Exercise 12.2

Answer 1E.

(A) The cost of a theater ticket is a scalar because the sum of cost of tickets is obtained by simple algebraic sum of cost of two individual tickets. i.e., it has magnitude only.

(B) The current in a river means the speed of the water in the river and the direction in which the water is flowing. So it is a vector quantity.

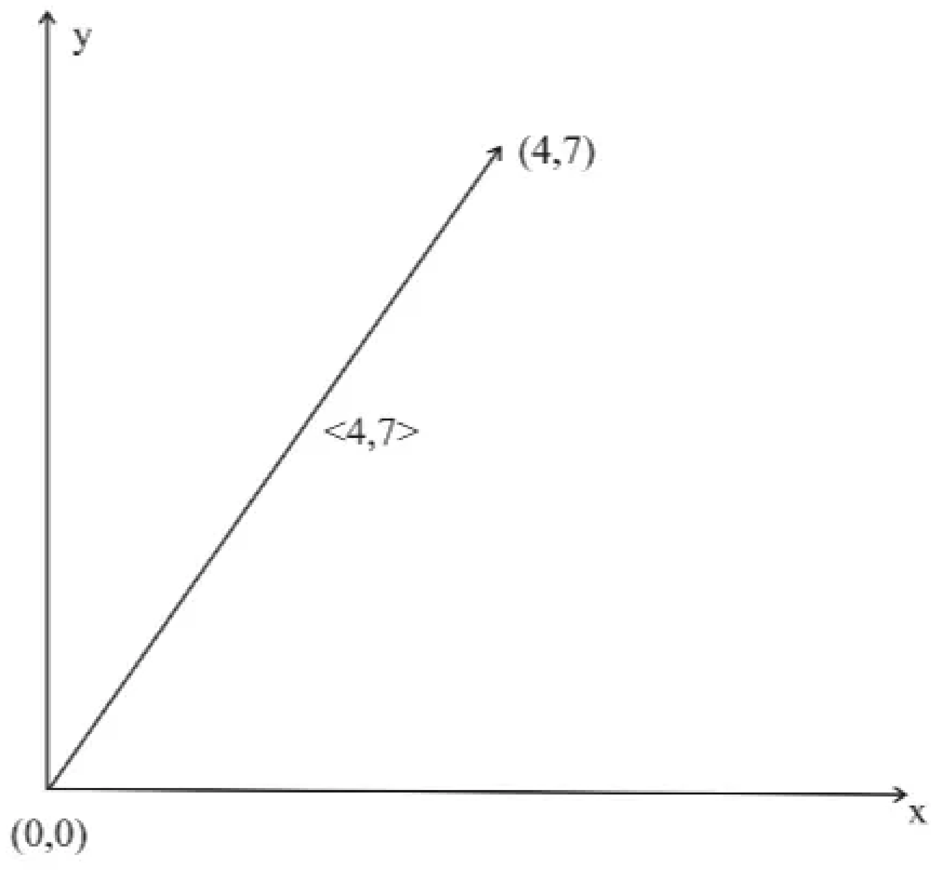
(C) The initial flight path from Houston to Dallas is a vector as the direction of path from Houston to Dallas is clearly mentioned.

(D) The population of the world is a scalar. Since the sum of population at two different places can be obtained by their Algebraic sum. i.e., it has magnitude only.

Answer 2E.

The point (4, 7) represents a point which has x-coordinate 4 and y – coordinate 7. And the vector <4, 7> is a vector with component 4 and 7 i.e. it represents the vector with initial point at the origin of rectangular co-ordinate system and giving the terminal point at point (4, 7)

i.e. the point $\boxed{\text{(4,7) represents the terminal point of vector <4,7>}}$



Answer 3E.

Two vectors are said to be equal if they have same magnitude and direction. In a parallelogram opposite sides are parallel and equal and diagonals bisect each other.

Therefore,

$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$\overrightarrow{DA} = \overrightarrow{CB}$$

$$\overrightarrow{DE} = \overrightarrow{EB}$$

$$\overrightarrow{CE} = \overrightarrow{EA}$$

Answer 4E.

The visual representation of the vectors is shown in Figure-1. The resultant vector $\overrightarrow{AC}$ is shown by red color.

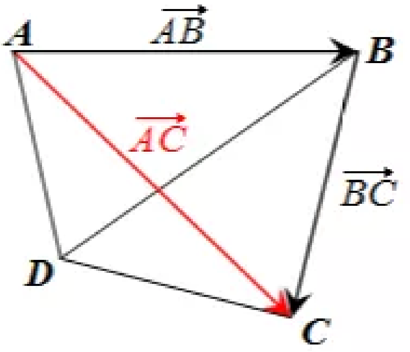


Figure-1

(b)

Consider the following combination of vectors:

$$\overrightarrow{CD} + \overrightarrow{DB}$$

The objective is to write the above combination of vectors as a single vector.

Notice that, the terminal point of first vector $\overrightarrow{CD}$ is D which is the initial point of the second vector $\overrightarrow{DB}$.

The initial point of the first vector $\overrightarrow{CD}$ is C, and the terminal point of second vector $\overrightarrow{DB}$ is B, so the resultant vector is $\overrightarrow{CB}$.

Therefore, the combination of above vectors as a single vector will be,

$$\overrightarrow{CD} + \overrightarrow{DB} = \boxed{\overrightarrow{CB}}$$

The visual representation of the vectors is shown in Figure-2. The resultant vector $\overrightarrow{CB}$ is shown by red color.

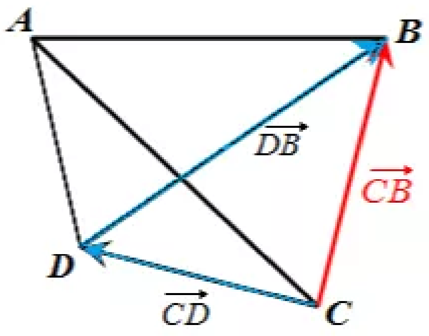


Figure-2

(c)

Consider the following combination of vectors:

$\overrightarrow{DB} - \overrightarrow{AB}$.

The objective is to write the above combination of vectors as a single vector.

The difference $\overrightarrow{DB} - \overrightarrow{AB}$ of two vectors implies that,

$$\overrightarrow{DB} - \overrightarrow{AB} = \overrightarrow{DB} + \left(-\overrightarrow{AB}\right).$$

That is, the resultant vector is the sum of the vectors $\overrightarrow{DB}$ and $-\overrightarrow{AB}$.

The vector $-\overrightarrow{AB}$ has the same length as $\overrightarrow{AB}$ but points in the opposite direction. So, the initial point of this vector is *B* and terminal point is *A*.

Notice that, the terminal point of first vector $\overrightarrow{DB}$ is *B* which is the initial point of the second vector $-\overrightarrow{AB}$.

The initial point of the first vector $\overrightarrow{DB}$ is *D*, and the terminal point of second vector $-\overrightarrow{AB}$ is *A*, so the resultant vector is $\overrightarrow{DA}$.

Therefore, the combination of above vectors as a single vector will be,

$$\begin{aligned}\overrightarrow{DB} - \overrightarrow{AB} &= \overrightarrow{DB} + \left(-\overrightarrow{AB}\right) \\ &= \boxed{\overrightarrow{DA}}.\end{aligned}$$

The visual representation of the vectors is shown in Figure-3. The resultant vector $\overrightarrow{DA}$ is shown by red color.

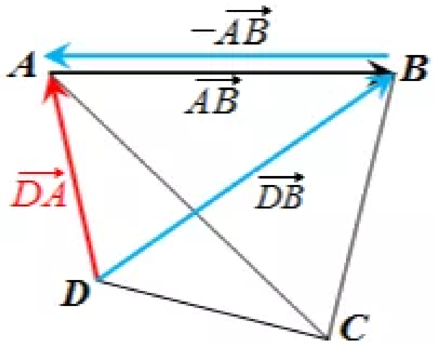


Figure-3

(d)

Consider the following combination of vectors:

$\overrightarrow{DC}+\overrightarrow{CA}+\overrightarrow{AB}$.

The objective is to write the above combination of vectors as a single vector.

Notice that, the terminal point of first vector $\overrightarrow{DC}$ is C which is the initial point of second vector $\overrightarrow{CA}$, and the terminal point of second vector $\overrightarrow{CA}$ is A which is the initial point of third vector $\overrightarrow{AB}$.

The initial point of the first vector $\overrightarrow{DC}$ is D, and the terminal point of second vector $\overrightarrow{CA}$ is A, so the resultant vector of $\overrightarrow{DC}+\overrightarrow{CA}$ is $\overrightarrow{DA}$. So, the combination of $\overrightarrow{DC}+\overrightarrow{CA}$ will be,

$\overrightarrow{DC}+\overrightarrow{CA}=\overrightarrow{DA}$.

The initial point of the above resultant vector $\overrightarrow{DA}$ is D, and the terminal point of third vector $\overrightarrow{AB}$ is B, so the resultant vector of $\overrightarrow{DA}+\overrightarrow{AB}$ is $\overrightarrow{DB}$. So, the combination of $\overrightarrow{DA}+\overrightarrow{AB}$ will be,

$\overrightarrow{DA}+\overrightarrow{AB}=\overrightarrow{DB}$.

Hence, finally, the combination of above vectors as a single vector will be,

$$\begin{aligned}\overrightarrow{DC}+\overrightarrow{CA}+\overrightarrow{AB}&=\left(\overrightarrow{DC}+\overrightarrow{CA}\right)+\overrightarrow{AB}\\&=\overrightarrow{DA}+\overrightarrow{AB}\\&=\boxed{\overrightarrow{DB}}.\end{aligned}$$

Therefore, the combination of above vectors as a single vector will be,

$\overrightarrow{DC}+\overrightarrow{CA}+\overrightarrow{AB}=\boxed{\overrightarrow{DB}}$.

The visual representation of the vectors is shown in Figure-4. The resultant vector $\overrightarrow{DB}$ is shown by red color.

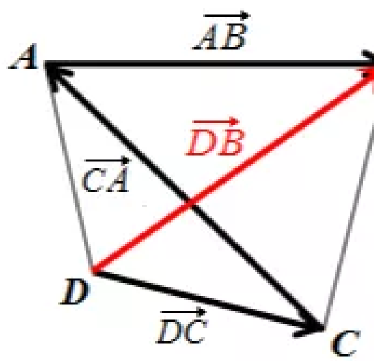


Figure-4

Answer 5E.

(a)

The objective is to draw the vectors $\mathbf{u}+\mathbf{v}$.

Arrange the vectors **u** and **v** such that the terminal point of **u** is at the initial point of **v**.

Then, the sum $\mathbf{u}+\mathbf{v}$ is the vector from the initial point of **u** to the terminal point of **v**.

The sketch of the resultant vector $\mathbf{u}+\mathbf{v}$ is shown by black color in Figure-2.

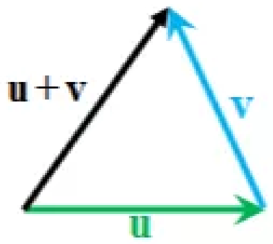


Figure-2

(b)

The objective is to draw the vectors $\mathbf{u}+\mathbf{w}$.

Arrange the vectors **u** and **w** such that the terminal point of **u** is at the initial point of **w**.

Then, the sum $\mathbf{u}+\mathbf{w}$ is the vector from the initial point of **u** to the terminal point of **w**.

The sketch of the resultant vector $\mathbf{u}+\mathbf{w}$ is shown by black color in Figure-3.

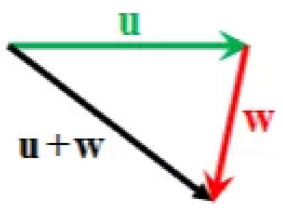


Figure-3

(c)

The objective is to draw the vectors $\mathbf{v}+\mathbf{w}$.

Arrange the vectors **v** and **w** such that the terminal point of **v** is at the initial point of **w**.

Then, the sum $\mathbf{v}+\mathbf{w}$ is the vector from the initial point of **v** to the terminal point of **w**.

The sketch of the resultant vector $\mathbf{v}+\mathbf{w}$ is shown by black color in Figure-4.

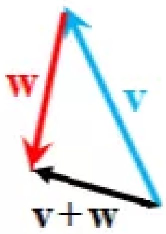


Figure-4

(d)

The objective is to draw the vectors $\mathbf{u}-\mathbf{v}$.

The difference $\mathbf{u}-\mathbf{v}$ of two vectors implies that,

$\mathbf{u}-\mathbf{v}=\mathbf{u}+(-\mathbf{v})$.

That is, the resultant vector is the sum of the vectors $\mathbf{u}$ and $-\mathbf{v}$.

The vector $-\mathbf{v}$ has the same length as **v** but points in the opposite direction.

So, to draw the vectors $\mathbf{u}-\mathbf{v}$, first draw the negative of **v**, $-\mathbf{v}$, and then add **u**.

Arrange the vectors **u** and $-\mathbf{v}$ such that the terminal point of **u** is at the initial point of $-\mathbf{v}$.

Then, the vector $\mathbf{u}-\mathbf{v}$ is the vector from the initial point of **u** to the terminal point of $-\mathbf{v}$.

The sketch of the resultant vector $\mathbf{u}-\mathbf{v}$ is shown by black color in Figure-5.

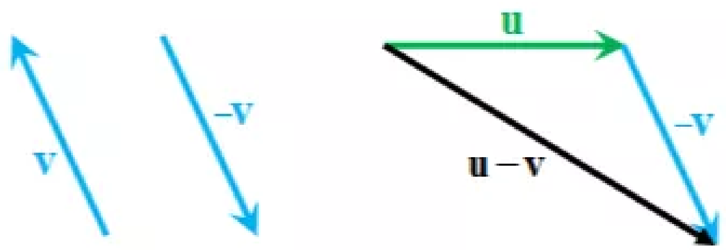


Figure-5

(e)

The objective is to draw the vectors $\mathbf{v}+\mathbf{u}+\mathbf{w}$.

Arrange the vectors **v**, **u** and **w** such that the terminal point of **v** is at the initial point of **u**, and, the terminal point of **u** is at the initial point of **w**.

Then, the sum $\mathbf{v}+\mathbf{u}+\mathbf{w}$ is the vector from the initial point of **v** to the terminal point of **w**.

The sketch of the resultant vector $\mathbf{v}+\mathbf{u}+\mathbf{w}$ is shown by black color in Figure-6.

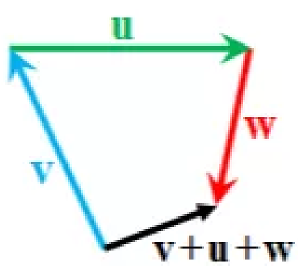


Figure-6

(f)

The objective is to draw the vectors $\mathbf{u}-\mathbf{w}-\mathbf{v}$.

The difference $\mathbf{u}-\mathbf{w}-\mathbf{v}$ implies that,

$$\mathbf{u}-\mathbf{w}-\mathbf{v}=\mathbf{u}+(-\mathbf{w})+(-\mathbf{v}).$$

That is, the resultant vector is the sum of the vectors $\mathbf{u},\ -\mathbf{w}$ and $-\mathbf{v}$.

The vector $-\mathbf{w}$ has the same length as **w** but points in the opposite direction.

The vector $-\mathbf{v}$ has the same length as **v** but points in the opposite direction.

So, to draw the vectors $\mathbf{u}-\mathbf{w}-\mathbf{v}$, first draw the negative of **w**, $-\mathbf{w}$, then draw the negative of **v**, $-\mathbf{v}$, and then add **u**.

Arrange the vectors $\mathbf{u},\ -\mathbf{w}$ and $-\mathbf{v}$ such that the terminal point of **u** is at the initial point of $-\mathbf{w}$, and, the terminal point of $-\mathbf{w}$ is at the initial point of $-\mathbf{v}$.

Then, the vector $\mathbf{u}-\mathbf{w}-\mathbf{v}$ is the vector from the initial point of **u** to the terminal point of $-\mathbf{v}$.

The sketch of the resultant vector $\mathbf{u}-\mathbf{w}-\mathbf{v}$ is shown by black color in Figure-7.

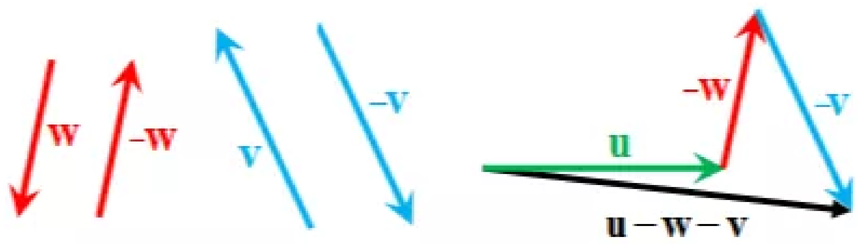
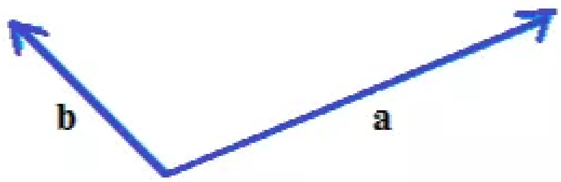


Figure-7

Answer 6E.

Consider the figure:



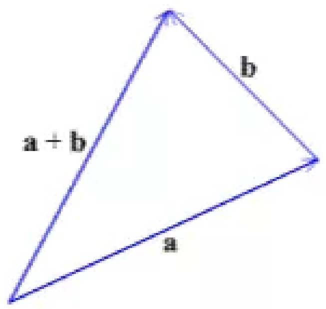
(a)

Consider the vector $\mathbf{a}+\mathbf{b}.$

To draw this vector, use the given figure.

Start by translating **b** and place its tail at the tip of **a**. Then, draw vector **a** + **b** starting at the initial point of **a,** and ending at the terminal point of the copy of **b**.

The below figure shows the vector $\mathbf{a}+\mathbf{b}.$



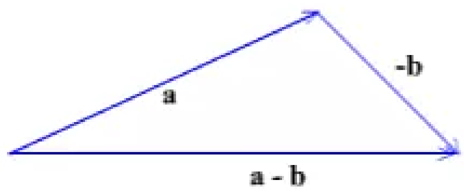
(b)

Consider the vector $\mathbf{a}-\mathbf{b}.$

To draw this given vector, use the given figure.

We know that **a** – **b** is equivalent to **a** + (–**b**). So, we start by drawing the negative of **b** and place its tail at the tip of **a**. Then, draw vector **a** + (–**b**) starting at the initial point of **a** and ending at the terminal point of the copy of **b**.

The below figure shows the vector $\mathbf{a}-\mathbf{b}.$



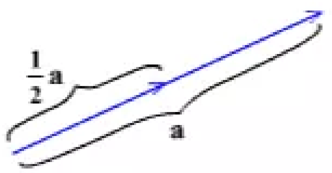
(c)

Consider the vector $\frac{1}{2}\mathbf{a}$.

To draw this given vector, use the given figure.

If *c* is a scalar and **a** is a vector, then the scalar multiple *c***a** is the vector whose length is $|c|$ times the length of **a** and whose direction is the same as **a** if $c > 0$ and is opposite to **a** if $c < 0$.

The below figure shows the vector $\frac{1}{2}\mathbf{a}$.



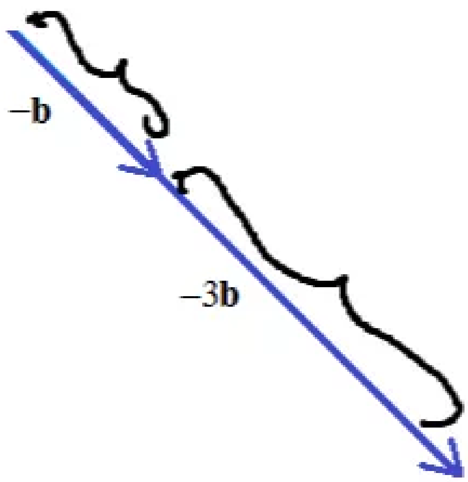
(d)

Consider the vector $-3\mathbf{b}$.

To draw this given vector, use the given figure.

If *c* is a scalar and **b** is a vector, then the scalar multiple *c***b** is the vector whose length is $|c|$ times the length of **b** and whose direction is the same as **b** if $c > 0$ and is opposite to **b** if $c < 0$. Here, we note that $c < 0$ and so we have *c***b** in the direction opposite to **b**.

The below figure shows the vector $-3\mathbf{b}$.



