EXERCISE 7.5

Q No 1 $\displaystyle\int \frac{x}{(x+2)(x+1)}\,dx.$

Sol. Resolving into partial fractions.

Let $\displaystyle \frac{x}{(x+2)(x+1)} = \frac{A}{x+1} + \frac{B}{x+2} \Rightarrow x = A(x+2)+B(x+1)$

When $x+1=0$ ie $x=-1$

$-1 = A(-1+2)+0 \Longrightarrow A=-1$

When $x+2=0$ ie $x=-2$

$-2 = 0+B(-2+1) \Longrightarrow -2=-B \Rightarrow B=2$

$\displaystyle \therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$

$$\therefore \int \frac{x}{(x+1)(x+2)}\,dx = \int \frac{-1}{x+1}\,dx + \int \frac{2}{x+2}\,dx$$

$$= -\log|x+1| + 2\log|x+2| + c.$$

$$= \log\left(\frac{|x+2|^2}{|x+1|}\right) + c.$$

Q No 2. $\displaystyle\int \frac{1}{x^2-9}\,dx$

Sol. $\displaystyle \frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)}$

Let $\displaystyle \frac{1}{(x+3)(x-3)} = \frac{A}{x+3} + \frac{B}{x-3}$

$\Longrightarrow 1 = A(x-3)+B(x+3)$

$\therefore$ When $x-3=0$ ie $x=3$.

$1 = 6B \Rightarrow B = \frac{1}{6}$

When $x+3=0$ ie $x=-3$

$1 = A(-6) \Rightarrow A = -\frac{1}{6}$

$$\therefore \frac{1}{x^2-9} = -\frac{1}{6}\,\frac{1}{x+3} + \frac{1}{6}\cdot\frac{1}{x-3}$$

$$\therefore \int \frac{1}{x^2-9}\,dx = -\frac{1}{6}\int \frac{1}{x+3}\,dx + \frac{1}{6}\int \frac{1}{x-3}\,dx$$

$$= \frac{1}{6}\left[\log(x-3) - \log(x+3)\right] = \frac{1}{6}\log\left|\frac{x-3}{x+3}\right| + c$$

Q No 3 $\displaystyle\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx.$

Sol. Resolving into Partial fractions using short cut method.

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{3(1)-1}{(x-1)(1-2)(1-3)} + \frac{3(2)-1}{(2-1)(x-2)(2-3)} + \frac{3(3)-1}{(3-1)(3-2)(x-3)}$$

$$= \frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3}$$

$$\therefore \int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx = \int \frac{1}{x-1}\,dx - 5\int \frac{1}{x-2}\,dx + 4\int \frac{1}{x-3}\,dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + c$$

Q No 4. $\displaystyle\int \frac{x}{(x-1)(x-2)(x-3}\,dx.$

Sol. Resolving into partial fractions using short cut method.

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)(1-2)(1-3)} + \frac{2}{(x-2)(2-1)(2-3)} + \frac{3}{(x-3)(3-1)(3-2)}$$

$$= \frac{1}{2(x-1)} - \frac{2}{x-2} + \frac{3}{2(x-3)}$$

$$\therefore \int \frac{x}{(x-1)(x-2)(x-3)}\,dx = \frac{1}{2}\int \frac{1}{x-1}\,dx - 2\int \frac{1}{x-2}\,dx + \frac{3}{2}\int \frac{1}{x-3}\,dx$$

$$= \frac{1}{2}\log|x-1| - 2\log|x-2| + 3\log|x-3| + C$$

Q No 5. $\displaystyle\int \frac{2x}{x^2+3x+2}$

Sol. $$\frac{2x}{x^2+3x+2} = \frac{2x}{x^2+2x+x+2} = \frac{2x}{x(x+2)+1(x+2)} = \frac{2x}{(x+1)(x+2)}$$

Resolving into partial fractions by short cut method.

$$\frac{2x}{(x+1)(x+2)} = \frac{-2\times 1}{(x+1)(-1+2)} + \frac{2\times(-2)}{(-2+1)(x+2)}$$

$$= \frac{-2}{x+1} + \frac{4}{x+2}$$

$$\therefore \int \frac{2x}{(x+1)(x+2)}dx = -2\int \frac{1}{x+1}dx + 4\int \frac{1}{x+2}dx.$$

$$= -2\log|x+1| + 4\log|x+2| + C.$$

Q No 6 $\frac{1-x^2}{x(1-2x)} = \frac{-(x^2-1)}{-(2x^2-x)} = \frac{x^2-1}{2x^2-x} = \frac{1-x^2}{x-2x^2}.$

$$= \frac{1}{2} + \frac{-\frac{1}{2}x+1}{x(1-2x)} \quad \text{[By Long division]}$$

