Class-X Session 2022-23

Subject - Mathematics (Standard)

Sample Question Paper - 39

With Solution

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Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Real Number	6	2(Q6, 17)	1(Q19)		1(Q31)			6
2	Polynomials	20	1(Q1)			1(Q27)			4
3	Pair of Linear Equations in Two Variables		1(Q5)		1(Q21)	1(Q26)			6
4	Quadratic Equations		1(Q2)				1(Q32)		6
5	Arithmetic Progression							1(Q36)	4
6	Triangles	15	1(Q7, 9)		1(Q23)		1(Q33)		9
7	Circles		1(Q10)		1(Q24)	1(Q29)			6
8	Coordinate Geometry	6	1(Q3, 8)					1(Q37)	6
9	Introduction to Trigonometry	12	1(Q4)	1(Q20)	1(Q22)	1(Q28)			7
10	Some Applications of Trigonometry		1(Q18)					1(Q38)	5
11	Areas Related to Circles	10	2(Q11, 14)		1(Q25)				4
12	Surface Areas and Volumes		1(Q13)				1(Q34)		6
13	Statistics	11	1(Q12, 16)				1(Q35)		7
14	Probability		1(Q15)			1(Q30)			4
Total Marks (Total Questions)		80	18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 case based/integrated units of assessment (4 marks each) with sub parts of values of 1, 1 and 2 marks each respectively.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. The zeroes of the polynomial are
$p(x) = x^2 - 10x - 75$
(a) 5, – 15 (b) 5, 15 (c) 15, – 5 (d) – 5, – 15

2. If one root of $5x^2 + 13x + k = 0$ be the reciprocal of the other root, then the value of k is
(a) 0 (b) 1 (c) 2 (d) 5

3. If P (x, y) is any point on the line joining the points A $(a, 0)$ and B $(0, b)$, then
(a) $\frac{x}{b} + \frac{y}{a} = 1$ (b) $\frac{x}{a} - \frac{y}{b} = 1$ (c) $\frac{x}{a} + \frac{y}{b} = 1$ (d) $\frac{x}{b} - \frac{y}{a} = 1$

4. $\sin^2\theta + \text{cosec}^2\theta$ is always
(a) greater than 1 (b) less than 1 (c) greater than or equal to 2 (d) equal to 2

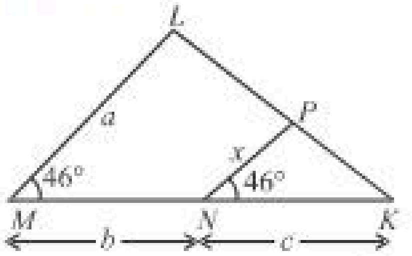
5. The pair of linear equations $x + 2y = 5$ and $3x + 12y = 10$ has
(a) unique solution (b) no solution (c) more than two solutions (d) infinitely many solutions

6. What is the largest number that divides 70 and 125, leaving remainders 5 and 8 respectively?
(a) 13 (b) 9 (c) 3 (d) 585

7. Which of the following statement is false?
(a) All isosceles triangles are similar. (b) All quadrilateral triangles are similar.
(c) All circles are similar. (d) None of the above

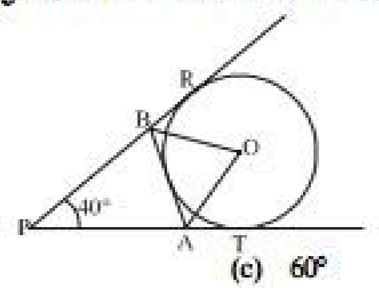
8. The point P on x-axis equidistant from the points A(–1, 0) and B(5, 0) is
(a) (2, 0) (b) (0, 2) (c) (3, 0) (d) (2, 2)

9. In the given figure, express x in terms of a, b and c.



(a) $x = \frac{ab}{a+b}$ (b) $x = \frac{ac}{b+c}$ (c) $x = \frac{bc}{b+c}$ (d) $x = \frac{ac}{a+c}$

10. In the figure, ΔAPB is formed by three tangents to the circle with centre O. If ∠APB = 40°, then the measure of ∠BOA is



(a) 50° (b) 55° (c) 60° (d) 70°

11. The area of a sector of angle p (in degrees) of a circle with radius R is

(a) $\frac{p}{360°} \times 2\pi R$ (b) $\frac{p}{180°} \times \pi R^2$ (c) $\frac{p}{720°} \times 2\pi R$ (d) $\frac{p}{720°} \times 2\pi R^2$

