Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. Find the value of sec $\left(\frac{-19\pi}{3}\right)$. **[1]**

a) $\frac{1}{2}$ b) -2

c) 2 d) $\frac{-1}{2}$

2. Number of relations that can be defined on the set A = {a, b, c, d} is **[1]**

a) 24 b) 4^4

c) 16 d) 2^{16}

3. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is: **[1]**

a) 38 b) 30

c) 35 d) 28

4. If $f(x)=\frac{x^n-a^n}{x-a}$ for some constant, a, then f'(a) is equal to **[1]**

a) 1/2 b) does not exist

c) 1 d) 0

5. A line passes through P (1, 2) such that its intercept between the axes is bisected at P. The equation of the line is **[1]**

a) x + 2y = 5 b) 2x + y – 4 = 0

c) x + y – 3 = 0 d) x – y + 1 = 0

6. Distance of the point (α, β, γ) from y-axis is **[1]**

a) $\sqrt{\alpha^2+\gamma^2}$ b) $|\beta|+|\gamma|$

c) $|\beta|$ d) β

7. Mark the correct answer for $(1 + i)^{-1}$ =? **[1]**

a) $\left(\frac{-1}{2}+\frac{1}{2}i\right)$

b) (2 - 3i)

c) $\left(\frac{1}{2}-\frac{1}{2}i\right)$

d) (2 - i)

8. If ${}^{20}C_{r+1} = {}^{20}C_{r-1}$, then r is equal to **[1]**

a) 19

b) 10

c) 12

d) 11

9. If $y=\frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{-4x}{x^2-1}$

b) $\frac{1-x^2}{4x}$

c) $\frac{-4x}{(x^2-1)^2}$

d) $\frac{4x}{x^2-1}$

10. Which is smaller, sin 64° or cos 64°? **[1]**

a) cos 64°

b) sin 64°

c) cannot be compared

d) both are equal

11. Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to **[1]**

a) $F_2 \cap F_3$

b) $F_3 \cap F_4$

c) $F_2 \cup F_5$

d) $F_2 \cup F_3 \cup F_4 \cup F_1$

12. $\left\{\frac{c_1}{c_0}+2\frac{C_2}{C_1}+3\frac{C_3}{C_2}+\ldots+n\cdot\frac{C_n}{C_{n-1}}\right\}$ = ? **[1]**

a) $\frac{1}{2}n(n+1)$

b) 2n

c) 2^{n-1}

d) 2^n

13. $\left(\sqrt{5}+1\right)^4+\left(\sqrt{5}-1\right)^4$ is **[1]**

a) an irrational number

b) a negative real number

c) a rational number

d) a negative integer

14. The solution set of 6x - 1 > 5 is : **[1]**

a) $\{x : x > 1, x \in N\}$

b) $\{x : x > 1, x \in R\}$

c) $\{x : x < 1, x \in N\}$

d) $\{x : x < 1, x \in W\}$

15. If A = {1, 3, 5, B} and B = {2, 4}, then **[1]**

a) $\{4\} \subset A$

b) None of these

c) $B \subset A$

d) $4 \in A$

16. The value of $\frac{2\left(\sin 2x+2\cos^2 x-1\right)}{\cos x-\sin x-\cos 3x+\sin 3x}$ is **[1]**

a) sin x

b) cos x

c) cosec x

d) sec x

[1]

17. $\lim_{x \to 3} \frac{x-3}{|x-3|}$ is equal to:

a) 1 | b) -1
c) 0 | d) None of these

18. In how many ways can 5 persons occupy 3 seats? **[1]**

a) 30 | b) 15
c) 60 | d) 20

19. **Assertion (A):** The set A = {a, b, c, d, e, g} is finite set. **[1]**
Reason (R): The set B = {men living presently in different parts of the world} is finite set.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If the numbers $\frac{-2}{7}$, K, $\frac{-7}{2}$ are in GP, then k = ± 1. **[1]**
Reason (R): If a_1, a_2, a_3 are in GP, then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Let R = {(x, x^2) : x is a prime number less than 10} **[2]**

i. Write R in roster form.
ii. Find dom (R) and range (R).

OR

Let R be a relation from N to N defined by R = {(a, b) : a, b $\in N$ and a = b^2}. Are the following true?

(i) (a, a) $\in R$ for all $a \in N$

(ii) (a, b) $\in R$implies (b, a) $\in R$

(iii) (a, b) $\in R$, (b, c) $\in R$ implies (a, c) $\in R$

Justify your answer in each case.

22. Differentiate: (x^2 - 4x + 5)(x^3 - 2). **[2]**

23. If $\frac{5}{14}$ is the probability of occurrence of an event, find **[2]**

i. the odds in favour of its occurrence.
ii. the odds against its occurrence.

OR

If A and B are two events associated with a random experiment for which P(A) = 0.60, P(A or B) = 0.85 and P(A and B) = 0.42, find P(B).

24. Is B = {x : x^2 + 2x + 1 = 0, x $\in$ N} a singleton set? **[2]**

25. Find a point on the y-axis which is equidistant from A (-4, 3) and B(5, 2). **[2]**

Section C

26. If ${}^{22}P_{r+1}$: ${}^{20}P_{r+2} = 11 : 52$. find r. **[3]**

27. Three vertices of a parallelogram ABCD are A(3, -1, 2), B(1, 2, -4) and C(-1, 1, 2). Find the coordinates of the **[3]**

fourth vertex.

28. Using binomial theorem, expand: $\left(x^2-\frac{2}{x}\right)^7$. **[3]**

OR

Show that $2^{4n+4}-15n-16$ where $n \in \mathbf{N}$ is divisible by 225

29. Find the derivative of $x^{-4}(3 - 4x^{-5})$ **[3]**

OR

Find the derivative of $\frac{(x-1)(x-2)}{(x-3)(x-4)}$.

