MATTER WAVES, RADIATION AND PHOTOELECTRIC EFFECT [JEE ADVANCED PREVIOUS YEAR SOLVED PAPERS]

JEE Advanced

Single Correct Answer Type

1. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential in volts is

a. 2 **b.** 4 c. 6 **d.** 10

(IIT-JEE 1997)

2. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately

a. 540 nm **b.** 400 nm c. 310 nm **d.** 220 nm

(IIT-JEE 1998)

3. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de Broglie wavelengths of the particles λ_1/λ_2 is

a. $\frac{m_1}{m_2}$ **b.** $\frac{m_2}{m_1}$ c. 1.0 **d.** $\frac{\sqrt{m_2}}{\sqrt{m_1}}$

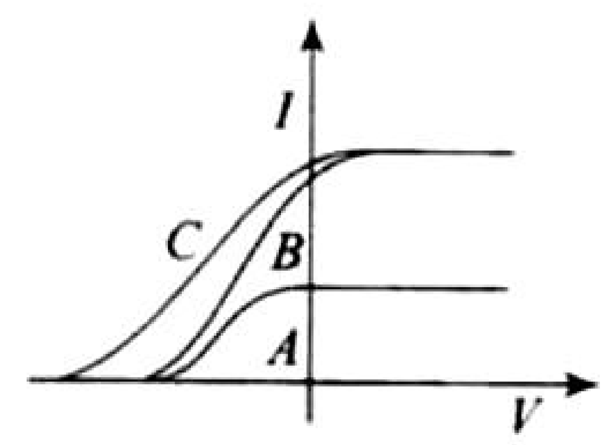
(IIT-JEE 1999)

4. A proton has kinetic energy E = 100 keV which is equal to that of a proton. The wavelength of photon is λ_2 and that of proton is λ_1. The ratio of λ_2/λ_1 is proportional to

a. E^2 **b.** $E^{-1/2}$

c. E^{-1} **d.** $E^{1/2}$ **(IIT-JEE 2004)**

5. In a photoelectric experiment, anode potential is plotted against plate current. Select the correct statement from the following.



a. A and B will have different intensities while B and C will have different frequencies

b. B and C will have different intensities while A and C will have different frequencies

c. A and B will have different intensities while A and C will have equal frequencies

d. A and B will have equal intensities while B and C have different frequencies **(IIT-JEE 2004)**

6. A beam of electron is used in a YDSE experiment. The slit width is d. When the velocity of electron is increased, then
   a. no interference is observed
   b. fringe width increases
   c. fringe width decreases
   d. fringe width remains same **(IIT-JEE 2005)**

7. If a star can convert all the He nuclei completely into oxygen nuclei, the energy released per oxygen nuclei is [Mass of He nucleus is 4.0026 amu and mass of oxygen nucleus is 15.9994 amu]
   a. 7.6 MeV b. 56.12 MeV
   c. 10.24 MeV d. 23.9 MeV
   (IIT-JEE 2005)

8. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 m/s. The final momentum of the object is
   a. 0.3×10^{-17} kg ms^{-1} b. 1.0×10^{-17} kg ms^{-1}
   c. 0.3×10^{-17} kg ms^{-1} d. 9.0×10^{-17} kg ms^{-1}
   (JEE Advanced 2013)

9. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240$ eV nm, the work function of the metal is nearly
   a. 3.7 eV b. 3.2 eV c. 2.8 eV d. 2.5 eV
   (JEE Advanced 2014)

