Chapter 5
Introduction to Euclid's Geometry

Exercise 5.1

Question: 1 Which of the following statements are true and which are false? Give reasons for your answers.

(i) Only one line can pass through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

(iii) A terminated line can be produced indefinitely on both the sides.

(iv) If two circles are equal, then their radii are equal.

(v) In Fig. 5.9, if AB = PQ and PQ = XY, then AB = XY.

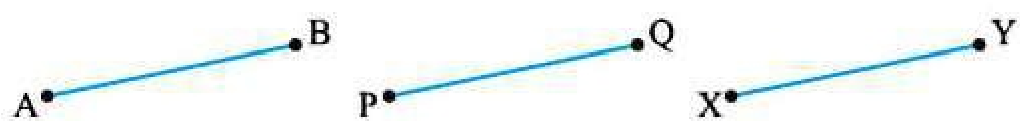
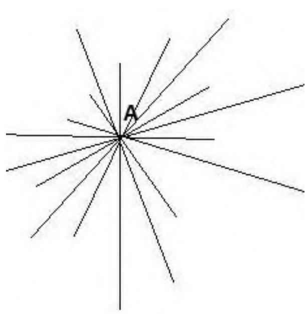


Fig. 5.9

Answer:

(i) False

Since, there can be infinite lines that can be drawn passing through a single point.



As in the diagram from point A, infinitely many lines can be drawn.

(ii) False

Only and only one line can be drawn which passes through two distinct points.

As you can see only one line can be drawn through two points.

(iii) True

A terminated line can be produced indefinitely on both the sides. A line can be extended in both directions. A line refers to infinite long length

(iv) True

If two circles are equal, then their radii are equal.

By superposition, we will find that the center and circumference of the both circles coincide. Hence, their radius ought to be equal.

(v) True

By Euclid's first axiom things which are equal to the same thing, are equal to one another.

Question: 2 Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

(i) Parallel lines

(ii) Perpendicular lines

(iii) Line segment

(iv) Radius of a circle

(v) Square

Ans.:

Yes, there are other terms that are needed to be defined first which are:

Plane: A plane is a flat surface on which geometric figures are drawn.

Point: A point is a dimensionless dot which is drawn on a plane surface.

Line: A line is collection of n number of points which can extend in both the directions and has only one dimension.

Now,

(i) Parallel lines: When two or more lines do not intersect each other in a plane and perpendicular distance between them remains constant then they are said to be parallel lines.

(ii) Perpendicular lines: When two lines intersect each other making a right angle in a plane then they are said to be perpendicular to each other.

(iii) Line segment: A line segment is a part of a line with two end points and cannot be extended further. It has a definite length or breadth.

(iv) Radius of circle: The fixed distance between the centre and the circumference of the circle is called radius of the circle.

(v) Square: A square is a quadrilateral in which all the four sides are equal and each internal angle is a right angle.

Question: 3 Consider two 'postulates' given below:

(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.

(ii) There exist at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent?

Do they follow from Euclid's postulates? Explain.

Answer.:

Undefined terms in the postulates:

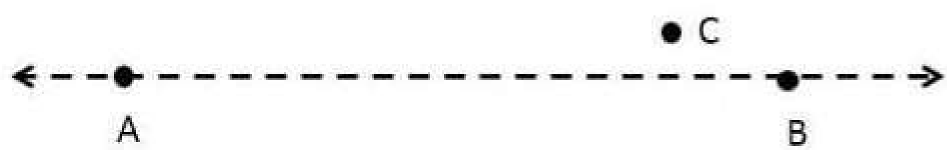
•Here, there is no information given about the plane whether the points are in same plane or not.

•Also, n number of points lie in a plane. But here the position of the point C is not specified that whether it lies on the line segment joining AB or not.

Yes, these postulates are consistent when we deal with these two situations:

(i) Point C is lying in between and on the line segment joining A and B.

(ii) Point C not lies on the line segment joining A and B.



No, they don't follow from Euclid's postulates. They follow the axioms.

Question: 4 If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}AB$ Explain by drawing the figure.

Answer.:

Here,

AC = BC

Now,

After adding AC both side, we get

AC + AC = BC + AC

2AC = AB (Since, If equals are added to equals, the wholes are equal.)

Therefore,

AC = $\frac{1}{2}$AB

Question: 5 In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

Answer:

Method 1:

Let A and B be the line segment and points P and Q be two different midpoints of AB.

