CHAPTER

6.2

STABILITY

1. Consider the system shown in fig. P6.2.1. The range of K for the stable system is

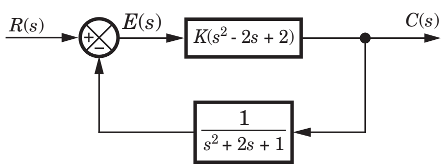

Fig. P6.2.1

(A) $-1 < K < -\frac{1}{2}$ (B) $-\frac{1}{2} < K < 1$

(C) $-1 < K < 1$ (D) Unstable

2. The forward transfer function of a *ufb* system is

$$G(s) = \frac{K(s^2 + 1)}{(s+1)(s+2)}$$

The system is stable for

(A) $K < -1$ (B) $K > -1$

(C) $K < -2$ (D) $K > -2$

3. The open-loop transfer function with *ufb* are given below for different systems. The unstable system is

(A) $\frac{2}{s+2}$ (B) $\frac{2}{s^2(s+2)}$

(C) $\frac{2}{s(s+2)}$ (D) $\frac{2(s+1)}{s(s+2)}$

4. Consider a *ufb* system with forward-path transfer function

$$G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$$

The range of K to ensure stability is

(A) $K > \frac{6}{8}$ (B) $K < -1$ or $K > \frac{3}{4}$

(C) $K < -1$ (D) $-1 < K < \frac{3}{4}$

5. Consider a *ufb* system with forward-path transfer function

$$G(s) = \frac{K(s+3)}{s^4(s+2)}$$

The system is stable for the range of K

(A) $K > 0$ (B) $K < 0$

(C) $K > 1$ (D) Always unstable

6. The open-loop transfer function of a *ufb* control system is

$$G(s) = \frac{K(s+2)}{(s+1)(s-7)}$$

For $K > 6$, the stability characteristic of the open-loop and closed-loop configurations of the system are respectively

(A) stable and unstable

(B) stable and stable

(C) unstable and stable

(D) unstable and unstable

7. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{K(s^2 - 4)}{s^2 + 3}$$

For the system to be stable the range of K is

(A) $K > -1$ (B) $K < \frac{3}{4}$

(C) $-1 < K < \frac{3}{4}$ (D) marginal stable

8. A *ufb* system have the forward-path transfer function

$$G(s) = \frac{K(s+6)}{s(s+1)(s+3)}$$

The system is stable for

(A) $K < 6$ (B) $-6 < K < 0$

(C) $0 < K < 6$ (D) $K > 6$

9. The feedback control system shown in the fig. P6.2.8.

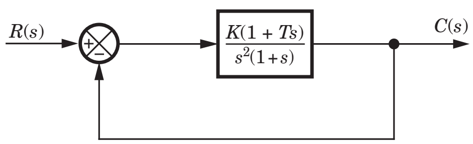

Fig. P6.2.9

is stable for all positive value of K, if

(A) $T = 0$ (B) $T < 0$

(C) $T > 1$ (D) $0 < T < 1$

10. Consider a *ufb* system with forward-path transfer function

$$G(s) = \frac{K}{(s+15)(s+27)(s+38)}$$

The system will oscillate for the value of K equal to

(A) 23690 (B) 2369

(C) 144690 (D) 14469

11. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{K(s-2)(s+4)(s+5)}{(s^2+3)}$$

For system to be stable, the range of K is

(A) $K > \frac{1}{54}$ (B) $K < \frac{3}{40}$

(C) $\frac{1}{54} < K < \frac{3}{40}$ (D) Unstable

12. The closed loop system shown in fig. P6.2.12 become marginally stable if the constant K is chosen to be

(A) 30 (B) –30

(C) 10 (D) –10

13. The open-loop transfer function of a *ufb* system is

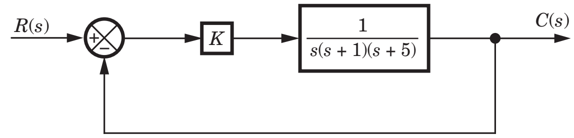

Fig. P6.2.12

$$G(s) = \frac{K(s+10)(s+20)}{s^2(s+2)}$$

The closed loop system will be stable if the value of K is

(A) 2 (B) 3

(C) 4 (D) 5

Statement for Q.14–15:

A feedback system is shown in fig. P6.14-15.

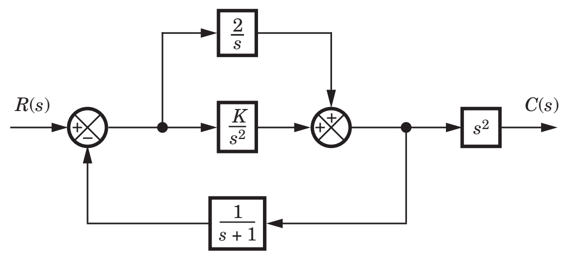

Fig. P6.2.14–15

14. The closed loop transfer function for this system is

(A) $\dfrac{s^5 + s^4 + 2s^3 + (K+2)s^2 + (K+2)s + K}{s^3 + s^2 + 2s + K}$

(B) $\dfrac{2s^4 + (K+2)s^3 + Ks^2}{s^3 + s^2 + 2s + K}$

(C) $\dfrac{s^3 + s^2 + 2s + K}{s^5 + s^4 + 2s^3 + (K+2)s^2 + (K+2)s + K}$

(D) $\dfrac{s^3 + s^2 + 2s + K}{2s^4 + (K+2)s^3 + Ks^2}$

15. The poles location for this system is shown in fig. P6.2.15. The value of K is

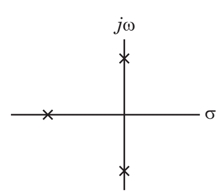

Fig. P6.2.15

(A) 4 (B) –4

(C) 2 (D) –2

(A) stable (B) unstable

(C) marginally stable

(D) More information is required.