30. If the AM and GM of two positive numbers a and b are in the ratio m : n, show that **[3]**
a: b = $\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$

OR

Evaluate: $\sum_{k=1}^{11}\left(2+3^k\right)$

31. In a class, 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics of these 13 took both Chemistry and Mathematics, 12 took both Physics and Chemistry and 11 took both Physics an Mathematics. If 6 students offered all the three subjects, find: **[3]**
    i. The total number of students.
    ii. How many took Maths but not Chemistry.
    iii. How many took exactly one of the three subjects.

Section D

32. Find the mean deviation about the mean for the following data: **[5]**

x_i	3	5	7	9	11	13
f_i	6	8	15	3	8	4

33. Find the equation of a circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point (5, 4). **[5]**

OR

Find the equation of the hyperbola, the length of whose latus rectum is 4 and the eccentricity is 3.

34. Solve the following system of linear inequalities **[5]**
$\frac{4x}{3}-\frac{9}{4} < x + \frac{3}{4}$ and $\frac{7x-1}{3} - \frac{7x+2}{6} > x$.

35. Prove that: $\sin 20^o \sin 40^o \sin 80^o = \frac{\sqrt{3}}{8}$ **[5]**

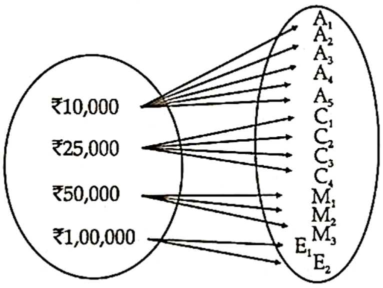
OR

Prove that $\cos 12^o + \cos 60^o + \cos 84^o = \cos 24^o + \cos 48^o$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**
A Relation R from A to B can be depicted pictorially using arrow diagram. In arrow diagram, we write down the elements of two sets A and B in two disjoint circles. Then we draw arrow from set A to set B whenever (A, B) ∈ R. An example of information depicted through an arrow diagram is shown below. For example:
A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provides ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 to the people who work in the categories A, C, M and E respectively. Here A_1, A_2, A_3, A_4 and A_5 are Assistants; C_1, C_2, C_3, C_4 are Clerks; M_1, M_2, M_3 are Managers and E_1, E_2 are Executive Officers then the relation R is defined by xRy,

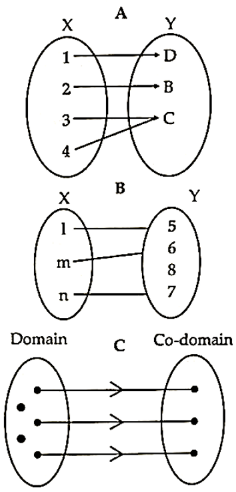
where x is the salary given to person y.



i. If the number of elements in set A and set B are p and q then find the number of functions from A to B. (1)

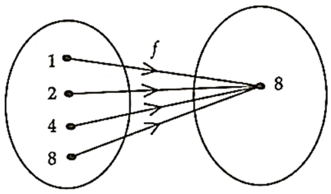
ii. If the number of elements in set A and set B are p and q, then find the number of relations from A to B. (1)

iii. Which figures shows a relation between the two non-empty sets? (2)



OR

Show the relation defined in the below arrow diagram from set A to set B. (2)



37. **Read the following text carefully and answer the questions that follow:** **[4]**

There are 4 red, 5 blue and 3 green marbles in a basket.

i. If two marbles are picked at randomly, find the probability that both red marbles. (1)

ii. If three marbles are picked at randomly, find the probability that all green marbles. (1)

iii. If two marbles are picked at randomly then find the probability that both are not blue marbles. (2)

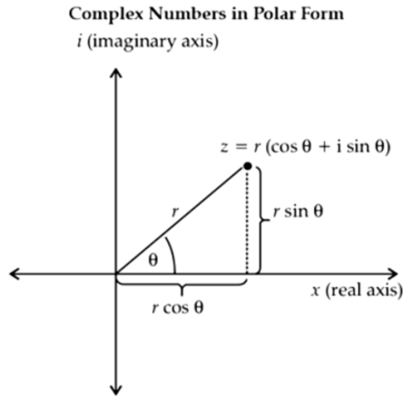
OR

If three marbles are picked at randomly, then find the probability that atleast one of them is blue. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Consider the complex number Z = 2 - 2i.

Complex Number in Polar Form



i. Find the principal argument of Z. (1)

ii. Find the value of $\mathbf{z\bar{z}}$? (1)

iii. Find the value of |Z|. (2)

OR

Find the real part of Z. (2)

Solution

Section A

1.

(c) 2

Explanation: $\sec\left(\frac{-19\pi}{3}\right) = \sec\frac{19\pi}{3}$ [$\because \sec(-\theta) = \sec\theta$]

$= \sec\left(6\pi + \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$ [$\because \sec(2n\pi + \theta) = \sec\theta$]

2.

(d) 2^{16}

Explanation: No. of elements in the set A = 4 . Therefore , the no. of elements in $A \times A = 4 \times 4 = 16.$ As, the no. of relations in $A \times A$ = no. of subsets of $A \times A = 2^{16}$.

3. **(a)** 38

Explanation: L et the numbers are x_1, x_2, x_3, x and x_5. Then,

we have, $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$ = 30

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 150$ ...(i)

Now, suppose x_1 is excluded, then $\frac{x_2 + x_3 + x_4 + x_5}{4}$ = 28 [given]

$\Rightarrow x_2 + x_3 + x_4 + x_5 = 112$ ...(i)

From Eqs. (i) and (ii), we get x_1 = 150 - 112 = 38

4.