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{-x+2}{x(1-2x)}\right)$$

Now $\frac{2-x}{x(1-2x)} = \frac{2-0}{x} + \frac{2-\frac{1}{2}}{\frac{1}{2}(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$

$$\therefore \int \frac{1-x^2}{x(1-2x)}dx = \int \left(\frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} + \frac{3}{1-2x}\right)\right)dx$$

$$= \frac{1}{2}\int dx + \int \frac{1}{x}dx + \frac{3}{2}\int \frac{1}{1-2x}dx.$$

$$= \frac{1}{2}x + \log|x| + \frac{3}{2}\frac{\log|1-2x|}{-2} + C$$

$$= \frac{1}{2}x + \log|x| - \frac{3}{4}\log|1-2x| + C$$

Q No. 7. $\int \frac{x}{(x^2+1)(x-1)}dx$

Sol : Let $\frac{x}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$$\Rightarrow \quad x = A(x^2+1) + (Bx+C)(x-1)$$

$$\Rightarrow \quad x = Ax^2 + A + Bx^2 - Bx + Cx - C.$$

$$\Rightarrow \quad x = (A+B)x^2 + (-B+C)x + (A-C)$$

Comparing the coeff. of x^2, x and Constant term, we get

$$A+B=0 \quad ; \quad -B+C=1 \quad \quad A-C=0$$

Solving for A, B, C we get

$$A=C=\frac{1}{2} \quad \text{and} \quad B=-A=-1/2$$

$$\therefore \frac{x}{(x^2+1)(x-1)} = \frac{1}{2(x-1)} + \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}$$

$$= \frac{1}{2(x-1)} - \frac{1}{2}\left(\frac{x+1}{x^2+1}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{x-1} - \frac{1}{2}\frac{x}{x^2+1} - \frac{1}{2}\cdot\frac{1}{x^2+1}.$$

$$\therefore \int\frac{x}{(x^2+1)(x-1)}dx = \frac{1}{2}\int\frac{1}{x-1}dx - \frac{1}{2\times2}\int\frac{2x}{x^2+1}dx - \frac{1}{2}\int\frac{1}{x^2+1}dx$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\cdot\tan^{-1}x + c$$

Q No. 8. $\int\frac{x}{(x-1)^2(x+2)}dx$

Sol. Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$

$$\Rightarrow \quad x = A(x-1)(x+2) + B(x+2) + C(x-1)^2 \quad -(i)$$

$$\Rightarrow \quad x = A(x^2+x-2) + Bx + 2B + (x^2+1-2x)C$$

$$\Rightarrow \quad x = (A+C)x^2 + (A+B-2C)x + (-2A+2B+C) \quad -(ii)$$

Now when $x-1=0$ ie $x=1$ from (i)

$$1 = 3B \Rightarrow B = 1/3$$

When $x+2=0$ ie $x=-2$

$$-2 = (-2-1)^2C \Rightarrow -2 = 9C \Rightarrow C = -2/9 \text{ from (i)}$$

Also from (ii) $A+C=0 \Rightarrow A=-C=\frac{2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2/9}{x-1} + \frac{1/3}{(x-1)^2} + \frac{-2/9}{x+2}$$

$$\therefore \int \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9}\int \frac{1}{x-1}dx + \frac{1}{3}\int (x-1)^{-2}dx - \frac{2}{9}\int \frac{1}{x+2}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\frac{(x-1)^{-2+1}}{-2+1} - \frac{2}{9}\log|x+2| + c.$$

$$= \frac{2}{9}\log|x-1| - \frac{1}{3}\left(\frac{1}{x-1}\right) - \frac{2}{9}\log|x+2| + c$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3}\left(\frac{1}{x-1}\right) + c$$

Q No: 9 $\int \frac{3x+5}{x^3-x^2-x+1}dx$

Sol. $\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)(x^2-1)} = \frac{3x+5}{(x-1)(x-1)(x+1)} = \frac{3x+5}{(x-1)^2(x+1)}$

Let $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

$\Rightarrow \quad 3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \quad \cdots (i)$

i.e. $\quad 3x+5 = A(x^2-1) + B(x+1) + C(x^2+1-2x)$

i.e. $\quad 3x+5 = (A+C)x^2 + (B-2C)x + (-A+B+C) \quad \cdots (ii)$

From (i) When $x-1=0$ i.e. $x=1$.