12. For the data $(2, 9, x+6, 2x+3, 5, 10, 5)$ if mean is 7, then mode is

(a) 3 (b) 5 (c) 9 (d) 10

13. If a sphere and a cube have equal surface areas, then the ratio of the diameter of the sphere to the edge of the cube is

(a) 1 : 2 (b) 2 : 1 (c) $\sqrt{\pi} : \sqrt{6}$ (d) $\sqrt{6} : \sqrt{\pi}$

14. If the sector of a circle of diameter 10 cm subtends an angle of 144° at the centre, then the length of the arc of the sector is

(a) 2π cm (b) 4π cm (c) 5π cm (d) 6π cm

15. An unbiased die is rolled twice. Find the probability of getting the sum of two numbers as a prime

(a) $\frac{3}{5}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{4}{5}$

16. Find the mean of the following frequency distribution.

Class Interval	0–10	10–20	20–30	30–40	40–50
Frequency	8	12	10	11	9

(a) 25.3 (b) 25.2 (c) 24 (d) 25.5

17. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then LCM (p, q) is equal to

(a) ab (b) a^2b^2 (c) a^3b^2 (d) a^3b^3

18. If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the sun's elevation is

(a) 60° (b) 45° (c) 30° (d) 90°

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion :** Denominator of 34.12345. When expressed in the form $\frac{p}{q}, q \neq 0$, is of the form $2^m \times 5^n$, where m, n are non-negative integers.

Reason : 34.12345 is a terminating decimal fraction.

20. **Assertion:** In a right angled triangle, if $\tan\theta = \frac{3}{4}$, the greatest side of the triangle is 5 units.

Reason: $(\text{greatest side})^2 = (\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

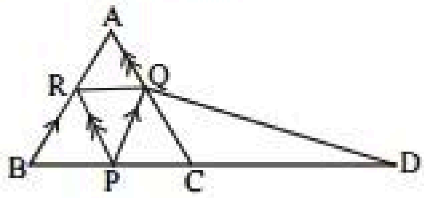
21. Solve the system of equations : $ax + by = 1$, $bx + ay = \frac{2ab}{a^2 + b^2}$.

22. Prove that $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\,\text{cosec}^2\theta$

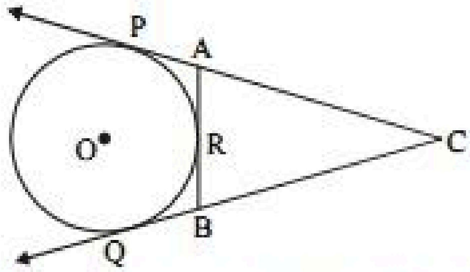
OR

If $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$, then find the value of θ.

23. In the given figure, $PQ \parallel BA$ and $PR \parallel CA$. If $PD = 12$ cm, find $BD \times CD$.



24. In the given figure, CP and CQ are tangents to a circle wih centre O. ARB is another tangent touching the cirlce at R. If CP = 11 cm and BC = 7 cm, then find the length of BR.



25. The area enclosed by the circumferences of two concentric circle is 346.5 cm^2. If the circumference of the inner circle is 88 cm, calculate the radius of the outer circle.

SECTION-C

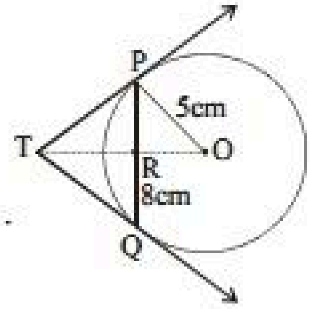
This section comprises of short answer type questions (SA) of 3 marks each.

26. Determine the values of m and n so that the following system of linear equations have infinite number of solutions :
$(2m-1)x + 3y - 5 = 0$
$3x + (n-1)y - 2 = 0$

27. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$.

28. Solve $\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1; (\theta < 90^\circ)$.

29. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see Fig.). Find the length of TP.



30. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

OR

Two different dice are tossed together. Find the probability that the product of the two numbers on the top of the dice is 6.

31. Find how many integers between 200 and 500 are divisible by 8.

OR

Find the greatest number of six digits exactly divisible by 18, 24 and 36.

