CHAPTER 17 HEIGHT AND DISTANCE

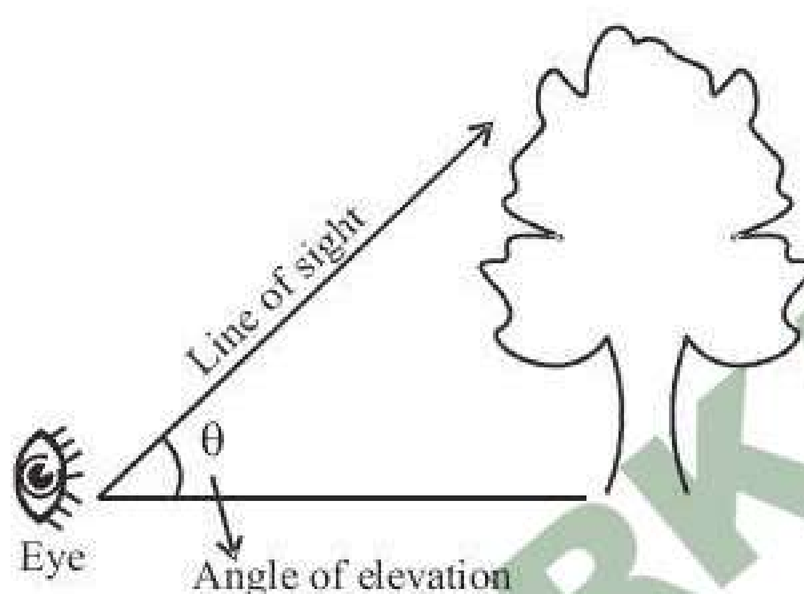
HEIGHT AND DISTANCE

Sometimes, we have to find the height of a tower, building, tree, distance of a ship, width of a river, etc. Though we cannot measure them easily, we can determine these by using trigonometric ratios.

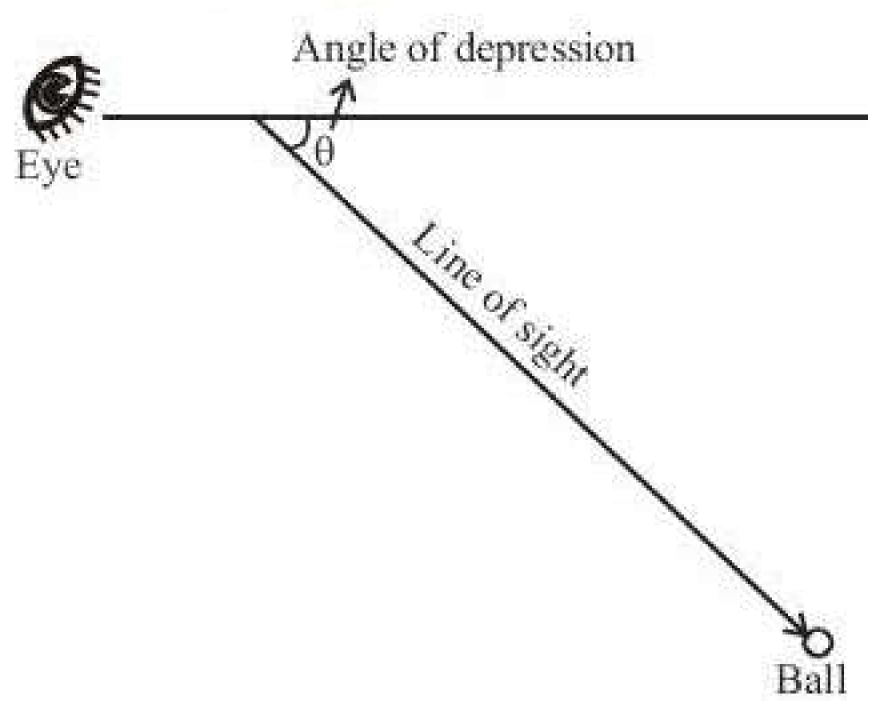
Line of Sight

The line of sight or the line of vision is a straight line from our eye to the object we are viewing.

If the object is above the horizontal from the eye, we have to lift up our head to view the object. In this process, our eye move through an angle. This angle is called **the angle of elevation** of the object.



If the object is below the horizontal from the eye, then we have to turn our head downwards to view the object. In this process, our eye move through an angle. This angle is called **the angle of depression** of the object.

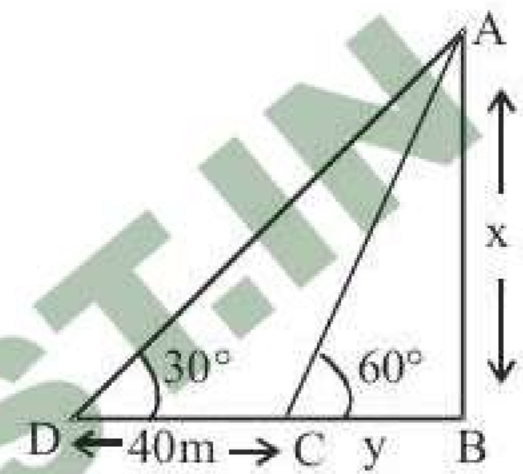


Example 1: A person observed the angle of elevation of the top of a tower is 30°. He walked 40 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60° Find the height of tower.

Solution :

Let height of tower AB = x m and BC = y m, DC = 40 m.

In ΔABC,



$$\frac{AB}{BC} = \tan 60° \Rightarrow \frac{x}{y} = \sqrt{3} \Rightarrow y = \frac{x}{\sqrt{3}} \quad ....(i)$$

Now In rt Δ ABD, $\frac{AB}{BD} = \tan 30°$

$$\Rightarrow \frac{x}{40+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}x = 40 + y \Rightarrow \sqrt{3}x = 40 + \frac{x}{\sqrt{3}} \quad \text{[using (i)]}$$

$$\Rightarrow 3x = 40\sqrt{3} + x \Rightarrow 3x - x = 40\sqrt{3} \Rightarrow 2x = 40\sqrt{3}$$

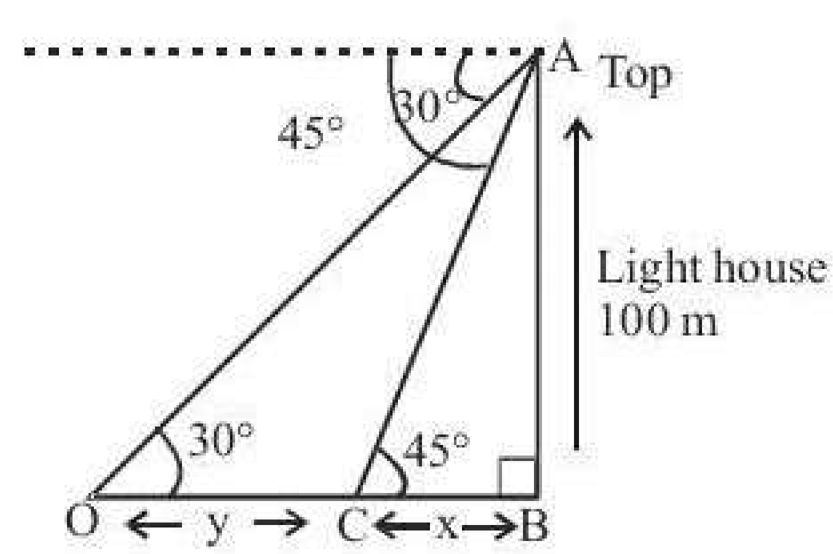
$$x = 20\sqrt{3}m$$

Example 2: As observed from top of a light house 100 m. high above sea level, the angle of depression of a ship sailing directly toward it changes from 30° to 45°. The distance travelled by the ship during the period of observation is

Solution :

Let 'y' be the required distance between two positions O and C of the ship

In rt. ΔABC,



$\cot 45° = \frac{x}{100} \Rightarrow x = 100$ ...(i)

In ΔAOB, $\frac{y+x}{100} = \cot 30°$

$\Rightarrow y + x = 100\sqrt{3} \Rightarrow y = 100\sqrt{3} - x$

$\Rightarrow y = 100\sqrt{3} - 100$ [using (i)]

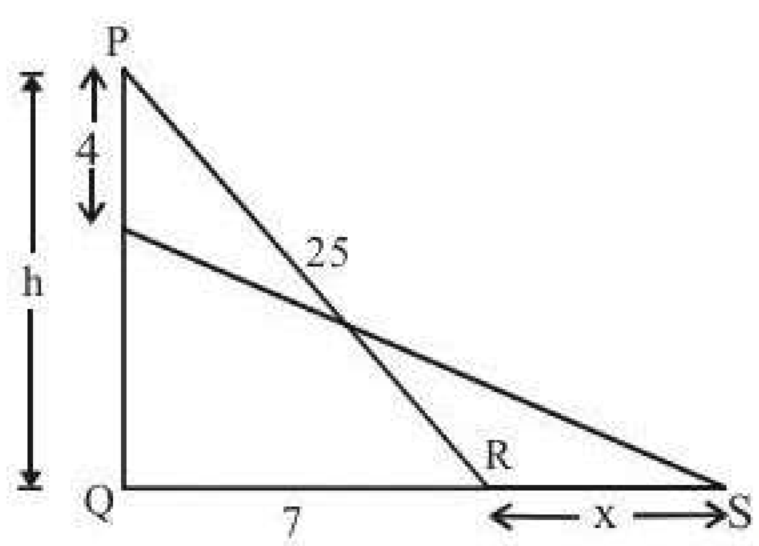
$\Rightarrow y = 100(\sqrt{3} - 1)$

$\Rightarrow y = 100(1.732 - 1) = 100 \times 0.732 = 73.20\,m.$

Example 3 : A 25 m long ladder is placed against a vertical wall of a building. The foot of the ladder is 7 m from the base of the building. If the top of the ladder slips 4 m, then the foot of the ladder will slide by how much distance.

Solution :

Let the height of the wall be h.



Now, $h = \sqrt{25^2 - 7^2}$

$= \sqrt{576} = 24m$

$QS = \sqrt{625 - 400}$

$= \sqrt{225} = 15m$

Required distance, $x = (15 - 7) = 8$ m

Example 4: A man is standing on the deck of a ship, which is 8m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

Solution :

Let x be the distance of hill from man and $h + 8$ be height of hill which is required.

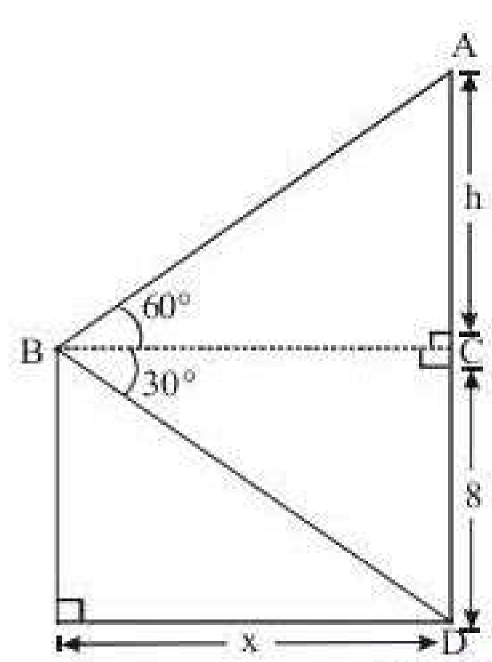
In rt. ΔACB,

$\tan 60° = \frac{AC}{BC} = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

In rt. ΔBCD,

$\tan 30° = \frac{CD}{BC} = \frac{8}{x}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{x} \Rightarrow x = 8\sqrt{3}$

$\therefore$ Height of hill $= h + 8 = \sqrt{3}x + 8 = (\sqrt{3})(8\sqrt{3}) + 8 = 32$ m

Distance of ship from hill $= x = 8\sqrt{3}$ m

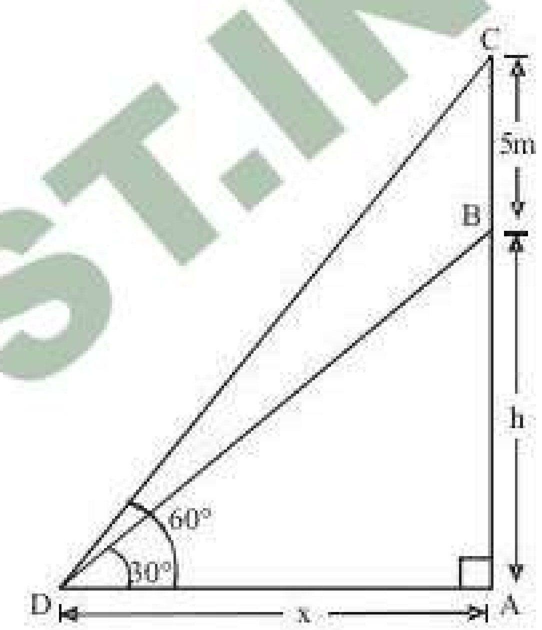
Example 5: A vertical to stands on a horizontal plane and is surmounted by a vertical flag staff of height 6 meters. At point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60°. Find the height of tower.

