Chapter 16
Mensuration
Exercise 16.1

1 . find the area of a triangle whose base is 6 cm and corresponding Height is 4 cm .

Solution :

It is given that

Base of triangle $=$ 6cm

Height of triangle $=$ 4 cm

We know that

Area of triangle $= \frac{1}{2} \times base \times height$

Substituting the values

$= \frac{1}{2} \times 6 \times 4$

By further calculation

$= 6 \times 2$

$= 12\text{cm}^2$

2. find the area of a triangle whose sides are

(i) 3 cm , 4 cm and 5 cm

(ii) 29 cm , 20 cm and 21 cm

(iii) 12 cm , 9.6 cm and 7.2 cm

Solution

(i) consider a = 3 cm , b = 4 cm and c = 5 cm

We know that

S = semi perimeter $= \frac{a+b+c}{2}$

Substituting the values

$= \frac{3+4+5}{2}$

$= \frac{12}{2}$

= 6 cm

Here

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{6(6-3)(6-4)(6-5)}$

By further calculation

$= \sqrt{6 \times 3 \times 2 \times 1}$

$= \sqrt{6 \times 6}$

$= 6 \text{ cm}^2$

(ii) consider a = 29 cm , b = 20 cm and c = 21cm

We know that

S = semi perimeter = $\dfrac{a + b + c}{2}$

Substituting the values

$= \dfrac{29+20+21}{2}$

$= \dfrac{70}{2}$

$= 35 \text{ cm}$

Here

Area of triangle $= \sqrt{s(s - a)(s - b)(s - c)}$

Substituting the values

$= \sqrt{35(35 - 29)(35 - 20)(35 - 21)}$

By further calculation

$= \sqrt{35 \times 6 \times 15 \times 14}$

$= \sqrt{7 \times 5 \times 3 \times 2 \times 5 \times 3 \times 7 \times 2}$

We can write it as

$= \sqrt{7 \times 7 \times 5 \times 5 \times 3 \times 3 \times 2 \times 2}$

So we get

$= 7 \times 5 \times 3 \times 2$

$= 210 \text{ cm}^2$

(iii) consider a = 12 cm , b = 9.6 cm and c = 7.2 cm

We know that

S = semi perimeter $= \dfrac{a+b+c}{2}$

Substituting the values

$= \dfrac{12+9.6+7.2}{2}$

$= \dfrac{28.8}{2}$

$= 14.4$ cm

Here

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{14.4(14.4-12)(14.4-9.6)(14.4-7.2)}$

By further calculation

$= \sqrt{14.4 \times 2.4 \ \times 4.8 \times 7.2}$

$= \sqrt{6 \times 2.4 \times 2.4 \times 2 \times 2.4 \times 3 \times 2.4}$

We can write it as

$= 2.4 \times 2.4 \times \sqrt{6 \ \times 6}$

So we get

$2.4 \times 2.4 \times 6$

$= 34.56 \text{ cm}^2$

3. find the area of a triangle whose sides are 34 cm , 20 cm and 42 cm. Hence ,find the length of the altitude corresponding to the shortest side.

Solution

Consider 34 cm , 20 cm and 42 cm as the sides of triangle

A = 34 cm , b = 20 cm and c = 42 cm

We know that

$$S = \text{semi perimeter} = \frac{a+b+c}{2}$$

Substituting the values

$$= \frac{34+20+42}{2}$$

$$= \frac{96}{2}$$

$$= 48 \text{ cm}$$

Here

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$$= \sqrt{48(48-34)(48-20)(48-42)}$$

By further calculation

$$= \sqrt{48 \times 14 \times 28 \times 6}$$

$$= \sqrt{8 \times 6 \times 14 \times 14 \times 2 \times 6}$$

We can write it as

$= 14 \times 6 \times \sqrt{8 \times 2}$

$= 14 \times 6 \times \sqrt{16}$

So we get

$= 14 \times 6 \times 4$

$= 336 \text{ cm}^2$

Here the shortest side of the triangle is 20 cm

Consider h cm as the corresponding altitude

Area of triangle $= \frac{1}{2} \times base \times Height$

Substituting the values

$336 = \frac{1}{2} \times 20 \times h$

By further calculation

$h = \frac{336 \times 2}{20}$

so we get

$h = \frac{336}{10}$

$h = 33.6 \text{ cm}$

hence the required altitude of the triangle is 33.6 cm .

4 . the sides of a triangle field are 975 m , 1050 m and 1125 m . if this field is sold at the rate of Rs 1000 Per hectare, find its selling price .(1 hectare = 10000 m²)

Solution

It is given that

A = 975 m , b = 1050 m and c = 1125 m

We know that

S = semi perimeter = $\frac{a+b+c}{2}$

Substituting the values

$= \frac{975+1050+1125}{2}$

$= \frac{3150}{2}$

= 1575 cm

Here

Area of triangle field = $\sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{1575(1575-975)(1575-1050)(1575-1125)}$

By further calculation

$= \sqrt{1575 \times 600 \times 525 \times 450}$

$= \sqrt{525 \times 3 \times 150 \times 4 \times 525 \times 150 \times 3}$

We can write it as

$= 525 \times 3 \times 150 \times \sqrt{4}$

So we get

$= 525 \times 450 \times 2$

It is given that

1 hectare $= 10000$ m^2

$= \dfrac{525 \times 900}{10000}$

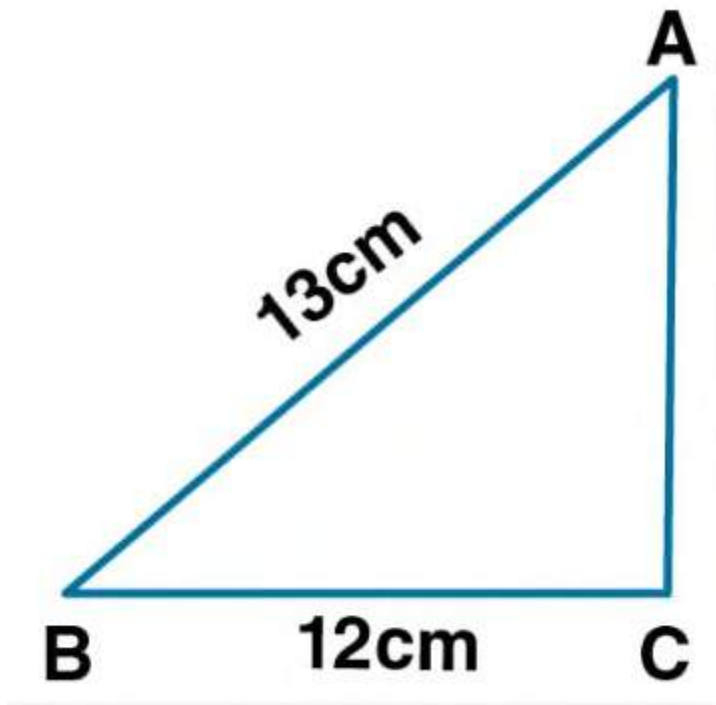By further calculation

$= \dfrac{525 \times 9}{100}$

$= \dfrac{4725}{100}$

$= 47.25$ hectares

We know that

Selling price of 1 hectare field $=$ Rs 1000

Selling price of 47.25 hectare field $= 1000 \times 47.25 =$ Rs 47250

5. the base of a right angled triangle is 12 cm and its hypotenuse is 13cm long . find its area and the perimeter .

Solution

It is given that

ABC is a right angled triangle

BC =12cm and AB = 13cm

Using the Pythagoras theorem

$AB^2 = AC^2 + BC^2$

Substituting the values

$13^2 = AC^2 + 12^2$

By further calculation

$AC^2 = 13^2 - 12^2$

So we get

$AC^2 = 169 - 144 = 25$ '

$AC = \sqrt{25} = 5$ cm

We know that

Area of triangle ABC $= \frac{1}{2} \times base \times height$

Substituting the values

$= \frac{1}{2} \times 12 \times 5$

$= 30 \text{ cm}^2$

Similarly

Perimeter of triangle ABC $= AB + BC + CA$

Substituting the values

$= 13 + 12 + 5$

$= 30$ cm

6 . find the area of an equilateral triangle whose side is 8 m . give your answer correct to two decimal places .

Solution :

It is given that

Side of equilateral triangle = 8 m

We know that

Area of equilateral triangle $= \dfrac{\sqrt{3}}{4}$ (side)^2

Substituting the values

$= \dfrac{\sqrt{3}}{4} \times 8 \times 8$

By further calculation

$= \sqrt{3} \times 2 \times 8$

$= 1.73 \times 16$

$= 27.71 \text{ m}^2$

7. if the area of an equilateral triangle is $81\sqrt{3}\text{cm}^2$ find its perimeter.

Solution

We know that

Area of equilateral triangle $= \dfrac{\sqrt{3}}{4}$ (side)^2

Substituting the values

$81\sqrt{3} = \frac{\sqrt{3}}{4}$ (side)²

By further calculation

Side² $= \frac{81\sqrt{3} \times 4}{\sqrt{3}}$

So we get

(side)² = 81× 4

Side = 9 × 2 = 18 cm

So the perimeter of equilateral triangle = 3× side

= 3 × 18

= 54 cm

8 . if the perimeter of an equilateral triangle is 36 cm ,

calculate its area and height .

Solution
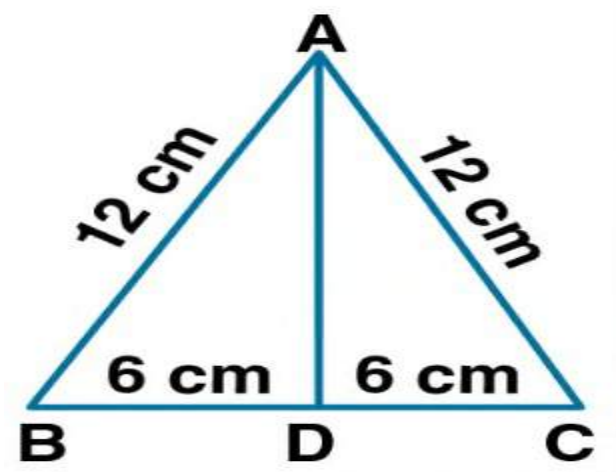
We know that

Perimeter of an equilateral triangle $= 3 \times$ side

Substituting the values

$36 = 3 \times$ side

By further calculation

Side $= \dfrac{36}{3} = 12$cm

So AB $=$ BC $=$ CA $= 12$cm

Here

Area of equilateral triangle $= \dfrac{\sqrt{3}}{4}(\text{side})^2$

Substituting the values

$= \dfrac{\sqrt{3}}{4}(12)^2$

By further calculation

$= \dfrac{\sqrt{3}}{4} \times 12 \times 12$

So we get

$= \sqrt{3} \times 3 \times 12$

$= 1.73 \times 36$

$= 62.4$ cm^2

In triangle ABD

Using Pythagoras theorem

$AB^2 = AD^2 + BD^2$

Here BD $= \dfrac{12}{2} = 6$cm

Substituting he values

$12^2 = AD^2 + 6^2$

By further calculation
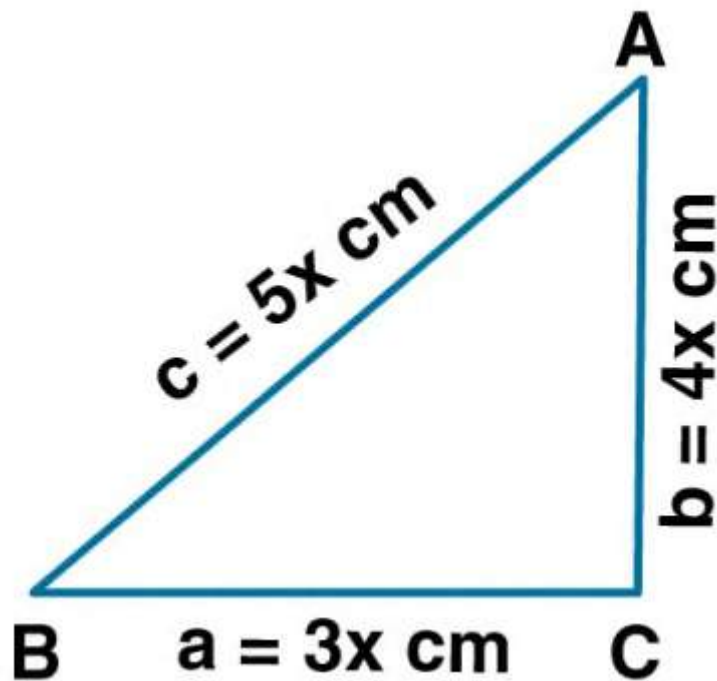
$144 = AD^2 + 36$

$AD^2 = 144 - 36 = 108$

So we get

$AD = \sqrt{108} = 10.4$ cm

Therefore , the required height is 10.4 cm .

9 (i) if the length of the sides of a triangle are in the ratio 3:4:5   and its perimeter is 48 cm , find its area .

(ii) the sides of a triangle plot are in the ratio 3 :5:7 and its perimeter is 300 m . find its area

Solution :

(i) consider ABC as the triangle

Ratio of the sides are 3x, 4x and 5x

Take a = 3x cm , b = 4x cm and c = 5x cm

We know that

a + b + c = 48

substituting the values

3x + 4x + 5x = 48

12x = 48

So we get

$X = \dfrac{48}{12} = 4$

Here

a = 3x = 3 × 4 = 12cm

b = 4x = 4 × 4 = 16 cm

c = 5x = 5 × 4 = 20 cm

we know that

s = semi perimeter = $\frac{a+b+c}{2}$

substituting the values

$= \frac{12+16+20}{2}$

$= \frac{48}{2}$

= 24 cm

Here

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{24(24-12)(24-16)(24-20)}$

By further calculation

$= \sqrt{24 \times 12 \times 8 \times 4}$

$= \sqrt{12 \times 2 \times 12 \times 4 \times 2 \times 4}$

So we get

$= 12 \times 4 \times 2$

= 96 cm²

(ii) it is given that

Sides of a triangle are in the ratio = 3 :5 :7

Perimeter = 300 m

We know that

First side $= \dfrac{300×3}{sum\,of\,ration}$

Substituting the values

$= \dfrac{300×3}{3+5+7}$

By further calculation

$= \dfrac{300×3}{15}$

= 60 m

Second side $= \dfrac{300 ×15}{15} = 100$ m

Third side $= \dfrac{300 ×7}{15} = 140$ m

Here

$S = \dfrac{perimeter}{2} = \dfrac{300}{2} = 150$m

So we get

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$$= \sqrt{150(150-60)(150-100)(150-140)}$$

By further calculation

$$= \sqrt{150 \times 90 \times 50 \times 10}$$

$$= \sqrt{6750000}$$

We can write it as

$$= \sqrt{225 \times 10000 \times 3}$$

$$= 15 \times 100 \times \sqrt{3}$$

$$= 1500 \times \sqrt{3}$$

We get

$$= 1500 \times 1.732$$

$$= 2598 \text{ m}^2$$

10. ABC is a triangle in which AB = AC = 4cm and ∠A = 90° . calculate the area of Δ ABC . also find the length of perpendicular from A to BC.

Solution

It is given that

AB = AC = 4cm
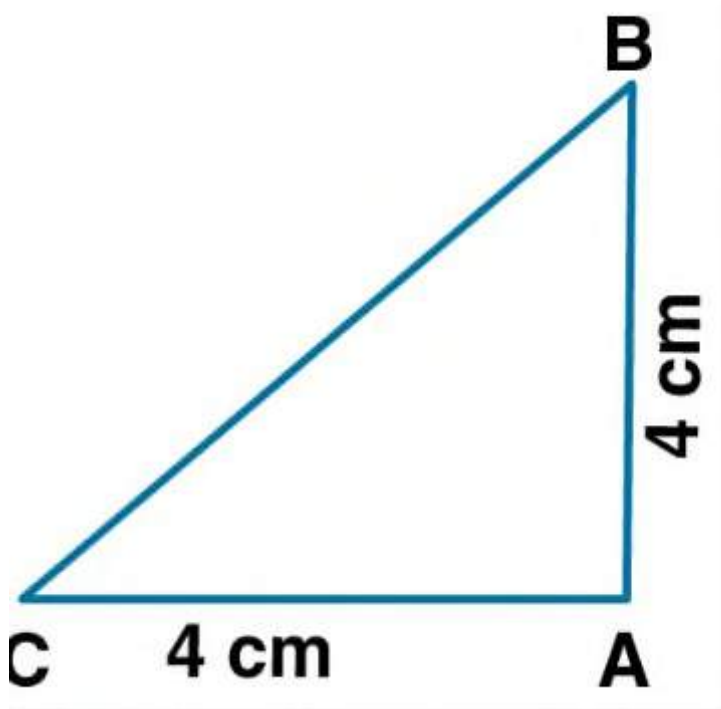
Using the Pythagoras theorem

$BC^2 = AB^2 + AC^2$

Substituting the values

$BC^2 = 4^2 + 4^2$

By further calculation

$BC^2 = 16 + 16 = 32$

$BC = \sqrt{32} = 4\sqrt{2}$ cm

We know that

Area of Δ ABC $= \dfrac{1}{2} \times BC \times H$

Substituting the values

$8 = \dfrac{1}{2} \times 4\sqrt{2} \times h$

By further calculation

$$h = \frac{8 \times 2}{4\sqrt{2}}$$

we can write it as

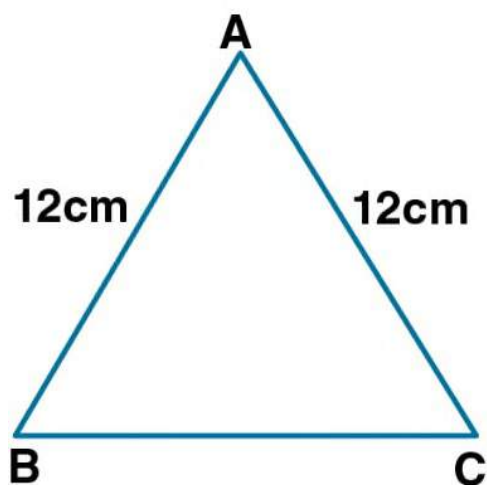$$h = \frac{2 \times 2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

so we get

$$h = \frac{4\sqrt{2}}{2} = 2 \times \sqrt{2}$$

$$h = 2 \times 1.41 = 2.82 \text{ cm}$$

11. find the area of an isosceles triangle whose equal sides are 12 cm each and the perimeter is 30 cm .

Solution :

Consider ABC as the isosceles triangle

Here AB =AC = 12cm

Perimeter = 30 cm

So BC = 30 –(12 +12) = 30 -24 = 6 cm

We know that

S = semi perimeter = $\dfrac{a+b+c}{2}$

Substituting the values

$= \dfrac{30}{2}$

= 15 cm

Here

Area of triangle ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{15(15-12)(15-12)(15-6)}$

By further calculation

$= \sqrt{15 \times 3 \times 3 \times 9}$

$= \sqrt{81 \times 15}$

$= 9\sqrt{15}$

We can write it as

= 9 × 3.873

= 34.857

= 34.86 cm²

12. find the area of an isosceles triangle whose base is 6 cm and perimeter is 16 cm .
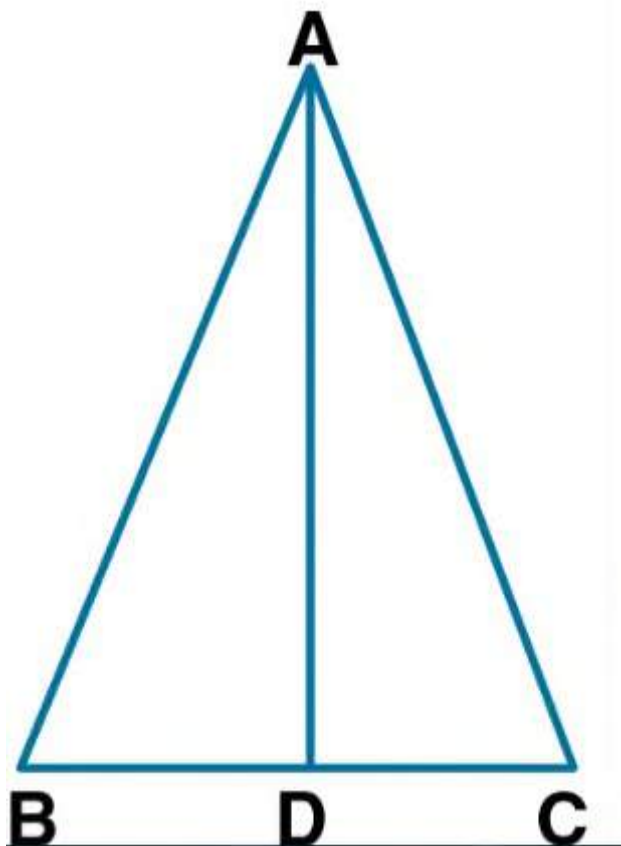
Solution :

It is given that

Base = 6cm

Perimeter = 16 cm

Consider ABC as an isosceles triangle in which

AB = AC = x

So BC = 6cm

We know that

Perimeter of Δ ABC $=$ AB + BC + AC

Substituting the values

16 = x + 6 + x

By further calculation

16 = 2x + 6

16 − 6 = 2x

10 = 2x

So we get

$X = \frac{10}{2} = 5$

Here AB $=$ AC = 5 cm

$BC = \frac{1}{2} \times 6 = 3cm$

In Δ ABD

$AB^2 = AD^2 + BD^2$

Substituting the values

$5^2 = AD^2 + 3^2$

$25 = AD^2 + 9$

By further calculation

$AD^2 = 25 − 9 = 16$

So we get

AD = 4cm

Here

Area of Δ ABC = $\frac{1}{2}$ × $base$ × $height$

Substituting the values

= $\frac{1}{2}$ × 6 × 4

= 3 × 4
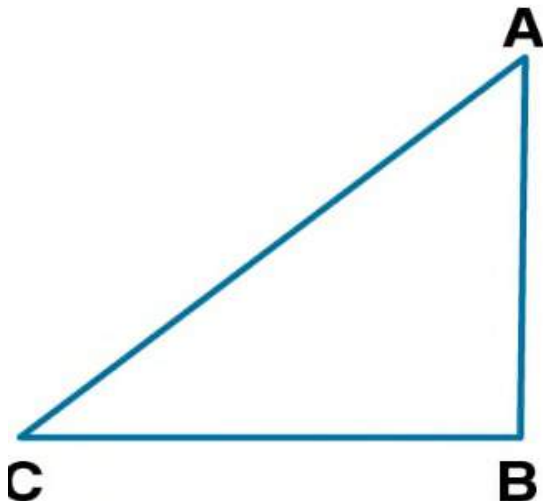
= 12 cm²

13 . the sides of a right angled triangle containing the right angle are 5x cm and (3x -1) cm . calculate the length of the hypotenuse of the triangle if its area is 60 cm².

Solution :

Consider ABC as a right angled triangle

AB = 5x cm and BC = (3x -1) cm

We know that

Area of $\triangle$ ABC $= \frac{1}{2} \times AB \times BC$

Substituting the values

$60 = \frac{1}{2} \times 5x(3x - 1)$

By further calculation

$120 = 5x (3x - 1)$

$120 = 15x^2 - 5x$

It can be written as

$15x^2 - 5x - 120 = 0$

Taking out the common terms

$5(3x^2 - x - 24) = 0$

$3x^2 - x - 24 = 0$

$3x^2 - 9x + 8x - 24 = 0$

Taking out the common terms

$3x(x - 3) + 8(x - 3) = 0$

$(3x + 8)(x - 3) = 0$

Here

$3x + 8 = 0$ or $x - 3 = 0$

We can write it as

$3x = -8$ or $x = 3$

$X = -\dfrac{8}{3}$ or $x = 3$

$X = -\dfrac{8}{3}$ is not possible

So $x = 3$

AB $= 5 \times 3 = 15$cm

BC $= (3 \times 3 - 1) = 9 - 1 = 8$ cm

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 15^2 + 8^2$

By further calculation

$AC^2 = 225 + 64 = 289$

$AC^2 = 17^2$

So AC $= 17$ cm

Therefore, the hypotenuse of the right angled triangle is 17 cm .

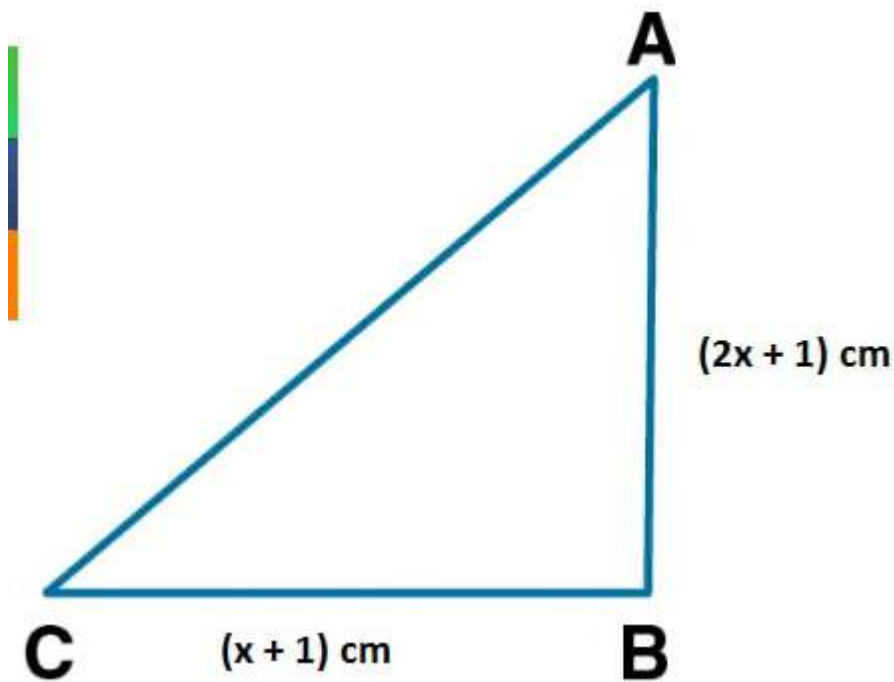**14.** in Δ ABC , ∠B = 90⁰ , AB = (2A +1) cm and BC = (A +1) cm . if the area of the Δ ABC is 60 cm² , find its perimeter .

Solution

It is given that

AB = (2x +1) cm

BC = (x +1) cm

A

(2x + 1) cm

C   (x + 1) cm   B

We know that

Area of Δ ABC $= \frac{1}{2} \times AB \times BC$

Substituting the values

$$60 = \frac{1}{2} \times (2x + 1)(x + 1)$$

By cross multiplication

$60 \times 2 = (2x + 1)(x + 1)$

By further calculation

$120 = 2x^2 + 3x + 1$

We can write it as

$0 = 2x^2 + 3x + 1 - 120$

$0 = 2x^2 + 3x - 119$

So we get

$2x^2 + 3x - 119 = 0$

$2x^2 + 17x - 14x - 119 = 0$

Taking out the common terms

$X(2x + 17) - 7(2x + 17) = 0$

$(x - 7)(2x + 17) = 0$

Here

$X - 7 = 0$ or $2x + 17 = 0$

$X = 7$ or $2x = -17$

$X = 7$ or $x = -\frac{17}{2}$

AB $= (2x + 1) = 2 \times 7 + 1$

AB $= 14 + 1 = 15$ cm

BC $= (x + 1) = 7 + 1 = 8$ cm

In right angled Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

So we get

AC $= 17$ cm

So the perimeter $=$ AB $+$ BC $+$ AC

Substituting the values

$= 15 + 8 + 17$

$= 40$ cm

15. if the perimeter of a right angled triangle is 60 cm and its hypotenuse is 25 cm , find its area .

Solution

We know that

Perimeter of a right angled triangle = 60 cm

Hypotenuse = 25 cm

Here the sum of two sides = 60 – 25 = 35 cm

Consider base = x cm

Altitude = (35 – x) cm

Using the Pythagoras theorem

$X^2 + (35 – x)^2 = 25^2$

By further calculation

$X^2 + 1225 + x^2 – 70 x = 625$

$2x^2 – 70x + 1225 – 625 = 0$

$2x^2 - 70x + 600 = 0$

Dividing by 2

$X^2 – 35x + 300 = 0$

$X^2 -15x -20x +300 = 0$

Taking out the common terms

$X(x -15) -20(x -15) = 0$

$(x – 15) (x – 20) = 0$

Here

X – 15 = 0

So we get

X = 15

Similarly

X – 20 = 0

So we get

X = 20 cm

So 15 cm and 20 cm are the sides of the triangle

Area $= \frac{1}{2} \times base \times altitude$

Substituting the values

$= \frac{1}{2} \times 15 \times 20$

$= 150 cm^2$

16. the perimeter of an isosceles triangle is 40 cm . the base is two third of the sum of equal sides . find the length of each side.

Solution :

It is given that

Perimeter of an isosceles triangle = 40 cm

Consider x cm as each equal sides

We know that

Base $= \frac{2}{3}(2x) = \frac{4}{3}x$

So according to the sun

$2x + \frac{4}{3}x = 40$

By further calculation

$6x + 4x = 120$

$10x = 120$

By division

$X = \frac{120}{10} = 12$

Therefore , the length of each sides is 12 cm .

17 . if the area of an isosceles triangle is 60 cm² and the length of each of its equal sides is 13 cm , find its base .
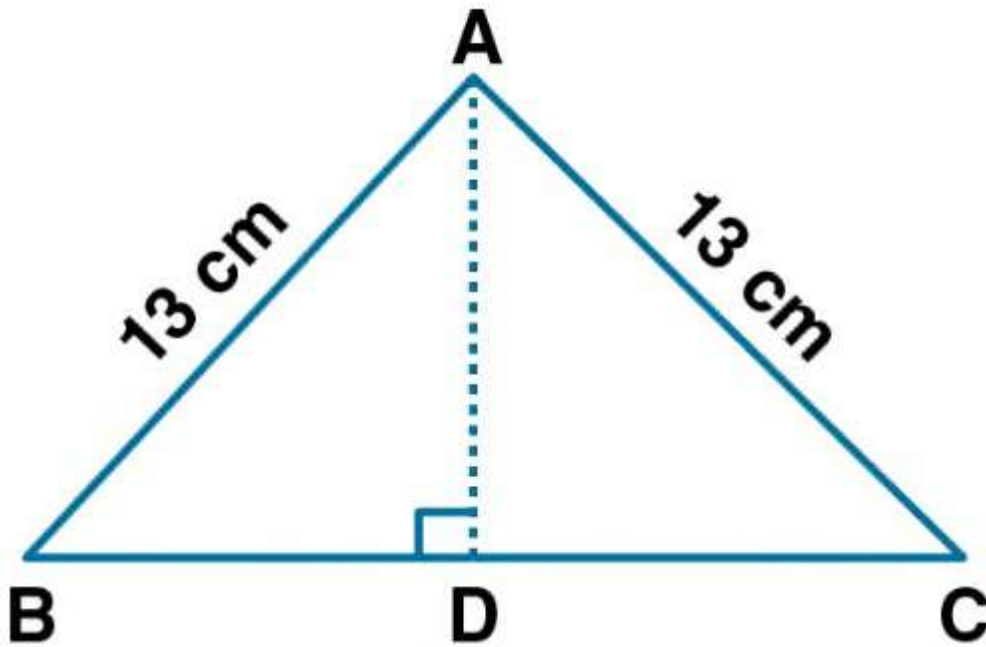

Solution :

It is given that

Area of isosceles triangle $= 60$ cm²

Length of each equal side $= 13$cm

Consider base BC $= x$ cm

Construct AD perpendicular to BC which bisects BC at D

So BD = DC = $\frac{x}{2}$ cm

In right Δ ABD

$AB^2 = BD^2 + AD^2$

Substituting the values

$13^2 = \left(\frac{x}{2}\right)^2 + AD^2$

By further calculation

$169 = \frac{x^2}{4} + AD^2$

$AD^2 = 169 - \frac{x^2}{4}$ ......(1)

We know that

Area $= 60$ cm^2

$AD = \frac{area \times 2}{base}$

Substituting the values

$AD = \frac{60 \times 2}{x} = \frac{120}{x}$ .....(2)

Using both the equations

$169 - \frac{x^2}{4} = \left(\frac{120}{x}\right)^2$

By further calculation

$\frac{676 - x^2}{4} = \frac{14400}{x^2}$

By cross multiplication

$676x^2 - x^4 = 57600$

We can write it as

$X^4 - 676x^2 + 57600 = 0$

$X^4 - 576x^2 - 100x^2 + 57600 = 0$

Taking out the common terms

$X^2 (x^2 - 576) - 100(x^2 - 576) = 0$

$(X^2 - 576) (x^2 - 100) = 0$

Here

$X^2 - 576 = 0$ where $x^2 = 576$

So x = 24

Similarly

$X^2 - 100 = 0$ where $x^2 = 100$

So x = 10

Hence , the base is 10 cm or 24 cm .

18. the base of a triangle field is 3 times its height if the cost of cultivating the field at the rate of Rs 25 per 100 m² is Rs 60000 ; find its base and height .

Solution :

It is given that

Cost of cultivating the field at the rate of Rs 25 per 100 m² = Rs 60000

Here the cost of cultivating the field of Rs 25 for 100m²

So the cost of cultivating the field of Rs 60000 $= \frac{100}{25} \times 60000$

$= 4 \times 60000$

$= 240000 \ m^2$

So the area of field $= 240000 m^2$

$\frac{1}{2} \times base \times height = 240000$ ......(1)

Consider

Height of triangular field $= h\ m^2$

Base of triangular field $= 3h\ m^2$

Substituting the values in equation (1)

$\frac{1}{2} \times 3h \times h = 240000$

By further calculation

$\frac{1}{2} \times 3h^2 = 240000$

$h^2 = \frac{240000 \times 2}{3}$

$h^2 = 80000 \times 2$

$h^2 = 160000$

so we get

$h = \sqrt{160000} = 400$

here the height of triangular field $= 400\ m$

base of triangular field $= 3 \times 400 = 1200\ m^2$

19. **A triangular park ABC has sides 120 m , 80 m and 50 m (as shown in the given figure). A garden dhania has to put a fence around it and also plant grass inside . how much area does she need to plant ? find the cost of fencing it with barbed wire at the rate of Rs 20 per meter leaving a space 3 m wide for a gate on one side .**
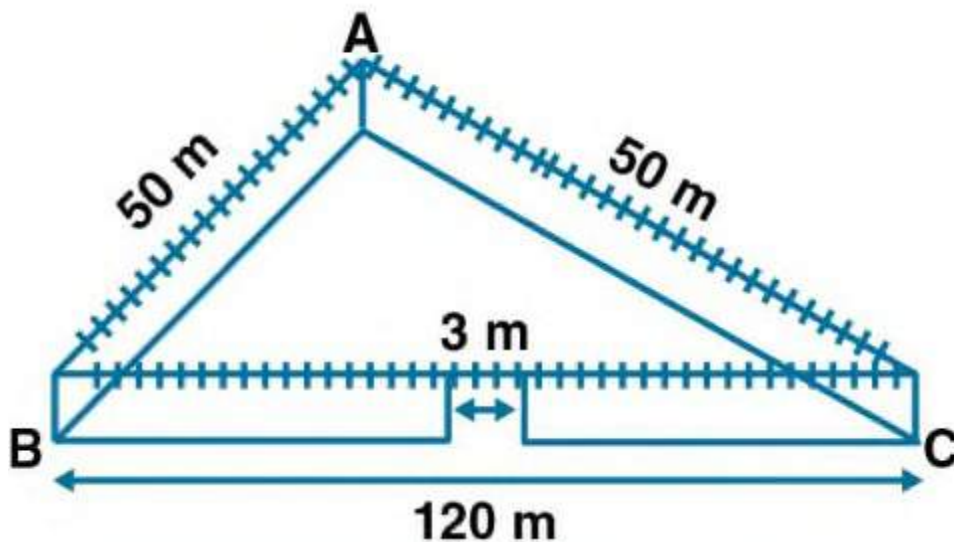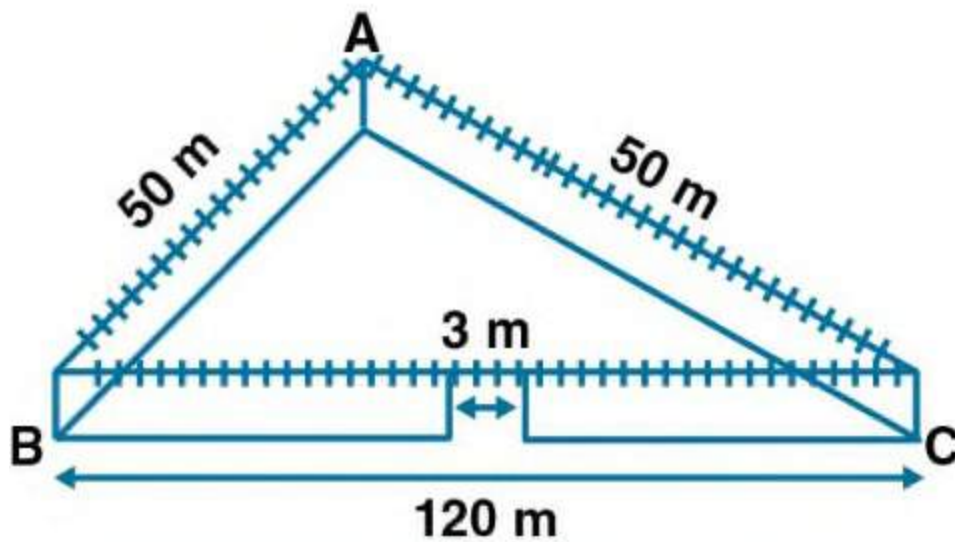

Solution

It is given that

ABC is a triangular park with sides 120 m , 80 m and 50 m

Here the perimeter of triangle ABC $= 120 + 80 + 50 = 250$ m

We know that

Portion at which a gate is build = 3m

Remaining perimeter = 250 – 3 = 247 m

So the length of fence around it = 247 m

Rate of fencing = Rs 20 per m

Total cost of fencing = 20 × 247 = Rs 4940

We know that

S = semi perimeter = $\frac{a+b+c}{2}$

Substituting the values

$= \frac{250}{2}$

= 125 cm

Here

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{125(125-50)(125-80)(125-120)}$

By further calculation

$= \sqrt{125 \times 75 \times 45 \times 5}$

$= \sqrt{5 \times 5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 5 \times 5}$

$= 5 \times 5 \times 3 \times 5\sqrt{15}$

So we get

$= 375\sqrt{15} \text{ m}^2$

20. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (show in the give figure) each piece measuring 20 cm , 50 cm and 50 cm . how much cloth of each colour is required for the umbrella ?

Solution :

It is given that

An umbrella is made by stitching 10 triangular piece of cloth of

two different colours

20 cm , 50 cm , 50 cm are the measurement of each triangle

So we get

$$\frac{s}{2} = \frac{20+50+50}{2}$$

$$\frac{s}{2} = \frac{120}{2} = 60$$

We know that

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$$= \sqrt{60(60-20)(60-50)(60-50)}$$

By further calculation

$$= \sqrt{60 \times 40 \times 10 \times 10}$$

So we get

$= \sqrt{240000}$

$= 100\sqrt{24}$

$= 100 \times 2\sqrt{6}$

$= 200\sqrt{6} \text{ cm}^2$

Here the area of 5 triangular piece of first colour $= 5 \times 200\sqrt{6}$

$= 1000\sqrt{6} \text{ cm}^2$

Area of triangular piece of second colour $= 1000\sqrt{6} \text{ cm}^2$

21. (a) in the figure (1) given below ABC is an equilateral triangle with each side of length 10 cm . in Δ BCD , $\angle$D = 90º and CD = 6 cm

Find the area of the shaded region . given your answer correct to one decimal place .
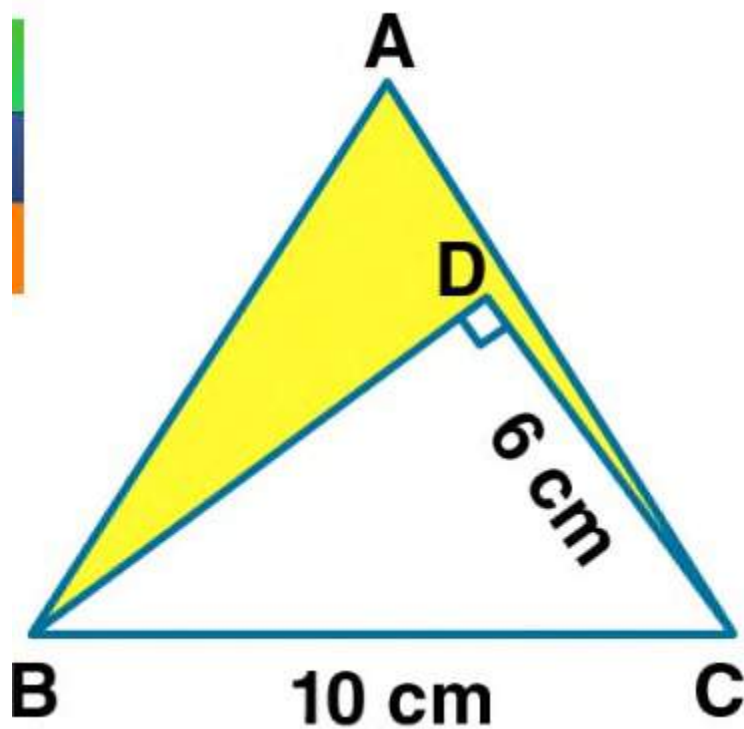
(b) in the figure (ii) given ABC is an isosceles right angled triangle and DEFG is a rectangle . if AD = AE = 3cm and DB = EC = 4 cm , find the area of the shaded region .
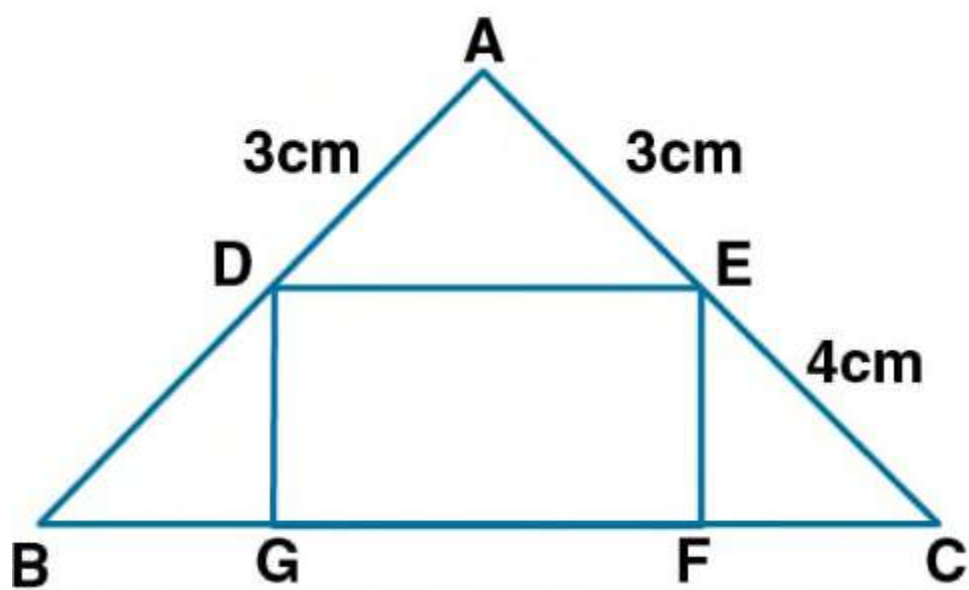
Solution

(a) it is given that

ABC is an equilateral triangle of side = 10 cm

We know that

Area of equilateral triangle ABC $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

Substituting the values

$= \frac{\sqrt{3}}{4} \times 10^2$

$= \frac{\sqrt{3}}{4} \times 100$

So we get

$= \sqrt{3} \times 25$

$= 1.73 \times 25$

$= 43.3 \text{ cm}^2$

In right angled triangle BDC

$\angle D = 90^\circ$

$BC = 10 \text{ cm}$

$CD = 6 \text{ cm}$

Using Pythagoras theorem

$BD^2 + DC^2 = BC^2$

Substituting the values

$BD^2 + 6^2 = 10^2$

$BD^2 + 36 = 100$

So we get

$BD^2 = 100 - 36 = 64$

$BD = \sqrt{64} = 8$ cm

We know that

Area of triangle BDC $= \frac{1}{2} \times base \times height$

So we get

$= \frac{1}{2} \times 8 \times 6$

$= 4 \times 6$

$= 24$ cm^2

Here the area of shaded portion = area of triangle ABC - area of triangle BDC

Substituting the values

$= 43.3 - 24$

$= 19.3$ cm^2

(b) it is given that

AD = AE = 3cm

DB = EC = 4 cm

By addition we get

AD + DB = AE + EC = (3 + 4) cm

AB = AC = 7 cm

$\angle$ A = 90°

We know that

Area of right triangle ADE $= \frac{1}{2} \times AD \times AE$

Substituting the values

$= \frac{1}{2} \times 3 \times 3$

$= \frac{9}{2}$ cm²

So triangle BDG is an isosceles right triangle

Similarly

DG² + BG² = BD²

DG² + DG² = 4²

By further calculation

2DG² = 16

DG² $= \frac{16}{2} = 8$

DG = √8 cm

Area of triangle BDG $= \frac{1}{2} \times BG \times DG$

We can write it as

$= \frac{1}{2} \times DG \times DG$

Substituting the values

$= \frac{1}{2} \left(\sqrt{8}\right)^2$

$= \dfrac{1}{2} \times 8$

$= 4 \text{ cm}^2$

Area of isosceles right triangle EFC = 4cm^2

So the area of shaded portion $= \dfrac{9}{2} + 4 + 4$

Taking LCM

$= \dfrac{9+8+8}{2}$

$= \dfrac{25}{2}$

$= 12.5 \text{ cm}^2$

Exercise 16.2

1 . (i) find the area of quadrilateral whose one diagonal is 20 cm long and the perpendicular to this diagonal from other vertices are of length 9 cm and 15 cm .
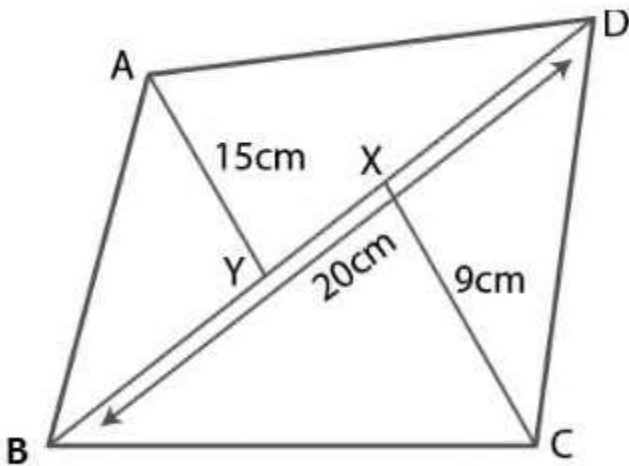
(ii) find the area of a quadrilateral whose diagonals are of length 18 cm and 12 cm and they intersect each other at right angles .