(b) does not exist

Explanation: Given $f(x) = \frac{x^n - a^n}{x - a}$

$f'(x) = \frac{(x-a)\left(n \cdot x^{n-1}\right) - \left(x^n - a^n\right) \cdot 1}{(x-a)^2}$

$\therefore \quad f'(a) = \frac{(a-a)\left(n \cdot a^{n-1}\right) - \left(a^n - a^n\right)}{(a-a)^2}$

So $f'(a) = \frac{0}{0}$ = does not exist

5.

(b) 2x + y – 4 = 0

Explanation: We know that the equation of a line making intercepts a and b with x-axis and y-axis, respectively, is given by $\frac{x}{a} + \frac{y}{b} = 1$

Here we have $1 = \frac{a+0}{2}$ and $2 = \frac{0+b}{2}$

which give a = 2 and b = 4.

Thus,now we have to find the required equation of the line is given by $\frac{x}{2} + \frac{y}{4} = 1$ or 2x + y – 4 = 0

6. **(a)** $\sqrt{\alpha^2 + \gamma^2}$

Explanation: The foot of perpendicular from point $P(\alpha, \beta, \gamma)$ on y-axis is $Q(0, \beta, 0)$

$\therefore$ Required distance, $PQ = \sqrt{(\alpha - 0)^2 + (\beta - \beta)^2 + (\gamma - 0)^2} = \sqrt{\alpha^2 + \gamma^2}$

7.

(c) $\left(\frac{1}{2} - \frac{1}{2}i\right)$

Explanation: $(1 + i)^{-1} = \frac{1}{(1+i)} = \frac{1}{(1+i)} \times \frac{(1-i)}{(1-i)} = \frac{(1-i)}{\left(1^2 - i^2\right)} = \frac{(1-i)}{2} = \left(\frac{1}{2} - \frac{1}{2}i\right)$

8.

(b) 10

Explanation: r + 1 + r - 1 = 20 [$\because {}^nC_x = {}^nC_y \Rightarrow$ n = x + y or x = y]

$\Rightarrow$ 2r = 20

$\Rightarrow$ r = 10.

9.

(c) $\frac{-4x}{\left(x^2-1\right)^2}$

Explanation: Given $y=\frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}} \Rightarrow y=\frac{x^2+1}{x^2-1}$

$\therefore \quad \frac{dy}{dx}=\frac{\left(x^2-1\right)\cdot 2x-\left(x^2+1\right)\cdot 2x}{\left(x^2-1\right)^2}$

$=\frac{2x\left(x^2-1-x^2-1\right)}{\left(x^2-1\right)^2}=\frac{2x(-2)}{\left(x^2-1\right)^2}=\frac{-4x}{\left(x^2-1\right)^2}$

10. **(a)** cos 64°

Explanation: In quadrant I, sin θ is increasing.

Now, cos 64° = cos (90° - 26°) = sin 26°.

Clearly, sin 26° < sin 64° ⇒ cos 64° < sin 64°

11.

(d) $F_2 \cup F_3 \cup F_4 \cup F_1$

Explanation: We know that

Every rectangle, square and rhombus is a parallelogram

But, no trapezium is a paralleogrm

Thus, $F_1 = F_2 \cup F_3 \cup F_4 \cup F_1$

12. **(a)** $\frac{1}{2}n(n+1)$

Explanation: We know that $\frac{C_r}{C_{r1}}=\frac{n-r+1}{r}$,

Substituting r = 1,2,3,...,n, we obtain

$\frac{C_1}{C_0}+2\cdot\frac{C_2}{C_1}+3\cdot\frac{C_3}{C_2}$ + ... + $n\cdot\frac{C_n}{C_{n-1}}$ = n + (n -1) + (n - 2) + ... + 1= $\frac{1}{2}n(n+1)$.

13.

(c) a rational number

Explanation: We have $(a + b)^n + (a - b)^n$

= $\left[{}^nC_0a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+{}^nC_3\quad a^{n-3}b^3\right.$ + ... + $\left.{}^nC_nb^n\right]$ +

$\left[{}^nC_0a^n-{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2-{}^nC_3a^{n-3}b^3\right.$ + ... + $\left.(-1)^n\cdot{}^nC_n\quad b^n\right]$

= 2[${}^nC_0\quad a^n+{}^nC_2\quad a^{n-2}b^2+$ ...]

Let a = $\sqrt{5}$ and b = 1 and n = 4

Now we get $(\sqrt{5}+1)^4+(\sqrt{5}-1)^4$ = $2\left[{}^4C_0(\sqrt{5})^4+{}^4C_2(\sqrt{5})^21^2+{}^4C_4(\sqrt{5})^01^4\right]$

= 2[25 + 30 + 1] = 112

14.

(b) {x : x > 1, x ∈ R}

Explanation: 6x - 1 > 5

⇒ 6x - 1 + 1 > 5 + 1

⇒ 6x > 6

⇒ x > 1

Hence the solution set is {x : x > 1, x ∈ R}

15.

(b) None of these

Explanation: 4 ∉ A

{4} ⊄ A

B ⊄ A

Therefore, we can say that none of these options satisfy the given relation.

16.

(c) cosec x

Explanation: We have,

$\frac{2\left(\sin 2x+2\cos^2 x-1\right)}{\cos x-\sin x-\cos 3x+\sin 3x}$

$=\frac{2(\sin 2x+\cos 2x)}{\cos x-\sin x-4\cos^3 x+3\cos x+3\sin x-4\sin^3 x}$

$= \frac{2(\sin 2x+\cos 2x)}{4\cos x-4\cos^3 x+2\sin x-4\sin^3 x}$
$= \frac{2(\sin 2x+\cos 2x)}{4\cos x(1-\cos^2 x)+2\sin x(1-2\sin^2 x)}$
$= \frac{2(\sin 2x+\cos 2x)}{4\cos x\sin^2 x+2\sin x\cos 2x}$
$= \frac{2(\sin 2x+\cos 2x)}{2\times 2\sin x\cos x\sin x+2\sin x\cos 2x}$
$= \frac{2(\sin 2x+\cos 2x)}{2\sin 2x\sin x+2\sin x\cos 2x}$
$= \frac{2(\sin 2x+\cos 2x)}{2\sin x(\sin 2x+\cos 2x)}$
$= \frac{1}{\sin x}$
= cosec x

17.