Solution :

Let AB be the tower of height h meter and BC be the height of flag staff surmounted on the tower.

Let the point of the plane be D at a distance m meter from the foot of the tower.

In ΔABD,



$\tan 30° = \frac{AB}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h$ .... (i)

In Δ ADC,

$\tan 60° = \frac{AC}{AD} \Rightarrow \sqrt{3} = \frac{5+h}{x} \Rightarrow x = \frac{5+h}{\sqrt{3}}$ .... (ii)

From (i) and (ii), $\sqrt{3}h = \frac{5+h}{\sqrt{3}} \Rightarrow 3h = 5 + h \Rightarrow 2h = 5 \Rightarrow$

$h = \frac{5}{2} = 2.5$ m

So, the height of tower = 2.5 m

Example 6: The angles of depressions of the top and bottom of 8m tall building fron the top of a multistoried building are 30° and 45° respectively. Find the height of multistoried building and the distance between the two buildings.

Solution :

Let AB be the multistoried building of height h and let the distance between two buildings be x meters.

$\angle XAC = \angle ACB = 45°$ (Alternate angles)

$\angle XAD = \angle ADE = 30°$ (Alternate angles)

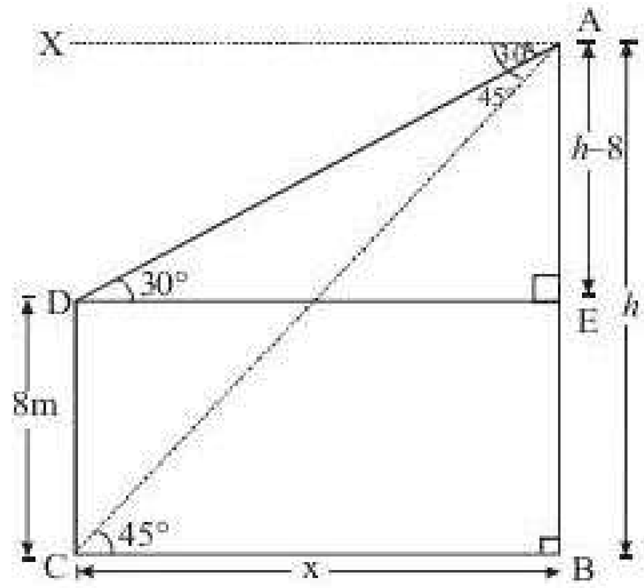
In ΔADE, $\tan 30° = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$

[$\because CB = DE = x$]

$\Rightarrow x = \sqrt{3}(h - 8)$ .... (i)

In ΔACB,

$\tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \Rightarrow = h$ .... (ii)



From (i) and (ii),

$$\sqrt{3}\,(h-8)=h \Rightarrow \sqrt{3}h-8\sqrt{3}=h$$

$$\Rightarrow \sqrt{3}h-h=8\sqrt{3} \Rightarrow h(\sqrt{3}-1)=8\sqrt{3}$$

$$\Rightarrow h=\frac{8\sqrt{3}}{\sqrt{3}-1}\times\frac{(\sqrt{3}+1)}{\sqrt{3}+1} \Rightarrow h=\frac{8\sqrt{3}(\sqrt{3}+1)}{2}$$

$$\Rightarrow h=4\sqrt{3}(\sqrt{3}+1) \Rightarrow h=4(3+\sqrt{3}) \text{ metres}$$

From (ii), $x=h$

So, $x=4(3+\sqrt{3})$ metres

Hence, height of multistoried building = $4\,(3+\sqrt{3})$ metres

distance between two building = $4\,(3+\sqrt{3})$ metres.

Example 7: The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 sec, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Solution :

Let the point on the ground is E which is y metres from point B and let after 15 sec. flight it covers x metres distance

In ΔAEB, $\tan 45^\circ=\frac{AB}{EB}$

$$\Rightarrow 1=\frac{3000}{y} \Rightarrow y=3000\text{m} \quad \text{.............(i)}$$

In ΔCED, $\tan 30^\circ=\frac{CD}{ED} \Rightarrow \frac{1}{\sqrt{3}}=\frac{3000}{x+y}$

$(\because AB=CD)$

$$\Rightarrow x+y=3000\sqrt{3} \quad \text{.............(ii)}$$

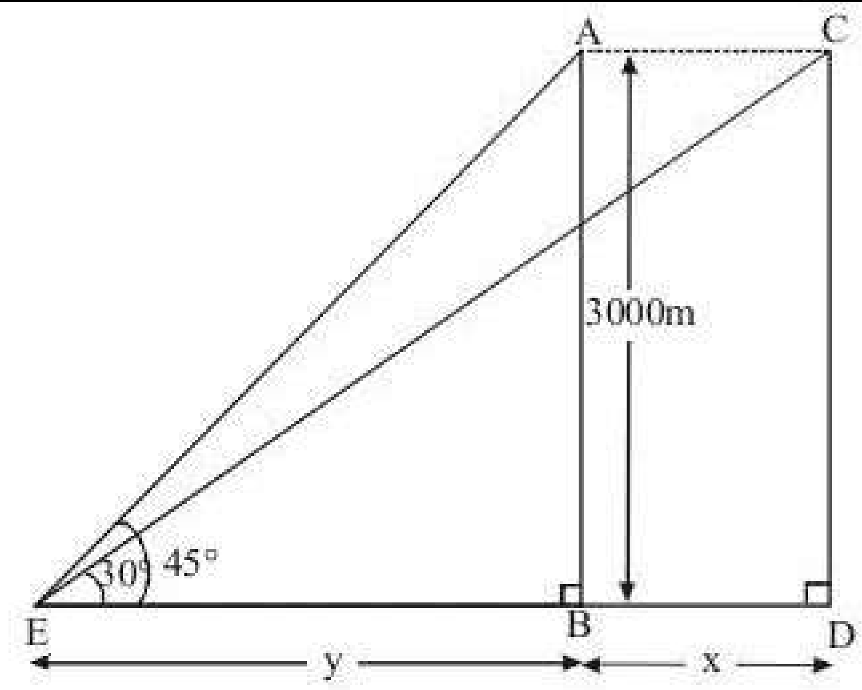
From eqs. (i) and (ii)

$$x+3000=3000\sqrt{3} \Rightarrow x=3000\sqrt{3}-3000$$

$$\Rightarrow x=3000\,(\sqrt{3}-1) \Rightarrow x=3000\times(1.732-1)$$

$$\Rightarrow x=3000\times 0.732 \Rightarrow x=2196\text{m}$$

Speed of aeroplane $=\dfrac{\text{Distance covered}}{\text{Time taken}}$



$$=\frac{2196}{15}\text{m/sec}=146.4\text{ m/sec}$$

$$=\frac{2196}{15}\times\frac{18}{5}\text{km/hr}=527.04\text{ km/hr}$$

Hence, the speed of aeroplane is 527.04 km/hr.

Example 8: A boy is standing on the ground and flying a kite with 100m of string at an elevation of 30°. Another boy is standing on the roof of a 10m high building and is flying his at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Solution :

Let the length of second string be x m.

In ΔABC,

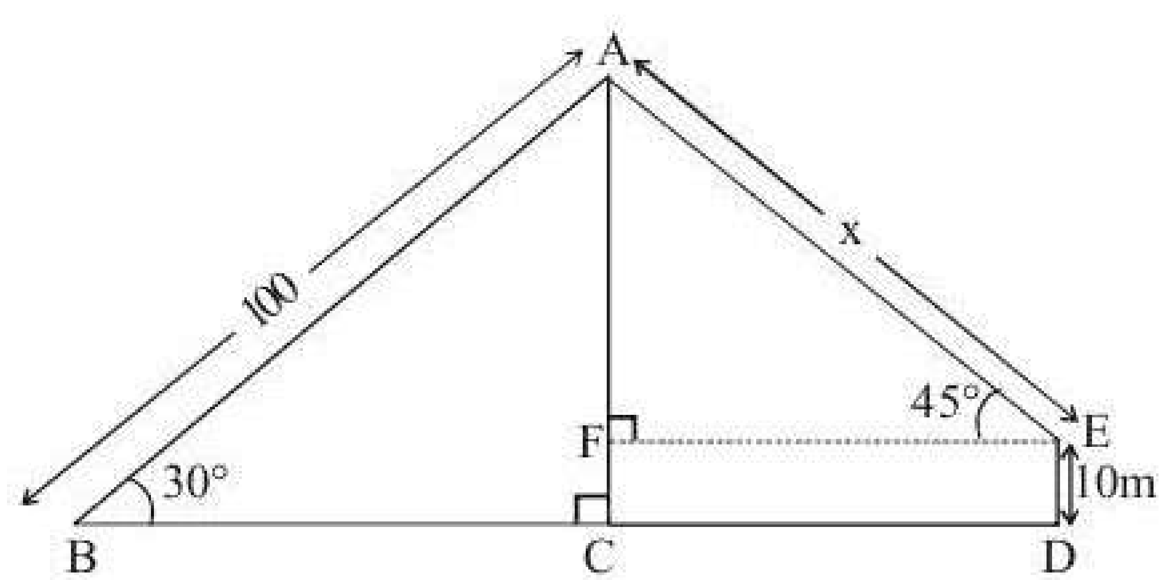
$$\sin 30^\circ=\frac{AC}{AB} \quad \text{or} \quad \frac{1}{2}=\frac{AC}{100} \Rightarrow AC=50\text{m}$$

In ΔAEF,

$$\sin 45^\circ=\frac{AF}{AE} \Rightarrow \frac{1}{\sqrt{2}}=\frac{AF-FC}{x}$$

$$\Rightarrow \frac{1}{\sqrt{2}}=\frac{50-10}{x} \quad [\because AC=50\text{m}, FC=ED=10\text{m}]$$

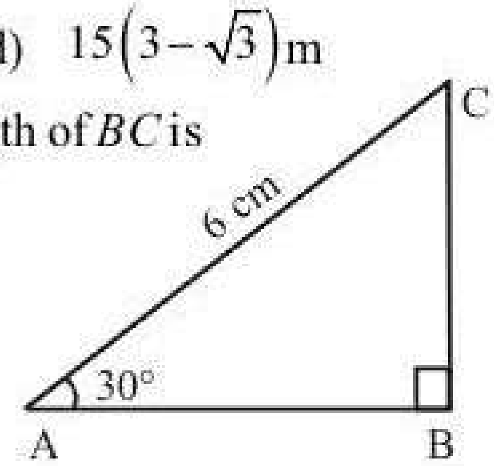
$$\Rightarrow \frac{1}{\sqrt{2}}=\frac{40}{x} \Rightarrow x=40\sqrt{2}\text{m}$$



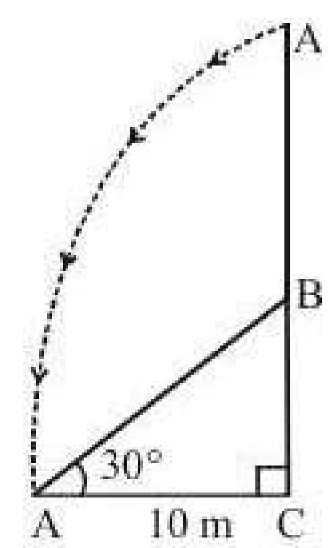
So the length of string that the second boy must have so that the two kites meet = $40\sqrt{2}$m

EXERCISE

1. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is 60°; when he retreates 20m from the bank, he finds the angle to be 30°. The height of the tree and the breadth of the river are –
   (a) $10\sqrt{3}$ m, 10 m (b) 10 m, $10\sqrt{3}$ m
   (c) 20 m, 30 m (d) None of these
2. What should be the height of a flag where a 20 feet long ladder reaches 20 feet below the flag (The angle of elevation of the top of the flag at the foot of the ladder is 60°)?
   (a) 20 feet (b) 30 feet
   (c) 40 feet (d) $20\sqrt{2}$ feet
3. A vertical pole with height more than 100 m consists of two parts, the lower being one-third of the whole. At a point on a horizontal plane through the foot and 40 m from it, the upper part subtends an angle whose tangent is $\frac{1}{2}$. What is the height of the pole ?
   (a) 110m (b) 200m
   (c) 120m (d) 150m
4. The angular elevation of a tower CD at a place A due south of it is 60° ; and at a place B due west of A, the elevation is 30°. If AB = 3 km, the height of the tower is
   (a) $2\sqrt{3}$ km (b) $3\sqrt{6}$ km
   (c) $\frac{3\sqrt{3}}{2}$ km (d) $\frac{3\sqrt{6}}{4}$ km
5. The angles of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance 9 ft and 16 ft respectively are complementary angles. Then the height of the tower is
   (a) 9 ft (b) 12 ft
   (c) 16 ft (d) 144 ft
6. From 125 metre high towers, the angle of depression of a car is 45°. Then how far the car is from the tower ?
   (a) 125 metre (b) 60 metre
   (c) 75 metre (d) 95 metre
7. The shadow of a tower standing on a level plane is found to be 30 m longer when the Sun's altitude changes from 60° to 45°. The height of the tower is
   (a) $15\left(3+\sqrt{3}\right)$m (b) $15\left(\sqrt{3}+1\right)$m
   (c) $15\left(\sqrt{3}-1\right)$m (d) $15\left(3-\sqrt{3}\right)$m
8. In the adjoining figure, the length of BC is
   (a) $2\sqrt{3}$ cm
   (b) $3\sqrt{3}$ cm
   (c) $4\sqrt{3}$ cm
   (d) 3 cm



9. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is
   (a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m
   (c) $75\sqrt{3}$ m (d) 150 m
10. The angle of elevation of the top of a tower at point on the ground is 30°. If on walking 20 metres toward the tower, the angle of elevation become 60°, then the height of the tower is
    (a) 10 metre (b) $\frac{10}{\sqrt{3}}$ metre
    (c) $10\sqrt{3}$ metre (d) None of these
11. The top of a broken tree has its top touching the ground (shown in the adjoining figure) at a distance of 10 *m* from the bottom. If the angle made by the broken part with ground is 30°, then the length of the broken part is
    (a) $10\sqrt{3}$ cm (b) $\frac{20}{\sqrt{3}}$ m
    (c) 20 cm (d) $20\sqrt{3}$ m



12. An aeroplane flying horizontally 1 km. above the ground is observed at an elevation of 60° and after 10 seconds the elevation is observed to be 30°. The uniform speed of the aeroplane in km/h is
    (a) 240 (b) $240\sqrt{3}$
    (c) $60\sqrt{3}$ (d) None of these
13. If the length of the shadow of a tower is $\sqrt{3}$ times that of its height, then the angle of elevation of the sun is
    (a) 15° (b) 30°
    (c) 45° (d) 60°
14. An aeroplane at a height of 600 m passes vertically above another aeroplane at an instant when their angles of elevation at the same observing point are 60° and 45° respectively. How many metres higher is the one from the other ?
    (a) 286.53 m (b) 274.53 m
    (c) 253.58 m (d) 263.83 m
15. A person of height 2m wants to get a fruit which is on a pole of height $\left(\frac{10}{3}\right)$m. If he stands at a distance of $\left(\frac{4}{\sqrt{3}}\right)$m

from the foot of the pole, then the angle at which he should throw the stone, so that it hits the fruit is

(a) 60° (b) 45°
(c) 90° (d) 30°

16. A straight highway leads to the foot of a 50m tall tower. From the top of the tower the angles of depression of two cars on the highway are 30° and 60°. What is the distance between the two cars?

(a) $\frac{100}{\sqrt{3}}$m (b) $100\sqrt{3}$ m
(c) 86.50m (d) None of these

17. From the top of a pillar of height 20m the angles of elevation and depression of the top and bottom of another pillar are 30° and 45° respectively. The height of the second pillar (in metre) is:

(a) $\frac{20}{\sqrt{3}}\left(\sqrt{3}-1\right)$ m (b) $\frac{20}{\sqrt{3}}\left(\sqrt{3}+1\right)$ m
(c) $20\sqrt{3}$ m (d) $\frac{20}{\sqrt{3}}$ m

18. At a point P on the ground, the angle of elevation of the top of a 10m tall building and of a helicopter hovering some distance over the top of the building are 30° and 60° respectively. Then, the height of the helicopter above the ground is

(a) $\frac{10}{\sqrt{3}}$m (b) $10\sqrt{3}$ m
(c) $\frac{20}{\sqrt{3}}$ m (d) 30m

19. There is a small island in the middle of a 100 m wide river. There is a tall tree on the island. Points P and Q are points directly opposite to each other on the two banks and in line with the tree. If the angles of elevation of the top of the tree at P and Q are 30° and 45°, then the height of tree is :

(a) $50(\sqrt{3}-1)$m (b) $50\sqrt{3}$ m
(c) $50(\sqrt{3}+1)$m (d) $\frac{100}{\sqrt{3}-1}$m

20. If the angles of a tower from two points distant a and b ($b > a$) from its foot and in the same straight line from it are 60° and 30°, then the height of the tower is:

(a) $\sqrt{a-b}$ (b) $\sqrt{b-a}$
(c) $\sqrt{ab}$ (d) $\sqrt{\frac{a}{b}}$

21. The angles of depression of two ships from the top of the light house are 45° and 30° towards east. If the ships are 100 m apart. then the height of the light house is :

(a) $50(\sqrt{3}-1)$m (b) $50(\sqrt{3}+1)$m
(c) $50(\sqrt{3})$m (d) $\frac{50}{\sqrt{3}-1}$m

22. A balloon of radius r makes an angle α at the eye of an observer and the angle of elevation of its centre is β. The height of its centre from the ground level is given by:

(a) $r \sin\beta \operatorname{cosec}\alpha/2$ (b) $r \operatorname{cosec}\alpha/2 \sin\alpha$
(c) $r \operatorname{cosec}\alpha \sin\beta$ (d) None of these

23. The angle of elevation 'θ' of the top of a lilght house at a point 'A' on the ground is such that $\tan\theta = \frac{5}{12}$. When the point is moved 240m towards the light house, the angle of elevation becomes ϕ such that $\tan\phi = \frac{3}{4}$. Then the height of light house is:

(a) 225m (b) 200m
(c) 215m (d) 235m

24. At the foot of a mountain the elevation of its summit is 45°; after ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°, Find the height of the mountain:

(a) $\frac{\sqrt{3}+1}{2}$km (b) $\frac{\sqrt{3}-1}{2}$km
(c) $\frac{\sqrt{3}}{2}$km (d) $\frac{1}{\sqrt{3}}$km

25. The angles of elevation of the top of a building and the top of the chimney on the roof of the building from a point on the ground are x and 45° respectively. The height of building is h metre. Then the height of the chimney, in metre, is :

(a) $h \cot x + h$ (b) $h \cot x - h$
(c) $h \tan x - h$ (d) $h \tan x + h$

26. From a lighthouse the angles of depression of two ships on opposite sides of the lighthouse are observed to 30° and 45°. If the height of lighthouse is h, what is the distance between the ships?

(a) $(\sqrt{3}+1)h$ (b) $(\sqrt{3}-1)h$
(c) $\sqrt{3}h$ (d) $\left(1+\frac{1}{\sqrt{3}}\right)h$

27. On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30° to 45°. The height of the chimney is.

(a) 120m (b) $60(\sqrt{3}-1)$m
(c) $60(\sqrt{3}+1)$m (d) None of these

DIRECTIONS (Q. Nos. 28-32) : *Read the following information carefully to answer the questions that follow.*

As seen from the top and bottom of a building of height h m, the angles of elevation of the top of a tower of height $\frac{(3+\sqrt{3})h}{2}$ m are α and β, respectively.

28. If $\beta = 30°$, then what is the value of $\tan\alpha$?

(a) 1/2 (b) 1/3
(c) 1/4 (d) None of these

29. If $\alpha = 30^o$, then what is the value of $\tan \beta$?
(a) 1 (b) 1/2
(c) 1/3 (d) None of these

30. If $\alpha = 30^o$ and $h = 30$ m, then what is the distance between the base of the building and the base of the tower?
(a) $15 + 15\sqrt{3}$ m (b) $30 + 15\sqrt{3}$ m
(c) $45 + 15\sqrt{3}$ m (d) None of these

31. If $\beta = 30^o$ and if θ is the angle of depression of the foot of the tower as seen from the top of the building, then what is the value of $\tan \theta$?
(a) $\frac{(3-\sqrt{3})}{3\sqrt{3}}$ (b) $\frac{(3+\sqrt{3})}{3\sqrt{3}}$
(c) $\frac{(2-\sqrt{3})}{3\sqrt{3}}$ (d) None of these

32. A spherical balloon of radius r subtends angle 60^o at the eye of an observer. If the angle of elevation of its centre is 60^o and h is the height of the centre of the balloon, then which one of the following is correct?
(a) $h = r$ (b) $h = \sqrt{2}r$
(c) $h = \sqrt{3}r$ (d) $h = 2r$

33. What is the angle of elevation of the Sun, when the shadow of a pole of height x m is $\frac{x}{\sqrt{3}}$ m?
(a) 30^o (b) 45^o
(c) 60^o (d) 75^o

34. The heights of two trees are x and y, where $x > y$. The tops of the trees are at a distance z apart. If s is the shortest distance between the trees, then what is s^2 equal to?
(a) $x^2 + y^2 - z^2 - 2xy$
(b) $x^2 + y^2 - z^2$
(c) $x^2 + y^2 + z^2 - 2xy$
(d) $z^2 - x^2 - y^2 + 2xy$

35. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30^o, then when it is 60^o. What is the height of the tower?
(a) 25m (b) $25\sqrt{3}$ m
(c) $\frac{25}{\sqrt{3}}$ m (d) 30m

36. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60^o and the angle of elevation of the top of the second tower from the foot of the first tower is 30^o. The distance between the two towers in m times the height of the shorter tower. What is m equal to?
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

37. From a certain point on a straight road, a person observe a tower in the West direction at a distance of 200 m. He walks some distance along the road and finds that the same tower is 300 m South of him. What is the shortest distance of the tower from the road ?
(a) $\frac{300}{\sqrt{13}}m$ (b) $\frac{500}{\sqrt{13}}m$
(c) $\frac{600}{\sqrt{13}}m$ (d) $\frac{900}{\sqrt{13}}m$

38. The angles of elevation of the top of a tower from two points P and Q at distances m^2 and n^2 respectively, from the base and in the same straight line with it are complementary. The height of the tower is **(CDS)**
(a) $(mn)^{1/2}$ (b) $mn^{1/2}$
(c) $m^{1/2}n$ (d) mn

39. The angle of elevation of a cloud from a point 200 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. The height of the cloud is **(CDS)**
(a) 200 m (b) 300 m
(c) 400 m (d) 600 m

40. From the top of a tower, the angles of depression of two objects P and Q (situated on the ground on the same side of the tower) separated at a distance of $100(3-\sqrt{3})m$ are 45° and 60° respectively. The height of the tower is **(CDS)**
(a) 200 m (b) 250 m
(c) 300 m (d) None of the above

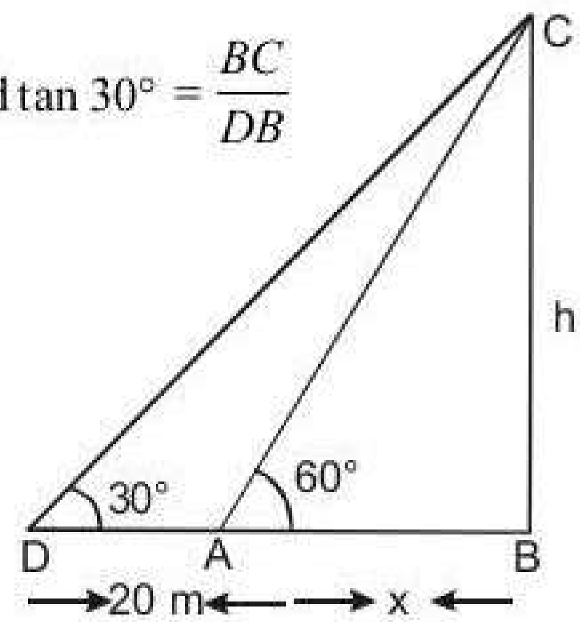
41. An aeroplane flying at a height of 3000 m passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from some point on the ground are 60° and 45° respectively. Then the vertical distance between the two planes is **(CDS)**
(a) $1000(\sqrt{3}-1)$m (b) $1000\sqrt{3}$m
(c) $1000(3-\sqrt{3})$m (d) $3000\sqrt{3}$m

42. A pole is standing erect on the ground which is horizontal. The tip of the pole is tied tight with a rope of length $\sqrt{12}$ m to a point on the ground. If the rope is making 30° with the horizontal, then the height of the pole is **(CDS)**
(a) $2\sqrt{3}$m (b) $3\sqrt{2}$m
(c) 3m (d) $\sqrt{3}$m

43. Two observers are stationed due north of a tower (of height x metre) at a distance y metre from each other. The angles of elevation of the tower observed by them are 30° and 45° respectively. Then x/y is equal to **(CDS)**
(a) $\frac{\sqrt{2}-1}{2}$ (b) $\frac{\sqrt{3}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) 1

HINTS & SOLUTIONS

1. (a) From right angled Δs ABC and DBC, we have



$$\tan 60° = \frac{BC}{AB} \text{ and } \tan 30° = \frac{BC}{DB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

and $\frac{1}{\sqrt{3}} = \frac{h}{x+20}$

$\Rightarrow h = x\sqrt{3}$

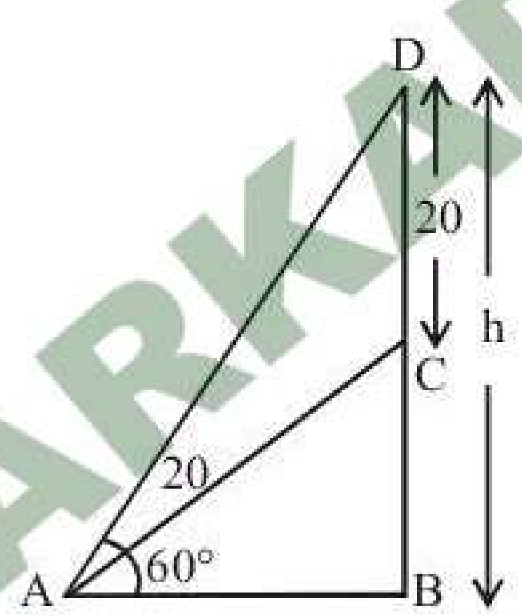
and $h = \frac{x+20}{\sqrt{3}}$

$$\Rightarrow x\sqrt{3} = \frac{x+20}{\sqrt{3}} \Rightarrow 3x = x + 20 \Rightarrow x = 10 \text{ m}$$

Putting x = 10 in $h = \sqrt{3}\,x$, we get $h = 10\sqrt{3}$

Hence, height of the tree = $10\sqrt{3}$ m and the breadth of the river = 10 m.