Solution :

(i) consider ABCD as a quadrilateral in which AC = 20 cm

We know that

BY = 9 cm and DY = 15 cm

Here

Area of quadrilateral ABCD = area of triangle ABC + area of triangle ACD

We can write it as

$$= \frac{1}{2} \times base \times height + \frac{1}{2} \times base \times height$$

So we get

$$= \frac{1}{2} \times AC \times BX + \frac{1}{2} \times AC \times DY$$

Substituting the values

$$= \left(\frac{1}{2} \times 20 \times 9\right) + \left(\frac{1}{2} \times 20 \times 15\right)$$
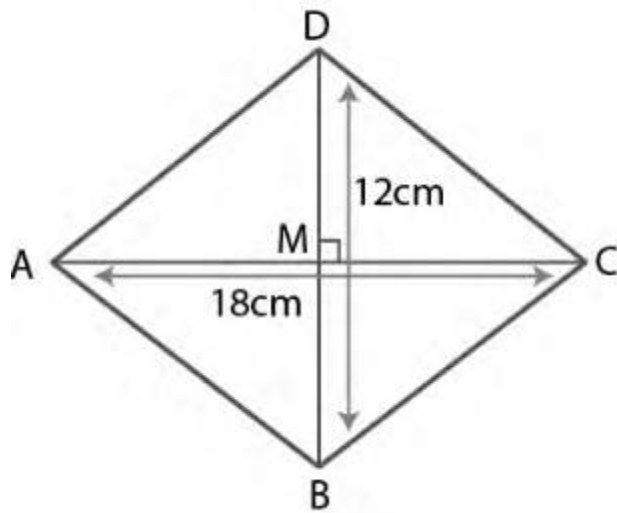
By further calculation

$$= (10 \times 9 + 10 \times 15)$$

$$= 90 + 150$$

$$= 240 \text{ cm}^2$$

(ii) consider ABCD as a quadrilateral in which the diagonal AC and BD intersect each other at M at right angles

AC = 18 cm and BD = 12 cm

We know that

Area of quadrilateral ABCD $= \frac{1}{2} \times diagonal\ AC \times diagonal\ BD$

Substituting the values

$= \frac{1}{2} \times 18 \times 12$

By further calculation

$= 9 \times 12$

$= 108\ cm^2$

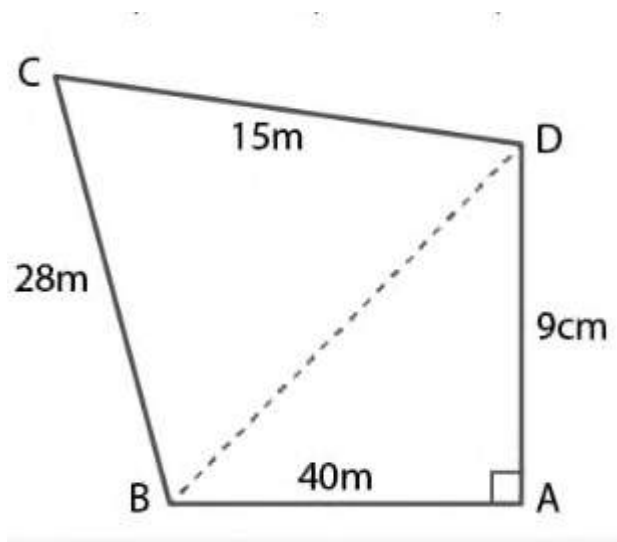**2. find the area of the quadrilateral field ABCD whose sides AB = 40m , BC = 28m , CD = 15 m , AD = 9 m and ∠A = 90⁰**

Solution

It is given that

ABCD is a quadrilateral field

AB = 40m , BC = 28 m , CD = 15 m , AD = 9 m and ∠A = 90º

In triangle BAD

∠A = 90º

Using the Pythagoras theorem

$BD^2 = BA^2 + AD^2$

Substituting the values

$BD^2 = 40^2 + 9^2$

By further calculation

BD^2 = 1600 + 81 = 1681

So we get

BD = 41

We know that

Area of quadrilateral ABCD = area of Δ BAD + area of ΔBDC

It can be written as

$= \frac{1}{2} \times base \times height + area\ of\ \Delta\ BDC$

Substituting the values

$= \frac{1}{2} \times 40 \times 9 + area\ of\ \Delta BDC$

By further calculation

= 180m^2 + area of Δ BDC

Determining the area of Δ BDC

Consider a = BD = 41m , b = CD = 15 m , c = BC = 28 m

We know that

S = semi perimeter $= \frac{a+b+c}{2}$

Substituting the values

$= \frac{41+15+28}{2}$

= 42 cm

Here

Area of Δ BDC $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$$= \sqrt{42(42-41)(42-15)(42-28)}$$

By further calculation

$$= \sqrt{42 \times 1 \times 27 \times 14}$$

We can write it as

$$= \sqrt{2 \times 3 \times 7 \times 3 \times 3 \times 3 \times 2 \times 7}$$

So we get

$$= 2 \times 7 \times 3 \times 3$$

$$= 126 \ m^2$$

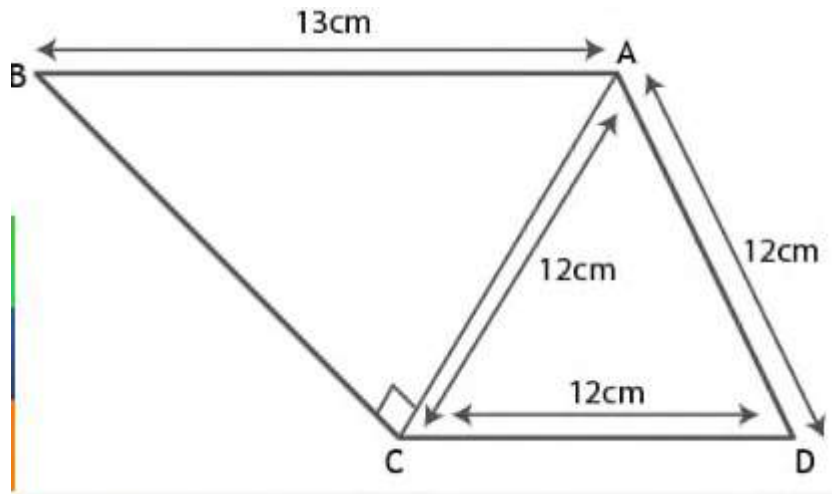So the area of quadrilateral ABCD $= 180m^2 +$ area of Δ BDC

Substituting the values

$$= 180 + 126$$

$$= 306 \ m^2$$

3. find the area of the quadrilateral ABCD in which ∠BCA = 90⁰ , AB = 13 cm and ACD is an equilateral triangle of side 12 cm .

13cm

B

A

12cm

12cm

12cm

C

D

Solution

It is given that

ABCD is a quadrilateral in which ∠BCA = 90° and AB = 13cm

ABCD is an equilateral triangle in which AC = CD =AD = 12cm

In right angled Δ ABC

Using Pythagoras theorem ,

$AB^2 = AC^2 + BC^2$

Substituting the values

$13^2 = 12^2 + BC^2$

By further calculation

$BC^2 = 13^2 - 12^2$

$BC^2 = 169 - 144 = 25$

So we get

BC = $\sqrt{25}$ = 5 cm

We know that

Area of quadrilateral ABCD = area of Δ ABC + area of Δ ACD

It can be written as

$= \frac{1}{2} \times base \times height \times \frac{\sqrt{3}}{4} \times side^2$

$= \frac{1}{2} \times AC \times BC + \frac{\sqrt{3}}{4} \times 12^2$

Substituting the values

$= \frac{1}{2} \times 12 \times 5 + \frac{\sqrt{3}}{4} \times 12 \times 12$

So we get

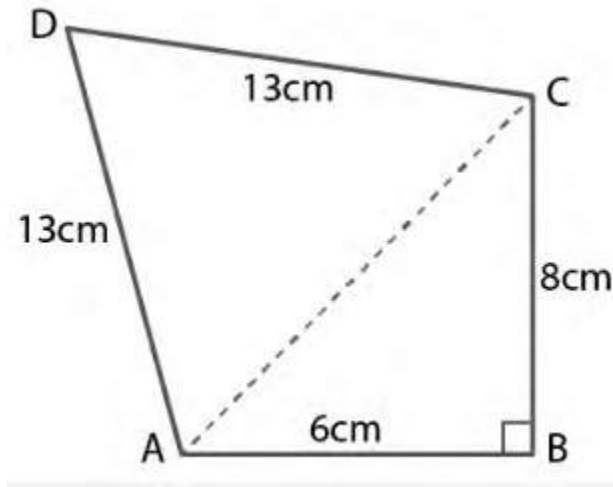$6 \times 5 + \sqrt{3} \times 3 \times 12$

$= 30 + 36\sqrt{3}$

Substituting the value of $\sqrt{3}$

$= 30 + 36 \times 1.732$

$= 30 + 62.28$

$= 92.28$ cm^2

4. find the area of quadrilateral ABCD in which ∠B = 90⁰ , AB = 6 cm , BC = 8 cm and CD = AD = 13cm

Solution

It is given that

ABCD is a quadrilateral in which ∠B = 90⁰ , AB = 6 cm , BC = 8 cm and CD = AD = 13cm

In Δ ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 6^2 + 8^2$

By further calculation

$AC^2 = 36 + 64 = 100$

So we get

AC² = 10²

AC = 10 cm

We know that

Area of quadrilateral ABCD = area of Δ ABC + area of Δ ACD

It can be written as

$$= \frac{1}{2} \times base \times height + area\ of\ \Delta ACD$$

$$= \frac{1}{2} \times AB \times BC + Area\ of\ \Delta\ ACD$$

$$= \frac{1}{2} \times AB \times BC + area\ of\ \Delta\ ACD$$

Substituting the values

$$= \frac{1}{2} \times 6 \times 8 + area\ of\ \Delta ACD$$

By further calculation

= 24 cm² + area of Δ ACD ....(1)

Finding the area of Δ ACD

Consider a = AC = 10cm , b = CD = 13 cm , c = AD =13cm

We know that

$$S = semi\ perimeter = \frac{a+b+c}{2}$$

Substituting the values

$$= \frac{10+13+13}{2}$$

$= \frac{10+26}{2}$

$= \frac{36}{2}$

$= 18$ cm

Here

Area of Δ ACD $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{18(18-10)(18-13)(18-13)}$

By further calculation

$= \sqrt{18 \times 8 \times 5 \times 5}$

We can write it as

$= \sqrt{6 \times 3 \times 8 \times 5 \times 5}$

$= \sqrt{3 \times 2 \times 3 \times 2 \times 2 \times 2 \times 5 \times 5}$

So we get

$= 3 \times 2 \times 2 \times 5$

$= 60$ cm^2

Using equation (1)

Area of quadrilateral ABCD $= 24$cm^2 + area of Δ ACD

Substituting the values

$= 24 + 60$

$= 84$ cm^2

5. the perimeter of a rectangular cardboard is 96 cm ; if its breadth is 18 cm , find the length and the area of the cardboard .
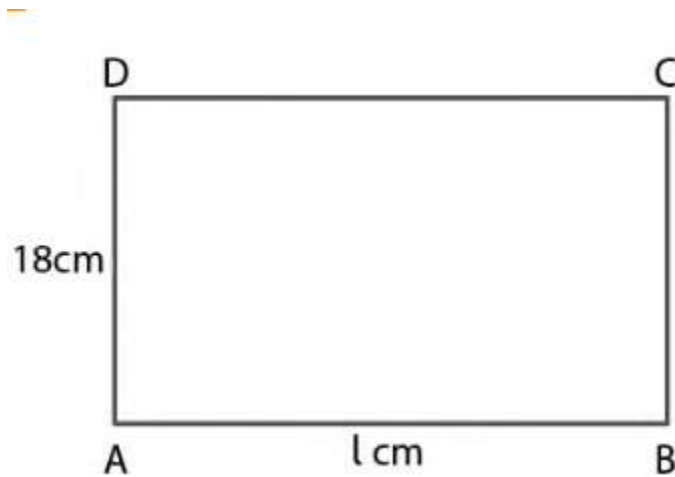
Solution :

Consider ABCD as a rectangle

Take length = 1 cm

Breath = 18 cm

Perimeter = 96 cm

```
      D                    C
      ┌────────────────────┐
      │                    │
18cm  │                    │
      │                    │
      └────────────────────┘
      A         l cm        B
```

We know that

$2\times (l + b) = 96$ cm

Substituting the values

$2\times (l + 18) = 96$ cm

By further calculation

$(1 + 18) = \dfrac{96}{2}$

$l + 18 = 48$

so we get

$l = 48 - 18 = 30$ cm

here

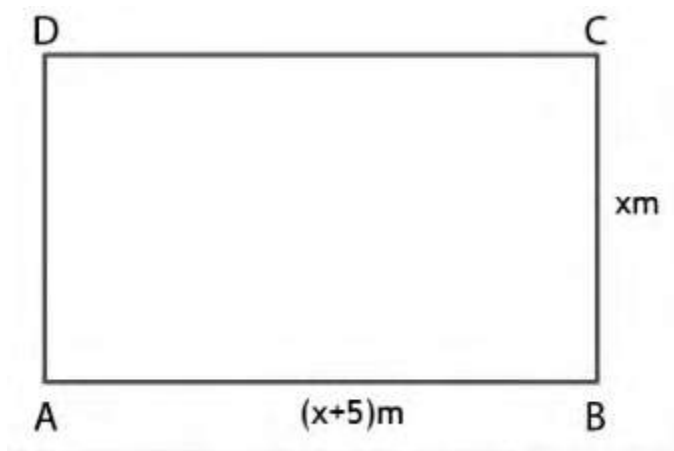area of rectangle cardboard $= l \times b$

substituting the values

$= 30 \times 18$

$= 540$ cm^2

**6 . the length of a rectangle hall is 5 cm   more than its breath ,
if the area of the hall is 594 m^2 , find its perimeter .**

Solution

Consider ABCD is a rectangular hall

Take breath $= x$ m

Length $= (x + 5)$ m

We know that

Area of rectangular field $= l \times b$

 Substituting the values

$594 = x(x + 5)$

By further calculation

$594 = x^2 + 5x$

$0 = x^2 + 5x - 594$

$x^2 + 5x - 594 = 0$

It can be written as

$x^2 + 27x - 22x - 594 = 0$

Taking out the common terms

$x(x+27) - 22(x+27) = 0$

So we get

$(x - 22)(x + 27) = 0$

Here

$x - 22 = 0$ or $x + 27 = 0$

We get

$x = 22$m or $x = -27$ which is not possible

We know that

Breath $= 22$m

Length $= (x + 5) = 22 + 5 = 27$ m

Perimeter = 2(1 +b)

Substituting the values

= 2(27 +22)
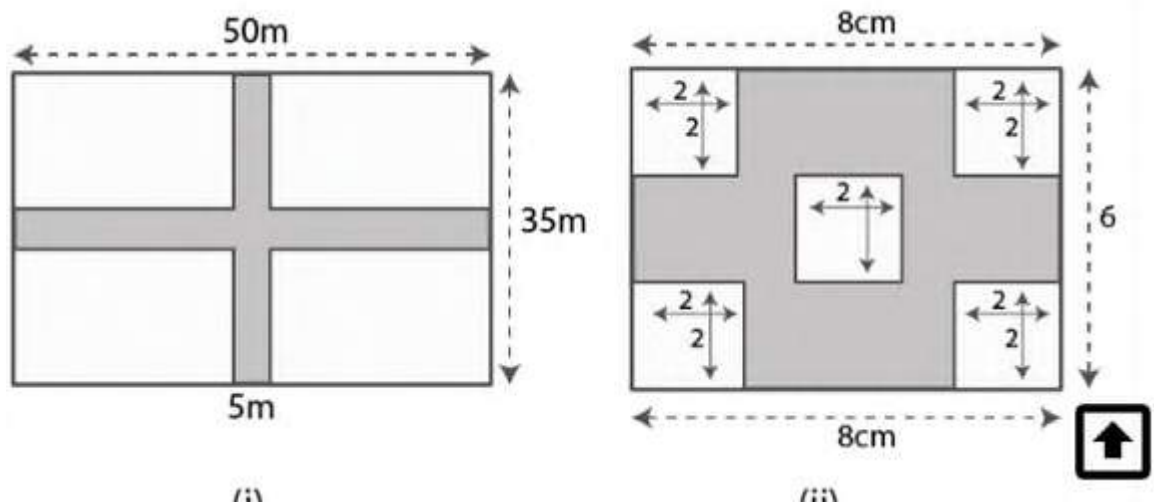
By further calculation

= 2 × 49

= 98 m

7 . (a) the diagram (i) given below shows two paths drawn inside a rectangular field 50m long and 35 m wide . the width of each path is 5 meters . find the area of the shaded portion.
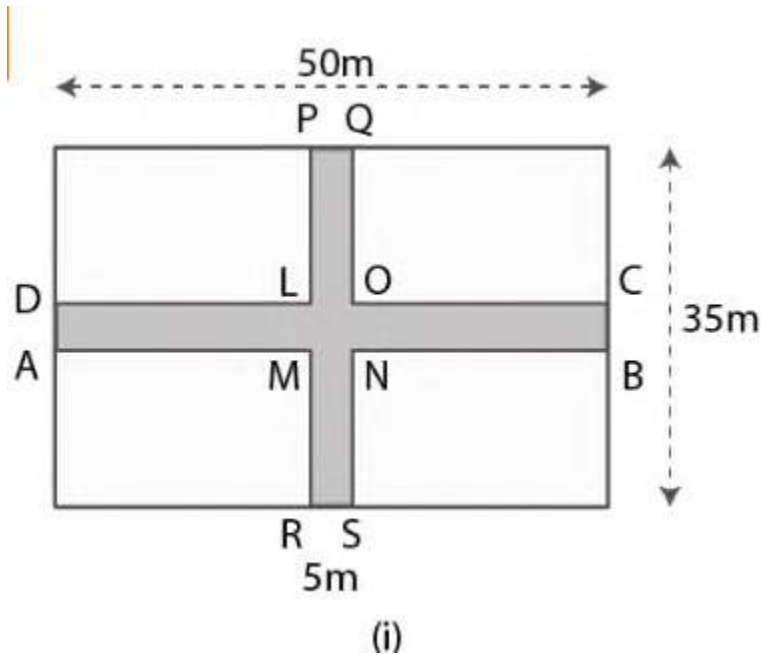
(b) in the diagram (ii) given below , calculate the area of the shaded portion . all measurements are in centimetres .

Solution :

(a) we know that

Area of shaded portion = area of rectangle ABCD + area of rectangle PQRS - area of square LMNO

(i)

Substituting the values

$= 50 \times 5 + 5 \times 35 - 5 \times 5$
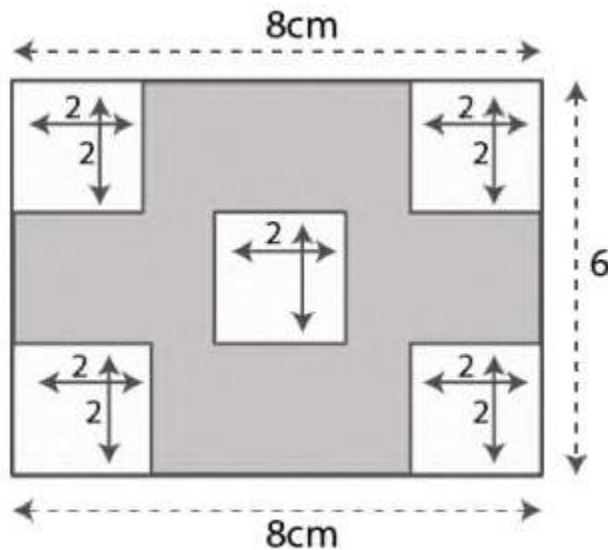
By further calculation

$= 250 + 175 - 25$

So we get

$= 250 + 150$

$= 400 \text{ m}^2$

(b) we know that

Area of shaded portion = area of ABCD - 5 × Area of any small square

It can be written as

$= 1 \times b - 5 \times side \times side$

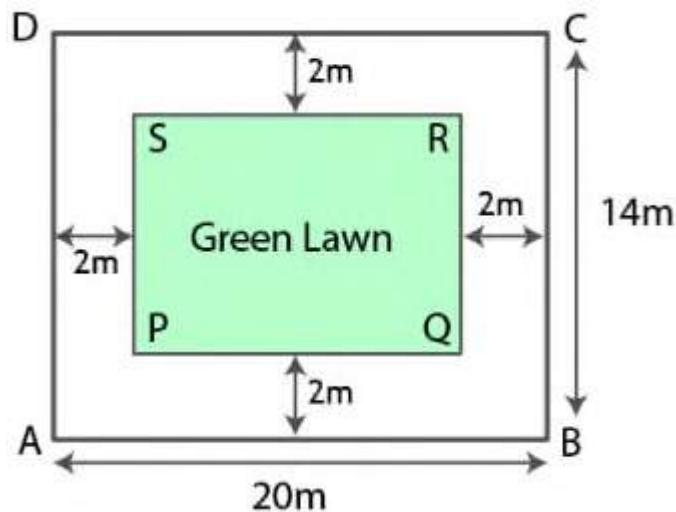Substituting the values

$= 8 \times 6 - 5 \times 2 \times 2$

By further calculation

$= 48 - 20$

$= 28 \text{ cm}^2$

8. A rectangular plot 20 m long and 14 m wide is to covered with grass leaving 2 m all around . find the area to be laid with grass

Solution

Consider ABCD as a plot

Length of plot = 20 m

Breadth of plot = 14 m

Take PQRS as the grassy plot

Here

Length of grassy lawn = 20 − 2 × 2

By further calculation

= 20 − 4

= 16 m

Breadth of grassy lawn $= 14 - 2 \times 2$

By further calculation

$= 14 - 4$

$= 10$ m

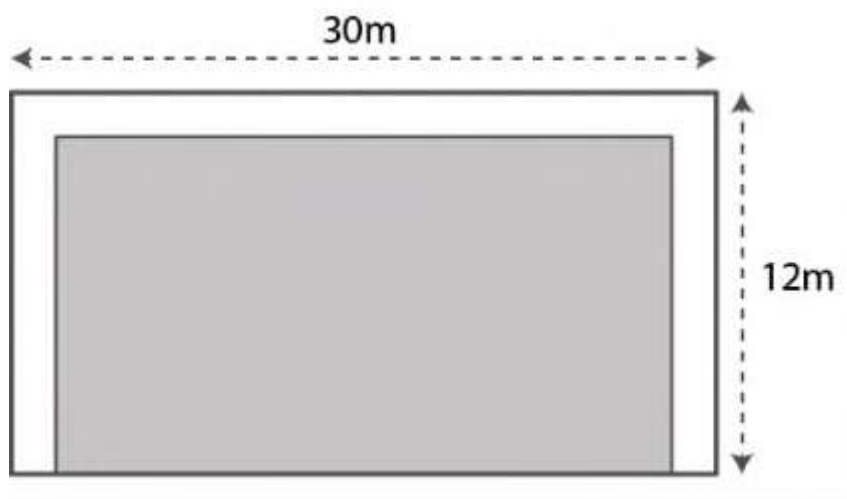Area of grassy lawn $= length \times breath$

Substituting the values

$= 16 \times 10$

$= 160$ m^2

9. the shaded region of the given diagram represents the lawn in front of a house . on three sides of the lawn three are flower beds of width 2 m .
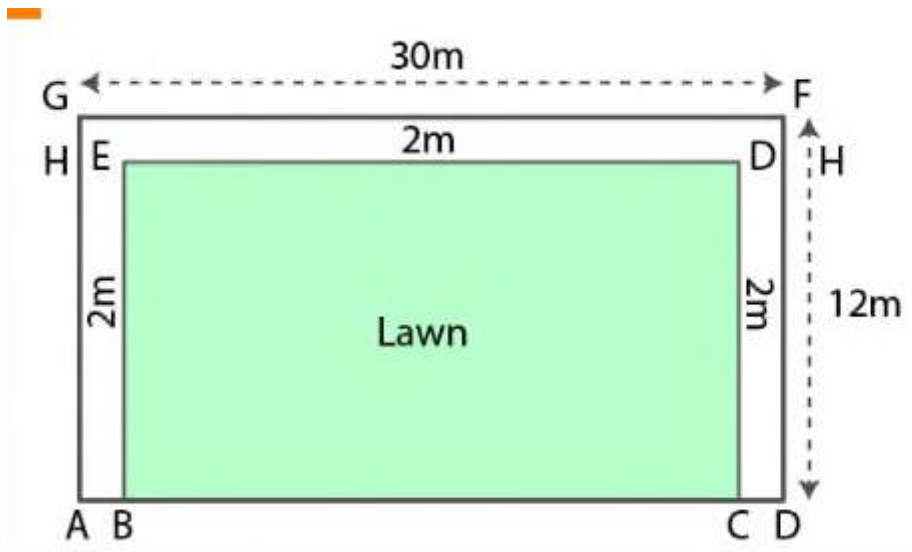
(i) find the length and the breath of the lawn .

(ii) hence or otherwise , find the area of the flower – beds .

Solution

Consider BCDE as the lawn

(i) we know that

Length of lawn BCDE = BC

It can be written as

= AD – AB – CD

Substituting the values

= 30 – 2 – 2

By further calculation

= 30 – 4

= 26 m

Breath of lawn BDCE = BE

It can be written as

= AG – GH

Substituting the values

= 12 – 2

= 10 m

(ii) we know that

Area of flower beds = area of rectangle ADFG - area of lawn BCDE

It can be written as

= AD × AG – BC × BE

Substituting the values

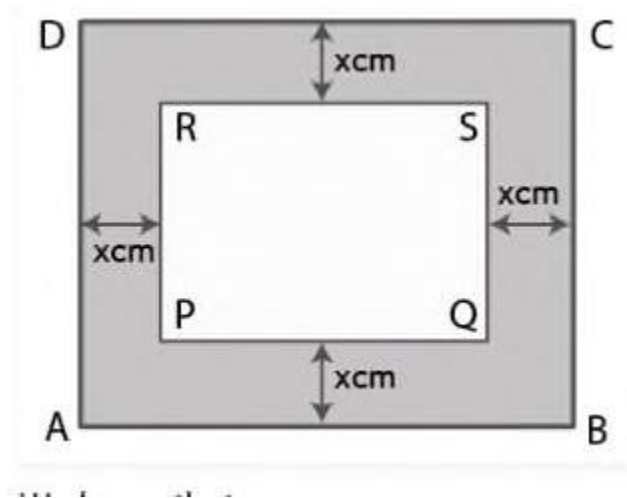= 30 × 12 – 26 × 10

By further calculation

= 360 – 260

= 100m^2

10. A foot path of uniform widths runs all around the inside of a rectangular field 50m long and 38m wide . if the area of the path is 492 m^2 , find its width .

Solution

Consider ABCD as a rectangular field having

Length = 50 m

Breadth = 38 m

We know that

Area of rectangular filed ABCD = 1 × b

Substituting the values

= 50 × 38

= 1900 m²

Let x m as the width of foot path all around the inside of a rectangular field

Length of rectangular field PQRS = (50 – x –x) = (50 – 2x) m

Breath of rectangular field PQRS = (38 – x – x) = (38 – 2x)m

Here

Area of foot path = area of rectangular field ABCD - area of rectangular field PQRS

Substituting the values

$492 = 1900 - (50 - 2x)(38 - 2x)$

It can be written as

$492 = 1900 - [50(38 - 2x) - 2x(38 - 2x)]$

By further calculation

$492 = 1900 - (1900 - 100x - 76x + 4x^2)$

$492 = 1900 - 1900 + 100x + 76x - 4x^2$

On further simplification

$492 = 176x - 4x^2$

Taking out 4 as common

$492 = 4(44x - x^2)$

$44x - x^2 = \frac{492}{4} = 123$

We get

$X^2 - 44x + 123 = 0$

It can be written as

$X^2 - 41x - 3x + 123 = 0$

Taking out the common terms

$X(x - 41) - 3(x - 41) = 0$

$(x - 3)(x - 41) = 0$

Here

$X - 3 = 0$ or $x - 41 = 0$

So x = 3 m or x = 41 m which is not possible

Therefore , width is 3 m .

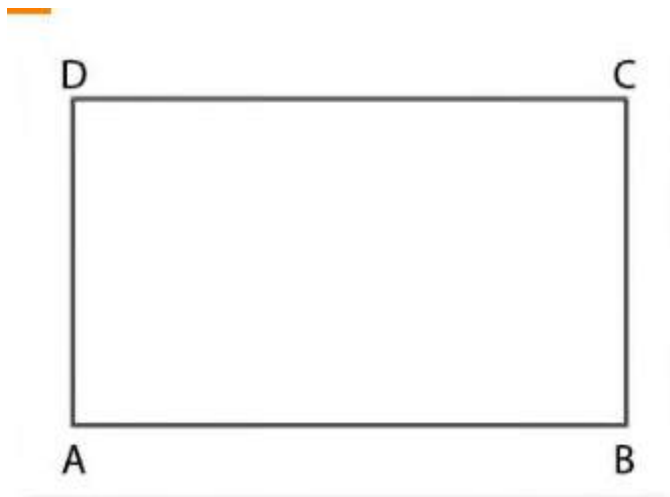**11.** **the cost of enclosing a rectangular garden with a fence all around at the rate of Rs 15 per meter is Rs 5400 . if the length of the garden is 100m , find the area of the garden .**

Solution :

Consider ABCD as a rectangular garden

Length = 100 m

Take breadth = x m

We know that

Perimeter of the garden $= 2(1 + b)$

Substituting the values

$= 2(100 + x)$

$= (200 + 2x)$ m

We know that

Cost of 1 m to enclosing a rectangular garden $=$ Rs 15

So the cost of $(200 + 2x)$ m to enclosing a rectangular garden $=$

$15(200 + 2x)$

$= 3000 + 30x$

Given cost $=$ Rs 5400

We get

$3000 + 30x = 5400$

It can be written as

$30x = 5400 - 3000$

$X = \frac{2400}{30} = 80$ m

Breadth of garden $= 80$ m

So the area of rectangular filed $= 1 \times b$

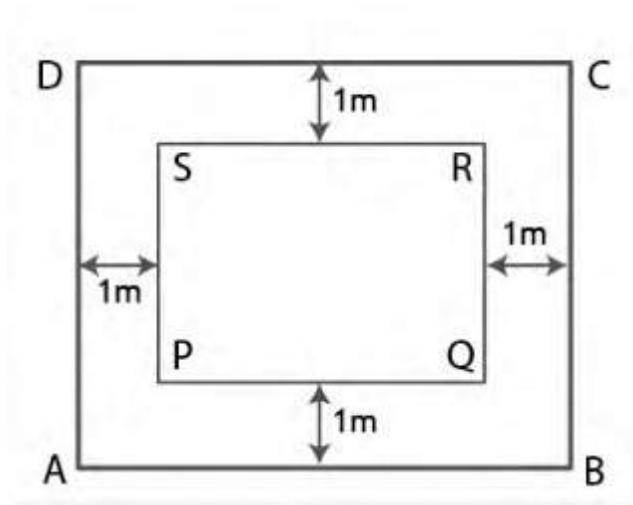Substituting the values

$= 100 \times 80$

$= 8000 \text{ m}^2$

12. a rectangular floor which measure 15m × 8m is to be laid with tiles measuring 50cm × 25 cm find the number of tiles required further , if a carpet is laid on the floor so that a space of 1 m exits between its edges and the edges of the floor , what fraction of the floor is uncovered ?

Solution

Consider ABCD as a rectangular field of measurement 15m × 8m

Length = 15 m

Breadth = 8m

Here the area $= l \times b = 15 \times 8 = 120 \text{ m}^2$

Measurement of tiles $= 50 \text{ cm} \times 25 \text{ cm}$

Length $= 50$ cm $= \dfrac{50}{100} = \dfrac{1}{2}$ m

Breadth $= 25$ cm $= \dfrac{25}{100} = \dfrac{1}{4}$ m

So the area of one tile $= \dfrac{1}{2} \times \dfrac{1}{4} = \dfrac{1}{8}$ m^2

No. of required tiles $= \dfrac{\text{area of rectangular field}}{\text{area of one tile}}$

Substituting the values

$= \dfrac{120}{\frac{1}{8}}$

By further calculation

$= \dfrac{120 \times 8}{1}$

$= 960$ tiles

Length of carpet $= 15 - 1 - 1$

$= 15 - 2$

$= 13$ m

Breadth of carpet $= 8 - 1 - 1$

$= 8 - 2$

$= 6$ m

Area of carpet $= l \times b$

$= 13 \times 6$

$= 78 \text{ m}^2$

We know that

Area of floor which is uncovered by carpet = area of floor – area of carpet

Substituting the values

$= 120 - 78$

$= 42 \text{ m}^2$

$\text{Fraction} = \dfrac{\text{area of floor which is uncovered by carpet}}{\text{area of floor}}$

Substituting the values

$= \dfrac{42}{120}$

$= \dfrac{7}{20}$

13. the width of a rectangular room is $\dfrac{3}{5}$ of its length x meters . if its perimeter is y meters , write an equation connecting x and y . find the floor area of the room if its perimeter is 32 m .

Solution

It is given that

Length of rectangular room = x m

Width of rectangular room $= \dfrac{3}{5}$ of its length

$$= \frac{3x}{5} \text{ m}$$

Perimeter = y m

We know that

Perimeter $= 2(1 + b)$

Substituting the values

$$Y = 2 \left[\frac{5x + 3x}{5} \right]$$

By further calculation

$$Y = 2 \times \frac{8x}{5}$$

$$Y = \frac{16x}{5}$$

We get

5y = 16x

16x = 5y .....(1)

Equation (1) is the required relation between x and y

Given perimeter = 32 m

So y = 32 m

Now substituting the value of y in equation (1)

16x = 5 × 32

By further calculation

$$X = \frac{5 \times 32}{16}$$

$X = \dfrac{5 \times 2}{1}$

X = 10 m

Breadth $= \dfrac{3}{5} \times x$

Substituting the value of x

$= \dfrac{3}{5} \times 10$

$= 3 \times 2$

$= 6$ m

Here the floor area of the room $= 1 \times b$
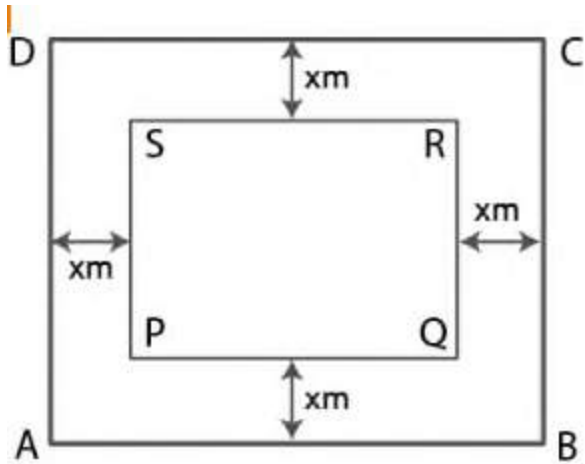
Substituting the values

$= 10 \times 6$

$= 60$ m^2

14. A rectangular garden 10 m by 16 m is to be surrounded by a concrete walk of uniform width . given that the area of the walk is 120 square meters, assuming the width of the walk to be x , form an equation in x and solve it to find the value of x.

Solution

Consider ABCD as a rectangular garden

Length = 10 m

Breath = 16 m

So the area of ABCD = l × b

Substituting the values

= 10 × 16

= 160 m²

Consider x m as the width of the walk

Length of rectangular garden PQRS = 10 – x – x = (10 – 2x) m

Breadth of rectangular garden PQRS = 16 – x – x = (16 – 2x) m

15 . A rectangular room is 6 m long , 4. 8 m wide and 3.5 m high . find the inner surface of the four walls.

Solution

It is given that

Length of rectangular room = 6m

Breadth of rectangular room = 4.8 m

Height of rectangular room = 3.5 m

Here

Inner surface area of four wall = 2(l + b) × h

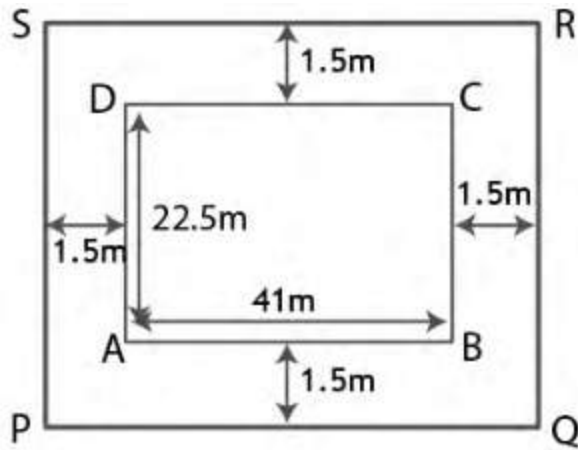Substituting the values

= 2(6 + 4.8) × 3.5

By further calculation

= 2 × 10.8 × 3.5

= 21.6 × 3.5

= 75.6 m^2

16. a rectangular plot of land measures 41 meters in length and 22.5 meters in width . A boundary wall 2 meters high is built all around the plot at a distance of 1.5 m from the plot . find the inner surface the boundary wall.

Solution

It is given that

Length of rectangular plot = 41 meters

Breadth of rectangular plot = 22. 5 meters

Height of boundary wall = 2meters

Here

Boundary wall is built at a distance of 1.5 m

New length = 41 + 1.5 + 1.5

= 41 + 3

= 44 m

New breath = 22.5 +1.5 +1.5

= 22.5 + 3

= 25.5 m

We know that

Inner surface area of the boundary wall = 2(l + b) × h

Substituting the values

= 2 (44 + 25.5) × 2

By further calculation

= 2 × 69.5 × 2

= 2 × 139

= 278 m^2

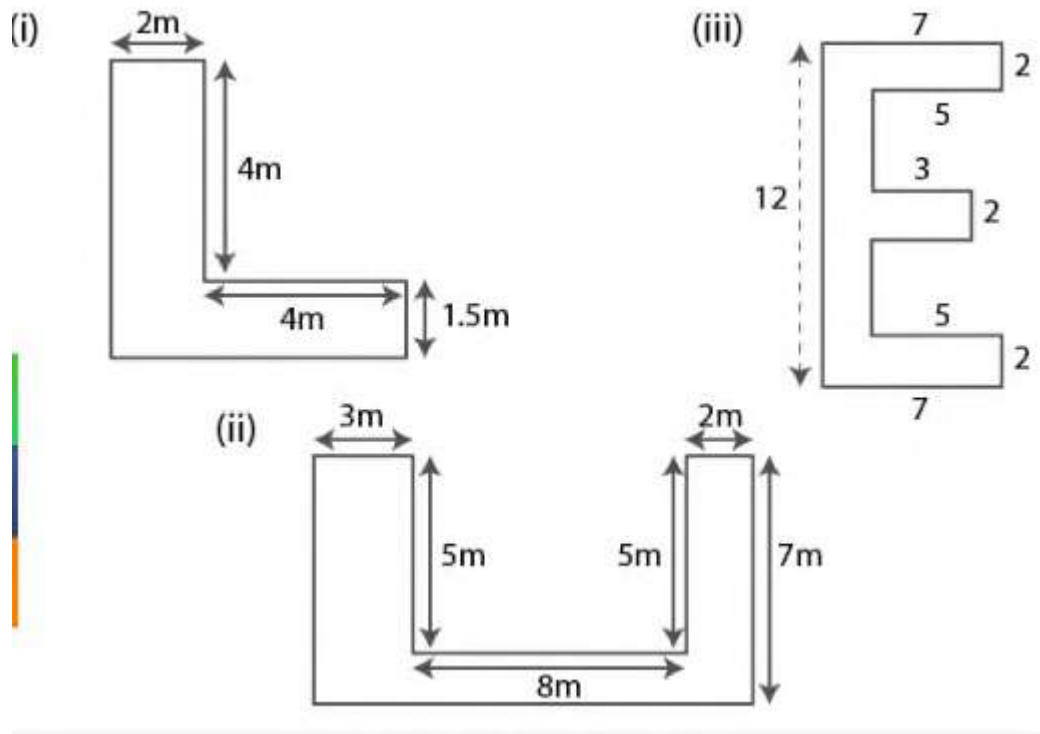**17. (a) find the perimeter and area of the figure**

(i) given below in which all corners are right angled .

(b) find the perimeter and area of the figure

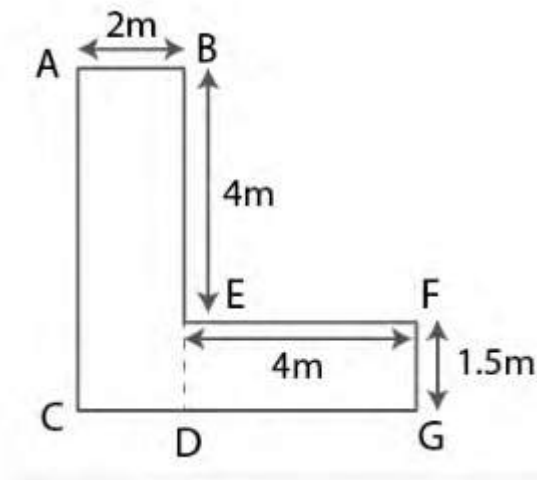**(ii) given below in which all concerns are right angles .**

(c) find the area and perimeter of the figure

(iii) given below in which all corners are right angled and all measurement in centimetres .