So,

AP = PB

And,

AQ = QB

And,

PB + AP = AB (It coincides with line segment AB)

Similarly,

QB + AQ = AB

Now,

AP + AP = AP + BP (Since, If equals are added to equals, the wholes are equal.)

2AP = AB ...... (i)

Similarly,

2AQ = AB ...... (ii)

From (i) and (ii),

2AP = 2AQ (Since things which are equal to the same thing are equal to one another)

And as we know:

Things which are double of the same thing are equal to one another.

Therefore,

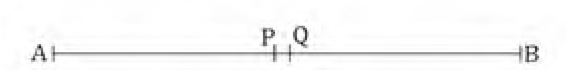
AP = AQ

Thus, P and Q are the same points.

This contradicts the fact that P and Q are two different midpoints AB.

Thus, it is proved that every line segment has one and only one midpoint.

Method 2:



From the Figure, AP + PB = AB.......................eq (i)

AQ + QB = AB.....................eq (ii)

From eq (i) and eq(ii)

AP + PB = AQ + QB Now let AP = PB, and AQ = QB (as they are the midpoints, then)

2 AP = 2 AQAP = AQP = Q Hence, there is only one midpoint.

Question: 6. In Fig. 5.10, if AC = BD, then prove that AB = CD.

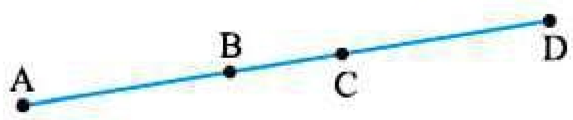


Fig. 5.10

Answer:

Given: AC = BD

From the figure,

AC = AB + BC

BD = BC + CD

AB + BC = BC + CD

According to Euclid's axiom,

When two equals are subtracted from equals, remainders are also equal.

Subtracting BC both sides,

AB + BC – BC = BC + CD – BC

AB = CD

Question: 7 Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate)

Answer:

Axiom 5: The whole is always greater than the part.

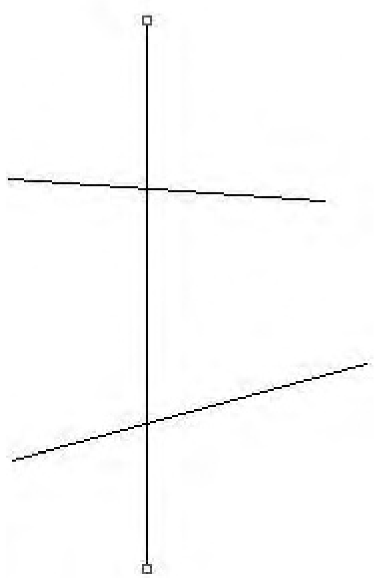
Let us take an example of the cake. When it is whole it will measure 3 pound but when we took out a part from it and measure its weight then it would come out less than the previous one. Hence, the fifth axiom of Euclid is true for all the universal things. That is the reason that it is considered as a 'universal truth'.

Exercise 5.2

Question: 1 How would you rewrite Euclid's fifth postulate so that it would be easier to understand?

Answer:

Euclid's Fifth Postulate: If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.



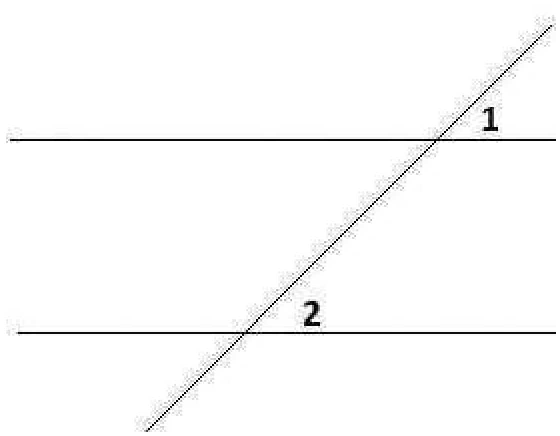
From the figure, we can see that the two lines that are not parallel and make acute angles, will intersect at the side which is towards the acute angle. Now we can look at this Postulate from a different point of view, we can say that the lines that are not parallel to each other, intersect with each other where the inclination is towards meeting them.

Question: 2 Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Answer:

Fifth postulate of Euclid geometry: If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Now according to this postulate if the straight lines will not meet when the angles made are equal, the lines will not intersect and hence the lines are parallel.



From the figure if angles 1 and 2 are equal then the lines when extended to infinity will not intersect and hence will be parallel.