$3(1)+5 = B(1+1) \Rightarrow B = 4$

When $x+1=0$ i.e. $x=-1$

$3(-1)+5 = C(-1-1)^2 \Rightarrow 2 = 4C \Rightarrow C = \frac{1}{2}$

From (ii) $\quad A+C=0 \Rightarrow A = -C = -\frac{1}{2}$

$$\therefore \int \frac{3x+5}{x^3-x^2+x+1}dx = \int \frac{-\frac{1}{2}}{x-1}dx + \int \frac{4}{(x-1)^2}dx + \int \frac{\frac{1}{2}}{x+1}dx$$

$$= -\frac{1}{2}\log|x-1| + 4\frac{(x-1)^{-2+1}}{-2+1} + \frac{1}{2}\log|x+1| + c$$

$$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{x-1} + C.$$

Q.No.10. $\int \frac{2x-3}{(x^2-1)(2x+3)} dx$

Sol.. $\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x-1)(x+1)(2x+3)}$

Resolving in to partial fractions by short cut method

$$\frac{2x-3}{(x-1)(x+1)(2x+3)} = \frac{2(1)-3}{(x-1)(2)(5)} + \frac{2(-1)-3}{(-2)(x+1)(-2+3)} + \frac{2(-3/2)-3}{(-\frac{3}{2}-1)(-\frac{3}{2}+1)(2x+3)}$$

$$= \frac{-1}{10(x-1)} + \frac{5}{2(x+1)} - \frac{24}{5(2x+3)}$$

$$\therefore \int \frac{2x-3}{(x^2-1)(2x+3)} dx = -\frac{1}{10}\int \frac{1}{x-1} dx + \frac{5}{2}\int \frac{1}{x+1} dx - \frac{24}{5}\int \frac{1}{2x+3} dx.$$

$$= -\frac{1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{24}{5} \log \frac{|2x+3|}{2} + c$$

$$= -\frac{1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{12}{5} \log|2x+3| + c$$

Q.No. 11. $\int \frac{5x}{(x+1)(x^2-4)} dx.$

Sol. $\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$

Resolving into partial fractions by short cut method.

$$\frac{5x}{(x+1)(x+2)(x-2)} = \frac{5(-1)}{(x+1)(-1+2)(-1-2)} + \frac{5(-2)}{(-2+1)(x+2)(-2-2)} + \frac{5(2)}{(2+1)(2+2)(x-2)}$$

$$= \frac{5}{3(x+1)} + \frac{5}{6(x-2)} - \frac{5}{2(x+2)}$$

$$\therefore \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3}\int \frac{1}{x+1} dx + \frac{5}{6}\int \frac{1}{x-2} dx - \frac{5}{2}\int \frac{1}{x+2} dx$$

$$= \frac{5}{3} \log|x+1| + \frac{5}{6} \log|x-2| - \frac{5}{2} \log|x+2| + c$$

QNo.12. $\int \frac{x^3+x+1}{x^2-1}\,dx$

Sol. Now $\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$ (By Long division)

$$= x + \frac{2x+1}{(x-1)(x+1)}$$

Now Resolving $\frac{2x+1}{(x-1)(x+1)}$ by by short cut method into partial fractions.

$$\frac{2x+1}{(x-1)(x+1)} = \frac{2(1)+1}{(x-1)(2)} + \frac{2(-1)+1}{(-1-1)(x+1)} = \frac{3}{2(x-1)} + \frac{1}{2(x+1)}$$

$$\therefore \int \frac{x^3+x+1}{x^2-1}\,dx = \int x\,dx + \frac{3}{2}\int \frac{1}{x-1}\,dx + \frac{1}{2}\int \frac{1}{x+1}\,dx$$

$$= \frac{x^2}{2} + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$$

QNo.13. $\int \frac{2}{(1-x)(1+x^2)}$

Sol. Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{1+x^2}$

$$\Rightarrow \quad 2 = A(1+x^2) + (Bx+C)(1-x) \quad \cdots\cdots (i)$$

$$\Rightarrow \quad 2 = A(1+x^2) + Bx + C - Bx^2 - Cx$$

$$\Rightarrow \quad 2 = (A-B)x^2 + (B-C)x + (A+C) \quad \cdots (ii)$$

By putting $1-x=0$ ie $x=1$ in (i) we get.

$$2 = A(1+1) \Rightarrow A = 1$$

from (ii) Comparing coeff of x, x^2.

$$A-B=0 \Rightarrow A=B \Rightarrow B=1$$

and $A+C=2 \Rightarrow C=1$

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$= \frac{1}{(1-x)} + \frac{x}{1+x^2} + \frac{1}{1+x^2}$$

$$\therefore \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \frac{1}{2}\int \frac{2x}{x^2+1}dx + \int \frac{1}{1+x^2}dx$$

$$= \frac{\log|1-x|}{-1} + \frac{1}{2}\log|x^2+1| + \tan^{-1}x + C$$

Q No. 14 $\int \frac{3x-1}{(x+2)^2}dx.$

Sol. Let $\frac{3x-1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$.