27. The forward path transfer of *ufb* system is

$$G(s) = \frac{1}{4s^2(s^2+1)}$$

The system is

(A) stable (B) unstable

(C) marginally stable

(D) More information is required

28. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{G(s)}{2s^4+5s^3+s^2+2s}$$

The system is

(A) stable (B) unstable

(C) marginally stable

(D) more information is required.

29. The open loop transfer function of a system is as

$$G(s)H(s) = \frac{K(s+0.1)}{s(s-0.2)(s^2+s+0.6)}$$

The range of K for stable system will be

(A) $K > 0.355$ (B) $0.149 < K < 0.355$

(C) $0.236 < K < 0.44$ (D) $K > 0.44$

30. The open-loop transfer function of a *ufb* control system is given by

$$G(s) = \frac{K}{s(sT_1+1)(sT_2+1)}$$

For the system to be stable the range of K is

(A) $0 < K < \left(\frac{1}{T_1}+\frac{1}{T_2}\right)$ (B) $K > \left(\frac{1}{T_1}+\frac{1}{T_2}\right)$

(C) $0 < K < T_1T_2$ (D) $K > T_1T_2$

31. The closed loop transfer function of a system is

$$T(s) = \frac{s^3+4s^2+8s+16}{s^5+3s^4+5s^2+s+3}$$

The number of poles in right half-plane and in left half-plane are

(A) 3, 2 (B) 2, 3

(C) 1, 4 (D) 4, 1

32. The closed loop transfer function of a system is

$$T(s) = \frac{(s+8)(s+6)}{s^5-s^4+4s^3-4s^2+3s-2}$$

The number of poles in RHP and in LHP are

(A) 4, 1 (B) 1, 4

(C) 3, 2 (D) 2, 3

33. The closed loop transfer function of a system is

$$T(s) = \frac{s^3+3s^2+7s+24}{s^5-2s^4+3s^3-6s^2+2s-4}$$

The number of poles in LHP, in RHP, and on $j\omega$ axis are

(A) 2, 1, 2 (B) 0, 1, 4

(C) 1, 0, 4 (D) 1, 2, 2

34. For the system shown in fig. P6.2.34. the number of poles on RHP, LHP, and imaginary axis are

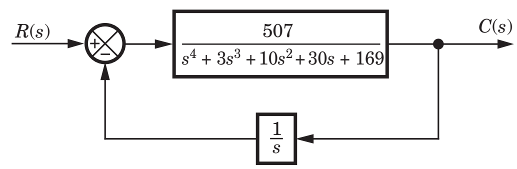

Fig. P6.2.34

(A) 2, 3, 0 (B) 3, 2, 0

(C) 2, 1, 2 (D) 1, 2, 2

35. A Routh table is shown in fig. P6.2.36. The location of pole on RHP, LHP and imaginary axis are

s^7	1	2
s^5	1	2
s^5	3	4
s^4	1	–1

Fig. P6.2.35

(A) 1, 2, 4 (B) 1, 6, 0

(C) 1, 0, 6 (D) None of the above

36. For the open loop system of fig. P6.2.35 location of poles on RHP, LHP, and an $j\omega$-axis are

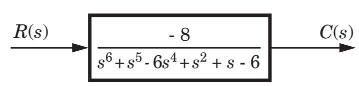

Fig. P6.2.35

(A) 3, 3, 0 (B) 1, 3, 2

(C) 1, 1, 4 (D) 3, 1, 2

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11. (C) $T(s) = \dfrac{G(s)}{1+G(s)}$

$$= \frac{K(s-2)(s+4)(s+5)}{Ks^3+(7K+1)s^2+2Ks+(3-40K)}$$

Routh table is as shown in fig. S.6.211

s^3	K	$2K$
s^2	$7K+1$	$3-40K$
s^1	$\frac{54K^2-K}{7K+1}$	
s^0	$3-40K$	

Fig. S.6.2.11

$K > 0,$

$$\left.\begin{aligned} 7K+1>0 \quad &\Rightarrow \quad K > -\frac{1}{7} \\ \frac{54K^2-K}{7K+1}>0 \quad &\Rightarrow \quad K > \frac{1}{54} \\ 3-40K>0 \quad &\Rightarrow \quad K < \frac{3}{40} \end{aligned}\right\} \Rightarrow \quad \frac{1}{54} < K < \frac{3}{40}$$

12. (A) $T(s) = \dfrac{1}{s^3+6s^2+5s+K}$

Routh table is as shown in fig. S.6.212

s^3	1	5
s^2	6	K
s^1	$30-K$	
s^0	K	

Fig. S.6.2.12

13. (D) $T(s) = \dfrac{K(s+10)(s+20)}{s^3+(K+2)s^2+30Ks+200K}$

Routh table is as shown in fig. S.6.2.13

s^3	1	$30K$
s^2	$K+2$	$200K$
s^1	$30K^2-140K$	
s^0	$200K$	

Fig. S.6.2.13

$200K > 0 \quad \to \quad K > 0, \quad 30K^2 - 140K > 0$

$\Rightarrow \quad K > \dfrac{14}{3},\quad$ 5 satisfy this condition.