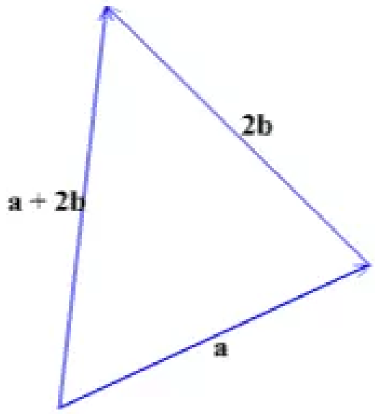
(e)

Consider the vector $\mathbf{a}+2\mathbf{b}$.

To draw this given vector, use the given figure.

Start by drawing 2**b.** Now, draw vector **a** + 2**b** starting at the initial point of **a** and ending at the terminal point of the copy of 2**b**.

The below figure shows the vector $\mathbf{a}+2\mathbf{b}$.



(f)

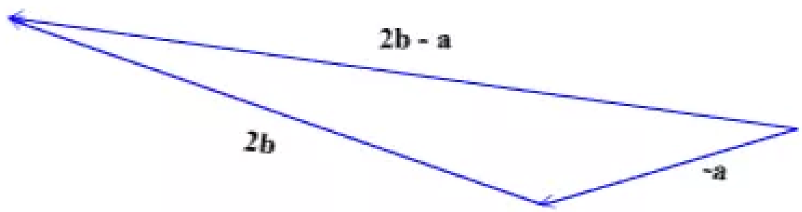
Consider the vector $2\mathbf{b}-\mathbf{a}$.

To draw this given vector, use the given figure.

Start by drawing 2**b**. We know that that $2\mathbf{b}-\mathbf{a}$ is equivalent to 2**b** + (–**a)**.

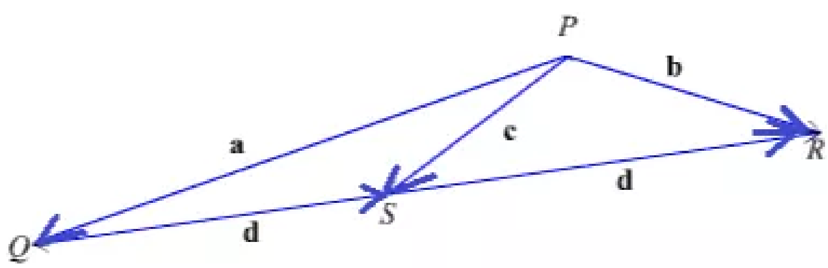
Now, draw vector 2**b** + (–**a)** starting at the initial point of 2**b** and ending at the terminal point of the copy of –**a**.

The below figure shows the vector $2\mathbf{b}-\mathbf{a}$.



Answer 7E.

Consider the figure:



In the figure the tip of $\mathbf{c}$ and the tail of $\mathbf{d}$ are both the midpoint of QR.

To express $\mathbf{c}$ and $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, use the definition of Vector Addition.

Definition of Vector Addition:

If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the sum $\mathbf{u}+\mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.

Consider triangle *PQS*.

By the definition of Vector Addition, get $\overrightarrow{PQ} + \overrightarrow{QS} = \overrightarrow{PS}$.

That is the initial point of $\mathbf{d}$ is at the terminal point of $\mathbf{a}$, then the sum $\mathbf{a}+\mathbf{d}$ is the vector from the initial point of $\mathbf{a}$ to the terminal point of $\mathbf{d}$.

Then

a + **d** = **c**. ...... (1)

Since $\overrightarrow{QR}$ is divided into two equal halves, and $\overrightarrow{QS} = \mathbf{d}$.

Now, consider the triangle *PSR*.

On applying the definition of Vector Addition, get

c + **d** = **b**. ...... (2)

That is the initial point of $\mathbf{d}$ is at the terminal point of $\mathbf{c}$, then the sum $\mathbf{c}+\mathbf{d}$ is the vector from the initial point of $\mathbf{c}$ to the terminal point of $\mathbf{d}$.

From equation (1), get

$\mathbf{d} = \mathbf{c} - \mathbf{a}$. ...... (3)

Substitute the value of $\mathbf{d}$ in the equation (2), get

$$\begin{aligned} \mathbf{c}+\mathbf{d} &= \mathbf{b} \\ \mathbf{c}+\mathbf{c}-\mathbf{a} &= \mathbf{b} \\ 2\mathbf{c} &= \mathbf{a}+\mathbf{b} \\ \mathbf{c} &= \frac{1}{2}(\mathbf{a}+\mathbf{b}) \end{aligned}$$

Therefore, $\mathbf{c} = \frac{1}{2}(\mathbf{a}+\mathbf{b})$.

Now substitute the value of $\mathbf{c}$ in the equation (3), get

$$\begin{aligned}\mathbf{d} &= \mathbf{c}-\mathbf{a} \\ &= \frac{1}{2}(\mathbf{a}+\mathbf{b})-\mathbf{a} \\ &= -\frac{1}{2}\mathbf{a}+\frac{1}{2}\mathbf{b} \\ &= \frac{1}{2}(\mathbf{b}-\mathbf{a})\end{aligned}$$

Therefore, $\mathbf{d}=\frac{1}{2}(\mathbf{b}-\mathbf{a})$.

Hence the required values of $\mathbf{c}$ and $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ are

$$\boxed{\mathbf{c}=\frac{1}{2}(\mathbf{a}+\mathbf{b}),\mathbf{d}=\frac{1}{2}(\mathbf{b}-\mathbf{a})}.$$

Answer 8E.

We have $|\mathbf{u}|=|\mathbf{v}|=1$. From the given figure, we can say that $\mathbf{u}$ is perpendicular to $\mathbf{v}$. As $\mathbf{u}+\mathbf{v}+\mathbf{w}=0$, we can say that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ form the three sides of a triangle. Since $\mathbf{u}$ is perpendicular to $\mathbf{v}$, by Pythagorean rule we get $|\mathbf{w}| = \sqrt{|\mathbf{u}|^2+|\mathbf{v}|^2}$ or $\boxed{|\mathbf{w}| = \sqrt{2}}$.

Answer 11E.

Consider the vectors $A(-1,3)B(2,2)$.

Find a vector $\mathbf{a}$ with representation given by the directed line segment $\overrightarrow{AB}$.

The points $A(x_1,y_1)$ and $B(x_2,y_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ are as follows:

$a=\langle x_2-x_1,y_2-y_1\rangle$

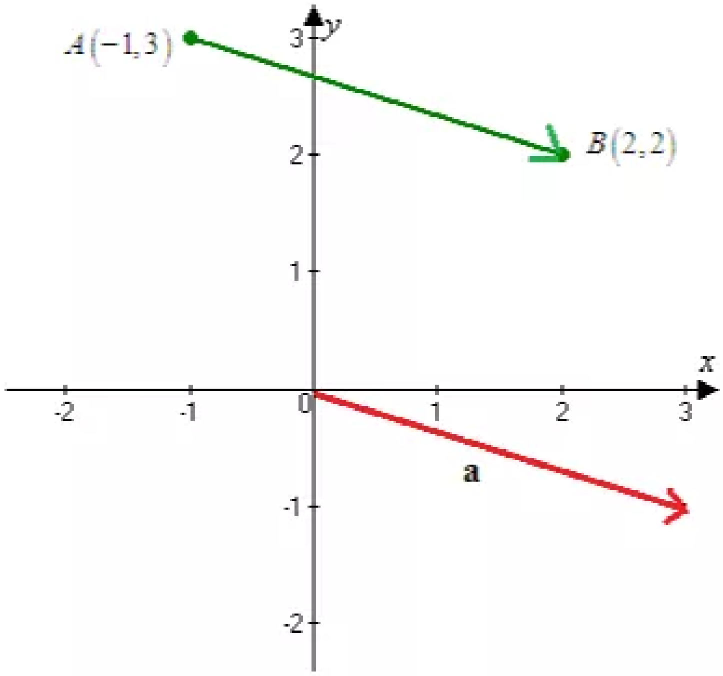
$=\langle 2-(-1),2-3\rangle$ Since $A(x_1,y_1)=A(-1,3)$ and $B(x_2,y_2)=B(2,2)$

$=\langle 3,-1\rangle$

$=3\mathbf{i}-\mathbf{j}$

Thus, the vector $\mathbf{a}$ with representation given by the directed line segment $\overrightarrow{AB}$ is $\boxed{\langle 3,-1\rangle}$.

Sketch the graph of the vector $\mathbf{a} = \langle 3, -1 \rangle$ as shown below:



Answer 12E.

Consider the vectors $A(2,1)$ and $B(0,6)$

The objective is to find the vector **a** with representation given by the directed line segment $\overrightarrow{AB}$ and draw line segment $\overrightarrow{AB}$.

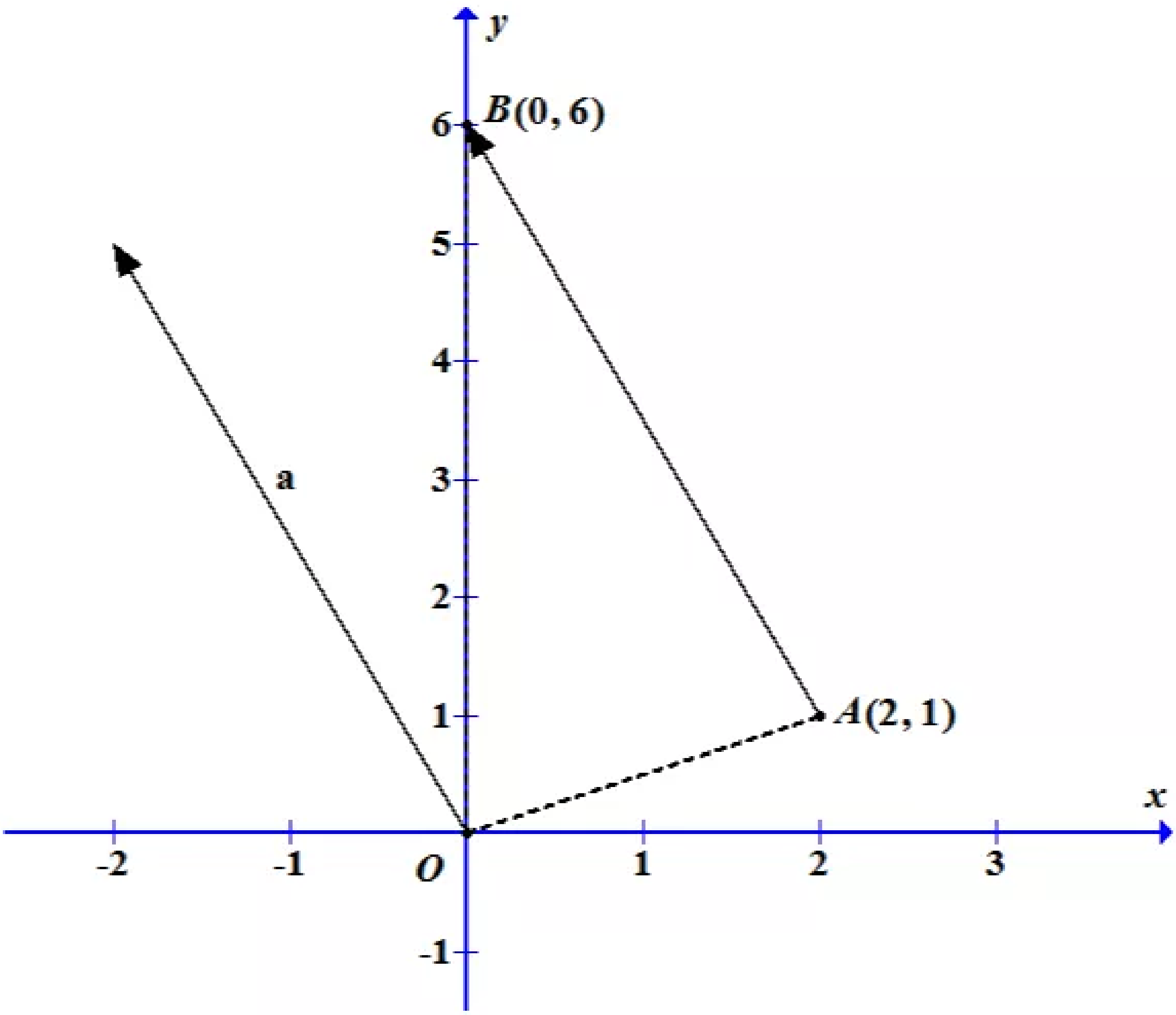
The vector **a** with representation given by the directed line segment $\overrightarrow{AB}$ is given by,

$$\begin{aligned}\mathbf{a} &= \langle x_2 - x_1 \quad y_2 - y_1 \quad z_2 - z_1 \rangle \\ &= (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}\end{aligned}$$

The vector corresponding to $\overrightarrow{AB}$ is given by,

$$\begin{aligned}\mathbf{a} &= \overrightarrow{AB} \\ &= \langle 0 - 2 \quad 6 - 1 \rangle \\ &= \langle -2 \quad 5 \rangle \\ &= -2\mathbf{i} + 5\mathbf{j}\end{aligned}$$

A vector **a** represented by the directed line segment $\overrightarrow{AB}$ joining the points $A(2,1)$ and $B(0,6)$, and starting at the origin is as shown below.



Thus, the vector corresponding to $\overrightarrow{AB}$ is $\boxed{\mathbf{a} = -2\mathbf{i} + 5\mathbf{j}}$.

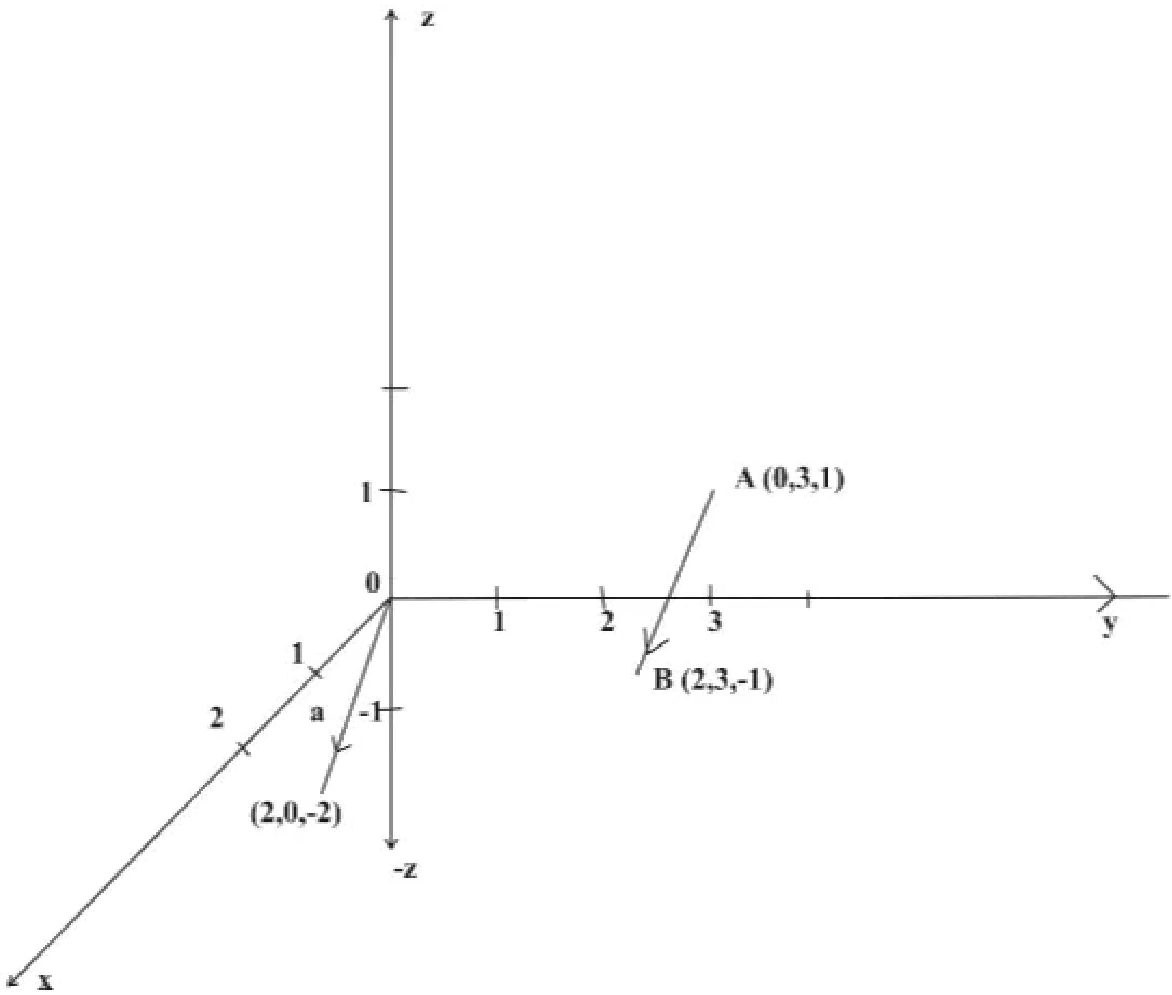
Answer 13E.

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is $\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

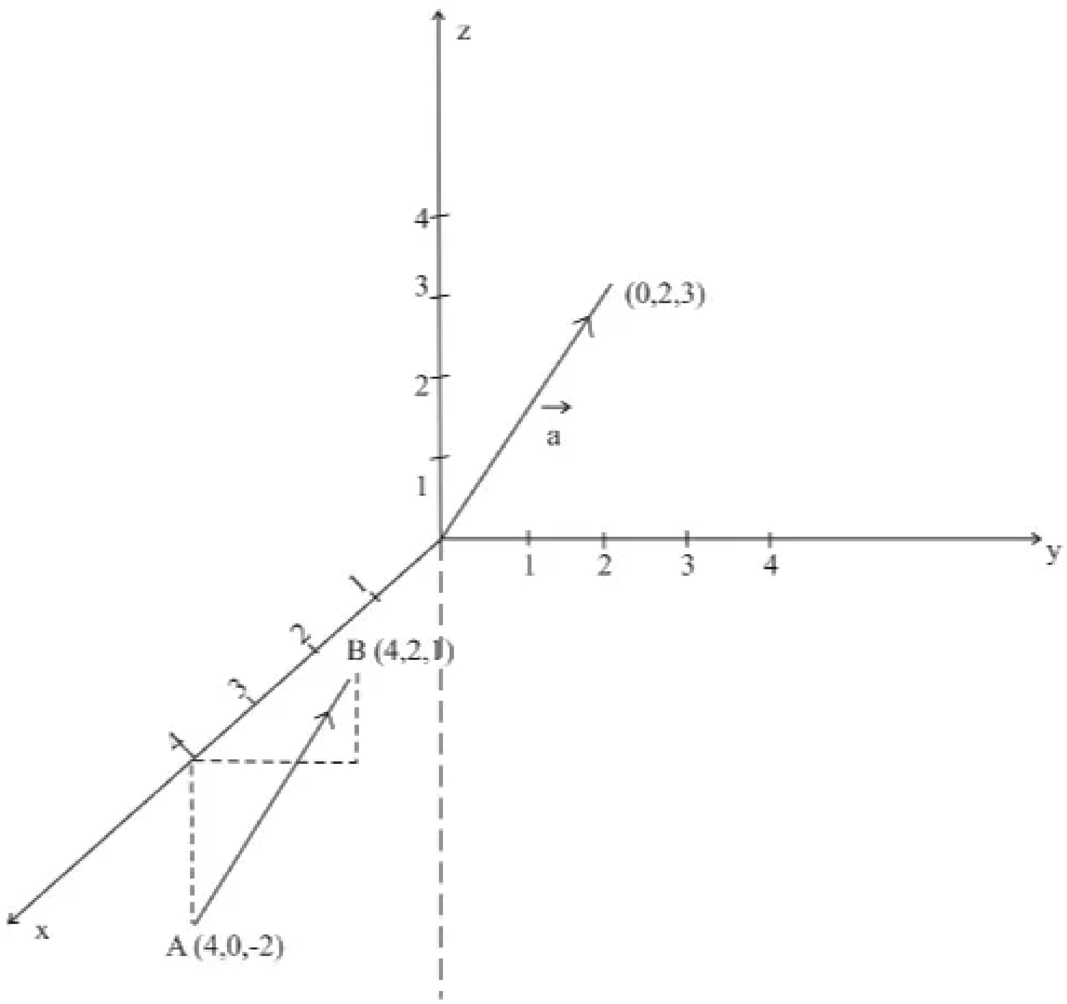
Given $A=(0,3,1)$ and $B=(2,3,-1)$

The vector **a** with representation $\overrightarrow{AB}$ is

$\mathbf{a}=\langle 2-0,3-3,-1-1\rangle=\langle 2,0,-2\rangle$



Answer 14E.