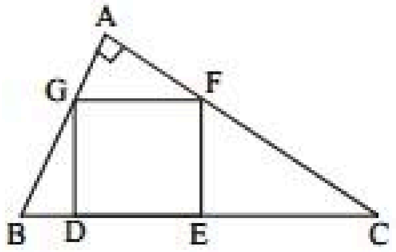
SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Solve the following quadratic equation for x :

$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

33. ΔABC is right-angled at A. $DEFG$ is a square as shown in the figure. Prove that $DE^2 = BD \times EC$.



OR

In ΔABC, D is the midpoint of BC and $AE \perp BC$. If $AC > AB$, show that

$$AB^2 = AD^2 - BC \cdot DE + \frac{1}{4}BC^2$$

34. If h, c, v are respectively the height, the curved surface area and the volume of a cone, prove that $3\pi vh^3 - c^2h^2 + 9v^2 = 0$.

35. **Find the mean of the following frequency distribution by Assumed Mean Method.**

Class Interval	Frequency
0–4	6
4–8	3
8–12	6
12–16	16
16–20	3
20–24	14
24–28	10
28–32	8

OR

Calculate the mean for the following frequency distribution (By step deviation method).

Class interval	0 – 80	80 – 160	160 – 240	240 – 320	320 – 400
frequency	22	35	44	25	24

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36. **Case - Study 1:** Read the following passage and answer the questions given below.
Your friend Veer wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less.

(i) Which day he takes 31 seconds to complete

(ii) Which day he taks 27 seconds to complete

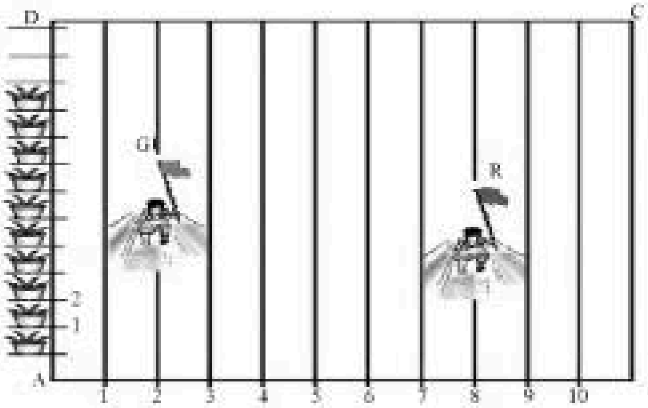
(iii) Find the time in 10th day

OR

Find the time in 15th day.

37. Case - Study 2: Read the following passage and answer the questions given below.

In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flowerpots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs 1/4 th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5 th distance AD on the eighth line and posts a red flag.



(i) Find the position of green flag

(ii) Find the position of red flag

(iii) What is the distance between both the flags?

OR

If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

38. Case - Study 3: Read the following passage and answer the questions given below.

A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

(i) What is the angle of elevation if they are standing at a distance of 42m away from the monument?

(ii) They want to see the tower at an angle of 60°. So, they want to know the distance where they should stand and hence find the distance.

(iii) If the altitude of the Sun is at 60°, then find the height of the vertical tower that will cast a shadow of length 20 m.

OR

The ratio of the length of a rod and its shadow is 1:1. Find the angle of elevation of the Sun.

Solution

SAMPLE PAPER-7

1. (c) We have, $p(x) = x^2 - 10x - 75 = x^2 - 15x + 5x - 75$
$= x(x-15) + 5(x-15) = (x-15)(x+5)$
$\therefore\ p(x) = (x-15)(x+5)$
So, $p(x) = 0$ when $x = 15$ or $x = -5$. Therefore required zeroes are 15 and –5.

2. (d) Let the roots be α and $\frac{1}{\alpha}$. Then, product of roots
$= \left(\alpha \times \frac{1}{\alpha}\right) = 1.$
So, $\frac{k}{5} = 1 \Rightarrow k = 5.$

3. (c) As the point P (x, y) lies on the line joining the points A (a, 0) and B (0, b), the points A, B and P are collinear
$\Rightarrow\ a(b-y) + 0(y-0) + x(0-b) = 0$
$\Rightarrow\ ab - ay - bx = 0 \Rightarrow bx + ay = ab$
$\Rightarrow\ \frac{x}{a} + \frac{y}{b} = 1$

4. (c) Hint: [at $\theta = 90°$]

5. (a) The pair of linear equations are
$x + 2y - 5 = 0$ and
$3x + 12y - 10 = 0$
Here, $a_1 = 1, b_1 = 2, c_1 = -5$ and $a_2 = 3, b_2 = 12,$
$c_2 = -10$. Now, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{2}{12} = \frac{1}{6}.$
As $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
So, pair of equations has a unique solution.