14. (B) First combine the parallel loop $\dfrac{K}{s^2}$ and $\dfrac{2}{s}$ giving $\dfrac{K}{s^2}+\dfrac{2}{s}$. Then apply feedback formula with $\left(\dfrac{K}{s^2}+\dfrac{2}{s}\right)$ and $\dfrac{1}{(s+1)}$, and then multiply with s^2.

$$T(s) = \frac{s^2\left(\dfrac{K}{s^2}+\dfrac{2}{s}\right)}{1+\dfrac{1}{s+1}\left(\dfrac{K}{s^2}+\dfrac{2}{s}\right)} = \frac{2s^4+(K+2)s^3+Ks^2}{s^3+s^2+2s+K}$$

15. (C) Denominator $= s^3+s^2+2s+K$

Routh table is as shown in fig. S.6.2.15

s^3	1	5
s^2	1	K
s^1	$2-K$	
s^0	K	

Fig. S.6.2.15

Row of zeros when $K = 2$,

$s^2+2=0, \quad \Rightarrow \quad s = -1,\ j\sqrt{2},\ -j\sqrt{2}$

16. (D) Applying the feedback formula on the inner loop and multiplying by K yield

$$G_e(s) = \frac{K}{s(s^2+5s+7)},$$

$$T(s) = \frac{K}{s^3+5s^2+7s+K}$$

17. (B) Routh table is as shown in fig. S.6.2.17

s^3	1	7
s^2	5	K
s^1	$\frac{35-K}{5}$	
s^0	K	

Fig. S.6.2.17

$K > 0, \quad \dfrac{35-K}{5} > 0 \quad \Rightarrow \quad K < 35$

18. (C) At $K = 35$ system will oscillate.

Auxiliary equation $5s^2+35=0, \quad \Rightarrow \quad s = \pm j\sqrt{7}$

19. (B) For inner loop

$$G_i(s) = \frac{K}{(s-a)(s+3a)(s+4a)} = \frac{K}{P(s)}, \qquad T_i(s) = \frac{K}{P(s)+K}$$

For outer loop, $G_o(s) = T_i(s) = \dfrac{K}{P(s)+K}$,

$$T_o(s) = \frac{K}{P(s) + 2K},$$

Therefore if inner loop is stable for $X < K < Y$, then outer loop will be stable for $X < 2K < Y$

$$\Rightarrow \frac{X}{2} < K < \frac{Y}{2}.$$

20. (D) $T(s) = \dfrac{K(s+2)}{s^4 + 3s^3 - 3s^2 + (K+3)s + (2K-4)}$

Routh table is as shown in fig. S.6.2.20

s^4	1	-3	$2K-4$
s^3	3	$K+3$	
s^2	$-\frac{(K+12)}{3}$	$2K-4$	
s^1	$\frac{K(K+33)}{K+12}$		
s^0	$2K-4$		

Fig. S.6.2.20

$$\frac{-(K+12)}{3} > 0 \Rightarrow K < -12, \quad 2K - 4 > 0$$

$\Rightarrow K > 2$ and $K > -33$, These condition can not be met simultaneously. System is unstable for any value of K

21. (D) Routh table is as shown in fig. S.6.2.21

s^4	1	1	1
s^3	K	1	
s^2	$\frac{K-1}{K}$	1	
s^1	$\frac{K-1-K^2}{K-1}$		
s^0	1		

Fig. S.6.2.21

$$K > 0, \quad K - 1 > 0 \Rightarrow K > 1, \quad \frac{K-1-K^2}{K-1} > 0,$$

But for $K > 1$ third term is always -ive. Thus the three condition cannot be fulfilled simultaneously.

22. (D) Routh table is as shown in fig. S.6.2.22

s^4	1	$4+K$	25
s^3	2	9	
s^2	$\frac{2K-1}{2}$	25	
s^1	$\frac{18K-109}{2K-1}$		
s^0	25		

Fig. S.6.2.22

$$\left.\begin{array}{l} \dfrac{2K-1}{2} > 0 \quad \Rightarrow \quad K > \dfrac{1}{2} \\ \dfrac{18K-109}{2K-1} > 0 \quad \Rightarrow \quad K > \dfrac{109}{18} \end{array}\right\} \Rightarrow K > \frac{109}{18}$$

23. (B) Characteristic equation

$$s^4 + 9s^3 + 20s^2 + Ks + K = 0$$

Routh table is as shown in fig. S.6.2.23

s^4	1	20	K
s^3	9	K	
s^2	$\frac{180-K}{9}$	K	
s^1	$\frac{K(K-99)}{K-180}$		
s^0	K		

Fig. S.6.2.23

For stability $0 < K < 99$

24. (C) $T(s) = \dfrac{K(s+2)}{s^4 + 3s^2 - 3s^2 + (K+3)s + (2K-4)}$

Routh table is as shown in fig. S.6.2.24

s^4	1	-3	$2K-4$
s^3	3	$K+3$	
s^2	$\frac{-K+12}{3}$	$2K-4$	
s^1	$\frac{K(K+33)}{K+12}$		
s^0	$2K-4$		

Fig. S.6.2.24

For $K < -33$, 1 sign change

For $-33 < K < -12$, 1 sign change

For $-12 < K < 0$, 1 sign change

For $0 < K < 2$, 3 sign change

For $K > 2$, 2 sign change

Therefore $K > 2$ yield two RHP pole.

25. (B) Routh table is as shown in fig. S.6.2.25

s^4	1	8	15
s^3	4	20	
s^2	3	15	
s^1	6		ROZ
s^0	15		

Fig. S.6.2.25

$P(s) = 3s^2 + 15$, $\frac{d\,P(s)}{ds} = 6s$, No sign change from s^2 to s^0

on $j\omega$-axis 2 roots, RHP 0,   LHP 2.

26. (B) Closed-loop transfer function is

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{240}{s^4 + 10\,s^3 + 35\,s^2 + 50s + 264}$$

Routh table is as shown in fig. S.6.2.26

s^4	1	35	264
s^3	10	50	
s^2	30	264	
s^1	–386		ROZ
s^0	264		

Fig. S.6.2.26

Two sign change. RHP-2 poles. System is not stable.

27. (C) Closed loop transfer function

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{4s^4 + 4s^2 + 1}$$

Routh table is as shown in fig. S.6.2.27

s^4	4	4	1
s^3	16	8	ROZ
s^2	2	1	
s^1	46		ROZ
s^0	1		

Fig. S.6.2.27

$P(s) = 4s^4 + 4s + 1$, $\frac{dp(s)}{ds} = 16s^3 + 3s$

There is no sign change. So all pole are on $j\omega$–axis. So system is marginally stable.