(d) None of these
Explanation: $\lim\limits_{x\to 3}\frac{x-3}{|x-3|}$
LHL at x = 3
$\lim\limits_{x\to 3^-}\frac{x-3}{-(x-3)}$ [∵ |x - 3| = -(x - 3) x < 3]
= -1
RHL at x = 3
$\lim\limits_{x\to 3^+}\frac{x-3}{x-3}$ [∴ |x - 3| = x - 3, when x > 3]
= 1
LHL $\neq$ RHL

18.

(c) 60
Explanation: Required number of ways = ${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5\times4\times3\times2\times1}{2\times1} = 60$

19.

(b) Both A and R are true but R is not the correct explanation of A.
Explanation: Assertion: We know that, a set which is empty or consists of a definite number of elements, is called finite, otherwise the set is called infinite. Since, set A contains finite number of elements. So, it is a finite set.
Reason: We do not know the number of elements in B, but it is some natural number. So, B is also finite.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Assertion: If $\frac{-2}{7}$, K, $\frac{-7}{2}$ are in G.P.
Then, $\frac{a_2}{a_1} = \frac{a_3}{a_2}$
[∵ common ratio (r) = $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3}$ = ...]
$\therefore \frac{k}{\frac{-2}{7}} = \frac{\frac{-7}{2}}{k}$
$\Rightarrow \frac{7}{-2}k = \frac{-7}{2} \times \frac{1}{k}$
$\Rightarrow$ 7k $\times$ 2k = -7 $\times$ (-2)
$\Rightarrow$ 14k^2 = 14
$\Rightarrow$ k^2 = 1 $\Rightarrow$ k = $\pm$1
Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21. i. Here we have, {(x, x^2): x is a prime number less than 10}.
Roster form of R = {(1, 1), (2, 4), (3, 9), (5, 25), (7, 49)}
ii. The domain of R is the set of first co-ordinates of R
Domain of R = {1, 2, 3, 5, 7}
The domain of R is the set of first co-ordinates of R
Range(R) = {1, 4, 9, 25, 49}

OR

Here R = {(a, b): a, $b \in N$ and a = b^2}
(i) No (3, 3) $\in R$ because $3 \neq 3^2$
(ii) No. (9, 3) $\in R$ but (3, 9) $\in R$
(iii) No. (81, 9) $\in R$ (9, 3) $\in R$ but (81, 3) $\neq R$

22. To find: Differentiation of $(x^2 – 4x + 5) (x^3 – 2)$
Formula used: (i) (uv)′ = u′v + uv′ (Using Leibnitz or product rule)
(ii) $\frac{dx^n}{dx} = nx^{n-1}$
Let u = $(x^2 – 4x + 5)$ and v = $(x^3 – 2)$
$u' = \frac{du}{dx} = \frac{d(x^2-4x+5)}{dx} = 2x - 4$
$v' = \frac{dv}{dx} = \frac{d(x^3-2)}{dx} = 3x^2$
Put the above obtained values in the formula:-
(uv)′ = u′v + uv′
$[(x^2 – 4x + 5) (x^3 – 2)]' = (2x – 4)\times(x^3 – 2) + (x^2 – 4x + 5)\times(3x^2)$
$= 2x^4 – 4x - 4x^3 + 8 + 3x^4 – 12x^3 + 15x^2$
$= 5x^4 - 16x^3 + 15x^2 – 4x + 8.$

23. i. We know that,
If odds in favor of the occurrence an event are a:b, then the probability of an event to occur is $\frac{a}{a+b}$
Given, probability = $\frac{5}{14}$
We know, probability of an event to occur = $\frac{a}{a+b}$
Here,a = 5 and a + b = 14 i.e. b = 9
So, $\frac{a}{a+b}$ = $\frac{5}{14}$
odds in favor of its occurrence = a : b = 5 : 9
Conclusion: Odds in favor of its occurrence is 5: 9
ii. As we solved in part (i), a = 5 and b = 9
Also, we know, odds against its occurrence is b: a = 9: 5
Conclusion: Odds against its occurrence is 9 : 5

OR

We have given that: P(A) = 0.60, P(A or B) = 0.85 and P(A and B) = 0.42
To find : P(B)
Formula used : P(A or B) = P(A) + P(B) - P(A and B)
Substituting the values in the above formula we get,
0.85 = 0.60 + P(B) - 0.42
0.85 = 0.18 + P(B)
0.85 - 0.18 = P(B)
0.67 = P(B)
P(B) = 0.67

24. We have, $B = \{x : x^2 + 2x + 1 = 0, x \in N$]
Now, $x^2 + 2x + 1 = 0$
$\Rightarrow (x + 1)^2 = 0$
$\Rightarrow x = -1$ which is not a natural number.
Thus, B = {} = ϕ
Hence, B is not a singleton set.

25. Let the point on the y-axis be P(0, y)
Here, it is given that P is equidistant from A(-4, 3) and B(5, 2).
i.e., PA = PB
$\Rightarrow \sqrt{(-4-0)^2 + (3-y)^2} = \sqrt{(5-0)^2 + (2-y)^2}$
Squaring both sides, we obtain
$\Rightarrow (-4 – 0)^2 + (3 – y)^2 = (5 – 0)^2 + (2 – y)^2$
$\Rightarrow 16 + 9 – 6y + y^2 = 25 + 4 – 4y + y^2$
$\Rightarrow 25 – 6y = 29 – 4y$

$\Rightarrow$ 2y = -4

$\Rightarrow$ y = -2

Thus, the required point on the y-axis is (0, -2).