2. (b) In ΔABD,



$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AB}$$

$$\Rightarrow AB = \frac{h}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{h}{3}\sqrt{3}$$

Now, in ΔABC

$AC^2 = AB^2 + BC^2$

$$\Rightarrow 20^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (h-20)^2$$

$\Rightarrow h^2 + 3h^2 - 120h = 0$

$\Rightarrow 4h^2 - 120h = 0$

$\Rightarrow h(h-30) = 0$

$h = 0$ or 30

$h = 0$ not possible

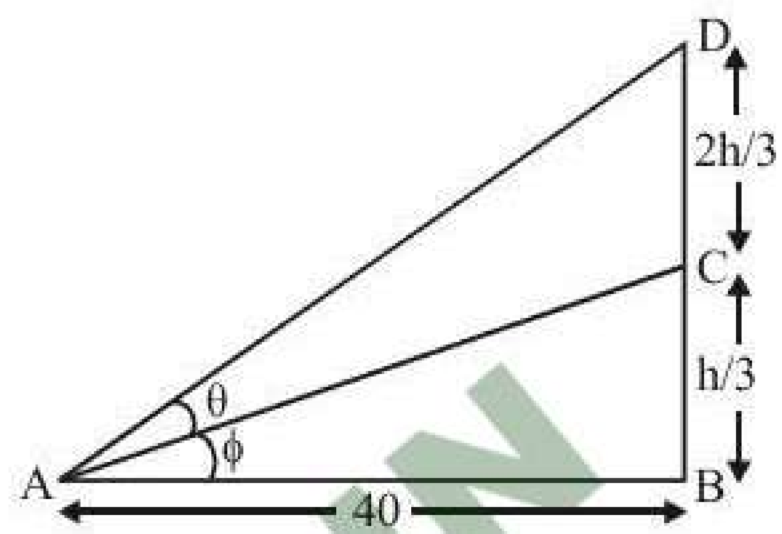
$\Rightarrow h = 30$ ft

3. (c) Let h be the height of pole, upper portion CD subtend angle θ at A.

Then, $\tan\theta = \frac{1}{2}$

Let lower part BC subtend angle ϕ at A then

In ΔABC,



$$\tan\phi = \frac{BC}{AB} = \frac{h/3}{40} = \frac{h}{120}$$

In ΔABD,

$$\tan(\theta+\phi) = \frac{BD}{AB}$$

$$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{h}{40}$$

$$\Rightarrow \frac{\frac{1}{2} + \frac{h}{120}}{1 - \frac{h}{240}} = \frac{h}{40}$$

$$\Rightarrow \frac{2(60+h)}{(240-h)} = \frac{h}{40}$$

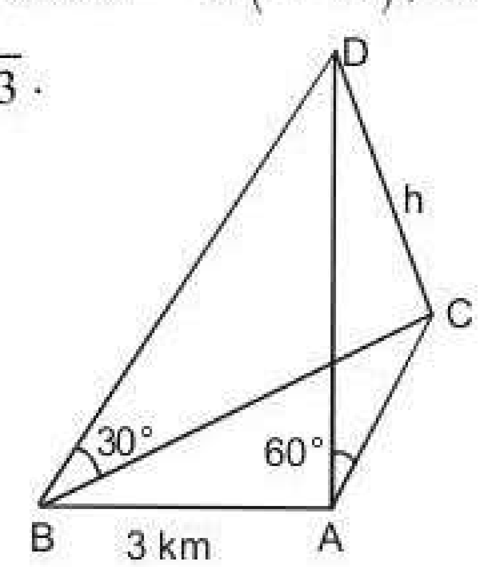
$\Rightarrow 80(60+h) = 240h - h^2 \Rightarrow 4800 + 80h = 240h - h^2$

$\Rightarrow h^2 - 160h + 4800 = 0 \Rightarrow (h-120)(h-40) = 0$

$\Rightarrow h = 120$

4. (d) In Δ ACD, we get AC = h cot 60° = $h.(1/\sqrt{3})$, In ΔBCD, BC = h cot 30° = $h\sqrt{3}$.

Therefore, from right-angled triangle BAC, we have



$BC^2 = AB^2 + AC^2$

$$\Rightarrow \left(h\sqrt{3}\right)^2 = (3)^2 + \left(\frac{h}{\sqrt{3}}\right)^2$$

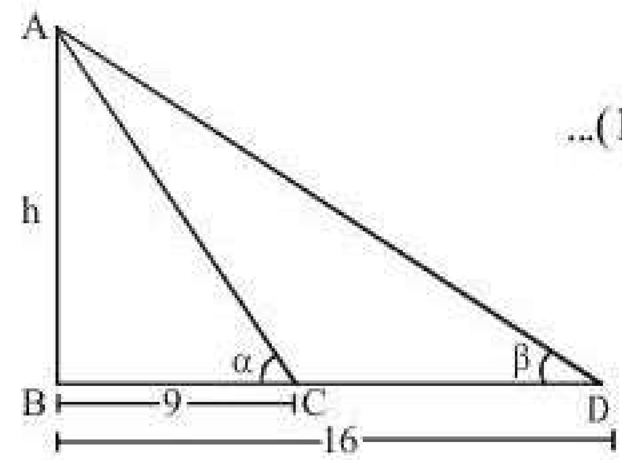
$$\Rightarrow 3h^2 = 9 + \frac{h^2}{3} \Rightarrow \frac{8}{3}h^2 = 9$$

$$\Rightarrow h^2 = \frac{27}{8}$$

$$\Rightarrow h = \frac{3\sqrt{3}}{2\sqrt{2}}\text{ km} = \frac{3\sqrt{6}}{4}\text{ km}$$

5. (b) In ΔABC

$\tan\alpha = \frac{h}{9}$ ...(1)



In ΔABD

$\tan\beta = \frac{h}{16}$

$\alpha + \beta = 90°$ (given)
$\beta = 90 - \alpha$

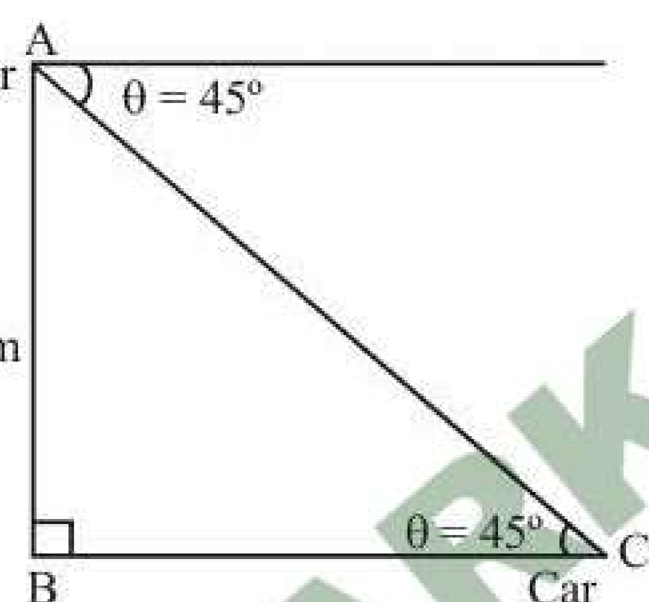
since $\tan\beta = \frac{h}{16}$

$\tan(90-\alpha) = \frac{h}{16} \Rightarrow \cot\alpha = \frac{h}{16}$ or $\tan\alpha = \frac{16}{h}$ ...(2)

From eqn. (1) and (2)

$\frac{h}{9} = \frac{16}{h} \Rightarrow h^2 = 16\times 9 \Rightarrow h = 12$ feet.

6. (a)

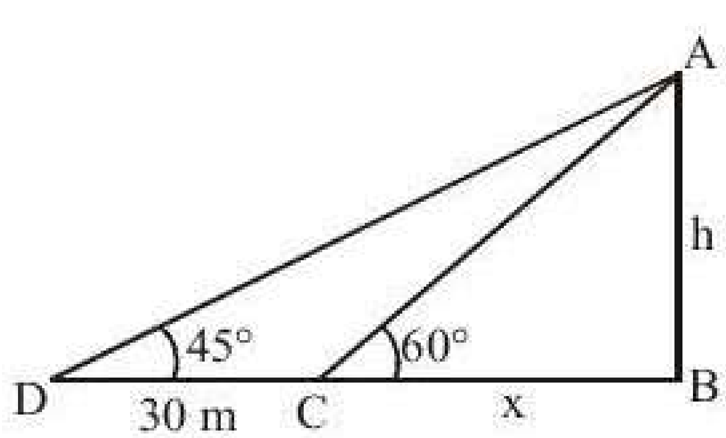


In ΔABC

$\tan\theta = \frac{AB}{BC} \Rightarrow \tan 45° = \frac{125}{BC} \Rightarrow 1 = \frac{125}{BC}$

BC = 125 m
Hence, car is 125 m from the tower.

7. (a)



In ΔABC, $\tan 60° = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$ ...(1)

In ΔABD, $\tan 45° = \frac{h}{30+x}$

$1 = \frac{h}{30+x}$ or $h = 30 + x$

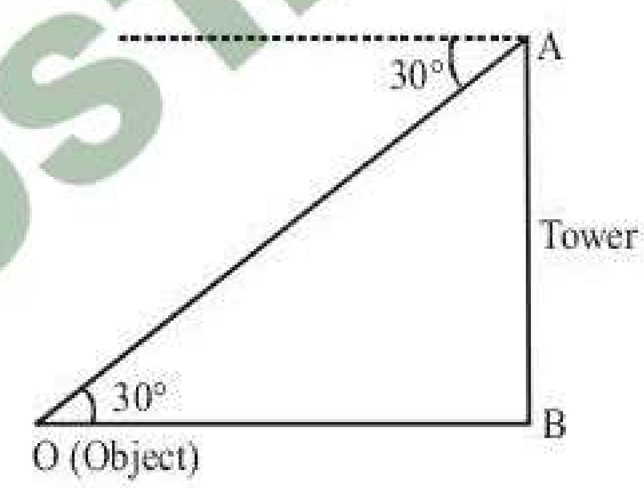
Putting value of x from (1)

$h = 30 + \frac{h}{\sqrt{3}}$

or $h\frac{(\sqrt{3}-1)}{\sqrt{3}} = 30 \Rightarrow h = 15\,(3+\sqrt{3})$ m

8. (d) $\sin 30° = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{BC}{6\text{ cm}} \Rightarrow BC = 3$ cm.

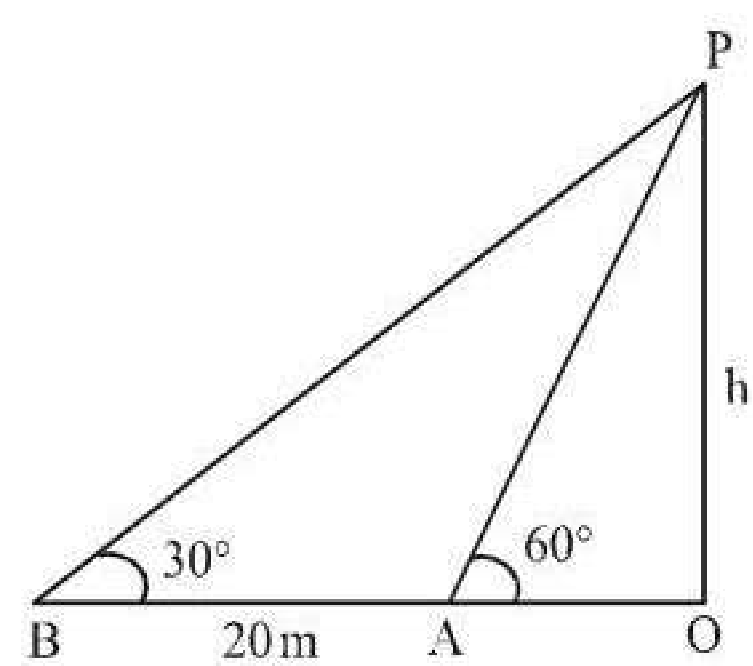
9. (c) $\tan 30° = \frac{AB}{OB}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75\text{ m}}{OB}$

$\Rightarrow OB = 75\sqrt{3}$ m

10. (c)



$OA = h\cot 60°$, $OB = h\cot 30°$
$OB - OA = 20 = h(\cot 30° - \cot 60°)$

$$\Rightarrow h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}$$

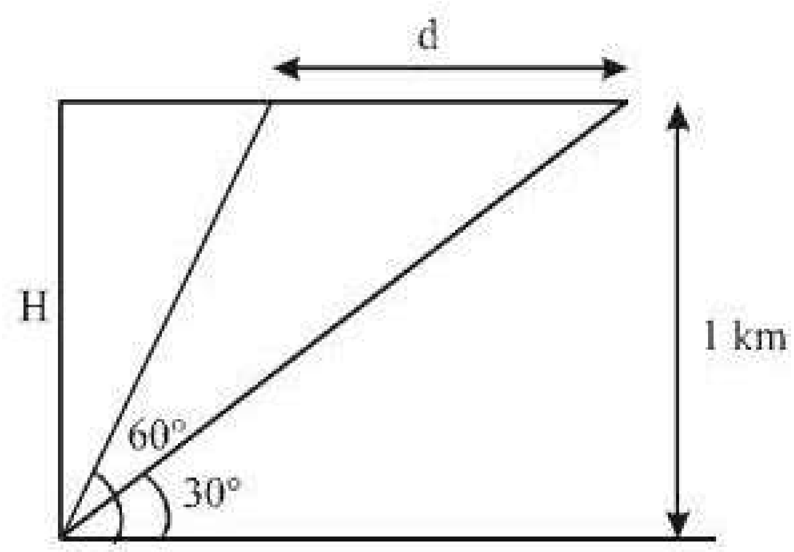
11. (b) $\cos 30° = \frac{AC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{10\text{ m}}{AB}$

$\Rightarrow AB = \frac{20}{\sqrt{3}}$ m.