28. (B) Closed loop transfer function

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{2s^4 + 5s^3 + s^2 + 2s + 1}$$

Routh table is as shown in fig. S.6.2.28

s^4	2	1	1
s^3	5	2	
s^2	$\frac{1}{5}$	1	
s^1	–23		
s^0	1		

Fig. S.6.2.28

2 RHP poles so unstable.

29. (B) The characteristic equation is $1 + G(s)H(s) = 0$

$\Rightarrow \quad s(s-0.2)(s^2 + s + 0.6) + K(s+0.1) = 0$

$s^4 + 0.8\,s^3 + 0.4s^2 + (K - 0.12)s + 0.1K = 0$

Routh table is as shown in fig. S.6.2.29

s^4	2	0.4	1
s^3	0.8	$K - 0.12$	
s^2	$0.55 - 1.25K$	$0.1K$	
s^1	$\frac{-1.25K^2 + 0.63K - 0.066}{0.55 - 1.25K}$		
s^0	$0.1K$		

Fig. S.6.2.29

$K > 0$, $0.55 - 1.25K > 0 \;\Rightarrow\; K < 0.44$

$-1.25K^2 + 0.63K - 0.066 > 0$

$(K - 0.149)(K - 0.355) < 0$, $0.149 < K < 0.355$

30. (A) Characteristic equation

$s(sT_1 + 1)(sT_2 + 1) + K = 0$

$T_1T_2s^3 + (T_1 + T_2)s^2 + s + K = 0$

Routh table is as shown in fig. S.6.2.30

s^3	T_1T_2	1
s^2	$T_1 + T_2$	K
s^1	$\frac{(T_1+T_2) - T_1T_2K}{T_1+T_2}$	
s^0	K	

Fig. S.6.2.30

$K > 0$, $(T_1 + T_2) - T_1T_2K > 0 \quad \Rightarrow \quad 0 < K < \left(\frac{1}{T_1} + \frac{1}{T_2}\right)$

31. (B) Routh table is as shown in fig. S.6.2.31

s^5	1	5	1
s^4	3	4	3
s^3	3.67	0	
s^2	4	3	
s^1	–2.75		
s^0	3		

In RHP –2 poles. In LHP –3 poles.

32. (C) Routh table is as shown in fig. S.6.2.32

ε + −
+ +
− −
+ −
+ −
+ +
− −

s^5	1	4	3
s^4	–1	–4	–2
s^3	ε	1	
s^2	$\frac{1-4\varepsilon}{\varepsilon}$	–2	
s^1	$\frac{2\varepsilon^2+1-4\varepsilon}{1-4\varepsilon}$		
s^0	–2		

Fig. S.6.2.32

3 RHP, 2 LHP poles.

33. (B) Routh table is as shown in fig. S.6.2.33

s^5	1	3	2
s^4	–2	–6	–4
s^3	–2	–3	ROZ
s^2	–3	–4	
s^1	$-\frac{1}{3}$		
s^0	–4		

Fig. S.6.2.33

$P(s) = -2s^4 - 6s^2$, $\frac{dP(s)}{ds} = -8s^3 - 12s$, –2 , –3

No sign change exist from the s^4 row down to the s^0 row. Thus, the even polynomial does not have RHP poles. Therefore because of symmetry all four poles must be on $j\omega$-axis.

$j\omega$-axis 4 pole

RHP 1 pole (1 sign change)

LHP 0 pole

34. (D) Closed loop transfer function

$$T(s) = \frac{G(s)}{1 + G(s) + H(s)}$$

$$= \frac{507s}{s^5 + 3s^4 + 10s^3 + 30s^2 + 169s + 507}$$

Routh table is as shown in fig. S.6.2.34

s^5	1	10	69
s^4	3	30	57
s^3	12	60	ROZ
s^2	15	507	
s^1	–345.6		
s^0	507		

Fig. S.6.2.33

$P(s) = 3s^4 + 30s^2 + 507$, $\frac{dP(s)}{ds} = 12s^3 + 60$

From s^4 row down to s^0 there is one sign change. So LHP–1 + 1 = 2 pole. RHP–1 pole, $j\omega$-axis –2 pole.

35. (A) Notice that in s^5 row there would be zero. In this row coefficient of $\frac{dP(s)}{ds}$, where $P(s) = s^6 + 2s^4 - s^2 - 2$ have been entered. From s^6 to row down to the s^0 row, there is one sign change. So there is one pole on RHP. Corresponding to this pole there is a pole on LHP. Corresponding to this pole there is a pole on LHP. Rest 4 out of 6 poles are on imaginary axis. Rest 1 pole is on LHP.

36. (A) Routh table is as shown in fig. S.6.2.36

ε + −
+ +
+ +
− −
− −
+ −
− +
− −

s^6	1	–6	–6
s^5	1	0	
s^4	–6	0	
s^3	–24	0	ROZ
s^2	ε		
s^1	$-\frac{144}{\varepsilon}$		
s^0	–6		

Fig. S.6.2.36

$P(s) = -6s^4 - 6$, $\frac{dP(s)}{ds} = -24s^3$,

There is two sign change from the s^4 row down to the s^0 row. So two roots are on RHS. Because of symmetry rest two roots must be in LHP. From s^6 to s^4 there is 1 sign change so 1 on RHP and 1 on LHP.

Total LHP 3 root, RHP 3 root.