$\Rightarrow \quad 3x-1 = A(x+2)+B$

$\Rightarrow \quad 3x-1 = Ax+(2A+B)$

Comparing the coeff. of x and constant term.

$A = 3$ and $2A+B = -1 \Rightarrow B = -7$

$$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{x+2} + \frac{-7}{(x+2)^2}$$

$$\therefore \int \frac{3x-1}{(x+2)^2}dx = 3\int \frac{1}{x+2}dx - 7\int (x+2)^{-2}dx$$

$$= 3\log|x+2| - 7\frac{(x+2)^{-2+1}}{-2+1} + C$$

$$= 3\log|x+2| + \frac{7}{x+2} + C$$

Q No 15. $\int \frac{1}{x^4-1}dx.$

Sol. $\frac{1}{x^4-1} = \frac{1}{(x^2-1)(x^2+1)} = \frac{1}{(x-1)(x+1)(x^2+1)}$

Let $\frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$

$\Rightarrow \quad 1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + ((Cx+D))(x^2-1)$

When $x = -1 \Rightarrow 1 = B(-2)(2) \Rightarrow B = -\frac{1}{4}$

When $x = +1 \Rightarrow 1 = (2)(2)A \Rightarrow A = \frac{1}{4}$

Comparing coeff of x^3 we get $C = 0$

Comparing Constant term we get

$$A - B - D = 1. \Rightarrow D = \frac{1}{4} - \left(-\frac{1}{4}\right) - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\therefore \int \frac{1}{x^4-1}\,dx = \frac{1}{4}\int \frac{1}{x-1}\,dx - \frac{1}{4}\int \frac{1}{x+1}\,dx - \frac{1}{2}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{1}{4}\log|x-1| - \frac{1}{4}\log|x+1| - \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$$

QNo. 16 $\int \frac{1}{x(x^n+1)}\,dx$

Soln: $I = \int \frac{1}{x(x^n+1)}\,dx = \int \frac{x^{n-1}}{x \cdot x^{n-1}(x^n+1)}\,dx = \int \frac{x^{n-1}}{x^n(x^n+1)}\,dx$

Put $x^n = y$

$$\Rightarrow n x^{n-1}\,dx = dy. \Rightarrow x^{n-1}\,dx = \frac{dy}{n}.$$

$$\therefore I = \int \frac{dy/n}{y(y+1)} = \frac{1}{n}\int \frac{1}{y(y+1)}\,dy.$$

Now $\frac{1}{y(y+1)} = \frac{A}{y} + \frac{B}{y+1}$

$$\Rightarrow 1 = A(y+1) + By$$

$\Rightarrow$ when $y = 0 \Rightarrow A = 1$

$y = -1 \Rightarrow B = -1$

$$\therefore \frac{1}{y(y+1)} = \frac{1}{y} - \frac{1}{y+1}$$

$$\therefore I = \frac{1}{n}\int \frac{1}{y(y+1)}\,dy = \frac{1}{n}\left[\int \frac{1}{y}\,dy - \int \frac{1}{y+1}\,dy\right]$$

$$= \frac{1}{n}\log y - \frac{1}{n}\log|y+1| + C$$

$$= \frac{1}{n}\log\left|\frac{y}{y+1}\right| + C = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

Q No. 17. $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$

Sol. Put $\sin x = t \Rightarrow \cos x \cdot dx = dt$

$$\therefore I = \int \frac{dt}{(1-t)(2-t)}$$

Now Resolving $\frac{1}{(1-t)(2-t)}$ into partial fractions using short cut method

$$\frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)(2-1)} + \frac{1}{(1-2)(2-t)} = \frac{1}{1-t} - \frac{1}{2-t}$$

$$\therefore I = \int \frac{1}{1-t} dt - \int \frac{1}{2-t} dt$$

$$= \frac{\log|1-t|}{-1} - \frac{\log|2-t|}{-1} + c$$

$$= \log|2-t| - \log|1-t| + c = \log\left|\frac{2-t}{1-t}\right| + c$$

$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + c$$

Q No 18. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$

Sol. $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(y+1)(y+2)}{(y+3)(y+4)}$ where $y = x^2$

$$= \frac{y^2+3y+2}{y^2+7y+12} = 1 + \frac{-4y-10}{y^2+7y+12} = 1 - 2\frac{2y+5}{(y+3)(y+4)}$$