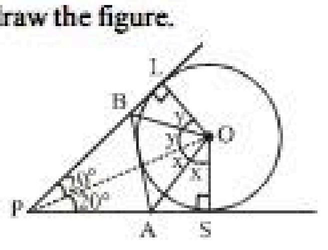
6. (a) Required number = H.C.F. {(70 – 5), (125 – 8)}
= H.C.F. (65, 117) = 13.

7. (a) Statement given in option (a) is false.

8. (a) $P(x, 0) = \left(\frac{5-1}{2}, 0\right) = (2, 0)$
[$\because$ A and B both lies on x-axis]
Three or more points lies in same line are called collinear.

9. (b) In ΔKPN and ΔKLM, we have
$\angle KNP = \angle KML = 46°$
$\angle K = \angle K$ (Common)
$\therefore\ \Delta KNP \sim \Delta KML$ (By $A\ A$ criterion of similarity)
$\Rightarrow \frac{KN}{KM} = \frac{NP}{ML} \Rightarrow \frac{c}{b+c} = \frac{x}{a}$

10. (d) We redraw the figure.



In ΔOPS, using Pythagoras theorem, $\angle POS = 70°$
and In ΔPOL, $\angle POL = 70°$
From figure, $2x + 2y = 140°$
$\angle BOA = x + y = 70°$

11. (d) Hint: Area of sector $= \frac{1}{2} r^2 \theta$

12. (c) Mean $= \frac{2+9+x+6+2x+3+5+10+5}{7} = 7$
$3x + 40 = 49 \Rightarrow x = 3$
$x + 6 = 9 \Rightarrow 2x + 3 = 9$
Data (2, 9, 9, 9, 5, 10, 5) and Mode = 9

13. (d) Let the diameter of the sphere be d units and the edge of the cube be a units, then
$4\pi\left(\frac{d}{2}\right)^2 = 6a^2 \Rightarrow \frac{d^2}{a^2} = \frac{6}{\pi} \Rightarrow \frac{d}{a} = \frac{\sqrt{6}}{\sqrt{\pi}}.$

14. (b) Length of arc $= \frac{114}{369}\pi 10 = 4\pi$

15. (b) The sum of the two numbers lies between 2 and 12.
So, the primes are 2, 3, 5, 7, 11.
No. of ways for getting 2 = (1, 1) = 1
No. of ways of getting 3 = (1, 2), (2, 1) = 2
No. of ways of getting 5
= (1, 4), (4, 1), (2, 3), (3, 2) = 4
No. of ways of getting 7
= (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3) = 6
No. of ways of getting 11 = (5, 6), (6, 5) = 2
No. of favourable ways = 1 + 2 + 4 + 6 + 2 = 15
No. of exhaustive ways = 6 × 6 = 36
$\therefore$ Probability of getting the sum as a prime
$= \frac{15}{36} = \frac{5}{12}$

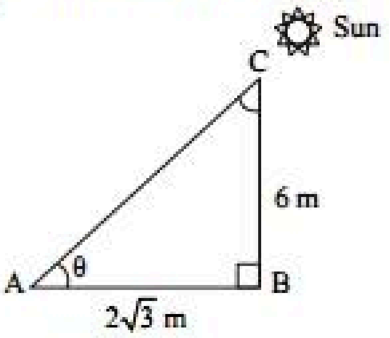
16. (b)

C.I.	x_i	f_i	$f_i x_i$
0 – 10	5	8	40
10 – 20	15	12	180
20 – 30	25	10	250
30 – 40	35	11	385
40 – 50	45	9	405
		50	1260

We have $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1260}{50} = 25.2$

17. (c) Given that, $p = ab^2 = a \times b \times b$
and $q = a^3b = a \times a \times a \times b$
$\therefore$ LCM (p, q) = LCM $(ab^2, a^3b) = a \times b \times b \times a \times a = a^3b^2$

18. **(a)** Suppose BC = 6 m be the height of the pole and AB = $2\sqrt{3}$ m be the length of the shadow on the ground. Suppose the sun's makes an angle θ on the ground.



Now, in ΔBAC

$$\tan\theta = \frac{BC}{AB}$$

$$\Rightarrow \quad \tan\theta = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60^\circ$$

$$\therefore \quad \theta = 60^\circ$$

So, sun's elevation is 60°.