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8. A system is shown in fig. P6.3.8. The rise time and settling time for this system is

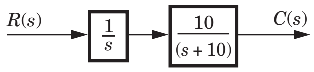

Fig. P6.3.8

(A) 0.22s, 0.4s (B) 0.4s, 0.22s

(C) 0.12s, 0.4s (D) 0.4s, 0.12s

9. For a second order system settling time is $T_s = 7$ s and peak time is $T_p = 3$ s. The location of poles are

(A) $-0.97 \pm j0.69$ (B) $-0.69 \pm j0.97$

(C) $-1.047 \pm j0.571$ (D) $-0.571 \pm j1.047$

10. For a second order system overshoot $= 10\%$ and peak time $T_p = 5$ s. The location of poles are

(A) $-0.46 \pm j0.63$ (B) $-0.63 \pm j0.46$

(C) $-0.74 \pm j0.92$ (D) $-0.92 \pm j0.74$

11. For a second-order system overshoot $= 12\ \%$ and settling time $= 0.6$ s. The location of poles are

(A) $-9.88 \pm j6.67$ (B) $-6.67 \pm j9.88$

(C) $-4.38 \pm j6.46$ (D) $-6.46 \pm j4.38$

Statement for Q.12–13:

A system has a damping ratio of 1.25, a natural frequency of 200 rad/s and DC gain of 1.

12. The response of the system to a unit step input is

(A) $1 + \frac{5}{3}e^{-50t} - \frac{2}{3}e^{-150t}$ (B) $1 - \frac{4}{3}e^{-100t} + \frac{1}{3}e^{-400t}$

(C) $1 + \frac{1}{3}e^{-100t} - \frac{4}{3}e^{-400t}$ (D) $1 + \frac{2}{3}e^{-50t} - \frac{5}{3}e^{-150t}$

13. The system is

(A) overdamped (B) under damped

(C) critically damped (D) None of the above

14. Consider the following system

a. $T(s) = \dfrac{5}{(s+3)(s+6)}$

b. $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

c. $T(s) = \dfrac{20}{s^2 + 6s + 144}$

d. $T(s) = \dfrac{s+2}{s^2 + 9}$

e. $T(s) = \dfrac{(s+5)}{(s+10)^2}$

Consider the following response

1. Overdamped 2. Under damped

3. Undamped 4. Critically damped.

The correct match is

	1	2	3	4
(A)	a	c	d	e
(B)	b	a	d	e
(C)	c	a	e	d
(D)	c	b	e	d

15. The forward-path transfer of a *ufb* control system is

$$G(s) = \frac{1000}{(1+0.1s)(1+10s)}$$

The step, ramp, and parabolic error constants are

(A) 0, 1000, 0 (B) 1000, 0, 0

(C) 0, 0, 0 (D) 0, 0, 1000

16. The open-loop transfer function of a *ufb* control system is

$$G(s) = \frac{K(1+2s)(1+4s)}{s^2(s^2+2s+8)}$$

The position, velocity and acceleration error constants are respectively

(A) 0, 0, $4K$ (B) ∞, $\frac{K}{8}$, 0

(C) 0, $4K$, ∞ (D) ∞, ∞, $\frac{K}{8}$

17. The open-loop transfer function of a unit feedback system is

$$G(s) = \frac{50}{(1+0.1s)(1+2s)}$$

The position, velocity and acceleration error constants are respectively

(A) 0, 0, 250 (B) 50, 0, 0

(C) 0, 250, ∞ (D) ∞, 50, 0

Statement for Q.18–19:

The forward-path transfer function of a unity feedback system is

$$G(s) = \frac{K}{s^n(s+a)}$$

The system has 10% overshoot and velocity error constant $K_v = 100$.

18. The value of K is

(A) 237×10^3 (B) 144

(C) 14.4×10^3 (D) 237

19. The value of a is

(A) 23.7×10^3 (B) 237

(C) 14.4×10^3 (D) 144

20. For the system shown in fig. P6.3.20 the steady state error component due to unit step disturbance is 0.000012 and steady state error component due to unit ramp input is 0.003. The values of K_1 and K_2 are respectively