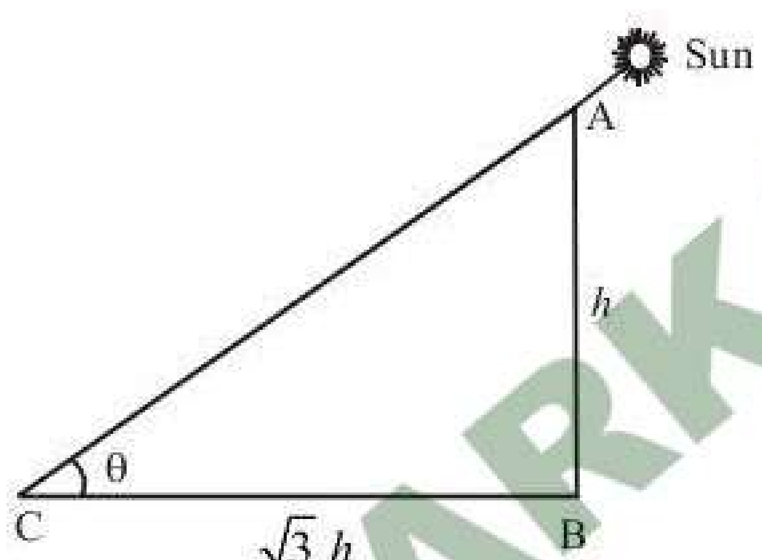
12. (b) $d = H\cot 30° - H\cot 60°$

Time taken = 10 second

$$\text{speed} = \frac{\cot 30° - \cot 60°}{10} \times 60 \times 60 = 240\sqrt{3}$$



13. (b) Let height of tower (AB) be h metres, then length of its shadow $(BC) = \sqrt{3}\,h$ metres.

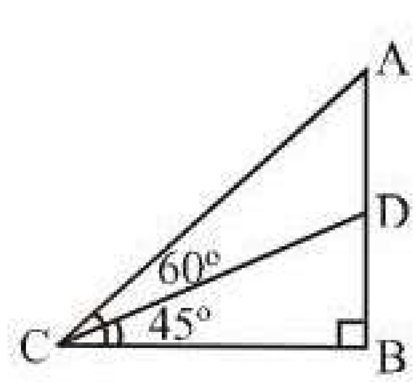


Let angle of elevation be θ,

then $\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

14. (c) Let the aeroplanes are at point A and D respectively. Aeroplane A is flying 600m above the ground.



So, AB = 600.

$\angle ACB = 60°, \angle DCB = 45°$

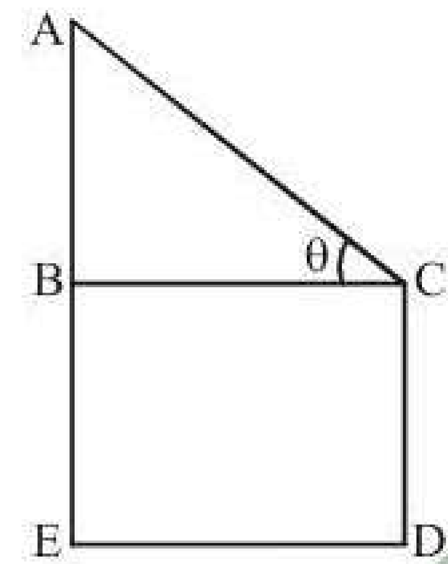
From ΔABC, $\frac{AB}{BC} = \tan 60° \Rightarrow BC = \frac{600}{\sqrt{3}} = 200\sqrt{3}$.

From ΔDCB, $\frac{DB}{BC} = \tan 45° \Rightarrow DB = 200\sqrt{3}$.

So, the distance $AD = AB - BD = 600 - 200\sqrt{3}$

$= 200(3 - \sqrt{3}) = 200(3 - 1.7321) = 253.58$m.

15. (d)



BE = CD = height of man = 2m

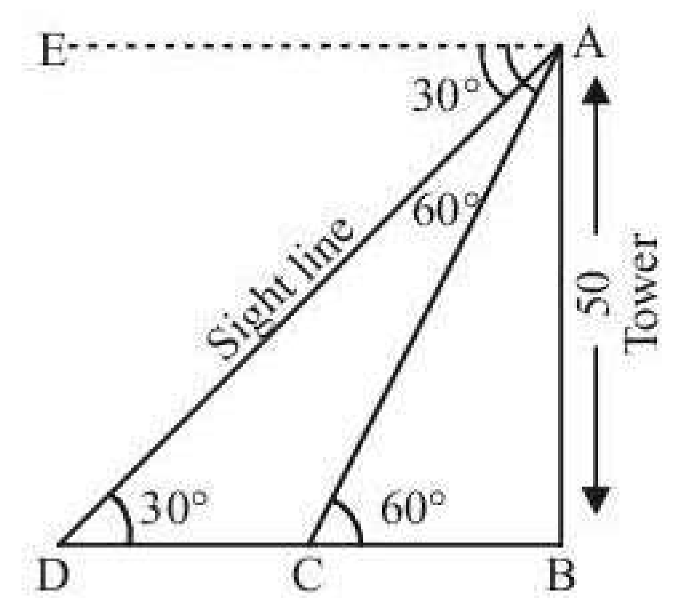
$$\tan\theta = \frac{AB}{BC} = \frac{\frac{10}{3} - 2}{\frac{4}{\sqrt{3}}} = \frac{4}{3} \times \frac{\sqrt{3}}{4} = \frac{1}{\sqrt{3}}$$

$= \tan\theta = 30° \Rightarrow \theta = 30°$

16. (a) Here, $\theta_1 = 60°$, $\theta_2 = 30°$

$h = 50$m

$DC = x$

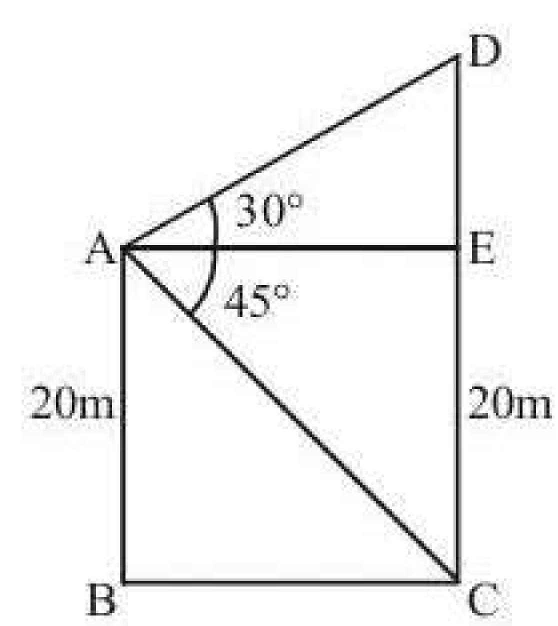


$$h = \frac{x}{\cot\theta_2 - \cot\theta_1}$$

$x = h(\cot\theta_2 - \cot\theta_1)$

$$= 50\left(\sqrt{3} - \frac{1}{3}\right) = \frac{50(2)}{\sqrt{3}} = \frac{100}{\sqrt{3}}\text{m}$$

17. (b)



Let AB and CD are pillars.
Let DE = h
From ΔADE, tan 30°

$$= \frac{h}{AE} = \frac{1}{\sqrt{3}} \Rightarrow AE = h\sqrt{3} \quad ...(i)$$

From ΔACE,

$$\tan 45° = \frac{20}{AE} \Rightarrow AE = 20$$

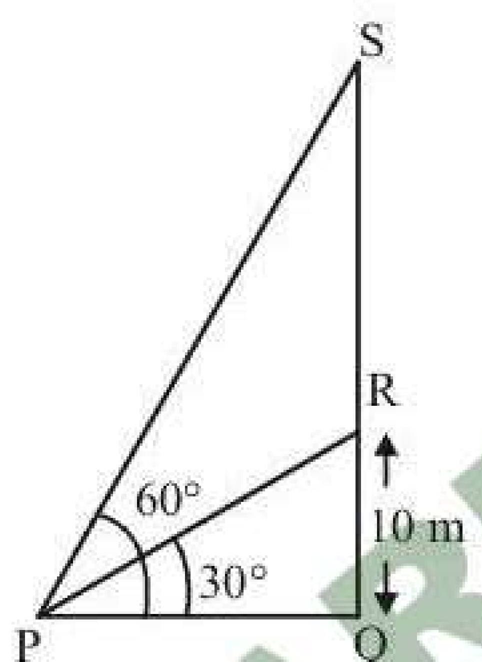
From equation (i),

$$20 = h\sqrt{3} \Rightarrow h = \frac{20}{\sqrt{3}}\text{m}$$

$\therefore$ Required height $= 20 + \frac{20}{\sqrt{3}} = \frac{20}{\sqrt{3}}\left(\sqrt{3}+1\right)$m

18. (d) Let RQ be the height of building, then RQ = 10m, S be the position of helicopter. Then
In ΔPQR,

$$\frac{RQ}{PQ} = \tan 30° \Rightarrow PQ = \frac{RQ}{\tan 30°} = 10\sqrt{3}$$



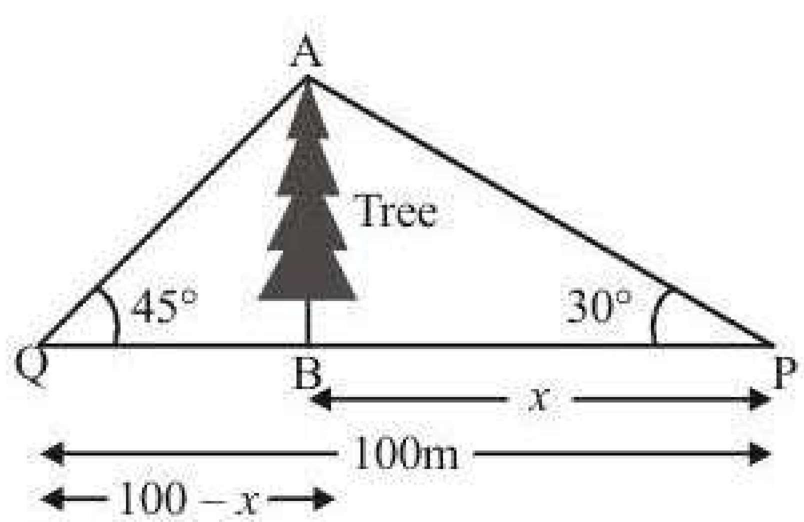
$\therefore$ In ΔSPQ, $\tan 60° = \frac{SQ}{PQ}$

$$\Rightarrow \frac{SQ}{PQ} = \sqrt{3}$$

$$\Rightarrow SQ = PQ \times \sqrt{3}$$

$$= 10\sqrt{3} \times \sqrt{3} = 30 \text{ m}$$

19. (a) Here height of tree = AB
In ΔAPB



$$\tan 30° = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{x}$$

or $x = \sqrt{3}$ AB

In ΔAQB, $\tan 45° = \frac{AB}{BQ} \Rightarrow \frac{AB}{100-x} = 1$

$\Rightarrow x = 100 - AB$
So from (i) and (ii)

$$\sqrt{3}\text{ AB} = 100 - AB \Rightarrow AB\left(\sqrt{3}+1\right) = 100$$

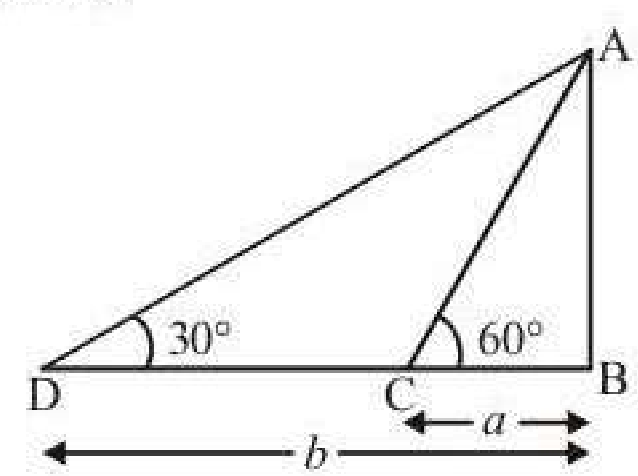
$$\Rightarrow AB = \frac{100}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 50(\sqrt{3}-1)$$

$\therefore$ Height of tree $= 50(\sqrt{3}-1)$ metre.

