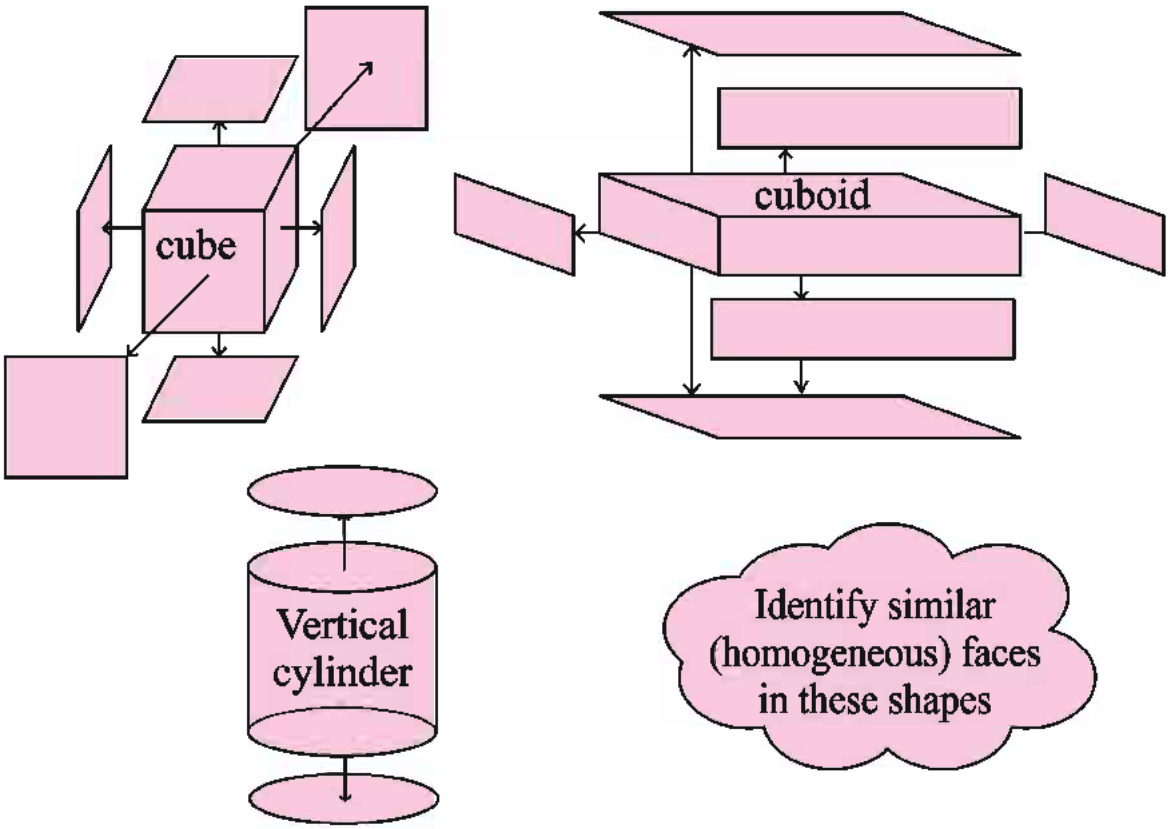
CHAPTER

15 Surface Area and Volume

15.1 You have learnt to identify three dimensional shapes in form of two dimensional. You have also learnt to form cube, cuboid, cylinders etc. through two dimensional lattice.

You have seen in some shapes there are two or more faces or surfaces as same (Congruent).



Note: This is compulsory that in vertical cylinder line segment joining centre of circular faces is perpendicular on base.

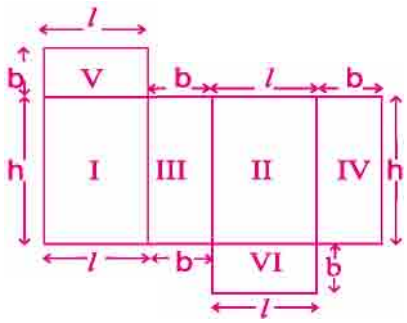
15.2 Surface Area of Cube and Cuboids –

Gopal and Ramesh want to make colourful cube and cuboids for decoration in house. Gopal prepared cube of side 4 cm. Ramesh prepared a cuboid which is 5 cm long, 4 cm broad and 3 cm high.