(i)

(iii)

(ii)

Solution

(a) it is given that

AB = 2m , BE =4 m , FE = 4m and FG = 1.5 m

So BD = 4 + 1.5 = 5.5 m

AC = BD = 5.5 m

CG = (4 + 2) = 6 m

We know that

Perimeter of figure (i) = AC + CG + GF + FE + EB + BA

Substituting the values

= 5.5 + 6 + 1.5 + 4 + 4+2

= 23 m

Here

Area of given figure = $area\ of\ ABEDC + area\ of\ FEDG$

It can be written as

= length × breadth + length × breadth

Substituting the values

= 2 × 5.5 + 4 × 1.5

= 11 + 6

= 17 m^2

(b) it is given that

AB = CD = 3m

HI = AC = 7m

JE = BE = 5m

GF = DE = 2m

DG = EF = 8m

GH = JI = 2 m
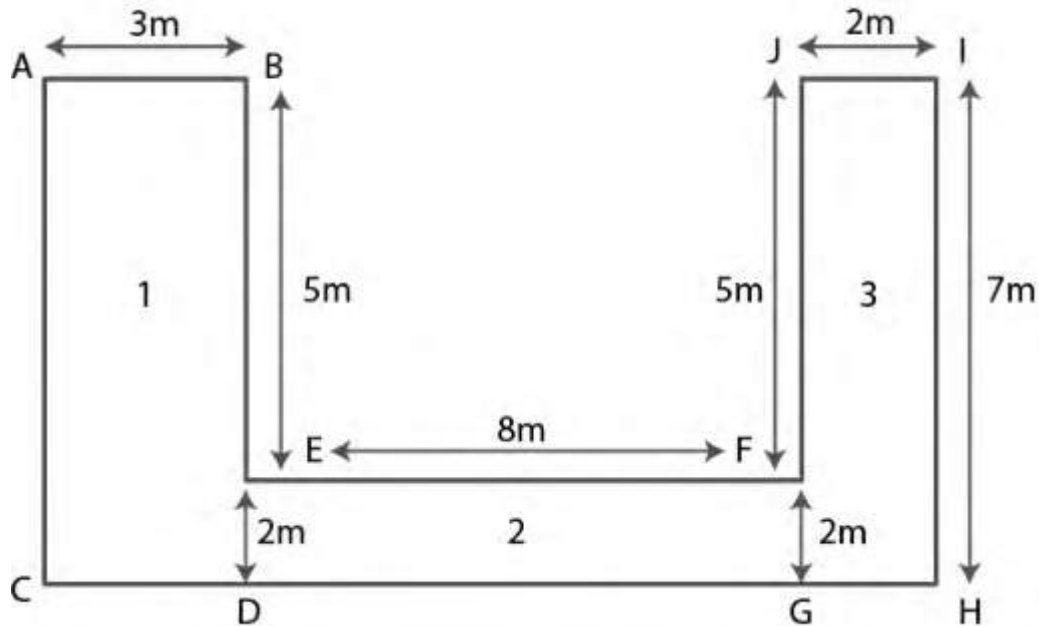
We know that

CH = CD + DG +GH

Substituting the values

= 3 + 8 + 2

= 13 m

Perimeter of the given figure = AB + AC + CH + HI + IJ + JF + FE + BE

Substituting he values

= 3 + 7+ 13 +7 + 2 + 5 + 8 + 5

= 50 m

Here

Area of given figure = area of first figure + area of second figure + area of third figure

Substituting the values

$= 7 \times 3 + 8 \times 2 + 7 \times 2$

$= 21 + 16 + 14$

$= 51 \text{ m}^2$

(c) it is given that

AB $= 12$cm

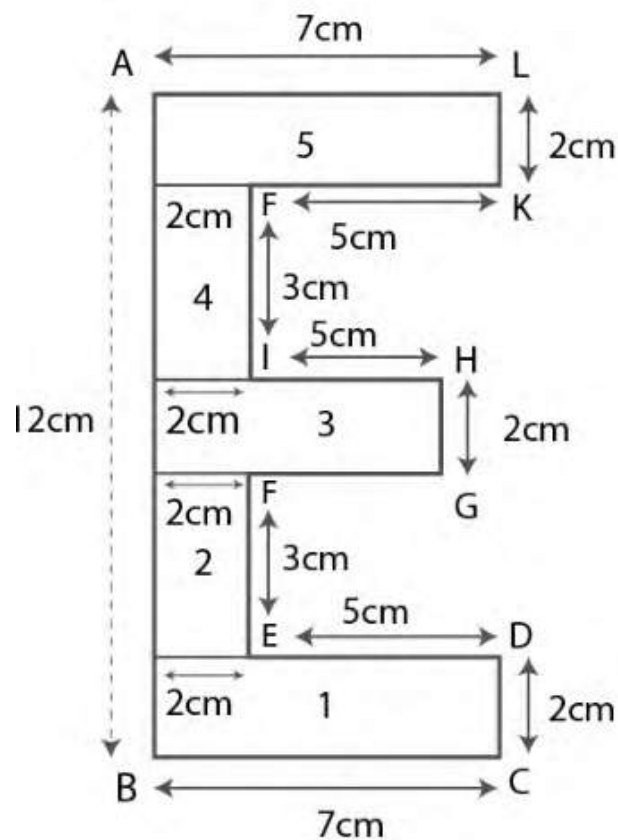AL $=$ BC $= 7$cm

JK $=$ DE $= 5$ cm

HJ $=$ GF $= 3$cm

LK $=$ HG $=$ CD $= 2$ cm

We know that

Perimeter of given figure = AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK + KL + LA

Substituting the values

= 12 + 7 + 2 + 5 + 3 + 3 + 2 + 3 + 3 + 5 + 2 + 7

= 54 cm

Here

Area of given figure = area of first part + area of second part + area of third part + area of fourth part + area of fifth part

Substituting the values

$= 7 \times 2 + 2 \times 3 + (2 + 3) \times 2 + 2 \times 3 + 7 \times 2$

By further calculation

$= 14 + 6 + 10 + 6 + 14$

$= 50 \text{ cm}^2$

18. the length and the breadth of a rectangle are 12 cm and 9 cm respectively . find the height of a triangle whose base is 9 cm and whose area is one third that of rectangle.

Solution

It is given that
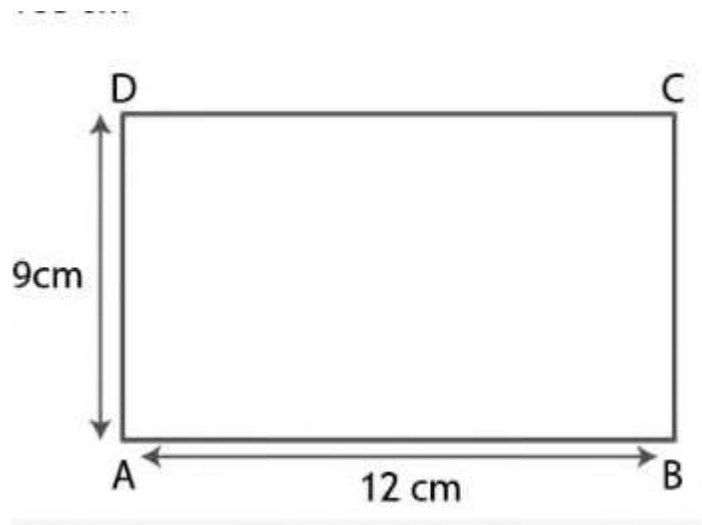
Length of a rectangle = 12cm

Breadth of a rectangle = 9 cm

So the area $= l \times b$

Substituting the values

$= 12 \times 9$

$= 108 \text{ cm}^2$

Using the condition

Area of triangle ABC $= \frac{1}{3} \times area\ of\ rectangle$

Substituting the values

$= \frac{1}{3} \times 108$

$= 36\ \text{cm}^2$

Consider h cm as the height of triangle ABC

Area of triangle ABC $= \frac{1}{2} \times base \times height$

Substituting the values

$36 = \frac{1}{2} \times 9 \times h$

$h = 36 \times \frac{2}{9}$

so we get
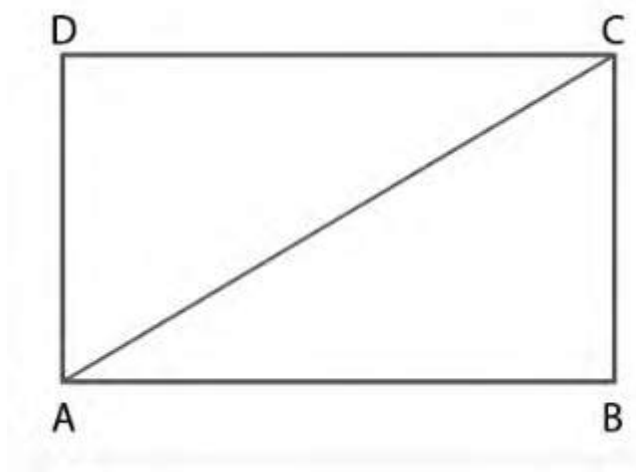
$h = 4 \times 2$

$h = 8$ cm

therefore , height of triangle ABC is 8 cm

19. the area of a square plot is 484 m^2 . find the length of its one side and the length of its one diagonal .

Solution

It is given that

ABCD is a square plot having area $= 484$m^2

Sides of square are AB , BC , CD and AD

We know that

Area of square $=$ side $\times$ side

Substituting the values

$484 = (\text{side})^2$

So we get

$\text{Side} = \sqrt{484} = 22\text{m}$

$AB = BC = 22\text{m}$

In triangle ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 22^2 + 22^2$

$AC^2 = 484 + 484 = 968$

By further calculation

$AC = \sqrt{968} = \sqrt{484 \times 2}$

$AC = 22 \times \sqrt{2}$

So we get

$AC = 22 \times 1.414 = 31.11 \text{ m}$

Therefore , length of side is 22 m and length of diagonal is 31.11 m

.

20 . A square has the perimeter 56 m . find its area and the length of one diagonal correct up to two decimal places .

Solution
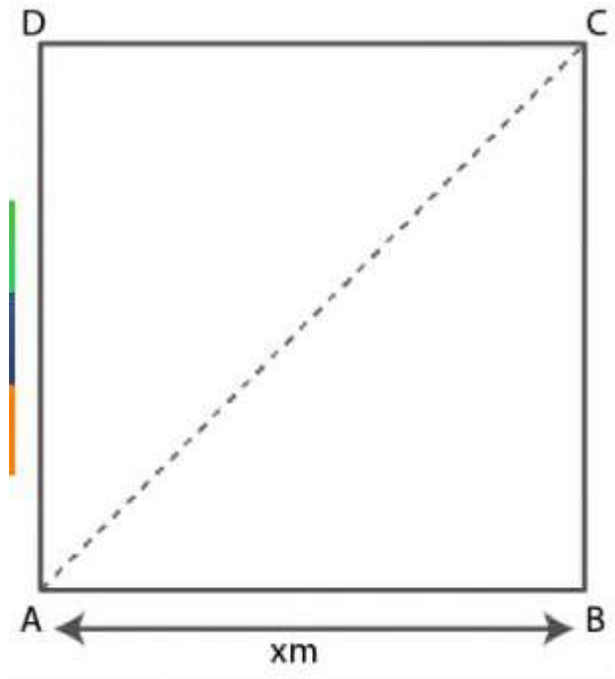
Consider ABCD as a square with side x m

Perimeter of square $= 4 \times$ side

Substituting the values

$56 = 4x$

By further calculation

$X = \dfrac{56}{4} = 14$ m

In triangle ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 14^2 + 14^2$

By further calculation

$AC^2 = 196 + 196 = 392$

So we get

$AC = \sqrt{392}$

$AC = \sqrt{196 \times 2}$

$AC = 14\sqrt{2}$

Substituting the value of $\sqrt{2}$

$AC = 14 \times 1.414$

$AC = 19.80$ m

Therefore , the side of square is 14 m and diagonal is 19. 80 m

21. A wire when bent in the form of an equilateral triangle encloses an area of $36\sqrt{3}$ cm^2 . find the area enclosed by the same wire when bent to form :

(i) a square , and

(ii) a rectangle whose length is 2 cm more than its width .

Solution

It is given that

Area of equilateral triangle = $36\sqrt{3}$ cm^2

Consider x cm as the side of equilateral triangle

We know that

Area = $\frac{\sqrt{3}}{4}$ (side)^2

Substituting the values

$36\sqrt{3} = \frac{\sqrt{3}}{4} \times (x)^2$

By further calculation

$X^2 = \frac{(36\sqrt{3} \times 4)}{\sqrt{3}}$

$X^2 = 36 \times 4$

So we get

$X = \sqrt{36 \times 4}$

$X = 6 \times 2$

$X = 12$ cm

Here

Perimeter of equilateral triangle = 3× side

$= 3 \times 12$

$= 36$ cm

(i) we know that

Perimeter of equilateral triangle = perimeter of square

It can be written as

36 = 4 × side

So we get

Side = $\frac{36}{4}$ = 9cm

Area of square = side × side

$$= 9 \times 9$$

$$= 81 \text{ cm}^2$$

(ii) we know that

Perimeter of triangle = perimeter of rectangle .....(1)

According to the condition of rectangle

Length is 2 cm more than its width

Width of rectangle = x cm

Length of rectangle = (x+ 2) cm

Perimeter of rectangle = 2(1 + b)

Substituting the values

= 2[(x + 2) + x]

By further calculation

= 2(2x + 2)

= 4x + 4

Using equation (1)

4x + 4 = perimeter of triangle

4x + 4 = 36

By further calculation

4x = 36 – 4 = 32 cm

So we get

$X = \frac{32}{4} = 8$ cm

Here

Length of rectangle = 8 + 2 = 10cm

Breadth of rectangle = 8 cm

Area of rectangle = length × breadth

= 10 × 8

= 80 cm^2

22. two adjacent sides of a parallelogram are 15 cm and 10 cm . if the distance between the longer sides is 8 cm , find the area of the parallelogram . also find the distance between shorter sides.
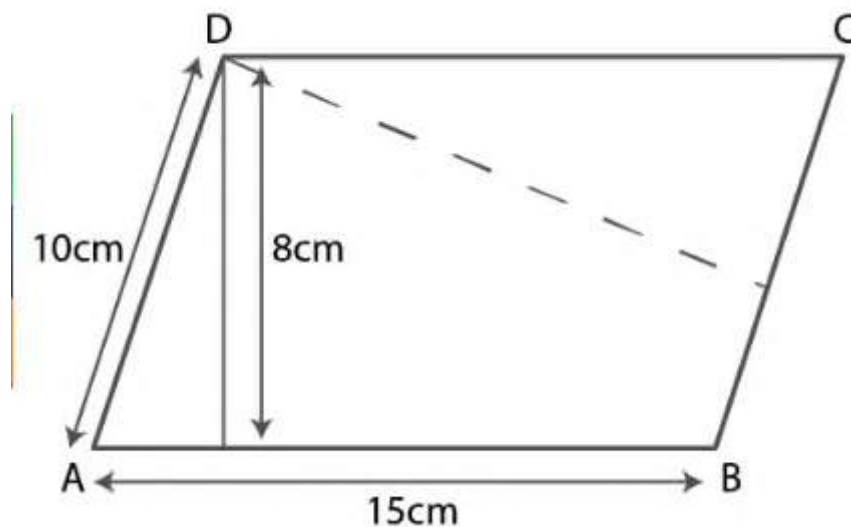
Solution

Consider ABCD as a parallelogram

Longer side AB = 15 cm

Shorter side = 10 cm

Distance between longer side DM = 8 cm

Consider DN as the distance between the shorter side

Area of parallelogram ABCD $= base \times height$

We can write it as

$= AB \times DM$

Substituting the values

$= 15 \times 8$

$= 120 \text{ cm}^2$

If base is AD

Area of parallelogram $= AD \times DN$

Substituting the values

$120 = 10 \times DN$

So we get

$DN = \dfrac{120}{10} = 12 cm$

Therefore , the area of parallelogram is 120 cm² and the distance

Between shorter side is 12 cm .
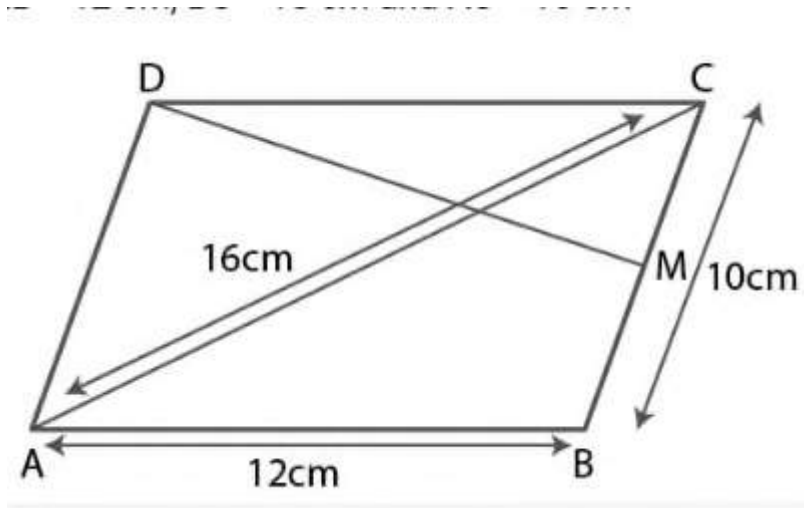

23. ABCD is a parallelogram with sides AB = 12 cm , BC = 10 cm and diagonal AC = 16 cm . find the area of the parallelogram Also find the distance between its shorter sides.

Solution

It is given that

ABCD is a parallelogram

AB = 12cm , BC = 10 cm and AC = 16cm

Area of triangle ABC

BC = a = 10 cm

AC = B = 16 cm

AB = C = 12 cm

We know that

$S = \dfrac{a+b+c}{2}$

Substituting the values

$S = \dfrac{10+16+12}{2}$

By further calculation

$S = \dfrac{38}{2} = 19$ cm

Here

Area of Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{19\,(19-10)(19-16)(19-12)}$

By further calculation

$= \sqrt{19 \times 9 \times 3 \times 7}$

We can write it as

$= \sqrt{19 \times 3 \times 3 \times 3 \times 7}$

$= 3\sqrt{19 \times 3 \times 7}$

$= 3\sqrt{19 \times 21}$

So we get

$= 3\sqrt{399}$ cm^2

We know that

Area of parallelogram $= 2 \times$ area of triangle ABC

Substituting the values

$= 2 \times 3\sqrt{399}$

$= 6\sqrt{399}$

So we get

$= 6 \times 19.96$

$= 119.8$ cm^2

Consider DM as the distance between the shorter lines

Base $= AD = BC = 10$ cm

Area of parallelogram $= AD \times DM$

Substituting the values

$119.8 = 10 \times DM$

By further calculation

$DM = \frac{119.8}{10}$

$DM = 11.98$ cm

Therefore , the distance between shorter lines is 11.98 cm

24. diagonal AC and BD of a perpendicular ABCD intersect at o . given that AB =12cm and perpendicular distance between AB and DC is 6cm . calculate the area of the triangle AOD .
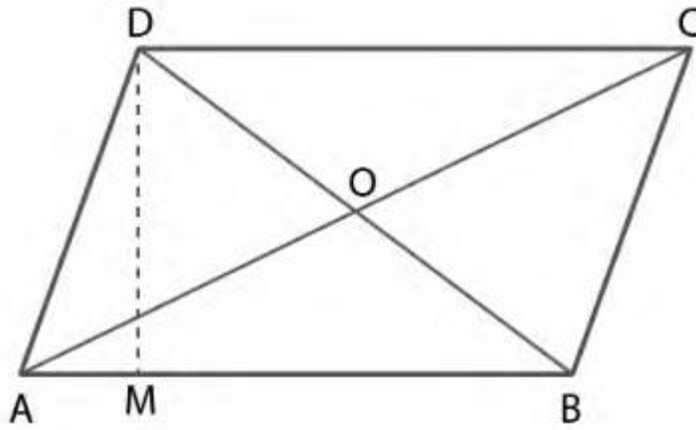

Solution

It is given that

ABCD is a parallelogram

AC and BD are the diagonal which intersect at o

$AB = 12cm$ and $DM = 6$ cm

We know that

Area of parallelogram ABCD $= AB \times DM$

Substituting the values

$= 12 \times 6$

$= 72 cm^2$

Similarly

Area of triangle AOD $= \dfrac{1}{4} \times$ area of parallelogram

Substituting the values

$= \dfrac{1}{4} \times 72$

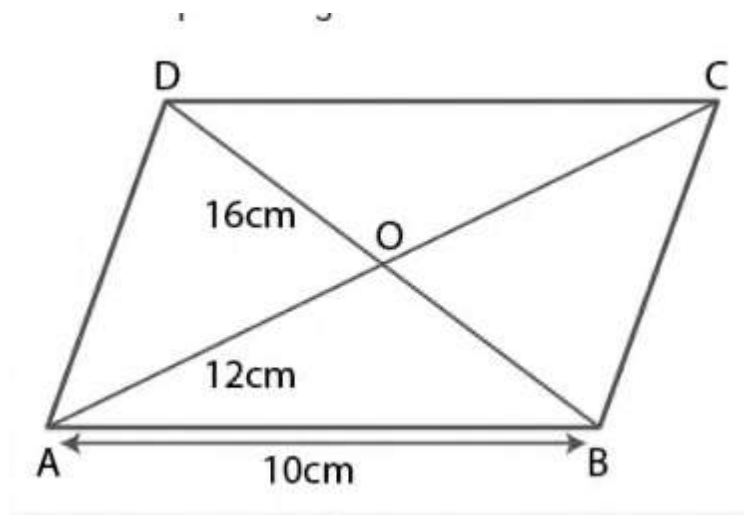$= 18 \ cm^2$

25. ABCD is a parallelogram with side AB = 10 cm . its diagonals AC and BD are of length 12 cm and 16 cm respectively . find the area of the parallelogram ABCD .

Solution

It is given that

ABCD is a parallelogram

AB = 10cm , AC = 12cm

$AO = CO = \dfrac{12}{2} = 6$ cm

BD = 16 cm

$BO = OD = \dfrac{16}{2} = 8$ cm

In triangle AOB

a = 10 cm , b = AO = 6cm , C = BO = 8 cm

we know that

$$s = \frac{a+b+c}{2}$$

substituting the values

$$= \frac{10+6+8}{2}$$

By further calculation

$$S = \frac{24}{2} = 12 \text{ cm}$$

Area of Δ AOB $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$$= \sqrt{12(12-10)(12-6)(12-8)}$$

By further calculation

$$= \sqrt{12 \times 2 \times 6 \times 4}$$

$$= \sqrt{12 \times 2 \times 4}$$

So we get

$$= 12 \times 2$$

$$= 24 \text{ cm}^2$$

We know that

Area of parallelogram ABCD = 4 × area of triangle AOB

Substituting the values

$$= 4 \times 24$$

$$= 96 \text{ cm}^2$$

26. the area of a parallelogram is p cm² and its height is q cm . A second parallelogram has equal area but its base is r cm more than that of the first . obtain an expression in terms of p , q and r for the height h of the second parallelogram .

Solution

It is given that

Area of a parallelogram $= p$ cm²

Height of the first parallelogram $= q$ cm

We know that

Area of parallelogram $=$ base $\times$ height

Substituting the values

$P = base \times q$

Base $= \dfrac{p}{q}$

Here

Base of second parallelogram $= \left(\dfrac{p}{q} + r\right)$

Taking LCM

$= \dfrac{p+qr}{q}$ cm

Area of second parallelogram $=$ area of first parallelogram $= p$ cm²

It can be written as

$base \times height = \text{p cm}^2$

Substituting the values

$$\left[\frac{p+qr}{q}\right] \times h = p$$

So we get

$h = \dfrac{pq}{p+qr}$ cm

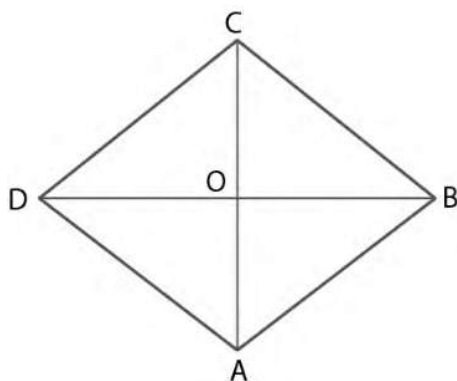therefore , the height of second parallelogram is $h = \dfrac{pq}{p+qr}$ cm

27 . what is the area of a rhombus whose diagonals are 12cm and 16 cm ?

Solution

It is given that

ABCD is a rhombus

BD =12 cm and AC = 16 cm are diagonals

We know that

Area of rhombus ABCD $= \frac{1}{2} \times AC \times BD$

Substituting the values

$= \frac{1}{2} \times 16 \times 12$

By further calculation

$= 8 \times 12$

$= 96 \text{ cm}^2$

28. the area of a rhombus is 98cm^2 . if one of its diagonal 14 cm ,That is the length of the other diagonal ?

Solution

It is given that

Area of rhombus $= 98\text{cm}^2$

One of its diagonal $= 14\text{cm}$

We know that

Area of rhombus $= \frac{1}{2} \times product\ of\ diagonals$

Substituting the values

$98 = \frac{1}{2} \times 14 \times$ other diagonal

By further calculation

Other diagonal $= \dfrac{98 \times 2}{14}$

$= 7 \times 2$

$= 14 \text{ cm}$

Therefore , the other diagonal is 14 cm .

29 . the perimeter of a rhombus is 45 cm . if its height is 8cm , calculate its area.

Solution

It is given that

ABCD is a rhombus

Consider x cm as each side
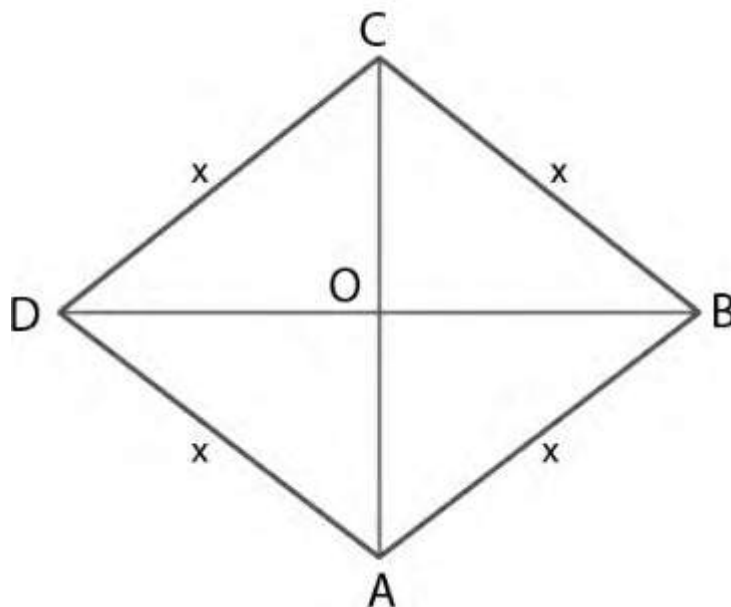
Perimeter = 45 cm

AB + BC + CD + AD = 45 cm

Substituting the values

x + x + x + x = 45

4x = 45

By division

$X = \frac{45}{4}$ cm

We know that

Height = 8 cm

Area of rhombus = base × height

Substituting the values

$= \frac{45}{4} \times 8$

$= 45 \times 2$

$= 90 \text{ cm}^2$

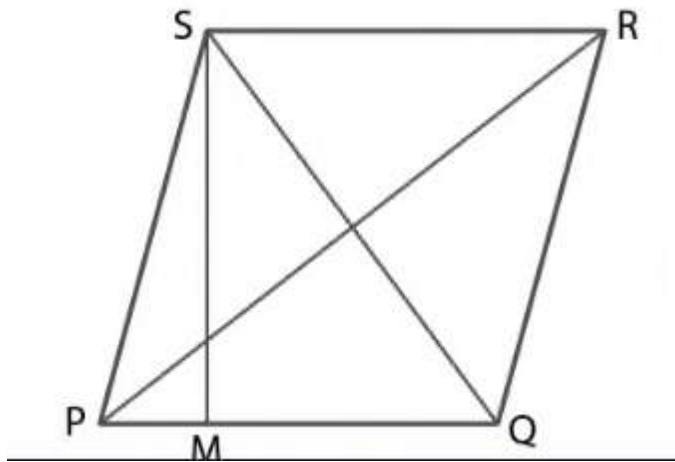**30. PQRS is a rhombus . if it is given that PQ = 3 cm and the height of the rhombus is 2.5 cm , calculate its area .**

Solution :

It is given that

PQRS is a rhombus

PQ = 3cm

Height = 2.5 cm

Consider PQ as the base of rhombus PQRS .

SM = 2.5 cm is the height of rhombus

We know that

Area of rhombus PQRS = base × height

Substituting the values

= 3 × 2.5

= 7.5 cm²

31. if the diagonals of a rhombus are 8 cm and 6 cm , find its perimeter .

Solution

Consider ABCD as a rhombus with AC and BD as two diagonals

Here

AC = 8 cm and BD = 6 cm

AO = 4cm and BO = 3cm
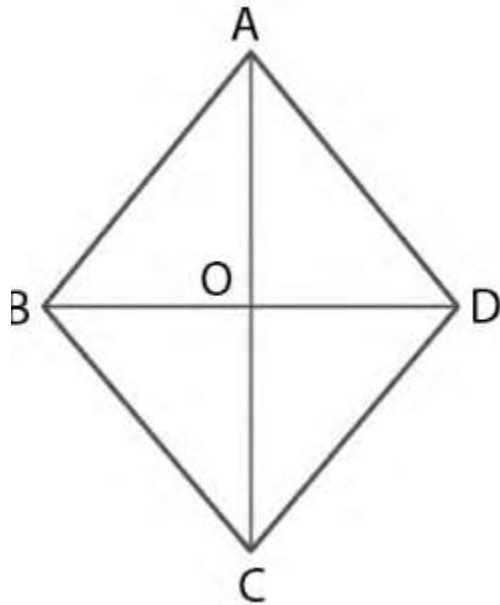
In triangle ABC

Using Pythagoras theorem

$$AB^2 = AO^2 + BO^2$$

Substituting the values

$$AB^2 = 4^2 + 3^2$$

By further calculation

$AB^2 = 16 + 9 = 25$

So we get

$AB = \sqrt{25} = 5$ cm

Side of rhombus ABCD = 5 cm

Here

Perimeter of rhombus = 4 × side

Substituting he values

= 4 × 5

= 20 cm

32. if the sides of a rhombus are 5 cm each and one diagonal is 8 cm , calculate

(i) the length of the other diagonal , and

(ii) the area of the rhombus

Solution

It is given that

ABCD is a rhombus with side AB , BC , CD and AD

AB = BC = CD = AD = 5 cm

AC =8 cm and AO = 4 cm
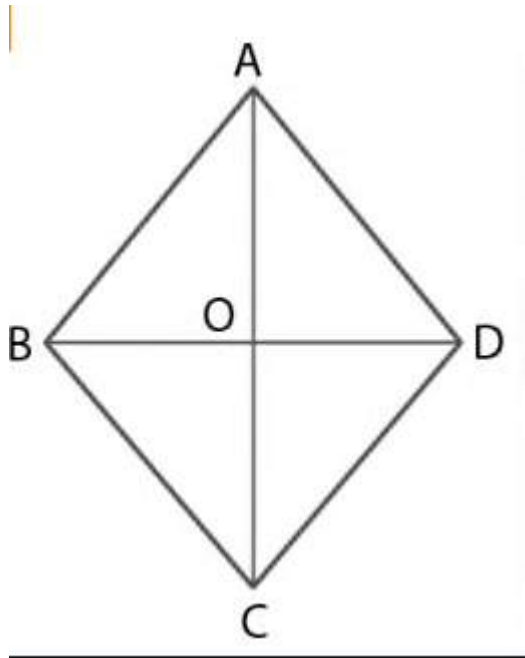
In triangle AOB

Using Pythagoras theorem

$AB^2 = AO^2 + BO^2$

Substituting the values

$5^2 = 4^2 + BO^2$

By further calculation

$25 = 16 + BO^2$

$BO^2 = 25 - 16 = 9$

So we get

$BO = \sqrt{9} = 3$ cm

$BD = 2 \times BO = 2 \times 3 = 6$ cm

Length of other diagonal = 6 cm

Area of rhombus $= \frac{1}{2} \times product\ of\ diagonals$

Substituting the values

$= \frac{1}{2} \times 8 \times 6$

By further calculation

$= 4 \times 6$

$= 24$ cm^2

33. (a) the diagram (i) given below is a trapezium . find the length of BC and the area of the trapezium . Assume AB = 5cm , AD = 4 cm , CD = 8 cm .

(b) the diagram (ii) given below is a trapezium . find

(i) AB

(ii) area of trapezium ABCD

(c) the cross – section of a canal is shown in figure (iii) given below. If the canal is 8 m wide at the top and 6 m wide at the bottom and the area of the cross – section is 16.8 m^2 , calculate its depth .

(i) B — 5 — A
4
C — 8 — D

(ii) 8m / 6m

(iii) D, 10, 8, C, 2, A, B
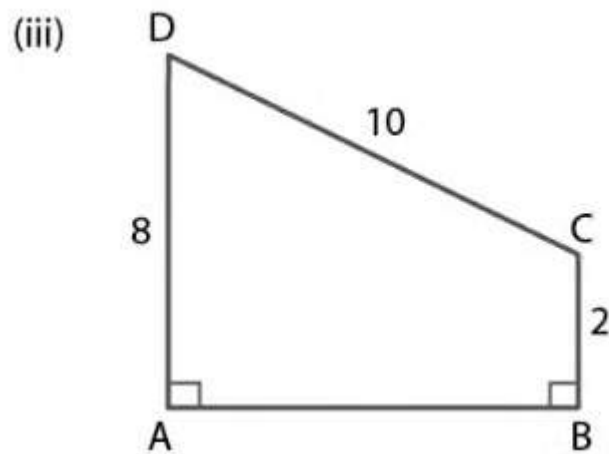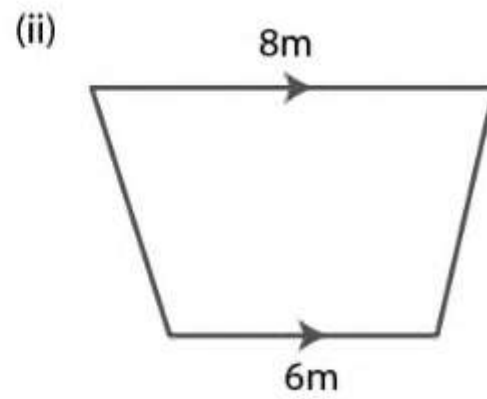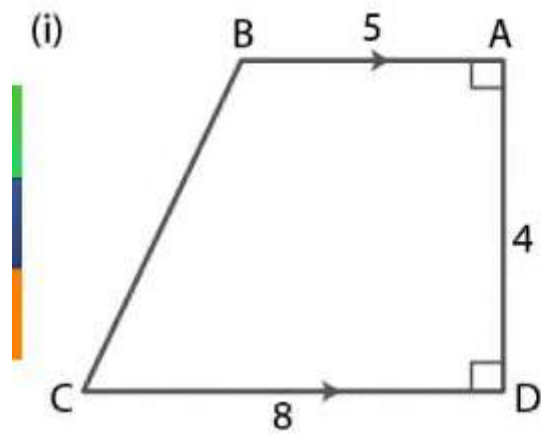
Solution

(a) it is given that

ABCD is a trapezium

AB = 5cm , AD = 4 cm and CD = 8 cm

Construct BN perpendicular to CD
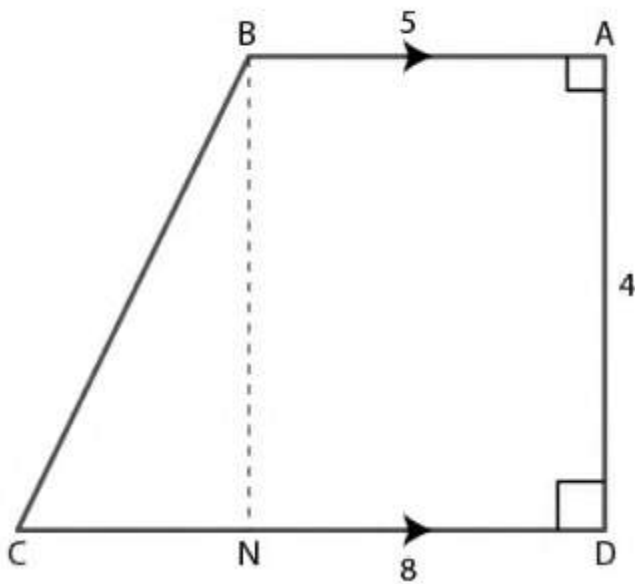
Here

BN = 4cm

CN = CD − ND

CN = CD − AO

CN = 8 − 5 = 3 cm

In triangle BCN

Using Pythagoras theorem

$BC^2 = BN^2 + CN^2$

Substituting the values

$BC^2 = 4^2 + 3^2$

By further calculation

$BC^2 = 16 + 9 = 25$

BC = $\sqrt{25}$ = 5 cm

Length of BC = 5 cm

Area of trapezium = $\frac{1}{2} \times$ *sum of parallel sides* $\times$ *height*

It can be written as

= $\frac{1}{2} \times (AB + CD) \times AD$

Substituting the values

= $\frac{1}{2} \times (5 + 8) \times 4$

By further calculation

= $\frac{1}{2} \times 13 \times 4$

So we get

= 13 × 2

= 26 cm²

Area of trapezium = 26cm²

(b) from the figure (ii)

AD = 8 units

BC = 2 units

CD = 10 units

Construct CN perpendicular to AD

AN = 2 units

(ii)

We know that

DN = AD − DN
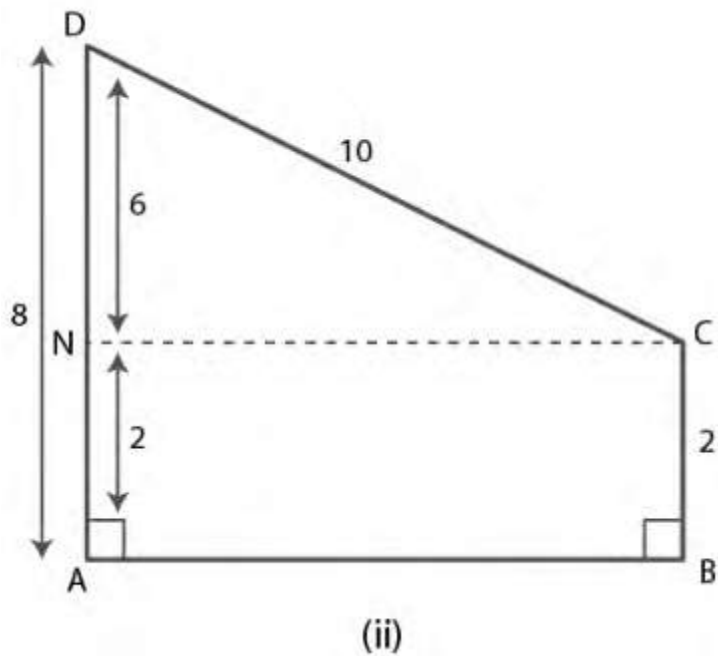
Substituting the values

= 8 − 6

= 2 units

In triangle CDN

Using Pythagoras theorem

$CD^2 = DN^2 + NC^2$

Substituting the values

$10^2 = 6^2 + NC^2$

By further calculation

$NC^2 = 10^2 - 6^2$

$NC^2 = 100 \ -36 \ = 64$

So we get

$NC = \sqrt{64} \ = 8$ units

From the figure NC = AB = 8 units

We know that

Area of trapezium $= \frac{1}{2} \times sum\ of\ parallel\ sides \ \times height$

It can be written as

$= \frac{1}{2} \times (BC + AD) \times AB$

Substituting the values

$= \frac{1}{2} \times (2 + 8\ ) \times 8$

By further calculation

$= \frac{1}{2} \times 10 \times 8$

$= 5 \ \times 8$

$= 40$ sq. Units

(c) consider ABCD as the cross section of canal in the shape of trapezium .

AB = 6m , DC = 8m

Take AL as the depth of canal

(iii)
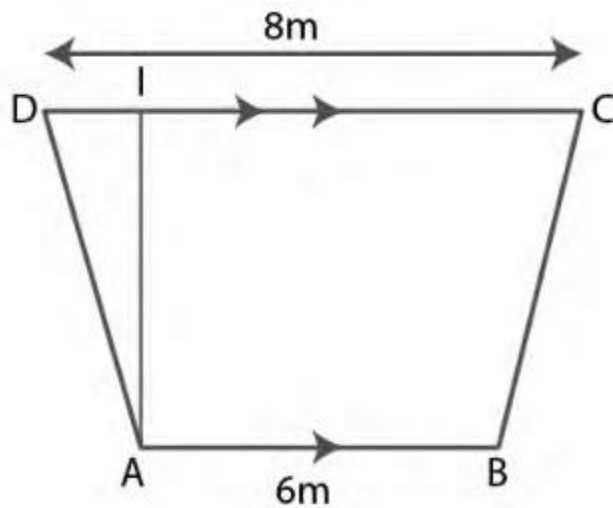
So the area of cross section $= 16.8 \text{ m}^2$

It can be written as

$\frac{1}{2} \times sum\ of\ parallel\ sides\ \times depth = 16.8$

$\frac{1}{2} \times (AB + DC) \times AL = 16.8$

Substituting the values

$\frac{1}{2} \times (6 + 8) \times AL = 16.8$

By further calculation

$\frac{1}{2} \times 14 \times AL = 16.8$

$AL = \frac{16.8 \times 2}{14}$

So we get

$AL = \frac{16.8 \times 1}{7}$

AL = 2.4 m

34. the distance between parallel sides of trapezium is 12 cm and the distance between mid – points of other sides is 18 cm . find the area of the trapezium .

Solution

Consider ABCD as a trapezium in which AB ‖ DC

Height CL = 12 cm

E and F are the mid points of sides AD and BC

EF = 18 cm
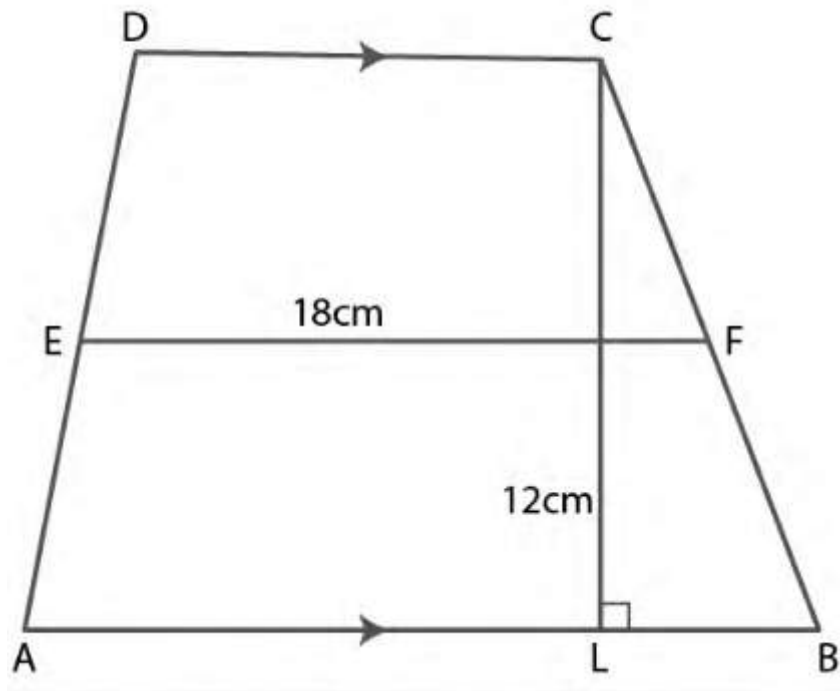
We know that

$EF = \dfrac{1}{2}(AB + DC) = 18cm$

Here

Area of trapezium ABCD $= \dfrac{1}{2}(AB + DC) \times$ height

Substituting he values

$= 18 \times 12$

$= 216 \text{ cm}^2$

35. the area of a trapezium is 540 cm^2 . if the ratio of parallel sides is 7 : 5 and the distance between them is 18 cm , find the length of parallel sides .

Solution

It is given hat

Area of trapezium $= 540 \text{ cm}^2$

Ratio of parallel sides $= 7 : 5$

Consider 7x cm as one parallel side

Other parallel side $= 5x$ cm

Distance between the parallel sides $=$ height $= 18cm$

We know that

Area of trapezium $= \dfrac{1}{2} \times sum\ of\ parallel\ sides \times height$

Substituting the values

$540 = \frac{1}{2} \times (7x + 5x) \times 18$

By further calculation

$540 = \frac{1}{2} \times 12x \times 18$

$540 = 6x \times 18$

$540 = 108x$

$X = \frac{540}{108} = 5$

Here

First parallel side = $7x = 7 \times 5 = 35$ cm

Second parallel side = $5x = 5 \times 5 = 25$ cm

36. the parallel sides of an isosceles trapezium are in the ratio 2 :3 . if its height is 4 cm and area is 60 cm² , find the perimeter .