$\vec{a}$ is the equivalent representation of $\overrightarrow{AB}$ starting at origin where $\boxed{\vec{a} = \langle 0, 2, 3 \rangle}$

Answer 15E.

Consider the vectors $\langle -1, 4 \rangle, \langle 6, -2 \rangle$.

The objective is to find the sum of these two vectors.

Take $\mathbf{a} = \langle -1, 4 \rangle, \mathbf{b} = \langle 6, -2 \rangle$.

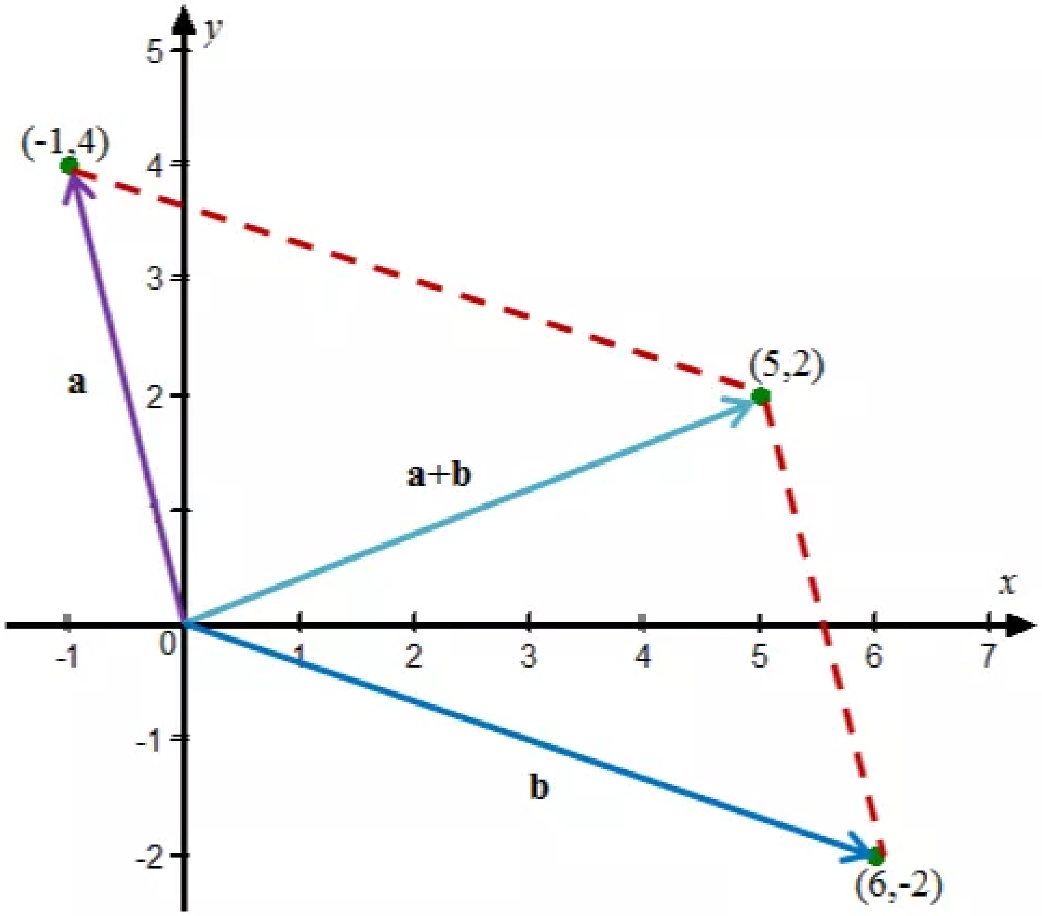
Now their sum

$$\begin{aligned} \mathbf{a} + \mathbf{b} &= \langle -1, 4 \rangle + \langle 6, -2 \rangle \\ &= \langle -1 + 6, 4 - 2 \rangle \\ &= \langle 5, 2 \rangle \end{aligned}$$

Hence, the sum of the vectors $\mathbf{a} = \langle -1, 4 \rangle$ and $\mathbf{b} = \langle 6, -2 \rangle$ is $\boxed{\langle 5, 2 \rangle}$.

Now geometrically this sum is the diagonal of a parallelogram whose adjacent sides are

$\mathbf{a} = \langle -1, 4 \rangle, \mathbf{b} = \langle 6, -2 \rangle.$



Answer 16E.

The geometric representation of the sum of vectors is shown in Figure-1. The vector $\langle 3,-1\rangle$ is shown by red color, the vector $\langle -1,5\rangle$ is shown by blue color, and their vector sum $\langle 2,4\rangle$ is represented by black color.

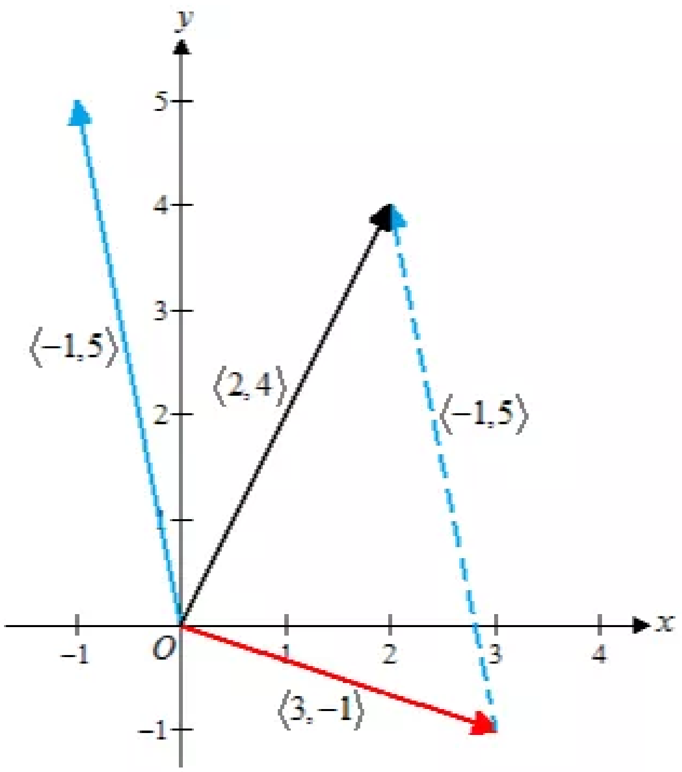


Figure-1

Answer 17E.

The geometric representation of the sum of vectors is shown in Figure-1. The vector $\langle 3,0,1\rangle$ is shown by red color, the vector $\langle 0,8,0\rangle$ is shown by blue color, and their vector sum $\langle 3,8,1\rangle$ is represented by black color.

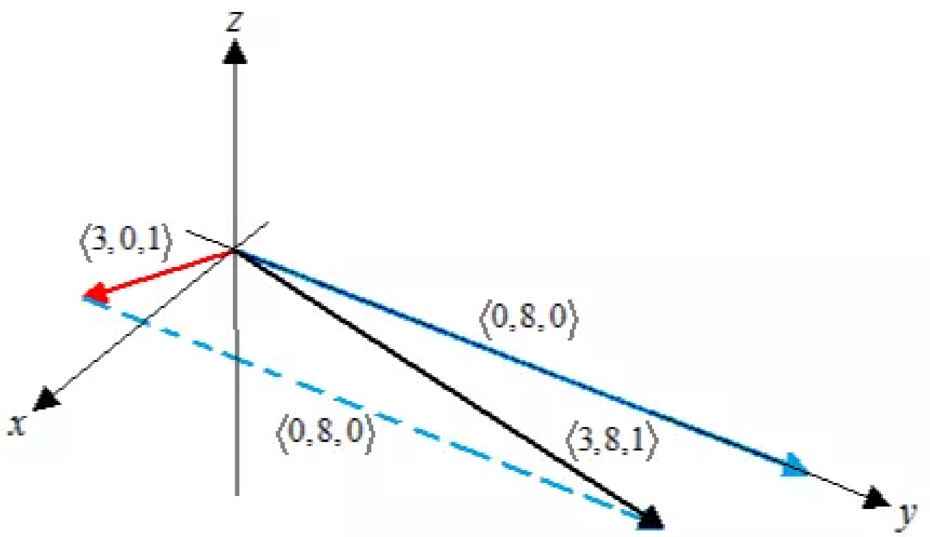


Figure-1

Answer 18E.

The geometric representation of the sum of vectors is shown in Figure-1. The vector $\langle 1,3,-2\rangle$ is shown by red color, the vector $\langle 0,0,6\rangle$ is shown by blue color, and their vector sum $\langle 1,3,4\rangle$ is represented by black color.

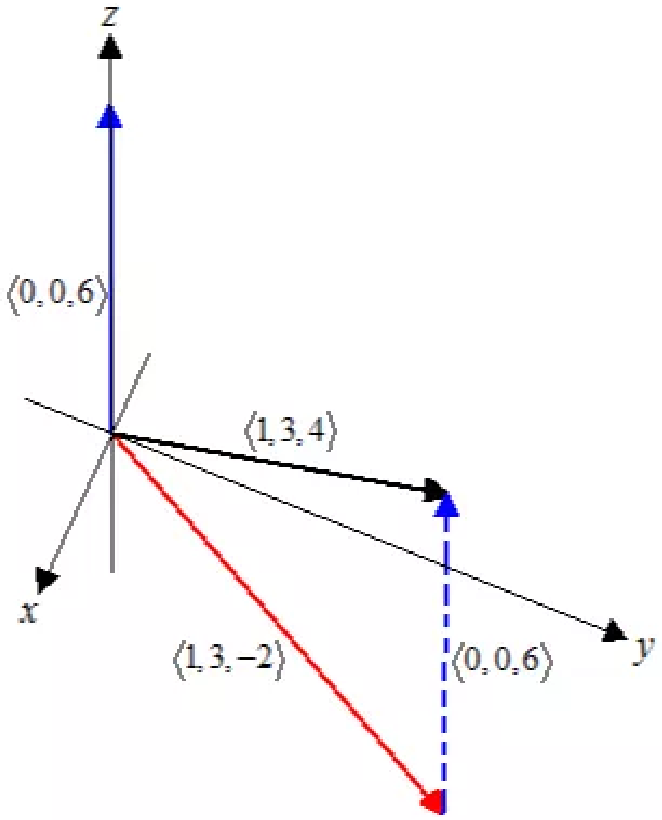


Figure-1

Answer 19E.

Consider the vectors $\mathbf{a}=\langle 5,-12\rangle$ and $\mathbf{b}=\langle -3,-6\rangle$.

Find $\mathbf{a}+\mathbf{b}$.

If $\mathbf{a}=\langle a_1,a_2\rangle$ and $\mathbf{b}=\langle b_1,b_2\rangle$ then $\mathbf{a}+\mathbf{b}=\langle a_1+b_1,a_2+b_2\rangle$.

Here, $\langle a_1,a_2\rangle=\langle 5,-12\rangle$ and $\langle b_1,b_2\rangle=\langle -3,-6\rangle$.

$$\begin{aligned}\mathbf{a}+\mathbf{b}&=\langle 5,-12\rangle+\langle -3,-6\rangle\\&=\langle 5-3,-12-6\rangle\\&=\langle 2,-18\rangle\end{aligned}$$

Therefore, $\mathbf{a}+\mathbf{b}=\boxed{\langle 2,-18\rangle}$.

Find $2\mathbf{a}+3\mathbf{b}$.

First find $2\mathbf{a}$.

If c be any constant then $c\mathbf{a}=c\langle a_1,a_2\rangle=\langle ca_1,ca_2\rangle$.

Here, $c=2$ and $\langle a_1,a_2\rangle=\langle 5,-12\rangle$.

Then,

$$\begin{aligned}2\mathbf{a}&=2\langle 5,-12\rangle\\&=\langle (2)(5),(2)(-12)\rangle\\&=\langle 10,-24\rangle\end{aligned}$$

Similarly, find $3\mathbf{b}$.

$$\begin{aligned}3\mathbf{b}&=3\langle -3,-6\rangle\\&=\langle (3)(-3),(3)(-6)\rangle\\&=\langle -9,-18\rangle\end{aligned}$$

Find $2\mathbf{a}+3\mathbf{b}$.

$$\begin{aligned}2\mathbf{a}+3\mathbf{b}&=\langle 10,-24\rangle+\langle -9,-18\rangle\\&=\langle 10-9,-24-18\rangle\\&=\langle 1,-42\rangle\end{aligned}$$

Therefore, $2\mathbf{a}+3\mathbf{b}=\boxed{\langle 1,-42\rangle}$.

Find $|\mathbf{a}|$.

If $\mathbf{a}=\langle a_1,a_2\rangle$ then $|\mathbf{a}|=\sqrt{a_1^{\,2}+a_2^{\,2}}$.

Here, $\mathbf{a}=\langle a_1,a_2\rangle=\langle 5,-12\rangle$.

$$\begin{aligned}|\mathbf{a}|&=\sqrt{a_1^{\,2}+a_2^{\,2}}\\&=\sqrt{(5)^2+(-12)^2}\\&=\sqrt{25+144}\\&=\sqrt{169}\\&=13\end{aligned}$$

Therefore, $|\mathbf{a}|=\boxed{13}$.

Find $|\mathbf{a}-\mathbf{b}|$.

First find $\mathbf{a}-\mathbf{b}$.

If $\mathbf{a}=\langle a_1,a_2\rangle$ and $\mathbf{b}=\langle b_1,b_2\rangle$ then $\mathbf{a}-\mathbf{b}=\langle a_1-b_1,a_2-b_2\rangle$.

Here, $\langle a_1,a_2\rangle=\langle 5,-12\rangle$ and $\langle b_1,b_2\rangle=\langle -3,-6\rangle$.

$$\begin{aligned}\mathbf{a}-\mathbf{b}&=\langle 5,-12\rangle-\langle -3,-6\rangle\\&=\langle 5-(-3),-12-(-6)\rangle\\&=\langle 5+3,-12+6\rangle\end{aligned}$$

$$=\langle 8,-6\rangle$$
$$=\langle k_1,k_2\rangle$$

$$\begin{aligned}|\mathbf{a}-\mathbf{b}|&=\sqrt{k_1^{\,2}+k_2^{\,2}}\\&=\sqrt{(8)^2+(-6)^2}\end{aligned}$$

$$=\sqrt{64+36}$$
$$=\sqrt{100}$$
$$=10$$

Therefore, $|\mathbf{a}-\mathbf{b}|=\boxed{10}$.

Answer 20E.

Consider the following vectors:

$$\mathbf{a}=4\mathbf{i}+\mathbf{j}$$
$$\mathbf{b}=\mathbf{i}-2\mathbf{j}$$

Compute the values of $\mathbf{a}+\mathbf{b}, 2\mathbf{a}+3\mathbf{b}, |\mathbf{a}|$ and $|\mathbf{a}-\mathbf{b}|$ by addition or subtraction of the components or the components multiplied by a scalar, of the given vectors along the direction of $\mathbf{i}$ and $\mathbf{j}$.

Determine $\mathbf{a}+\mathbf{b}$ by adding the corresponding components of each vector.

$$\begin{aligned}\mathbf{a}+\mathbf{b}&=(4\mathbf{i}+\mathbf{j})+(\mathbf{i}-2\mathbf{j})\\&=4\mathbf{i}+\mathbf{j}+\mathbf{i}-2\mathbf{j} \quad \text{Remove the parenthesis}\\&=\boxed{5\mathbf{i}-\mathbf{j}}\end{aligned}$$

Determine $2\mathbf{a}+3\mathbf{b}$ as follows:

$2\mathbf{a}+3\mathbf{b}=2(4\mathbf{i}+\mathbf{j})+3(\mathbf{i}-2\mathbf{j})$ Use the property: $c(\mathbf{a}+\mathbf{b})=c\mathbf{a}+c\mathbf{b}$

$$=2(4)\mathbf{i}+2\mathbf{j}+3\mathbf{i}+3(-2)\mathbf{j}$$
$$=8\mathbf{i}+2\mathbf{j}+3\mathbf{i}-6\mathbf{j}$$
$$=\boxed{11\mathbf{i}-4\mathbf{j}}$$

Determine $|\mathbf{a}|$ as follows:

$|\mathbf{a}| = \sqrt{4^2 + 1^2}$ Use the formula $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$ for the vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$

$= \boxed{\sqrt{17}}$

Determine $|\mathbf{a} - \mathbf{b}|$ as follows:

$$\begin{aligned}
|\mathbf{a} - \mathbf{b}| &= |(4\mathbf{i} + \mathbf{j}) - (\mathbf{i} - 2\mathbf{j})| \\
&= |4\mathbf{i} + \mathbf{j} - \mathbf{i} + 2\mathbf{j}| \\
&= |3\mathbf{i} + 3\mathbf{j}| \\
&= \sqrt{3^2 + 3^2} \\
&= \boxed{3\sqrt{2}}
\end{aligned}$$

Answer 21E.

Consider the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$.

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ then it can be written as $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$.

If $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then it can be written as $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$.

Find $\mathbf{a} + \mathbf{b}$.

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ then $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$.

Here, $\langle a_1, a_2, a_3 \rangle = \langle 1, 2, -3 \rangle$ and $\langle b_1, b_2, b_3 \rangle = \langle -2, -1, 5 \rangle$.

$$\begin{aligned}
\mathbf{a} + \mathbf{b} &= \langle 1, 2, -3 \rangle + \langle -2, -1, 5 \rangle \\
&= \langle 1 - 2, 2 - 1, -3 + 5 \rangle \\
&= \langle -1, 1, 2 \rangle \\
&= -\mathbf{i} + \mathbf{j} + 2\mathbf{k}
\end{aligned}$$

Therefore, $\mathbf{a} + \mathbf{b} = \boxed{-\mathbf{i} + \mathbf{j} + 2\mathbf{k}}$.

Find $\mathbf{2a+3b}$.

First find $\mathbf{2a}$.

If c be any constant then $c\mathbf{a} = c\langle a_1, a_2, a_3\rangle = \langle ca_1, ca_2, ca_3\rangle$.

Here, $c = 2$ and $\langle a_1, a_2, a_3\rangle = \langle 1, 2, -3\rangle$.

Then,

$$\begin{aligned}\mathbf{2a} &= 2\langle 1, 2, -3\rangle \\ &= \langle (2)(1), (2)(2), (2)(-3)\rangle \\ &= \langle 2, 4, -6\rangle\end{aligned}$$

Similarly, find $\mathbf{3b}$.

$$\begin{aligned}\mathbf{3b} &= 3\langle -2, -1, 5\rangle \\ &= \langle (3)(-2), (3)(-1), (3)(5)\rangle \\ &= \langle -6, -3, 15\rangle\end{aligned}$$

Find $\mathbf{2a+3b}$.

$$\begin{aligned}\mathbf{2a+3b} &= \langle 2, 4, -6\rangle + \langle -6, -3, 15\rangle \\ &= \langle 2-6, 4-3, -6+15\rangle \\ &= \langle -4, 1, 9\rangle \\ &= -4\mathbf{i} + \mathbf{j} + 9\mathbf{k}\end{aligned}$$

Therefore, $\mathbf{2a+3b} = \boxed{-4\mathbf{i} + \mathbf{j} + 9\mathbf{k}}$.

Find $|\mathbf{a}|$.

If $\mathbf{a} = \langle a_1, a_2\rangle$ then $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$.

Here, $\mathbf{a} = \langle a_1, a_2, a_3\rangle = \langle 1, 2, -3\rangle$.

$$\begin{aligned}|\mathbf{a}| &= \sqrt{a_1^2 + a_2^2 + a_3^2} \\ &= \sqrt{(1)^2 + (2)^2 + (-3)^2} \\ &= \sqrt{1+4+9} \\ &= \sqrt{14}\end{aligned}$$

Therefore, $|\mathbf{a}| = \boxed{\sqrt{14}}$.

Find $|\mathbf{a}-\mathbf{b}|$.

First find $\mathbf{a}-\mathbf{b}$.

If $\mathbf{a}=\langle a_1,a_2,a_3\rangle$ and $\mathbf{b}=\langle b_1,b_2,b_3\rangle$ then $\mathbf{a}-\mathbf{b}=\langle a_1-b_1,a_2-b_2,a_3-b_3\rangle$.