20. (c) Let AB be the tower such that
CB = a and BD = b
In ΔABC

$$\tan 60° \frac{AB}{BC} = \frac{AB}{a}$$

$\Rightarrow AB = a\sqrt{3}$
In ΔABD,

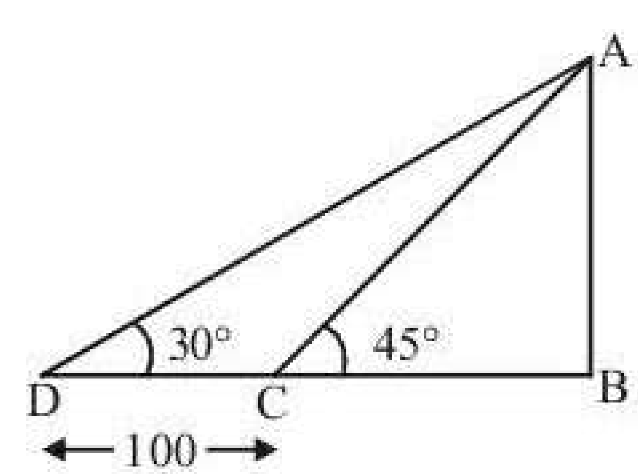


$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{b}$$

From equations (i) and (ii)
$(AB)^2 = ab$
$AB = \sqrt{ab}$

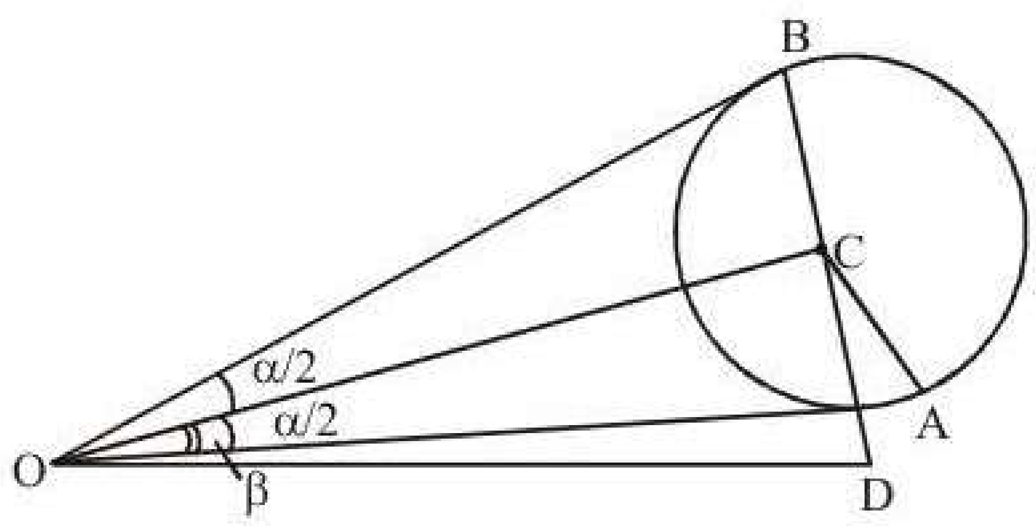
21. (b) Here let AB be the height of light house and D, C are the ships such that CD = 100 m.
Applying short cut method.



$$AB = \frac{100}{\cot 30° - \cot 45°} \Rightarrow AB = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$AB = \frac{100(\sqrt{3}+1)}{2} = 50(\sqrt{3}+1)\text{m}$$

22. (a) Let O be the position of the man's eye and C be the centre of the balloon.
In Δ COA



$\angle BOC = \angle COA = \frac{\alpha}{2}$ Here CA = BC = r

In right angled ΔCOD

$$\sin\beta = \frac{CD}{OC}$$

$\therefore$ CD = OC sin β ...(i)
In right angled ΔCOA. we have

$$\sin\beta = \frac{CA}{OC} \quad \therefore OC = \frac{r}{\sin\frac{\alpha}{2}} = r\ \text{cosec}\frac{\alpha}{2}$$

$\therefore$ So (i) become

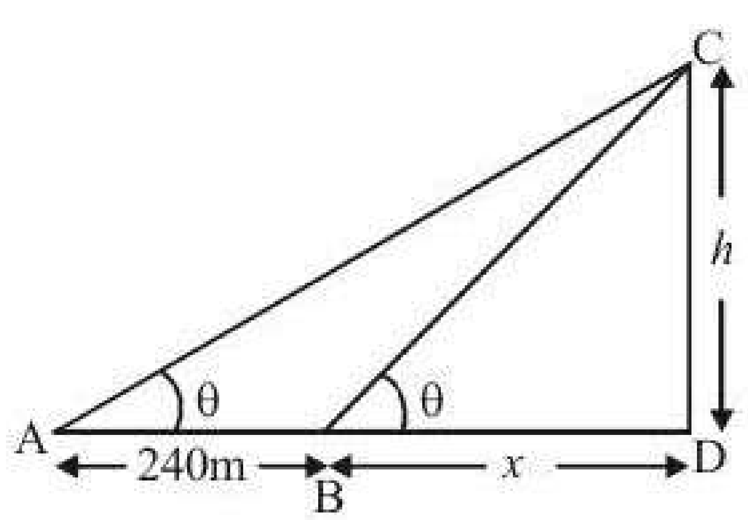
$$CD = r\ \text{cosec}\frac{\alpha}{2}\sin\beta$$

$\therefore$ Height of centre of the balloon is

$$r\sin\beta\ \text{cosec}\ \frac{\alpha}{2}$$

23. (a) Let CD be the height = h

In ΔADC $\tan\theta = \frac{h}{240+x}$



$$\frac{5}{12} = \frac{h}{240+x} \qquad \left\{\because \tan\theta = \frac{5}{12}\right\}$$

$\therefore 12h = 5(240+x)$ ...(i)
In ΔBDC, we have

$$\tan\phi = \frac{h}{x} \Rightarrow \frac{h}{x} = \frac{3}{4} \Rightarrow x = \frac{4}{3}h$$

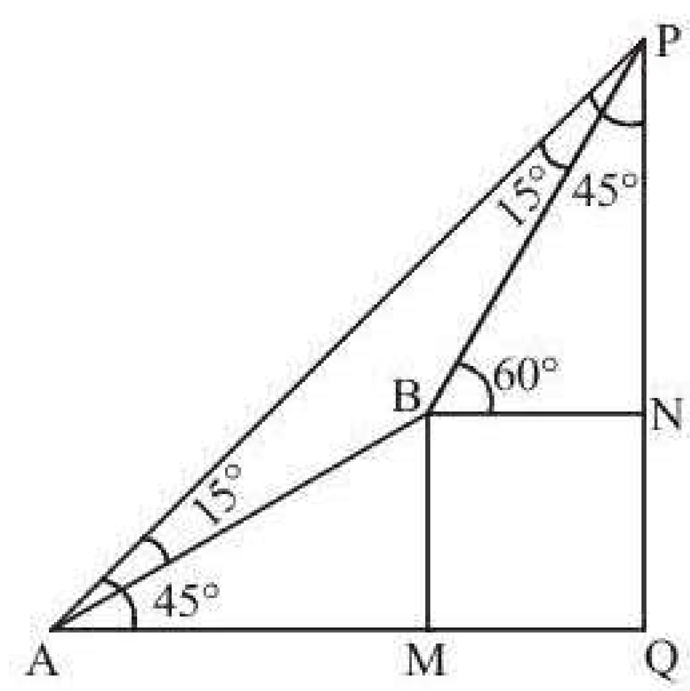
So (i) becomes $12h = 5\left(240 + \frac{4}{3}h\right)$

$$\Rightarrow 12h = 1200 + \frac{20h}{3} \Rightarrow 16h = 3600$$

$$\Rightarrow h = \frac{3600}{16} = 225\text{m}$$

$\therefore$ Height of the light house = 225m

24. (a) Suppose P be the summit of the mountain and Q be the foot. Here BN and BM are perpendiculars from B to PQ and AQ respectively.
Here AB = 100 m = 1 km
$\angle$MAB = 30°, $\angle$MAP = 45°,
$\angle$NBP = 60°, $\angle$BAP = 15°
$\angle$APB = 15°
$\therefore$ ΔABP is isosceles and AP = BP

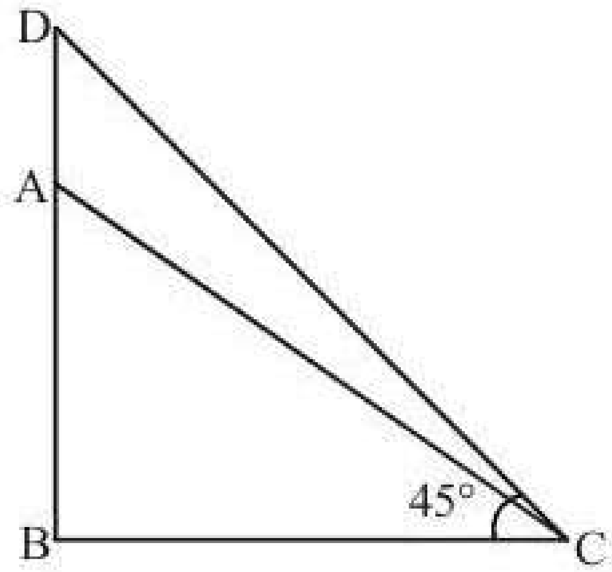


But AB = 1 km = PB
In ΔPBN
PN = BP sin 60°
In ΔABM
BM = AB sin 30°
PQ = PN + NQ
= PN + BM

$$= BP\sin 60° + AB\sin 30° = 1\frac{\sqrt{3}}{2} + 1.\frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

$\therefore$ Height of the mountain is $\frac{\sqrt{3}+1}{2}$ km.

25. (b)



AB = Building = h metre
AD = Chimney = y metre
From ΔBCD,

$$\tan 45° = \frac{BD}{BC} \Rightarrow 1 = \frac{h+y}{BC}$$

$\Rightarrow BC = h + y$
From ΔABC,

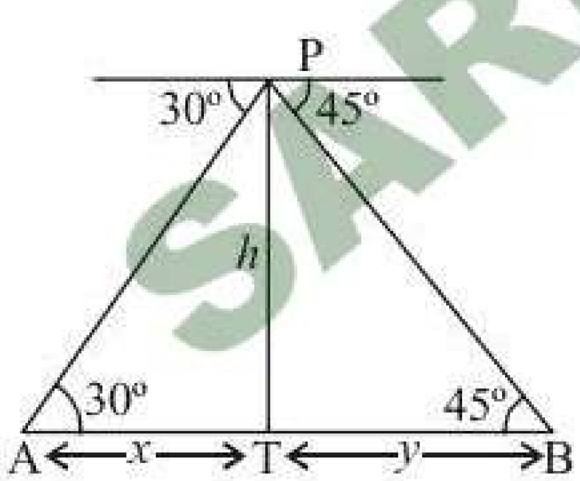
$$\tan x = \frac{AB}{BC} \Rightarrow \tan x = \frac{h}{BC}$$

$\Rightarrow BC = h \cot x$ ...(ii)
From equations (i) and (ii),
$h + y = h \cot x$
$\Rightarrow y = (h \cot x - h)$ metre

26. (a) In ΔPBT,

$$\tan 45^o = \frac{h}{y} = 1$$

$\therefore \quad y = h$ ...(*i*)



Now, in ΔPTA

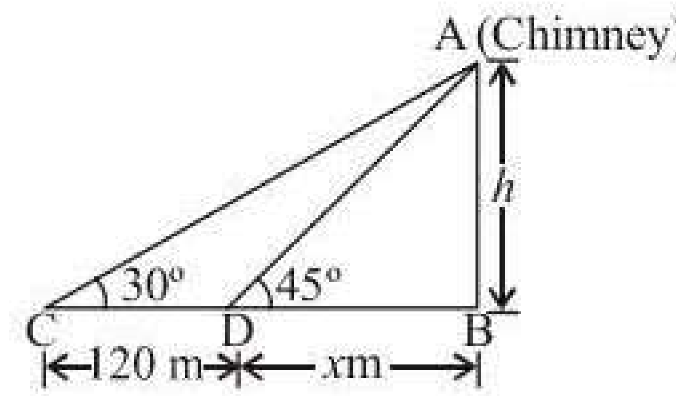
$$\tan 30^o = \frac{h}{x} \Rightarrow x = \sqrt{3}h$$ ...(*ii*)