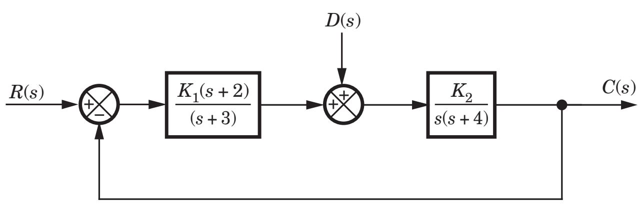

Fig. P6.3.20

(A) 16.4, 1684 (B) 1250, 2.4

(C) 125×10^3, 0.016 (D) 463, 3981

21. The transfer function for a single loop nonunity feedback control system is

$$G(s) = \frac{1}{s^2 + s + 2}, \quad H(s) = \frac{1}{(s+1)}$$

The steady state error due to unit step input is

(A) $\frac{6}{7}$ (B) $\frac{6}{5}$

(C) $\frac{2}{3}$ (D) 0

22. For the system of fig. P6.3.22 the total steady state error due to a unit step input and a unit step disturbance is

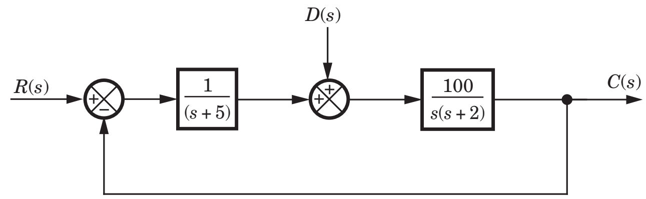

Fig. P6.3.22

(A) $-\frac{49}{11}$ (B) $\frac{49}{11}$

(C) $-\frac{63}{11}$ (D) $\frac{63}{11}$

23. The forward path transfer function of a *ufb* system is

$$G(s) = \frac{K}{s(s+4)(s+8)(s+10)}$$

If a unit ramp is applied, the minimum possible steady-state error is

(A) 0.16 (B) 6.25

(C) 0.14 (D) 7.25

24. The forward-path transfer function of a *ufb* system is

$$G(s) = \frac{1000(s^2 + 4s + 20)(s^2 + 20s + 15)}{s^3(s+2)(s+10)}$$

The system has $r(t) = t^3$ applied to its input. The steady state error is

(A) 4×10^{-4} (B) 0

(C) ∞ (D) 2×10^{-5}

25. The transfer function of a *ufb* system is

$$G(s) = \frac{10^5(s+3)(s+10)(s+20)}{s(s+25)(s+a)(s+30)}$$

The value of a to yield velocity error constant $K_v = 10^4$ is

(A) 4 (B) 0

(C) 8 (D) 16

26. A system has position error constant $K_p = 3$. The steady state error for input of $8tu(t)$ is

(A) 2.67 (B) 2

(C) ∞ (D) 0

27. The forward path transfer function of a unity feedback system is

$$G(s)=\frac{1000}{(s+20)(s^2+4s+10)}$$

For input of $60u(t)$ steady state error is

(A) 0 (B) 300

(C) ∞ (D) 10

28. For *ufb* system shown in fig. P6.3.28 the transfer function is

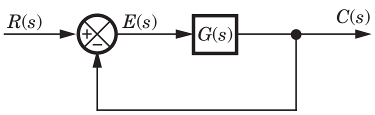

Fig. P6.3.28

$$G(s)=\frac{20(s+3)(s+4)(s+8)}{s^2(s+2)(s+15)}$$

If input is $30t^2$, then steady state error is

(A) 0.9375 (B) 0

(C) ∞ (D) 64

29. The forward-path transfer function of a *ufb* control system is

$$G(s)=\frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

The steady state errors for the test input $37tu(t)$ is

(A) 0 (B) 0.061

(C) ∞ (D) 609

30. In the system shown in fig. P6.3.30, $r(t)=1+2t$, $t>0$. The steady state error $e(t)$ is equal to

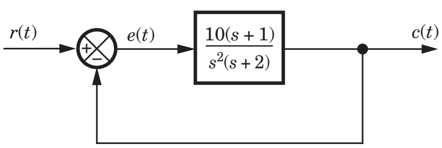

Fig. P6.3.30

(A) $\frac{1}{5}$ (B) 5

(C) 0 (D) ∞

31. A *ufb* control system has a forward path transfer function

$$G(s)=\frac{10(1+4s)}{s^2(1+s)}$$

If the system is subjected to an input $r(t)=1+t+\frac{1}{2}t^2$, $t>0$ the steady state error of the system will be

(A) 0 (B) 0.1

(C) 10 (D) ∞

32. The system shown in fig. P6.3.32 has steady-state error 0.1 to unit step input. The value of K is

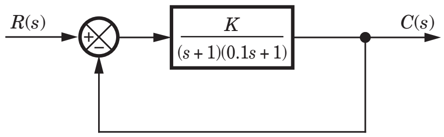

Fig. P6.3.32

(A) 0.1 (B) 0.9

(C) 1.0 (D) 9.0

Statement for Q.33–34:

Block diagram of a position control system is shown in fig.P6.3.33–34.

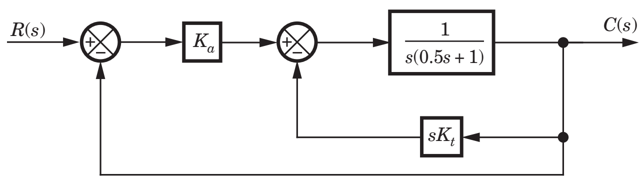

Fig. P6.3.33–34

33. If $K_t=0$ and $K_a=5$, then the steady state error to unit ramp input is

(A) 5 (B) 0.2

(C) ∞ (D) 0

34. If the damping ratio of the system is increased to 0.7 without affecting the steady state error, then the value of K_a and K_t are

(A) 86, 12.8 (B) 49, 9.3

(C) 24.5, 3.9 (D) 43, 6.4

35. A system has the following transfer function

$$G(s)=\frac{100(s+15)(s+50)}{s^4(s+12)(s^2+3s+10)}$$

The type and order of the system are respectively

(A) 7 and 5 (B) 4 and 5

(C) 4 and 7 (D) 7 and 4

SOLUTIONS

1. (D) Characteristic equation is $s^2+9s+18$.

$\omega_n^2=18,\ 2\xi\omega_n=9$

Therefore $\xi=1.06,\quad \omega_n=4.24$ rad/s

2. (A) $T(s)=\frac{1}{6}\frac{0.6}{(s+0.8)^2+(0.6)^2}$

$\omega_n\sqrt{1-\xi^2}=0.6\ ,\quad \xi\omega_n=0.8$

Hence $\omega_n=1,\quad \xi=0.8$

3. (A) Characteristic equation is

$\Delta s=\{s-(-3+j4)\}\{s-(-3-j4)\}=(s+3)^2+4^2.$

$=s^2+6s+25,\ \omega_n^2=25\ \Rightarrow\ \omega_n=5$ rad/s

$2\xi\omega_n=6\ ,\quad \xi=\frac{6}{2\times 5}=0.6$

4. (A) $T(s)=\frac{16}{(4s^2+8s+16)}=\frac{4}{(s^2+2s+4)}$

$\omega_n^2=4\ \Rightarrow\ \omega_n=2\ ,\quad 2\xi\omega_n=2,\quad \xi=0.5$

5. (D) $M_p=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}=\frac{5}{100}=0.05,$

$\frac{\xi\pi}{\sqrt{1-\xi^2}}=3\ \Rightarrow\ \xi=0.69,$

$T(s)=\frac{1}{1+G(s)}=\frac{K}{s^2+2s+K}$

$2\xi\omega_n=2,\quad \omega_n=\frac{1}{0.69}=1.45$

Peak time,

$T_p=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{-\xi^2}}=\frac{\pi}{1.45\sqrt{(1-0.69^2)}}=3$ sec

But the peak time T_p given is 1 sec. Hence these two specification cannot be met.