Here, $\mathbf{a}=\langle a_1,a_2,a_3\rangle=\langle 1,2,-3\rangle$ and $\langle b_1,b_2,b_3\rangle=\langle -2,-1,5\rangle$.

$$\begin{aligned}\mathbf{a}-\mathbf{b}&=\langle 1,2,-3\rangle-\langle -2,-1,5\rangle\\&=\langle 1-(-2),2-(-1),-3-5\rangle\\&=\langle 3,3,-8\rangle\end{aligned}$$

$$=\langle k_1,k_2,k_3\rangle$$

$$\begin{aligned}|\mathbf{a}-\mathbf{b}|&=\sqrt{k_1^2+k_2^2+k_3^2}\\&=\sqrt{(3)^2+(3)^2+(-8)^2}\\&=\sqrt{9+9+64}\\&=\sqrt{82}\end{aligned}$$

Therefore, $|\mathbf{a}-\mathbf{b}|=\boxed{\sqrt{82}}$.

Answer 22E.

Consider the following vectors:

$$\mathbf{a}=2\mathbf{i}-4\mathbf{j}+4\mathbf{k}$$
$$\mathbf{b}=2\mathbf{j}-\mathbf{k}$$

The objective is to find $\mathbf{a}+\mathbf{b}$, $2\mathbf{a}+3\mathbf{b}$, $|\mathbf{a}|$ and $|\mathbf{a}-\mathbf{b}|$

If $\mathbf{a}=\langle a_1,a_2,a_3\rangle$ and $\mathbf{b}=\langle a_1,a_2,a_3\rangle$,then $\mathbf{a}+\mathbf{b}=\langle a_1+b_1,a_2+b_2,a_3+b_3\rangle$

$\mathbf{a}-\mathbf{b}=\langle a_1-b_1,a_2-b_2,a_3-b_3\rangle$ and $c\langle a_1,a_2,a_3\rangle=\langle ca_1,ca_2,ca_3\rangle$

To find $\mathbf{a}+\mathbf{b}$ proceed as follows:

$$\begin{aligned}\mathbf{a}+\mathbf{b}&=(2\mathbf{i}-4\mathbf{j}+4\mathbf{k})+(2\mathbf{j}-\mathbf{k})\\&=2\mathbf{i}-4\mathbf{j}+4\mathbf{k}+2\mathbf{j}-\mathbf{k}\\&=\boxed{2\mathbf{i}-2\mathbf{j}+3\mathbf{k}}\end{aligned}$$

Find $2\mathbf{a}+3\mathbf{b}$ as follows:

$$\begin{aligned}2\mathbf{a}+3\mathbf{b} &= 2(2\mathbf{i}-4\mathbf{j}+4\mathbf{k})+3(2\mathbf{j}-\mathbf{k})\\ &= 4\mathbf{i}-8\mathbf{j}+8\mathbf{k}+6\mathbf{j}-3\mathbf{k}\\ &= \boxed{4\mathbf{i}-2\mathbf{j}+5\mathbf{k}}\end{aligned}$$

Find $|\mathbf{a}|$ as follows:

The length of the three-dimensional vector $\mathbf{a}=\langle a_1,a_2,a_3\rangle$ is $|\mathbf{a}|=\sqrt{a_1^{\ 2}+a_2^{\ 2}+a_3^{\ 2}}$

Here $\mathbf{a}=\langle 2,-4,4\rangle$

$$\begin{aligned}|\mathbf{a}| &= \sqrt{2^2+(-4)^2+4^2}\\ &= \boxed{6}\end{aligned}$$

Find $|\mathbf{a}-\mathbf{b}|$ as follows:

$$\begin{aligned}|\mathbf{a}-\mathbf{b}| &= |(2\mathbf{i}-4\mathbf{j}+4\mathbf{k})-(2\mathbf{j}-\mathbf{k})|\\ &= |2\mathbf{i}-4\mathbf{j}+4\mathbf{k}-2\mathbf{j}+\mathbf{k}|\\ &= |2\mathbf{i}-6\mathbf{j}+5\mathbf{k}|\\ &= \sqrt{2^2+(-6)^2+(5)^2}\\ &= \boxed{\sqrt{65}}\end{aligned}$$

Answer 23E.

The given vector is $-3i+7j$

and the vector $-3i+7j$ it has length

$$|-3i+7j| = \sqrt{(-3)^2+7^2} = \sqrt{58}$$

So, by the equation $\vec{u} = \frac{1}{|\vec{a}|}\vec{a} = \frac{\vec{a}}{|\vec{a}|}$, the vector with the same direction is

$$\frac{1}{\sqrt{58}}\left(-3i+7j\right) = \frac{-3}{\sqrt{58}}i+\frac{7}{\sqrt{58}}j$$

Answer 24E.

The given vector is $\langle -4, 2, 4\rangle$

This vector $\langle -4, 2, 4\rangle$ has a length

$$|\langle -4, 2, 4\rangle| = \sqrt{(-4)^2 + 2^2 + 4^2} = \sqrt{36} = 6$$

So, by the equation $\vec{u} = \frac{1}{|\vec{a}|}\vec{a} = \frac{\vec{a}}{|\vec{a}|}$, the vector with the same direction is

$$\frac{1}{6}\langle -4, 2, 4\rangle = \left\langle \frac{-4}{6}, \frac{2}{6}, \frac{4}{6}\right\rangle = \left\langle \frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle$$

Answer 25E.

Consider the vector $\mathbf{a} = 8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

Therefore, the length of the vector $\mathbf{a}$ is

$$\begin{aligned}|\mathbf{a}| &= |8\mathbf{i} - \mathbf{j} + 4\mathbf{k}| \\ &= \sqrt{8^2 + (-1)^2 + 4^2} \\ &= \sqrt{64 + 1 + 16} \\ &= \sqrt{81} \\ &= 9\end{aligned}$$

Note that, the unit vector that has the same direction as **a** is

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Therefore, the unit vector in the same direction as $\mathbf{a} = 8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ is

$$\begin{aligned}\mathbf{u} &= \frac{1}{9}(8\mathbf{i} - \mathbf{j} + 4\mathbf{k}) \\ &= \boxed{\frac{8}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}}\end{aligned}$$

Answer 26E.

Let $\vec{a} = \langle -2, 4, 2 \rangle$

Then $|\vec{a}| = \sqrt{(-2)^2 + (4)^2 + (2)^2}$

$= \sqrt{4+16+4}$

$= \sqrt{24}$

$= 2\sqrt{6}$

Then the vector having same direction as of $\vec{a}$ and unit magnitude is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$= \langle -2, 4, 2 \rangle \frac{1}{2\sqrt{6}}$$

$$= \left\langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$$

Therefore the vector having same direction as of $\vec{a}$ and of length 6 i.e magnitude 6 is

$$\vec{b} = 6\left\langle \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$$

i.e. $\boxed{\vec{b} = \langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle}$

Answer 27E.

From figure-1, it is obvious that, the angle between the above vector and the positive direction of the *x*-axis is represented by θ. So, find the value of θ.

In Figure-1, the vector, the *x*- and *y*-components of the vector make a right triangle, so,

$$\tan\theta = \frac{y\text{-component of the vector}}{x\text{-component of the vector}}$$

$$\tan\theta = \frac{\sqrt{3}}{1}$$

$$\tan\theta = \sqrt{3}$$

$$\tan\theta = \tan 60^\circ$$

$$\theta = 60^\circ.$$

Therefore, the angle between the vector $\mathbf{a} = \mathbf{i} + \sqrt{3}\mathbf{j}$ and the positive direction of the *x*-axis is

$$\theta = \boxed{60^\circ}.$$

Answer 28E.

From figure-1, it is obvious that, the angle between the above vector and the positive direction of the *x*-axis is represented by θ. So, find the value of θ.

In Figure-1, the vector, the *x*- and *y*-components of the vector make a right triangle, so,

$$\tan\theta = \frac{y\text{-component of the vector}}{x\text{-component of the vector}}$$

$$\tan\theta = \frac{6}{8}$$

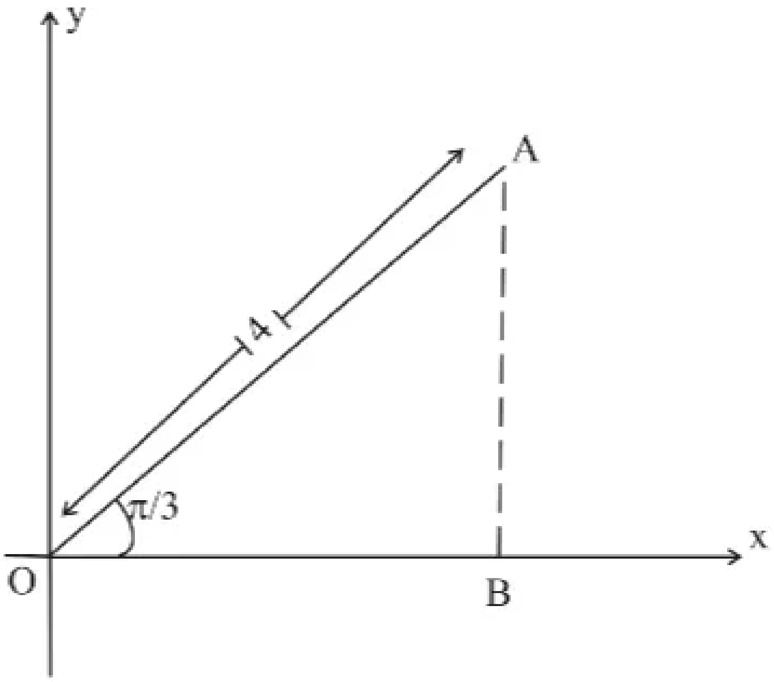
$$\tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right).$$

Therefore, the angle between the vector $\mathbf{a} = 8\mathbf{i} + 6\mathbf{j}$ and the positive direction of the *x*-axis is

$$\theta = \boxed{\tan^{-1}\left(\frac{3}{4}\right)}.$$

Answer 29E.



Let $\vec{v} = \overrightarrow{OA}$

such that $|\vec{v}| = 4$

From A draw AB perpendicular on x-axis meeting x-axis in B. Then $< AOB = \frac{\pi}{3}$

Now in triangle ΔABO,

$$\sin\frac{\pi}{3}=\frac{AB}{OA}$$

i.e. $$\frac{\sqrt{3}}{2}=\frac{AB}{4}$$

$$\Rightarrow AB=2\sqrt{3}$$

and $$\cos\frac{\pi}{3}=\frac{OB}{OA}$$

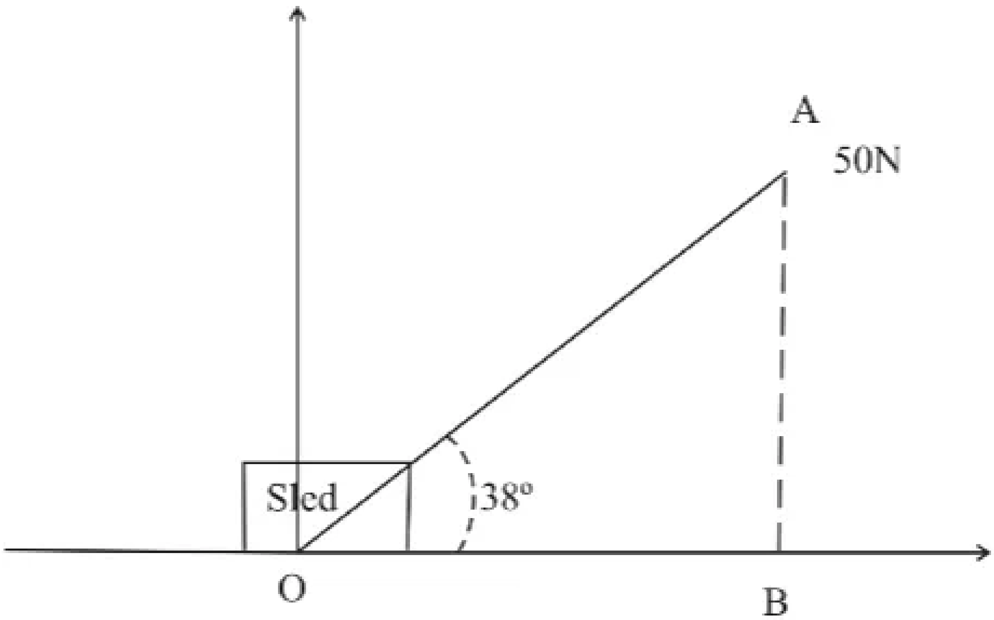
i.e. $$\frac{1}{2}=\frac{OB}{4}$$

$$\Rightarrow OB=2$$

Then $A\equiv\left(2,2\sqrt{3}\right)$

and hence $\overrightarrow{OA}=\boxed{\vec{v}=\left\langle 2,2\sqrt{3}\right\rangle}$

Answer 30E.



Let the child is pulling the sled at O with a force of 50N exerted at an angle 38^{o} above the horizontal. Let OA= 50N .form A draw AB perpendicular on the horizontal. Then $<AOB=38^{o}$

Now in triangle ΔABO,

$$\sin\ 38^o = \frac{AB}{OA}$$

i.e. $0.6156 = \frac{AB}{50}$

i.e. $AB = 30.78$

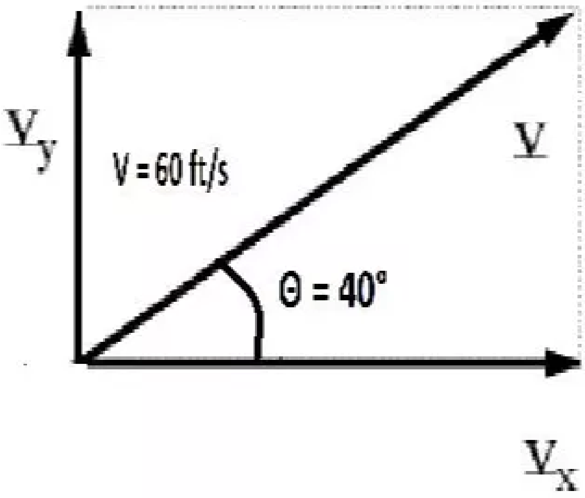
and $\cos 38^o = \frac{OB}{OA}$

i.e. $0.788 = \frac{OB}{50}$

i.e. $OB = 39.40$

Hence horizontal component of force is 39.40N and vertical component of force is 30.78N.

Answer 31E.



The velocity vector, **V**, consists of two components, the x-component Vx (horizontal) and the y-component Vy (vertical). These two components are what need to be determined in this problem.

To find these, draw the parallelogram with sides parallel to Vx and Vy and with the velocity vector **V** being the resultant. Then use simple trigonometric identities to determine the answers.

We are given V, which is the resultant, and for our use, it is the hypotenuse of two right triangles.

To find the horizontal component, we use the trig identity:

$$\cos\left(\theta\right) = \frac{\text{adjacent}}{\text{hypotenuse}} \therefore \cos\left(40^\circ\right) = \frac{V_x}{V} = \frac{V_x}{60} \therefore V_x = 60 \times \cos\left(40^\circ\right)$$

Horizontal Component: Vx ~ 45.96 ft/sec

To find the vertical component, we use the trig identity:

$$\sin\left(\theta\right) = \frac{\text{opposite}}{\text{hypotenuse}} \therefore \sin\left(40^{\circ}\right) = \frac{V_y}{V} = \frac{V_y}{60} \therefore V_y = 60 \times \sin\left(40^{\circ}\right)$$

Vertical Component: Vy ~ 38.57 ft/sec

Note: To find the components, you also could have used the identities:

sin (50°) = $\frac{\text{Vx}}{V} = \frac{\text{Vx}}{60}$

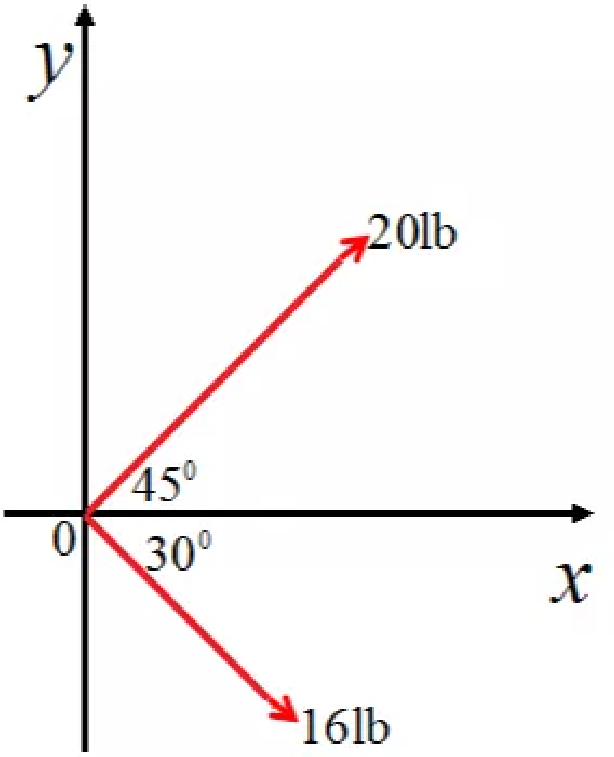
and

cos(50°) = $\frac{\text{Vy}}{V} = \frac{\text{Vy}}{60}$

because the angle between the vertical component and the velocity vector (resultant) is 50°. (90°-40°=50°)

Answer 32E.

Consider the following diagram depicted along the positive x – axis:



From the above diagram, resolve each vector into its x and y components along the positive x – axis.

The x-component of the forces and the angle it makes with the positive x – direction is towards the right. So, both will be positive.

This is calculated as follows:

$$= 20\cos(45) + 16\cos(30)$$

$$= 20\left(\frac{1}{\sqrt{2}}\right) + 16\left(\frac{\sqrt{3}}{2}\right)$$

$$= 20\left(\frac{1}{1.414}\right) + 16\left(\frac{1.732}{2}\right)$$

$$= 20\left(\frac{1}{1.414}\right) + 16\left(\frac{1.732}{2}\right)$$

$$= 20(0.7072) + 16(0.866)$$

$$= 14.144 + 13.856$$

$$= 28$$

Now, resolve the y-component of 20lb force which is directed upwards along the positive y axis. The y-component of 16lb force will be downwards so it is negative.

Solve this as follows:

$$= 20\sin(45) - 16\sin(30)$$
$$= 20\left(\frac{1}{\sqrt{2}}\right) - 16\left(\frac{1}{2}\right)$$
$$= 6.142136$$

The magnitude of the resultant force at $\langle 28, 6.142136 \rangle$ is calculated as follows:

$$\left|\langle 28, 6.142136 \rangle\right| = \sqrt{28^2 + (6.142136)^2}$$
$$= \boxed{28.665761 \text{ lbs}}$$

Angle of the resultant force at $\langle 28, 6.142136 \rangle$ is calculated as follows:

$$= \tan^{-1}\left(\frac{y}{x}\right)$$
$$= \tan^{-1}\left(\frac{6.142136}{28}\right)$$
$$= \boxed{12.372547 \text{ degrees.}}$$

Answer 33E.

Consider the figure as shown in the following.

In order to find the resultant for the two given forces, we must write them in their component form, which can be found using $\vec{u} = \|\vec{u}\|\cos\theta\vec{i} + \|\vec{u}\|\sin\theta\vec{j}$, where θ is the angle made with the positive x-axis.

Thus, for the force of 200 N into the first quadrant, the component form of the vector is:

$\mathbf{u} = (200)\cos(60^\circ)\mathbf{i} + (200)\sin(60^\circ)\mathbf{j}$ $= 200\frac{1}{2}\mathbf{i} + 200\frac{\sqrt{3}}{2}\mathbf{j}$ $= 100\mathbf{i} + 100\sqrt{3}\mathbf{j}$	Plug into the formula above. The magnitude is 200 N Use the unit circle to evaluate the trig functions. Simplify.

For the force of 300 N along the negative *x*-axis, the component form of the vector is:

$\mathbf{v} = (300)\cos(180^\circ)\mathbf{i} + (300)\sin(180^\circ)\mathbf{j}$	Plug into the formula above. The magnitude is 16 lbs and the angle is in the fourth quadrant.
$= 300(-1)\mathbf{i} + 300(0)\mathbf{j}$	
$= -300\mathbf{i} + 0\mathbf{j}$	Use the unit circle to evaluate the trig functions.
$= -300\mathbf{i}$	Simplify.