Now to make them attractive they want to stick colourful papers on them. Both are in this dilemma that how much paper should be bought from the market? Gopal said to Ramesh "We can add area of each face." Listening to their conversation Gopal's elder brother said this is called surface area means area covered by all faces of a solid is called its surface area.

15.2.1Cuboids

After cutting and opening a cuboidal box its lattice appears as shown in the figure. Write dimensions on each of the face. You will see three pairs of two-two congruent faces of a cuboid.



Surface area of the whole cuboid

= Area I + Area II + Area III + Area IV + Area V + Area VI

= h x l + h x l + b x h + b x h + l x b + l x b
= 2hl + 2bh + 2lb
= 2(lb + bh + hl)

Where l, b and h are length, breadth and height.

In this way total surface area of cuboid of Ramesh is 2 (5×4 +4×3 +3×5)

=2 (20+ 12 + 15)

=2 (47)

=94 square cm.

For painting all four walls of a cuboid ceiling and bottom (floor) of cuboid are to be left. Area of only four walls is to be calculated. This is called lateral surface area of a cuboid.

Means lateral surface area of a cuboid = total surface area – area of ceiling and floor

= 2 (lb + bh + hl) - 2lb
= 2lb + 2bh + 2hl - 2lb
= 2bh + 2hl
= 2x (l + b) x h Square unit

= Perimeter of base (floor) × height

Area of ceiling and floor is independent of height. It depends only on length and breadth.

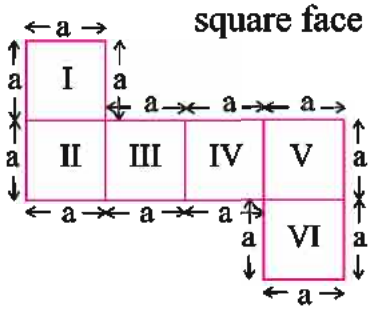
15.2.2 Total surface area of a cube

You know a cube is such cuboid whose all dimensions length, breadth and height are same. Area of one face of a cube is a^2. All six faces are squares. Each side is represented by a.

So total Surface area of a cube = 6 × area of square face

$= 6 \times a^2$

$= 6a^2$



In this way total surface area of cube of Gopal $= 6 \times 4^2$

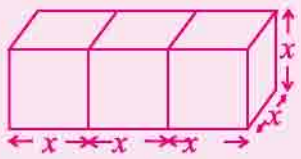
$= 6 \times 16$

$= 96$ square cm.

Do and learn

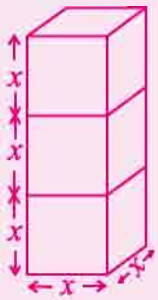
If a cuboid is formed by sticking three cubes of side x then what will be the dimensions of cuboid.

Condition I



Length = ---------
Breadth = ---------
Height = ---------

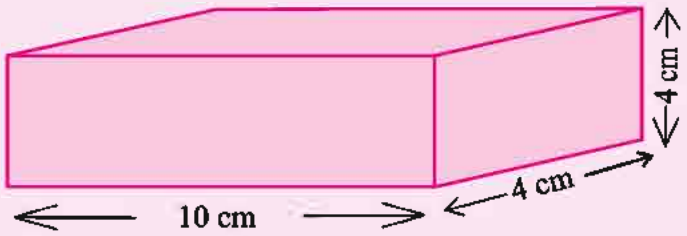
Condition II



Length = ---------
Breadth = ---------
Height = ---------

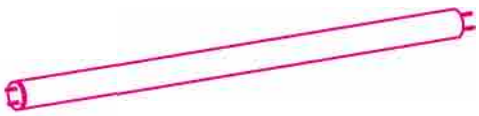
Do and learn

1. Find out surface area of a cube having side 3 cm. What will be the surface area of 5 such cubes?
2. What will be the total surface area of 5 cubes having side 3 cm? if they are sticked one after one then how much surface area will be decreased.
3. Find out lateral surface area of the given cuboid

