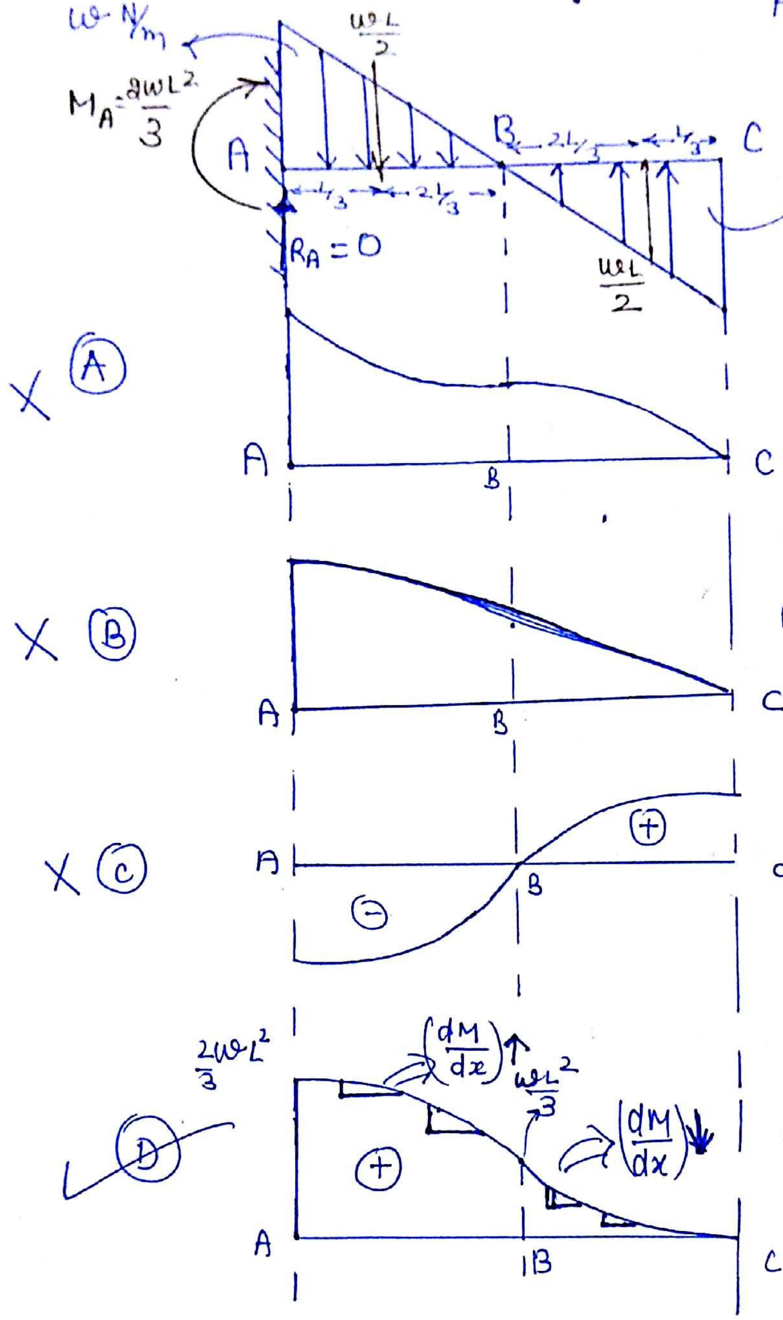
$$w_{x-x} = -\frac{wx}{L}$$

$$(S.F.)_{x-x} = \frac{wx^2}{2L}$$

$$(B.M.)_{x-x} = -\frac{wx^2}{2L} \times \frac{x}{3} = -\frac{wx^3}{6L}$$

Question For the Cantilever beam as shown in Fig. identify which B.M.D. is correct.

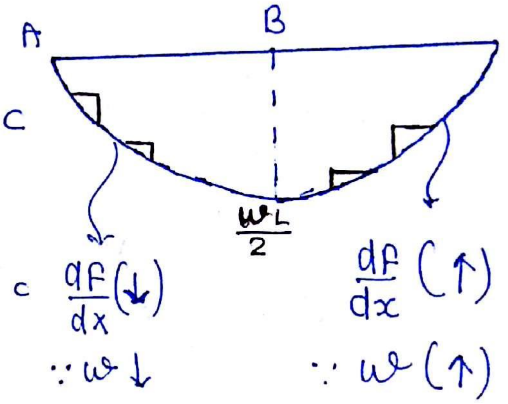
At free end B.M. = 0
i.e. No external load



$$M_A = \left(\frac{wL}{2}\right)\left\{L + \frac{2L}{3} - \frac{L}{3}\right)$$

$$\boxed{M_A = \frac{2wL^2}{3}}$$

⇒ SFD



$(SF)_A = R_A = 0$

$(SF)_B = R_A - \frac{WL}{2} = -\frac{wL}{2}$

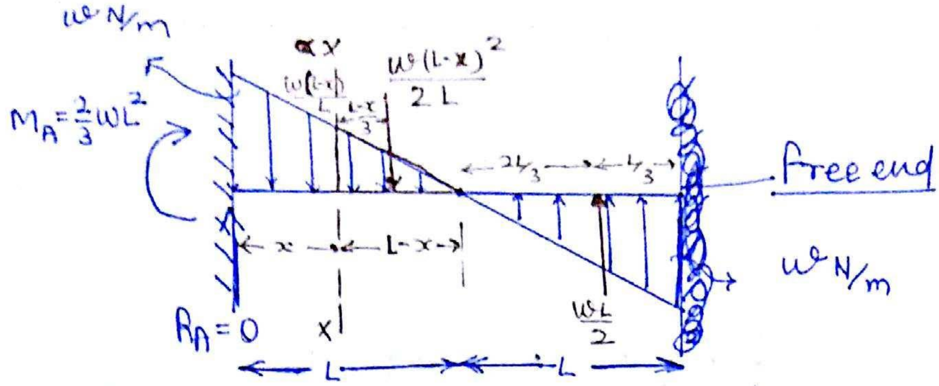
$(SF)_C = R_A - \frac{wL}{2} + \frac{wL}{2} = 0$