The resultant force is simply the sum of the two vectors, component-wise.

$$\mathbf{u} + \mathbf{v} = (100 - 300)\mathbf{i} + \left(100\sqrt{3} + 0\right)\mathbf{j}$$
$$= -200\mathbf{i} + 100\sqrt{3}\mathbf{j}$$

In order to determine the magnitude and the direction of the resultant vector, we use the formulas $\|\mathbf{u}\| = \sqrt{u_1^{\,2} + u_2^{\,2}}$ and $\tan(\theta) = \frac{u_2}{u_1}$.

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(-200)^2 + \left(100\sqrt{3}\right)^2}$$
$$= \sqrt{40000 + 30000}$$
$$= \sqrt{70000} \approx 264.6$$

$$\tan(\theta) = \frac{100\sqrt{3}}{-200}$$
$$\tan(\theta) \approx -\frac{\sqrt{3}}{2}$$
$$\theta \approx -40.9^\circ$$

However, because the resultant has to occur "between" the two forces, we know that the angle must be in either the first or second quadrant. Thus, the resultant force is approximately 264.6 N at an angle of $180^\circ - 40.9^\circ = 139.1^\circ$ counterclockwise to the *x*-axis.

Answer 34E.

The objective is to find the resultant for the two given forces, we must write them in their component form, which can be found using $\vec{u} = \|\vec{u}\|\cos\theta\vec{i} + \|\vec{u}\|\sin\theta\vec{j}$, where θ is the angle made with the positive *x*-axis.

The wind with a speed of 50 km/h is blowing at a direction of N45oW. This means that the wind is blowing at an angle 45o into the second quadrant, measured from the y-axis. In other words, the angle measured from the positive *x*-axis is 135o and thus the component form of the wind speed vector is:

$\mathbf{u} = (50)\cos(135^\circ)\mathbf{i} + (50)\sin(135^\circ)\mathbf{j}$ $= 50\left(-\frac{\sqrt{2}}{2}\right)\mathbf{i} + 50\frac{\sqrt{2}}{2}\mathbf{j}$ $= -25\sqrt{2}\mathbf{i} + 25\sqrt{2}\mathbf{j}$	Plug into the formula above. The magnitude is 50. Use the unit circle to evaluate the trig functions. Simplify.

The airplane has a speed of 250 km/h at a direction of N60oE. This means that the airplane is moving (in still air) at an angle 60o into the first quadrant, measured from the y-axis. In other words, the angle measured from the positive *x*-axis is 30o and thus the component form of the wind speed vector is:

$\mathbf{v} = (250)\cos(30^\circ)\mathbf{i} + (250)\sin(30^\circ)\mathbf{j}$ $= 250\left(\frac{\sqrt{3}}{2}\right)\mathbf{i} + 250\left(\frac{1}{2}\right)\mathbf{j}$ $= 125\sqrt{3}\mathbf{i} + 125\mathbf{j}$	Plug into the formula above. The magnitude is 250 and the angle is in the fourth quadrant. Use the unit circle to evaluate the trig functions. Simplify.

The resultant force is simply the sum of the two vectors, component-wise.

$$\mathbf{u}+\mathbf{v}=\left(-25\sqrt{2}+125\sqrt{3}\right)\mathbf{i}+\left(25\sqrt{2}+125\right)\mathbf{j}$$

In order to determine the magnitude and the direction of the resultant vector, we use the formulas $\|\mathbf{u}\|=\sqrt{u_1^{\,2}+u_2^{\,2}}$ and $\tan(\theta)=\frac{u_2}{u_1}$.

$$\begin{aligned}\|\mathbf{u}+\mathbf{v}\| &= \sqrt{\left(-25\sqrt{2}+125\sqrt{3}\right)^2+\left(25\sqrt{2}+125\right)^2} \\ &\approx \sqrt{32815.7+25713.8} \\ &\approx 241.9\end{aligned}$$

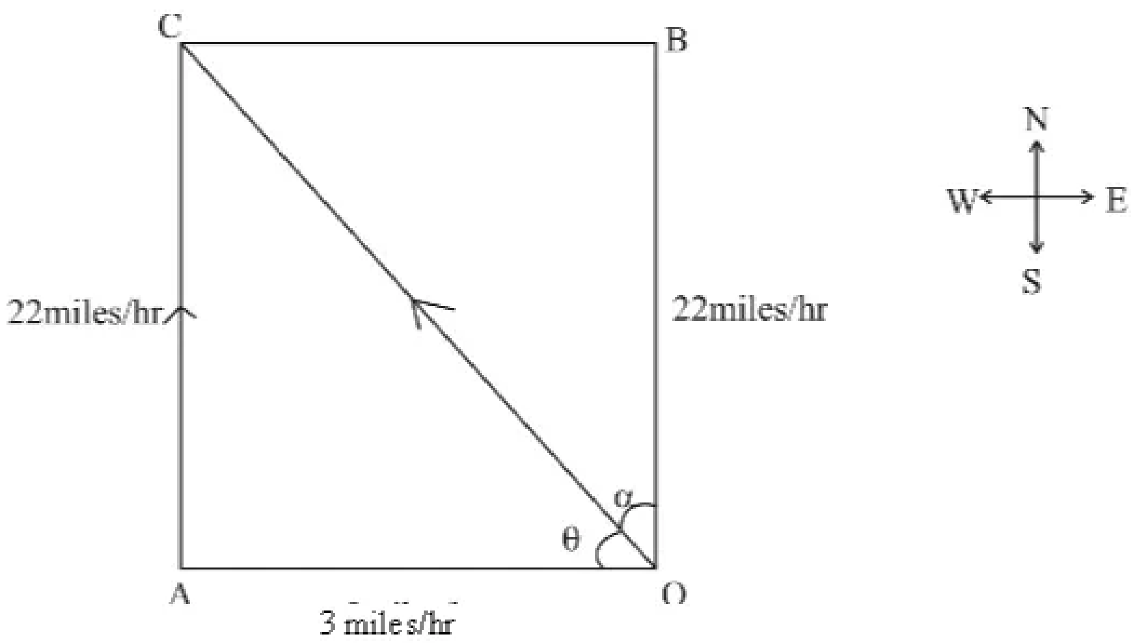
$$\tan(\theta)=\frac{25\sqrt{2}+125}{-25\sqrt{2}+125\sqrt{3}}$$

$$\tan(\theta)\approx .8852$$

$$\theta\approx 41.5^{\circ}$$

The ground speed of the plane is 241.9 km/h and the true course of the plane is at an angle which is 41.5o above the x-axis, meaning that, since $90-41.5=48.5$, the true course of the plane is N48.5oE.

Answer 35E.



Let the speed of woman due west on the deck of the ship is represented by OA, and the speed of ship due north is represented by OB. Form A draw AC equal and parallel to OB, i.e

AC = OB = 22 miles /hr. Join OC. let $<COA=\theta^{\circ}$

In right angled triangle CAO, using Pythagoras Theorem,

$$OC^2 = OA^2 + AC^2$$

i.e. $OC = \sqrt{9 + 484}$

$= \sqrt{493}$

$= 22.2$

i.e. the speed of woman relative to the surface of water is 22.2 miles/hr.

Now in ΔCAO,

$$\tan\theta = \frac{AC}{AO}$$

i.e. $\tan\theta = \frac{22}{3}$

i.e. $\tan\theta = 7.33$

i.e. $\theta = 82.23^\circ$

i.e. $\theta \approx 82^\circ$

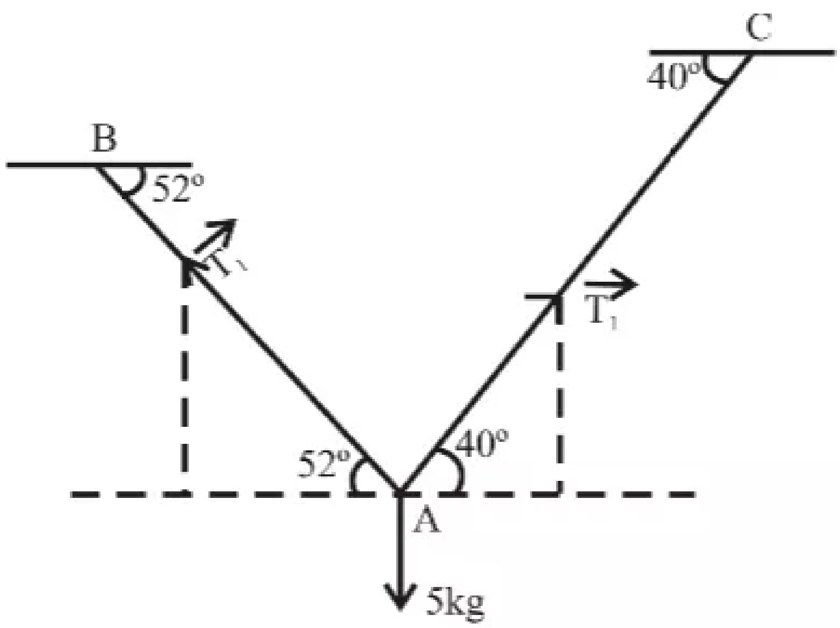
Then angle with vertical $\alpha = 90^\circ - \theta^\circ$

$= 90^\circ - 82^\circ$

$= 8^\circ$

Hence woman has a speed of 22.2 miles/hr relative to the surface of water and its direction is $N8^\circ W$.

Answer 36E.



Let $\vec{T}_1$ and $\vec{T}_2$ be the tensions in wires AB and AC respectively. Expressing $\vec{T}_1$ and $\vec{T}_2$ in terms of their horizontal and vertical components.

$$\vec{T}_1 = -\left|\vec{T}_1\right|\cos 52^\circ \hat{i} + \left|\vec{T}_1\right|\sin 52^\circ \hat{j}$$

$$\vec{T}_2 = \left|\vec{T}_2\right|\cos 40^\circ \hat{i} + \left|\vec{T}_2\right|\sin 40^\circ \hat{j}$$

The resultant $\vec{T}_1$ and $\vec{T}_2$ i.e., $\vec{T}_1 + \vec{T}_2$ will balance the weight 5 kg of the decoration..

Therefore, $\vec{T}_1 + \vec{T}_2 = 5\times 9.8\hat{j}$

Equating horizontal and vertical components,

$$-\left|\vec{T}_1\right|\cos 52^\circ + \left|\vec{T}_2\right|\cos 40^\circ = 0 \qquad \text{----- (1)}$$

$$\left|\vec{T}_1\right|\sin 52^\circ + \left|\vec{T}_2\right|\sin 40^\circ = 49 \qquad \text{----- (2)}$$

From Equation (1) $\left|\vec{T}_2\right| = \dfrac{\left|\vec{T}_1\right|\cos 52^\circ}{\cos 40^\circ}$ putting in equation (2) we get

$$\left|\vec{T}_1\right|\sin 52^\circ + \frac{\left|\vec{T}_1\right|\cos 52^\circ}{\cos 40^\circ}\sin 40^\circ = 49$$

$$\left|\vec{T}_1\right|\left(\sin 52^\circ + \cos 52^\circ \tan 40^\circ\right) = 49$$

$$\left|\vec{T}_1\right| = \frac{49}{\sin 52^\circ + \cos 52^\circ \tan 40^\circ}$$

$$= 37.559 \text{ N.}$$

Now, $\left|\vec{T}_2\right| = \dfrac{\left|\vec{T}_1\right|\cos 52^\circ}{\cos 40}$

$$= \frac{37.559\times 0.6156}{0.7660}$$

$$= 30.18 \text{ N.}$$

Therefore, Tension in the wire AB is

$$\vec{T}_1 = -\left|\vec{T}_1\right|\cos 52^\circ \hat{i} + \left|\vec{T}_1\right|\sin 52^\circ \hat{j}$$

$$= -37.559\times 0.6156\hat{i} + 37.559\times 0.7880\hat{j}$$

$$= -23.123\hat{i} + 29.596\hat{j}$$

$$\approx -23\hat{i} + 30\hat{j}$$

And tension in wire AC is

$$\vec{T} = \left|\vec{T}_2\right|\cos 40°\hat{i} + \left|\vec{T}_2\right|\sin 40°\hat{j}$$
$$= 30.18\times0.7660\hat{i} + 30.18\times0.6427\hat{j}$$
$$= 23.11\hat{i} + 19.39\hat{j}$$
$$\approx 23\hat{i} + 19\hat{j}$$

Hence,

Tension in AB is $\vec{T}_1 = -23\hat{i} + 30\hat{j}$
with magnitude $\left|\vec{T}_1\right| \approx 38\,\text{N}$
And tension in AC is $\vec{T}_2 = 23\hat{i} + 19\hat{j}$
With magnitude $\left|\vec{T}_2\right| \approx 30\,\text{N}$

Answer 37E.

Observe the figure 1; clothesline is tied between two poles, 8 meter apart with mass of 0.8 kg is hung at the middle of the line and the midpoint is pulled down 8 cm or 0.08 m.

Let $\mathbf{T}_1$ and $\mathbf{T}_2$ be the tensions in both parts, then let $\mathbf{T}$ be the tension in the both parts of the clothesline.

Take O as origin and $OA = OC = 4m$.

Write the following tension vectors as,

$$\mathbf{T}_1 = -a\mathbf{i} + b\mathbf{j}$$
$$\mathbf{T}_2 = a\mathbf{i} + b\mathbf{j}$$

Use the property of similar triangle, we have

$$\frac{a}{b} = \frac{4}{0.08} \quad \text{...... (1)}$$

Plugging in and creating an equation for the forces counterbalancing as,

$$\mathbf{T}_1 + \mathbf{T}_2 = \mathbf{w}$$

$$-a\mathbf{i} + b\mathbf{j} + a\mathbf{i} + b\mathbf{j} = mg\mathbf{j} \qquad \text{Use } \mathbf{T}_1 = -a\mathbf{i} + b\mathbf{j} \text{ and } \mathbf{T}_2 = a\mathbf{i} + b\mathbf{j}$$

$$2b\mathbf{j} = (0.8)(9.8)\mathbf{j} \qquad \text{Use } g = 9.8$$

$$2b\mathbf{j} = 7.84\mathbf{j} \qquad \text{Simplify}$$

$$2b\mathbf{j} = 7.84\mathbf{j} \qquad \text{Cancel out the common factors}$$

$$b = 3.92 \qquad \text{Divide each side by 2}$$

Substitute the value in equation (1) then the equation becomes,

$$\frac{a}{b} = \frac{4}{0.08}$$

$$\frac{a}{b} = 50$$

$$a = 50(3.92)$$

$$a = 196$$

Substitute $a = 196, b = 3.92$ in $\mathbf{T}_1 = -a\mathbf{i} + b\mathbf{j}$ and $\mathbf{T}_2 = a\mathbf{i} + b\mathbf{j}$, we have

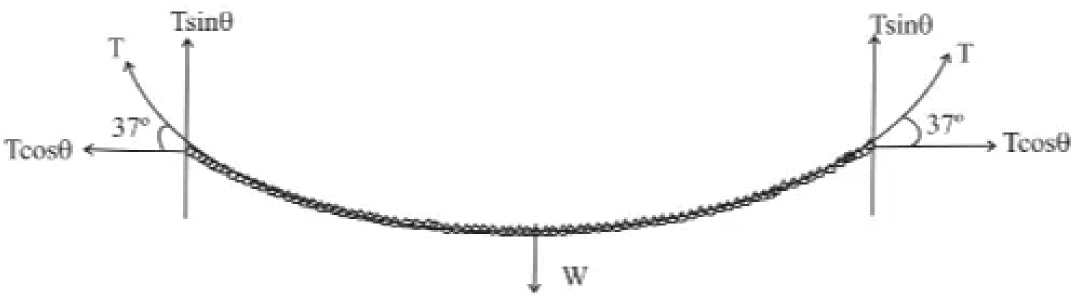
$$\begin{aligned}\mathbf{T}_1 &= -a\mathbf{i} + b\mathbf{j} \\ &= -196\mathbf{i} + 3.92\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{T}_2 &= a\mathbf{i} + b\mathbf{j} \\ &= 196\mathbf{i} + 3.92\mathbf{j}\end{aligned}$$

Therefore, the two tensions are given by

$$\mathbf{T}_1 = \boxed{-196\mathbf{i} + 3.92\mathbf{j}} \text{ and } \mathbf{T}_2 = \boxed{196\mathbf{i} + 3.92\mathbf{j}}.$$

Answer 38E.



Let W be the weight of the chain acting downwards. Resolving the components of tensions at both the ends horizontally and vertically we find,

$$T\cos 37^\circ = T\cos 37^\circ$$

$$T\sin 37^\circ + T\sin 37^\circ = W$$

or $\quad W = 2T\sin 37^0$

But $T = 25N$ (given)

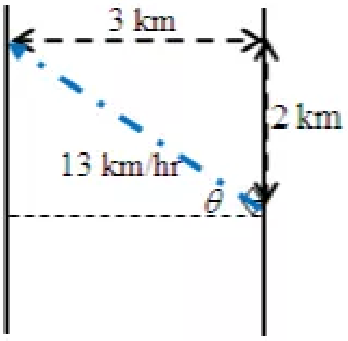
Then $W = 2\times 25\times \sin 37^o$

i.e. W= 30.09 N

or $\boxed{W \approx 30N}$.

Answer 39E.

In order to better understand our problem, a picture would probably be helpful.



So, the boat is moving across the river at an angle of θ above the horizontal. Thus, as a vector, the boat can be represented as $\vec{v} = \langle 13t\cos\theta, 13t\sin\theta\rangle$, where *t* represents the number of hours that the boat is in motion.

(Note that the 13*t* comes from the 13 km/hr, so that when we multiply 13*t* together, get the magnitude of the vector in km.)

Since the boat is travelling upstream, the movement of the current is operating in the opposite direction as the boat is moving. For us, then, represent the current negatively, as the vector $\vec{w} = \langle 0, -3.5t\rangle$, where *t* is the same as above.

Because the current is the only force acting upon the boat, easily find the speed of the boat relative to the current by adding the two vectors, component-wise.

$$\vec{v} + \vec{w} = \langle 13t\cos\theta, 13t\sin\theta - 3.5t\rangle$$

Know that the vertical distance traveled up the canal is to be 2 km, and that the horizontal distance across is 3 km. Set each of the above components equal to the appropriate distance to create a system of equations.

$$\begin{cases} 13t\cos\theta = 3 \\ 13t\sin\theta - 3.5t = 2 \end{cases}$$

Solve for the variable θ and t by using substitution

$$13t\cos\theta = 3 \rightarrow t = \frac{3}{13\cos\theta}$$

$$13\left(\frac{3}{13\cos\theta}\right)\sin\theta - 3.5\left(\frac{3}{13\cos\theta}\right) = 2$$ Substitute fortin the 2nd equation

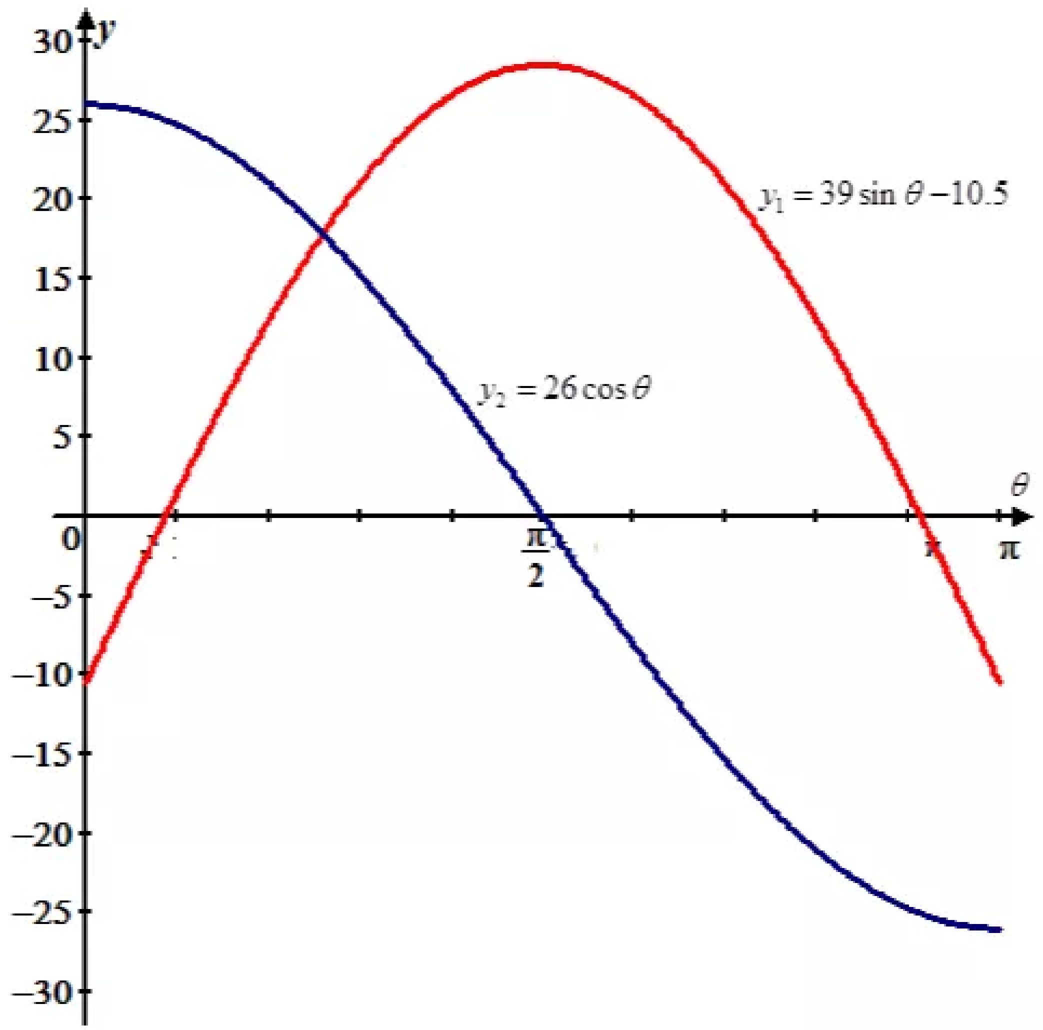
$$\frac{39\sin\theta}{13\cos\theta} - \frac{10.5}{13\cos\theta} = 2$$ Multiply the numerators

$$39\sin\theta - 10.5 = 26\cos\theta$$ Multiply both sides by the denominator of $13\cos\theta$.