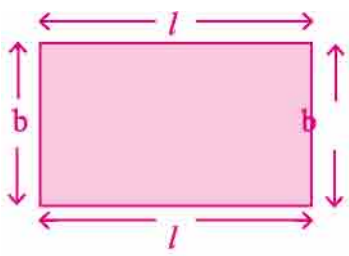


15.3 Surface area of cylinder

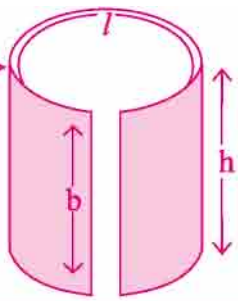
A tin of cold drink, pipe, tube of tube light, round pillar etc. are example of vertical cylinder. Area left after excluding area of both circular faces is called curved surface area. To determine its area we will do an activity.



Activity – Take a rectangular paper and mark length and breadth on its corner. According to the figure fold the given paper in direction of its length and stick its end with help of a tape without overlapping. In this way a curved surface of cylinder will be formed.

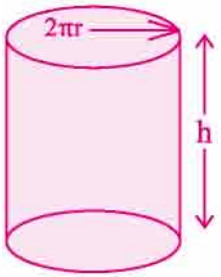


2πr (circumference of circular face of cylinder)



Height of cylinder

Length of rectangular paper (l) is transformed into circumference ($2\pi r$) of circular base of cylinder and breadth (b) has taken form of height (h) of cylinder.



In this way curved surface area of cylinder= area of rectangular paper $= L \times b$

$= 2\pi r \times h \quad = 2\pi rh$ square units

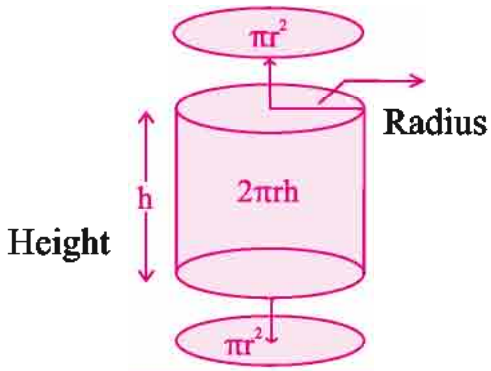
Now find curved surface area of cylinder by folding rectangular paper along its breadth.

If two congruent circular faces of cylinder are included with curved faces then it is called total surface area of cylinder. How its area will be calculated?

Total surface area of cylinder = area of curved surface + area of both circular faces

$= 2\pi rh + 2 \times \pi r^2$

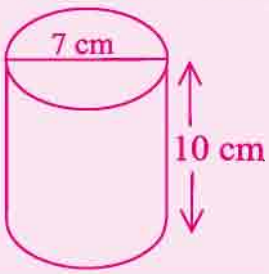
$= 2\pi r (h + r)$ Square unit



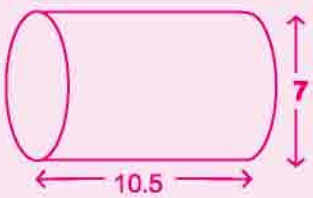
Do and learn

Determine total surface areas of following cylinders

(i)



(ii)



Example 1 Internal dimensions of cubical room of Vijay are 12m, 8m and 4m. He wishes to paint its all four walls. Determine cost of painting with Rs 5 per meter square. Tell him if he also wish to get the ceiling painted then cost will increase by what amount?

Solution Length of room (l) = 12m

Breadth (b) = 8m

Height (h) = 4m

lateral surface area (four walls) = perimeter of base × height

$= 2(l + b) \times h$

$= 2(12 + 8) \times 4$

$= 2 \times 20 \times 4$

$= 160\ m^2$

Expenditure of painting per meter square = Rs 5

Total expenditure of painting four walls of room = 160 × 5 = Rs 800

Area of ceiling = l × b = 12 × 8 = Rs 96

Extra expenses for painting ceiling = 96 × 5 = Rs 480

Means if Vijay will get the ceiling painted the expense will increase by Rs 480 and becomes Rs 1280.