Solution

It is given that

ABCD is an isosceles trapezium

BC = AD

Height = 4 cm

Consider CD = 2x and AB = 3x
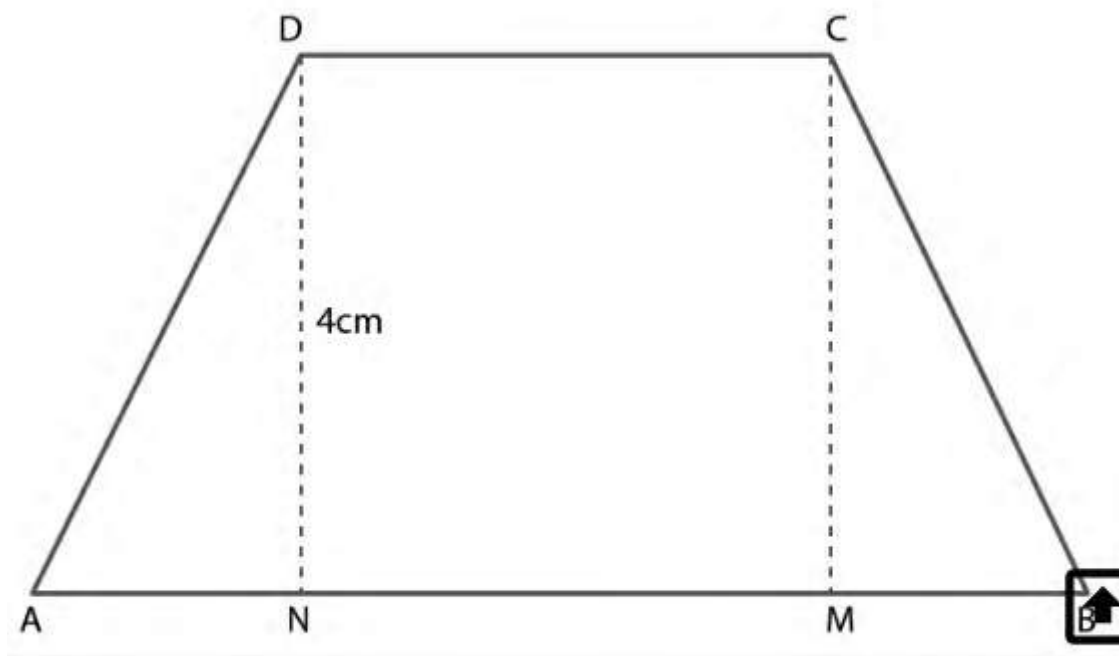
We know that

Area of trapezium $= \frac{1}{2}$ (*sum of parallel sides*) × *height*

Substituting the values

$60 = \frac{1}{2} \times (2x + 3x) \times 4$

By further calculation

$60 = \frac{1}{2} \times 5x \times 4$

$60 = 5x \times 2$

$60 = 10x$

So we get

$X = \dfrac{60}{10} = 6$

Here

CD = 2x = 2 × 6 = 12 cm

AB = 3x = 3 × 6 = 18 cm

AN = BM

We can write it as

AN = AB − BN

AN = AB − (MN + BM)

We know that

MN = CD

AN = AB − (CD + BM)

Similarly BM = AN

AN = AB − (CD + AN)

Substituting the values

AN = 18 − (12 + AN)

AN = 18 − 12 − AN

AN + AN = 6

2AN = 6

By division

$AN = \dfrac{6}{2} = 3$

In triangle AND

Using Pythagoras theorem

$AD^2 = DN^2 + AN^2$

Here DN = 4 cm

$AD^2 = 4^2 + 3^2$

By further calculation

$AD^2 = 16 + 9 = 25$

So we get

$AD = \sqrt{25} = 5cm$

Here AD = BC = 5cm

Perimeter of trapezium = AB + BC + CD + AD

Substituting the values

= 18 + 5 + 12 + 5

= 40 cm

37. the area of a parallelogram is 98 cm^2 . if one altitude is half the corresponding base , determine the base and the altitude of the parallelogram .

Solution

It is given that

Area of parallelogram = 98 cm^2

Condition – if one altitude is half the corresponding base

Take base = x cm

Corresponding altitude $= \frac{x}{2}$ cm

We know that

Area of parallelogram = base × altitude

Substituting the values

$98 = x \times \frac{x}{2}$

$98 = \frac{x^2}{2}$

By cross multiplication

$X^2 = 98 \times 2 = 196$

So we get

$X = \sqrt{196} = 14$ cm

Base = 14 cm

Here altitude $= \frac{14}{2} = 7$ cm

38 . the length of a rectangular garden is 12 m more than its breadth . the numerical value of its area is equal to 4 times the numerical value of its perimeter . find the dimension of the garden .

Solution

The dimension of rectangular garden are

Breadth = x m

Length = (x + 12) m

We know that

Area = l × b

 Substituting the values

Area = (x + 12) × x

Area = $(x^2 + 12x)$m²

Perimeter = 2(l + b)

Substituting the values

= 2[(x + 12) + x]

= 2[x + 12 + x]

= 2 [2x + 12]

= (4x + 24) m
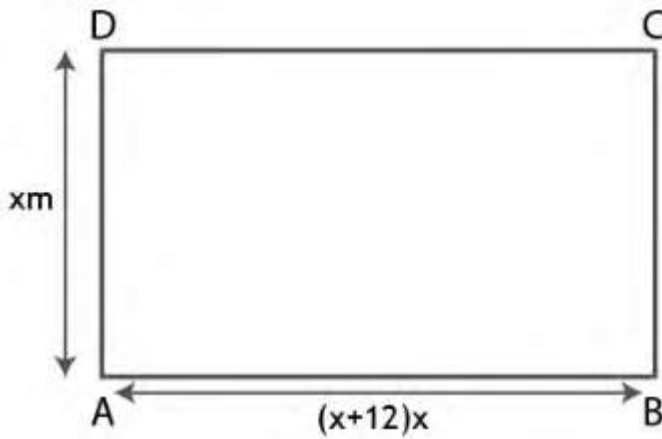
Base on the equation

Numerical value of area = 4 × numerical value of perimeter

$X^2 + 12x = 4 \times (4x + 24)$

By further calculation

$X^2 + 12x = 16x + 96$

$X^2 + 12x - 16x - 96 = 0$

$X^2 - 4x - 96 = 0$

Taking out the common terms

$X(x - 12) + 8(x - 12) = 0$

$(x + 8)(x - 12) = 0$

Here

$X + 8 = 0$ or $x - 12 = 0$

So we get

$X = -8$ (not possible) or $x = 12$

Breadth of rectangular garden = 12 m

Length of rectangular garden = 12 + 12 = 24 m

39. if the perimeter of a rectangular plot is 68 m and length of its diagonal is 26 m , find its area

Solution :

It is given that

Perimeter of a rectangular plot = 68 m

Length of its diagonal = 26 m

ABCD is a rectangular plot of length x m and breadth y m
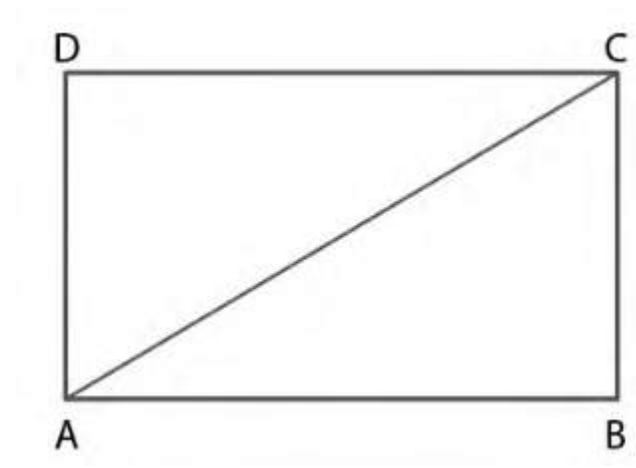
Perimeter = 2(length + breadth)

Substituting the values

68 = 2(x + y)

By further calculation

$\frac{68}{2} = x + y$

34 = x + y

So we get

X = 34 − y ...(1)

In triangle ABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting he values

$26^2 = x^2 + y^2$

$X^2 + y^2 = 676$

Now substituting the value of x in equation (1)

$(34 − y)^2 + y^2 = 676$

$1156 + y^2 − 68y + y^2 = 676$

By further calculation

$2y^2 − 68y + 1156 − 676 = 0$

$2y^2 − 68y − 480 = 0$

Taking 2 as common

$2(y^2 − 34y − 240) = 0$

$Y^2 − 34y − 240 = 0$

It can be written as

$Y^2 − 24y − 10y − 240 = 0$

Taking out the common terms

$Y(y -24) – 10(y – 24) = 0$

$(y -10) (y – 24) = 0$

Here

$Y – 10 = 0$ or $y – 24 = 0$

$Y = 10$ m or $y = 24$ m

Now substituting the value of y in equation (1)

$Y = 10$ m , $x = 34 – 10 = 24$ m

$Y = 24$ m , $x = 34 – 24 = 10$ m

Area in both cases $= xy$

$= 24 \times 10$ or 10×24

$= 240$ m²

Therefore , the area of the rectangular block is 240 m²

40 . A rectangle has twice the area of a square . the length of the rectangle is 12 cm greater and the width is 8 cm greater than 2 side of a square . find the perimeter of the square .

Solution

Consider

Side of a square = x cm

Length of rectangle $= (x + 12)$ cm

Breadth of rectangle $= (x + 8)$ cm

We know that

Area of square = side $\times$ side = x $\times$ x = x^2 cm^2

Area of rectangle $= 1 \times b$

Substituting the values

$= (x + 12)(x + 8)$ cm^2

Based on the question

Area of rectangle $= 2 \times$ area of square

Substituting the values

$(x + 12)(x + 8) = 2 \times x^2$

It can be written as

$X(x+8) + 12(x + 8) = 2x^2$

$X^2 + 8x + 12x + 96 = 2x^2$

$X^2 - 2x^2 + 8x + 12x + 96 = 0$

By further calculation

$-x^2 + 20x + 96 = 0$

$-(x^2 - 20x - 96) = 0$

$X^2 - 20x - 96 = 0$

We can write it as

$X^2 - 24x + 4x - 96 = 0$

X(x -24) + 4(x – 24) = 0

Here

X + 4 = 0 or x – 24 = 0

X = - 4 or x = 24 cm

Side of square = 24 cm

Perimeter of square = 4 × side

Substituting the values

= 4 × 24

= 96 cm

41. the perimeter of a square is 48 cm . the area of a rectangle is 4 cm² less than the area of the square . if the length of the rectangle is 4 cm greater than its breadth , find the perimeter of the rectangle .

Solution

It is given that

Perimeter of a square = 48 cm

$$\text{Sides} = \frac{perimeter}{4} = \frac{48}{4} = 12 \text{ cm}$$

We know that

Area = side² = 12² = 144 cm²

Area of rectangle $= 144 - 4 = 140$ cm^2

Take breadth of rectangle $= x$ cm

Length of rectangle $= x + 4$ cm

So the area $= (x + 4) \times x$ cm^2

Substituting the values

$(x + 4)x = 140$

By further calculation

$X^2 + 4x - 140 = 0$

$X^2 + 14x - 10x - 140 = 0$

Taking out the common terms

$X(x + 14) - 10(x + 14) = 0$

$(x + 14) (x - 10) = 0$

Here

$X + 14 = 0$ where $x = - 14$

$X - 10 = 0$ where $x = 10$

Breadth $= 10$ cm

Length $= 10 + 4 = 14$ cm

Perimeter $= 2(1 + b)$

Substituting the values

$= 2(14 + 10)$

$= 2 \times 24$

= 48 cm

42 . in the adjoin figure ,ABCD is a rectangle with sides AB = 10cm and BC = 8 cm . HAD and BFC are equilateral triangle , AEB and DCG are right angled isosceles triangles . find the area of the shaded region and the perimeter of the figure .

Solution
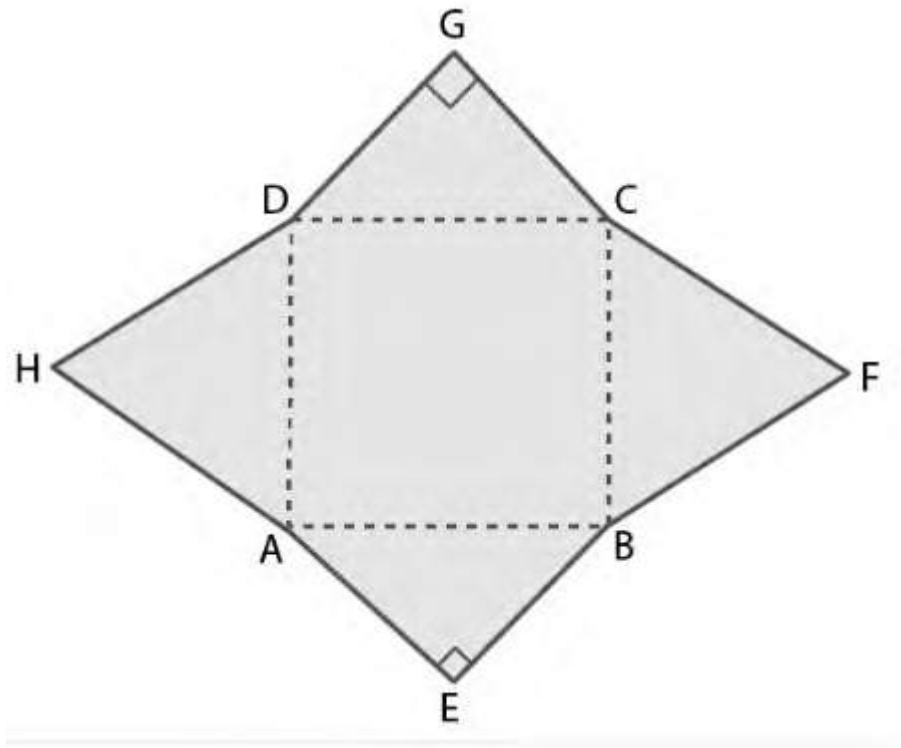
It is given that

ABCD is a rectangle with sides AB = 10 cm and BC = 8 cm

HAD and BFC are equilateral triangle with each side = 8 cm

AEB and DCG are right angled isosceles triangle with hypotenuse
= 10 cm

Consider AE = EB = x cm

In triangle ABE

$AE^2 + EB^2 = AB^2$

Substituting the values

$X^2 + x^2 = 10^2$

$2x^2 = 100$

By further calculation

$X^2 = \frac{100}{2} = 50$

$X = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$ cm

We know that

Area of triangle AEB = area of triangle GCD

It can be written as

$= \frac{1}{2} \times x \times x$

$= \frac{1}{2} x^2$ cm^2

Substituting the value of x

$= \frac{1}{2} \times 50$

$= 25$ cm^2

Area of triangle HAD = area of BFC

It can be written as

$$= \frac{\sqrt{3}}{4} \times 8^2$$

$$= \frac{\sqrt{3}}{4} \times 64$$

$$= 16\sqrt{3} \text{ cm}^2$$

Area of shaded portion = area of rectangle ABCD + 2 area of triangle AEB + 2 area of triangle BFC

Substituting the values

$$= (10 \times 8 + 2 \times 25 + 2 \times 16\sqrt{3})$$

By further calculation

$$= 80 + 50 + 32\sqrt{3}$$

So we get

$$= (130 + 32\sqrt{3}) \text{ cm}^2$$

Here

Perimeter of the figure = AE + EB + BF + FC + CD + GD + DH + HA

It can be written as

$$= 4AE + 4BF$$

Substituting the values

$$= 4 \times 5\sqrt{2} + 4 \times 8$$

So we get

$$= 20\sqrt{2} + 32$$

$$= (\, 32 + 20\sqrt{2} \,) \text{ cm}$$

43 . (a) find the area enclosed by figure (i) given below , where ABC is an equilateral triangle and DEFG is an isosceles trapezium . All measurements are in centimetres .

(b) find the area enclosed by the figure (ii) given below . AH measurements are in centimetres .

(c) in the figure (iii) given below , from a 24 cm × 24 cm piece of cardboard , a block in the shape of letter M is cut off . find the area of the cardboard left over , all measurements are in centimetres.
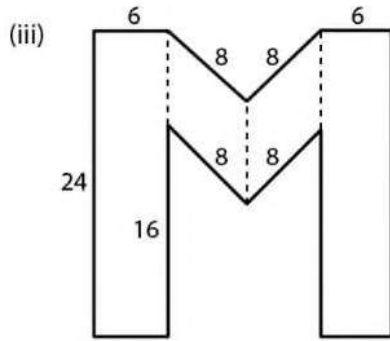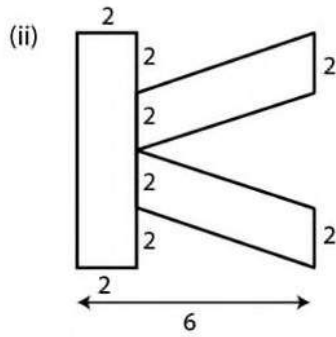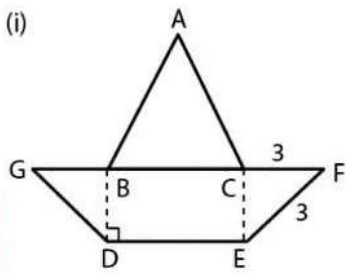
Solution :

(a) it is given that

ABC is an equilateral triangle and DEFG is an isosceles trapezium

EF = GD = 5 cm

DE = 6cm

GF = GB + BC + CF

Substituting the values

= 3 + 6 + 3

= 12 cm

AB = AC = BC = 6 cm

Join BD and CE

In right triangle CEF

$CE^2 = EF^2 - CF^2$

Substituting the values

$= 5^2 - 3^2$

$= 25 - 9$

$= 16$

So we get

CE $= \sqrt{16} = 4$ cm

Area of triangle ABC $= \dfrac{\sqrt{3}}{4} \times 6^2$

By further calculation

$= \dfrac{\sqrt{3}}{4} \times 36$

$= 9\sqrt{3}$ cm²

Area of trapezium DEFG $= \dfrac{1}{2}(DE + GF) \times CE$

Substituting the values

$= \dfrac{1}{2} \times (6 + 12) \times 4$

By further calculation

$= \dfrac{1}{2} \times 18 \times 4$

$= 36$ cm²

So the area of figure $= 9\sqrt{3} + 36$

Substituting the values

$= 9 \times 1.732 + 36$

= 15.59 + 36

= 51.59 cm²

(b) we know that

Length of rectangle = 2 + 2 + 2 + 2 = 8 cm

Width of rectangle = 2 cm

Area of rectangle = l × b

Substituting the values

= 8 × 2

= 16 cm²

Here

Area of each trap $= \frac{1}{2} \, (2 + 2) \times (6 - 2)$

By further calculation

$= \frac{1}{2} \times 4 \times 4$

= 8 cm²

So the total area = area of rectangle + area of 2 trapezium

Substituting the values

= 16 + 8 + 8

= 32 cm²

(c) we know that

Length of each rectangle = 24 cm

Width of each rectangle = 6 cm

Area of each rectangle = l × b

Substituting the values

= 24 × 6

= 144 cm^2

Base of each parallelogram = 8 cm

Height of each parallelogram = 6 cm

So the area of each parallelogram = 8 × 6 = 48 cm^2

Here

Area of the M – shaped figure = 2 × 144 + 2 × 48

So we get

= 288 + 96

= 384 cm^2

Area of the square cardboard = 24 × 24 = 576 cm^2

Area of the removing cardboard = 567 – 384 = 192cm^2

44. (a) The figure (i) given below shows the cross – section of the concrete structure with the measurements as given . calculate the area of cross – section .

(b) the figure (ii) given below show a field with the measurements given in metres . find the area of the field .

(c) Calculate the area of the pentagon ABCDE shown in fig (iii) below , given that AX = BX = 6cm , EY = CY = 4 cm , DE = DC = 5 cm , DX = 9 cm and DX is perpendicular to EC and AB .

(i)

6m

1.2m

.3m

2.4m

1.8m

(iii)

D

E

Y

C

A

X

B

(ii)

A

12

X

30

B

E

15

Y

25

Z

C

10

D
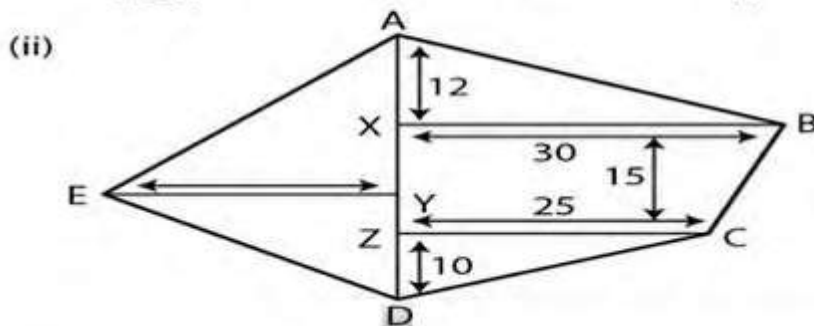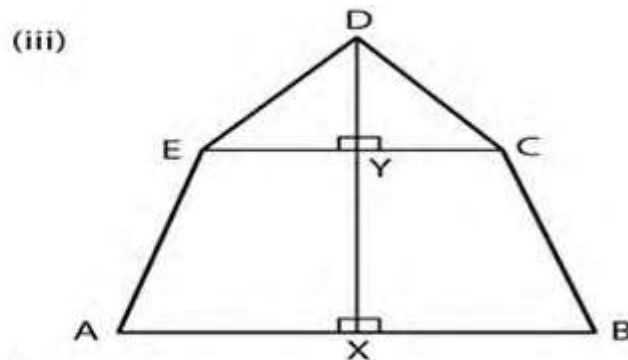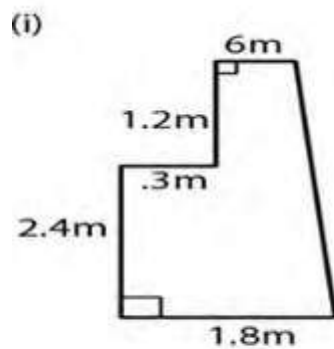
Solution :

(a) from the figure (i)

AB = 1.8 m , CD = 0.6 m , DE = 1.2 m

EF = 0.3 m ,AF = 2.4 m

Construct DE to meet AB in G

∠ FEG = ∠GAF = 90°

So , AGEF is a rectangle

(i)
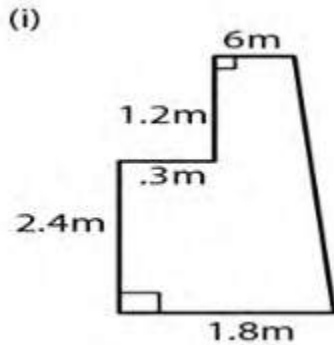
We know that

Area of given figure = area of rectangle AGEF + area of trapezium GBCD

It can be written as

$$= l \times b + \frac{1}{2} \, (\text{ sum of parallel sides} \times \text{height })$$

$$= AF \times AG + \frac{1}{2} \, (GB + CD) \times DG$$

Substituting the values

$$= 2.4 \times 0.3 + \frac{1}{2} \, [(AB - AG) + CD] \times (DE + EG)$$

Here AG = FE and using EG = AF

$$= 0.72 + \frac{1}{2} \, [(1.8 - 0.3) + 0.6] \times (1.2 + 2.4)$$

By further calculation

$= 0.72 + \frac{1}{2} [1.5 + 0.6] \times 3.6$

$= 0.72 + \frac{1}{2} \times 2.1 \times 3.6$

So we get

$= 0.72 + 2.1 \times 1.8$

$= 0.72 + 3.78$

$= 4.5 \text{ m}^2$

(b) it is given that

ABCD is a pentagonal field

AX = 12m , BX = 30 m , XZ = 15 m , CZ = 25m

DZ = 10 m , AD = 12 + 15 + 10 = 37 m , EY = 20 m
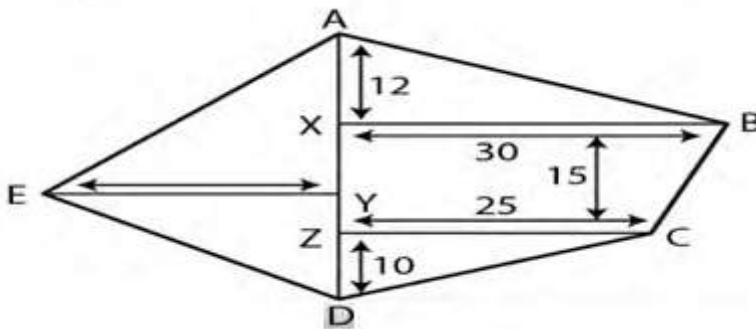
We know that

Area of pentagonal field ABCDE = area of triangle ABX + area of trapezium BCZX + area of triangle CDZ + area of triangle AED

It can be written as

$$= \frac{1}{2} \times base \times height + \frac{1}{2} (sum\ of\ parallel\ sides) \times height + \frac{1}{2} \times base \times height + \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times BX \times AX + \frac{1}{2}(BX + CZ) \times XZ + \frac{1}{2} \times CZ \times DZ + \frac{1}{2} \times AD \times EY$$

Substituting the values

$$= \frac{1}{2} \times 30 \times 12 + \frac{1}{2} (30 + 25) \times 15 + \frac{1}{2} \times 25 \times 10 + \frac{1}{2} \times 37 \times 20$$

By further calculation

$$= 15 \times 12 + 7.5 \times 55 + 25 \times 5 + 37 \times 10$$

So we get

$$= 180 + 412.5 + 125 + 370$$

$$= 1087.5 \text{ m}^2$$

(c) it is given that

ABCDE is a pentagon

AX = BX = 6 cm , EY = CY = 4 cm

DE = DC = 5 cm , DX = 9 cm

Construct DX perpendicular to EC and AB
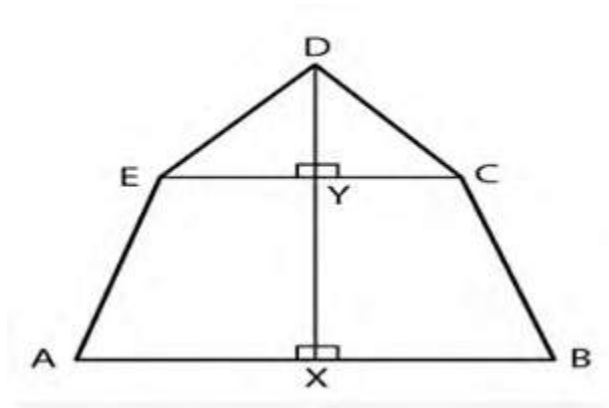
In triangle DEY

Using Pythagoras theorem

$$DE^2 = DY^2 + EY^2$$

Substituting the values

$$5^2 = DY^2 + 4^2$$

$$25 = DY^2 + 16$$

$$DY^2 = 25 - 16 = 9$$

So we get

$$DY = \sqrt{9} = 3cm$$

Here

Area of pentagonal field ABCDE = area of triangle DEY + area of triangle DCY + area of trapezium EYXA + area of trapezium CYXB

It can be written as

$$= \frac{1}{2} \times base \times height + \frac{1}{2} \times base \times height + \frac{1}{2} \times$$

$$(sum\ of\ parallel\ sides\ ) \times height + \frac{1}{2} \times$$

$$(sum\ of\ parallel\ sides) \times height$$

$$= \frac{1}{2} \times EY \times DY + \frac{1}{2} \times CY \times DY + \frac{1}{2} \times (EY + AX) \times XY + \frac{1}{2} \times$$

$$(CY + BX) \times XY$$

Substituting the values

$$= \frac{1}{2} \times 4 \times 3 + \frac{1}{2} \times 4 \times 3 + \frac{1}{2} \times (4 + 6) \times (DX - DY) +$$

$$\frac{1}{2}(4 + 6) \times (DX - DY)$$

By further calculation

$$= 2 \times 3 + 2 \times 2 + \frac{1}{2} \times 10 \times (9 - 3) + \frac{1}{2} \times 10 \times (9 - 3)$$

So we get

$$= 6 + 6 + 5 \times 6 + 5 \times 6$$

$$= 6 + 6 + 30 + 30$$

$$= 72 \text{ cm}^2$$

45. if the length and the breadth of a room are increased by 1 meter the area is increased by 21 square metres . if the length is increased by 1 meter and breadth is decreased by 1 meter the area is   decreased by 5 square meters. Find the perimeter of the room .

Solution :

Taken length of room = x m

Breadth of room = y m

Here

Area of room = l × b = xy m^2

We know that

Length is increased by 1 m then new length = (x +1) m

Breadth is increased by 1 m then new breadth = (y +1) m

So the new area = new length × new breadth

Substituting the values

=(x +1) (y +1) m^2

Based on the question

Xy   = (x + 1) (y +1) − 21

By further calculation

Xy = x(y + 1) + 1(y + 1) − 21

Xy = xy + x + y + 1 − 21

So we get

$0 = x + y + 1 - 21$

$0 = x + y - 20$

$X + y - 20 = 0$

$X + y = 20$ ....(1)

Similarly

Length is increased by 1 meter than new length = (x + 1) meter

Breadth is decreased by 1 meter than new breadth = (y -1) meter

So the new area = new length × new breadth

$= (x+ 1) (y - 1) \, m^2$

Based on the question

$Xy = (x + 1) (y -1) + 5$

By further calculation

$Xy = x(y -1) + 1(y -1) + 5$

$Xy = xy - x + y - 1 + 5$

$0 = -x + y + 4$

So we get

$X - y = 4$ ....(2)

By adding equation (1) and (2)

$2x = 24$

$X = \frac{24}{2} = 12m$

Now substituting the value of x in equation (1)

12 + y = 20

Y = 20 – 12 = 8 m

Here

Length of room = 12 m

Breadth of room = 8 m

So the perimeter = 2(1 + b)

Substituting the values

= 2(12 + 8)

= 2 × 20 = 40 m

46 . a triangle and a parallelogram have the same base and same area . if the sides of the triangle are 26cm , 28 cm and 30 cm and the parallelogram stands on the base 28 cm , find the height of the parallelogram .

Solution

It is given that

Sides of a triangle = 26 cm , 28 cm and 30 cm

We know that

$S = \frac{a+b+c}{2}$

Substituting the values

$$S = \frac{26+28+30}{2}$$

$$S = \frac{84}{2}$$

$$S = 42 \text{ cm}$$

Here

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

By further calculation

$$= \sqrt{42 \times 16 \times 14 \times 12}$$

$$= \sqrt{7 \times 6 \times 4 \times 4 \times 7 \times 2 \times 6 \times 2}$$

So we get

$$= 2 \times 4 \times 6 \times 7$$

$$= 336 \text{ cm}^2$$

We know that

Base = 28 cm

Height $= \dfrac{area}{base}$

Substituting the values

$$= \dfrac{336}{28}$$

$$= 12 \text{ cm}$$

47. A rectangle of area 105cm² has its length equal to x cm . write down its breadth in terms of x . given that its perimeter is 44 cm , write down an equation in x and solve it to determine the dimension of the rectangle

Solution :

It is given that

Area of rectangle = 105 cm²

Length of rectangle = x cm

We know that

Area = length × breadth

Substituting the values

105 = x × breadth

$X = \dfrac{105}{x}$ cm

Perimeter of rectangle = 44 cm

So we get

2(l + b) =44

$2 \left(x + \dfrac{105}{x} \right) = 44$

By further calculation

$\dfrac{x^2 + 105}{x} = 22$

By cross multiplication

$X^2 + 105 = 22x$

$X^2 - 22x + 105 = 0$

We can write it as

$X^2 - 15x - 7x + 105 = 0$

X(x -15) – 7(x -15) = 0

(x -7)(x -15) = 0

Here

X – 7 = 0 or x – 15 = 0

X = 7 cm or x = 15 cm

If x = 7 cm

Breadth = $\frac{105}{7}$ = 15 cm

If x = 15 cm

Breadth = $\frac{105}{15}$ = 7 cm

Therefore the required dimension of rectangle are 15cm and 7 cm .

48 . the perimeter of a rectangular plot is 180 m and its area is 1800 m^2 . take the length of plot as x m . use the perimeter 180 m to write the value of the breadth in terms of x . use the value of the length , breadth and the area to write an equation in x . solve the equation to calculate the length and breadth of the plot .

Solution :

It is given that

Perimeter of a rectangular plot = 180 m

Area of a rectangular plot = 1800 m^2

Take length of rectangle = x m

Here

Perimeter = 2(length + breadth)

Substituting the values

$180 = 2(x + \text{breadth})$

$\frac{180}{2} = x + breadth$

$x + \text{breadth} = 90$

$\text{breadth} = 90 - x \text{ m}$

we know that

$\text{area of rectangle} = l \times b$

substituting the values

$1800 = x \times (90 - x)$

$90x - x^2 = 1800$

It can be written as

$-(x^2 - 90x) = 1800$

$X^2 - 90x = -1800$

By further calculation

$X^2 - 90x + 1800 = 0$

$X^2 - 60x - 30x + 1800 = 0$

$X(x - 60) - 30(x - 60) = 0$

$(x - 30)(x - 60) = 0$

Here

$X - 30 = 0 \text{ or } x - 60 = 0$

$X = 30 \text{ or } x = 60$

If x = 30m

Breadth = 90 – 30 = 60 m

If x= 60 m

Breadth = 90 – 60 = 30 m

Therefore the required length of rectangle is 60 m and the breadth of rectangle is 30 m .

Exercise 16.3

1. find the length of the diameter of a circle whose circumference is 44 cm .

Solution

Consider radius of the circle = r cm

Circumference = $2\pi r$

We know that

$2\pi r = 44$

So we get

$\frac{2 \times 22}{7} r = 44$

By further calculation

$r = \frac{44 \times 7}{2 \times 22} = 7$ cm

diameter = 2r = 2 × 7 = 14 cm

2. find the radius and area of a circle if its circumference is 18 π cm.

Solution :

Consider the radius of the circle = r

Circumference = $2\pi r$

We know that

$2\pi r = 18\pi$

So we get

$2r = 18$

$r = \dfrac{18}{2} = 9$cm

here

area $= \pi r^2$

substituting the value of r

$= \pi \times 9 \times 9$

$= 81\pi$ cm^2

3. find the perimeter of a semicircular plate of radius 3. 85 cm

Solution :

It is given that

Radius of semicircle plate = 3. 85 cm
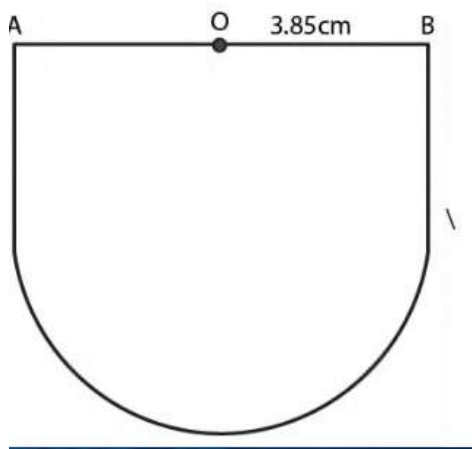
We know that

Length of semicircular plat $= \pi r$

Perimeter $= \pi r + 2r = r(\pi + 2)$

Substituting the values

$= 3.85 \left(\frac{22}{7} + 2 \right)$

By further calculation

$= 3.85 \times \frac{36}{7}$

$= 0.55 \times 36$

$= 19.8$ cm

4. find the radius and circumference of a circle whose area is 144π cm².

Solution

It is given that

Area of a circle $= 144\pi$ cm²

Consider radius $= r$

$\pi r^2 = 144 \pi$

$r^2 = 144$

so we get

$r = \sqrt{144} = 12$ cm

here

circumference $= 2\pi r$

so we get

$= 2 \times 12 \times \pi$

$= 24\pi$ cm

5. A sheet is 11 cm long and 2 cm wide . circular pieces 0.5 cm in diameter are cut from it to prepare discs. Calculate the number of discs that can be prepared .

Solution

It is given that

Length of sheet $= 11$ cm

Width of sheet $= 2$ cm

We have to cut the sheet to a square of side 0.5 cm

Here

Number of square $= \dfrac{11}{0.5} \times \dfrac{2}{0.5}$

Multiply and divide by 10

$= \dfrac{11 \times 10}{5} \times \dfrac{2 \times 10}{5}$

By further calculation

$= 22 \times 4$

$= 88$

Hence the number of discs will be equal to number of square cut out

= 88.

6. if the area of a semicircular region is 77 cm^2 , find its perimeter .

Solution

It is given that

Area of a semicircle region = 77 cm^2

Consider r as the radius of the region
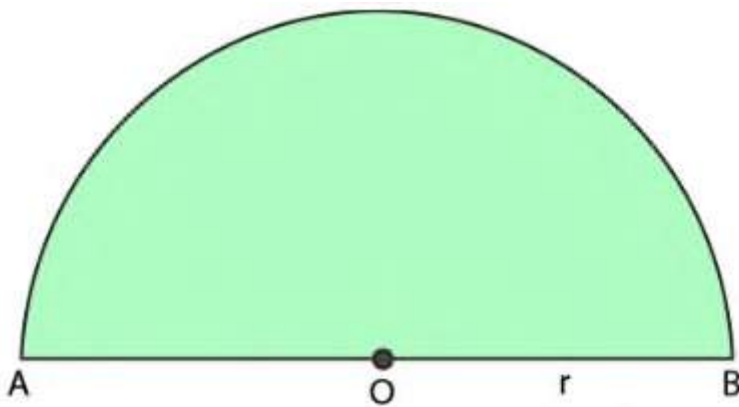
We know that

$$\text{Area} = \frac{1}{2} \pi r^2$$

$$\frac{1}{2} \pi r^2 = 77$$

By further calculation

$$\frac{1}{2} \times \frac{22}{7} r^2 = 77$$

So we get

$$r^2 = \frac{77 \times 2 \times 7}{22} = 49 = 7^2$$

r = 7 cm

here

perimeter of the region $= \pi r + 2r$

by further calculation

$$= \frac{22}{7} \times 7 + 2 \times 7$$

So we get

= 22 + 14

= 36 cm

7. (a) in the figure (i) given below AC and BD are two perpendicular diameter of a circle ABCD . given that the area of shaded portion is 308 cm^2 , calculate

(i) the length of AC and

(ii) the circumference of the circle

(b) in the figure (ii) given below AC and BD are two perpendicular diameters of a circle with centre O. If AC = 16

cm, calculate the area and perimeter of the shaded part . (take π = 3.14)

A

B   D   B   D
      O   O

C   C
(i)   (ii)
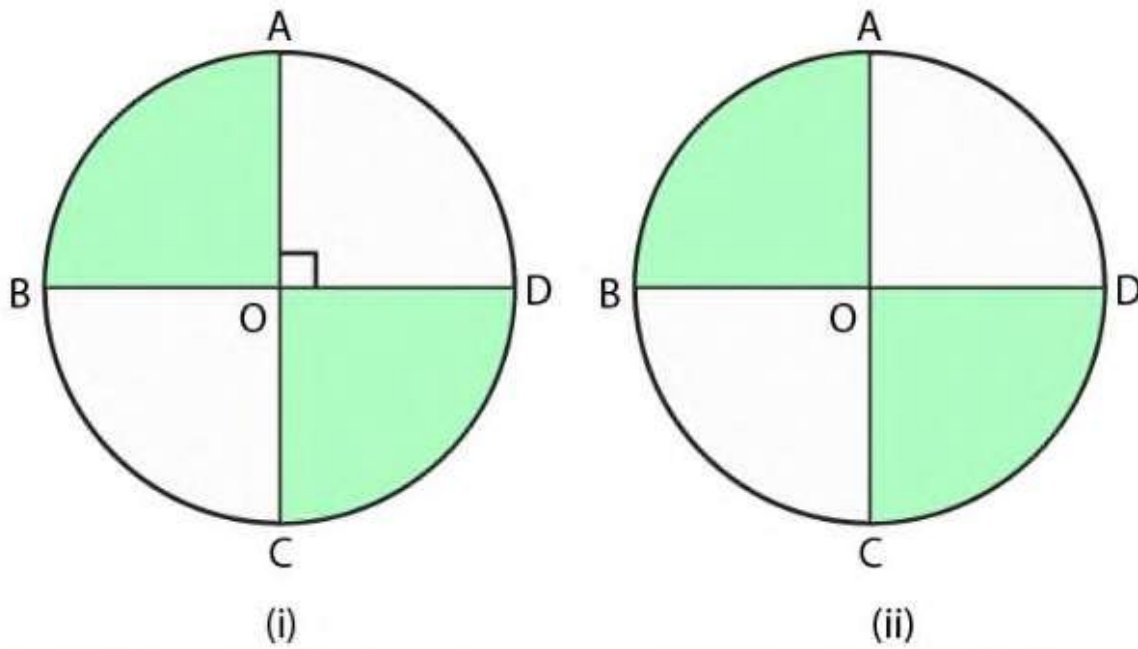
Solution

(a) it is given that

Area of shaded portion = area of semicircle = 308 cm^2

Consider r as the radius of circle

$\frac{1}{2} \pi r^2 = 308$

By further calculation

$\frac{1}{2} \times \frac{22}{7} r^2 = 308$

$r^2 = \frac{308 \times 2 \times 7}{22}$

$r^2 = 196 = 14^2$

so r =14 cm

(i) AC = 2r = 2 × 14 = 28cm

(ii) we know that

Circumference of the circle = 2 πr

Substituting the values

$= 28 \times \dfrac{22}{7}$

So we get

$= 4 \times 22$

$= 88$ cm

(b) we know that

Diameter of circle = 16 cm

Radius of circle $= \dfrac{16}{2} = 8$ cm

Here

Area of shaded part = 2 × area of one quadrant

So we get

$= \dfrac{1}{2} \pi r^2$

Substituting the values

$= \dfrac{1}{2} \times 3.14 \times 8 \times 8$

$= 100.48$ cm^2

We know that

Perimeter of shaded part $= \frac{1}{2}$ of circumference $+ 4r$

It can be written as

$= \frac{1}{2} \times 2\pi r + 4r$

$= \pi r + 4r$

Taking r as common

$= r\,(\pi + 4)$

Substituting the values

$= 8(3.14 + 4)$

$= 8 \times 7.14$

$= 57.12$ cm

8. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm . given that the bucket ascends in 1 minute 28 seconds with a uniform speed of 1.1 m/sec, calculate the number of complete revolutions the number of complete revolutions the wheel makes in raising the bucket .

Solution

It is given that

Diameter of wheel = 77 cm

So radius of wheel $= \dfrac{77}{2}$ cm

We know that

Circumference of wheel $= 2\,\pi r$

Substituting the values

$= 2 \ \times \dfrac{22}{7} \ \times \dfrac{77}{2}$

$= 242$ cm

9 . the wheel of a cart is making 5 revolution per second . if The diameter of the wheel is 84 cm , find its speed in km/hr. Give your answer correct to the nearest km .

Solution

It is given that

Diameter of wheel $= 84$ cm

Radius of wheel $= \dfrac{84}{2} = 42$cm

Here

Circumference of wheel $= 2\pi r$

Substituting the values

$= 2 \times \dfrac{22}{7} \ \times 42$

$= 264$ cm

So the distance covered in 5 reductions = 264 × 5 = 1320 cm

Time = 1 second

We know that

Speed of wheel = $\dfrac{1320}{1} \times \dfrac{60 \times 60}{100 \times 1000}$

= 47 .25 km/hr

= 48 km/hr

10. the circumference of a circle is 123.2 cm . calculate :

(i) the radius of the circle in cm .

(ii) the area of the circle in cm² , correct to the nearest cm²

(iii) the effect on the area of the circle if the radius is doubled .

Solution

It is given that

Circumference of a circle = 123.2 cm

Consider radius = r cm

(i) we know that

$2\,\pi r = 123.2$

By further calculation

$\dfrac{2 \times 22}{7}\,r = \dfrac{1232}{10}$

So we get

$$r = \frac{(1232 \times 7)}{10 \times 2 \times 22}$$

r = 19.6 cm

therefore , radius of the circle is 19.6 cm

(ii) here

Area of the circle = πr^2

Substituting the values

$= \frac{22}{7} \times 19.6 \times 19.6$

= 1207.36

= 1207 cm²

(iii) we know that

If radius is doubled = 19.6 × 2 = 39.2 cm

So the area of circle = πr^2

Substituting the values

$= \frac{22}{7} \times 39.2 \times 39.2$

= 4829.44 cm²

Effect on area $= \frac{4829.44}{1207} = 4$ times

11. (a) in the figure (i) given below , the area enclosed between the concentric circles is 770 cm² . given that the radius of the outer circle is 21 cm , calculate the radius of the inner circle .

(b) in the figure (ii) given below , the area enclosed between the circumference of two concentric circle is 346.5 cm ^{2} . the circumference of the inner circle is 88 cm . calculate the radius of the outer circle .