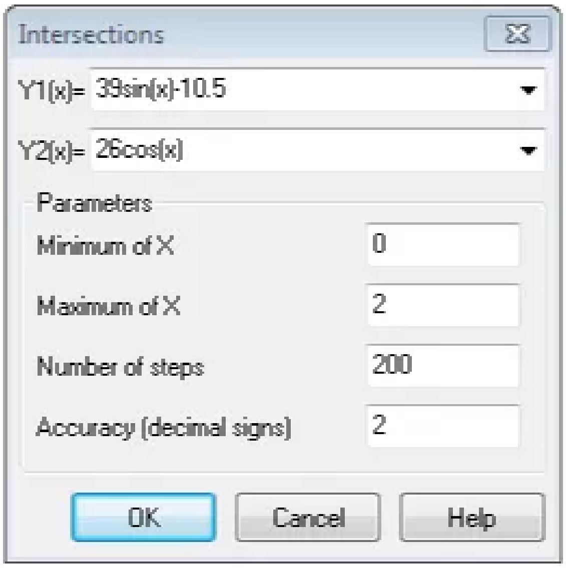
This type of equation is extremely difficult to solve using conventional methods. However, using a calculator can make it very doable. Sketch the graph of the two functions with respect to θ, and find their intersection point (in terms of positive θ).

(a) Sketch

$y_1 = 39\sin\theta - 10.5$
$y_2 = 26\cos\theta$



Using advanced grapher future to find the intersection point of the two curves.



Intersection points: 1

X	Y
0.81	17.86

The above value is in radians but convert into degrees then obtain $\boxed{46.6}$

So the boatman should point his boat about 46 degrees from the shore, pointing upstream.

(b) Now, plug in the value of theta to find the time it takes to cross the canal, *t*.

$$t = \frac{3}{13\cos\left(46.6^{\circ}\right)} \approx .336 \text{ hours.}$$

Thus, it takes the boat just over 20 minutes $\boxed{20.2}$ minutes to make the trip across the canal.

Answer 40E.

Three forces act on an object. Two of the forces are at an angle of $100°$ to each other and have magnitudes 25 N and 12 N.

The third is perpendicular to the plane of these two forces and has magnitude 4 N.

Now need to calculate the magnitude of the force that would exactly counterbalance these three forces.

The text book approach must be using vector notation $\mathbf{i}, \mathbf{j}, \mathbf{k}$ and basic trigonometric properties but not the advanced.

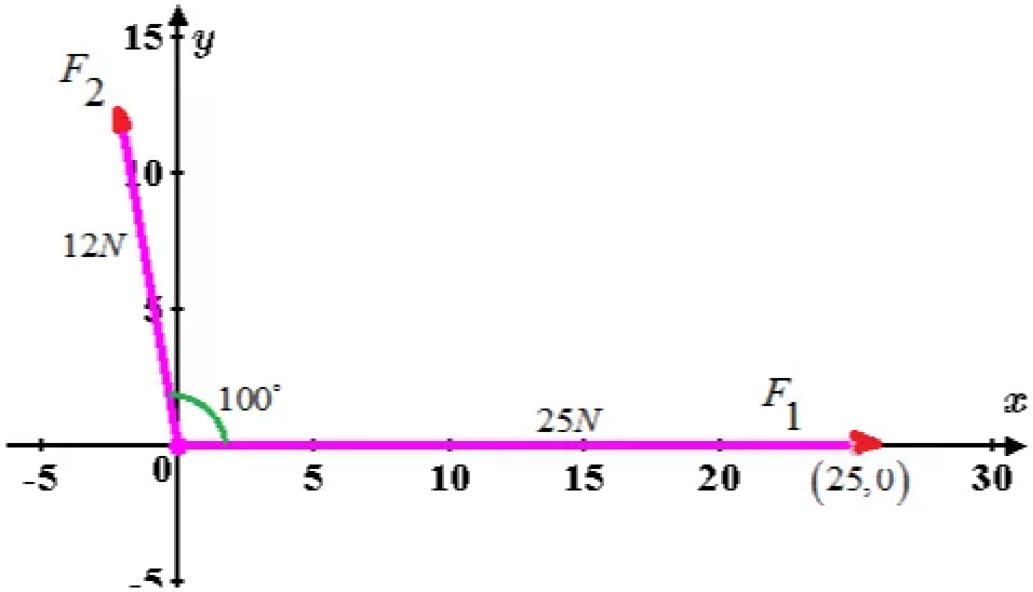
A vector is a physical quantity having both magnitude and direction.

In view of this definition, assume the force $F_1 = 25N$ is acting on the object placed at the origin along the positive direction of x – axis.

Consider the usual rotation (anti – clock wise) at the origin to show the second force

$F_2 = 12N$.

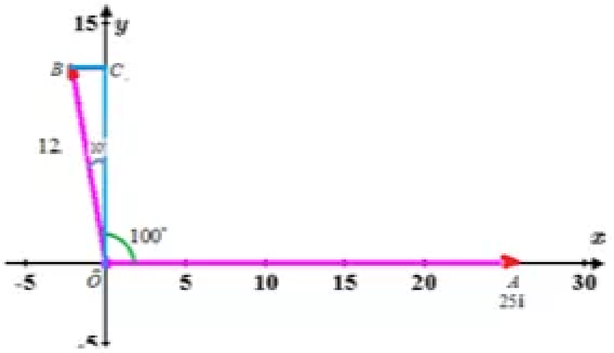
While the angle between these forces is $100°$, it is easy to see that the second force is in the 2nd quadrant of the xy – plane.



In view of the text book approach, use the unit vectors along the coordinate axes.

But, the 2nd force is not along the coordinate axis.

So, let us construct a triangle with $10°$ at the origin and use the sine and cosine angles at that point.



The triangle OBC is having the angle $\angle BOC = 10^{\circ}$

$$\begin{aligned} BC &= OB \sin 10^{\circ} \\ &= 12 \sin 10^{\circ} \\ &= 2.08 \end{aligned}$$

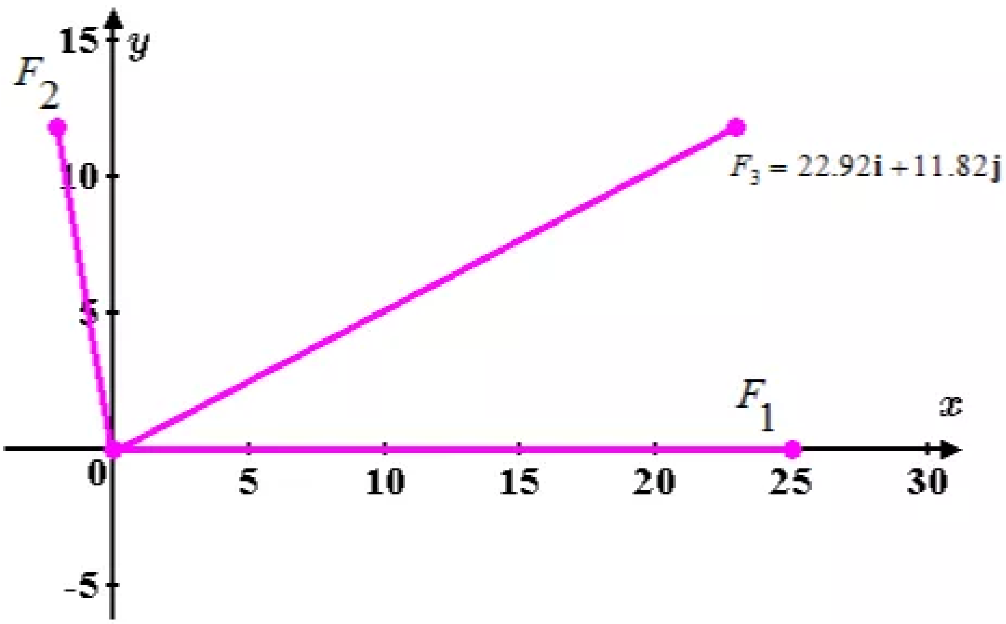
$$\begin{aligned} OC &= OB \cos 10^{\circ} \\ &= 12 \cos 10^{\circ} \\ &= 11.82 \end{aligned}$$

Observe that *BC* is in the negative direction of *x* – axis and *OC* is in the positive direction of *y* – axis.

Therefore, $F_2 = -2.08\mathbf{i} + 11.82\mathbf{j}$.

Use the addition of vectors, the resultant force of the two forces F_1, F_2 is

$$\begin{aligned} F_3 &= F_1 + F_2 \\ &= 25\mathbf{i} - 2.08\mathbf{i} + 11.82\mathbf{j} \\ &= 22.92\mathbf{i} + 11.82\mathbf{j} \end{aligned}$$



Suppose another force F_4 with 4*N* is acting perpendicular to the plane upon which F_1, F_2 are acting.

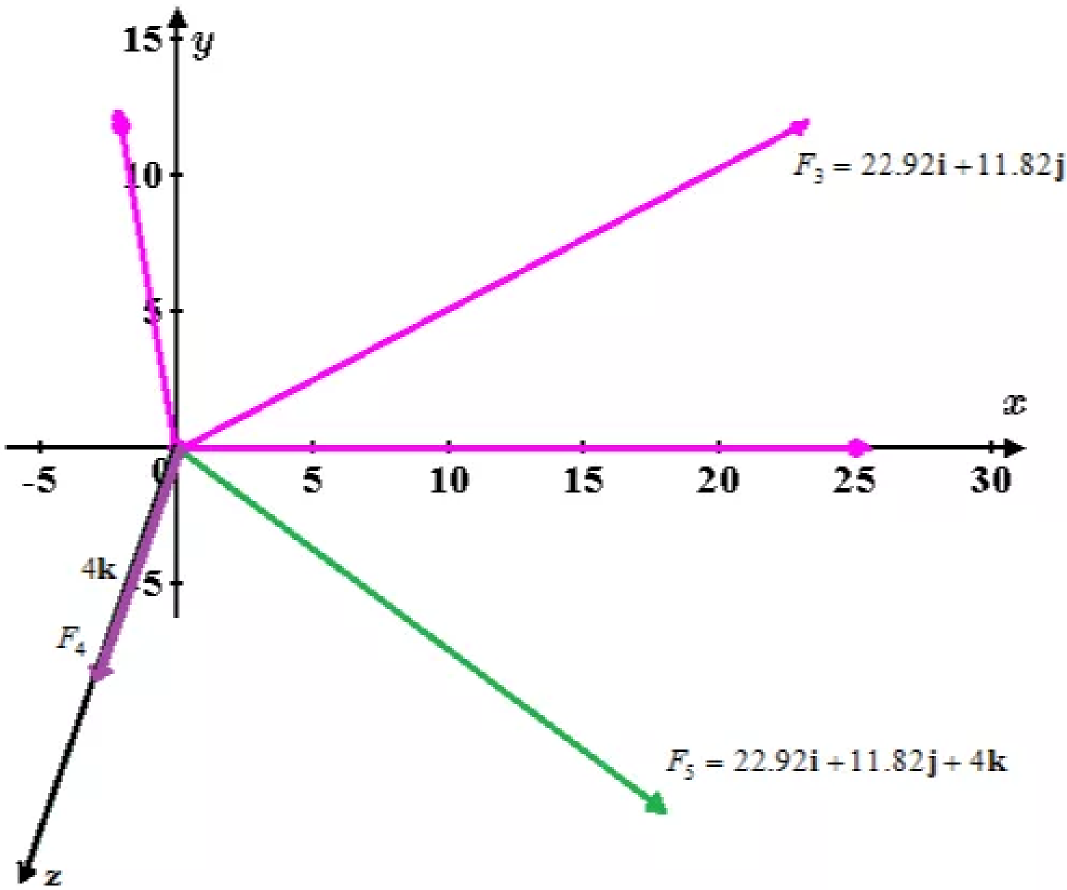
While F_1, F_2 are upon *xy* – plane, it is clear that the force F_4 is acting along the positive direction of *z* – axis with 4 Newton.

The unit vector along the direction of *z* – axis is **k**.

Therefore, $F_4 = 4\mathbf{k}.$

So, using the addition of vectors property, the resultant force of the two forces F_3, F_4 is

$$\begin{aligned} F_5 &= F_3 + F_4 \\ &= 22.92\mathbf{i} + 11.82\mathbf{j} + 4\mathbf{k} \end{aligned}$$



So, the magnitude of the resultant force of the three forces F_1, F_2, and F_4 is

$$F_5 = \sqrt{(22.92)^2 + (11.82)^2 + 4^2}$$
$$= \sqrt{525.3264 + 139.7124 + 16}$$
$$= \sqrt{681.0388}$$
$$\approx 26.0967201$$
$$\approx 26.1N$$

Therefore, the required force to counterbalance the three forces F_1, F_2, and F_4 is 26.1*N* in the opposite direction of F_5 at the object.

Answer 41E.

To find the unit vectors that are parallel to the tangent line to the parabola y = x2 at the point (2,4), we need to first at the derivative of y = x2.

y = x2

y' = 2x   at the point (2,4), the derivative can be obtained by substituting in x = 2

y' = 4   This allows us to find the vectors.

This means the tangent line, or the vector, as a rise of 4 and run of 1, since the slope of a function is rise over run.Therefore, the vector is $(i + 4j)$. However, there are two of these vectors, since one would be in the other direction.Therefore, the vectors are $a = \pm(i + 4j)$. Let's denote the vectors by **a**. These are **not** the **unit** vectors. We still need to find the length of the vector and divide the vector we obtained earlier by its length to get the **unit** vector.

The formula to find the length of the vector is $|a| = \sqrt{x^2 + y^2}$.

$$|a| = \sqrt{x^2 + y^2}$$

$$|a| = \sqrt{1^2 + 4^2}$$

$|a| = \sqrt{17}$ This is the length of the vector.

A unit vector is written as $u = \frac{a}{|a|}$. Therefore, our unit vector is: $u = \pm \frac{1}{\sqrt{17}}\left(i + 4j\right)$

Answer 42E.

First, find a vector that is parallel to the above tangent line.

Use the following fact to find the vector parallel to the tangent line:

"If a line has slope *m*, then a vector $\mathbf{a}$ parallel to the line is $\mathbf{a} = \mathbf{i} + m\mathbf{j}$".

To find a vector that is parallel to the tangent line, first find the slope of the tangent line by following definition:

"The slope $m_{\tan}$ of the tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ is defined as follows:

$$m_{\tan} = \left.\frac{dy}{dx}\right|_{x=a}."$$

The derivative of the function $y = 2\sin x$ is,

$$\frac{dy}{dx} = \frac{d}{dx}(2\sin x) \\ = 2\cos x.$$

So, the slope of the tangent line at the point $\left(\frac{\pi}{6}, 1\right)$, that is, for $x = \frac{\pi}{6}$ will be,

$$m = \left.\frac{dy}{dx}\right|_{x=a} \\ = \left.(2\cos x)\right|_{x=\frac{\pi}{6}} \\ = 2\cos\frac{\pi}{6} \\ = 2\cdot\frac{\sqrt{3}}{2} \\ = \sqrt{3}.$$

So, by above fact, a parallel vector to the tangent line is,

$$\mathbf{a} = \mathbf{i} + m\mathbf{j} \\ = \mathbf{i} + \sqrt{3}\mathbf{j}.$$

Next, find the unit vectors that are parallel to the tangent line.

"The unit vectors that have the same direction as the vector $\mathbf{a}$ is defined by;

$$\mathbf{u} = \pm\frac{1}{|\mathbf{a}|}\mathbf{a}."$$

By above step, a parallel vector to the tangent line is $\mathbf{a} = \mathbf{i} + \sqrt{3}\mathbf{j}$, hence, the unit vectors parallel to the tangent line are,

$$\mathbf{u} = \pm\frac{1}{\left|\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)\right|}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)$$

$$= \pm\frac{1}{\sqrt{1^2+\left(\sqrt{3}\right)^2}}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)$$ Use the formula; $\left|\left(a_1\mathbf{i}+a_2\mathbf{j}\right)\right| = \sqrt{a_1^2+a_2^2}$

$$= \pm\frac{1}{\sqrt{4}}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)$$

$$= \pm\frac{1}{2}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right).$$

Therefore, the unit vectors that are parallel to the tangent line to the curve $y = 2\sin x$ at the point $\left(\frac{\pi}{6},1\right)$ are $\boxed{\pm\frac{1}{2}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)}$.

(b)

Consider the following curve and point:

Curve: $y = 2\sin x$,

Point: $\left(\frac{\pi}{6},1\right)$.

The objective is to find the unit vectors that are perpendicular to the tangent line to the curve $y = 2\sin x$ at the point $\left(\frac{\pi}{6},1\right)$.

First, find a vector that is perpendicular to the above tangent line.

By part (a), the slope of the tangent line is $m_{\tan} = \sqrt{3}$, so the slope of the line that is perpendicular to the tangent line is,

$$\begin{aligned} m_{\text{perpendicular}} &= -\frac{1}{m_{\tan}} \\ &= -\frac{1}{\sqrt{3}}. \end{aligned}$$

So, a vector that is perpendicular to the above tangent line is,

$$\begin{aligned} \mathbf{a} &= \mathbf{i} + m\mathbf{j} \\ &= \mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j}. \end{aligned}$$

Next, find the unit vectors that are perpendicular to the tangent line.

"The unit vectors that have the same direction as the vector $\mathbf{a}$ is defined by;

$$\mathbf{u} = \pm\frac{1}{|\mathbf{a}|}\mathbf{a}."$$

By above step, a vector that is perpendicular to the above tangent line is $\mathbf{a} = \mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j}$, hence, the unit vectors perpendicular to the tangent line are,

$$\mathbf{u} = \pm\frac{1}{\left|\left(\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j}\right)\right|}\left(\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j}\right)$$

$$= \pm\frac{1}{\sqrt{1^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}}\left(\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j}\right) \quad \text{Use the formula; } |(a_1\mathbf{i} + a_2\mathbf{j})| = \sqrt{a_1^2 + a_2^2}$$

$$= \pm\frac{1}{\sqrt{4/3}}\left(\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j}\right)$$

$$= \pm\frac{\sqrt{3}}{2}\left(\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j}\right)$$

$$= \pm\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right).$$

Therefore, the unit vectors that are perpendicular to the tangent line to the curve $y = 2\sin x$ at the point $\left(\frac{\pi}{6}, 1\right)$ are $\boxed{\pm\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)}$.

(c)

By part (a), the unit vectors parallel to the tangent line to the curve at the point are,

$\pm\frac{1}{2}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)$.

By part (b), the unit vectors perpendicular to the tangent line to the curve at the point are,

$\pm\left(\frac{\sqrt{3}}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}\right)$.