$\therefore$ Required distance = $x + 4$
$= \sqrt{3}h + h$
$= h(\sqrt{3}+1)$m

27. (c) Let the height of the Chemney be h

$$\text{In } \Delta ABD \tan 45^o = \frac{AB}{BD}$$

$$\tan 45^o = \frac{h}{x} = 1 \Rightarrow h = x$$



Now, in ΔABC

$$\tan 30^o = \frac{h}{120+x} = \frac{1}{\sqrt{3}}$$

Now put the value of $h = x$

$$\Rightarrow \frac{h}{120+h} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h = 120 + h$$

$$\sqrt{3}h - h = 120 \Rightarrow h(\sqrt{3}-1) = 120$$

$$\therefore \quad h = \frac{120}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{120(\sqrt{3}+1)}{2}$$

$\therefore$ Height of the chimney (h) = $60(\sqrt{3}+1)$ m

28. (b) Given that, $\beta = 30^o$

$$\text{In } \Delta AED, \tan\beta = \tan 30^o = \frac{AE}{DE} = \frac{1}{\sqrt{3}}$$

$$DE = \sqrt{3}\ AE = \sqrt{3}\left(\frac{3+\sqrt{3}}{2}\right)h$$

$$\Rightarrow BC = DE = \frac{3}{2}(1+\sqrt{3})h \quad (\because BC = DE)$$ ...(*i*)

Now, in ΔACB,

$$\Rightarrow \tan\alpha = \frac{AC}{BC}$$

$$\Rightarrow BC \tan\alpha = AE - CE = AE - BD \ (\because BD = CE)$$

$$\Rightarrow BC \tan\alpha = \left(\frac{3+\sqrt{3}}{2}\right)h - h = h\left(\frac{3+\sqrt{3}-2}{2}\right)$$

$$\Rightarrow \frac{3}{2}(1+\sqrt{3})h \tan\alpha = \left(\frac{1+\sqrt{3}}{2}\right)h$$ [from Eq. (*i*)]

$$\therefore \quad \tan\alpha = \frac{1}{3}$$

29. (a) Given that, $\alpha = 30^o$

$$\text{In } \Delta ACB, \tan\alpha = \tan 30^o = \frac{AC}{BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BC = \sqrt{3}\ AC = \sqrt{3}\ (AE - CE)$$

$$= \sqrt{3}\ (AE - BD) \quad (\because BD = CE)$$

$$= \sqrt{3}\left(\frac{3+\sqrt{3}}{2}-1\right)h = \frac{\sqrt{3}}{2}(1+\sqrt{3})h \quad ...(ii)$$

Now, in ΔAED,

$$\tan\beta = \frac{AE}{DE} \Rightarrow \tan\beta = \frac{AE}{BC} \quad (\because DE = BC)$$

$$= \frac{\left(\frac{3+\sqrt{3}}{2}\right)h}{\frac{\sqrt{3}}{2}(1+\sqrt{3})h} = \frac{\frac{\sqrt{3}(1+\sqrt{3})}{2}h}{\frac{\sqrt{3}(1+\sqrt{3})}{2}h}$$

$\therefore \quad \tan\beta = 1$

30. (c) Given, $\alpha = 30^o$ and $h = 30$ m

In ΔACB,

$$\tan\alpha = \tan 30^o = \frac{AC}{BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{BC}{\sqrt{3}} = (AE - CE) = (AE - BD) \quad (\because BD = CE)$$

$$\Rightarrow BC = \sqrt{3}\left(\frac{3+\sqrt{3}}{2}-1\right)h$$

$$\Rightarrow BC = \sqrt{3}\frac{(1+\sqrt{3})}{2}\cdot 30 = (\sqrt{3}+3)\cdot 15$$

$$\therefore DE = BC = (45+15\sqrt{3})\text{ m} \quad (\because DE = BC)$$

31. (a) Given that, $\beta = 30^o$

In ΔADE,

$$\tan\beta = \frac{AE}{DE} \Rightarrow \tan 30^o = \frac{\left(\frac{3+\sqrt{3}}{2}\right)h}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\left(\frac{1+\sqrt{3}}{2}\right)h}{DE}$$

$$\Rightarrow DE = \frac{3}{2}(1+\sqrt{3})h \quad ...(i)$$

In ΔBDE,

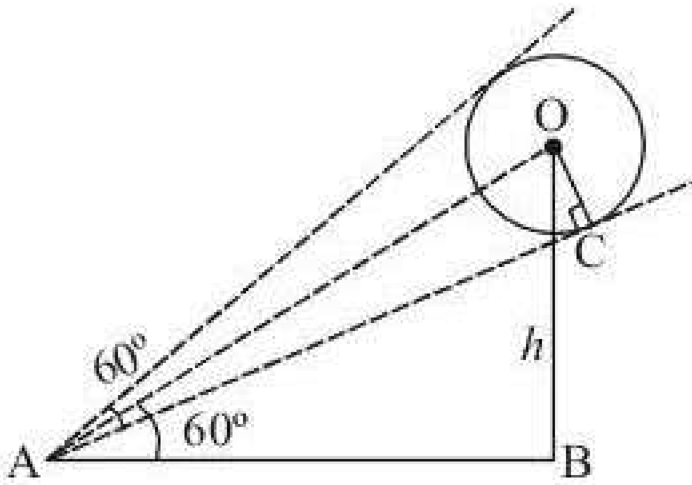
$$\tan\theta = \frac{BD}{DE} = \frac{h}{DE}$$

$$\Rightarrow \tan\theta = \frac{h}{\frac{3}{2}(1+\sqrt{3})h} \quad \text{[from Eq. (i)]}$$

$$= \frac{2}{3}\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{2(\sqrt{3}-1)}{3.2}$$

$$= \frac{(\sqrt{3}-1)}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{(3-\sqrt{3})}{3\sqrt{3}}$$

32. (c)



In ΔABO,

$$\sin 60^o = \frac{OB}{AO} \Rightarrow AO = \frac{OB}{\sin 60^o} \quad ...(i)$$

Now, in ΔACO

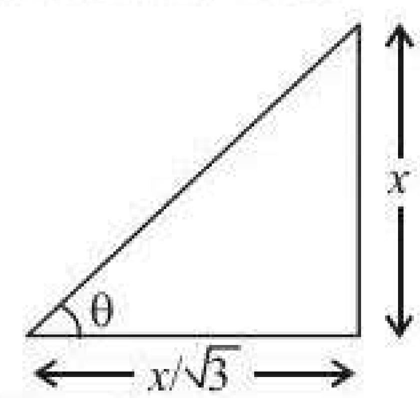
$$\sin\frac{60^o}{2} = \frac{OC}{AO} \Rightarrow AO = \frac{OC}{\sin 30^o} \quad ...(ii)$$

Comparing equation (*i*) and equation (*ii*)

$$\frac{OB}{\sin 60^o} = \frac{OC}{\sin 30^o} \Rightarrow \frac{h}{\frac{\sqrt{3}}{2}} = \frac{r}{\frac{1}{2}}$$

$$\therefore \quad h = \sqrt{3}r$$

33. (c) Let angle of elevation be θ.



Height of the pole = xm

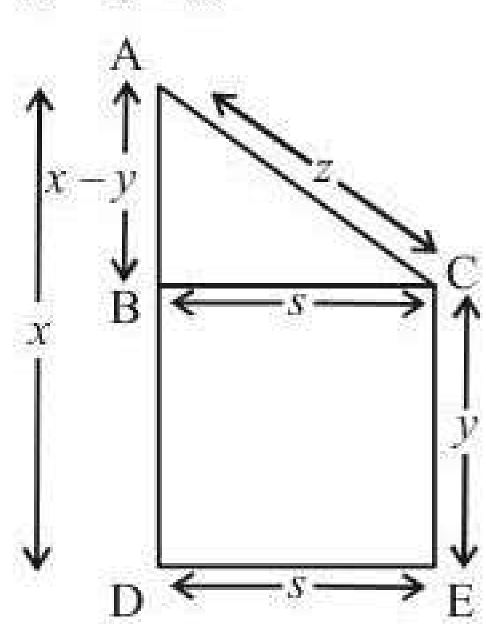
and length of shadow = $x/\sqrt{3}$ m

$$\tan\theta = \frac{x}{\frac{x}{\sqrt{3}}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

$$\tan\theta = \sqrt{3} = \tan 60^o$$

$\therefore \quad \theta = 60^o$

34. (d)



Here, BC is the shortest distance

In ΔABC, $AB^2 + BC^2 = AC^2$ (Use Pythagoras theorem)

$\Rightarrow z^2 = (x-y)^2 + s^2$

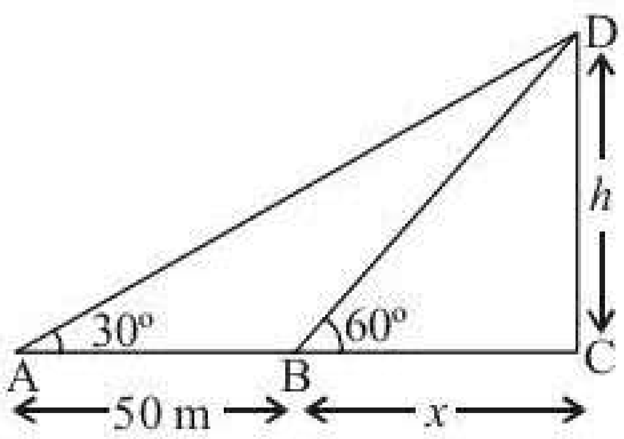
$\Rightarrow z^2 = x^2 + y^2 - 2xy + s^2$

$\Rightarrow s^2 = z^2 - x^2 - y^2 + 2xy$

35. (b) Let the height of tower be h and BC = xm.

In ΔBCD, $\tan 60^\circ = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = x\sqrt{3}$ ...(i)



In ΔACD, $\tan 30^\circ = \frac{h}{50+x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{50+x}$

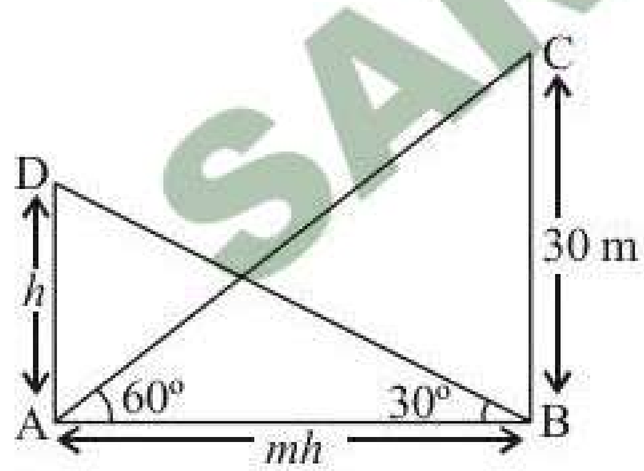
$\Rightarrow 50 + x = 3x$

$\Rightarrow x = 25\ m$

Now put the value of x in equation (i)

$\therefore h = 25\sqrt{3}\ m$

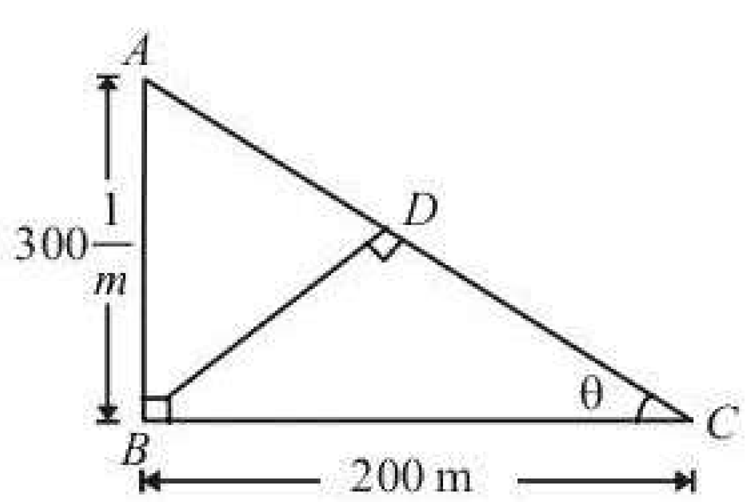
36. (b) Let the height of shorter tower be h then distance between two tower = hm.