Example 2 Manisha has a cuboidal tin having no lid whose side is 45cm. She wants to stick a colour paper on its external surface, find area of paper required?

Solution Total surface area of cube $= 6a^2$

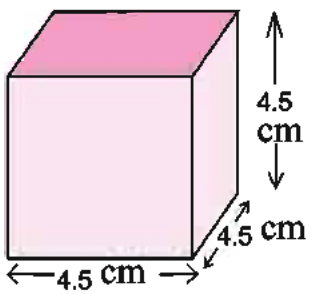
Area excluding top (lid) $= 6a^2 - a^2$

$= 5a^2$

$= 5 \times (45)^2$

$= 5 \times 2025$

$= 10125$ Square cm



Example 3 Find the height of such cylinder whose radius is 7 cm and total surface area is 968 square cm.

Solution Let the height of cylinder = h cm

Radius = 7cm

Total surface area of cylinder $= 2\ \pi r (h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (h + 7)$ Square cm

But according to the question total surface area is 968 cm^2
So

$$2 \times \frac{22}{7} \times 7 \times (h + 7) = 968$$

$$(h + 7) = \frac{968}{2 \times 22}$$

$$h + 7 = 22$$

$$h = 15 \text{ cm}$$

So height of cylinder = 15 cm

Example 4 A company labeled its product after packing it in a cylindrical tin of 20 cm height and 14 cm diameter. While labeling 2 cm distance left on both sides then determine area of label.

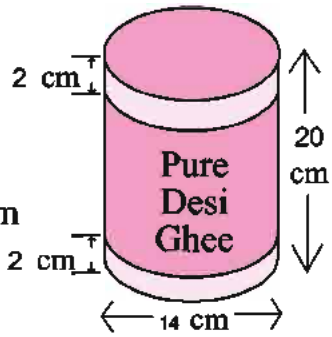
Solution Height of cylindrical tin = 20 cm

Height of applied label $= 20 - (2 \times 2)$
$= 16$ cm

Diameter of tin (2r) $= 14$cm

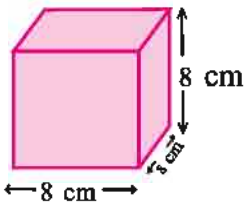
Area of label $= 2\pi rh$
$= 2 \times \frac{22}{7} \times 7 \text{ cm} \times 16 \text{ cm}$
$= 44 \times 16 \text{ cm}^2$
$= 704 \text{ cm}^2$

So area of label $= 704 \text{ cm}^2$

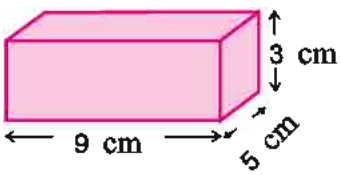


Exercise 15.1

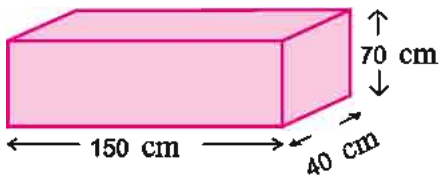
1. On the basis of given measure– determine surface area of cuboidal wooden log, cuboidal brick and box.