6. (C) $T(s)=\frac{K_1}{s^2+(K_2+s)+K_1}$,

$\omega_n^2=K_1\ ,\quad 2\xi\omega_n=1+K_2$

$\omega_d=0.10,\quad \xi=0.6,\quad \omega_d=\omega_n\sqrt{1-0.6^2}=10$

$\omega_n=12.5\ \Rightarrow\ K_1=156.25,$

$2\omega_n 3=K_2+1$

$2\times 12.5\times 0.6=K_2+1\ \Rightarrow\ K_2=14$

7. (A) $M_p=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$, At $\xi=0,\quad M_p=1=100\%$

8. (A) $C(s)=\frac{10}{s(s+10)}=\frac{1}{s}-\frac{1}{s+10}$

$\Rightarrow\ c(t)=1-e^{-10t}$

$a=10,\qquad$ Rise time $T_r=\frac{2.2}{a}=\frac{2.2}{10}=0.22$s

Settling time $T_s=\frac{4}{a}=0.4$s

9. (D) $\xi\omega_n=\frac{4}{T_s}=0.571,\ \omega_n\sqrt{1-\xi^2}=\frac{\pi}{T_p}=1.047$

Poles $=-0.571\pm j1.047$

10. (A) $0.1=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\ \Rightarrow\ \xi=0.59$

$\omega_n=\frac{\pi}{T_p}\sqrt{1-\xi^2}=0.779,$

Poles $=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2}=-0.46\ \pm\ j0.63$

11. (B) $0.12=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\ \Rightarrow\ \xi=0.56,\ \omega_n=\frac{4}{\xi T_s}=11.92$

Therefore Poles $=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2}=-6.67\pm j9.88$

Note :

$T_s=\frac{3.2}{\xi\omega_n}$, For $0<\xi<0.69$

$T_s=\frac{4.5}{\xi\omega_n}$, For $\xi>0.69$

12. (B) $T(s)=\frac{\omega_n^2}{s+2\xi\omega_n s+\omega_n^2}=\frac{40000}{s^2+500s+40000}$

$=\frac{40000}{(s+100)(s+400)}$

$R(s)=\frac{40000}{s(s+100)(s+400)}=\frac{1}{s}-\frac{4}{3(s+100)}+\frac{1}{3(s+400)}$

$r(t)=1-\frac{4}{3}e^{-100t}+\frac{1}{3}e^{-400t}$

13. (A) System has two different poles on negative real axis. So response is over damped.

14. (A) 1. Overdamped response (a, b)

Poles : Two real and different on negative real axis.

2. Underdamped response (c)

Poles : Two complex in left half plane

3.Undamped response (d)

Poles : Two imaginary.

4.Critically damped (e)

Poles : Two real and same on negative real axis.

15. (B) $K_p = \lim_{s\to 0} G(s) = 1000$

$K_v = \lim_{s\to 0} sG(s) = 0$

$K_a = \lim_{s\to 0} s^2 G(s) = 0$

16. (D) $H(s) = 1, \quad K_p = \lim_{s\to 0} G(s)\cdot H(s) = \infty$

$K_v = \lim_{s\to} sG(s)\cdot H(s) = \infty$

$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = \dfrac{K}{8}$

17. (B) $H(s) = 1, \quad K_p = \lim_{s\to 0} G(s)\cdot H(s) = 50$

$K_v = \lim_{s\to 0} sG(s)\cdot H(s) = 0$

$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = 0.$

18. (C) System type $= 1$, so $n = 1$

$K_v = \lim_{s\to 0} sG(s) = \dfrac{K}{a} = 100$

For 10% overshoot,

$0.1 = = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \quad \Rightarrow \quad \xi = 0.6$

$T(s) = \dfrac{G(s)}{1+G(s)} = \dfrac{K}{s^2 + as + K}$

$2\xi\omega_n = a, \quad \omega_n^2 = K \quad \Rightarrow \quad 2\times 0.6\sqrt{K} = a$

$\dfrac{K}{2}\times 0.6\sqrt{K} = 100 \quad \Rightarrow \quad K = 14400$

19. (D) $\dfrac{K}{a} = 100,\ K = 14400,$

$\dfrac{14400}{a} = 100 \quad \Rightarrow \quad a = 144$

20. (C) If $R(s) = 0$

$$T_D(s) = \frac{\dfrac{K_2}{s(s+4)}}{1 + \dfrac{K_1K_2(s+2)}{s(s+4)(s+3)}}$$

$$= \frac{K_2(s+3)}{s(s+3)(s+4) + K_1K_2(s+2)}$$

Error in output due to disturbance

$E(s) = T_D(s)D(s),$

If $D(s) = \dfrac{1}{s}$,

$$e_{ssD} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot T_D(s) = \lim_{s\to 0} T_D(s) = \frac{3}{2K_1}$$

$\dfrac{3}{2K_1} = 0.000012 \quad \Rightarrow \quad K_1 = 125\times 10^3$

Error due to ramp input

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)},$$

$R(s) = \dfrac{1}{s^2},\ G(s) = \dfrac{K_1K_2(s+2)}{s(s+3)(s+4)}$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s + \dfrac{K_1K_2(s+2)}{(s+3)(s+4)}} = \frac{6}{K_1K_2}$$

$\dfrac{6}{125\times 10^3 K_2} = 0.003 \quad \Rightarrow \quad K_2 = 0.016$

21. (C) $E(s) = R(s) - C(s)H(s)$

$$= R(s) - \frac{R(s)G(s)H(s)}{1+G(s)H(s)} = \frac{R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{\dfrac{s}{s}}{1 + \dfrac{1}{(s^2+s+2)}\dfrac{1}{(s+1)}} = \frac{2}{3}$$