$(BM)_A = \frac{2}{3}wL^2$

$(BM)_B = \frac{2}{3}wL^2 - \frac{wL}{2}\left(\frac{2L}{3}\right) + R_A(0) = \frac{wL^2}{3}$; $(BM)_C = 0$

From A to B → SF ↑ → $\frac{dM}{dx}$ ↑

From B to C → SF ↓ → $\frac{dM}{dx}$ ↓



$$P_{x-x} = \frac{w(L-x)^2}{2L}$$

$$w_{x-x} = -\frac{w(L-x)}{L} \quad -(\mathrm{I})$$

$$(SF)_{x-x} = \frac{w(L-x)^2}{2L} - \frac{wL}{2} \quad -(\mathrm{II})$$

$$(BM)_{x-x} = -\frac{w(L-x)^2}{2L} \times \left(\frac{L-x}{3}\right) + \frac{wL}{2}\left[(L-x) + \frac{2L}{3}\right]$$

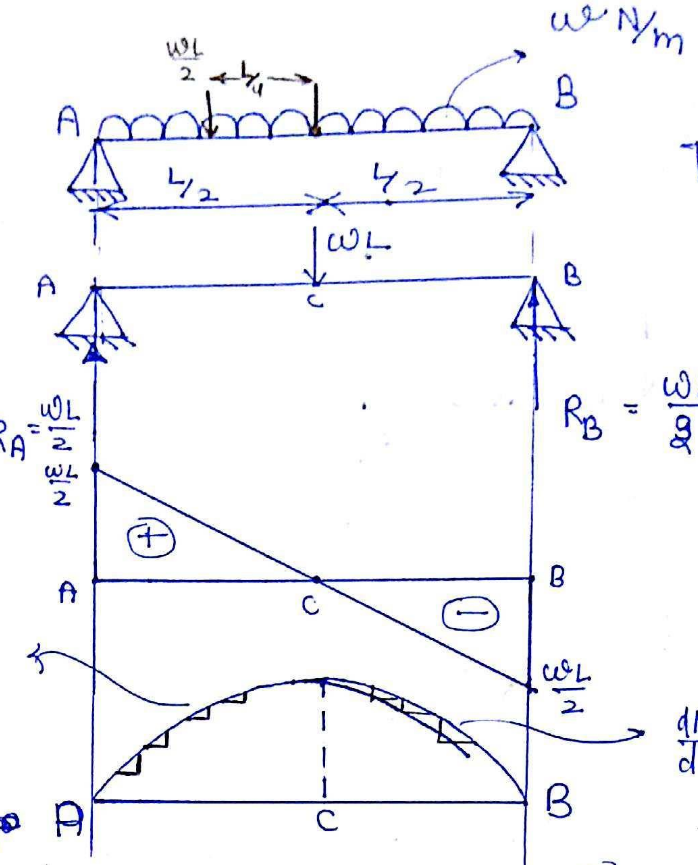
$$(BM)_{x-x} = -\frac{w(L-x)^3}{6L} + \frac{wL}{6}(5L-3x) \quad -(\mathrm{III})$$

$x = 0 \Rightarrow$
$(w)_A = -w$
$(SF)_A = 0$
$(BM)_A = \frac{2}{3}wL^2$

$x = L \Rightarrow$
$w_B = 0$
$(SF)_B = -\frac{wL}{2}$
$(BM)_B = \frac{wL^2}{3}$

$x = 2L \Rightarrow$
$w_c = w$
$SF_c = 0$
$BM_c = 0$

Question



Total load = wL

Act as Centroid

sol^n

SF ↓ $\quad \frac{dm}{dx}$ ↓

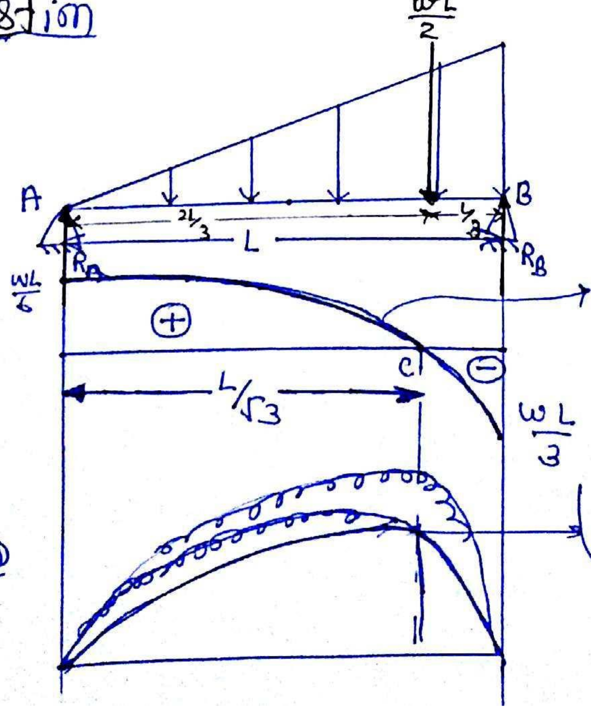
$\frac{dM}{dx}$ ↑ $\quad$ SF ↑

$(BM)_A = 0$

$$(BM)_C = (BM)_{max} = \frac{wL}{2}\left(\frac{L}{2}\right) - \frac{wL}{2}\left(\frac{L}{4}\right)$$