Wooden log



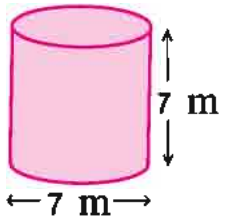
brick

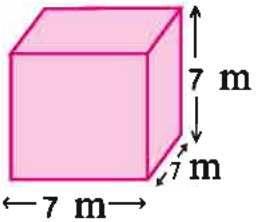


box

2. Determine side of a cube whose total surface area is 600 square cm.

3. In the given figure whose surface area is more? (π =22/7)





4. Find the area of curved surface if area of base of cylindrical tank is 176 Cm^2 and height is 30 cm.
5. Form a sheet of 8 square meter, a closed cylindrical tank is formed which has one meter height and 140 cm diameter. How much sheet will be left after making tank?
6. How many paint tins having spread capacity of 100 cm^2 will be required to paint external surface of box having dimensions 80 cm × 50 cm × 25 cm.
7. There are 25 cylindrical pillars in a building. Each pillar has radius of 28 cm and height of 4 m. find expenditure of painting curved surface area of all pillars at the rate of Rs. 8 per meter square.
8. Curved surface area of a hollow cylinder is 4224 cm^2. A rectangular sheet having width 33 cm is formed cutting it along its height. Find perimeter of sheet.
9. To make a road plain a roller has to complete 750 rounds. if the diameter of roller is 84 cm and length 1meter then find the area of road.
10. A cube is made by arranging 64 cubes having side of 1 cm, find total surface area of cube so formed.

15.4 Volume and Standard Unit

Place covered by a three dimensional object is called volume. Example volume of an almirah placed in a room is more than volume of flour tin.

You know we use squares of unit side to determine area of any shape. In this way to identify volume of a solid we have to determine number of unit cubes included in solid Volume is measured in unit given below.

1cm × 1cm × 1cm = 1 cm^3 or 1 cubic cm

1m × 1m × 1m = 1 m^3 or 1 cubic meter

1 cubic meter = 100 cm × 100 cm × 100cm = 1000000 cm^3

15.4.1 Volume of cube and cuboid

By arranging cubes of unit length find volume of cuboid so obtained-

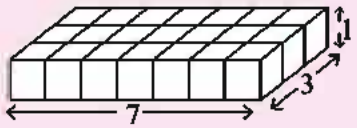
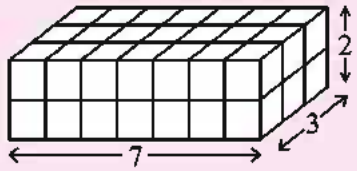
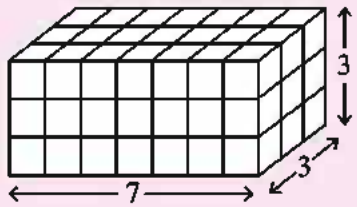
	Cuboid Volume	Length l	Breadth b	Height h	l×b×h
(i)	 21 cubic unit	7 unit	3 unit	1 unit	7×3×1 =21 cubic unit
(ii)	 42 cubic unit	7 unit	3 unit	2 unit	.....
(iii)	 63 cubic unit	7 unit	3 unit	3 unit	.....

Table 15.1

After completing table we reached to following conclusion

Volume of cuboid = length × breadth × height

In this way volume of 7 cm long, 5 cm broad and 2 cm high cuboid

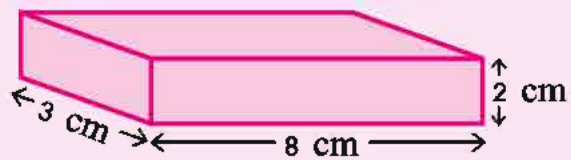
$$= \text{length} \times \text{breadth} \times \text{height}$$

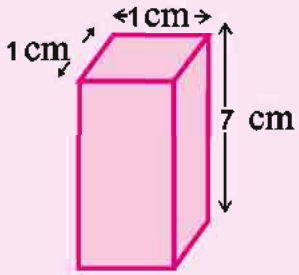
$$= 7\text{ cm} \times 5\text{ cm} \times 2\text{ cm}$$

$$= 70\text{ cm}^3$$

Do and learn

Find volume of cuboid on basis of given measures



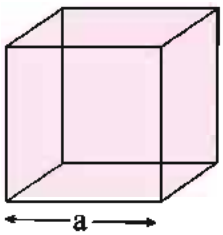


Volume of a cube

You know that cube is such cuboid whose length, breadth and height are same if each side is represented by a.