22. (A) $e_{ss} = \lim_{s\to 0} \dfrac{sR(s) - sD(s)G_2(s)}{1+G_1(s)G_2(s)}$

where $G_1(s) = \dfrac{1}{s+5}$ and $G_2(s) = \dfrac{100}{s+2}$

$R(s) = D(s) = \dfrac{1}{s}$

$$e_{ss} = \frac{1 - \dfrac{100}{2}}{1 + \dfrac{1}{5}\times\dfrac{100}{2}} = \frac{-49}{11}$$

23. (A) Using Routh-Hurwitz Criterion, system is stable for $0 < K < 2000$

maximum $K_v = \lim_{s\to 0} sG(s) = \dfrac{2000}{4\times 8\times 10} = 6.25$

minimum possible error $\dfrac{1}{K_v} = \dfrac{1}{6.25} = 0.16$

24. (A) $R(s) = \dfrac{6}{s^4}, \quad E(s) = \dfrac{R(s)}{1+G(s)}$

$e_{ss} = \lim_{s\to 0} sE(s)$

$$= \lim_{s\to 0} \frac{\dfrac{6s}{s^4}}{1 + \dfrac{1000(s^2+4s+20)(s^2+20s+15)}{s^3(s+2)(s+10)}}$$

$$= \lim_{s\to 0} \frac{6}{s^3 + \dfrac{1000(s^2+4s+20)(s^2+20s+15)}{(s+2)(s+10)}}$$

$$= \frac{6}{0 + \frac{1000\times 20\times 15}{2\times 10}} = 4\times 10^{-4}$$

25. (A) $K_v = \lim_{s\to 0} sG(s)$

$$10^4 = \frac{10^4 \times 3 \times 10 \times 20}{25 \times a \times 30} \Rightarrow a = 4$$

26. (C) System is zero type $K_v = 0$, $e_{ss} = \frac{1}{K_v} = \infty$

27. (D) $K_p = \lim_{s\to 0} G(s) = 5$

For input $60u(t)$, $e_{ss} = \frac{60}{1+K_p} = 10$

28. (A) $K_a = \lim_{s\to 0} s^2 G(s) = 64$

$$e_{ss} = \frac{30 \times 2}{64} = 0.9375$$

29. (B) $K_v = \lim_{s\to 0} sG(s) = 609.02$

$$e_{ss} = \frac{37}{K_v} = 0.0607$$

30. (C) The system is type 2. Thus to step and ramp input error will be zero.

$$E(s) = R(s) - C(s) = R(s) - \frac{G(s)R(s)}{1+G(s)} = \frac{R(s)}{1+G(s)}$$

$$R(s) = \frac{1}{s} + \frac{2}{s^2} = \frac{s+2}{s^2}$$

$$E(s) = \frac{s+2}{s^2 + \frac{10(s+1)}{(s+2)}}$$

$$e_{ss}(t) = \lim_{s\to 0} sE(s) = 0$$

31. (C) System is type 2. Therefore error due to $1+t$ would be zero and due to $\frac{t^2}{2}$ would be $\frac{1}{K_a}$.

$K_a = \lim_{s\to 0} s^2 G(s) = 10$, $e_{ss}(t) = \frac{1}{10} = 0.1$

Note that you may calculate error from the formula

$$e_{ss}(t) = \lim_{s\to 0} sE(s) = \frac{sR(s)}{1+G(s)}$$

32. (D) $K_p = \lim_{s\to 0} G(s) = K$

$$e_{ss}(t) = \frac{1}{1+K_p} = \frac{1}{1+K} = 0.1 \Rightarrow K = 9.$$

33. (B) $e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$

When $K_t = 0$ and $K_a = 5$

$$G(s) = \frac{5}{s(0.5\,s+1)}, \; H(s) = 1, \; R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{5}{s(0.5s+1)}} = \frac{1}{5} = 0.2$$

34. (C) The equivalent open-loop transfer function

$$G_e = \frac{\frac{K_a}{s(0.5\,s+1)}}{1 + \frac{sK_t}{s(0.5\,s+1)}} = \frac{K_a}{s(0.5\,s+1+K_t)}$$

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K_a}{0.5s^2 + s(1+K_t) + K_a}$$

$$= \frac{2K_a}{s^2 + 2s(1+K_t) + 2K_a}$$

$$\omega_n^2 = 2\,K_a \Rightarrow \omega_n = \sqrt{2K_a}$$

$$2\xi\,\omega_n = 2(1+K_t)$$

$$\xi = 1 + \frac{K_t}{\sqrt{2K_a}} = 0.7 \qquad \text{... (i)}$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G_e(s)}, \; R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{1}{s\left(1 + \frac{K_a}{s(0.5\,s+1+K_t)}\right)} = \frac{1+K_t}{K_a}$$

$$e_{ss} = \frac{1+K_t}{K_a} = 0.2 \qquad \text{...(ii)}$$

Solving (i) and (ii)

$K_a = 24.5$, $K_t = 3.9$

35. (C) The s has power of 4 and denominator has order of 7. So Type 4 and Order 7.

36. (D) For $8u(t)$, $e_{ss} = \frac{8}{1+K_p} = 2.$

For $8tu(t)$, $e_{ss} = \infty$, since the system is type 0.

37. (A) For equivalent unit feedback system the forward transfer function is

$$G_e = \frac{G(s)}{1 + G(s)H(s) - G(s)} = \frac{\frac{10(s+10)}{s(s+2)}}{1 + \frac{10(s+10)(s+3)}{s(s+2)}}$$

$$= \frac{10(s+10)}{11s^2 + 132\,s + 300}$$

The system is of Type 0. Hence step input will produce a constant error constant.