19. **(a)** Reason is clearly true.

Again, $34.12345 = \frac{3412345}{100000} = \frac{682469}{20000} = \frac{682469}{2^5 \times 5^4}$

Its denominator is of the form $2^m \times 5^n$, where

$m = 5, n = 4$ are non-negative integers

∴ Assertion is true. Since, reason gives assertion

∴ (a) holds.

20. **(a)** Both Assertion and Reason are correct and Reason is the correct explanation of the assertion.

greatest side = $\sqrt{(3)^2 + (4)^2} = 5$ units.

21. $ax + by = 1$ ...(i)

$bx + ay = \frac{2ab}{a^2 + b^2}$ ...(ii)

On adding (i) and (ii), we get

$$(a+b)x + (a+b)y = 1 + \frac{2ab}{a^2+b^2}$$

$$\Rightarrow (a+b)(x+y) = \frac{(a+b)^2}{a^2+b^2}$$

$$\Rightarrow x + y = \frac{a+b}{a^2+b^2}$$ ...(iii) [½ Mark]

Subtracting (ii) from (i)

$$(a-b)x + (b-a)y = 1 - \frac{2ab}{a^2+b^2}$$

$$\Rightarrow (a-b)(x-y) = \frac{(a-b)^2}{a^2+b^2}$$

$$\Rightarrow x - y = \frac{a-b}{a^2+b^2}$$ ...(iv) [1 Mark]

Adding (iii) and (iv),

$$2x = \frac{a+b}{a^2+b^2} + \frac{a-b}{a^2+b^2} = \frac{2a}{a^2+b^2} \Rightarrow x = \frac{a}{a^2+b^2}$$

Subtracting (iv) from (iii)

$$2y = \frac{a+b}{a^2+b^2} - \frac{a-b}{a^2+b^2} = \frac{2b}{a^2+b^2} \Rightarrow y = \frac{b}{a^2+b^2}$$

[½ Mark]

Therefore, the solution is : $x = \frac{a}{a^2+b^2},\ y = \frac{b}{a^2+b^2}$

22. LHS = $\tan^2\theta + \cot^2\theta + 2 = (\tan^2\theta + 1) + (\cot^2\theta + 1)$ [½ Mark]

$= \sec^2\theta + \operatorname{cosec}^2\theta$

[$\because \tan^2\theta + 1 = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$]

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$ [1 Mark]

$= \frac{1}{\sin^2\theta\cos^2\theta} = \operatorname{cosec}^2\theta\ \sec^2\theta$ [½ Mark]

= RHS (Hence Proved).

OR

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Dividing Nr. and Dr. in L.H.S. by cosθ

$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$ [1 Mark]

By comparison, we get $\tan\theta = \sqrt{3}$ [1 Mark]

23. $BA \parallel PQ$

$\Rightarrow \quad BR \parallel PQ$

and $\quad PR \parallel CA$

$\Rightarrow \quad PR \parallel CQ$

In ΔBRD, $\quad BR \parallel PQ$

$\Rightarrow \quad \frac{BD}{PD} = \frac{RD}{QD}$ (corr. of Thales) ...(1)

In ΔRPD, $\quad PR \parallel CQ$ [1 Mark]

$\Rightarrow \quad \frac{RD}{QD} = \frac{PD}{CD}$ (corr. of Thales) ...(2)

From (1) and (2),

$$\frac{BD}{PD} = \frac{PD}{CD}$$

$\Rightarrow \quad PD^2 = BD \times CD$

$\therefore \quad BD \times CD = 144$ cm. [1 Mark]

24. CQ = CP = 11

[∵ length of tangents from C to the circle are equal]

∴ CQ = 11cm [1 Mark]

But BC = 7 cm ∴ 7 + BQ = 11 [∵ CQ = BC + BQ]

∴ BQ = 11 − 7 = 4

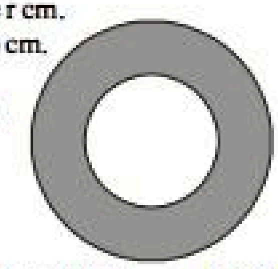
Since BR = BQ = 4

[∵ length of tangents from B to the circle are equal]

Thus BR = 4 cm. [1 Mark]

25. Let the radius of inner circle be r cm.

Then, its circumference = $(2\pi r)$ cm.



$\therefore 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = \left(88 \times \frac{7}{44}\right) = 14$cm.