Section C

26. Here ${}^{22}P_{r+1}\colon\ {}^{20}P_{r+2} = 11:52$

$\Rightarrow \frac{22!}{(21-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52}$

$\Rightarrow = \frac{22\times21\times20!}{(21-r)(20-r)(19-r)(18-r)!} \times \frac{(18-r)!}{20!} = \frac{11}{52}$

$\Rightarrow \frac{22\times21}{(21-r)(20-r)(19-r)} = \frac{11}{52}$

$\Rightarrow (21-r)(20-r)(19-r) = 2\times21\times52$

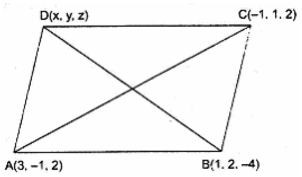
$\Rightarrow (21-r)(20-r)(19-r) = 14\times13\times12$

$\Rightarrow (21-r)(20-r)(19-r) = (21-7)(20-7)(19-7)$

$\Rightarrow r = 7$

27. Let D (x, y, z) be the fourth vertex of parallelogram ABCD.

We know that diagonals of a parallelogram bisect each other. So the mid points of AC and BD coincide.



$\therefore$ Coordinates of mid point of AC $\left(\frac{3-1}{2},\ \frac{-1+1}{2},\ \frac{2+2}{2}\right)$

= (1, 0, 2)

Also coordinates of mid point of BD $\left(\frac{x+1}{2},\ \frac{y+2}{2},\ \frac{z-4}{2}\right)$

$\therefore \frac{x+1}{2} = 1$ $\Rightarrow$ x + 1 = 2 $\Rightarrow$ x = 1

$\frac{y+2}{2} = 0$ $\Rightarrow$ y + 2= 0 $\Rightarrow$ y = -2

$\frac{z-4}{2} = 2$ $\Rightarrow$ z - 4 = 4 $\Rightarrow$ z = 8

Thus the coordinates of point D are (1, -2, 8)

28. To find: Expansion of $\left(x^2 - \frac{3x}{7}\right)^7$

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

We know that ,$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

Here We have, $(x^2 - \frac{3x}{7})^7$

$\Rightarrow \left[{}^7C_0\left(x^2\right)^{7-0}\right] + \left[7C_1\left(x^2\right)^{7-1}\left(-\frac{3x}{7}\right)^1\right] + \left[7c_2\left(x^2\right)^{7-2}\left(-\frac{3x}{7}\right)^2\right] + \left[7C_3\left(x^2\right)^{7-3}\left(-\frac{3x}{7}\right)^3\right] + \left[7C_4\left(x^2\right)^{7-4}\left(-\frac{3x}{7}\right)^4\right]$

$+ \left[7C_5\left(x^2\right)^{7-5}\left(-\frac{3x}{7}\right)^5\right] + \left[7C_6\left(x^2\right)^{7-6}\left(-\frac{3x}{7}\right)^6\right] + \left[7C_7\left(-\frac{3x}{7}\right)^7\right]$

$\Rightarrow \left[\frac{7!}{0!(7-0)!}\left(x^2\right)^7\right] - \left[\frac{7!}{1!(7-1)!}\left(x^2\right)^6\left(\frac{3x}{7}\right)\right] + \left[\frac{7!}{2!(7-2)!}\left(x^2\right)^5\left(\frac{9x^2}{49}\right)\right] - \left[\frac{7!}{3!(7-3)!}\left(x^2\right)^4\left(\frac{27x^3}{343}\right)\right]$

$+ \left[\frac{7!}{4!(7-4)!}\left(x^2\right)^3\left(\frac{81x^4}{2401}\right)\right] - \left[\frac{7!}{5!(7-5)!}\left(x^2\right)^2\left(\frac{243x^5}{16807}\right)\right] + \left[\frac{7!}{6!(7-6)!}\left(x^2\right)^1\left(\frac{729x^6}{117649}\right)\right] - \left[\frac{7!}{7!(7-7)!}\left(\frac{2187x^7}{823543}\right)\right]$

$- \left[\frac{7!}{7!(7-7)!}\left(\frac{2187x^7}{823543}\right)\right] + \left[21\left(x^{10}\right)\left(\frac{9x^2}{49}\right)\right] - \left[35\left(x^8\right)\left(\frac{27x^3}{343}\right)\right]$

$+ \left[35\left(x^6\right)\left(\frac{81x^4}{2401}\right)\right] - \left[21\left(x^4\right)\left(\frac{243x^5}{16807}\right)\right] + \left[7\left(x^2\right)\left(\frac{729x^6}{117649}\right)\right] - \left[1\left(\frac{2187x^7}{823543}\right)\right]$

$\Rightarrow x^{24} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7$

$x^{14} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7$

OR

From the given equation we have $2^{4n+4} - 15n - 16 = 2^{4(n+1)} - 15n - 16$

$= 16^{n+1} - 15n - 16$

$= (1+15)^{n+1} - 15n - 16$

Using binomial expression we have

$={}^{n+1}C_0 15^0 + {}^{n+1}C_1 15^1 + {}^{n+1}C_2 15^2 + {}^{n+1}C_3 15^3$

$+ \ldots + x + [C], (15)^{n+1} - 15n - 16$

$= 1 + (n+1)15 + {}^{n+1}C_2 15^2 + {}^{n+1}C_3 15^3$

$+ \ldots + n + 1C_{n+1}(15)^{n+1} - 15n - 16$
$= 1 + 15n + 15 +^{n+1} \mathrm{C}_2 15^2 +^{n+1} \mathrm{C}_3 15$
$+ \ldots +^{n+1} \mathbf{C}_{n+1}(15)^{n+1} - 15n - 16$
$= 15^2 \left[^{n+1}\mathrm{C}_2 +^{n+1} \mathrm{C}_3 15 + \ldots \text{ so on } \right]$
Thus, $2^{4n+4} - 15n - 16$ is divisible 225.