Volume of cube = side × side × side

Volume of cube = a × a × a = a^3 cubic unit



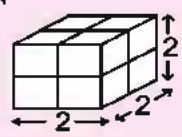
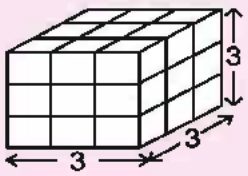
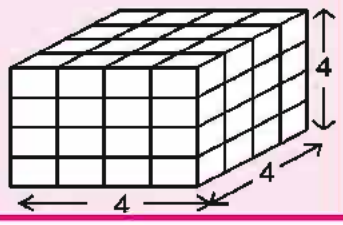
S.No	Cube	Number of unit cubes	Side	Side × side × side	$(side)^3$
(i)	N 	8 cubic unit	2 unit	2 unit × 2 unit × 2 unit	8 cubic Unit
(ii)		27 cubic unit	.....	.....	.....
(iii)		64 cubic unit	.....	.....	.....

Table 15.2

Do and learn

Determine volume of the cube having side

(i) 1.5 cm (ii) 4 m

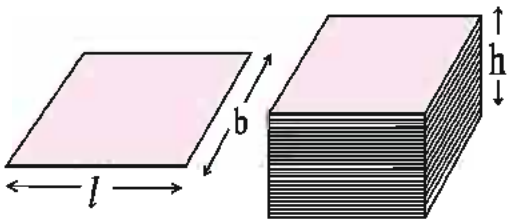
Take a rectangular paper sheet and measure its area. Now taking paper as base, place sheets of same measure on one another and make cuboid of desired height (to make it understandable paper rim can be used) multiply area of base and height and note down the result.

Determine volume of this cuboid by previously established formula. What conclusions can be drawn from both results?

Do you agree with this-

Volume of cuboid = area of base × height

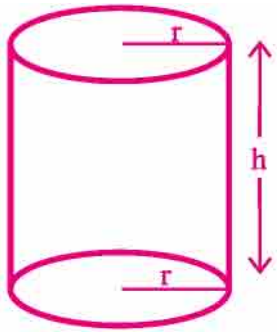
In this way volume of any shape can be determined



Note- here it is to be kept in mind that base and top of solid are congruent and line segment joining both centers are perpendicular to base and top.

15.4.2 Volume of a cylinder

Like cuboid in cylinder also base and top are congruent. And curved surface is perpendicular on base



Volume of cuboid = area of base × height

Volume of cylinder = area of base × height

$= \pi r^2 \times h$

$= \pi r^2 h$ cubic unit

Capacity

Place covered by a three dimensional object is called volume and quantity of liquid that can be filled in it is called capacity. In a cube of one cm, amount of liquid that can be filled is 1 ml. In this way its standard units are:

1 ml $= 1\ cm^3$

1litre $= 1000\ cm^3$ or 1000 milliliter

1 kilo litre = 1000 litre or $1\ m^3 = 1000$ litre

Example 5 determine height of a cuboid whose volume is 275 cm^3 and area of base is 25 cm^2

Solution volume of cuboid = area of base × height

$275\ cm^3 = 25\ cm^2 \times$ height

Height = $\frac{275\,cm^3}{25\,cm^2} = 11$ cm

Hint:
$m \times m \times m = m^3$

Example 6 Inside a cuboidal store house of dimension 60m × 40m × 30m how many cuboidal tin can be placed, if volume of tin is 0.8 m^3

Solution volume of a cuboidal tin $= 0.8\ m^3$

Volume of a store house $= 60m \times 40m \times 30m$

$= 72000\ m^3$

Number of tins that can be placed in store house are

$$= \frac{\text{volume of store house}}{\text{volume of a tin}}$$

$$= \frac{60m \times 40m \times 30m}{0.8m^3}$$

$$= \frac{72000\ m^3}{0.8\ m^3} = 90000$$

Example 7 if weight of one meter ice cube is 900 kilogram then what will be the weight of ice cube having side 50cm.