(i)   (ii)
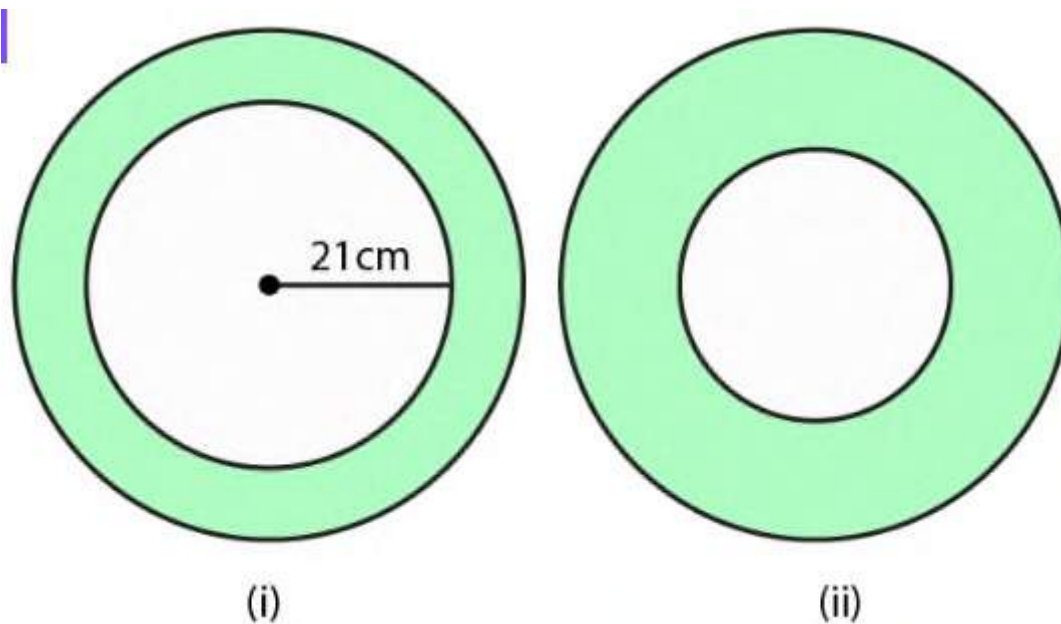
Solution

(a) it is given that

Radius of outer circle (r) = 21 cm

Consider r cm as the radius of inner circle

We know that

Area of the ring $= \pi(R^2 - r^2)$

Substituting the values

$= \frac{22}{7} (21^2 - r^2)$

$= \frac{22}{7} (441 - r^2)$

Area of the ring $= 770 \ cm^2$

So we get

$\frac{22}{7} (441 - r^2) = 770$

By further calculation

$441 - r^2 = \frac{770 \times 7}{22} = 245$

$r^2 = 441 - 245 = 196$

$r = \sqrt{196} = 14$

hence the radius of inner circle is 14 cm .

(b) it is given that

Area of ring $= 346.5 \ cm^2$

Circumference of inner circle $= 88$ cm

We know that

Radius $= \frac{88 \times 7}{2 \times 22} = 14 cm$

Consider R cm as the radius of outer circle

Area of ring $= \pi (R^2 - r^2)$

Substituting the values

$= \dfrac{22}{7}(R^2 - 14^2)$

$= \dfrac{22}{7}(R^2 - 196)$ cm^2

Area of ring $= 346.5$ cm^2

By equating we get

$\dfrac{22}{7}(R^2 - 196) = 346.5$

By further calculation

$R^2 - 196 = \dfrac{346.5 \times 7}{22} = 110.25$

$R^2 = 110.25 + 196 = 306.25$

So we get

$R = \sqrt{306.25} = 17.5$

Hence , the radius of outer circle is 17. 5 cm

12. A road 3.5 m wide surrounds a circular plot whose circumference is 44m . find the cost of paving the road at Rs 50 per m²

Solution

It is given that

Circumference of circular plot = 44m

Radius of circular plot $= \frac{44 \times 7}{22 \times 2} = 7\text{m}$

Width of the road $= 3.5$ m

So the radius of outer circle $= 7 + 3.5 = 10.5$ m

We know that

Area of road $= \pi(R^2 - r^2)$

Substituting the values

$= \frac{22}{7}(10.5^2 - 7^2)$

We can write it as

$= \frac{22}{7}(10.5 + 7)(10 - 7)$

$= \frac{22}{7} \times 17.5 \times 3.5$

$= 192.5$ m^2

Here

Rate of paving the road $=$ Rs 50 per m^2

Total cost $= 192.5 \times 50 =$ Rs 9625

13. the sum of diameter of two circle is 14 cm and the difference of their circumference is 8 cm . find the circumference of two circles.

Solution

It is given that

Sum of the diameters of two circle = 14 cm

Consider R and r as the radii of two circles

2R + 2r = 14

Dividing by 2

R + r = 7 .....(1)

We know that

Difference of their circumference = 8 cm

$2\pi R - 2\pi r = 8$

Taking out the common terms

$2\pi(R - r) = 8$

$\dfrac{2 \times 22}{7(R-r)} = 8$

By further calculation

$R - r = \dfrac{8 \times 7}{2 \times 22} = \dfrac{14}{11} .....(2)$

By adding both the equations

$2R = 7 + \dfrac{14}{11}$

By taking LCM

$2R = \dfrac{77+14}{11} = \dfrac{91}{11}$

By further calculation

$R = \dfrac{91}{11 \times 2} = \dfrac{91}{22}$

From equation (1)

R + r = 7

Substituting the value of R

$\frac{91}{22} + r = 7$

$r = 7 - \frac{91}{22}$

taking LCM

$r = \frac{154 - 91}{22} = \frac{63}{22}$

we know that

circumference of first circle = $2\pi r$

substituting the values

$= 2 \times \frac{22}{7} \times \frac{91}{22}$

$= 26$ cm

Circumference of second circle = $2\pi R$

Substituting the values

$= 2 \times \frac{22}{7} \times \frac{63}{22}$

$= 18$ cm

14. find the circumference of the circle whose area is equal to the sum of the areas of three circles with radius 2 cm , 3cm and 6 cm

Solution :

It is given that

Radius of first circle = 2cm

Area of first circle = πr^2

$= \pi (2)^2$

$= 4\pi$ cm^2

Radius of second circle = 3cm

Area of first circle = πr^2

$$= \pi (3)^2$$

$$= 9 \pi \text{ cm}^2$$

Radius of second circle = 6cm

Area of first circle $= \pi r^2$

$= \pi (6)^2$

$= 36 \pi$ cm^2

So the total area of the three circles $= 4\pi + 9\pi + 36\pi = 49\pi$ cm^2

Area of the given circle $= 49\pi$ cm^2

We know that

Radius $= \sqrt{\dfrac{49\pi}{\pi}} = \sqrt{49} = 7$cm

Circumference $= 2\pi r = 2 \times \dfrac{22}{7} \times 7 = 44$ cm

15. A copper wire when bent in the form of a square encloses an area of 121 cm^2 . if the same wire is bent into the form of a circle , find the area of the circle .

Solution

It is given that

Area of square = 121cm^2

So side = $\sqrt{121} = 11\text{cm}$

Perimeter = 4a = 4 × 11 = 44 cm

Circumference of circle = 44 cm

Radius of circle = $\frac{44 \times 7}{2 \times 22} = 7\text{cm}$

We know that

Area of the circle = πr^2

Substituting the values

$= \frac{22}{7} (7)^2$

So we get

$= \frac{22}{7} \times 7 \times 7$

$= 154 \text{ cm}^2$

16. a copper wire when bent into an equilateral triangle has area 121 $\sqrt{3}$ cm². If the same wire is bent into the form of circle , find the area enclosed by the wire .

Solution :

It is given that

Area of equilateral triangle = $121\sqrt{3}$ cm²

Consider a as the side of triangle

Area = $\frac{\sqrt{3}}{4}a^2$

It can be written as

$\frac{\sqrt{3}}{4}a^2 = 121\sqrt{3}$

By further calculation

$a^2 = \frac{121 \times \sqrt{3} \times 4}{\sqrt{3}}$

$a^2 = 484$

so we get

$a = \sqrt{484} = 22$ cm

here

length of the wire = 66 cm

radius of the circle = $\frac{66}{2}\pi$

by further calculation

$$= \frac{66 \times 7}{2 \times 22}$$

$$= \frac{21}{2} \text{ cm}$$

We know that
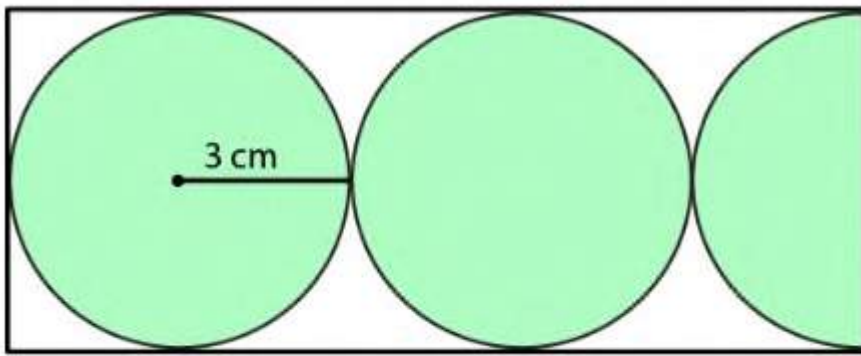
Area of the circle $= \pi r^2$

By further calculation

$$= \frac{22}{7} \times \left(\frac{21}{2}\right)^2$$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$

So we get

$$= \frac{693}{2}$$

$$= 346.5 \text{ cm}^2$$

17. (a) find the circumference of the circle whose area is 16 times the area of the circle with diameter 7 cm

(b) in the given figure, find the area of the unshaded portion within the rectangle .(take $\pi = 3.14$)

Solution :

(a) it is given that

Diameter of the circle = 7 cm

Radius of circle = $\frac{7}{2}$ cm

We know that

Area of the circle = πr^2

Substituting the values

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2}$ cm^2

Here

Area of bigger circle = $\frac{77}{2} \times 16 = 616$ cm^2

Consider r as the radius

$\pi r^2 = 616$

We can write it as

$\frac{22}{7} r^2 = 616$

By further calculation

$r^2 = \frac{616 \times 7}{22}$

$r^2 = 196 \text{ cm}^2$

so we get

$r = \sqrt{196} = 14 \text{ cm}$

circumference $= 2\pi r$

substituting the values

$= 2 \times \frac{22}{7} \times 14$

$= 88 \text{ cm}$

(b) it is given that

Radius of each circle $= 3\text{cm}$

Diameter of each circle $= 2 \times 3 = 6 \text{ cm}$

Here

Length of rectangle (l) $= 6 + 6 + 3 = 15 \text{ cm}$

Breadth of rectangle (b) $= 6 \text{ cm}$

So the area of rectangle $= l \times b$

 Substituting the values

$= 15 \times 6$

$= 90 \text{ cm}^2$

We know that

Area of $2\frac{1}{2}$ circle $= \frac{5}{2}\pi r^2$

Substituting the values

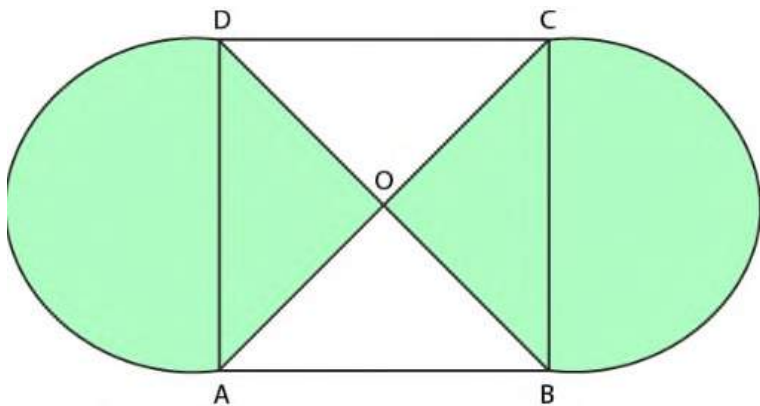$= \frac{5}{2} \times 3.14 \times 3 \times 3$

By further calculation

$= 5 \times 1.57 \times 9$

$= 70.65 \text{ cm}^2$

So the area of unshaded portion $= 90 - 70.65$

$= 19.35 \text{ cm}^2$

18 . in the adjoining figure, ABCD is a square of side 21 cm . AC and BD are two diagonals of the square . two semi circles are drawn with AD and BC as diameters . Find the area of the shaded region . take $\pi = \frac{22}{7}$

Solution :

It is given that

Side of square = 21 cm

So the area of square = side^2 = 21^2 = 441 cm^2

We know that

$\angle AOD + \angle COD + \angle AOB + \angle BOC = 441$

Substituting the values

x + x+ x+ x = 441

4x = 441

So we get

X = $\frac{441}{4}$ = 110.25 cm^2

Based on the question

We should find the area of shaded portion in square ABCD which is

$\angle$AOD and $\angle$BOC

$\angle$AOD + $\angle$BOC = 110.25 + 110.25 = 220.5 cm^2

Here

Area of two semicircle = πr^2

Substituting the values

= $\frac{22}{7}$ × 10.5 × 10.5

= 346.50 cm^2

So the area of shaded portion = 220.5 + 346.5 = 567 cm^2

19. (a) in the figure (i) given below , ABCD is a square of side 14cm and APD and BPC are semicircles. Find the area and the perimeter of the shaded region .
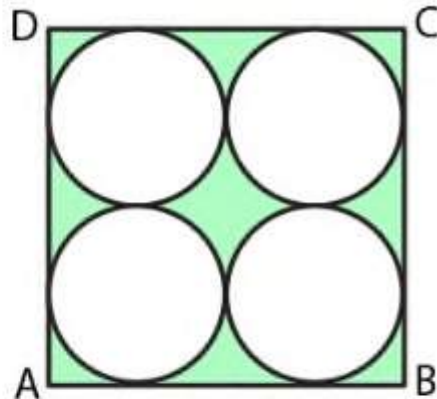
(b) in the figure (ii) given below , ABCD is a square of side 14 cm. Find the area of the shaded region .

(c) in the figure (iii) given below , the diameter of the semicircle is equal to 14 cm . calculate the area of the shaded region . take $\pi = \dfrac{22}{7}$
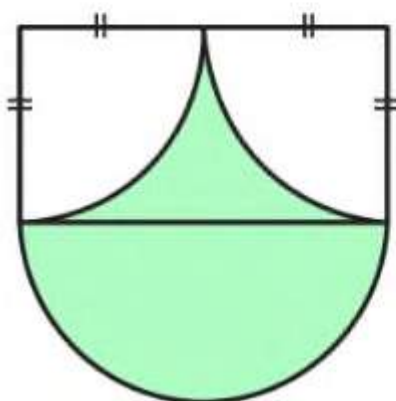
(i)

(ii)

(iii)
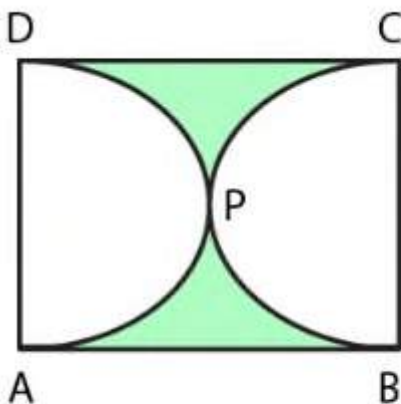
Solution

(a) it is given that

ABCD is a square of each side (a) = 14 cm

APD and BPC are semi circle with diameter = 14 cm

Radius of each semi circle (a) = $\frac{14}{2}$ = 7 cm

(i) we know that

Area of square = $a^2 = 14^2 = 196$ cm^2

Area of two semicircle = $2 \times \frac{1}{2} \pi r^2$

$= \pi r^2$

Substituting the values

$= \frac{22}{7} \times 7 \times 7$

$= 154$ cm^2

So the area of shaded portion = $196 - 154 = 42$ cm^2

(ii) here

Length of arcs of two semicircle = $2 \pi r$

Substituting the values
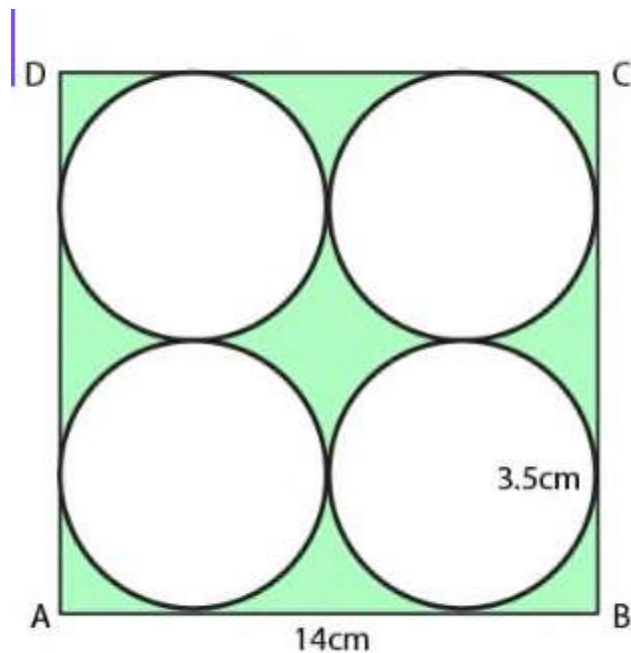
$= 2 \times \frac{22}{7} \times 7$

$= 44$ cm

So the perimeter of shaded portion = 44 + 14 + 14 = 72 cm

(b) it is given that

ABCD is a square whose each side (a) = 14 cm

4 circle are drawn which touch each other and the sides of squares

Radius of each circle (r) $= \dfrac{7}{2} = 3.5$cm

(i) we know that

Area of square ABCD = $a^2 = 14^2 = 196$ cm²

Area of 4 circle = $4 \times \pi r^2$

Substituting the values

$= 4 \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}$

$= 154$ cm²

So the area of shaded portion = 196 – 154 = 42 cm²

(ii) here

Perimeter of 4 circles = $4 \times 2\pi r$

Substituting the values

$= 4 \times 2 \times \frac{22}{7} \times \frac{7}{2}$

= 88 cm

So the perimeter of shaded portion = perimeter of 4 circles + perimeter of square

Substituting the values
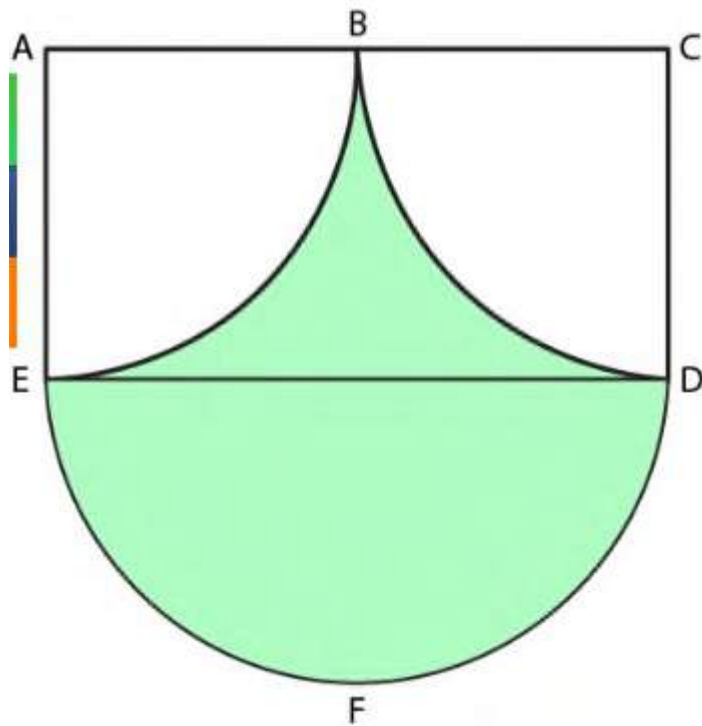
$= 88 + 4 \times 14$

So we get

= 88 + 56

= 144 cm

(c) we know that

Area of rectangle ACDE = ED × AE

Substituting the values

$= 14 \times 7$

= 98 cm²

Here

Area of semicircle DEF = $\dfrac{\pi r^2}{2}$

Substituting the values

$= \dfrac{22 \times 7 \times 7}{7 \times 2}$

$= 77 \text{ cm}^2$

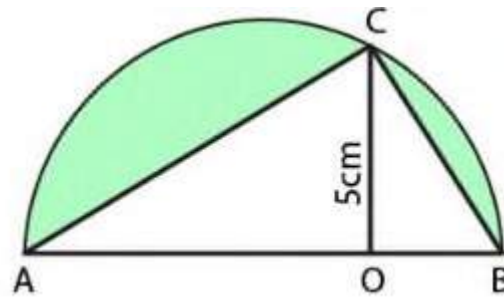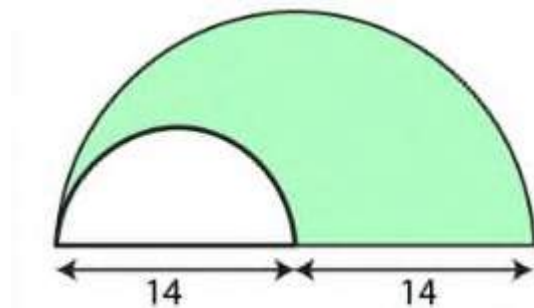So the area of shaded region $= 77 + (98 - 2 \times \dfrac{1}{4} \times \dfrac{22}{7} \times 7 \times 7)$

$= 77 + 21$

$= 98 \text{cm}^2$

20 . (a) find the area and the perimeter of the shaded region in figure (i) given below . the dimension are in centimetres.

(b) in the figure (ii) given below , area of ∆ABC = 35cm². Find the area of the shaded region.

Solution

(a) it is given that

There are 2 semicircle where the smaller is inside the larger

Radius of larger semicircle (R) = 14 cm

Radius of smaller circle (r) = $\dfrac{14}{2}$ = 7cm

(i) we know that

Area of shaded portion = area of large semicircle – area of smaller circle

$$= \frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2$$

We can write it as

$$= \frac{1}{2} \pi (R^2 - r^2)$$

Substituting the values

$$= \frac{1}{2} \times \frac{22}{7} (14^2 - 7^2)$$

By further calculation

$$= \frac{11}{7} (14 + 7)(14 - 7)$$

So we get

$$= \frac{11}{7} \times 21 \times 7$$

$$= 231 \text{ cm}^2$$

(ii) here

Perimeter of shaded portion = circumference of larger semicircle + circumference of smaller semicircle + radius of larger semicircle

$$= \pi R + \pi r + R$$

Substituting the values

$$= \frac{22}{7} \times 14 + \frac{22}{7} \times 7 + 14$$

By further calculation

$$= 44 + 22 + 14$$

$$= 80 \text{ cm}$$

(b) we know that

Area of Δ ABC formed in a semicircle = 3.5 cm

Altitude CD = 5 cm

So the base AB = $\frac{area \times 2}{altitude}$

Substituting the values

$= \frac{35 \times 2}{5}$

= 14 cm

Here

Diameter of semicircle = 14 cm

Radius of semicircle (R) = $\frac{14}{2}$ = 7cm

So the area of semicircle = $\frac{1}{2}\pi R^2$

Substituting the values

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

= 77 cm^2

Area of shaded portion = area of semicircle – area of triangle

$= 77 - 35$

$= 42$ cm^2

21. (a) in the figure (i) given below , AOBC is a quadrant of a circle of radius 10m . Calculate the area of the shaded portion. Take $\pi = 3.14$ and given your answer correct to two significant figures.

(b) In the figure (ii) given below OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. calculate the area of the shaded portion.

(i)   (ii)
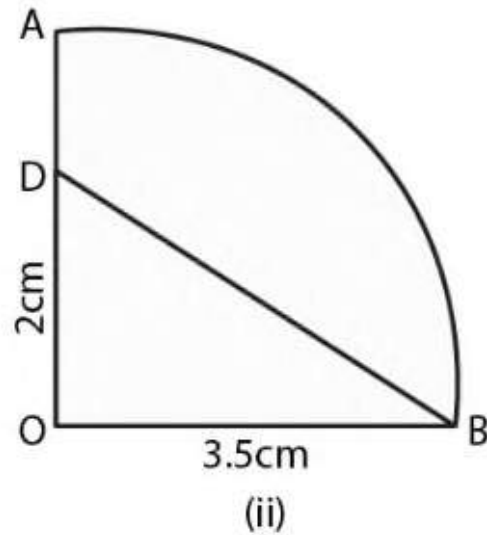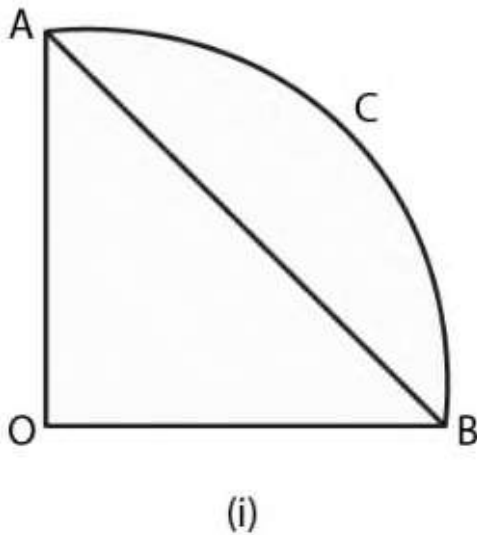
Solution

(a) In the figure

Shaded portion = quadrant - Δ AOB

Radius of the quadrant = 10 m

Here

Area of quadrant $= \frac{1}{2} \pi r^2$

Substituting the values

$= \frac{1}{4} \times 3.14 \times 10 \times 10$

By further calculation

$= \frac{3.14 \times 100}{4}$

$= \frac{314}{4}$

$= 78.5 \ m^2$

We know that

Area of Δ AOB $= \frac{1}{2} \times AO \times OB$

Substituting the values

$= \frac{1}{2} \times 10 \times 10$

$= 50 m^2$

So the area of shaded portion $= 78.5 - 50 = 28.5 m^2$

(b) in the figure

Radius of quadrant $= 3.5$ cm

(i) We know that

Area of quadrant $= \frac{1}{4} \pi r^2$

Substituting the values

$$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 9.625 \text{ cm}^2$$

(ii) Here

Area of Δ AOD $= \frac{1}{2} \times AO \times OD$

Substituting the values

$$= \frac{1}{2} \times 3.5 \times 2$$

$$= 3.5 \text{ cm}^2$$

So the area of shaded portion = area of quadrant – area of Δ AOD

Substituting the values

$$= 9.625 - 3.6$$

$$= 6.125 \text{ cm}^2$$

22. A student takes a rectangular piece of paper 30cm long and 21 cm wide. Find the area of the biggest circle that can be cut out from the paper. Also find the area of the paper left after cutting out the circle. (Take $\pi = \frac{22}{7}$)

Solution

It is given that
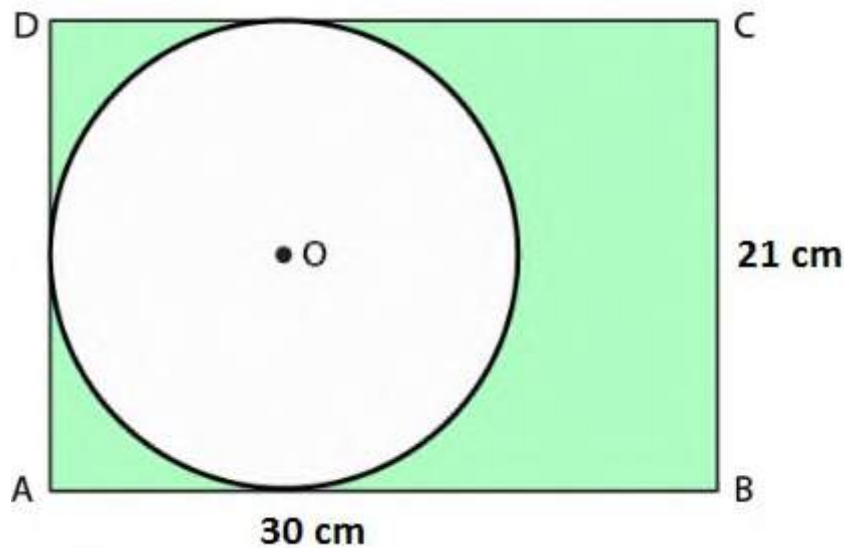
Length of rectangle = 30 cm

Width of rectangle = 21 cm

We know that

Area of rectangle = $l \times b$

$= 30 \times 21$

$= 630$ cm^2

So the radius of the biggest circle $= \dfrac{22}{2}$ cm

Here

Area of the circle = πr^2

Substituting the values

$= \dfrac{22}{7} \times \dfrac{21}{2} \times \dfrac{21}{2}$

So we get

$$= \frac{693}{2}$$

$$= 346.5 \text{ cm}^2$$

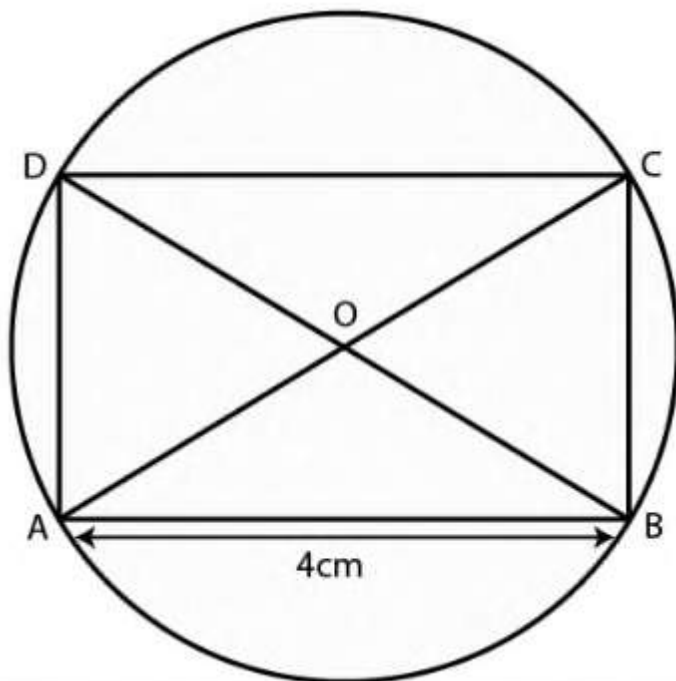So the area of remaining part = 630 – 346.5 = 283.5 cm²

23. A rectangle with one side 4 cm is inscribed in a circle of radius 2.5 cm. find the area of the rectangle.

Solution:

Consider ABCD as a rectangle

AB = 4 cm

Diameter of circle AC = 2.5 × 2 = 5 cm

Here

BC = $\sqrt{AC^2 - AB^2}$

Substituting the values

= $\sqrt{5^2 - 4^2}$

= $\sqrt{25 - 16}$

= $\sqrt{9}$

So we get

= 3 cm

We know that

Area of rectangle = $AB \times BC$

Substituting the values

= 4 × 3
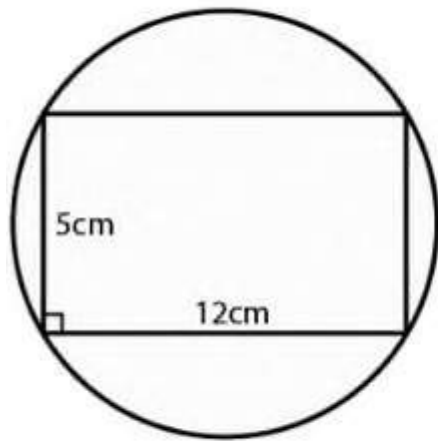
= 12 cm²

24. (a) in the figure (i) given below , calculate the area of the shaded region correct to two decimal places. (take π = 3.142)

(b) In the figure (ii) given below, ABC is an isosceles right angled triangle with ∠ABC = 90⁰. A semicircle is drawn with AC

as diameter. If AB = BC = 7cm, find the area of the shaded region. Take $\pi = \dfrac{22}{7}$

(i)

(ii)
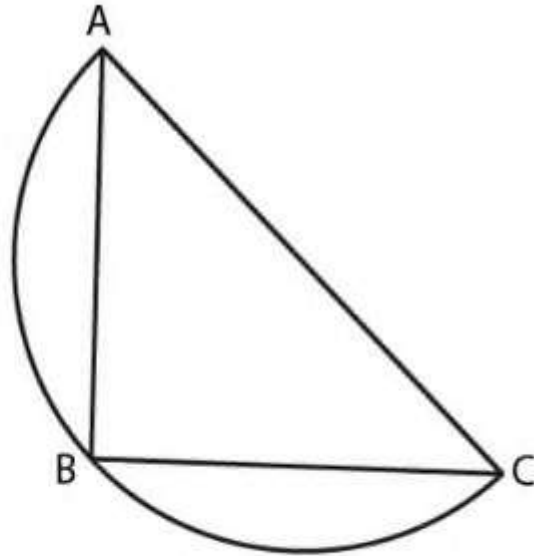
Solution:

(a) We know that

ABCD is a rectangle which is inscribed in a circle of length = 12cm

Width = 5 cm

$AC = \sqrt{AB^2 - BC^2}$

Substituting the values

$= \sqrt{12^2 - 5^2}$

$= \sqrt{144 - 25}$

$= \sqrt{169}$

= 13 cm

Diameter of circle = AC = 13cm

Radius of circle $= \dfrac{13}{2} = 6.5$ cm

Here

Area of circle $= \pi r^2$

Substituting the values

$= 3.142 \times (6.5)^2$

By further calculation

$= 3.142 \times 42.25$

$= 132.75$ cm^2

Area of rectangle $= l \times b$

Substituting the values

$= 12 \times 5$

$= 60$ cm^2

So the area of the shaded portion $= 132.75 - 60 = 72.75$ cm^2

(b) We know that

Area of Δ ABC $= \dfrac{1}{2} \times AB \times BC$

Substituting the values

$= \dfrac{1}{2} \times 7 \times 7$

$= \dfrac{49}{2} cm^2$

Here

$AC^2 = AB^2 + BC^2$

Substituting

$= 49 + 49$

So we get

$AC = 7\sqrt{2}$

Radius of semi circle $= \dfrac{7\sqrt{2}}{2} cm$

Area of semi circle $= \dfrac{\pi}{2} \times \dfrac{7\sqrt{2}^2}{2}$

By further calculation

$= \dfrac{1}{2} \times \dfrac{22}{7} \times \dfrac{98}{4}$

$= \dfrac{77}{2} cm^2$

Area of the shaded region = area of the semi circle − area of Δ ABC

Substituting the values

$= \dfrac{77}{2} - \dfrac{49}{2}$

$= \dfrac{28}{2}$

$= 14 \ cm^2$

25. A circular field has perimeter 660 m. A plot in the shape of a square having its vertices on the circumference is marked in the field. Calculate the area of the square field.

Solution

It is given that

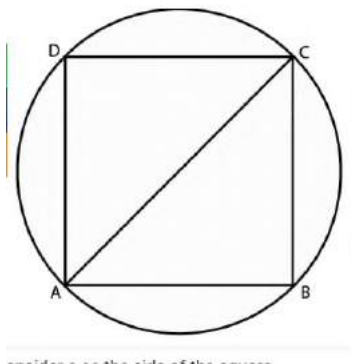Perimeter of circular field = 660 m

Radius of the field = $\dfrac{660}{2}\pi$

Substituting the values

$= \dfrac{660 \times 7}{2 \times 22}$

$= 105$ m

Here

ABCD is a square which is inscribed in the circle where AC is the diagonal which is the diameter of the circular field .

Consider a as the side of the square

$AC = \sqrt{2}\,a$

$a = \dfrac{AC}{\sqrt{2}}$

Substituting the values

$a = \dfrac{105 \times 2}{\sqrt{2}}$

Multiply and divide by $\sqrt{2}$

$a = \dfrac{105 \times 2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

By further calculation

$a = \dfrac{105 \times 2 \times \sqrt{2}}{2}$

$a = 105\sqrt{2}$ m

We know that

Area of the square $= a^2$
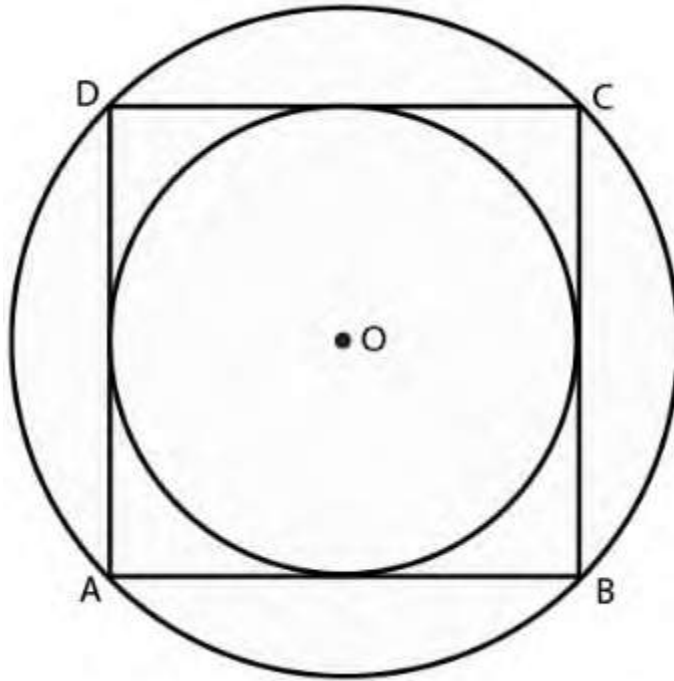
It can be written as

$= \left(\dfrac{105}{\sqrt{2}}\right)^2$

$= 105\sqrt{2} \times 105\sqrt{2}$

$= 22050$ m^2

26. In the adjoining figure, ABCD is a square . find the ratio between

(i) the circumference

(ii) the areas of the incircle and the circum circle of the square.

Solution

Consider side of the square = 2a

Area $= (2a)^2 = 4a^2$

Diagonal of $AC = \sqrt{2}AB$
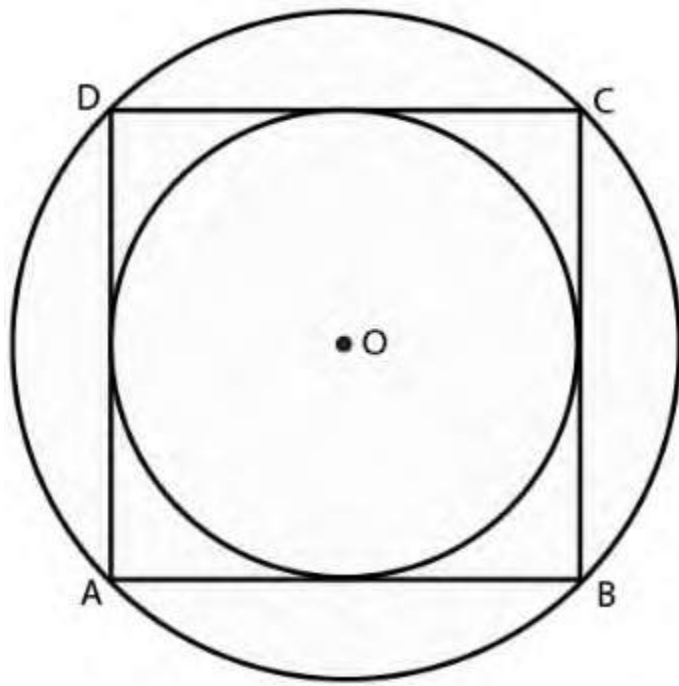
(i) we know that

Radius of the circum circle $= \frac{1}{2}$ AC

It can be written as

$= \frac{1}{2}(\sqrt{2} \times AB)$

Substituting the values

$= \frac{\sqrt{2}}{2} \times 2a$

$= \sqrt{2}\, a$

Circumference $= 2\pi r = 2 \times \pi \times \sqrt{2}a = 2\sqrt{2}\pi a$

Radius of in circle $= AB = \frac{1}{2} \times 2a = a$

Circumference $= 2\pi r = 2\pi a$

Here

Ratio between the circumference incircle and circumcircle $= 2\,\pi a :$

$2\sqrt{2}\pi a$

(ii) we know that

Area of incircle $= \pi r^2 = \pi a^2$

Area of circumcircle $= \pi R^2$
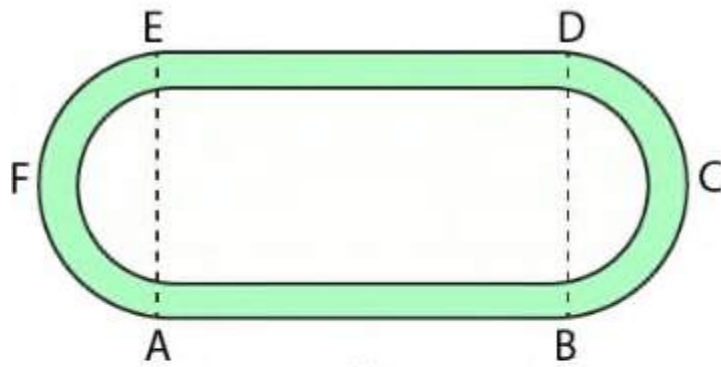
$= \pi(\sqrt{2}a)^2$

$= \pi 2a^2$

$= 2\pi a^2$

So the ratio $= \pi a^2 : 2\pi a^2 = 1:2$

27 .(a) the figure (i) given below shows a running track surrounding a grassed enclosure PQRSTU. The enclosure consists of a rectangle PQST with a semicircular region at each end .

PQ =200 m , PT = 70 m

(i)

(ii)

(i) calculate the area of the grassed enclosure in m^2 .

(ii) given that the track is of constant width 7 m , calculate the outer perimeter ABCDEF of the track .

(b) in the figure (ii) given below , the inside perimeter of a practice running track with semi circular ends and straight parallel sides is 312 m . the length of the straight portion of the track is 90 m. If the track has a uniform width of 2 m throughout, find its area

Solution :

(a) it is given that

Length of PQ = 200m

Width PT = 70 m

(i) we know that

Area of rectangle PQST = l × b

= 200 × 70

= 14000 m^2

Radius of each semi circular part on either side of rectangle = $\frac{70}{2}$ =

35 m

Area of both semi circular parts = $2 \times \frac{1}{2} \pi r^2$

Substituting the values

= $\frac{22}{7}$ × 35 × 35

= 3850 m^2

So the total area of grassed enclosure = 1400 + 3850 = 17850m^2

(ii) we know that

Width of track around the enclosure = 7 m

Outer length = 200 m

So the width $= 70 + 7 \times 2$

$$= 70 + 14$$

$$= 84 \text{ m}$$

Outer radius $= \dfrac{84}{2} = 42 \, m$

Here

Circumference of both semi circular part $= 2\pi r$

Substituting the values

$= 2 \times \dfrac{22}{7} \times 42$

$= 264$ m

Outer perimeter $= 264 + 200 \times 2$

$= 264 + 400$

$= 664$ m

(b) it is given that

Inside perimeter $= 312$ m

Total length of the parallel sides $= 90 + 90 = 180$ m

Circumference of two semi circle $= 312 - 180 = 132$ m

Here

Radius of each semi circle $= \dfrac{132}{2}\pi$

So we get

$= \dfrac{66}{3.14}$

$= 21.02$ m

Diameter of each semi circle $= \dfrac{66}{\pi} \times 2$

So we get

$= \dfrac{132}{\pi}$

$= \dfrac{132}{3.14}$

Multiply and divide by 100

$= \dfrac{132 \times 100}{314}$

$= 42.04$ m

Width of track $= 2$m

Outer diameter $= 42.04 + 4 = 46.04$ m

Radius $= \dfrac{46.04}{2} = 23.02$m

We know that

Area of two semi circle $= 2 \times \dfrac{1}{2} \times \pi R^2$

$= \pi R^2$

Substituting the values

$= 3.14 \times (23.02)^2$

$= 3.14 \times 23.02 \times 23.02$

$= 1663.95$ m^2

Area of rectangle = 90 × 46.04

= 4143.6 m²

Total area = 1663.95 + 4143.60 = 5807.55 m²

Area of two inner circles = $2 \times \frac{1}{2} \pi r^2$

Substituting the values

= 3.14 × 21.02 × 21.02

= 1387 .38 m²

Area of inner rectangle = 90 × 42.04

= 3783.6 m²

Total inner area =   3783.60 + 1387.38

= 5170.98 m²

Here

Area of path = 5807.55 – 5170.98

= 636.57 m²

28. (a) in the figure (i) given below , two circles with centres A and B touch each other at the point C. If AC = 8 cm and AB = 3cm , find the area of the shaded region .