29. Here f(x) = x^{-4} (3 - 4x^{-5})

f'(x) = $\frac{d}{dx}$[x^{-4} (3 - 4x^{-5})]

$= x^{-4} \frac{d}{dx}(3 - 4x^{-5}) + (3 - 4x^{-5}) \frac{d}{dx}(x^{-4})$

= x^{-4} (20x^{-6}) + (3 - 4x^{-5}) (-4x^{-5})

= 20x^{-10} - 12x^{-5} + 16x^{-10}

= 36x^{-10} - 12x^{-5} $= \frac{36}{x^{10}} - \frac{12}{x^5}$

OR

Let $y = \frac{(x-1)(x-2)}{(x-3)(x-4)}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\left[\begin{array}{r}(x-3)(x-4)\frac{d}{dx}[(x-1)(x-2)] - (x-1) \\ (x-2)\frac{d}{dx}[(x-3)(x-4)]\end{array}\right]}{[(x-3)(x-4)]^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= \frac{(x-3)(x-4)\left[(x-1)\frac{d}{dx}(x-2) + (x-2)\frac{d}{dx}(x-1)\right] - (x-1)(x-2)\left[(x-3)\frac{d}{dx}(x-4) + (x-4)\frac{d}{dx}(x-3)\right]}{(x-3)^2(x-4)^2}$$

$$= \frac{(x-3)(x-4)[(x-1)\cdot 1 + (x-2)\cdot 1] - (x-1)(x-2)[(x-3)\cdot 1 + (x-4)\cdot 1)]}{(x-3)^2(x-4)^2}$$

$$= \frac{(x-3)(x-4)[2x-3] - (x-1)(x-2)[2x-7]}{(x-3)^2(x-4)^2}$$

$$= \frac{(x^2-7x+12)(2x-3) - (x^2-3x+2)(2x-7)}{(x-3)^2(x-4)^2}$$

$$= \frac{2x^3 - 14x^2 + 24x - 3x^2 + 21x - 36 - 2x^3 + 6x^2 - 4x + 7x^2 - 21x + 14}{(x-3)^2(x-4)^2}$$

$$= \frac{-4x^2 + 20x - 22}{(x-3)^2(x-4)^2}$$

30. $\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$

$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$

By C and D

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$

By C and D

$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

Squaring both side

$\frac{a}{b} = \frac{m+n+m-n+2\sqrt{m^2-n^2}}{m+n+m-n-2\sqrt{m^2-n^2}}$

$\frac{a}{b} = \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

OR

Given: $\sum_{k=1}^{11}\left(2+3^k\right)$

= (2 + 3^1) + (2 + 3^2) + (2 + 3^3) + (2 + 3^{11})

= (2 + 2 + 2 +........11 times) + (3 + 3^2 + 3^3 +....... +3^{11})

= 22 + (3 + 3^2 + 3^3 +....... +3^{11}) ……….(i)

Here 3, 3^2,3^3 .... ,$3^{11}$is in G.P.

$\therefore$a = 3 and r = $\frac{3^2}{3} = 3$

$S_n = \frac{3(3^{11}-1)}{3-1} = \frac{3}{2}(3^{11}-1)$

Putting the value of S_n in eq. (i), we get $\sum_{k=1}^{11}(2+3^k) = 22 + \frac{3}{2}(3^{11}-1)$

31. Given, n(p) = 18, n(C) = 23, n(M) = 24, n(C ∩ M) = 13,
    n(P ∩ C) = 12, n(P ∩ M) = 11 and n(P ∩ C ∩ M) = 6
    i. Total no. of students in the class
       = n(P ∪ C ∪ M)
       = n(P) + n(C) + n(M) - n(P ∩ C) - n(P ∩ M) - n(C ∩ M) + n(P ∩ C ∩ M)
       = 18 + 23 + 24 - 12 - 11 - 13 + 6 = 35
    ii. No. of students who took Mathematics but not Chemistry
       = n(M - C)
       = n(M) - n(M ∩ C)
       = 24 - 13 = 11
    iii. No. of students who took exactly one of the three subjects
       = n(P) + n(C) + n(M) - 2n(M ∩ P) - 2n(P ∩ C) - 2n(M ∩ C) + 3n(P ∩ C ∩ M)
       = 18 + 23 + 24 - 2 × 11 - 2 × 12 - 2 × 13 + 3 × 6
       = 65 - 22 - 24 - 26 + 18
       = 83 - 72 = 11

Section D

32. We have

$N = \sum_{i=1}^{6} f_i$ = (6 + 8 + 1 5 + 3 + 8 + 4) = 44

$\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{N} = \frac{(6\times3)+(8\times5)+(15\times7)+(3\times9)+(8\times11)+(4\times13)}{44}$

$= \frac{(18+40+105+27+88+52)}{44} = \frac{330}{44} = \frac{15}{2}$ = 7.5

x_i	3	5	7	9	11	13
f_i	6	8	15	3	8	4
cf	6	14	29	32	40	44

Here we have, N = 44, which is even.

Therefore, median $= \frac{1}{2} \cdot \left\{\frac{N}{2}\right.$ th observation + $\left(\frac{N}{2}+1\right)$ th observation}

$= \frac{1}{2}$ (22nd observation + 23rd observation)

$= \frac{1}{2}$ (7 + 7) = 7

Thus, M = 7.

