10.BASICS

Intervals :

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows :

		Symbols Used
(i)	Open interval : $(a, b) = \{x : a < x < b\}$ i.e. end points are not included.	( ) or] [
(ii)	Closed interval : $[a, b] = \{x : a \le x \le b\}$ i.e. end points are also included. This is possible only when both a and b are finite.	[ ]
(iii)	Open-closed interval : $(a, b] = \{x : a < x \le b\}$	( ] or]]
(iv)	Closed - open interval : $[a, b) = x : a \le x < b\}$	[ ) or [[

The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$

(ii) $[a, \infty) = \{x : x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$

(iv) $(\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = \{x : x \in R\}$

Properties of Modulus :

For any $a, b \in R$

$|a| \ge 0$, $|a| = |-a|$, $|a| \ge a$, $|a| \ge -a$, $|ab| = |a|\,|b|$, $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$,

$|a + b| \le |a| + |b|$, $|a - b| \ge ||a| - |b||$

Trigonometric Functions of Sum or Difference of Two Angles:

(a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\therefore 2\sin A \cos B = \sin(A+B) + \sin(A-B)$ and and $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

(b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\therefore 2\cos A \cos B = \cos(A+B) + \cos(A-B)$ and $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

(c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B).\sin(A-B)$

(d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B).\cos(A-B)$

(e) $\cot(A \pm B) = \frac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

(f) $\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$.

Factorisation of the Sum or Difference of Two Sines or Cosines:

(a) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

Multiple and Sub-multiple Angles :

(a) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$; $2\cos^2\frac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$.

(c) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(d) $\sin 3A = 3\sin A - 4\sin^3 A$

(e) $\cos 3A = 4\cos^3 A - 3\cos A$

(f) $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Important Trigonometric Ratios:

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)$; $\tan n\pi = 0$, where $n \in I$

(b) $\sin 15°$ or $\sin \frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$ or $\cos \frac{5\pi}{12}$;

$\cos 15°$ or $\cos \frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$ or $\sin \frac{5\pi}{12}$;

$\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75°$; $\tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15°$

(c) $\sin \frac{\pi}{10}$ or $\sin 18° = \frac{\sqrt{5}-1}{4}$ & $\cos 36°$ or $\cos \frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$

Range of Trigonometric Expression:

$$-\sqrt{a^2+b^2} \le a \sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$$

Sine and Cosine Series :

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots + \sin\left(\alpha + \overline{n-1}\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left(\alpha + \frac{n-1}{2}\beta\right)$$

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \cos\left(\alpha + \overline{n-1}\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha + \frac{n-1}{2}\beta\right)$$

Trigonometric Equations

Principal Solutions: Solutions which lie in the interval $[0, 2\pi)$ are called **Principal solutions.**

General Solution :

(i) $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.

(ii) $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $\alpha \in [0, \pi]$, $n \in I$.

(iii) $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.

(iv) $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$.