$\therefore$ Radius of the inner circle is, r = 14 cm. [1 Mark]

Let the radius of the outer circle be R cm.

Then, area of the ring = $(\pi R^2 - \pi r^2)$ cm^2.

$= \left(\frac{22}{7}R^2 - 616\right)$ cm^2

$\therefore \frac{22}{7}R^2 - 616 = 346.5 \Rightarrow \frac{22}{7}R^2 = 962.5$

$\Rightarrow R^2 = \left(962.5 \times \frac{7}{22}\right) = 306.25 \Rightarrow R = \sqrt{306.25} = 17.5$cm.

[1 Mark]

26. $(2m-1)x + 3y - 5 = 0$ ...(i)

On comparing with $a_1x + b_1y + c_1 = 0$, we get :

$a_1 = 2m-1,\ b_1 = 3,\ c_1 = -5$

$3x + (n-1)y - 2 = 0$ ...(ii)

On comparing with $a_2x + b_2y + c_2 = 0$, we get :

$a_2 = 3,\ b_2 = n-1,\ c_2 = -2$ [1 Mark]

For a pair of linear equations to have infinite number of solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$ [1 Mark]

$\Rightarrow 2(2m-1) = 15$ and $5(n-1) = 6 \Rightarrow m = \frac{17}{4},\ n = \frac{11}{5}$

[1 Mark]

27. If α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$ then, $\alpha + \beta = -4$ and $\alpha\beta = 3$

Sum of zeroes $= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta} = \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$ [1 Mark]

Product of zeroes $= \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right) = 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\alpha\beta}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$ [1 Mark]

So, required polynomial

$= x^2 -$ (Sum of the zeroes) x + Product of the zeroes

$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3} = \frac{1}{3}(3x^2 - 16x + 16)$ [1 Mark]

28. $\frac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1 \Rightarrow \cos^2\theta - 3\cos\theta + 2 = \sin^2\theta$

[½ Mark]

$\Rightarrow \cos^2\theta - 3\cos\theta + 2 = 1 - \cos^2\theta$

$\Rightarrow 2\cos^2\theta - 3\cos\theta + 1 = 0$ [½ Mark]

$\Rightarrow 2\cos^2\theta - 2\cos\theta - \cos\theta + 1 = 0$

$\Rightarrow 2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$ [½ Mark]

$\Rightarrow (2\cos\theta - 1)(\cos\theta - 1) = 0$ [½ Mark]

Either $2\cos\theta - 1 = 0 \Rightarrow 2\cos\theta = 1 \Rightarrow \cos\theta = ½$

$\Rightarrow \cos\theta = \cos 60° \Rightarrow \theta = 60°$ or $\cos\theta - 1 = 0$

or $\cos\theta = 1 = \cos 0°$

$\theta = 0°$ (impossible)

$\therefore \theta = 60°$ [1 Mark]

29. Join OT. Let it intersect PQ at the point R. Then Δ TPQ is isosceles triangle and TO is the angle bisector of $\angle$ PTQ. So, OT $\perp$ PQ therefore, OT bisects PQ which gives PR = RQ = 4 cm.

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$ cm = 3 cm.

[1 Mark]

Now, $\angle$ TPR + $\angle$ RPO = 90° = $\angle$ TPR + $\angle$ PTR

[1 Mark]

So, $\angle$ RPO = $\angle$ PTR

Therefore, right triangle TRP is similar to the right triangle PRO (by AA similarity).

This gives $\frac{TP}{PO} = \frac{RP}{RO}$ i.e., $\frac{TP}{5} = \frac{4}{3}$ or $TP = \frac{20}{3}$ cm.

[1 Mark]

30. Total number of events = 52

In a pack of 52 playing cards, there are 2 red queens and 2 black queens, respectively. [1 Mark]

$\therefore$ Number of cards that are neither red nor queen $= 52 - (26 + 2) = 24$ [1 Mark]

Now, favourable number of events = 24

So, required probability $= \frac{24}{52} = \frac{6}{13}$ [1 Mark]

OR

P (Product = 6) = P [(1, 6), (2, 3), (3, 2), (6, 1)]

[1½ Marks]

Probability $= \frac{4}{6^2} = \frac{4}{36} = \frac{1}{9}$ [1½ Marks]

Hence, the probability that the product of the two numbers on the top of the dice is 6, will be $\frac{1}{9}$.

31. First number divisible by 8 between 200 and 500 is 208.

If form an A.P. = 208, 216, 224, ..., 496.