$$(BC)_{max} = \frac{wL^2}{8}$$

Question



$$R_A = \frac{\frac{wL}{2} \times \frac{L}{3}}{L} = \frac{wL}{6}$$

$$R_B = \frac{wL}{2} - R_A = \frac{wL}{3}$$

SFD

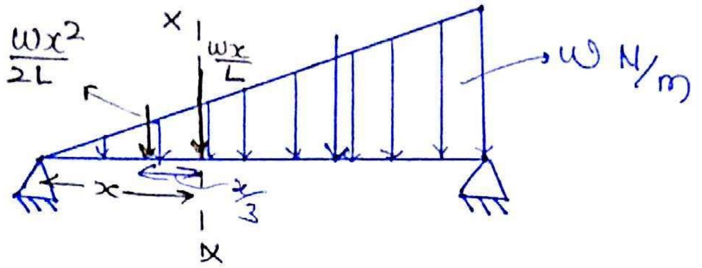
$\frac{dF}{dx}(\uparrow) \because w(\uparrow)$

BMD

$$(BM)_{max} = \frac{wL^2}{9\sqrt{3}}$$

zero shear force (or) max. BM location can be determine by (a) $(SF)_{x-x} = \text{———} = 0$

(or) (b) $\frac{d}{dx}\left[(BM)_{x-x}\right] = 0$



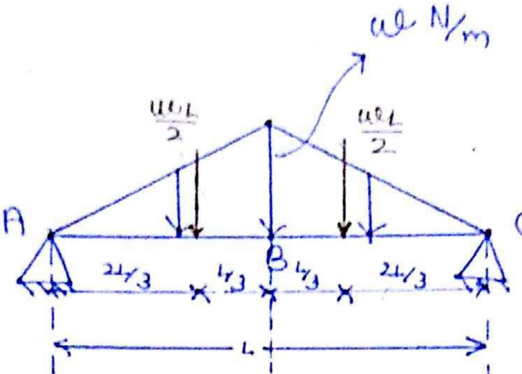
$$(SF)_{x-x} = \frac{wL}{6} - \frac{wx^2}{2L}$$

$$(BM)_{x-x} = \frac{wL}{6}(x) - \frac{wx^2}{2L}\left(\frac{x}{3}\right) = \frac{wLx}{6} - \frac{wx^6}{6L}$$

$$(SF)_{x-x} = \frac{wL}{6} - \frac{wx^2}{2L} = 0 \Rightarrow x = \frac{L}{\sqrt{3}} = 0.577L$$

$$\underline{\text{Max}}\ (BM)_{x-x} = \left[(BM)_{x-x}\right]_{x=\frac{L}{\sqrt{3}}} = \frac{wL^2}{9\sqrt{3}}$$