In ΔABD, $\tan 30^\circ = \frac{h}{mh} \Rightarrow \frac{1}{\sqrt{3}} = \frac{1}{m}$

$\therefore m = \sqrt{3}$

37. (c)



Let a person be at point C observe a tower in west direction at B. After walking some distance along road he observed same tower in south direction. Let angle C be = θ

In ΔABC

$h = \sqrt{(3)^2 + (2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

$\sin\theta = \frac{3}{\sqrt{13}}$

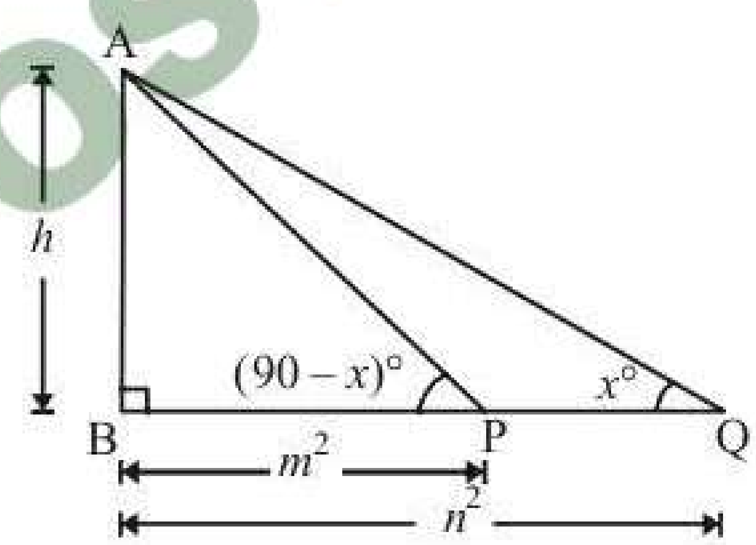
In ΔBDC

$\sin\theta = \frac{P}{h} = \frac{BD}{200}$

$\therefore$ BD = 200 sin θ

$= 200 \times \frac{3}{\sqrt{13}} = \frac{600}{\sqrt{13}}$ m

38. (d)



Let h be the height of tower

In Δ ABQ

$\tan x = \frac{AB}{BQ} = \frac{h}{n^2}$ ...(i)

In Δ ABP

$\tan(90 - x) = \frac{AB}{BP} = \frac{h}{m^2}$

$\cot x = \frac{h}{m^2}$; $\frac{1}{\tan x} = \frac{h}{m^2}$

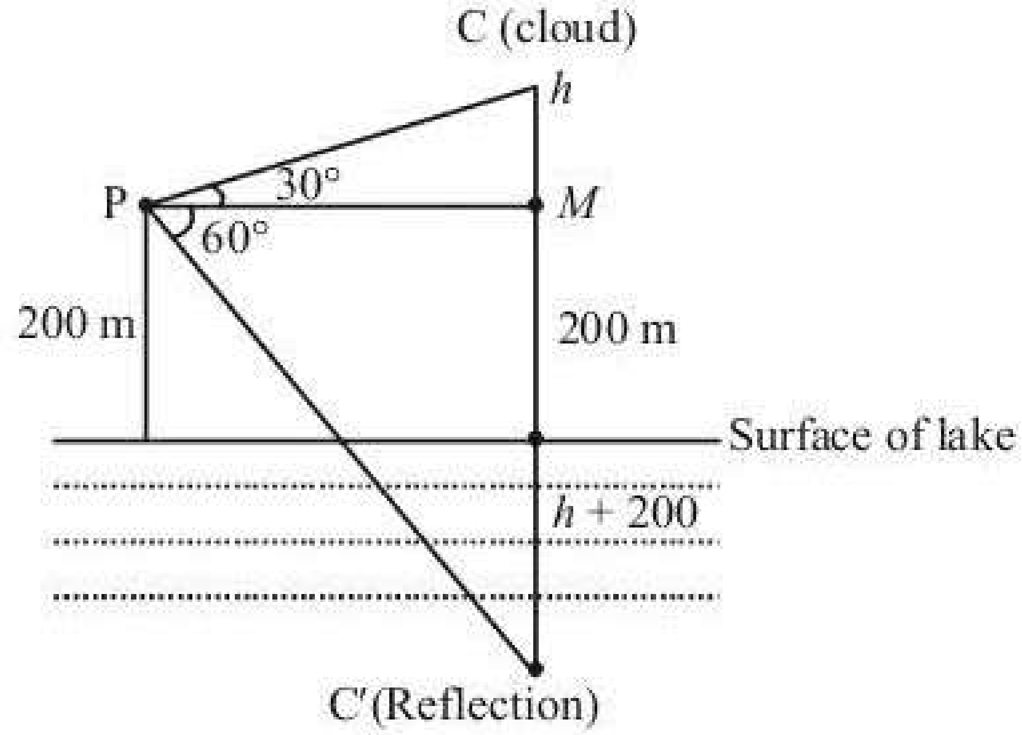
Now, from eqn (i),

$\Rightarrow \frac{1}{\frac{h}{n^2}} = \frac{h}{m^2}$

$\Rightarrow h^2 = m^2 n^2$

$\because h = mn$

39. (c)



$\tan 30° = \frac{h}{PM} \Rightarrow PM = \sqrt{3}h$

$\tan 60° = \frac{h+400}{PM} \Rightarrow PM = \frac{h+400}{\sqrt{3}}$

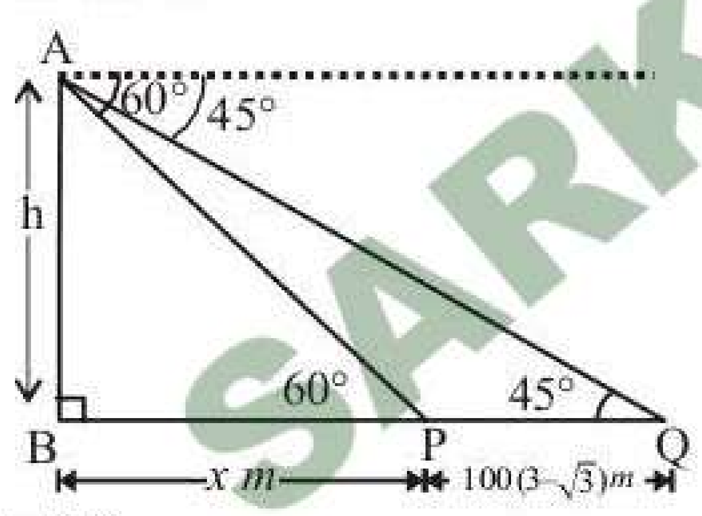
$\sqrt{3}h = \frac{h+400}{\sqrt{3}} = 3h - h = 400$

$\Rightarrow$ $2h = 400$

$\Rightarrow$ So, height of the cloud = 200 + 200 = 400m

40. (c) Let AB is tower whose height is h.

Distance between objects P and Q are $100(3-\sqrt{3})$ m and BP is x m.



In ΔABP

$\tan 60° = \frac{AB}{x} = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$ ...(i)

In ΔABQ

$\tan 45° = \frac{AB}{BQ} = \frac{h}{100(3-\sqrt{3})+x}$

$\Rightarrow$ $h = 100(3-\sqrt{3}) + x$

Now, from eqn (i), we put value of x

$h = 100\left(3-\sqrt{3}\right) + \frac{h}{\sqrt{3}}$

$h - \frac{h}{\sqrt{3}} = 100\left(3-\sqrt{3}\right)$

$h\frac{\left(\sqrt{3}-1\right)}{\sqrt{3}} = 100\left(3-\sqrt{3}\right)$

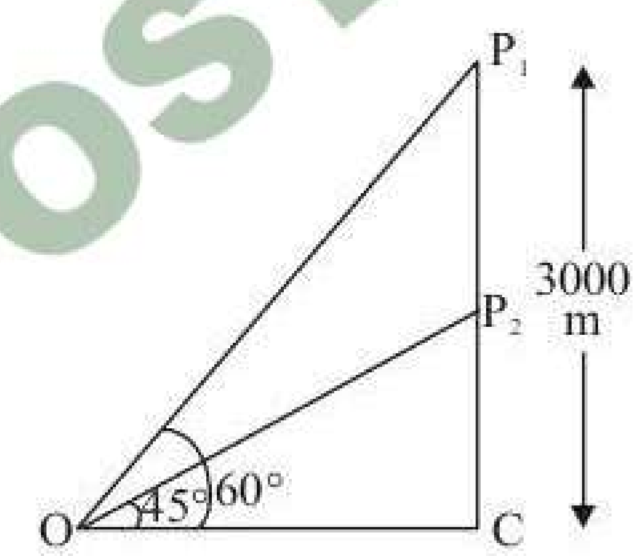
$h = \frac{100\sqrt{3}\left(3-\sqrt{3}\right)}{\left(\sqrt{3}-1\right)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{100\sqrt{3}\left(3\sqrt{3}+3-3-\sqrt{3}\right)}{3-1}$

$= \frac{100 \times 2\sqrt{3}\sqrt{3}}{2}$

$\therefore$ $h = 100 \times 3 = 300$ m

41. (c)



$CP_1 = 3000$ m

$OC = \frac{CP_1}{\tan 60°} = \frac{3000}{\sqrt{3}}$ m

$CP_2 = OC = \frac{3000}{\sqrt{3}}$ m

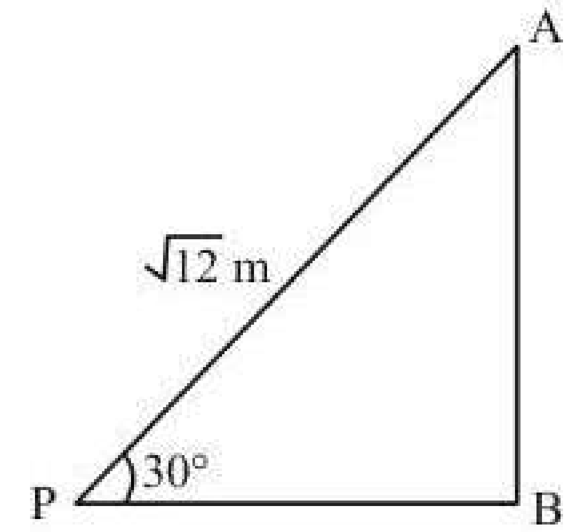
$P_1P_2 = CP_1 - CP_2 = \left(3000 - \frac{3000}{\sqrt{3}}\right)$

$= (3000 - 1000\sqrt{3})$ m

$= 1000(3-\sqrt{3})$ m

So, option (c) is correct.

42. (d) In ΔPAB



$\sin 30^\circ = \frac{AB}{AP}$

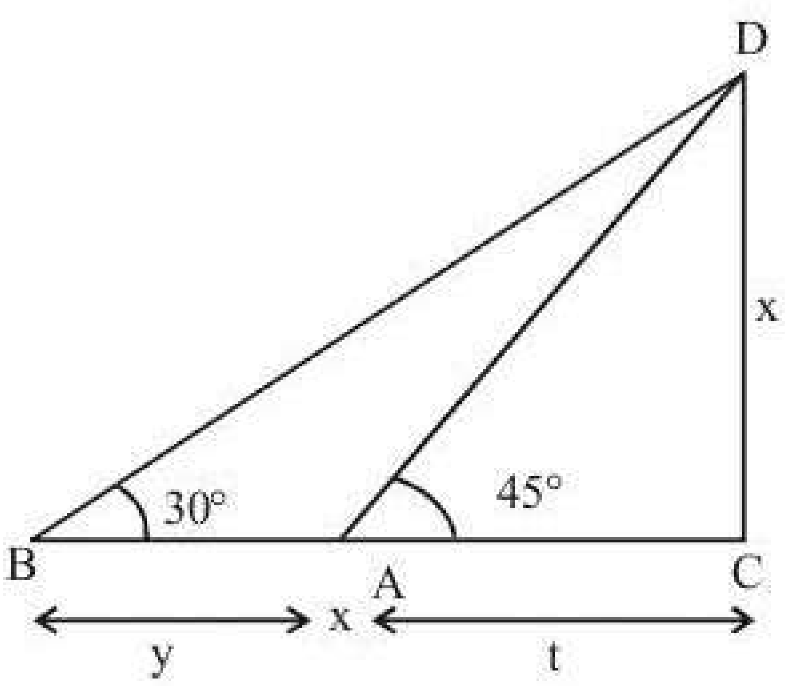
$\frac{1}{2} = \frac{AB}{\sqrt{12}}$

$AB = \frac{\sqrt{12}}{2} = \frac{2\sqrt{3}}{2}$

$AB = \sqrt{3}m$

So, option (d) is correct.

43. (c) Let two observers A and B are stationed at a distances y from each other.



Let CD be a tower of height x metre.
Also $\angle DAC = 45^\circ$ and $\angle DBC = 30^\circ$

In ΔACD, $\tan 45^\circ = \frac{x}{t}$ and In ΔDBC, $\tan 30^\circ = \frac{x}{y+t}$

$\Rightarrow 1 = \frac{x}{t}$ ...(1)

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y+t}$ ...(2)

From (1) and (2) we get

$\frac{t}{y+t} = \frac{1}{\sqrt{3}}$

$\Rightarrow \sqrt{3}t = y + t$

$\Rightarrow (\sqrt{3}-1)t = y$

$\Rightarrow t = \frac{1}{\sqrt{3}-1}y$

From (1) we get

$x = t$

$x = \frac{1}{\sqrt{3}-1} y \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2}$

∴ Option (c) is correct.