Here, $a_n = 496$, a = 208, d = 8 [1 Mark]

$a_n = a + (n-1)d$

$496 = 208 + (n-1)\,8$

$(n-1)8 = 496 - 208$

$(n-1) = \frac{288}{8}$ [1 Mark]

$n - 1 = 36$

$n = 37$ [1 Mark]

OR

LCM of 18, 24 and 36 is 72.

$72\overline{)999999}(13888$

$\underline{999936}$

63 [1 Mark]

$\therefore$ Required number = 9,99,936. [2 Marks]

32. $\because \quad 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$ [1 Mark]

$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$(\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$ [1½ Marks]

So, $\sqrt{3}x + 2 = 0$ or $4x - \sqrt{3} = 0$

Therefore, $x = -\dfrac{2\sqrt{3}}{3}, \dfrac{\sqrt{3}}{4}$ [1½ Marks]

Hence, the values of x are $\dfrac{-2\sqrt{3}}{3}$ and $\dfrac{\sqrt{3}}{4}$. [1 Mark]

33. **Given:** ΔABC is right-angled at A. $DEFG$ is a square [½ Mark]

To prove: $DE^2 = BD \times EC$.

Proof : In ΔAGF and ΔDBG

$\angle GAF = \angle BDG = 90°$

$\angle AGF = \angle DBG$ (corres. angles)

$\therefore \quad \Delta AGF \sim \Delta DBG$ ... (i) (AA similarity) [1½ Marks]

In ΔAGF and ΔEFC,

$\angle GAF = \angle CEF = 90°$

$\angle AFG = \angle FCE$ (corres. angles)

$\therefore \quad \Delta AFG \sim \Delta EFC$ ... (ii) (AA similarity)

From (i) and (ii) [1½ Marks]

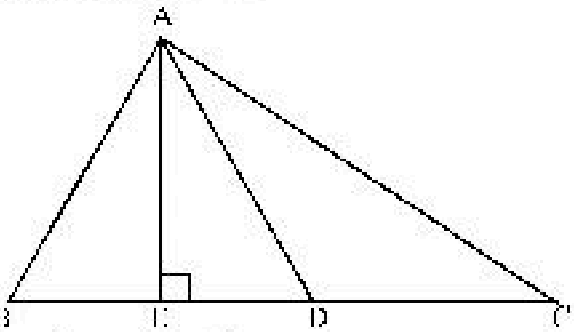
$\Delta DBG \sim \Delta EFC$

$\therefore \quad \dfrac{DB}{EF} = \dfrac{DG}{EC}$, But $EF = DG = DE$ (sides of a square)

$\therefore \quad \dfrac{DB}{DE} = \dfrac{DE}{EC} \quad \therefore \quad DE^2 = DB \times EC$ [1½ Marks]

OR

In ΔAEB, $\angle AEB = 90°$



$\therefore AB^2 = AE^2 + BE^2$ ... (i) [1 Mark]

In ΔAED, $\angle AED = 90°$.

$\therefore AD^2 = (AE^2 + DE^2)$

$\Rightarrow AE^2 = (AD^2 - DE^2)$ [½ Mark]

$\therefore AB^2 = (AD^2 - DE^2) + BE^2$ [using (i)] [1 Mark]

$= (AD^2 - DE^2) + (BD - DE)^2$ [½ Mark]

$= (AD^2 - DE^2) + \left(\dfrac{1}{2}BC - DE\right)^2$ [1 Mark]

$= AD^2 + \dfrac{1}{4}BC^2 - BC \cdot DE$ [1 Mark]

34. We know that $\ell = \sqrt{r^2 + h^2}$, $v = \dfrac{1}{3}\pi r^2 h$, $c = \pi r \ell$ [1½ Marks]

LHS. $= 3\pi v h^3 - c^2 h^2 + 9v^2$

$= 3\pi\left(\dfrac{1}{3}\pi r^2 h\right)h^3 - (\pi r \ell)^2 h^2 + 9\left(\dfrac{1}{3}\pi r^2 h\right)^2$ [1 Mark]

$= 3\pi\left(\dfrac{1}{3}\pi r^2 h\right)h^3 - \pi^2 r^2 (r^2 + h^2) \times h^2 + 9 \times \dfrac{1}{9}\pi^2 r^4 h^2$

$\left(\because \ \ell = \sqrt{h^2 + r^2}\right)$ [1½ Marks]

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0 =$ **RHS** [1 Mark]

35.