(b) the quadrants shown in the figure (ii) given below are each of radius 7 cm . calculate the area of the shaded portion

(i)

(ii)
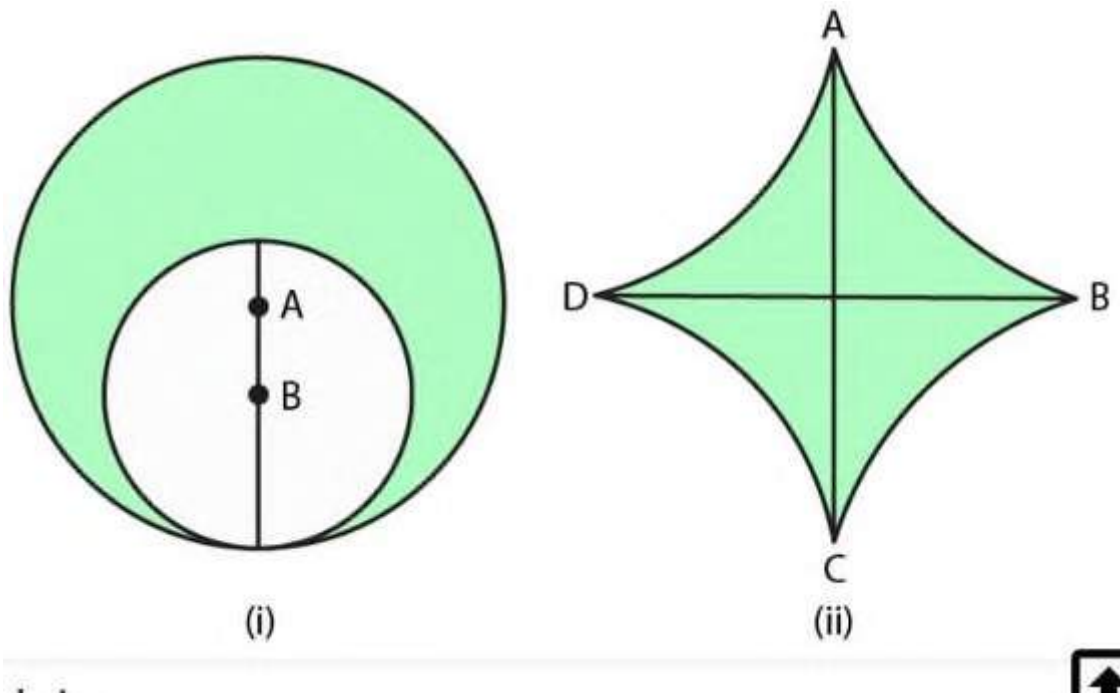
Solution

(a) it is given that

AC = 8 cm

BC = AC − AB = 8 − 3 = 5 cm

We know that

Area of big circle of radius AC = πR^2

Substituting the values

$= \frac{22}{7} \times 8 \times 8$

By further calculation

$= 64 \times \frac{22}{7}$ cm²

Area of smaller circle = πr^2

Substituting the values

$$= \frac{22}{7} \times 5 \times 5$$

By further calculation

$$= \frac{25 \times 22}{7} \text{ cm}^2$$

Here

Area of shaded portion $= \dfrac{64 \times 22}{7} - \dfrac{25 \times 22}{7}$

Taking out the common terms

$$= \frac{22}{7}(64 - 25)$$

By further calculation

$$= \frac{22}{7} \times 39$$

$$= 122.57 \text{ cm}^2$$

(b) we know that

Radius of each quadrant = 7 cm

(ii)
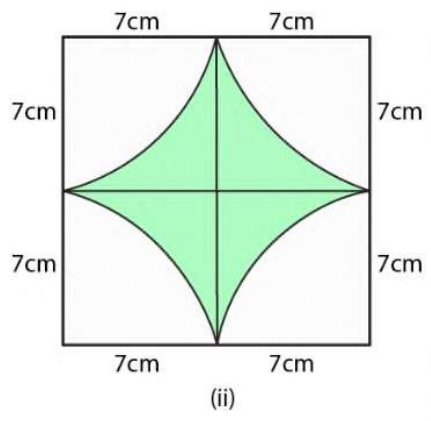
Here

Area of shaded region = Area of square – 4 area of the quadrant

It can be written as

$$= (\text{side})^2 - 4 \times \frac{1}{4}\pi r^2$$

Substituting the values

$$= 14^2 - \frac{22}{7} \times 7 \times 7$$

By further calculation

196 – 154

$$= 42 \text{ cm}^2$$

29. (a) in the figure (i) given below , two circular flower beds have been shown on the two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection o of the diagonals of the square lawn , find the sum of the areas of the lawn and the flower beds.

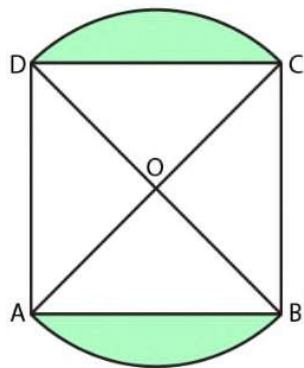**(b) in the figure (ii) given below a square OABC is inscribed in a quadrant OPBQ of a circle . if OA = 20 cm , find the area of the shaded region ($\pi = 3.14$)**
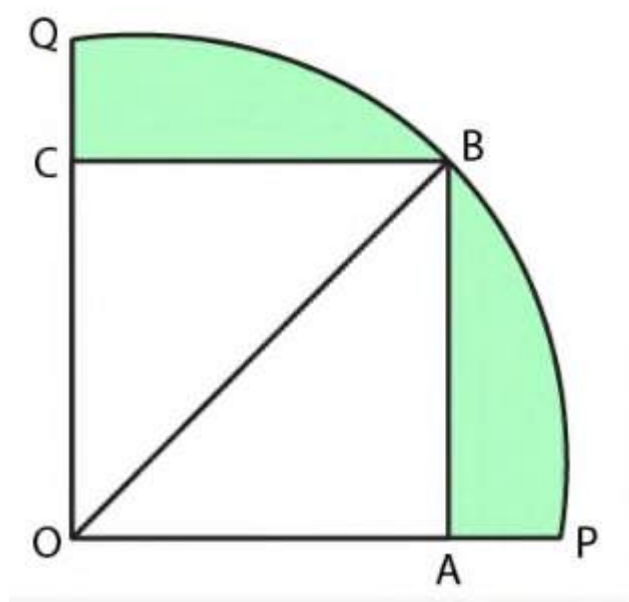
Solution

(a) it is given that

Side of square lawn ABCD (a) = 56 cm

Area $= a^2 = 5^2 = 3136$ cm^2

We know that

Length of the diagonal of the square $= \sqrt{2}a$

$\sqrt{2} \times 56$ cm

Radius of each quadrant $= \dfrac{\sqrt{2} \times 56}{2}$

$= 28\sqrt{2}$ cm
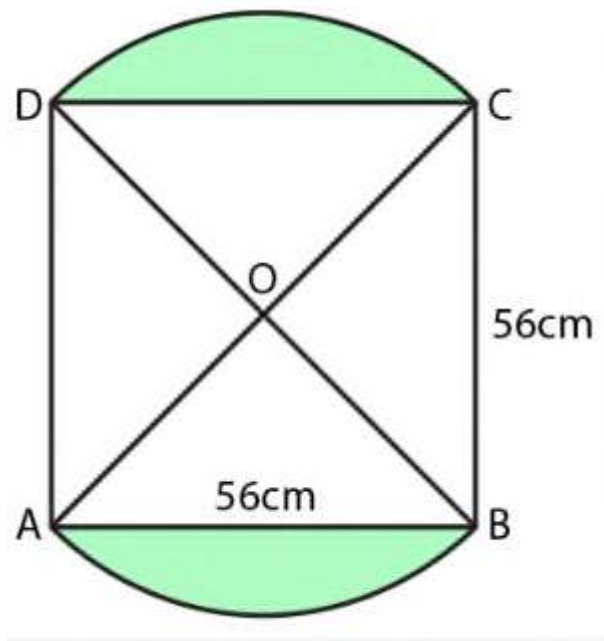
So the area of each segments $= \frac{1}{4}\pi r^2 - area\ of\ \Delta\ OBC$

Substituting the values

$= \frac{1}{4} \times \frac{22}{7} \times 28\sqrt{2} \times 28\sqrt{2} - \frac{1}{2} \times 28\sqrt{2} \times 28\sqrt{2}$

Taking out the common terms

$= 28\sqrt{2} \times 28\sqrt{2} \left(\frac{1}{4} \times \frac{22}{7} - \frac{1}{2}\right)$

By further calculation

$= 784 \times 2 \left(\frac{11}{14} - \frac{1}{2}\right)$

So we get

$= 784 \times 2 \times \frac{4}{14}$

$= 448\ cm^2$

Here

Area of two segments = 448 × 2 = 896 cm²

So the total area of the lawn and beds = 3136 + 896 = 4032 cm²

(b) in the figure

OPBQ is a quadrant and OABC is a square which is inscribed in a

side of square = 20 cm

OB is joined

Here

OB = $\sqrt{2}a = \sqrt{2} \times 20$cm

Radius of quadrant = OB = $20\sqrt{2}$ cm
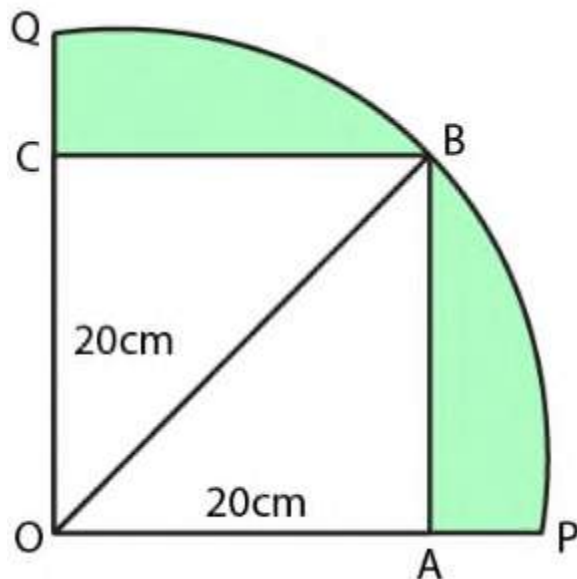
We know that

Area of quadrant = $\frac{1}{4}\pi r^2$

Substituting the values

$$= \frac{1}{4} \times 3.14 \times \left(20\sqrt{2}\right)^2$$

By further calculation

$$= \frac{1}{4} \times 3.14 \times 800$$

So we get

$$= 314 \times 2$$

$$= 628 \text{ cm}^2$$

Area of square $= a^2 = 20^2 = 400 \text{ cm}^2$

So the area of shaded portion $= 628 - 400 = 228 \text{cm}^2$

30. (a) in the figure (i) given below , ABCD is a rectangle , AB = 14 cm and BC = 7 cm . taking DC, BC and AD as diameter ,three semicircles are drawn as shown in the figure . find the area if the shaded portion .
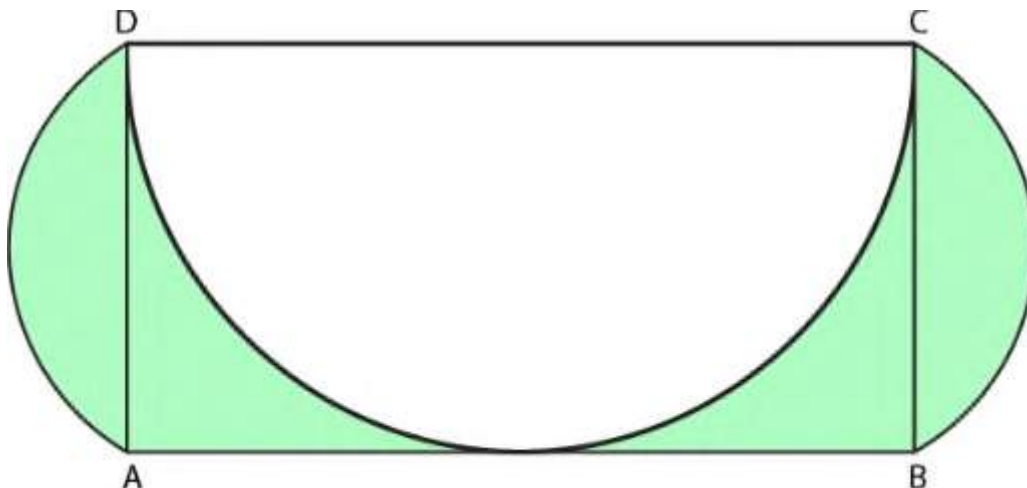
(b) in the figure (ii) given below , o is the centre of a circle with AC = 24 cm , AB = 7 cm and ∠BOD = 90⁰. find the area of the shaded region. (use π = 3.14)
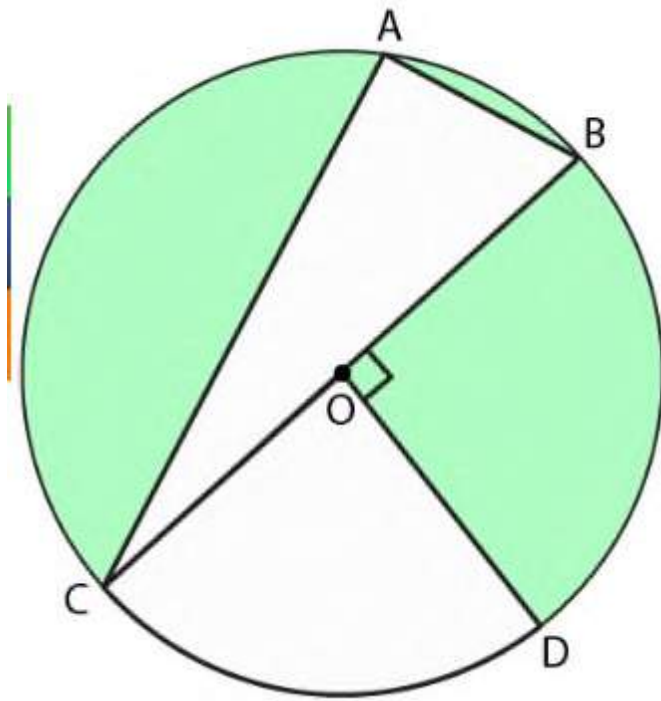
Solution :

(a) it is given that

ABCD is a rectangle

Three semicircle are drawn with AB = 14cm and BC = 7cm
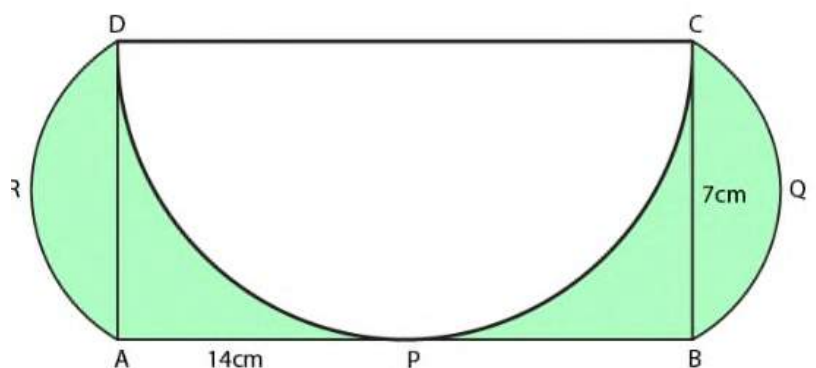
We know that

Area of rectangle ABCD $= l \times b$

Substituting the values

$= 14 \times 7$

$= 98 \text{ cm}^2$

Radius of each outer semicircle $= \dfrac{7}{2}$ cm

So the area $= 2 \times \dfrac{1}{2} \pi r^2$

Substituting the values

$= \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}$

$= \dfrac{77}{2}$

$= 38.5 \text{ cm}^2$

Here

Area of semicircle drawn on CD as diameter $= \dfrac{1}{2} \pi R^2$

Substituting the values

$= \dfrac{1}{2} \times \dfrac{22}{7} \times 7^2$

By further calculation

$= \dfrac{11}{7} \times 7 \times 7$

$= 77 \text{ cm}^2$

So the area of shaded region $= 98 + 38.5 - 77$

$= 59.5 \text{ cm}^2$

(b) in the figure

AC = 24 cm

AB = 7cm

∠BOD = 90°

In ΔABC
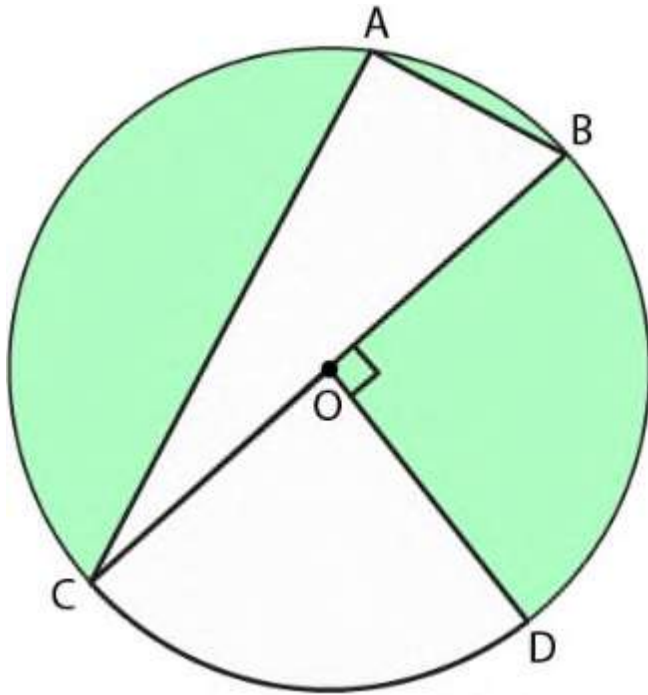
Using Pythagoras theorem

$BC^2 = AC^2 + AB^2$

Substituting the values

$BC^2 = 24^2 + 7^2$

By further calculation

$BC = \sqrt{576 + 49} = \sqrt{625} = 25$ cm

So the radius of circle $= \dfrac{25}{2}$ cm

We know that

Area of Δ ABC $= \frac{1}{2} \times AB \times AC$

Substituting the values

$= \frac{1}{2} \times 7 \times 24$

$= 84 \text{ cm}^2$

Area of quadrant COD $= \frac{1}{4} \pi r^2$

Substituting the values

$= \frac{1}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}$

By further calculation

$= \frac{1962.5}{16}$

$= 122.66 \text{ cm}^2$

Area of circle $= \pi r^2$

Substituting the values

$= 3.14 \times \frac{25}{2} \times \frac{25}{2}$

By further calculation

$= \frac{1962.5}{4}$

$= 490.63 \text{ cm}^2$

Area of shaded portion = area of circle − (area of Δ ABC + area of quadrilateral COD)

Substituting the values

$= 490.63 - (84 + 122.66)$

By further calculation

$= 490.63 - 206.66$

$= 283.97 \text{ cm}^2$

31.(a) in the figure given below ABCD is a square side 14 cm .

A,B,C and D are centres of the equal circle which touched

externally in pairs.

Find the area of the shaded region .
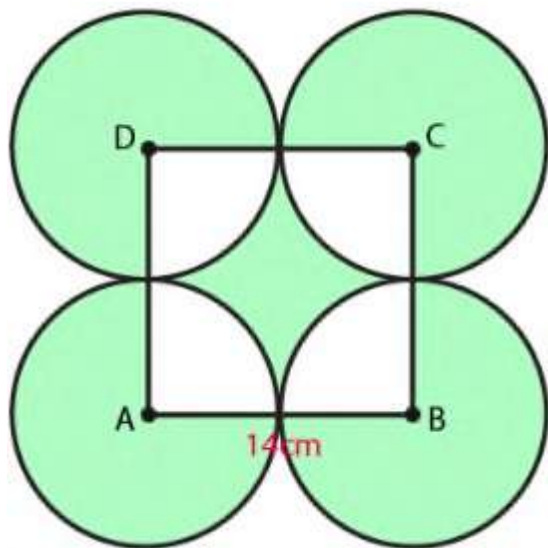
(b) in the figure (ii) given below , the boundary of the shaded

region in the given diagram consists of three semicircular arcs ,

the smaller being equal . if the diameter of the larger one is 10

cm , calculate .

(i) the length of the boundary

(ii) the area of the shaded region .(take π to be 3.14)
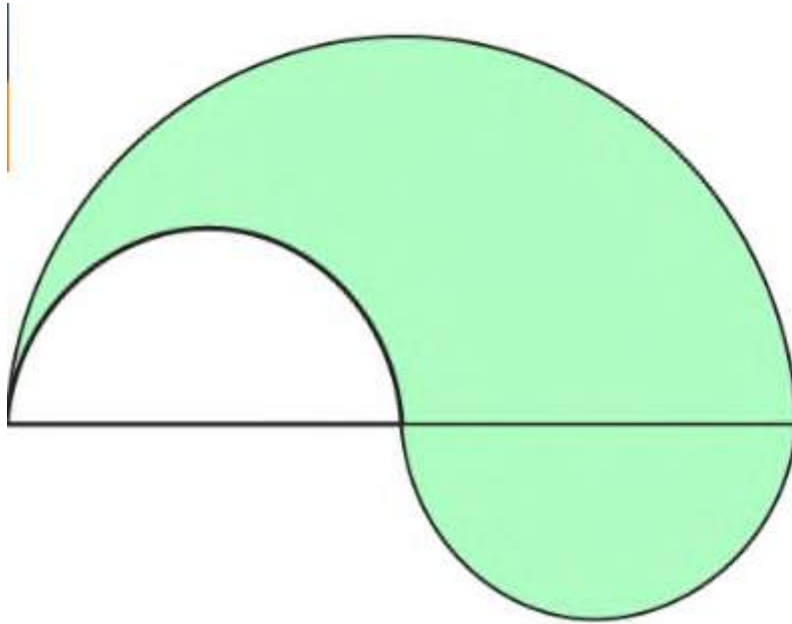
Solution

(a) it is given that

Side of square ABCD = 14 cm

Radius of each circle drawn from A,B,C and D and touching

externally in pairs $= \dfrac{14}{2} = 7$cm

We know that

Area of square $= a^2$

Substituting the values

$= 14 \times 14$

$= 196$ cm^2

Area of 4 sectors of 90° each $= 4 \times \pi \times r^2$

Substituting the values

$= 4 \times \frac{22}{7} \times 7 \times 7 \times \frac{1}{4}$

$= 154$ cm

Area of each sector of 270° angle $= \frac{3}{4}\pi r^2$

Substituting the values

$= \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$

$= \frac{231}{2}$

$= 115.5$ cm^2

Here

Area of 4sector $= 115.5 \times 4 = 462$cm^2

So the area of shaded portion = area of square + area of 4 bigger

sector – area of 4 smaller sector

Substituting the values

$= 196 + 462 - 154$

$= 658 -154$

$= 504$ cm^2

(b) we know that

Radius of big semi circle $= \frac{10}{2} = 5$cm

Radius of each smaller circle $= \dfrac{5}{2}$ cm

(i) length of boundary = circumference of bigger semi circle + 2 circumference of smaller semi circles

It can be written as

$= \pi R + \pi r + \pi r$

$= 3.14 (R + 2r)$

Substituting the values

$= 3.14 \left(5 + 2 \times \dfrac{5}{2}\right)$

By further calculation

$= 3.14 \times 10$

$= 31.4$ cm

(ii) here

Area of shaded region = area of bigger semi circle + area of one smaller semi circle − area of other smaller semi circle

We know that

Area of bigger semi circle $= \dfrac{1}{2} \pi R^2$

Substituting the values

$= \dfrac{3.14}{2} \times 5 \times 5$

$= 1.57 \times 25$

= 39.25 cm

32. (a) in the figure (i) given below , the points A,B and C are centres of arcs of circles of radii 5 cm , 3 cm and 2 cm respectively. Find the perimeter and the area of the shaded region. (take $\pi = 3.14$)

(b) in the figure(ii) given below ABCD is a square of side 4 cm . at each corner of the square a quarter circle of radius 1 cm , and at the centre a circle of diameter 2 cm are drawn . find the perimeter and the area of the shaded region . take $\pi = 3.14$
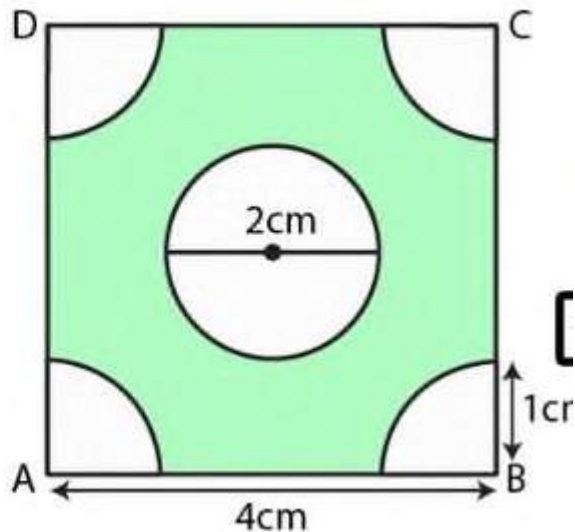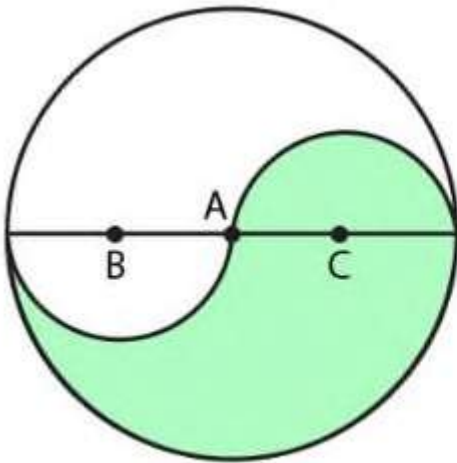
Solution

(a) it is given that

Radius of bigger circle = 5 cm

Radius of small circle (r_1) = 3 cm

Radius of smaller circle (r_2) = 2cm

(i) we know that

Perimeter of shaded region = circumference of bigger semi circle + circumference of small semi circle + circumference of smaller semi – circle

It can be written as

$= \pi R + \pi r_1 + \pi r_2$

$= \pi(R + r_1 + r_2)$

Substituting the values

$= \pi(5 + 3 + 2)$

$= 3.14 \times 10$

$= 31.4 \text{ cm}^2$

(ii) we know that

Area of shaded region = area of bigger semi circle + area of smaller semi circle – area of small semicircle

It can be written as

$= \frac{1}{2}\pi R^2 + \frac{1}{2}\pi r_2^2 - \frac{1}{2}\pi r_1^2$

$$= \frac{1}{2}\pi(R^2 + r_2^2 - r_1^2)$$

Substituting the values

$$= \frac{1}{2}\pi(5^2 + 2^2 - 3^2)$$

By further calculation

$$= \frac{1}{2}\pi(25 + 4 - 9)$$

$$= \frac{1}{2}\pi \times 20$$

So we get

$$= 10 \times 3.14$$

$$= 31.4 \text{ cm}^2$$

(b) we know that

Side of square ABCD = 4cm

Radius of each quadrant circle = 1 cm

Radius of circle in the square $= \frac{2}{2} = 1\text{cm}$

(i) here

Perimeter of shaded region = circumference of 4 quadrants + circumference of circle $+ 4 \times \frac{1}{2}$ side of square

It can be written as

$$= 4 \times \frac{1}{4}(2\pi r) + 2\pi r + 4 \times 2$$

By further calculation

$= 2\pi r + 2\pi r + 8$

$= 4\pi r + 8$

So we get

$= 4 \times 3.14 \times 1 + 8$

$= 12.56 + 8$

$= 20.56$ cm

(ii) area of shaded region $=$ area of square $-$ area of 4 quadrants $-$ area of circle

It can be written as

$= \text{side}^2 - 4 \times \frac{1}{4}\pi r^2 - \pi r^2$

$= 4^2 - \pi r^2 - \pi r^2$

By further calculation

$= 16 - 2\pi r^2$

So we get

$= 16 - 2 \times 3.14 \times 1^2$

$= 16 - 6.28$

$= 9.72$ cm^2

33. (a) in the figure given below , ABCD is a rectangle . AB = 14 cm , BC = 7 cm . from the rectangle , a quarter circle BFEC and a semicircle DGE are removed . calculate the area of the remaining piece of the rectangle . (take $\pi = \frac{22}{7}$)

(b) the figure (ii) given below shows a kite , in which BCD is in the shape of a quadrant of circle of radius 42 cm . ABCD is a square and $\triangle$ CEF is an isosceles right angled triangle whose equal sides are 6 cm long . find the area of the shaded region .
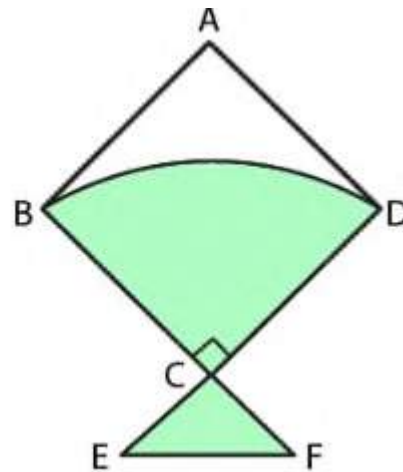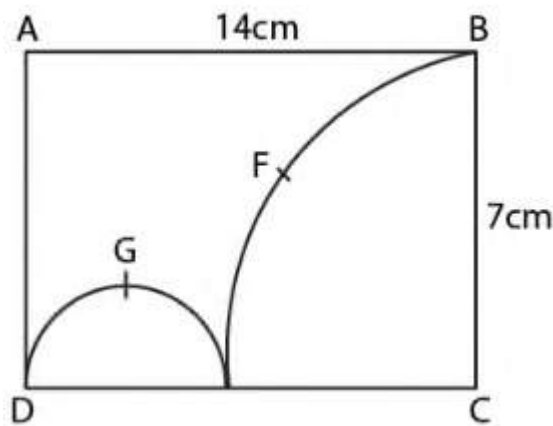
Solution

(a) here

Area of remaining piece = area of rectangle ABCD – area of semicircle DGE – area of quarter BFEC

Substituting the values

$$= 14 \times 7 - \frac{1}{2} \times \pi \left(\frac{7}{2}\right)^2 - \frac{1}{4}\pi \times 7^2$$

By further calculation

$$= 14 \times 7 - \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

So we get

$$= 98 - \frac{77}{4} - \frac{154}{4}$$

$$= 98 - 19.25 - 38.5$$

$$= 98 - 57.75$$

$$= 40.25 \text{ cm}^2$$

(b) in the given figure

ABCD is a square of side = radius of quadrant = 42 cm

Δ CEF is an isosceles right triangle with each side = 6cm

Area of shaded portion = area of quadrant + area of isosceles right triangle

It can be written as

$$= \frac{1}{4}\pi r^2 + \frac{1}{2}EC \times FC$$

Substituting the values

$$= \frac{1}{4} \times \frac{22}{7} \times 42 \times 42 + \frac{1}{2} \times 6 \times 6$$

By further calculation

$= 1386 + 18$

$= 1404 \text{ cm}^2$

34. (a) in the figure (i) given below , the boundary of the shaded region in the given diagram consists of four semicircular arcs , the smallest two being equal . if the diameter of the largest is 14 cm and of the smallest is 3.5 cm , calculate

(i) the length of the boundary

(ii) the area of the shaded region
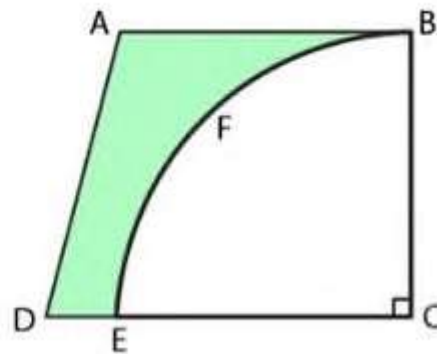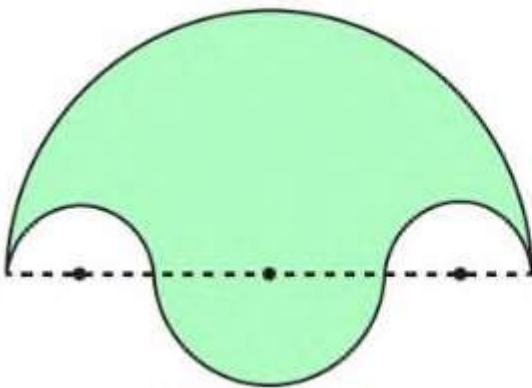
(b) in the figure (ii) given below , a piece of cardboard , in the shape of a trapezium ABCD , AB ∥CD and ∠BOD = 90⁰,

quarter circle BFEC is removed . given AB = BC = 3.5 cm and DE =2 cm . calculate the area of the remaining piece of the cardboard .

Solution

(a) (i) we know that

Length of boundary = circumference of bigger semi- circle + circumference of small semi circle + 2 × circumference of the smaller semi circles
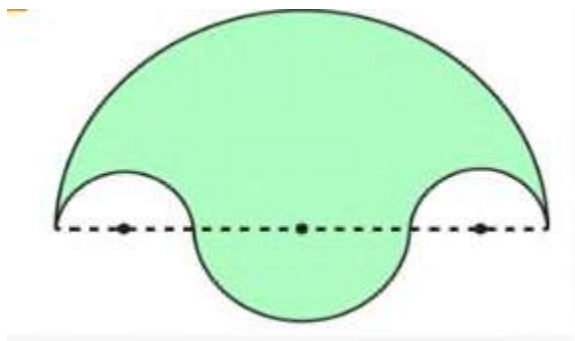
It can be written as

$$= \pi R + \pi r_1 + 2 \times \pi r_2$$

$$= \pi (R + r_1) + 2\pi r_2$$

Substituting the values

$$= \frac{22}{7}(7 + 3.5) + 2 \times \frac{22}{7} \times \frac{3.5}{2}$$

By further calculation

$$= \frac{22}{7} \times 10.5 + 11$$

So we get

= 33 +11

= 44 cm

(ii) we know that

Area of shaded region = area of bigger semicircle + area of small semicircle – 2 × area of smaller semicircle

It can be written as

$$= \frac{1}{2}\pi(7)^2 + \frac{1}{2}\pi(3.5)^2 - 2 \times \frac{1}{2}\pi(1.75)^2$$

By further calculation

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times 3.5 \times 3.5 - \frac{22}{7} \times 1.75 \times 1.75$$

So we get

= 77 + 19.25 – 9.625

= 86.625 cm^2

(b) here

ABCD is a trapezium in which AB ∥ DC and ∠c = 90°

3.5cm

A

B

3.5cm

3.5cm

D 2cm E 3.5cm C
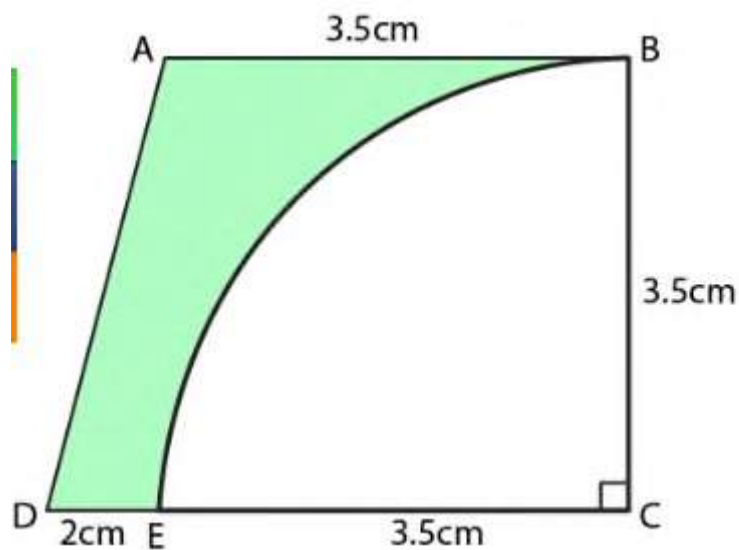
AB = BC = 3.5 cm and DE = 2 cm

Radius of quadrant = 3.5 cm

We know that

Area of trapezium = $\frac{1}{2}(AB + DC) \times BC$

Substituting the values

$= \frac{1}{2}(3.5 + 3.5 + 2) \times 3.5$

By further calculation

$= \frac{1}{2}(9 \times 3.5)$

$= 4.5 \times 3.5$

$= 15.75 \text{ cm}^2$

So the area of quadrant $= \frac{1}{4}\pi r^2$

Substituting the values

$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

= 9.625 cm²

Area of shaded portion = 15.75 – 9.625 = 6.125 cm²

35. (a) in the figure (i) given below , ABC is a right angled triangle, ∠B = 90⁰ , AB = 28cm and BC =21cm . with AC as diameter a semi circle is drawn and with BC as radius a quarter circle is drawn . find the area of the shaded region correct to two decimal places.

(b) in the figure (ii) given below , ABC is an equilateral triangle of side 8 cm . A, B and C are the centres of circular arcs of equal radius . find the area of the shaded region correct upto 2 decimal places . (take π = 3.142 and $\sqrt{3}$ = 1.732)

(i)   (ii)
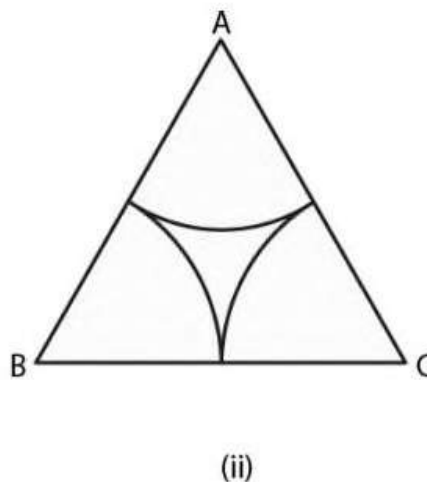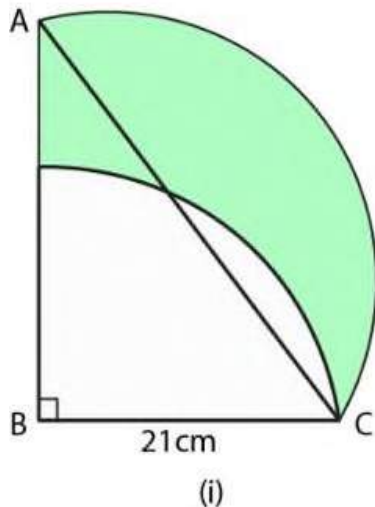
Solution

(a) in right Δ ABC

$\angle B = 90^\circ$

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$= 28^2 + 21^2$

$= 784 + 441$

$= 1225$

So we get

$AC = \sqrt{1225} = 35$ cm

Here

Radius of semi circle $(R) = \dfrac{35}{2}$

Radius of quadrant $(r) = 21$ cm

So the area of shaded region = area of ΔABC + area of semi circle – area of quadrant

$= \dfrac{1}{2} \times 28 \times 21 + \dfrac{1}{2} \pi R^2 - \dfrac{1}{4} r^2$

Substituting the values

$= 294 + \dfrac{1}{2} \times \dfrac{22}{7} \times \dfrac{35}{2} \times \dfrac{35}{2} - \dfrac{1}{4} \times \dfrac{22}{7} \times 21 \times 21$

By further calculation

$= 294 + \dfrac{1925}{4} - \dfrac{693}{2}$

$= 294 + 481.25 - 346.5$

So we get

$= 775.25 - 346.50$

$= 428.75 \text{ cm}^2$

(b) we know that

Δ ABC is an equilateral triangle of side 8 cm

A,B,C are the centres of three circular arcs of equal radius

Radius $= \dfrac{8}{2} = 4$ cm

Here

Area of Δ ABC $= \dfrac{\sqrt{3}}{4} a^2$

Substituting the values

$= \dfrac{\sqrt{3}}{4} \times 8 \times 8$

$= \dfrac{\sqrt{3}}{4} \times 64$

So we get

$= 16\sqrt{3} \text{ cm}^2$

Substituting the value of $\sqrt{3}$

$= 16 \times 1.732$

$= 27.712 \text{ cm}^2$

So the area of 3 equal sector of 60° whose radius is 4 cm $= 3 \times$

$\pi r^2 \times \dfrac{60}{360}$

By further calculation

$= 3 \times 3.142 \times 4 \times 4 \times \dfrac{1}{6}$

So we get

$= 3.142 \times 8$

$= 25.136 \text{ cm}^2$

Area of shaded region $= 27.712 - 25.136 = 2.576 = 2.58 \text{ cm}^2$

36. A circle is inscribed in a regular hexagon of side $2\sqrt{3}$ cm. find

(i) the circumference of the inscribed circle

(ii) the area of the inscribed circle

Solution

It is given that

ABCDEF is a regular hexagon of side $2\sqrt{3}$ cm and a circle is inscribed in it with o as the centre
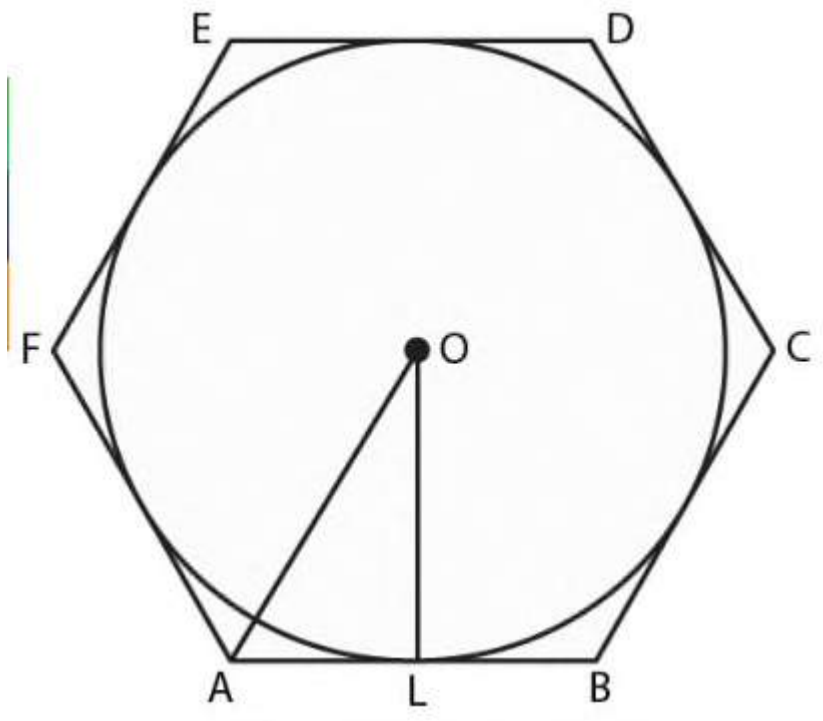
Radius of inscribed circle $= \dfrac{\sqrt{3}}{2} \times$ side of regular hexagon

Substituting the values

$= \dfrac{\sqrt{3}}{2} \times 2\sqrt{3}$

$= 3$ cm

(i) we know that

Circumference of the circle $= 2\pi r$

Substituting the values

$= 2\pi \times 3$

By further calculation

$= \dfrac{6 \times 22}{7}$

$= \dfrac{132}{7}$ cm

(ii) area of the circle $= \pi r^2$

Substituting the values

$= \pi \times 3 \times 3$

By further calculation

$= \dfrac{9 \times 22}{7}$

$= \dfrac{198}{7} \text{ cm}^2$

37. in the figure (i) given below , a chord AB of a circle of radius 10 cm subtends a right angle at the centre o . find the area of the sector OACB and of the major segment . take $\pi = 3.14$

Solution :
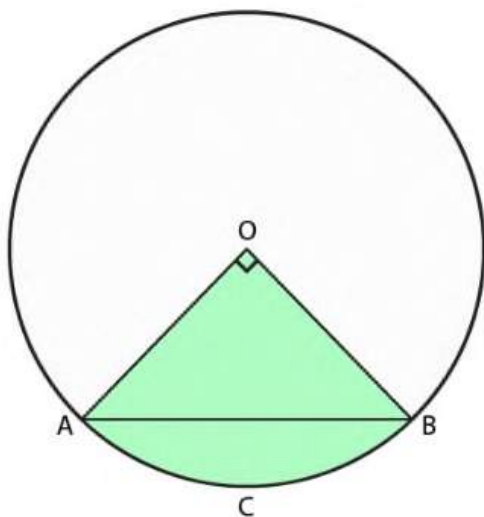
It is given that

Radius of the circle = 10cm

Angle at the centre subtended by a chord AB = 90°

We know that

Area of sector OACB = $\pi r^2 \times \dfrac{90}{360}$

Substituting the values

$= 3.14 \times 10 \times 10 \times \dfrac{90}{360}$

So we get

$= 314 \times \dfrac{1}{4}$

$= 78.5 \text{ cm}^2$

Here

Area of Δ OAB $= \dfrac{1}{2} \times 10 \times 10 = 50\text{cm}^2$

Area of minor segments = area of sector Δ ACB - area of Δ OAB

Substituting the values

$= 78.5 - 50$

$= 28.5 \text{ cm}^2$

Area of circle = πr^2

Substituting the values

$= 3.14 \times 10 \times 10$

$= 314 \text{ cm}^2$

Area of major segments = area of circle – area of minor segments

Substituting the values

$= 314 - 28.5$

$= 285.5 \text{ cm}^2$.