Now, we have:

$\lvert x_i - M\rvert$	4	2	0	2	4	6
f_i	6	8	15	3	8	4
$f_i\lvert x_i - M\rvert$	24	16	0	6	32	24

$\therefore \sum_{i=1}^{6} f_i = 44$ and $\sum_{i=1}^{6} f_i\lvert x_i - M\rvert = 102$

$\therefore \text{MD}(\bar{x}) = \frac{\sum_{i=1}^{6} f_i\lvert x_i-M\rvert}{N} = \frac{102}{44}$ = 2.32

33. Here, the equation of circle is $x^2 + y^2 + 4x + 6y + 11 = 0$

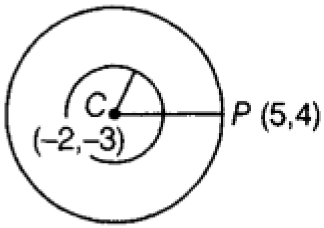
$\Rightarrow (x^2 + 4x) + (y^2 + 6y) = - 11$

On adding 4 and 9 both sides to make perfect squares, we get

$(x^2 + 4x + 4) + (y^2 + 6y + 9) = - 11 + 4 + 9$

$\Rightarrow (x + 2)^2 + (y + 3)^2 = (\sqrt{2})^2$ ...(i)

Its centre is (- 2, - 3)



The required circle is concentric with circle 1, therefore its centre is (- 2 , - 3) . Since, it passes through (5, 4), therefore radius is

r = CP = $\sqrt{(5+2)^2+(4+3)^2}$ [$\because$ distance = $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$]

= $\sqrt{49+49}=7\sqrt{2}$

Hence, the equation of required circle having centre (- 2, - 3) and radius $7\sqrt{2}$ is,

$(x + 2)^2 + (y + 3)^2 = (7\sqrt{2})^2$

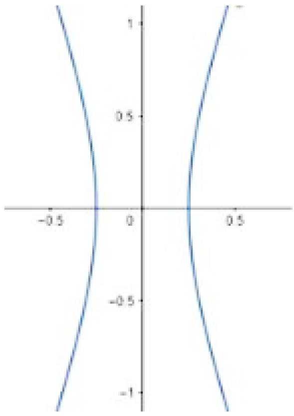
$\Rightarrow x^2 + 4x + 4 + y^2 + 6y + 9 = 98$

$\Rightarrow x^2 + 4x + y^2 + 6y - 85 = 0$

OR

Given: The length of latus rectum is 4, and the eccentricity is 3

Let, the equation of the hyperbola be: $\frac{x^2}{a^2}-\frac{y^2}{b^2}$ = 1



The length of the latus rectum is 4 units.

$\Rightarrow$ length of the latus rectum = $\frac{2b^2}{a}$ = 4

$\Rightarrow \frac{2b^2}{a}$ = 4 $\Rightarrow b^2 = 2a$ ...(i)

And also given, the eccentricity, e = 3

We know that, $e=\sqrt{1+\frac{b^2}{a^2}}$

$\Rightarrow \sqrt{1+\frac{b^2}{a^2}}$ = 3

$\Rightarrow 1+\frac{b^2}{a^2}$ = 9 [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2}$ = 8

$\Rightarrow b^2 = 8a^2$

$\Rightarrow 2a = 8a^2$ [From (i)]

$\Rightarrow a=\frac{1}{4} \Rightarrow a^2 = \frac{1}{16}$

From (i) $\Rightarrow b^2 = 2a = 2\times\frac{1}{4}=\frac{1}{2} \Rightarrow b^2 = \frac{1}{2}$

So, the equation of the hyperbola is,

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{1/16}-\frac{y^2}{1/2}$ = 1

$\Rightarrow 16x^2 - 2y^2 = 1$

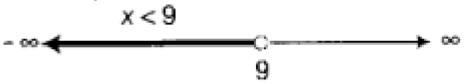
34. We have, $\frac{4x}{3}-\frac{9}{4} < x + \frac{3}{4}$ ... (i)

and $\frac{7x-1}{3}-\frac{7x+2}{6} > x$ ... (ii)

From inequality (i), we get

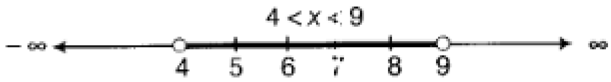
$\frac{4x}{3}-\frac{9}{4} < x + \frac{3}{4} \Rightarrow \frac{16x-27}{12} < \frac{4x+3}{4}$

$\Rightarrow$ 16x - 27 < 12x + 9 [multiplying both sides by 12]
$\Rightarrow$ 16x - 27 + 27 < 12x + 9 + 27 [adding 27 on both sides]
$\Rightarrow$ 16x < 12x +36
$\Rightarrow$ 16x - 12x < 12x + 36 - 12x [subtracting 12x from bot sides]
$\Rightarrow$ 4x < 36 $\Rightarrow$ x < 9 [dividing both sides by 4]
Thus, any value of x less than 9 satisfies the inequality. So, the solution of inequality (i) is given by $x \in (-\infty, 9)$



From inequality (ii) we get,
$\frac{7x-1}{3}$ - $\frac{7x+2}{6}$ > x $\Rightarrow$ $\frac{14x-2-7x-2}{6}$ > x
$\Rightarrow$ 7x - 4 > 6x [multiplying by 6 on both sides]
$\Rightarrow$ 7x - 4 + 4 > 6x + 4 [adding 4 on both sides]
$\Rightarrow$ 7x > 6x + 4
$\Rightarrow$ 7x - 6x > 6x + 4 - 6x [subtracting 6x from both sides]
$\therefore$ x > 4
Thus, any value of x greater than 4 satisfies the inequality.
So, the solution set is $x \in (4, \infty)$

The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



Clearly, the common value of x lie between 4 and 9.
Hence, the solution of the given system is, 4 < x < 9 i.e., $x \in (4, 9)$