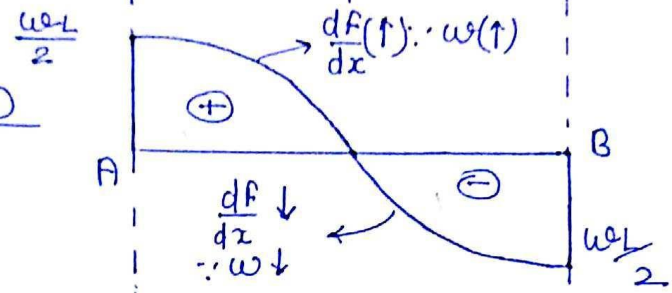
Question



$$R_A = \frac{wL}{2}$$

$$R_B = \frac{wL}{2}$$

SFD



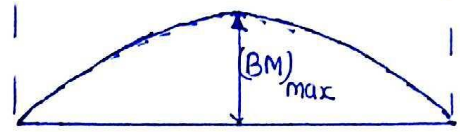
$$(BM)_A = 0$$

$$(BM)_{max} = (BM)_B$$

$$= \frac{w}{2}L(L)\ \frac{wL}{2}\left(\frac{L}{3}\right)$$

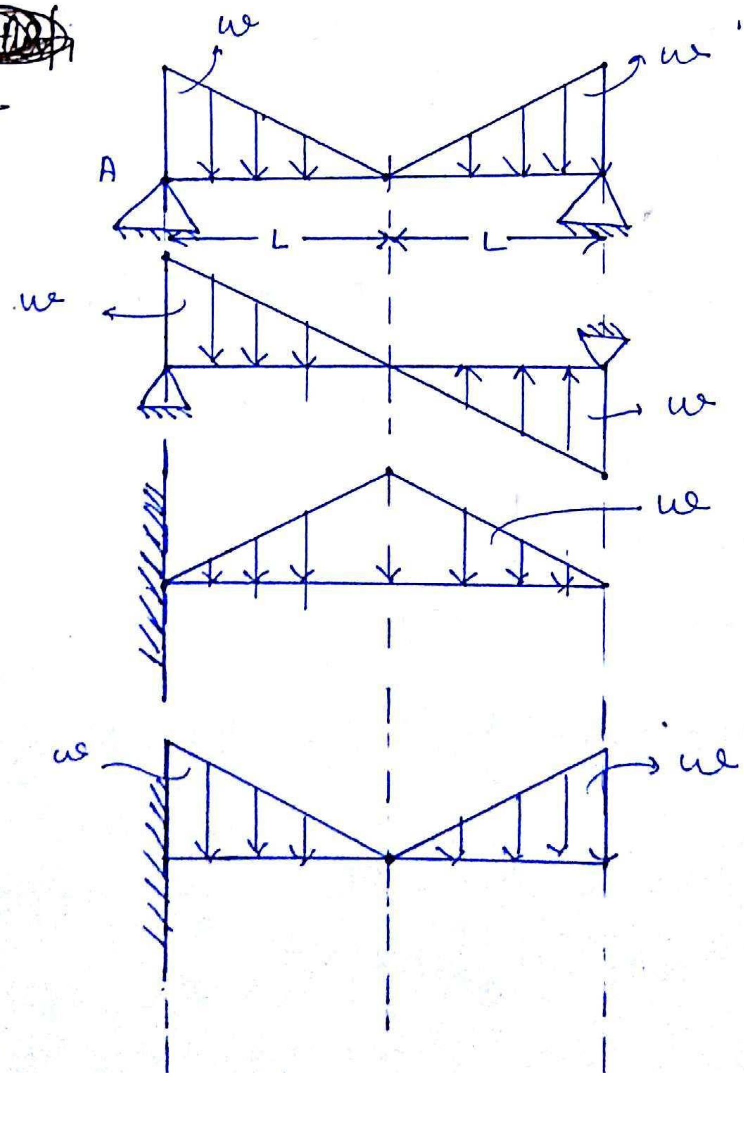
$$(BM)_B = \frac{wL^2}{3}$$

BMD



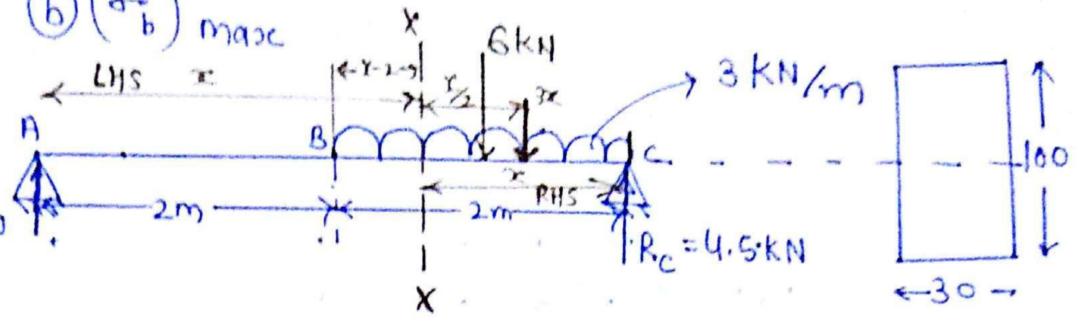
~~Question~~

Ex



Quest Find (a) location of zero S.F.

(b) $(\sigma_b)_{max}$



$R_A = \frac{6 \times 1}{4} = 1.5\,kN$

$R_C = 4.5\,kN$

From RHS

CB :- x [0 to 2m]

$$(SF)_{x-x} = -4.5 + 3x$$

$$(BM)_{x-x} = 4.5x - \frac{3}{2}x^2$$

$$(S.F)_{x-x} = -4.5 + 3x = 0$$

$$x = 1.5\,m \text{ from C}$$

From (LHS) BC [x = 2 to 4]

BC [x = 2 to 4m]

$$(SF)_{x-x} = 1.5 - 3(x-2)$$

$$(BM)_{x-x} = 1.5x - \frac{3(x-2)^2}{2}$$

AB

$$(SF)_{x-x} = 1.5\,KN$$

$$(BM)_{x-x} = 1.5x$$

ie from A = 4 − 1.5 = 2.5 m

$$\text{Max. BM} = [BM_{x-x}]_{x=1.5} = 4.5(1.5) - \frac{3}{2}(1.5)^2 = 3375\ N\text{-}m$$

$$(\sigma_b)_{max} = \frac{M_{max}}{Z_{N.A.}} = \frac{3375 \times 10^3}{\frac{1}{6} \times 30 \times 100^2} = 67.5\,MPa$$

Relationship between w, F & M:—

$$-w_{x-x} = \frac{dF_{x-x}}{dx} \quad -\text{(I)}$$

$$F_{x-x} = \frac{dM_{x-x}}{dx} \quad -\text{(II)}$$

$$\Rightarrow \int_A^B dF_{x-x} = \int_A^B -w_{x-x}\,dx$$

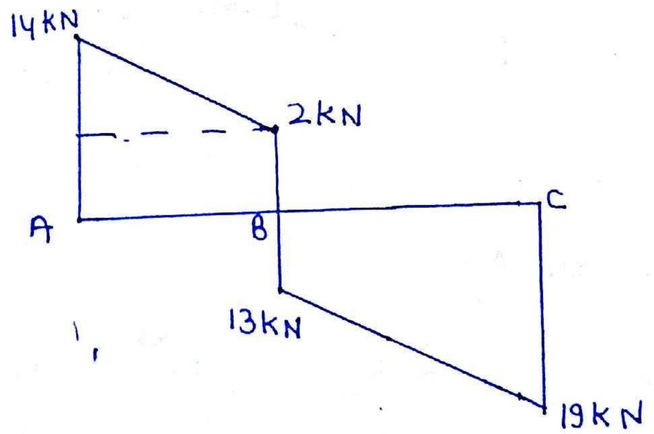
$$\boxed{F_B - F_A = \int_A^B w_{x-x}\,dx = \text{Area of a loading diag. of D.L. between B \& A}}$$

$$\Rightarrow \int dM_{x-x} = \int F_{x-x}\,dx$$

$$\boxed{M_B - M_A = \int_A^B F_{x-x}\,dx = \text{Area of SFD between B to A}}$$

Q.10

o.B.