Exercise 16.4

1. find the surface area and volume of a cube whose one edge is 7 cm .

Solution

It is given that

One edge of cube a = 7 cm

We know that

Surface area of cube = $6a^2$

Substituting the values

= $6(7)^2$

=$6 \times 7 \times 7$

= 294 cm^2

Volume of cube = a^3

Substituting the values

= 7^3

= $7 \times 7 \times 7$

= 343 cm^3

2. find the surface area and the volume of a rectangular solid measuring 5 m by 4 m by 3 m . also find the length of a diagonal.

Solution

In a rectangular solid

L = 5m , b = 4m and h = 3m

Here

Surface area of rectangular solid = 2(lb + bh + lh)

Substituting the values

= 2(5 × 4 + 4 × 3 + 5 × 3)

By further calculation

= 2(20 + 12 + 15)

= 2 × 47

= 94 sq. m

Volume of rectangular solid = $l \times b \times h$

Substituting the values

= 5 × 4 × 3

= 60 m^2

We know that

Length of diagonal = $\sqrt{l^2 + b^2 + h^2}$

Substituting the values

$= \sqrt{5^2 + 4^2 + 3^2}$

So we get

$= \sqrt{25 + 16 + 9}$

$= \sqrt{50}$

$= \sqrt{25 \times 2}$

It can be written as

$= 5\sqrt{2}$m

Substituting the value of $\sqrt{2}$

$= 5 \times 1.414$

$= 7.07$ m

Therefore , the length of diagonal is 7.07 m

3 . the length and breadth of a rectangular solid are respectively 25 cm and 20 cm . if the volume is 7000 cm^3 , find its height .

Solution

It is given that

Length of rectangular solid $= 25$ cm

Breadth of rectangular solid $= 20$ cm

Volume of rectangular solid $= 7000$ cm^2

Consider height of rectangular solid = h cm

Here

Volume $= l \times b \times h$

Substituting the values

$7000 = 25 \times 20 \times h$

By further calculation

$25 \times 20 \times h = 7000$

$H = \dfrac{7000}{25 \times 20}$

So we get

$H = \dfrac{700}{25 \times 2}$

$H = \dfrac{350}{25}$

By division

$H = \dfrac{70}{5}$

$H = 14$ cm

Therefore , height of rectangular solid is 14 cm .

4. A class room is 10 m long , 6m broad and 4 m high . how many students can it accommodate if one student needs 1.5 m² of floor area ? how many cubic meters of air will each students have?

 Solution :

The given dimensions of class room are

Length (l) = 10 m

Breadth (b) = 6m

Height(h) = 4 m

We know that

Floor area of class room = l×b

Substituting the values

= 10 × 6

= 60 m²

Here

One student needs 1.5m² floor area

So the number of students = $\frac{60}{1.5}$

Multiply and divide by 10

$= \frac{60 \times 10}{15}$

$= \frac{600}{15}$

= 40 students

Volume of class room = $l \times b \times h$

Substituting the values

= 10 × 6 × 4

= 240 m^3

So the cubic metres of air for each student = $\dfrac{\text{volume of classroom}}{\text{number of students}}$

Substituting the values

= $\dfrac{240}{40}$

= 6 m^3

5.(a) the volume of a cuboid is 1440 cm^2 . Its height is 10 cm and the cross – section is square. Find the side of the square.

(b) the perimeter of one face of a cube is 20 cm. Find the surface area and the volume of the cube.

Solution

(a) it is given that

Volume of cuboid = 1440 cm^3

Height of cuboid = 10 cm

We know that

Volume of cuboid = area of square × height

Substituting the values

1440 = area of square × 10

By further calculation

Area of square = $\frac{1440}{10}$ = 144cm²

Here

$side \times side = 144$

So we get

Side = $\sqrt{144}$ = 12 cm

Therefore, the side of square is 12cm .

(b) it is given that

Perimeter of one face of a cube = 20cm

Perimeter of one face of a cube = $4 \times side$

We can write it as

$20 = 4 \times side$

By further calculation

Side = $\frac{20}{4}$ = 5cm

Here

Area of one face = $side \times side$

Substituting the values

= 5 × 5

= 25 cm²

Area of 6 faces = 6 × 25 = 150cm²

So the volume of cube = $side \times side \times side$

Substituting the values

= 5 × 5 × 5

= 125 cm³

6 . mary wants to decorate her Christmas tree . she wants to place the tree on a wooden box covered with coloured papers with pictures of santa claus. She must know the exact quantity of paper to buy for this purpose. If the box has length 80 cm, breadth 40 cm and height 20 cm respectively, then how many square sheets of paper of side 40 cm would she require ?

Solution

It is given that

Length of box (l) = 80 cm

Breadth (b) = 40 cm

Height (h) = 20 cm

We know that

Surface area of the box $= 2(lh + bh + hl)$

Substituting the values

$= 2(80 \times 40 + 40 \times 20 + 20 \times 80)$

By further calculation

$= 2(320 + 800 + 1600)$

$= 2 \times 5600$

$= 11200 \text{ cm}^2$

So the area of square sheet $= \text{side}^2$

$= 40^2$

$= 1600 \text{ cm}^2$

Here

Number of sheets $= \dfrac{area\ of\ box}{area\ of\ one\ sheet}$

Substituting the values

$= \dfrac{11200}{1600}$

$= 7$

7. the volume of a cuboid is 3600 cm³ and its height is 12 cm. The cross – section is a rectangle whose length and breadth are in the ratio 4:3. Find the perimeter of cross – section .

Solution

It is given that

Volume of a cuboid = 3600cm³

Height of cuboid = 12cm

We know that

Volume of cuboid = area of rectangle × height

Substituting the values

3600 = area of rectangle × 12

By further calculation

Area of rectangle = $\frac{3600}{12}$

Area of rectangle = 300cm² .....(1)

Here

Ratio of length and breadth of rectangle = 4 : 3

Consider

Length of rectangle = 4x

Breadth of rectangle = 3x

Area of rectangle = $lenght \times breadth$

Substituting the values

Area of rectangle = 4x × 3x

So we get

Area of rectangle = $12x^2$ cm^2.....(2)

Using equation (1) and (2)

$12x^2 = 300$

$x^2 = \frac{300}{12}$

so we get

$x^2 = 25$

$x = \sqrt{25} = 5$

here

length of rectangle = 4 × 5 = 20cm

breadth of rectangle = 3 × 5 = 15cm

perimeter of the cross section = 2(l + b)

substituting the values

= 2(20 + 15)

= 2 × 35

= 70 cm

8. the volume of a cube is 729cm³. Find its surface area and the length of a diagonal.

Solution

It is given that

Volume of a cube $= 729cm^3$

We can write it as

$side \times side \times side = 729$

$(side)^3 = 729$

So we get

Side $= \sqrt[3]{729} = \sqrt[3]{9 \times 9 \times 9}$

Side $= 9cm$

We know that

Surface area of cube $= 6(side)^2$

Substituting the values

$= 6 \times (9)^2$

$= 6 \times 9 \times 9$

$= 486cm^2$

So the length of a diagonal $= \sqrt{3} \times side$

Substituting the values

$= \sqrt{3} \times 9$

$= 1.73 \times 9$

$= 15.57 \text{cm}$

9. the length of the longest rod which can be kept inside a rectangular box is 17 cm . if the inner length and breadth of box are 12cm and 8cm respectively , find its inner height .

Solution

Consider h m as the inner height

It is given that

Length of longest rod inside a rectangular box $= 17$ cm which is same as diagonal of rectangular box

$17 = \sqrt{l^2 + b^2 + h^2}$

Substituting the values

$17 = \sqrt{12^2 + 8^2 + h^2}$

By squaring on both sides

$17^2 = 12^2 + 8^2 + h^2$

By further calculation

$289 = 144 + 64 + h^2$

$289 = 208 + h^2$

So we get

$h^2 + 208 = 289$

$h^2 = 289 - 208 = 81$

$h = \sqrt{81} = 9cm$

therefore, the inner height of rectangular box is 9 cm.

10. A closed rectangular box has inner dimension 90cm by 80cm by 70cm. Calculate its capacity and the area of tin foil needed to line its inner surface.

Solution

It is given that

Inner length of rectangular box = 90cm

Inner breadth of rectangular box = 80cm

Inner height of rectangular box = 70 cm

We know that

Capacity of rectangular box = volume of rectangular box = $l \times b \times h$

Substituting the values

$= 90 \times 80 \times 70$

$= 504000 \text{ cm}^3$

Here

Required area of tin foil $= 2(lb + bh + lh)$

Substituting the values

$= 2(90 \times 80 + 80 \times 70 + 90 \times 70)$

By further calculation

$= 2(7200 + 5600 + 6300)$

So we get

$= 2 \times 19100$

$= 38200 \text{ cm}^2$

11. The internal measurements of a box are 20cm long, 16cm wide and 24 cm height. How many 4 cm cubes could be put into the box?

Solution

It is given that

Volume of box $= 20 \ cm \times 16cm \times 14cm$

Volume of cubes $= 4\text{cm} \times 4cm \times 4cm$

We know that

Number of cubes put into the box $= \dfrac{volume \ of box}{volume \ of \ cubes}$

Substituting the values

$$= \frac{20 \times 16 \times 24}{4 \times 4 \times 4}$$

$$= 5 \times 4 \times 6$$

$$= 120$$

Therefore, 120 cubes can be put into the box.

12. the internal measurements of a box are 10cm long, 8 cm wide and 7 cm high. How many cubes of side 2 cm can be put into the box ?

Solution

It is given that

Length of box = 10cm

Breadth of box = 8 cm

Height of box = 7cm

We know that

3 number of cubes of side 2cm can be put in box i.e. height of box is 7 cm so only 3 cubes can be put height wise.

13. A certain quantity of wood costs Rs 250 per m². A solid cubical block of such wood is bought for Rs 182.25. calculate the volume of the block and use the method of factors to find the length of one edge of the block.

Solution

It is given that

Cost of Rs 250 for 1m³ wood

Cost of Rs 1 for $\frac{1}{250}$ m³ wood

Cost of Rs 182.25 for $\frac{182.25}{250}$ m³ wood

Here

Quantity of wood $= \frac{182.25}{250}$ m³

Multiply and divide by 100

$= \frac{18225}{250 \times 100}$

So we get

$= \frac{18225}{25 \times 1000}$

$= \frac{729}{1000}$

$= 0.729 \text{ m}^3$

We know that

Volume of given block $= 0.729\text{m}^3$

Consider the length of one edge of block $= x$ m

So we get

$X^3 = 0.729 \text{ m}^3$

Now taking cube root on both sides

$$x = \sqrt[3]{0.729} = \sqrt[3]{\frac{729}{1000}}$$

it can be written as

$$= \sqrt{\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5}}$$

3	729
3	243
3	81
3	27
3	9
3	3
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

So we get

$$= \frac{3 \times 3}{2 \times 5}$$

$$= \frac{9}{10}$$

$$= 0.9 \text{ m}$$

Therefore, the length of one edge of the block is 0.9 m

14. A cube of 11cm edge is immersed completely in a rectangular vessal containing water. If the dimension of the base of the vessel are 15cm × 12 cm , find the rise in the water level in centimetres correct to 2 decimal places, assuming that no water over flows.

Solution

It is given that

Edge of cube = 11 cm

Volume of cube = edge^3

Substituting the values

= 11^3

= 11 × 11 × 11

=1331 cm^3

We know that

The dimension of the base of the vessel are 15cm × 12cm

Consider the rise in the water level = h cm

So the volume of cube = volume of vessel

Substituting the values

1331 = 15 × 12 × h

By further calculation

$h = \frac{1331}{15 \times 12}$

so we get

$$h = \frac{1331}{180} = 7.39 \text{ cm}$$

therefore the rise in the water level is 7.39 cm

15. A rectangle container , whose base is a square of side 6 cm, stands on a horizontal table and holds water upto 1 cm from the top. When a cube is placed in the water and is completely submerged , the water rises to the top and 2cm^3 of water over flows, calculate the volume of cube.

Solution

If the base of rectangle container is a square

l = 6cm and b = 6 cm

when a cube is placed in it, water rises to top i.e, through height 1 cm and 2 cm^3 of water overflows

We know that

Volume of cube = volume of water displaced

Substituting the values

$= 6 \times 6 \times 1 + 2$

$= 36 + 2$

$= 38 \text{ cm}^3$

16 .(a) two cubes, each with 12 cm edges are joined end to end. Find the surface area of the resulting cuboid.

(b) A solid cube of side 12cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between the surface area of the original cube and the sum of the surface areas of the new cubes.
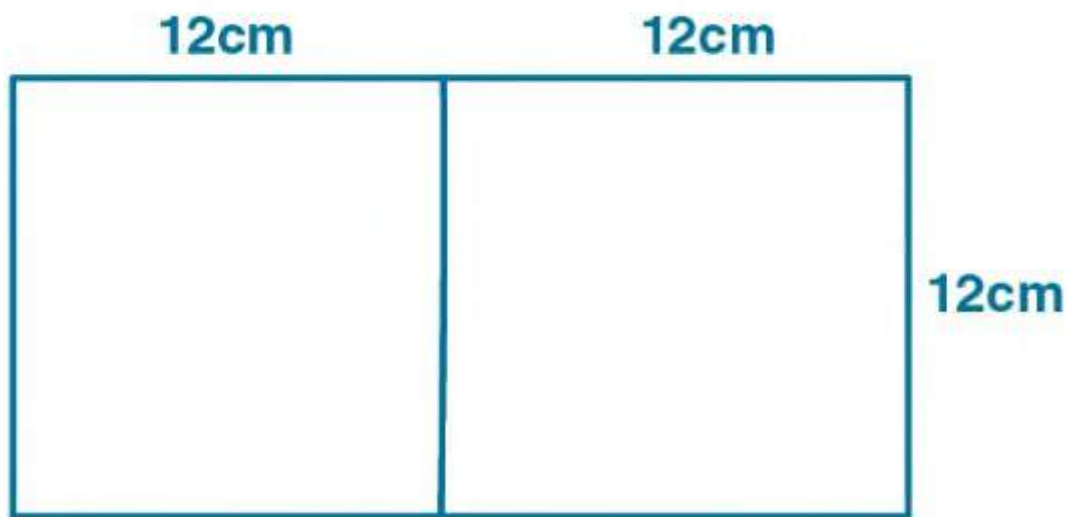
Solution

(a) we know that

By joining two cubes end to end a cuboid is formed whose dimensions are

$1 = 12 + 12 = 24 \text{ cm}$

$b = 12 \text{ cm}$

$h = 12 \text{ cm}$

Here

Total surface area of cuboid = 2(lb + bh + hl)

Substituting the values

= 2(24× 12 + 12 × 12 + 12 × 24)

By further calculation

= 2(288 +144 + 288)

So we get

= 2× 720

= 1440 cm^2

b) we know that

side of a cube = 12 cm

here

volume = side^3 = 12^3 = 1728cm^3
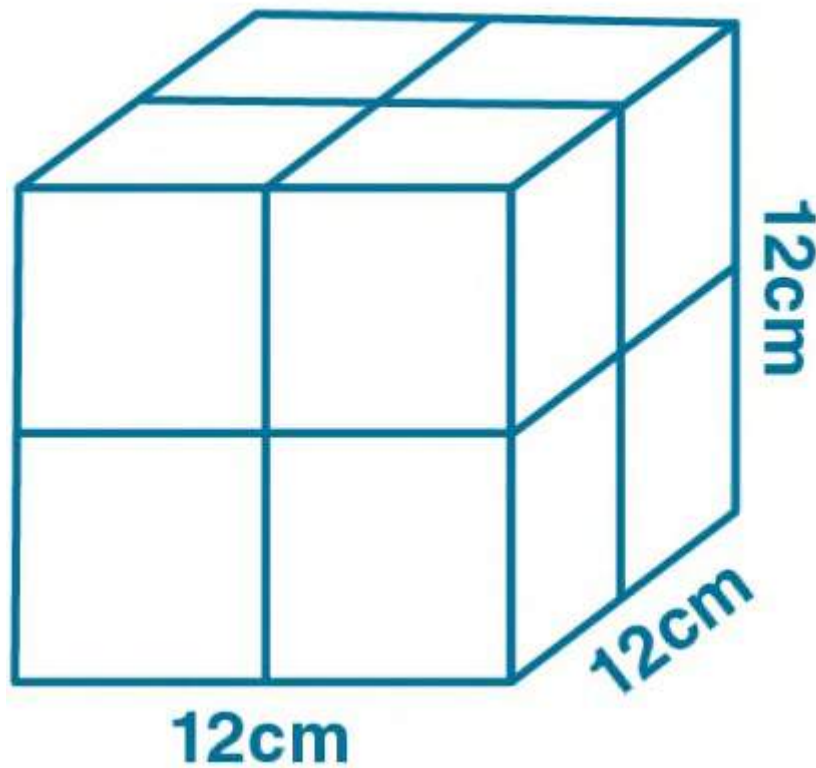
If cut into 8 equal cubes

Volume of each cube $= \dfrac{1728}{8} = 216 \text{cm}^3$

Side $= \sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6cm$

Surface area of original cube $= 6 \times side^2$

Substituting the values

$= 6 \times 12^2$

$= 6 \times 144$

$= 864 \text{ cm}^2$

Surface area of one smaller cube $= 6 \times 6^2$

So we get

= 6 × 36

= 216cm²

Surface area of 8 cube = 216 × 8

= 1728 cm²

So the ratio between their areas = 864 : 1728 = 1:2

17. A cube of a metal of 6 cm edge is melted and cast into a cuboid whose base is 9 cm × 8cm. find the height of the cuboid.

Solution

It is given that

Edge of melted cube = 6cm

Volume of melted cube = 6cm × 6cm × 6cm = 216 cm³

Dimensions of cuboid are

Length = 9cm

Breadth = 8cm

h cm is the height

we know that

volume of cuboid = $l \times b \times h$

substituting the values

$= 9 \times 8 \times h$

$= 72 \text{ h cm}^3$

Here

Volume of cuboid = volume of melted metal cube

Substituting the values

$72h = 216$

$h = \dfrac{216}{72} = 3 \text{cm}$

Therefore the height of cuboid is 3cm.

18. The area of a playground is 4800m². Find the cost of covering it with gravel 1cm deep, if the gravel costs Rs 260 per cubic meter.

Solution

It is given that

Area of playground $= 4800 \text{m}^2$

We can write it as

$l \times b = 4800$

We know that

Depth of level $= 1 \text{cm}$

$h = 1 \text{cm} = \dfrac{1}{100} \text{m}$

here

volume of gravel $= l \times b \times h$

substituting the values

$= 4800 \times \dfrac{1}{100}$

$= 48\text{m}^3$

Cost of gravel = Rs 260 per cubic metre

So the total cost $= 260 \times 48 = $ Rs 12480

19. A field is 30m long and 18m broad. A pit 6 m long, 4 m wide and 3 m deep is dug out from the middle of the field and the earth removed is evenly spread over the remaining area of the field . find the rise in the level of the remaining part of the field in centimetres correct to two decimal places.
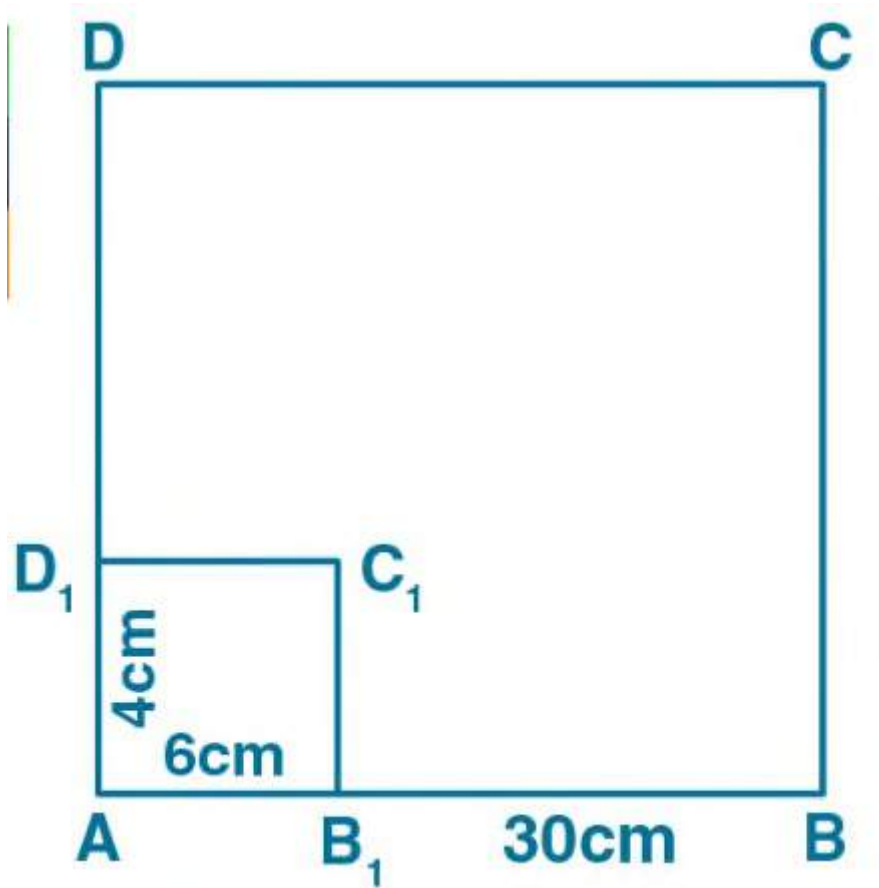
Solution

Consider ABCD is a field

Let ABCD be a part of the field where a pit is dug

Here

Volume of the earth dug out $= 6 \times 4 \times 3 = 72\text{m}^3$
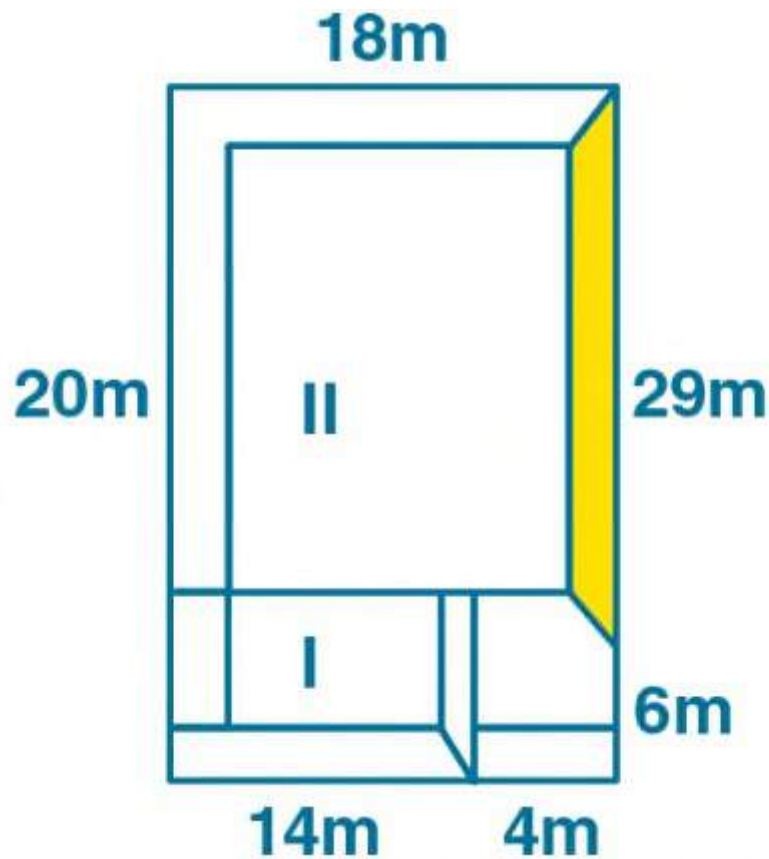
h m is the level raised over the field uniformly

Now divide the raised level of the field into parts I and II

Volume of part I $= 14 \times 6 \times h = 84h$ m^3

Volume of part II $= 24 \times 18 \times h = 432h$ m^3

Total volume of part I and II $= [84h + 432h] = 516h$ m^3

We know that

516 = volume of earth dug out

Substituting the values

516 = 72

So we get

$h = \frac{72}{516} = 0.1395 \text{m}$

multiply by 100

$h = 0.1395 \times 100$

$h = 13.95 \text{cm}$

Therefore, the level has been raised by 13.95 cm

20. A rectangular plot is 24m long and 20m wide. A cubical pit of edge 4m is dug at each of the four corners of the field and the soil removed is evenly spread over the remaining part of the plot. By what height does the remaining plot get raised?

Solution

It is given that

Length of plot (l) = 24m

Width of plot (b) = 20 m

So the area of plot = $l \times b$ = 24 × 20= 480m²

We know that

Side of cubical pit = 4m

Volume of each pit = 4^3 = 64m³

Here

Volume of 4 pits at the corners = 4 × 64 = 256m³

Area of surface of 4 pits = 4a²

= 4× 4²

= 64m²

So the area of remaining plot = 480 – 64 = 416m²

Height of the soil spread over the remaining plot = $\frac{256}{416} = \frac{8}{13}$ m

21. The inner dimensions of a closed wooden box are 2m , 1.2m and 0.75m. the thickness of the wood is 2.5cm. find the cost of wood required to make the box if 1m³ of wood Rs 5400.

Solution

It is given that

Inner dimension of wooden box are 2m , 1.2m and 0.75m

Thickness of wood $= 2.5$cm

So we get

$$= \frac{25}{10} cm$$

It can be written as

$$= \frac{25}{10} \times \frac{1}{100}$$

$$= \frac{1}{10} \times \frac{1}{4}$$

So we get

$$= \frac{1}{40}$$

$$= 0.025 \text{ m}$$

So the external dimensions of wooden box are

$$(2 + 2 \times 0.025), (1.2 + 2 \times 0.025), (0.75 + 2 \times 0.025)$$

By further calculation

$$= (2 + 0.05),(1.2 + 0.05) , (0.75 + 0.5)$$

$$= 2.05, 1.25, 0.80$$

Here

Volume of solid = external volume of box – internal volume of box

Substituting the values

$= 2.05 \times 1.25 \times 0.80 - 2 \times 1.2 \times 0.75$

By further calculation

$= 2.05 - 1.80$

$= 0.25 \text{m}^3$

Cost $= $ Rs 5400 for 1m^3

So the total cost $= 5400 \times 0.25$

Multiply and divide by 100

$= 5400 \times \dfrac{25}{100}$

$= 54 \times 25$

$= $ Rs 1350

22.A cubical wooden box of internal edge 1 m is made of 5 cm thick wood. The box is open at the top. If the wood costs Rs 9600 per cubic meter, find the cost of the wood required to make the box.

Solution

It is given that

Internal edge of cubical wooden box = 1m

Thickness of wood = 5cm

We know that

External length = 1m + 10 cm = 1.1m

Breadth = 1m + 10m = 1.1m

Height = 1m + 5 cm = 1.05m

Here

Volume of the wood used = outer volume – inner volume

Substituting the values

= 1.1 × 1.1 × 1.05 – 1 × 1 × 1

By further calculation

= 1.205 -1

= 0.2705m^3

Cost of 1 m^3 = Rs 9600

So the cost of 0.2705m^3 = 9600 × 0.2705

= Rs 2596.80

23. A square brass plate of side x cm is 1 mm thick and weight 4725g. If one cc of brass weights 8.4gm , find the value of x.

Solution

It is given that

Side of square brass plate = x cm

Here l = x cm and b = x cm

Thickness of plate = 1mm = $\frac{1}{10}$cm

We know that

Volume of plate = $l \times b \times h$

Substituting the values

$= x \times x \times \frac{1}{10}$

$= \frac{x^2}{10}$ cm^3 ....(1)

Here

8.4 gm weight brass having volume = 1cc

1 gm weight brass having volume = $\frac{1}{8.4}$ cc

So the 4725 gm weight brass having volume = $4725 \times \frac{1}{8.4} = 562.5$ cc

Volume of plate = 562.5 cc = 562.5 cm^3....(2)

Using both the equations

$$\frac{x^2}{10} = 562.5$$

By cross multiplication

$X^2 = 562.5 \times 10 = 5625$
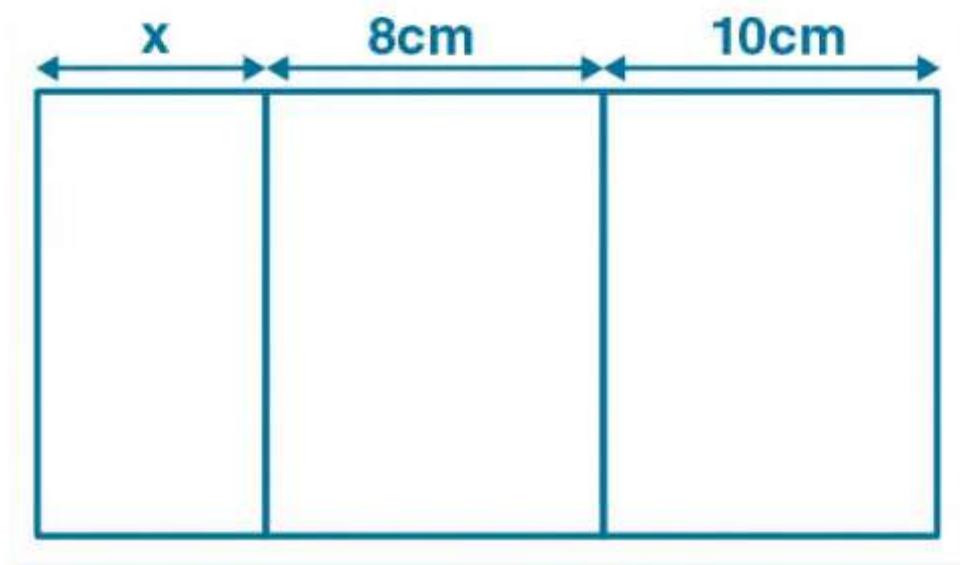
Here

$X = \sqrt{5625} = 75cm$

Therefore, the value of x is 75cm.

24. Three cubes whose edges are x cm , 8 cm and 10cm respectively are melted and recast into a single cube of edge 12 cm. Find x

Solution

It is given that

Edges of three cubes are x cm , 8 cm and 10 cm

So the volumes of these cubes are x^3, 8^3 and 10^3 i.e. x^3, 512 cm^3 and 1000cm^3

We know that

Edges of new cube formed = 12 cm

Volume of new cube = 12^3 = 1728 cm^3

Based on the question

X^3 + 512 + 1000 = 1728

By further calculation

X^3 + 1512 = 1728

X^3 = 216

It can be written as

X^3 = 6 × 6 × 6

So we get

X = 6 cm

25. The area of cross – section of a pipe is 3.5 cm^2 and water is flowing out of pipe at the rate of 40 cm/s. How much water is delivered by the pipe in one minute?

Solution

It is given that

Area of cross section of pipe = 3.5 cm^2

Speed of water = 40cm/sec

Length of water column in 1 sec = 40cm

We know that

Volume of water flowing in 1 second = area of cross section ×

length

Substituting the values

= 3.5 × 40

= 35 × 4

= 140cm^3

So the volume of water flowing in 1 minute i.e. 60 sec = 140 × 60

cm^3

Here

1 litre = 1000cm^3

So we get

Volume $= \frac{140 \times 60}{100}$

$= \frac{14 \times 6}{10}$

$= \frac{84}{10}$

= 8.4 litres

26. (a) The figure (i) given below shows a solid of uniform cross-section. Find the volume of the solid. All measurements are in cm and all angles in the figure are right angles.

(b) The figure (ii) given below shows the cross section of a concrete wall to be constructed. It is 2m wide at the top , 3.5 m wide at the bottom and its height is 6m and its length is 400 m . calculate

(i) the cross sectional area and

(ii) volume of concrete in the wall.

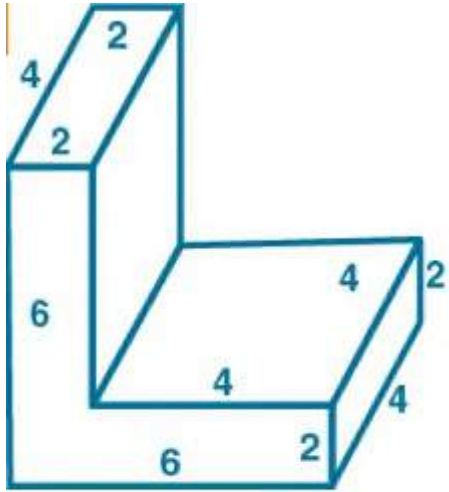**(c)** the figure(iii) given below show the cross section of a swimming pool 10 m broad, 2m deep at one end and 3 m deep at the other end. Calculate the volume of water it will hold when full, given that its length is 40 m.

(i)

(ii)

(iii)
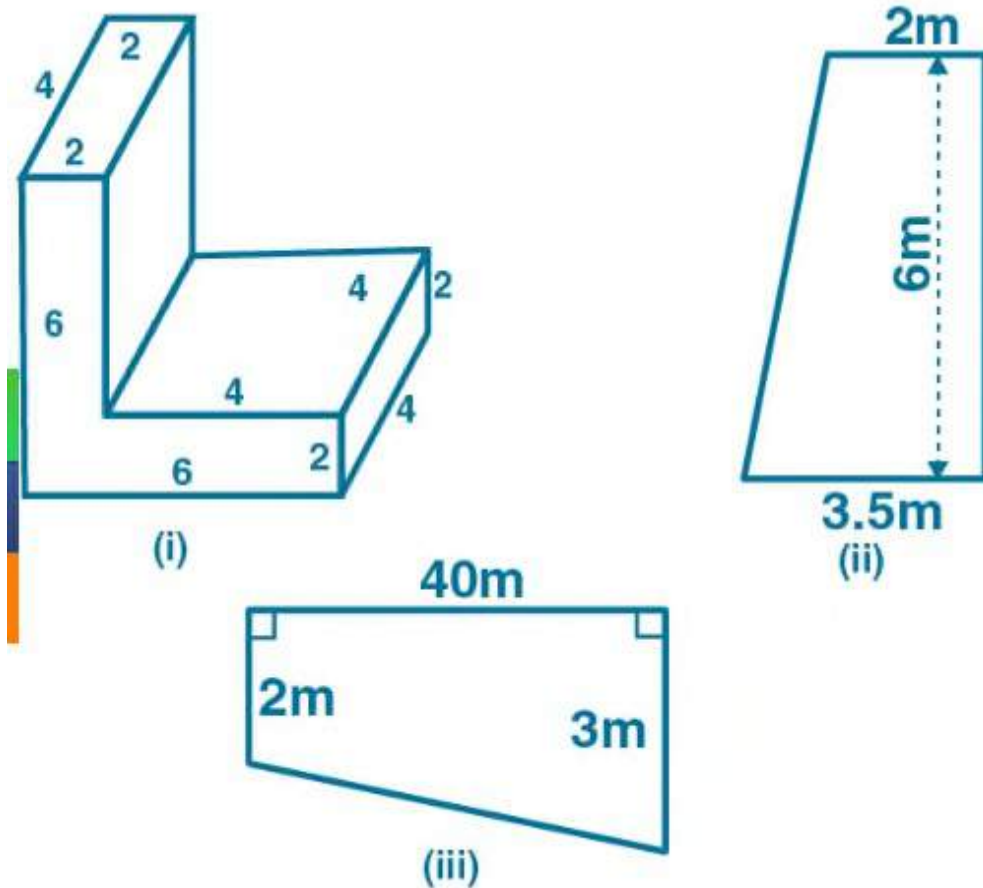
Solution

(a) we know that

The given figure can be divided into two cuboids of dimension 4cm , 4cm , 2cm , 4cm , 2cm and 6cm

Volume of solid $= 4 \times 4 \times 2 + 4 \times 2 \times 6$

$= 32 + 48$

$= 80 \text{ cm}^3$

(b) we know that

Figure (ii) is a trapezium with parallel sides 2m and 3.5 m

(i) area of cross section $= \dfrac{1}{2} \, (sum \; of \; parallel \; sides) \times height$

Substituting the values

$= \dfrac{1}{2} (2 + 3.5) \times 6$

By further calculation

$= \dfrac{1}{2} \times 5.5 \times 6$

So we get

$= 5.5 \times 3$

$= 16.5\text{m}^2$

(ii) volume of concrete in the wall = area of cross section × length

Substituting the values

$= 16.5 \times 400$

$= 165 \times 40$

$= 6600\text{m}^3$

(c) we know that

From figure(iii) we know that it is trapezium with parallel sides 2m and 3m

Here

Area of cross section $= \frac{1}{2}(sum\ of\ parallel\ sides) \times height$

Substituting the values

$= \frac{1}{2}(2+3) \times 10$

By further calculation

$= \frac{1}{2} \times 5 \times 10$

$= 5 \times 5$

$= 25\ \text{m}^2$

So the volume of water it will hold when full = area of cross section × height

$= 25 \times 40$

$= 1000 \text{ m}^3$

27. A swimming pool is 50 metres long and 15 meters wide. It shallow and deep ends are $1\frac{1}{2}$ metres and $14\frac{1}{2}$ metres deep respectively. If the bottom of pool slopes uniformly, find the amount of water required to fill the pool.

Solution

It is given that

Length of swimming pool = 50 m

Width of swimming pool = 15m

Its shallow and deep ends are $1\frac{1}{2}$ m and $5\frac{1}{2}$m deep

We know that

Area of cross section of swimming pool =

$\frac{1}{2}(sum\ of\ parallel\ sides) \times width$

Substituting the values

$= \frac{1}{2}\left(1\frac{1}{2} + 4\frac{1}{2}\right) \times 15$

By further calculation

$$= \frac{1}{2} \left(\frac{3}{2} + \frac{9}{2} \right) \times 15$$

So we get

$$= \frac{1}{2} \left[\frac{3+9}{2} \right] \times 15$$

$$= \frac{1}{2} \times \frac{12}{2} \times 15$$

$$= \frac{1}{2} \times 6 \times 15$$

$$= 3 \times 15$$

$$= 45\text{m}^2$$

Here

Amount of water required to fill pool = area of cross section × length

$$= 45 \times 50$$

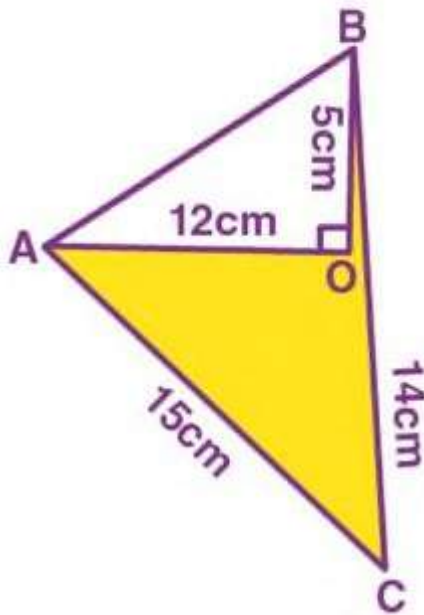$$= 2250\text{m}^3$$

Chapter test

Take $\pi = \dfrac{22}{7}$, useless stated otherwise

1(a) calculate the area of the shaded region.

(b) if the sides of a square are lengthened by 3cm , the area becomes 121cm². Find the perimeter of the original square.

Solution

(a) From the figure ,

OA is perpendicular to BC

It is given that

AC = 15cm , AO = 12cm , BO = 5cm , BC = 14cm

OC = BC – BO = 14 – 5 = 9 cm

Here

Area of right ΔAOC $= \frac{1}{2} \times base \times altitude$

Substituting the values

$= \frac{1}{2} \times 9 \times 12$

$= 54cm^2$

(b) consider the side of original square = x cm

So the length of given square = (x+3) cm

We know that

Area = side $\times$ side

Substituting the values

121 = (x +3)(x+3)

It can be written as
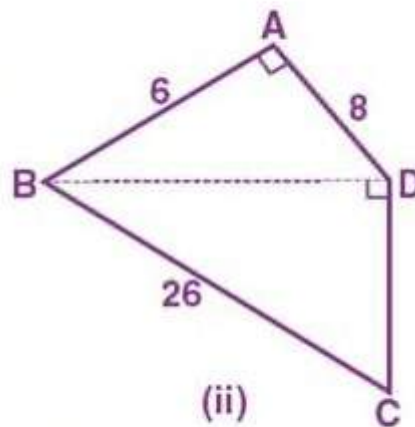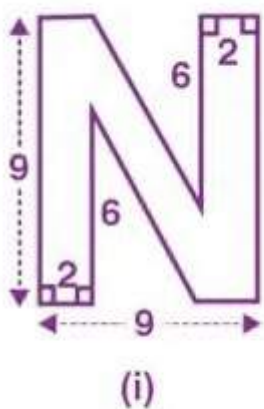
$11^2 = (x + 3)^2$

By further calculation

11 = x + 3

X = 11- 3

X = 8cm

2. (a) Find the area enclosed by the figure(i) given below . All measurements are in centimetres.
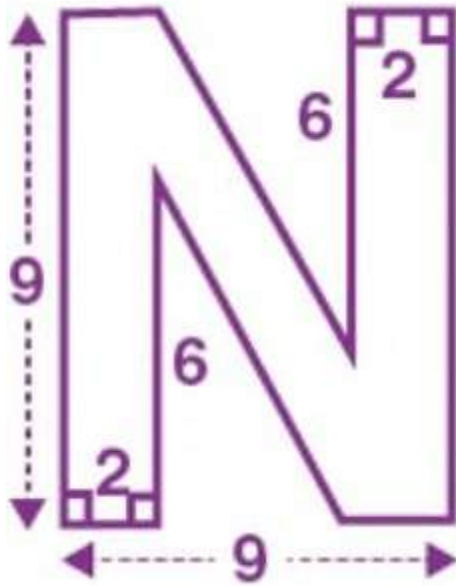
(b) Find the area of the quadrilateral ABCD shown in figure (ii) given below. All measurements are in centimetres.

(c) calculate the area of shaded region shown in figure(iii) given below. All measurements are in metres.