The objective is to sketch the curve $y=2\sin x$, and the unit vectors; $\pm\frac{1}{2}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)$ and $\pm\left(\frac{\sqrt{3}}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}\right)$, all starting at $\left(\frac{\pi}{6},1\right)$.

In the following figure, the sketch of the curve $y=2\sin x$ is shown by red color, and the point $(\pi/6,1)$ is shown by brown color.

The blue vectors are the unit tangent vectors:

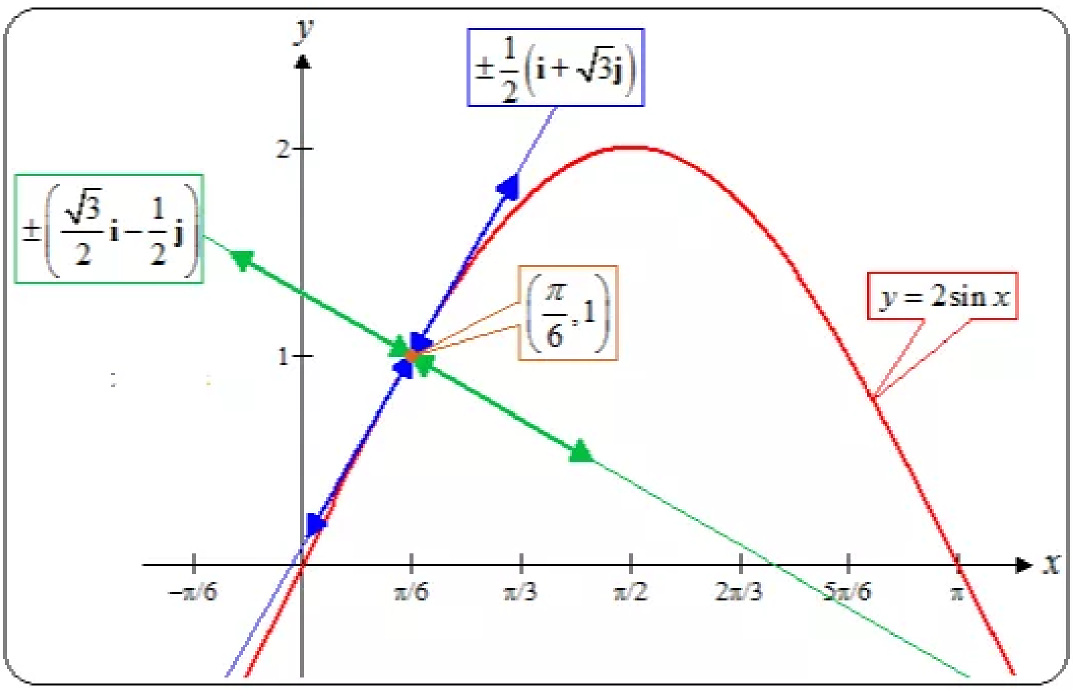
$-\frac{1}{2}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)$, and $+\frac{1}{2}\left(\mathbf{i}+\sqrt{3}\mathbf{j}\right)$,

starting at $(\pi/6,1)$, where the blue line is the tangent line at $(\pi/6,1)$.

The green vectors are the unit vectors that are perpendicular to the tangent line:

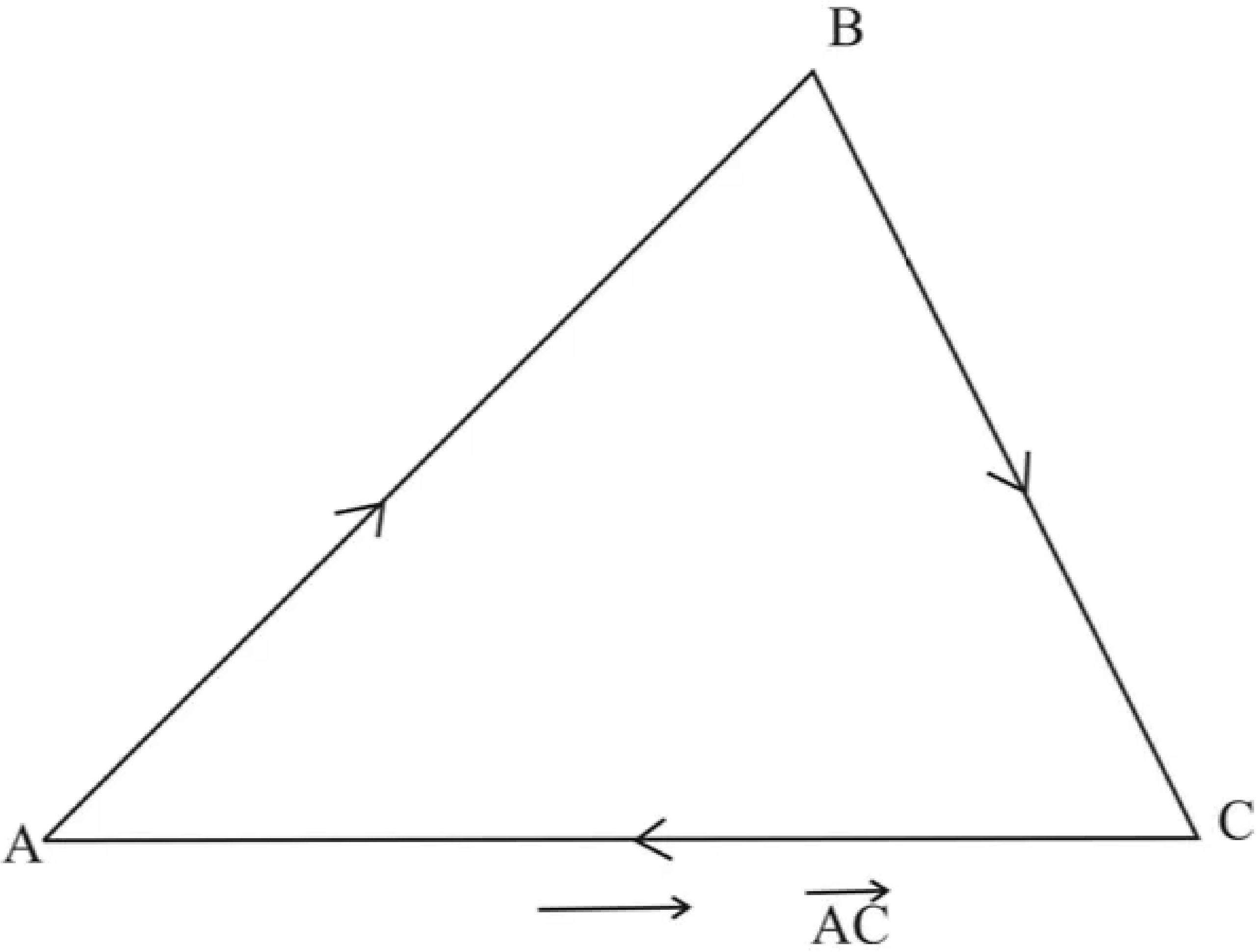
$-\left(\frac{\sqrt{3}}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}\right)$, and $+\left(\frac{\sqrt{3}}{2}\mathbf{i}-\frac{1}{2}\mathbf{j}\right)$,

starting at $(\pi/6,1)$, where the green line is the line perpendicular to the tangent line at $(\pi/6,1)$.



Answer 43E.

Consider A, B and C are the vertices of a triangle.



Use Triangle Law of Vector Addition.

From the above figure,

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$

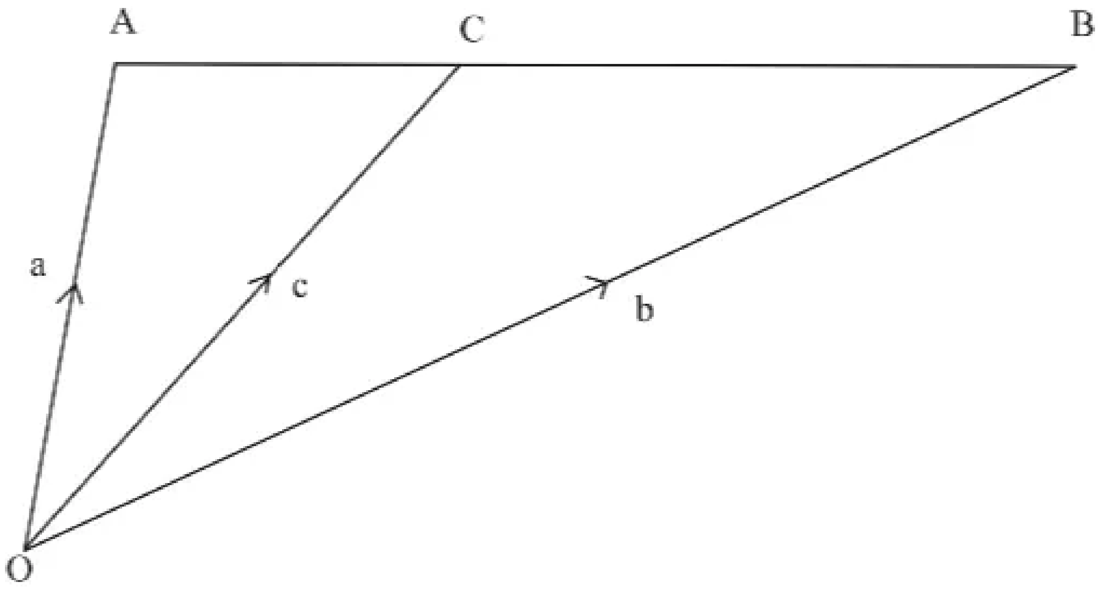
But, $\overrightarrow{AC}=-\overrightarrow{CA}$.

Therefore,

$$\begin{aligned}\overrightarrow{AB}+\overrightarrow{BC}&=-\overrightarrow{CA}\\ \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}&=-\overrightarrow{CA}+\overrightarrow{CA}\\ \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}&=0\end{aligned}$$

Hence, $\boxed{\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0}$.

Answer 44E.



In triangle OBC, by triangle law,

$$\overrightarrow{OB}+\overrightarrow{BC}=\overrightarrow{OC}$$

i.e. $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$

i.e. $\overrightarrow{BC}=\vec{c}-\vec{b}$

In ΔAOC, by triangle law,

$$\overrightarrow{OC}+\overrightarrow{CA}=\overrightarrow{OA}$$

i.e. $\overrightarrow{CA}=\overrightarrow{OA}-\overrightarrow{OC}$

i.e. $\overrightarrow{CA}=\vec{a}-\vec{c}$

But C is the point on segment AB, such that

$$2\ AC=CB$$

Or $2\overrightarrow{AC}=\overrightarrow{CB}$

Or $2\overrightarrow{CA}=\overrightarrow{BC}$

Or $2\left(\vec{a}-\vec{c}\right)=\vec{c}-\vec{b}$

Or $2\vec{a}-2\vec{c}=\vec{c}-\vec{b}$

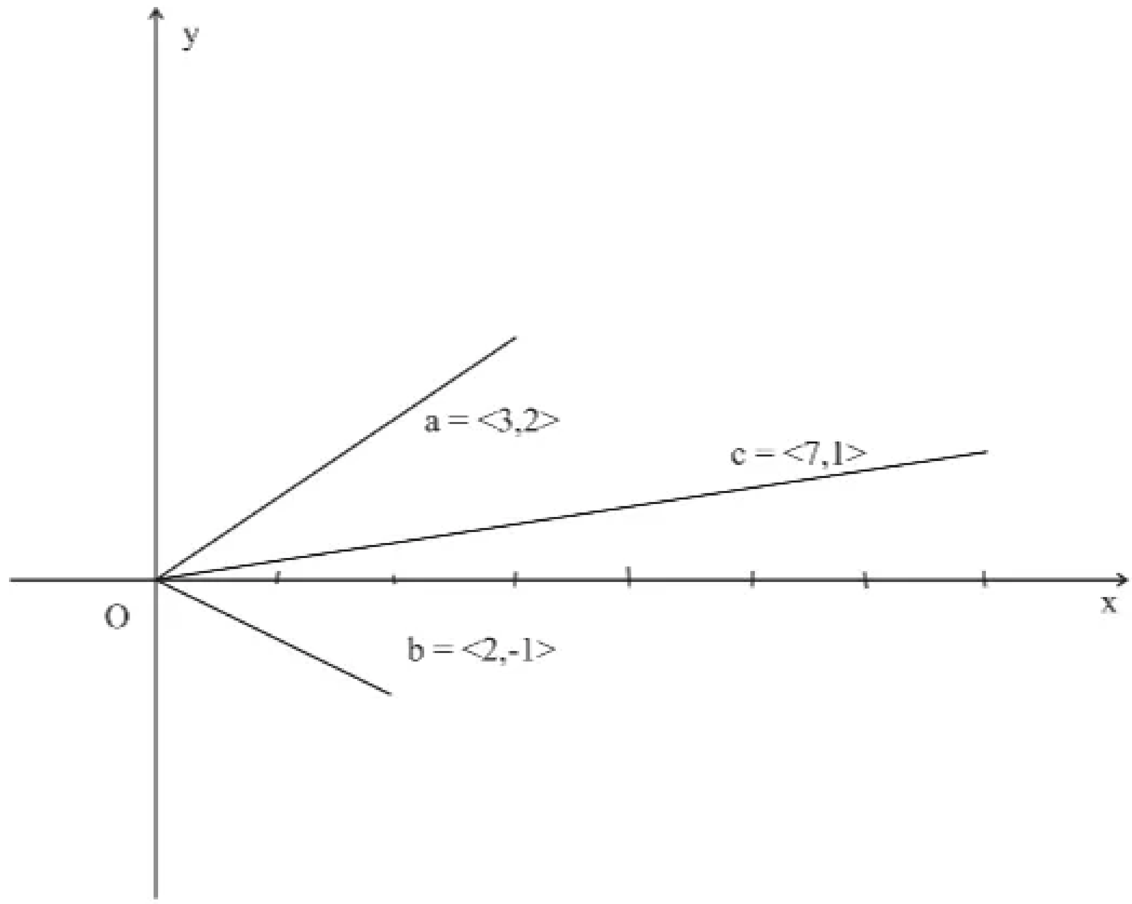
Or $2\vec{a}+\vec{b}=3\vec{c}$

Or $3\vec{c}=2\vec{a}+\vec{b}$

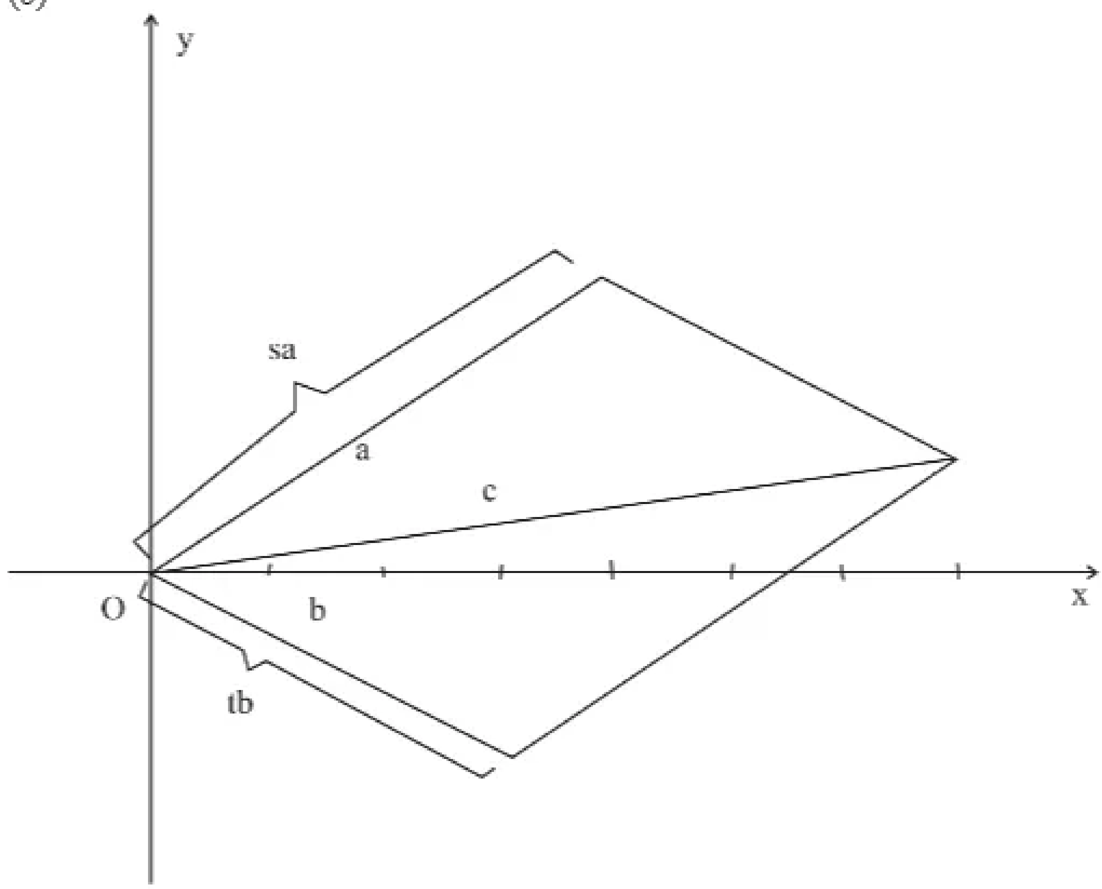
Or $\boxed{\vec{c}=\frac{2}{3}\vec{a}+\frac{1}{3}\vec{b}}$

Answer 45E.

(a)



(b)



(c)

From the sketch in part (b) we find that $s = 1.2$
And $t = 1.5$

(d)

Since $\vec{c} = s\vec{a} + t\vec{b}$
Then $\langle 7,1\rangle = s\langle 3,2\rangle + t\langle 2,-1\rangle$
i.e. $\langle 7,1\rangle = \langle 3s, 2s\rangle + \langle 2t, -1t\rangle$
i.e. $\langle 7,1\rangle = \langle 3s+2t, 2s-t\rangle$
$\Rightarrow 7 = 3s + 2t$
And $1 = 2s - t$
On solving these two equations simultaneously
We find $s = \frac{9}{7}$ and $t = \frac{11}{7}$

Hence $\boxed{s = \frac{9}{7} \quad \text{and} \quad t = \frac{11}{7}}$

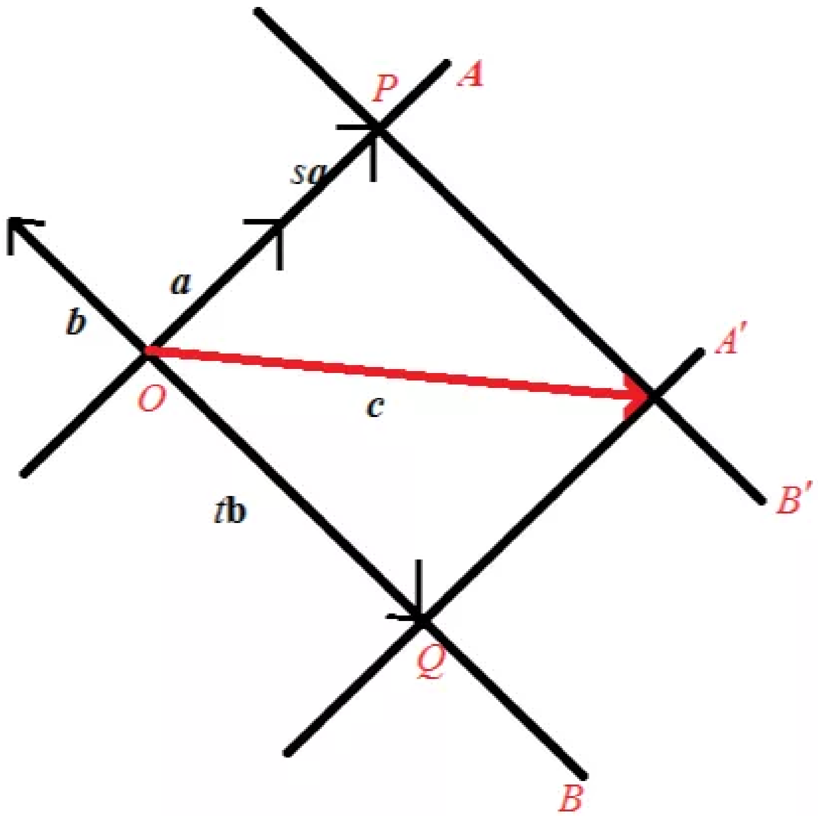
Answer 46E.