$$M_B - \cancel{M_A}^{0} = (2\times 2) + \frac{1}{2}\times 2\times 12$$

$$M_B = 4 + 12$$

$$M_B = 16 \text{ KNm}$$

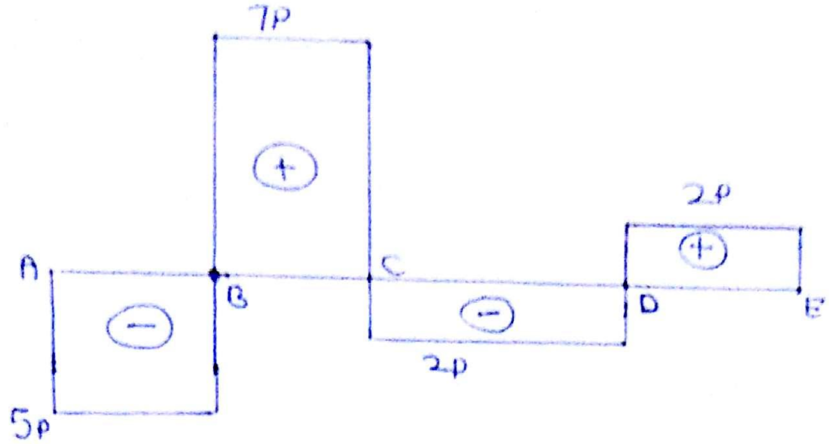
OR

$$M_C - M_B = (-13\times 1) + \left(\frac{1}{2}(-6)\times 1\right)$$

$$-M_{max} = -16$$

$$M_{max} = 16 \text{ KN-m}$$

Q.29



Assum SSB $M_A = 0$

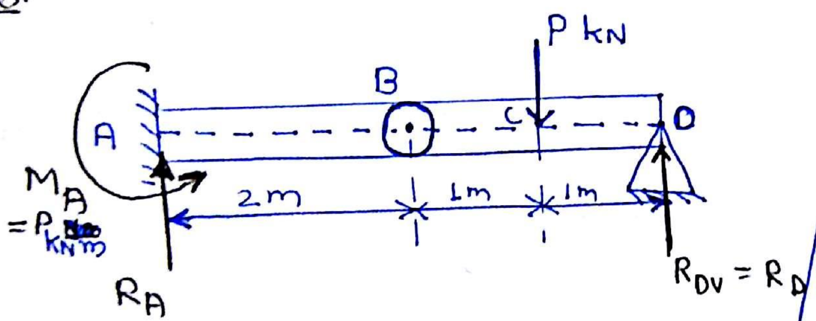
$M_B - M_A = -5Pa \Rightarrow M_B = -5Pa$ max.

$M_C - M_B = 7Pa \Rightarrow M_C = 2Pa$

$M_D - M_C = -4Pa \Rightarrow M_D = -2Pa$

$M_E - M_D = 2Pa \Rightarrow M_E = 0$

Que.



B ⇒ Internal (or) Unsupported Hinge
↳ become a P.O.C.F.
$(B.M.)_B = 0$

$(BM)_B = 0 = (R_D)2 - P(1) = 0$

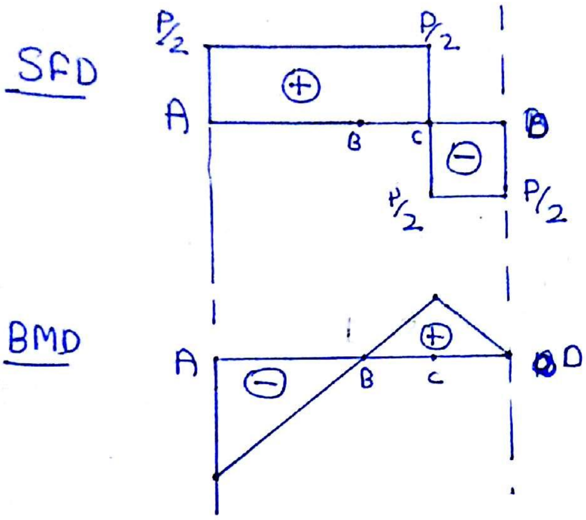
$R_D = P/2 \text{ kN} (\uparrow)$

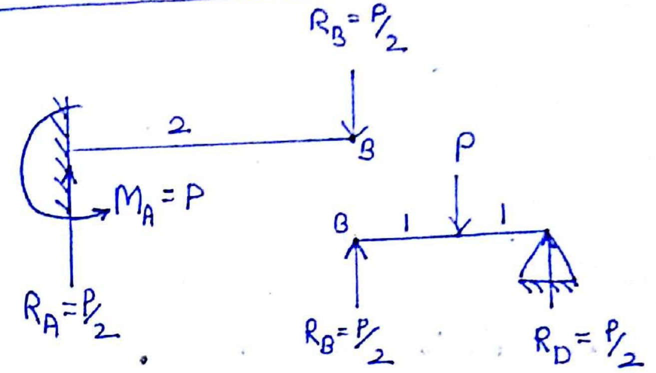
$\Sigma V = 0 \Rightarrow R_A - P + P/2 = 0 \Rightarrow R_A = P/2 \text{ kN} (\uparrow)$