(i)

(ii)

(iii)
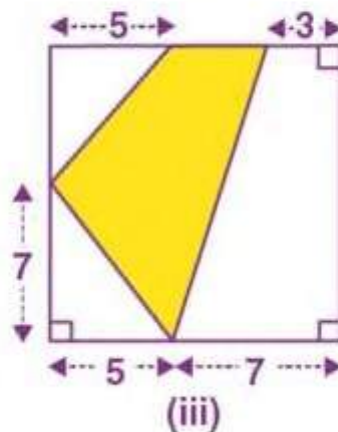
Solution

(a) we know that

Area of figure (i) = area of ABCD − area of both triangles

Substituting the values

$$= (9 \times 9) - \left(\frac{1}{2} \times 5 \times 6\right) \times 2$$
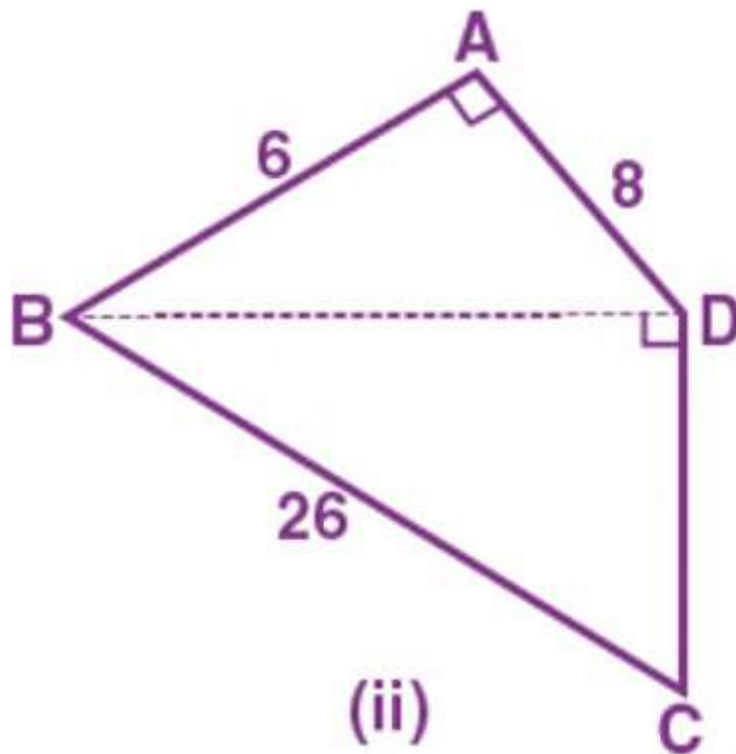
By further calculation

$$= 81 - (15 \times 2)$$

So we get

$$= 81 - 30$$

$$= 51 \text{ cm}^2$$

(b) in ΔABD

(ii)

Using Pythagoras theorem

$BD^2 = AB^2 + AD^2$

Substituting the values

$BD^2 = 6^2 + 8^2$

$BD^2 = 36 + 64$

So we get

$BD^2 = 100$

$BD = 10$ cm

In Δ BCD

Using Pythagoras theorem

$BC^2 = BD^2 + CD^2$

Substituting the values

$26^2 = 10^2 + CD^2$

$676 = 100 + CD^2$

By further calculation

$CD^2 = 676 - 100 = 576$

$CD = \sqrt{576}$

$CD = 24$ cm

Here

Area of the given figure = area of $\triangle$ABD + area of $\triangle$BCD

We can write it as

Area of the given figure = $\frac{1}{2} \times base \times height + \frac{1}{2} \times base \times height$

Area of the given figure = $\frac{1}{2} \times base \times height + \frac{1}{2} \times CD \times BD$

Substituting the values

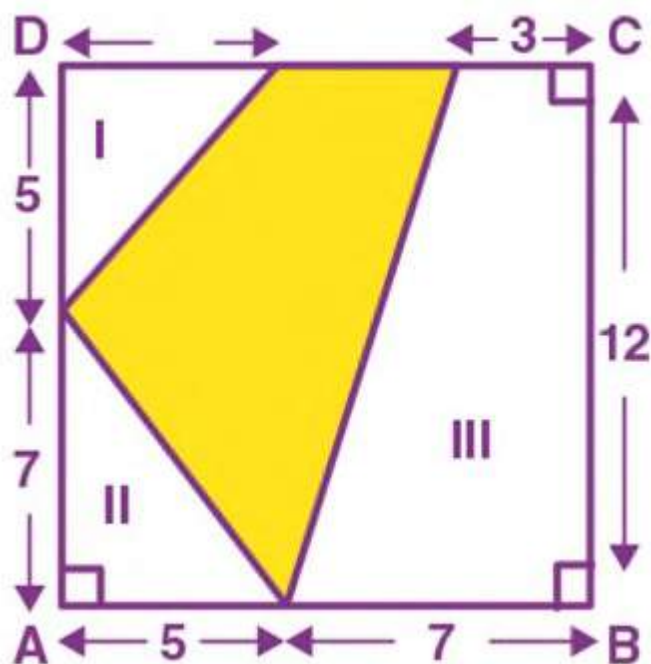Area of the given figure = $\frac{1}{2} \times 6 \times 8 + \frac{1}{2} \times 24 \times 10$

So we get

Area of the given figure = $3 \times 8 + 12 \times 10$

$= 24 + 120$

$= 144 cm^2$

(c) we know that

Area of the figure (iii) = area of ABCD – (area of 1st part + area of 2nd part + area of 3rd part)

It can be written as

$$= (\text{AB} \times \text{BC}) - \left[\left(\frac{1}{2} \times base \times height\right) + \left(\frac{1}{2} \times base \times height\right) + \frac{1}{2}(sum\ of\ parallel\ side\ \times height)\right]$$

Substituting the values

$$= (12 \times 12) - \left[\frac{1}{2} \times 5 \times 5 + \frac{1}{2} \times 5 \times 7 \times \frac{1}{2}(7 + 3) \times 12\right]$$

By further calculation

$$= 144 - \left[\frac{25}{2} + \frac{35}{2} + 10 \times 6\right]$$

$$= 144 - \left(\frac{60}{2} + 60 \right)$$

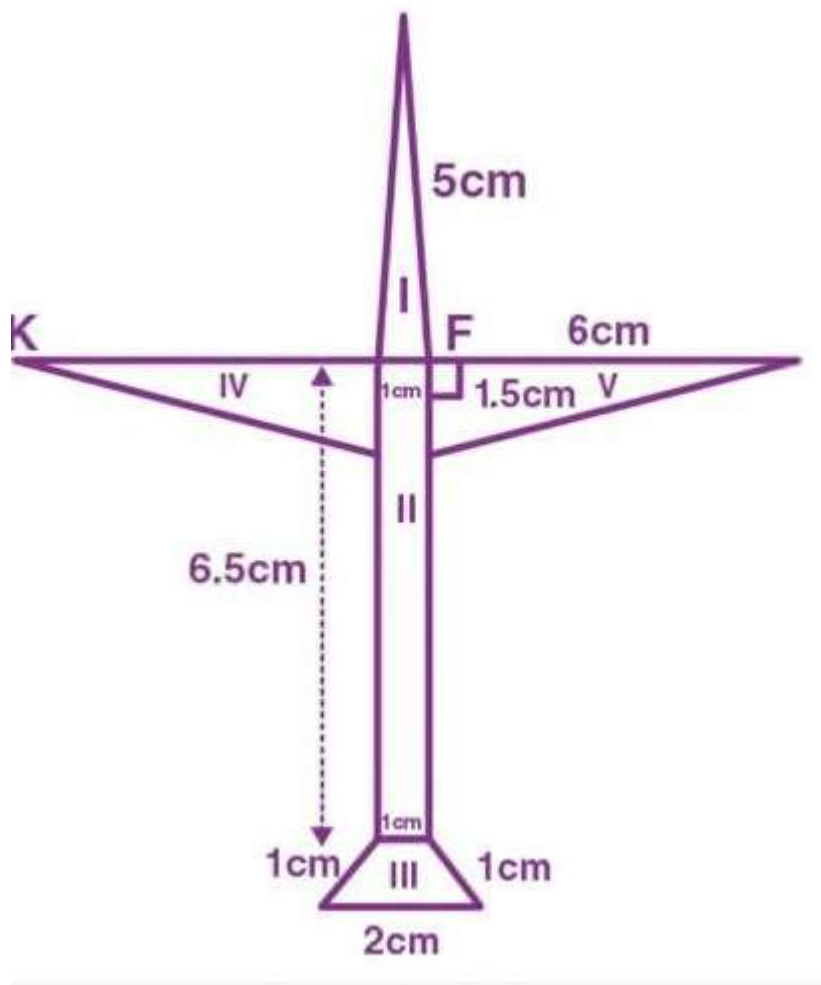$$= 144 - (30 + 60)$$

So we get

$$= 144 - 90$$

$$= 54 \text{ m}^2$$

Therefore, the required area of given figure $= 54\text{m}^2$

3. Asifa cut an aeroplane from a coloured chart paper (as shown in the adjoining figure). Find the total area of the chart paper used , correct to 1 decimal place.
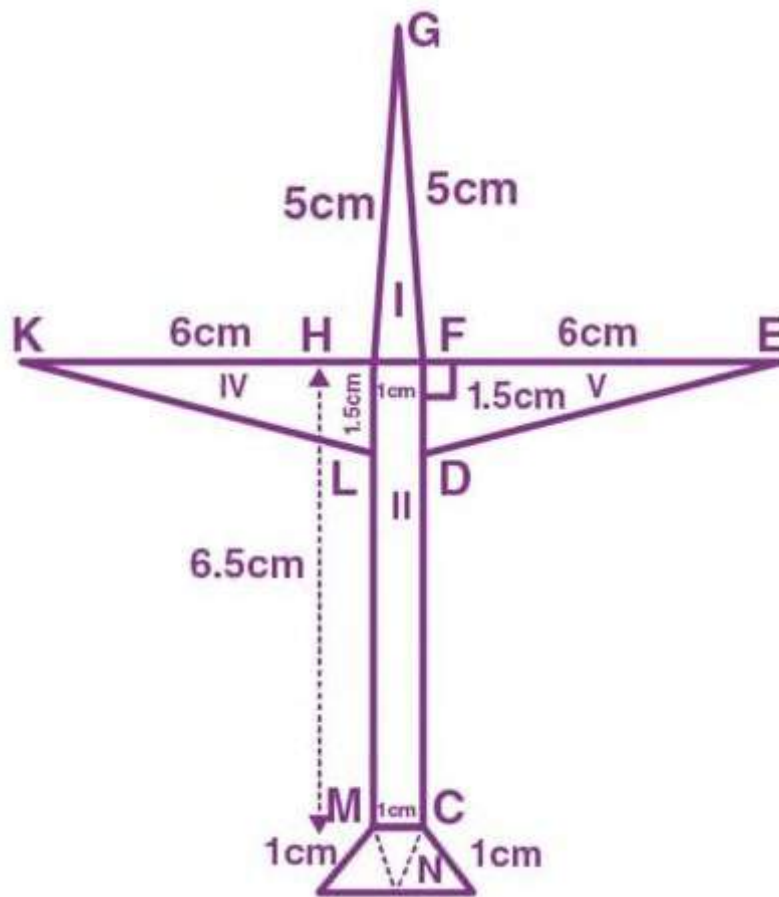
Solution

Here the whole figure of an aeroplane has 5 figures i.e. three

triangles, one rectangle and one trapezium

Consider M as the mid point of AB

AM = MB = 1cm

Now join MN and CN

Δ AMN , Δ NCB and ΔMNC are equilateral triangle having 1 cm

side each

Area of ΔGHF

$S = \frac{a+b}{c}$

$S = \frac{5+5+1}{2}$

$S = \frac{11}{2}$

We know that

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

Substituting the values

$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$

By further calculation

$= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}$

So we get

$= \sqrt{\frac{11 \times 9}{16}}$

$= \frac{3}{4}\sqrt{11}$

Here

$= \frac{3}{4} \times 3.316$

$= 3 \times 0.829$

$= 2.487$

$= 2.48 \text{cm}^2$

Area of rectangle $\|(MCFH) = l \times b$

Substituting the values

$= 6.5 \times 1$

$= 6.5 \text{cm}^2$

Area of Δ III +IV = $2 \times \dfrac{1}{2} \times 6 \times 1.5 = 9 \text{ cm}^2$

Area of three equilateral triangle formed trapezium III = $3 \times \dfrac{\sqrt{3}}{4} \times 1^2$

$= \dfrac{3}{4} \times 1.732$

$= 3 \times 0.433$

$= 1.299$

$= 1.3 \text{ cm}^2$

So we get

Total area = 2.48 + 6.50 + 9 + 1.30

= 19.28

= 19.3 cm²

4. if the area of a circle is 78.5 cm², find its circumference .(Take $\pi = 3.14$)

Solution

It is given that

Area of a circle = 78.5 cm²

Consider r as the radius

$r^2 = \dfrac{area}{\pi}$

substituting the values

$$r^2 = \frac{78.50}{3.14}$$

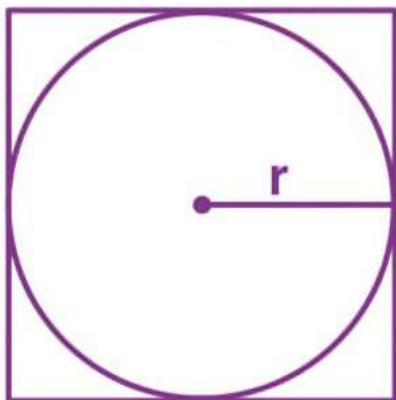$$r^2 = 25 = 5^2$$

so we get

r = 5 cm

here

circumference $= 2\pi r$

substituting the values

$= 2 \times 3.14 \times 5$

$= 31.4$ cm

5. From a square cardboard , a circle of biggest area was cut out. If the area of the circle is 154cm², calculate the original area of the cardboard.

solution

It is given that

Area of circle cut out from the square board $= 154 \text{cm}^2$

Consider r as the radius

$\pi r^2 = 154$

$\frac{22}{7} r^2 = 154$

By further calculation

$r^2 = \frac{154 \times 7}{22} = 49 = 7^2$

$r = 7 \text{cm}$

we know that

side of square $= 7 \times 2 = 14$ cm

so the area of the original cardboard $= a^2$

$= 14^2$

$= 196 \text{ cm}^2$

6.(a) From a sheet of paper of dimension 2m × 1.5m, how many circles of radius 5 cm can be cut ? also find the area of the paper wasted. Take $\pi = 3.14$
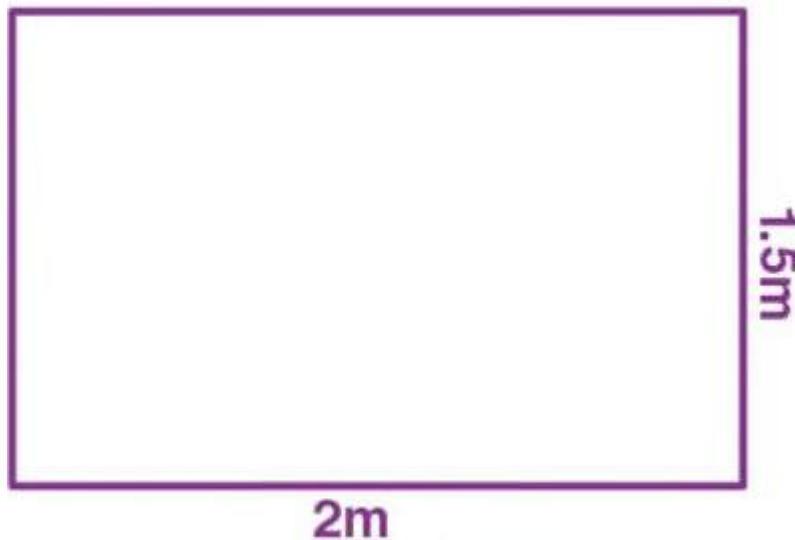
(b) if the diameter of a semi circular protractor is 14 cm , then find its perimeter.

Solution :

(a) it is given that

Length of sheet of paper = 2m = 200cm

Breadth of sheet = 1.5 m = 150 cm

Area = $l \times b$

Substituting the values

= 200 × 150

= 30000cm²

We know that

Radius of circle = 5 cm

Number of circle in lengthwise= $\frac{200}{5 \times 2}$ = 20

Number of circle in width wise = $\frac{150}{10}$ = 15

So the number of circle = 20 × 15 = 300

Here

Area of one circle = πr^2

Substituting the values

= 3.14 × 5 × 5cm²
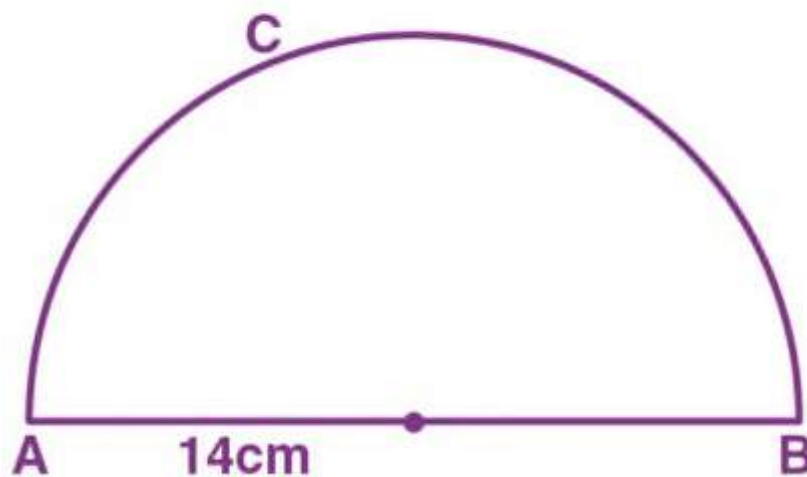
Area of 300 circle = 300 × $\frac{314}{100}$ × 25 = 23550cm²

So the area of remaining portion = area of square – area of 300 circle

= 30000 – 23550

= 6450 cm²

(b) it is given that

Diameter of semicircle protractor = 14 cm

We know that

Perimeter $= \frac{1}{2} \pi d + d$

Substituting the values

$= \frac{1}{2} \times \frac{22}{7} \times 14 + 14$

$= 22 + 14$

$= 36$ cm

7 . A road 3.5m wide surrounds a circular park whose circumference is 88m .find the cost of paving the road at the rate of Rs 60 per square meter.
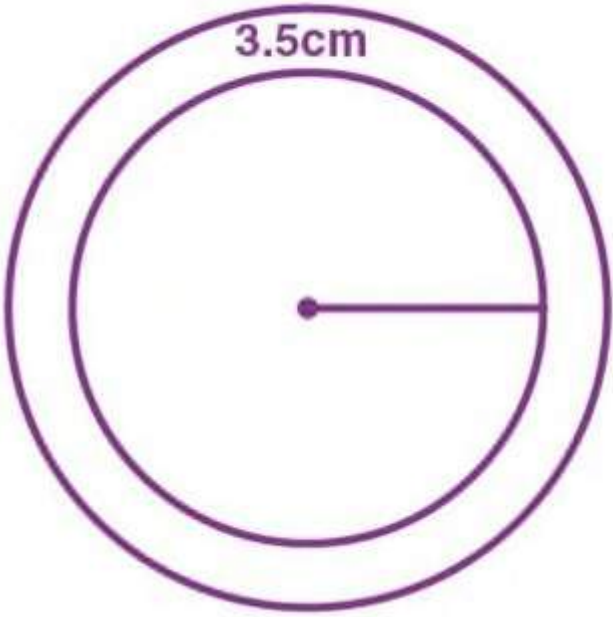
Solution

It is given that

Width of the road $= 3.5$ m

Circumference of the circular park $= 88$ m

Consider r as the radius of the park

2πr $= 88$

Substituting the values

$2 \times \frac{22}{7} r = 88$

By further calculation

$r = \frac{88 \times 7}{2 \times 22} = 14m$

here

outer radius (R) $= 14 + 3.5 = 17.5m$

area of the path $= \frac{22}{7} \times (17.5 + 14)(17.5 - 14)$

we know that

$\pi(R^2 - r^2) = \frac{22}{7}[(17.5)^2 - (14)^2]$

By further calculation

$= \frac{22}{7} (17.5 + 14)(17.5 - 14)$

$= \frac{22}{7} \times 31.5 \times 3.5$

$= 246.5 \text{ m}^2$

It is given that

Rate of paving the road = Rs 60 per m^2

So the total cost = 60 × 346.5 = Rs 20790

8. The adjoining sketch shows a running track 3.5 m wide all around which consists of two straight paths and two semicircular rings. Find the area of the track.

140m

42m

Solution

It is given that

Width of track = 3.5 m

Inner length of rectangular base = 140 m

Width = 42m

Outer length of rectangular base $= 140 + 2 \times 3.5$

$= 140 + 7$

$= 147$ m

Width $= 42 + 2 \times 3.5$

$= 42 + 7$

$= 49$ m

We know that

Radius of inner semicircle (r) $= \dfrac{42}{2} = 21$m

Outer radius (R) $= 21 + 3.5 = 24.5$ m

Here

Area of track $= 2(140 \times 3.5) + 2 \times \dfrac{1}{2}\pi(R^2 - r^2)$

Substituting the values

$= 2(490) + \dfrac{22}{7}[(24.5)^2 - (21)^2]$

By further calculation

$= 980 + \dfrac{22}{7}(24.5 + 21)(24.5 - 21)$
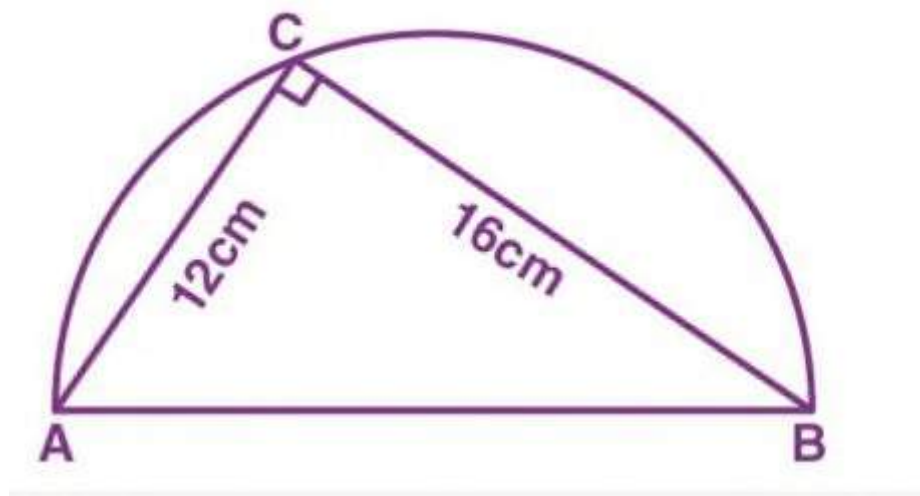
$980 + \dfrac{22}{7} \times 45.5 \times 3.5$

So we get

$= 980 + 500.5$

$= 1480.5$ m^2

9. in the adjoining figure, o is the centre of a circular arc and AOB is a line segment . Find the perimeter and the area of the shaded region correct to one decimal place. (take $\pi = 3.142$)

Hint. Angle in a semicircle is a right angle.

Solution

We know that

In a semicircle ∠ACB = 90º

Δ ABC is a right angled triangle

Using Pythagoras theorem

AB² = AC² + BC²

Substituting the values

$= 12^2 + 16^2$

$= 144 + 256$

= 400

$AB^2 = (20)^2$

AB = 20 cm

Radius of semicircle $= \dfrac{20}{2} = 10\ cm$

(i) we know that

Area of shaded portion = area of semicircle − area of ΔABC

It can be written as

$= \dfrac{1}{2}\pi r^2 - \dfrac{AC \times BC}{2}$

Substituting the values

$= \dfrac{1}{2} \times 3.142(10)^2 - \dfrac{12 \times 16}{2}$

By further calculation

$= \dfrac{314.2}{2} - 96$

$= 157.1 - 96$

$= 61.1\ cm^2$

(ii) here

Perimeter of shaded portion = circumference of semicircle + AC + BC

It can be written as

πr + 12 + 16

= 3.142 × 10 + 28

So we get

= 31.42 + 28

= 59.42 cm
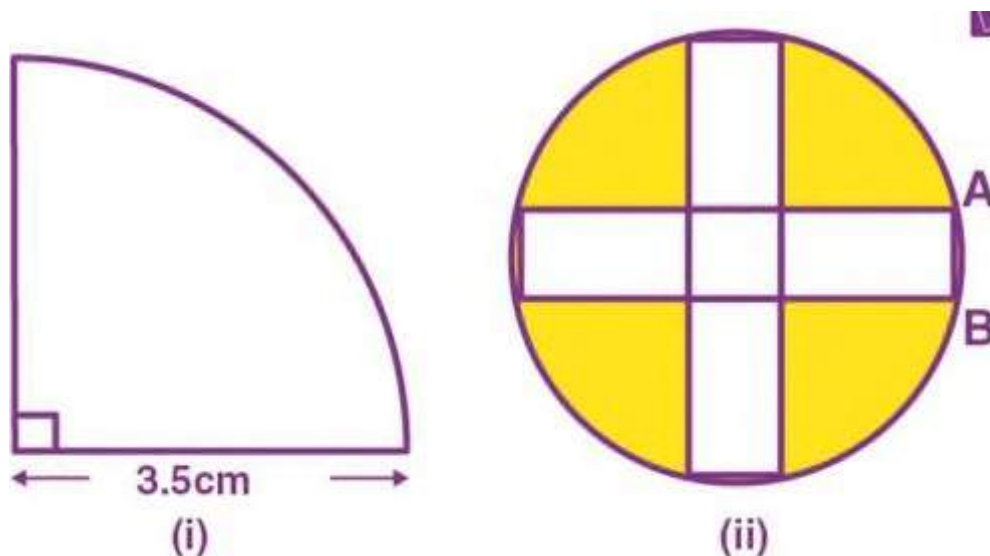
= 59.4 cm

10. (a) in the figure(i) given below the radius is 3.5cm . find the perimeter of the quarter of the circle.

(b) in the figure (ii) given below, there are five squares each of side 2 cm.

(i) Find the radius of the circle.

(ii) Find the area of the shaded region. (Take $\pi = 3.14$)

3.5cm
(i)   (ii)

Solution

(a) it is given that

Radius of quadrant = 3.5 cm

Perimeter $= 2r + \frac{1}{4} \times 2\pi r$
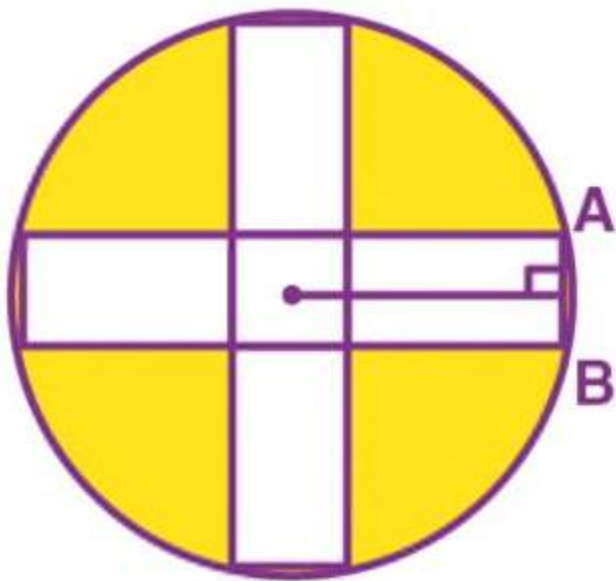
$= 2r + \frac{1}{2} \times \pi r$

Substituting the values

$= 2 \times 3.5 + \frac{1}{2} \times \frac{22}{7} \times 3.5$

So we get

$= 7 + 5.5$

$= 12.5$ cm

(b) from the figure

OB = 2 + 1 = 3 cm

AB = 1cm

Using Pythagoras theorem

OA = $\sqrt{OB^2 + AB^2}$

Substituting the values

OA = $\sqrt{3^2 + 1^2}$

= $\sqrt{9 + 1}$

= $\sqrt{10}$

So the radius of the circle = $\sqrt{10}$ cm

We know that

Area of the circle = πr^2

Substituting the values

= $3.14 \times (\sqrt{10})^2$

= 3.14×10

= 31.4 cm^2

Area of 5 square of side 2 cm each = $2^2 \times 5$

= 4×5

= 20 cm^2

So the area of shaded portion = 31.4 − 20 = 11.4 cm^2

11.(a) in the figure (i) given below , a piece of cardboard in the shape of quadrant of a circle of radius 7 cm is bounded by the perpendicular radii OX and OY. Points A and B lie on OX and OY respectively such that OA = 3 cm and OB = 4cm . triangular part OAB is removed. Calculate the area and the perimeter of the remaining piece.

(b) In the figure (ii) given below, ABCD is a square. Points A, B, C and D are centres of quadrants of circles of the same radius. If the area of the shaded portion is $21 \frac{3}{7}$ cm^2, find the radius of the quadrants .

(i)   (ii)
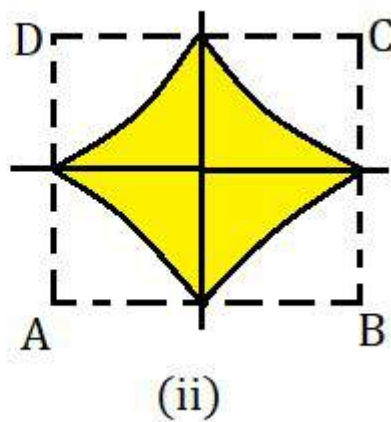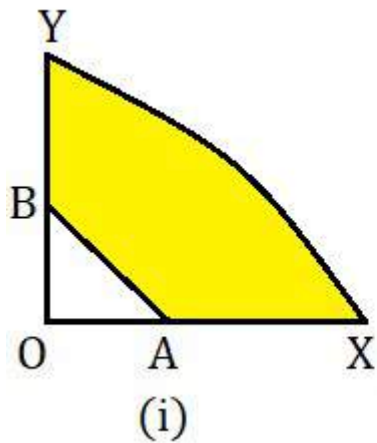
Solution

(a) it is given that

Radius of quadrant = 7 cm

OA = 3cm , OB = 4cm

AX = 7 -3 = 4 cm

BY = 7 – 4 = 3 cm

We know that

$AB^2 = OA^2 + OB^2$

Substituting the values

$= 3^2 + 4^2$

$= 9 + 16$

$= 25$

So we get

$AB = \sqrt{25} = 5$ cm

(i) area of shaded portion $= \frac{1}{4}\pi r^2 - \frac{1}{2}OA \times OB$

Substituting the values

$= \frac{1}{4} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 3 \times 4$

$= \frac{1}{4} \times \frac{22}{7} \times 49 - 6$

By further calculation

$$= \frac{77}{2} - 6$$

$$= \frac{65}{2}$$

$$= 32.5 \text{ cm}^2$$

(ii) perimeter of shaded portion $= \frac{1}{4} \times 2\pi r + AX + BY + AB$

Substituting the values

$$= \frac{1}{2} \times \frac{22}{7} \times 7 + 4 + 3 + 5$$

$$= 11 + 12$$

$$= 23 \text{ cm}$$

(b) we know that

ABCD is a square with centres A,B,C and D quadrants drawn.

Consider a as the side of the square

Radius of each quadrant $= \frac{a}{2}$

Here

Area of shaded portion $= a^2 - 4 \times \left[\frac{1}{4} \pi \left(\frac{a}{2} \right)^2 \right]$

We can write it as

$$= a^2 - 4 \times \frac{\frac{1}{4}\pi\ a^2}{4}$$

Substituting the values

$$= a^2 - \frac{22}{7} \times \frac{a^2}{4}$$

So we get

$$= a^2 - \frac{11a^2}{14}$$

$$= \frac{3a^2}{14}$$

Here

Area of shaded portion $= 21\frac{3}{7} = \frac{150}{7}$ cm^2

By equating both we get
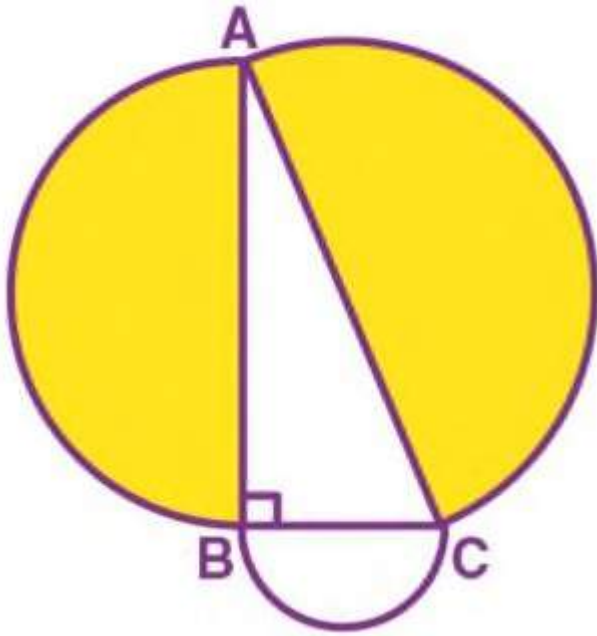
$$\frac{3a^2}{14} = \frac{150}{7}$$

On further calculation

$$a^2 = \frac{150}{7} \times \frac{14}{3}$$

$$a^2 = 100 = 10^2$$

$$a = 10 \text{ cm}$$

so the radius of each quadrant $= \frac{a}{2} = \frac{10}{2} = 5cm$

12. in the adjoining figure, ABC is a right angled triangle right angled at B. Semicircle are drawn on AB, BC and CA as diameter. Show that the sum of areas of semicircles drawn on AB and BC as diameter is equal to the area of semicircle drawn on CA as diameter.
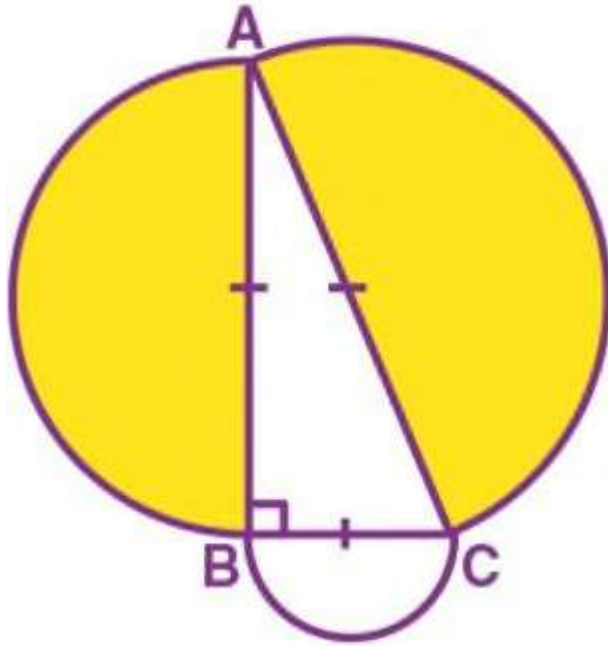
Solution

It is given that

ABC is right angled triangle right angled at B

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$ ......(i)

Area of semicircle on AC as diameter $= \frac{1}{2}\pi \left(\frac{AC}{2}\right)^2$

So we get

$= \frac{1}{2}\pi \times \frac{AC^2}{4}$

$= \frac{\pi AC^2}{8}$

Area of semicircle on AB as diameter $= \frac{1}{2}\pi \left(\frac{AB}{2}\right)^2$

So we get

$= \frac{1}{2}\pi \times \frac{AB^2}{4}$

$= \frac{\pi AB^2}{8}$

Area of semicircle on BC as diameter $= \frac{1}{2}\pi \left(\frac{BC}{2}\right)^2$

So we get

$$= \frac{1}{2}\pi \times \frac{BC^2}{4}$$

$$= \frac{\pi BC^2}{8}$$

We know that

$$\frac{\pi AB^2}{8} + \frac{\pi BC^2}{8} = \frac{\pi}{8}(AB^2 + BC^2)$$

From equation (i)

$$= \frac{\pi}{8}(AC)^2$$

$$= \frac{\pi AC^2}{8}$$

Therefore, it is proved

13. the length of minute hand of a clock is 14 cm . find the area swept by the minute hand in 15 minutes.

Solution

It is given that

Radius of hand = 14cm

So the area swept in 15 minutes $= \pi r^2 \times \frac{15}{60}$

By further calculation

$$= \frac{22}{7} \times 14 \times 14 \times \frac{1}{4}$$

= 154 cm²

14 . find the radius of circle if a 90⁰ arc has a length of 3.5 π cm

Hence, find the area of the sector formed by this arc.

Solution

It is given that

Length of arc of the sector of a circle = 3.5 π cm

Angle at the centre = 90⁰

We know that

Radius of the arc $= \dfrac{3.5\pi}{2} \times \dfrac{360}{90}$

So we get

$= \dfrac{3.5 \times 4}{2}$

$= 7$ cm

Area of the sector $= \pi r^2 \times \dfrac{90°}{360°}$

By further calculation

$= \dfrac{22}{7} \times 7 \times 7 \times \dfrac{1}{4}$

$= \dfrac{77}{2}$

$= 38.5$ cm²

15. A cube whose each edge is 28 cm long has a circle of maximum radius on each of its face painted red. Find the total area of the unpainted surface of the cube.

Solution

It is given that

Edge of cube = 28 cm

Surface area = 6 a²

Substituting the value

= 6 × (28)²

= 6 × 28 × 28

= 4704 cm²

Diameter of each circle = 28 cm

So the radius = $\frac{28}{2}$ = 14cm

We know that

Area of each circle = πr^2

Substituting the values

= $\frac{22}{7}$ × 14 × 14

= 616 cm²

Area of such 6 circles drawn on 6 faces of cube = 616 × 6 = 3696cm²

Here

Area of remaining portion of the cube = 4704 − 3696 = 1008 cm²

16. Can a pole 6.5 m long fit into the body of truck with internal dimensions of 3.5 m , 3m and 4m ?

Solution

No, a pole 6.5m long cannot fit into the body of a truck with internal dimension of 3.5 m , 3m and 4m

Length of pole = 6.5 m

Internal dimensions of truck are 3.5 m, 3m and 4 m

The internal dimensions are less than the length of pole. So the pole cannot fit into the body of truck with given dimensions.

17. A car has a petrol tank 40 cm long , 28cm wide and 25 cm deep . if the fuel consumption of the car averages 13.5 km per litre, how far can the car travel with a full tank of petrol ?

Solution

It is given that

Capacity of car tank = 40 cm $\times$ 28cm $\times$ 25cm = (40 $\times$ 28 $\times$ 25) cm^3

Here 1000cm^3 = 1 litre

So we get

$= \dfrac{40 \times 28 \times 25}{1000}$ litre

Average of car = 13.5 km per litre

We know that

Distance travelled by car $= \dfrac{40 \times 28 \times 25}{1000 \times 13.5}$

Multiply and divide by 10

$= \dfrac{(40 \times 25) \times 28}{1000} \times \dfrac{135}{10}$

So we get

$= \dfrac{1 \times 28}{1} \times \dfrac{135}{10}$

$= \dfrac{14 \times 135}{5}$

$= 14 \times 27$

$= 378$ km

Therefore, the car can travel 378 km with a full tank of petrol.

18. An aquarium took 96 minutes to completely fill with water. Water was filling the aquarium at a rate of 25 litres every 2 minutes. Given that the aquarium was 2 m long and 80 cm wide, compute the height of the aquarium.

Solution

We know that

Water filling in 2 minutes = 25 litres

Water filled in 1 minute = $\frac{25}{2}$ litres

Water filled in 96 minutes = $\frac{25}{2} \times 96$

= 25 × 48

= 1200 litres

So the capacity of aquarium = 1200 litres ....(1)

Here

Length of aquarium = 2m = 2 × 100 = 200 cm

Breadth of aquarium = 80 cm

Consider h cm as the height of aquarium

So the capacity of aquarium = 200 × 80 × h cm³

We can write it as

$= \frac{200 \times 80 \times h}{1000}$ litres

$= \frac{1}{5} \times 80 \times h$ litres

= 16 h litres ......(2)

Using equation (1) and (2)

16h = 1200

So we get

$h = \dfrac{1200}{16}$

h = 75 cm

therefore, height of aquarium = 75 cm.

19. The lateral surface area of a cuboid is 224 cm². Its height is 7 cm and the base is a square. find

(i) a side of the square, and

(ii) the volume of the cuboid

Solution

It is given that

Lateral surface area of a cuboid = 224cm²

Height of cuboid = 7 cm

Base is square

Consider x cm as the length of cuboid

X cm as the breadth of cuboid (since the base is square both length and breadth are same)

Here

Lateral surface area = 2(l + b)× h

Substituting the values

= 224 = 2$(x + x)$ × 7

By further calculation

$224 = 2 \times 2x \times 7$

$224 = 28x$

So we get

$28x = 224$

$X = \dfrac{224}{28} = 8cm$

(i) side of the square = 8 cm

(ii) we know that

Volume of the cuboid = $l \times b \times h$

Substituting the values

$= 8 \times 8 \times 7$

$= 448 \ cm^3$

20. if the volume of a cube is V m^3, its surface area is S m^2 and the length of a diagonal is d metres, prove that $6\sqrt{3} \ V = sd$

Solution

We know that volume of cube = (v) = (side)^3

Consider a as the side of cube
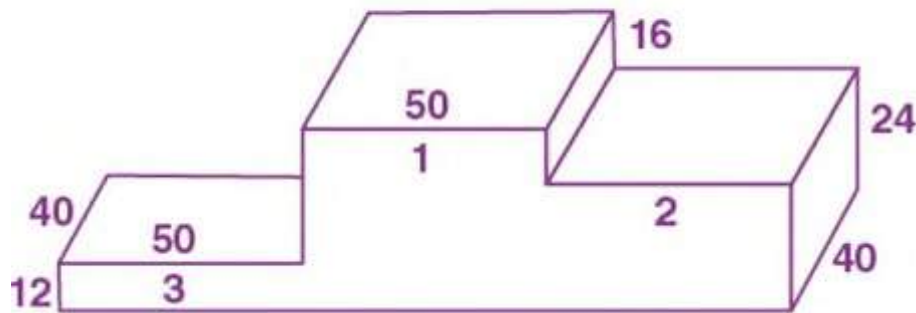
$V = a^3$ and $S = 6a^2$

Diagonal (d) = $\sqrt{3}.a$

$Sd = 6a^2 \times \sqrt{3}a = 6\sqrt{3}a^3$

Here $V = a^3$

So we get

$Sd = 6\sqrt{3}V = sd.$

21. The adjoining figure shows a victory stand, each face is rectangular. All measurements are in centimetres. Find its volume and surface area (the bottom of the stand is open).

Solution

Three parts are indicated as 3,1 and 2 in the figure

We know that

Volume of part $(3) = 50 \times 40 \times 12 = 24000 \text{cm}^3$

Volume of part $(1) = 50 \times 40 \times (16 + 24)$

By further calculation

$= 50 \times 40 \times 40$

= 80000cm^3

Volume of part (2) = 50 × 40 × 24 = 48000 cm^3

So the total volume = 24000 + 8000 + 48000 = 153000cm^3

We know that

Total surface area = area of front and back + area of vertical faces + area of top Faces

Substituting the values

= 2(50× 12 + 50 × 40 + 50 × 24)cm^2 + (12× 40 + 28 × 40 + 16 × 40 + 24 × 40) cm^2 + 3(50 × 40)cm^2

By further calculation

= 2(600 + 2000 + 1200) cm^2 + (480 + 1120 + 640 + 960)cm^2 +(3× 2000) cm^2

= 2(3800) + 3200 + 6000cm^2

So we get

= 7600 + 3200 + 6000

= 16800 cm^2

22. the external dimensions of an open rectangular wooden box are 98 cm by 84 cm by 77cm. If the wood is 2 cm thick all around, find

(i) the capacity of the box

(ii) the volume of the wood used in making the box , and

(iii) the weight of the box in kilograms correct to one decimal place, given that 1 cm^3 of wood weight 0.8g.

Solution

It is given that

External dimensions of open rectangular wooden box = 98cm, 84cm and 77cm

Thickness = 2cm

So the internal dimensions of open rectangular wooden box = (98 -2 × 2) cm ,(84 – 2 × 2)cm and (77-2) cm

= (98- 4)cm , (84 – 4)cm , 75 cm

= 94 cm , 80 cm , 75cm

(i) we know that

Capacity of the box = 94 cm × 80cm × 75cm

= 564000 cm^3

(ii) internal volume of box = 564000 cm^3

External volume of box = 98 cm × 84 cm × 77cm = 633864cm^3

So the volume of wood used in making the box = 633864 – 564000

= 69864 cm^3

(iii) weight of 1 cm³ wood = 0.8gm

So the weight of 69864 cm³ wood = 0.8×69864 gm.

By further calculation

$$= \frac{0.8 \times 69864}{1000} \text{ kg}$$

$$= \frac{55891.2}{1000} \text{ kg}$$

= 55.9 kg (correct to one decimal place)

23. A cuboidal block of metal has dimensions 36 cm by 32cm by 0.25 m .it is melted and recast into cubes with an edge of 4 cm.

(I) how many such cubes can be made?

(II) what is the cost of silver coating the surfaces of the cubes at the rate of Rs 1.25 per square centimetre ?

Solution

It is given that

Dimension of cuboidal block $= 36$ cm, 32cm and 0.25m

We can write it as

$= 36 \ cm \ \times 32cm \ \times (0.25 \ \times 100)\text{cm}$

$= (36 \ \times 32 \ \times 25)cm^3$

Volume of cube having edge 4 cm $= 4 \ \times 4 \times 4 = 64$ cm³

(i) we know that

Number of cubes $= \dfrac{\text{volume of cuboidalblock}}{\text{volume of one cube}}$

Substituting the values

$= \dfrac{36 \times 32 \times 25}{64}$

$= \dfrac{36 \ \times 25}{2}$

So we get

$= 18 \times 25$

$= 250$

(ii) here

Total surface area of one cube $= 6(a)^2$

Substituting the values

$= 6(4)^2$

$= 6 \ \times 4 \times 4$

$= 96$ cm²

So the total surface area of 450 cubes $= 450 \ \times 96 = 43200$ cm²

Given

Cost of silver coating the surface for 1 cm^2 = Rs 1.25

Cost of silver costing the surface for 43200 cm^2 = 43200 $\times$ 1.25 = Rs 54000

24. Three cubes of silver with edges 3 cm , 4 cm and 5 cm are melted and recast into a single cube . find the cost of coating the surface of the new cube with gold at the rate of Rs 3.50 per square centimetre.

Solution

We know that

Volume of first cube = edge^3

Substituting the values

= (3cm)^3

= 3 $\times$ 3 $\times$ 3

= 27cm^3

Volume of second cube = edge^3

Substituting the values

= (4cm)^3

= 4 $\times$ 4 $\times$ 4

= 64cm^3

Volume of third cube = edge³

Substituting the values

= (5 cm)³

= 5 × 5 × 5

= 125 cm³

So the total volume = 27 + 64 + 125 = 216 cm³

Make new cube whose volume = 216 cm³

So we get

Edge³ = 216 cm³

Edge³ = (6 cm)³

Edge = 6cm

Surface area of new cube = 6(edge)²

Substituting the values

= 6(6)²

= 6 × 6 × 6

= 216cm²

Given

Cost of coating the surface for 1 cm² = Rs 3.50

So the cost of coating the surface for 216cm² = 3.50 × 216 = Rs 756

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