Now $\frac{2y+5}{(y+3)(y+4)} = \frac{2(-3)+5}{(y+3)(-3+4)} + \frac{2(-4)+5}{(-4+3)(y+4)}$

$$= \frac{-1}{y+3} + \frac{3}{y+4}$$

$$\therefore \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 + \frac{2}{y+3} - \frac{6}{y+4} = 1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}$$

$$\therefore \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}\,dx = \int x\,dx + 2\int \frac{1}{(x)^2+(\sqrt{3})^2}\,dx - 6\int \frac{1}{x^2+2^2}\,dx$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{6}{2}\tan^{-1}\frac{x}{2} + C$$

QNo 19 $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$

Sol. Put $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore I = \int \frac{dt}{(t+1)(t+3)}$$

Now Resolving into partial fractions by short cut method.

$$\frac{1}{(t+1)(t+3)} = \frac{1}{(t+1)(-1+3)} + \frac{1}{(-3+1)(t+3)}$$

$$= \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\therefore \int \frac{dt}{(t+1)(t+3)} = \frac{1}{2}\int \frac{1}{t+1}\,dt - \frac{1}{2}\int \frac{1}{t+3}\,dt$$

$$= \frac{1}{2}\log|t+1| - \frac{1}{2}\log|t+3| + C$$

$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)}\,dx = \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

QNo 20 $\int \frac{1}{x(x^4-1)}\,dx$

Sol. $$\frac{1}{x(x^4-1)} = \frac{x^3}{x \cdot x^3(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$$

Put $x^4 = t \Rightarrow 4x^3\,dx = dt$

$$\therefore \int \frac{1}{x(x^4-1)}\,dx = \int \frac{dt/4}{t(t-1)} = \frac{1}{4}\int \frac{dt}{t(t-1)}$$

$$= \frac{1}{4}\int \left(\frac{1}{t(0-1)} + \frac{1}{1(t-1)}\right) dt =$$

$$= \frac{1}{4}\left(-\log|t| + \log|t-1|\right) + c$$

$$= +\frac{1}{4}\log\left|\frac{t-1}{t}\right| + c$$

$$\therefore \int \frac{1}{x(x^4-1)}\,dx = \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + c$$

Q No 21. $\int \frac{1}{e^x-1}\,dx$

Sol. Put $e^x = t \Rightarrow e^x dx = dt \Rightarrow dx = \frac{dt}{e^x} = \frac{dt}{t}$

$$\therefore I = \int \frac{dt}{t(t-1)} = \int\left(\frac{-1}{t} + \frac{1}{t-1}\right)dt$$

$$= -\log|t| + \log|t-1| + c$$

$$= \log\left|\frac{t-1}{t}\right| + c$$

$$\therefore \int \frac{1}{e^x-1}\,dx = \log\left|\frac{e^x-1}{e^x}\right| + c.$$

Choose the correct option in question No. 22, 23.

Q No 22. $\int \frac{x}{(x-1)(x-2)}\,dx$

Sol. $$\int \frac{x}{(x-1)(x-2)}\,dx = \int\left(\frac{1}{(x-1)(1-2)} + \frac{2}{(2-1)(x-2)}\right)dx.$$

$$= \int\left(\frac{-1}{x-1} + \frac{+2}{x-2}\right)dx$$

$$= -\log|x-1| + 2\log|x-2| + c.$$

$$= \log\left|\frac{(x-2)^2}{x-1}\right| + c.$$

So B is correct option.

Q No 23. $\int \frac{dx}{x(x^2+1)} = \int \frac{x}{x^2(x^2+1)} dx$

Sol..

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$$I = \int \frac{\frac{1}{2}dt}{t(t+1)} = \frac{1}{2}\int \frac{1}{t(t+1)} dt$$

$$= \frac{1}{2}\int\left(\frac{1}{t(0+1)} + \frac{1}{(-1)(t+1)}\right) dt = \frac{1}{2}\int\left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \frac{1}{2}\left(\log|t| - \log|t+1|\right) + c$$

$$= \log|t^{1/2}| - \frac{1}{2}\log|t+1| + c$$

$$= \log|x| - \frac{1}{2}\log|x^2+1| + c$$

$\therefore$ Option (A) is correct option.

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PREPARED By:
Rupinder Kaur.
Lect. Maths.
GSSS BHARI (FGS)