$\Sigma M_A = 0 \Rightarrow -M_A + P(3) - P/2(4) = 0$

$M_A = P \text{ kN-m}$

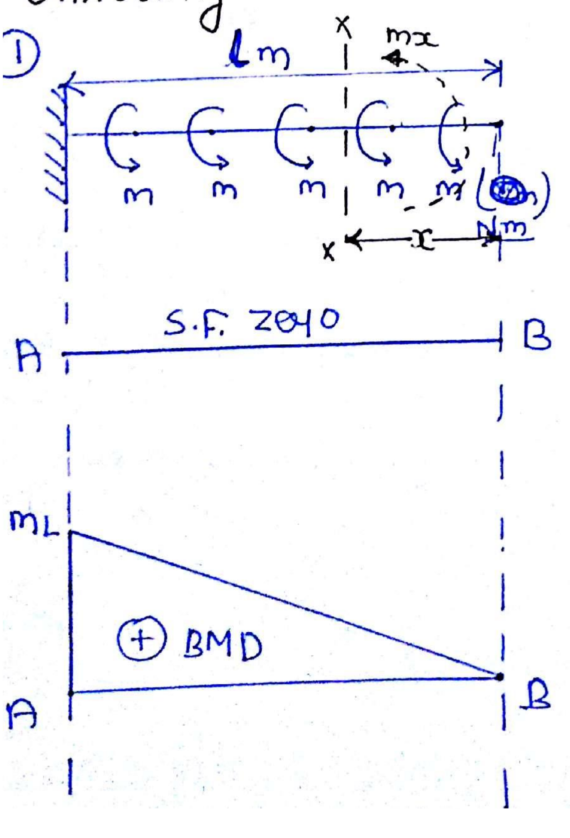
X-S/C	SF	BM
A	$P/2$	$-P$
B	$P/2$	0
C	$\pm P/2$	$P/2$
D	$-P/2$	0

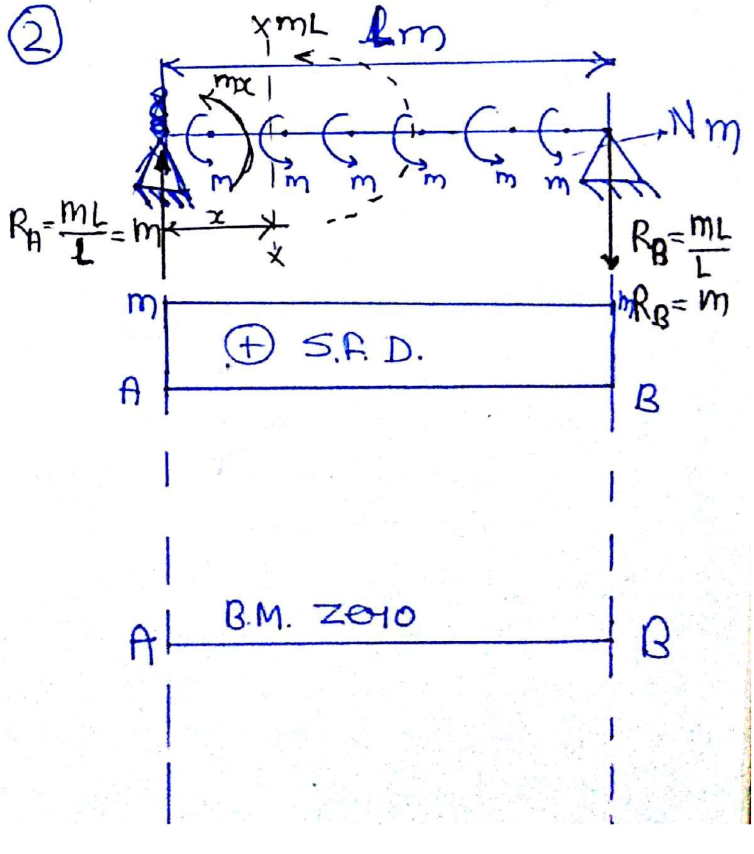




Seprate into two beam and calculate reaction & B.M.