Multiple Correct Answers Type

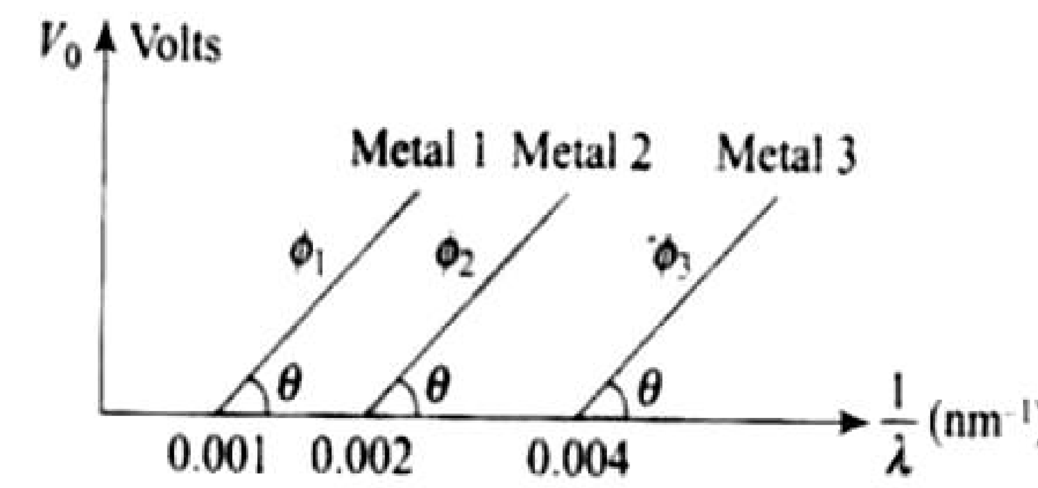
1. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a
   a. 50 W infrared lamp b. 1 W infrared lamp
   c. 50 W ultraviolet lamp d. 1 W ultraviolet lamp
   (IIT-JEE 1982)

2. Photoelectric effect supports quantum nature of light because
   a. there is a minimum frequency of light below which no photoelectrons are emitted
   b. the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
   c. even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
   d. electric charge of the photoelectrons is quantized
   (IIT-JEE 1987)

3. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are, respectively, 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then
   a. the stopping potential will be 0.2 V
   b. the stopping potential will be 0.6 V
   c. the saturation current will be 6.0 mA
   d. the saturation current will be 2.0 mA
   (IIT-JEE 1992)

4. When photons of energy 4.25 eV strike the surface of metal A, the ejected photoelectrons have maximum kinetic energy T_A and de Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then
   a. the work function of A is 2.25 eV
   b. the work function of B is 4.20 eV
   c. $T_A = 2.00$ eV
   d. $T_B = 2.75$ eV **(IIT-JEE 1994)**

5. The graph between the stopping potential (V_0) and ($1/\lambda$) is shown in the figure. ϕ_1, ϕ_2 and ϕ_3 are work functions. Then, which of the following is/are correct?



   a. $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
   b. $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
   c. $\tan \theta \propto (hc)/e$
   d. Ultraviolet light can be used to emit photoelectrons from metal 2 and metal 3 only
   (IIT-JEE 2006)

6. In Young's double-slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).
   a. If $d = \lambda$, the screen will contain only one maximum
   b. If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen
   c. If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase
   d. If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase
   (IIT-JEE 2008)

7. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0 where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck's constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are)
   a. $\frac{9}{32R}$ b. $\frac{9}{16R}$ c. $\frac{9}{5R}$ d. $\frac{4}{3R}$

Linked Comprehension Type

For Problems 1–3

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = p^2/2m$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ... (n = 1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving along the line from $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

(IIT-JEE 2009)

1. The allowed energy for the particle for a particular value of n is proportional to
   a. a^{-2} **b.** $a^{-3/2}$
   c. a^{-1} **d.** a^2
2. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and a = 6.6 nm, the energy of the particle in its ground state is closest to
   a. 0.8 meV **b.** 8 meV
   c. 80 meV **d.** 800 meV
3. The speed of the particle that can take discrete values is proportional to
   a. $n^{-3/2}$ **b.** n^{-1}
   c. $n^{1/2}$ **d.** n

Integer Answer Type

1. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p, respectively. The ratio λ_p/λ_α, to the nearest integer, is **(IIT-JEE 2010)**
2. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^z$ (where $1 < A < 10$). The value of "Z" is.