Solution volume of cube having side as 50 cm = $(50)^3\ cm^3$

$= 125000\ cm^3$

$= \frac{125000}{(100 \times 100 \times 100)}$ cubic meter

$= 0.125$ cubic meter

Weight of one cubic meter ice $= 900$ kg

Weight of 0.125 cubic meter ice $= 900 \times 0.125\ m^3 = 112.5$ kg

Example 8 A 4 cm high cylinder is formed after folding (without overlapping) an 11 cm × 4 cm rectangular paper. Find its volume.

Solution height of cylinder = 4cm

Circumference of base of cylinder = 11cm.

Means $2\pi r = 11$

$$2 \times \frac{22}{7} \times r = 11$$

$$r = \frac{7}{4} \text{ cm}$$

Volume of cylinder (v) = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4$$

$$= \frac{22 \times 7}{4} \text{ cm}^3 \qquad = 38.5 \text{ cm}^3$$

Example 9 curved surface areas of a cylinder is 440 m^2 whose height is 4 m. Find volume

Solution curved surface area of cylinder = 440 m^2

$$2\pi rh = 440 \text{ m}^2$$

$$2 \times \frac{22}{7} \times r \times 4 \text{ m} = 440 \text{ m}^2$$

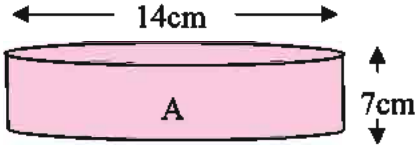
$$r = \frac{440 \times 7}{2 \times 22 \times 4} \text{ m} = \frac{35}{2} \text{ m}$$

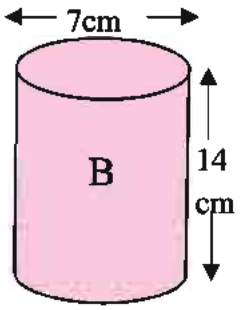
Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 4 \qquad = 3850 \text{ m}^3$$

Exercise 15.2

1. Dimensions of a cuboid are 60 cm × 54 cm × 30 cm. how many cubes of side 6cm can be placed in the cuboid?
2. How many wooden logs of side 6 cm can be cut from a 3m long, 50 cm broad and 25 cm high wooden pile.
3. Cylinder A has diameter of 14 cm and height of 7 cm. and cylinder B has diameter 7 cm and height 14cm. Without calculation tell volume of which cylinder is more? Verify the answer by calculation.





4. From a cylindrical milk tanker of radius 1.5 m and length 7m how many polythenes of one litre can be packed? (1 m^3= 1000 litre)
5. In what time a tap giving 60 litres of water per minute can fill a cylindrical tank of radius 3.5m and depth 3m.
6. Dimensions of a cuboidal ice is 50cm × 30cm × 20cm. Find its weight in kilogram. If weight of 1000cm^3 ice is 900 gram.
7. If side of a cube is doubled then
   (i) How many times its surface area will increase?
   (ii) How many times its volume will increase?
8. Volume of a cylindrical tank having 7 meter diameter is 770 cubic meters then find height of the tank.

We learnt

1. Area of a solid is sum total of areas of its faces.
2. Surface area of a cuboid = 2 (lb + bh + hl) where l, b, h are dimensions of cuboid
3. Surface area of cube = $6a^2$ where a = side of cube
4. Surface area of cylinder = $2\pi r(h+r)$
   R= radius of base of cylinder
   h = height of cylinder

If else not mentioned then surface area means total surface area.

5. Place covered by a three dimensional figure is called its volume.
6. Volume of a cuboid = length × breadth × height = l × b × h cubic unit (side of a cube = a unit)
7. Volume of cube = side × side × side = a^3 cubic unit
8. Volume of cylinder = $\pi r^3 h$ cubic unit
9. If base and top of a solid are congruent and line segment joining centers of top and base is perpendicular on base (cube, cuboid and cylinder) then
   Volume = area of base × height
10. (i) 1ml = 1cm^3
    (ii) 1 litre = 1000 ml
    (iii) 1kilolitre = 1000 litre or 1m^3= 1000 litre