Uniformly Distributed Moment





$(SF)_{x-x} = zero$

$(BM)_{x-x} = mx$

$x = 0 \Rightarrow (SF) = 0$

$(BM) = 0$

$x = L \Rightarrow (SF) = 0$

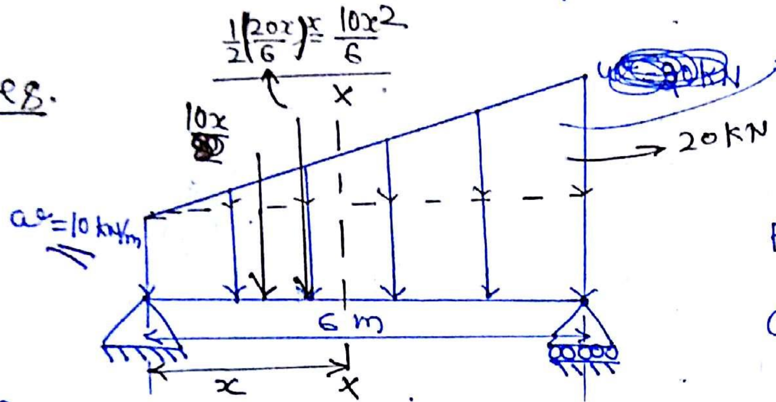
$(BM) = Ml$

$x = 0$ to L

$(SF)_{x-x} = m = \text{Constant}$

$(BM)_{x-x} = m(x) - m(x) = 0$

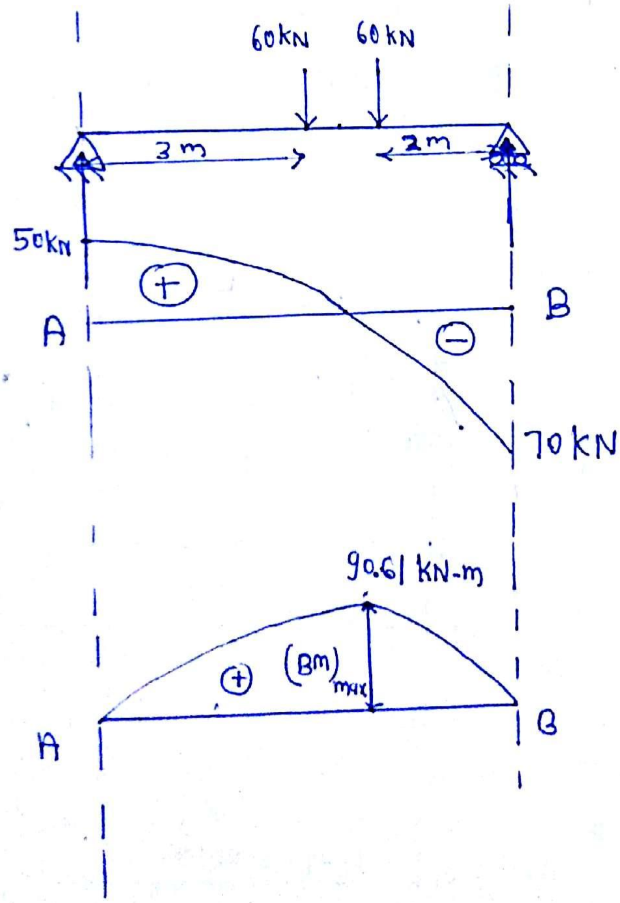
Ques.



$P_1 = 10 \times 6 = 60$ kN

$P_2 = \frac{1}{2} \times 6 \times 20 = 60$ kN

Sol^n



$$R_A = \frac{60 \times 3 + 60 \times 2}{6}$$

$R_A = 50$ kN

$R_B = 120 - 50 = 70$ kN

$x = 0$ to 6 m

$$(SF)_{x-x} = 50 - 10x - \frac{10x^2}{6} \quad -(1)$$

$$(BM)_{x-x} = 50x - 10x\left(\frac{x}{2}\right) - \frac{10x^2}{6}\left(\frac{2x}{3}\right)$$

$$(BM)_{x-x} = 50x - 5x^2 - \frac{5x^3}{9} \quad (2)$$

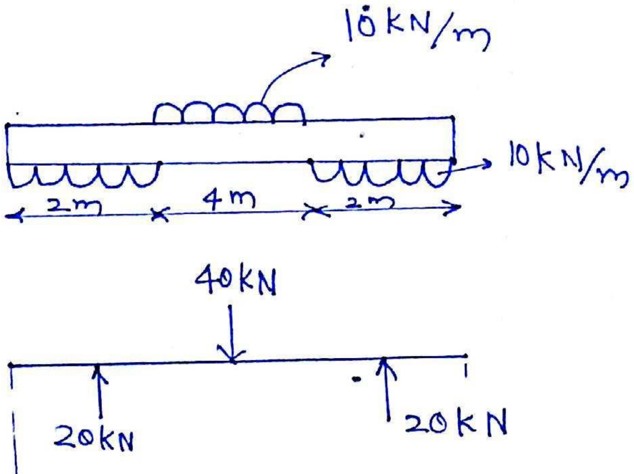
To det. zero shear force

$$(S.F.)_{x-x} = 50 - 10x - \frac{10x^2}{6} = 0$$

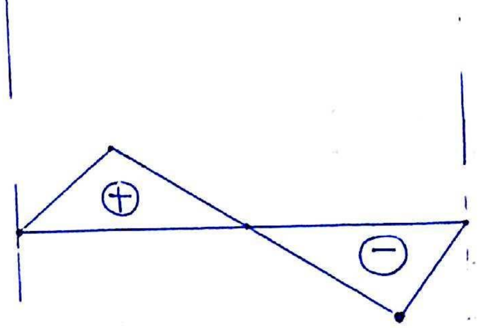
$$x = 3.24 \text{ m} \quad \text{from C}$$