35. Given, LHS $= sin20^o sin40^o sin80^o$
= $\frac{1}{2}$ [2 sin 20° $\cdot$ sin 40°] sin 80^o [multiplying and dividing by 2]
= $\frac{1}{2}[cos(20^o - 40^o) - cos(20^o + 40^o)] \cdot \sin 80^\circ$ [$\because 2\sin x \cdot \sin y$ = cos (x - y) - cos (x + y)]
= $\frac{1}{2}$ $[cos(-20^o) - cos60^o]sin80^o$
= $\frac{1}{2}$ [cos 20^o - $\frac{1}{2}$]$\cdot$ sin 80° [$\because$ cos (- θ) = cos θ and cos 60^o = $\frac{1}{2}$]
= $\frac{1}{2} \times \frac{1}{2}\left[2\left(\cos 20^\circ - \frac{1}{2}\right) \cdot \sin 80^\circ\right]$ [again multiplying and dividing by 2]
= $\frac{1}{4}[2\cos 20^\circ \cdot \sin 80^\circ - \sin 80^\circ]$
= $\frac{1}{4}[sin(20^o + 80^o) - sin(20^o - 80^o) - sin80^o]$ [$\because 2\cos x \cdot \sin y = sin(x+y) - sin(x-y)$]
= $\frac{1}{4}$ $[sin100^o - sin(-60^o) - sin80^o]$
= $\frac{1}{4}$ [sin 100^o + sin 60^o - sin 80^o] [$\because$ sin (- θ) = - sin θ]
= $\frac{1}{4}$ [sin (180^o - 80^o) + sin 60^o - sin 80^o] [$\because$ sin 100^o = sin (180^o - 80^o)]
= $\frac{1}{4}$ [sin 80^o + sin 60^o - sin 80^o] [$\because$ sin ($\pi - \theta$) = sin θ]
= $\frac{1}{4}$ $\times$ sin 60^o = $\frac{1}{4}$ $\times$ $\frac{\sqrt{3}}{2}$ [$\because$ sin 60^o = $\frac{\sqrt{3}}{2}$]
= $\frac{\sqrt{3}}{8}$ = RHS
Hence proved.

OR

LHS $= cos12^o + cos60^o + cos84^o$
$= cos12^o + (cos84^o + cos60^o)$
=$cos12^o + [2cos\left(\frac{84^\circ + 60^\circ}{2}\right) \times cos\left(\frac{84^\circ - 60^\circ}{2}\right)]$
[$\because$ cosx +cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]
$= cos12^o + [2cos\ \frac{144^\circ}{2} \times$ cos $\frac{24^\circ}{2}]$
$= cos12^o + [2cos72^o \times cos12^o] = cos12^o[1 + 2cos72^o]$
$= cos12^o[1 + 2cos(90^o - 18^o)]$
= cos 12^o [1 + 2 sin 18^o] [$\because$ cos (90^o - θ) = sin θ]
= cos 12^o [1 + 2 $\left(\frac{\sqrt{5}-1}{4}\right)$] [$\because$ sin 18^o = $\frac{\sqrt{5}-1}{4}$]

= $\left(1+\frac{\sqrt{5}-1}{2}\right)$ cos 12^o = $\left(\frac{\sqrt{5}+1}{2}\right)$ cos 12^o

RHS = cos 24^o + cos 48^o

= 2 cos $\left(\frac{24^\circ+48^\circ}{2}\right)$ cos $\left(\frac{24^\circ-48^\circ}{2}\right)$ [$\because$ cosx + cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$= 2cos36^o cos(-12^o)$

= 2 cos 36^o $\times$ cos 12^o [$\because$ cos (- θ) = cos θ]

= 2 $\times$ $\frac{\sqrt{5}+1}{4}$ $\times$ cos 12^o = $\frac{\sqrt{5}+1}{2}$ $\times$ cos 12^o [$\because$ cos 36^o = $\frac{\sqrt{5}+1}{4}$]

$\therefore$ **LHS = RHS**

Hence proved.

Section E

36. i. Number of functions from A to B are $n(B)^{n(A)} = q^p$

ii. Number of relations from A to B is $2^{n(A)n(B)} = 2^{pq}$.

iii. Figures A and B show relations. Figure C shows a function but not a relation.

OR

x is a factor of y.

1, 2, 4 and 8 are factors of 8.

37. i. Total marbles = 4 + 5 + 3 = 12

Required probability $= \frac{^4C_2}{^{12}C_2} = \frac{\frac{4\times3}{2\times1}}{\frac{12\times11}{2\times1}} = \frac{1}{11}$

ii. Total marbles = 4 + 5 + 3 = 12

Required probability = $\frac{^3C_3}{^{12}C_3} = \frac{1}{\frac{12\times11\times10}{3\times2}} = \frac{1}{220}$

iii. Total marbles = 4 + 5 + 3 = 12

Required probability $= \frac{^7C_2}{^{12}C_2} = \frac{\frac{7\times6}{2\times1}}{\frac{12\times11}{2\times1}} = \frac{21}{66} = \frac{7}{22}$

OR

Total marbles = 4 + 5 + 3 = 12

Required probability = 1 - P (None is blue)

$= 1 - \frac{^7C_3}{^{12}C_3}$

$= 1 - \frac{\frac{7\times6\times5}{3\times2}}{\frac{12\times11\times10}{3\times2}}$

$= 1 - \frac{7}{44} = \frac{37}{44}$

38. i. r = $|Z| = 2\sqrt{2}$

x = 2, y = -2

cosθ = $\frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

sinθ = $\frac{y}{r} = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}}$

Arg(Z) = $\frac{-\pi}{4}$

ii. $z\bar{z} = |z|^2 = (2\sqrt{2})^2$ = 8

iii. $|Z| = \sqrt{2^2+(-2)^2}$

= $\sqrt{8}$ = $2\sqrt{2}$

OR

Real part of 2 - 2i = 2