Consider the following vectors:

Vectors are non-zeros that are not parallel,

The vector is any vector in the plane determined by **a and b**.

Draw **a**, **b** and **c** emanating from the origin. Extend **a and b** to form lines A and B, and draw lines A' and B' parallel to these two lines through the terminal point of **c**.



As **a** and **b** are not parallel, A and B' must meet at P and A' and B must also meet at Q.

So

$$OP + OQ = c.$$

If $s = \frac{|OP|}{|\mathbf{a}|}$ and $t = \frac{|OQ|}{|\mathbf{b}|}$, then it follows that:

$$c = sa + tb.$$

Let the vectors **a**, **b** and **c** be $a = \langle a_1, a_2 \rangle, b = \langle b_1, b_2 \rangle$ and $c = \langle c_1, c_2 \rangle$.

So, the system of linear equations is $c_1 = sa_1 + tb_1$ and $c_2 = sa_2 + tb_2$.

$$c_1 = sa_1 + tb_1 \quad \rightarrow a_2c_1 = sa_1a_2 + tb_1a_2$$
$$c_2 = sa_2 + tb_2 \quad \rightarrow \underline{a_1c_2 = sa_1a_2 + ta_1b_2}$$
$$t = \frac{c_2a_1 - c_1a_2}{a_1b_2 - b_1a_2}, \quad a, b \neq 0$$

In same way:

$$c_1 = sa_1 + tb_1 \quad \rightarrow b_2c_1 = sa_1b_2 + tb_1b_2$$
$$c_2 = sa_2 + tb_2 \quad \rightarrow \underline{b_1c_2 = sb_1a_2 + tb_1b_2}$$
$$s = \frac{b_2c_1 - b_1c_2}{a_1b_2 - b_1a_2}$$

Hence, **a** is not scalar multiple of **b** and the denominate is not zero.

Answer 47E.

$$\vec{r} = \langle x, y, z \rangle$$
$$\vec{r_0} = \langle x_0, y_0, z_0 \rangle$$

Then $\quad \vec{r} - \vec{r_0} = \langle x - x_0, y - y_0, z - z_0 \rangle$

and therefore $\left|\vec{r} - \vec{r_0}\right| = 1$ *is*

$$\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} = 1$$

Squaring both sides,

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = 1$$

which is a sphere with centre (x_0, y_0, z_0) and radius 1.

Hence the set of all points (x,y,z) such that $\left|\vec{r} - \vec{r_0}\right| = 1$ is a sphere with radius 1 and centre at (x_0, y_0, z_0).

Answer 48E.

Consider the following vectors:

$\mathbf{r} = \langle x, y \rangle, \mathbf{r}_1 = \langle x_1, y_1 \rangle$, and $\mathbf{r}_2 = \langle x_2, y_2 \rangle$

Describe the set of all points (x, y) such that $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2| = k$, where $k > |\mathbf{r}_1 - \mathbf{r}_2|$.

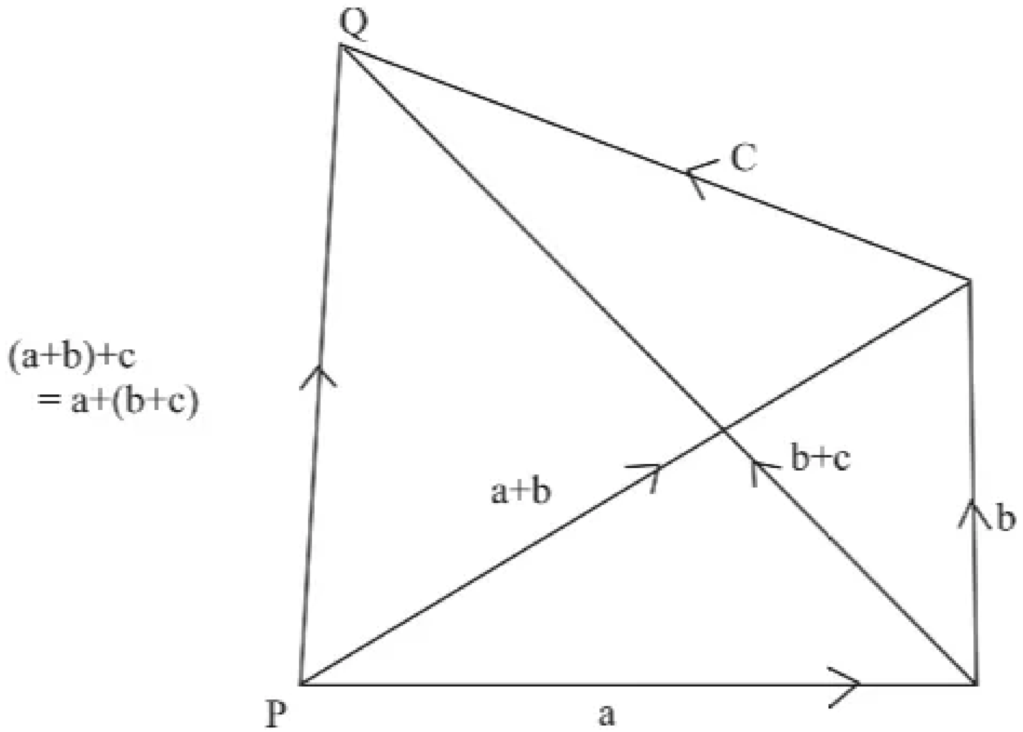
Let $\mathbf{r}_1$ and $\mathbf{r}_2$ represent the position vector of points *A* and *B*.

Then, $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2|$ is the sum of the distances from (x, y) to *A* and *B*.

Since this sum is constant, the set of points (x, y) represents an ellipse with foci *A* and *B*.

The condition $k > |\mathbf{r}_1 - \mathbf{r}_2|$ assures that the ellipse is not degenerate.

Answer 49E.



Let $\vec{a} = \langle a_1, a_2 \rangle$, $\vec{b} = \langle b_1, b_2 \rangle$ and $\vec{c} = \langle c_1, c_2 \rangle$

Then $\vec{a} + (\vec{b} + \vec{c}) = \langle a_1, a_2 \rangle + (\langle b_1, b_2 \rangle + \langle c_1, c_2 \rangle)$

$$= \langle a_1, a_2 \rangle + \langle b_1 + c_1, b_2 + c_2 \rangle$$

$$= \langle a_1 + (b_1 + c_1), a_2 + (b_2 + c_2) \rangle$$

$$= \langle (a_1 + b_1) + c_1, (a_2 + b_2) + c_2 \rangle$$

(Because of associative property in algebra)

$$= \langle a_1 + b_1, a_2 + b_2 \rangle + \langle c_1 + c_2 \rangle$$

$$= (\vec{a} + \vec{b}) + \vec{c}$$

Hence $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

Answer 50E.

Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$

And $\vec{b} = \langle b_1, b_2, b_3 \rangle$

Let c be a scalar, then consider

$$c(\vec{a} + \vec{b}) = c(\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle)$$

$$= c\langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$= \langle c(a_1 + b_1), c(a_2 + b_2), c(a_3 + b_3) \rangle$$

$$= \langle ca_1 + cb_1, ca_2 + cb_2, ca_3 + cb_3 \rangle$$

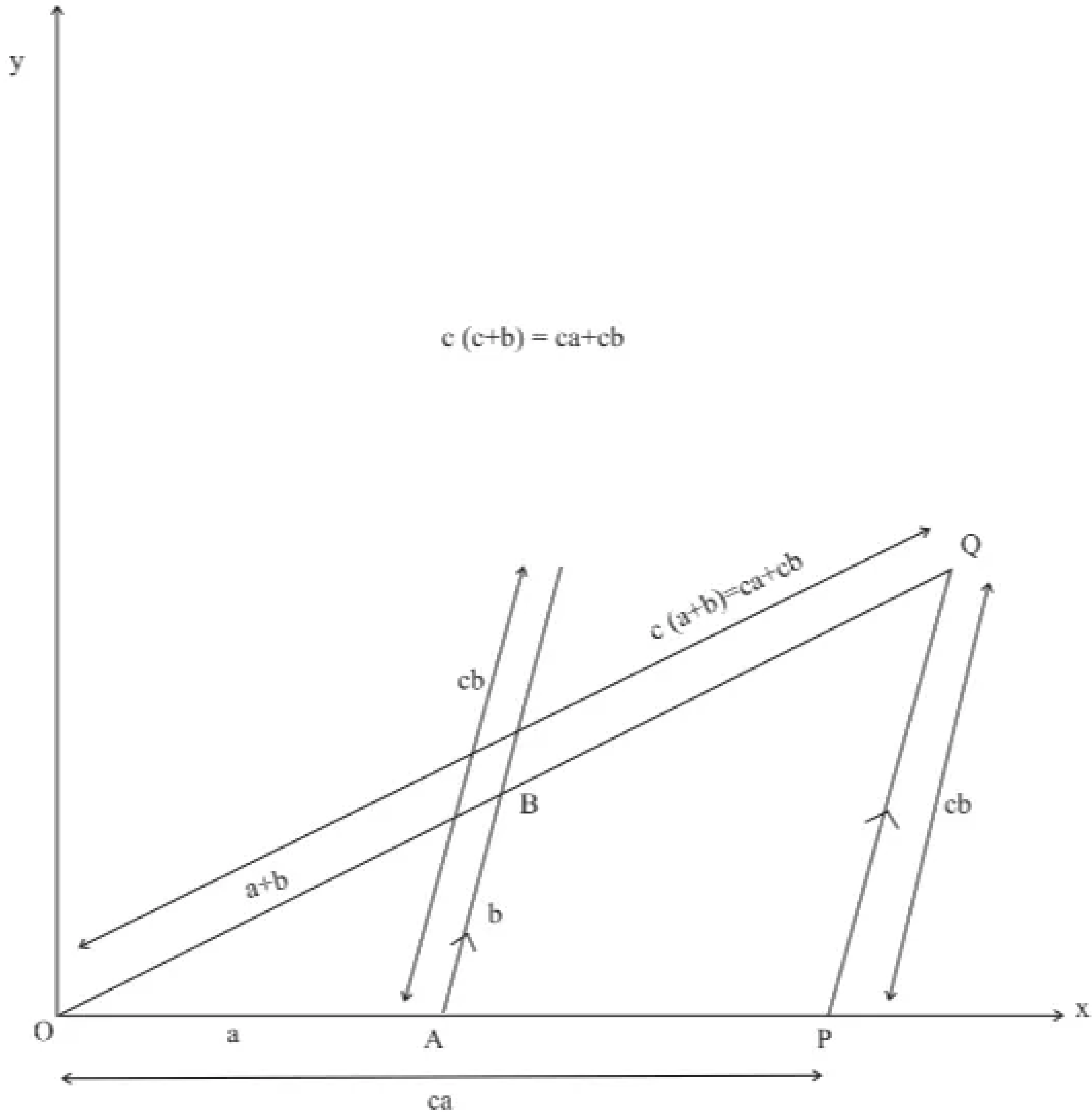
(by distributive property in algebra)

$$= \langle ca_1, ca_2, ca_3 \rangle + \langle cb_1, cb_2, cb_3 \rangle$$

$$= c\langle a_1, a_2, a_3 \rangle + c\langle b_1, b_2, b_3 \rangle$$

$$= c\vec{a} + c\vec{b}$$

Hence $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$



(when c = 2)

Let $\overrightarrow{OA} = \vec{a}, \qquad \overrightarrow{OB} = \vec{b}$

Then by triangle law

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

i.e. $\overrightarrow{OB} = \vec{a} + \vec{b}$

Let $\overrightarrow{OQ} = c\left(\vec{a} + \vec{b}\right)$, c is a scalar.

From Q draw QP parallel to AB meeting OA produced in P.

Now the triangles OAB and OPQ are similar. Then their sides are proportional.

i.e. $\frac{OP}{OA}=\frac{PQ}{AB}=\frac{PQ}{OB}=c$ $\left\{\begin{array}{l} as\ OQ=c(a+b) \\ Then\ \frac{OQ}{OB}=\frac{c(a+b)}{(a+b)}=c \end{array}\right.$

i.e. $\frac{OP}{a}=\frac{PQ}{b}=\frac{OQ}{a+b}=c$

i.e. $OP=ca,\quad PQ=cb,\quad OQ=c(a+b)$

Or $\overrightarrow{OP}=c\vec{a},\quad \overrightarrow{PQ}=c\vec{b},\quad \overrightarrow{OQ}=c(\vec{a}+\vec{b})$

Now in ΔOPQ, by triangle law,

$$\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}$$

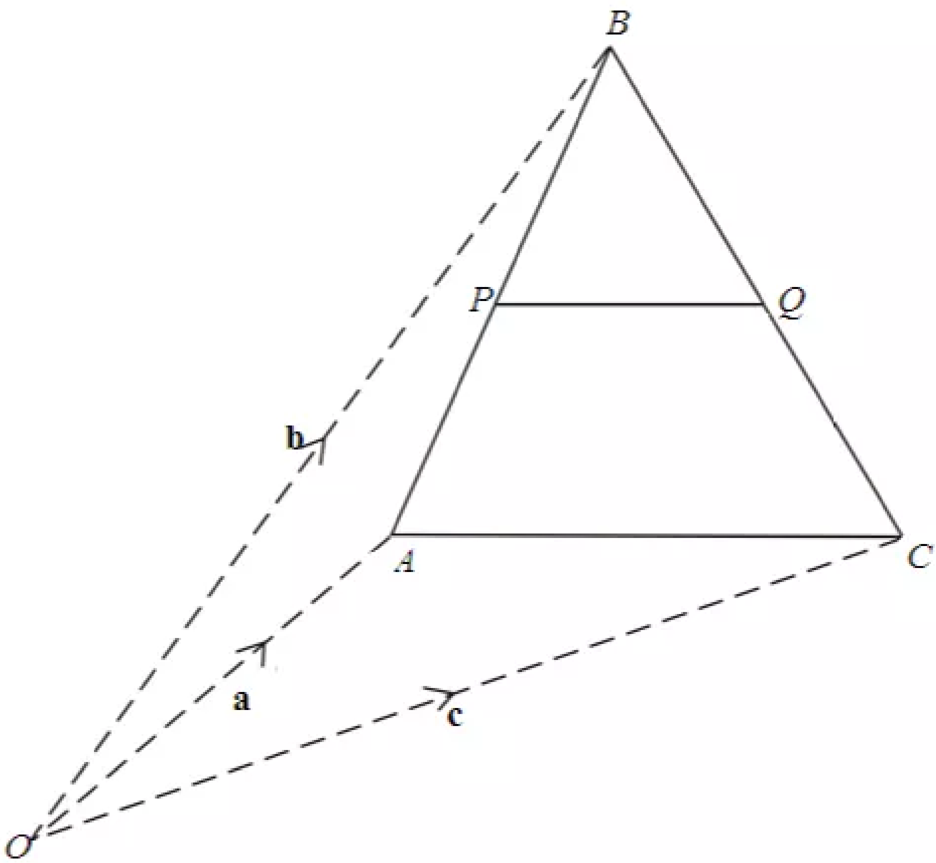
i.e. $$\boxed{c(\vec{a}+\vec{b})=c\vec{a}+c\vec{b}}$$

Answer 51E.

Let ΔABC be a triangle where the position vector of A that is $\overline{OA}=\mathbf{a}$, position vector of B is $\overline{OB}=\mathbf{b}$ and position vector of C is $\overline{OC}=\mathbf{c}$.

Suppose that $\overline{PQ}$ be the line joining the midpoints of the sides $\overline{AB}$ and $\overline{BC}$.

Sketch the triangle with line joining the midpoints with their position vectors.



Let P and Q be the mid points of sides AB and BC respectively

Then, the position vector of P is the midpoint of $\mathbf{a}$ and $\mathbf{b}$.

$$\overrightarrow{OP} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$$

And the position vector of Q is the midpoint of $\mathbf{b}$ and $\mathbf{c}$,

$$\overrightarrow{OQ} = \frac{1}{2}(\mathbf{b} + \mathbf{c})$$

Find the vector for the line joining P and Q, $\overline{PQ}$

Then, by triangle law,

$$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= \frac{1}{2}(\vec{b} + \vec{c}) - \frac{1}{2}(\vec{a} + \vec{b})$$

$$= \frac{1}{2}(\vec{c} - \vec{a})$$

As position vector of *C* is $\overline{OC} = \mathbf{c}$ and the position vector of *A* is $\overline{OA} = \mathbf{a}$, then the vector representation of $\overrightarrow{AC}$ is

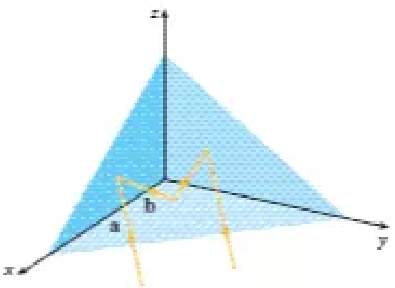
$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$ Distance from *A* to *C*

$$= \vec{c} - \vec{a}$$

As $\overrightarrow{AC}$ and $\overrightarrow{PQ}$ are proportional, $\overrightarrow{PQ}$ is parallel to $\overrightarrow{AC}$.

Thus, the line joining the mid-point of two sides of a triangle is parallel to third side and half its length.

Answer 52E.

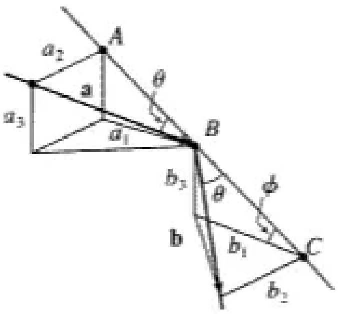


The three coordinate planes are all mirrored and a light ray given by the vector $a = \langle a_1, a_2, a_3 \rangle$ first strikes the xz-plane, as shown in the figure.

Since the light ray strikes all three mirrors, then it is not parallel to any of them and we have that $a \neq \langle 0, 0, 0 \rangle$, that is $a_1 \neq 0,\ a_2 \neq 0,\ a_3 \neq 0$.

Use the fact that the angle of incidence equals the angle of reflection to show that the direction of the reflected ray is given by $b = \langle a_1, -a_2, a_3 \rangle$

Let $b = \langle b_1, b_2, b_3 \rangle$ and $|b| = |a|$ because only the direction changes.



Since $|b| = |a|$ then $|AB| = |BC|$

$\sin\theta = \frac{|a_2|}{|a|}$ and $\sin\theta = \frac{|b_2|}{|b|}$

$$\frac{|a_2|}{|a|} = \frac{|b_2|}{|b|}$$

$\left|b_2\right| = \frac{|a_2||b|}{|a|}$, using $|b| = |a|$

$$|b_2| = |a_2|$$

$a_2 j$ and $b_2 j$ point in opposite directions.

Now

$\sin\phi = \frac{|b_3|}{|BC|}$ and $\sin\phi = \frac{|a_3|}{|AB|}$

$$\frac{|b_3|}{|BC|} = \frac{|a_3|}{|AB|}$$

$\cos\phi = \frac{|b_1|}{|BC|}$ and

$$\frac{|b_1|}{|BC|} = \frac{|a_1|}{|AB|}$$

Using $|AB| = |BC|$

$$|b_3| = |a_3|$$

$$|b_1| = |a_1|$$

$b_1 i$ and $a_1 i$ have the same direction.

$b_3 k$ and $a_3 k$ have the same direction.

So we have that

$|b_1| = |a_1|$ and $b_1 i$ and $a_1 i$ have the same direction.

$|b_2| = |a_2|$ and $a_2 j$ and $b_2 j$ have opposite directions.

$|b_3| = |a_3|$ and $b_3 k$ and $a_3 k$ have the same direction.

So, we conclude that $b = \langle b_1, b_2, b_3 \rangle = \langle a_1, -a_2, a_3 \rangle$

Deduce that, after being reflected by all three mutually perpendicular mirrors, the resulting ray is parallel to the initial ray.

When the ray hits the other mirrors, we have that

$|b_1| = |a_1|$ and $b_1 i$ and $a_1 i$ have opposite directions.

$|b_2| = |a_2|$ and $a_2 j$ and $b_2 j$ have opposite directions.

$|b_3| = |a_3|$ and $b_3 k$ and $a_3 k$ have opposite directions.

That is because the reflections revers the sings of b_1 and b_3.

So, we conclude that $b = \langle b_1, b_2, b_3 \rangle = \langle -a_1, -a_2, -a_3 \rangle = -a$

We know that **a** is parallel to -**a**, therefore **b** is parallel to **a**.