$$(BM)_{max} = \left[(BM)_{x-x}\right]_{x=3.24m} = 90.61 \text{ kN-m}$$

<u>Question</u>

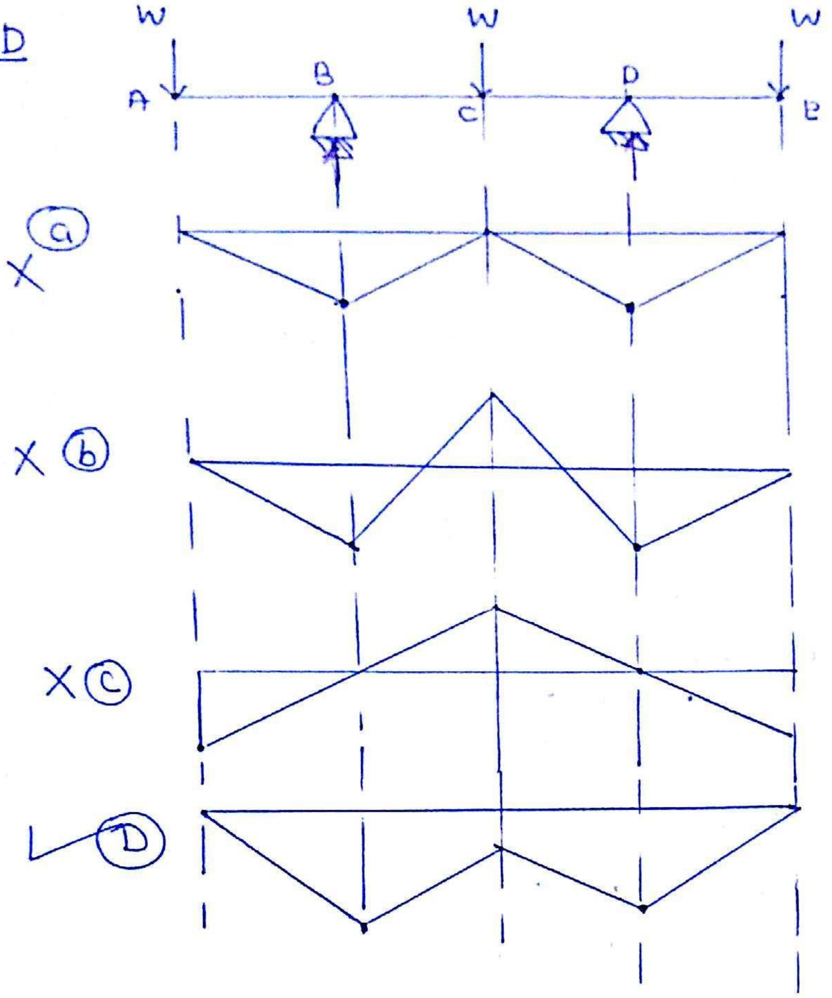


SFD



Quest

BMD



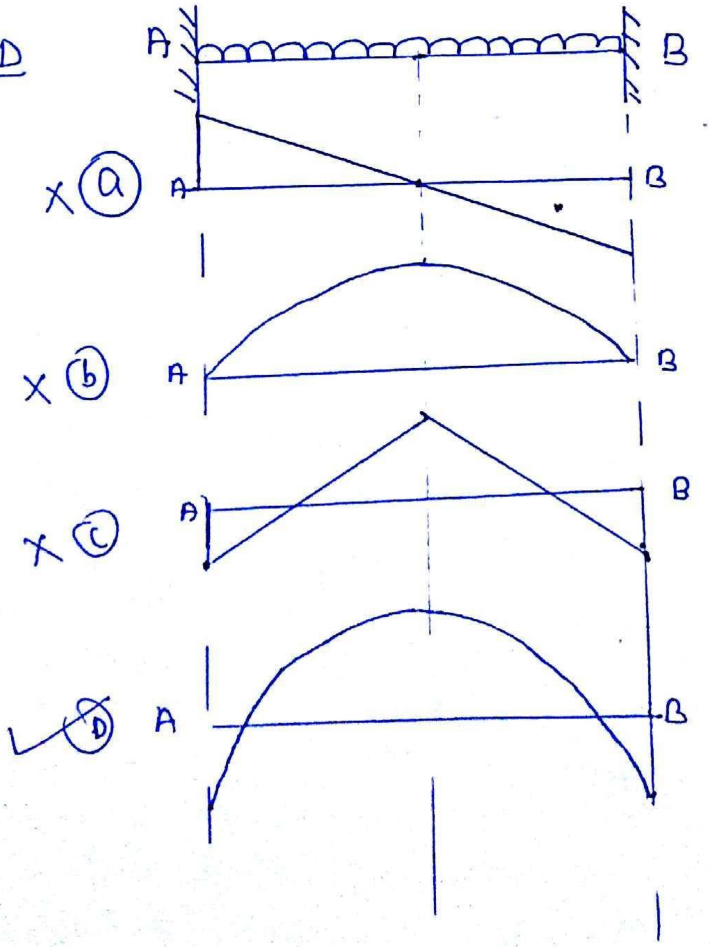
$$(BM)_C = -W(2L) + \frac{3}{2}WL$$

$$= -\frac{WL}{2}$$

(D) Ans

Quest

BMD

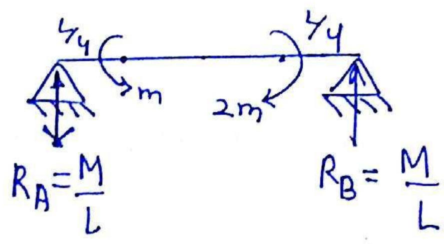


Conclusion

1. B.M. Variation is two order more than load intensity variation
2. at Fixed end B.M. Should be non-zero

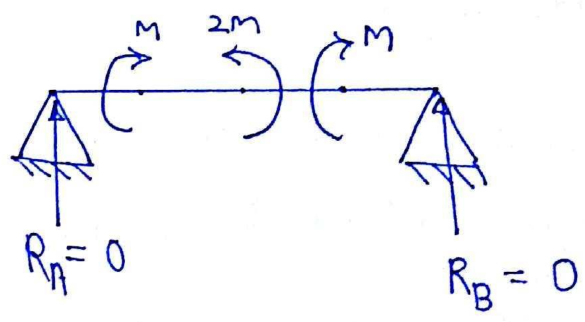
Load	S.F. Variation	B.M. variation
Zero (bet^n two Concentrated point loads)	Const (SFD Consists of Horizontal line)	linear (Inclined line)
Constant (UDL)	linear (Inclined line)	Parabolic ($M \propto x^2$)
linear Variation (UVL)	Parabolic ($F \propto x^2$)	Cubic Parabolic ($M \propto x^3$)
bet^n two Conc. moment (in the absence of Vertical loads on beam	Zero	Const. (Horizontal line

eg



$\boxed{SF \neq 0}$

eg



$\boxed{SF = 0}$