Class interval	Mid values (x_i)	Frequency (f_i)	Deviation (d_i) = $x_i - A$	$f_i d_i$
0–4	2	6	–12	–72
4–8	6	3	–8	–24
8–12	10	6	–4	–24
12–16	14 (A)	16	0	0
16–20	18	3	4	12
20–24	22	14	8	112
24–28	26	10	12	120
28–32	30	8	16	128
		$\Sigma f_i = 66$		$\Sigma f_i d_i = 252$

[2 Marks] [1 Mark]

Mean $= A + \dfrac{\sum f_i d_i}{\sum f_i} = 14 + \dfrac{252}{66} = 14 + 3.818 =$ **17.818** [2 Marks]

OR

Class Interval	Mid - value (x_i)	f_i	$u_i = (x_i - A)/h$	$f_i u_i$
0 – 80	40	22	–2	–44
80 – 160	120	35	–1	–35
160 – 240	200 (A)	44	0	0
240 – 320	280	25	1	25
320 – 400	360	24	2	48
Total		$\Sigma f_i = 150$		$\Sigma f_i u_i = -6$

Mean $(\bar{x}) = A + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times h = 200 + \left(\dfrac{-6}{150}\right) \times 80$

$= 200 - \dfrac{2 \times 8}{5} = 200 - \dfrac{16}{5} = \dfrac{1000 - 16}{5} = \dfrac{984}{5} = 196.8$

Remarks : (i) The mean obtained by all the there methods is the same.

(ii) The assumed mean method and step-deviation method are just simplified forms of the direct method.

(iii) If x_i and f_i are sufficiently small, then the direct method is an appropriate choice.

(iv) If x_i and f_i are numerically large numbers, then we can go for the assumed mean method or step-deviation method.

(v) If the class sizes are unequal, and x_i are large numberically, then we can go for the step-deviation method.

36. (i) $a_n = 51 - (n-1)2 = 31 \Rightarrow n = 11$ [1 Mark]

(ii) $a_n = 51\,(n-1)\,2 = 27 \Rightarrow n = 13$ [1 Mark]

(iii) $a_{10} = 51 - (10-1)2 = 33$ [2 Marks]

OR

$a_{15} = 51 - (15-1)2 = 23$ [2 Marks]

37. (i) (2, 25) $\left[\because x = 2, y = \dfrac{1}{4} \times 100 = 25\right]$ [1 Mark]

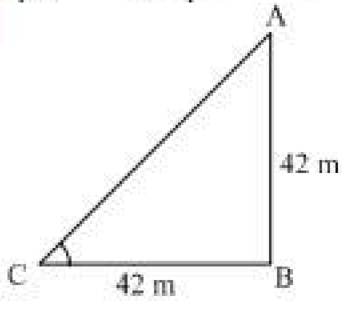
(ii) (8, 20) $\left[\because x = 8, y = \dfrac{1}{5} \times 100 = 20\right]$ [2 Marks]

(iii) $\sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36 + 25} = \sqrt{61}$

OR

$\left(\dfrac{8+2}{2}, \dfrac{25+20}{2}\right) = (5, 22.5)$ [2 Marks]

38. (i)



Since h = 42

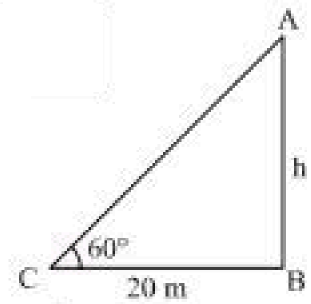
d = 42

So $\theta = 45°$ [1 Mark]

(ii) $\tan 60° = \dfrac{42}{x}$

$x = \dfrac{42}{\sqrt{3}} = \dfrac{42}{3}\sqrt{3} = 14\sqrt{3} = 24.24$ m [1 Mark]

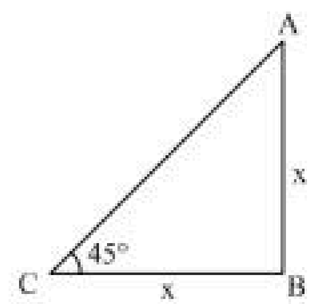
(iii)



$\tan 60° = \dfrac{h}{20}$

$h = 20\sqrt{3}$ m [2 Mark]

OR



Since ratio of length of a rod: Shadow of rod = 1:1

so Angle = 45° [2 Marks]