   (IIT-JEE 2011)
3. A proton is fired from very far away towards a nucleus with charge Q = 120e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, mp = (5/3) × 10^{-27} kg ; h/e = 4.2×10^{-15} J.s/C ; $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 m/F$; 1fm = 10^{-15} m)
4. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is **(JEE Advanced 2013)**

Fill in the Blanks Type

1. The maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the ______________ of the incident radiation.

   (IIT-JEE 1984)

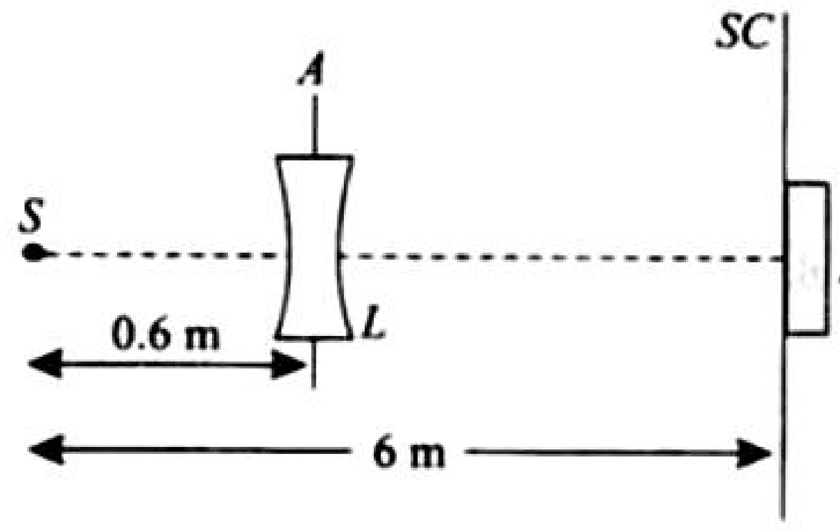
True/False Type

1. The kinetic energy of photoelectrons emitted by a photosensitive surface depends on the intensity of the incident radiation. **(IIT-JEE 1981)**
2. In a photoelectric emission process, the maximum energy of the photoelectrons increases with increasing intensity of the incident light. **(IIT-JEE 1986)**

Subjective Type

1. Light of wavelength 2000 Å falls on an aluminium surface. In aluminium, 4.2 eV of energy is required to remove an electron from its surface. What is the kinetic energy, in electron volt of (a) the fastest and (b) the slowest emitted photo-electrons. (c) What is the stopping potential? (d) What is the cut-off wavelength for aluminium? (Plack's constant $h = 6.6 \times 10^{-34}$ J-s and speed of light $c = 3 \times 10^8$ m s^{-1}). **(IIT-JEE 1986)**
2. Find the frequency of light which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photoelectric effect begins in this metal at a frequency of 6×10^{15} Hz. Find the work function for this metal. (Given $h = 6.63 \times 10^{-34}$ J-s). **(IIT-JEE 1987)**
3. A 40 W ultraviolet light source of wave-length 2480 Å illuminates a magnesium (Mg) surface placed 2 m away. Determine the number of photons emitted from the surface per second and the number incident on unit area of Mg surface per second. The photoelectric work function for Mg is 3.68 eV. Calculate the kinetic energy of the fastest electrons ejected from the surface. Determine the maximum wavelength for which the photoelectric effect can be observed with a Mg surface. **(IIT-JEE 1988)**
4. A beam of light has three wavelengths 4144 Å, 4972 Å, and 6216 Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed among the three wavelengths. The beams fall normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in 2 s. **(IIT-JEE 1990)**
5. A monochromatic point sources S radiating wavelength 6000 Å, with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in the figure. A photo-emissive detector D of surface area 0.5

cm^2 is placed at the center of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9.



a. Calculate the photon flux at the center of the screen and the photocurrent in the detector.
b. If a concave lens, L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.
c. If the work function of the photo-emissive surface is 1 eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture). **(IIT-JEE 1991)**

6. In a photoelectric effect set up a point source of light of power 3.2×10^{-3} W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the center of a stationary metallic sphere of work function 3.0 eV and of radius 8.0×10^{-3} m. The efficiency of photon-electron emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantaneously swept away after emission.
a. Calculate the number of photoelectrons emitted per second.
b. It is observed that photoelectron emission stops at a certain time t after the light source is switched on why?
c. Evaluate time t. **(IIT-JEE 1995)**

7. Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron, volt, and the least value of d for which the standing wave of the type described above can form. ($h = 6.63 \times 10^{-34}$ J-s) **(IIT-JEE 1997)**

8. Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α particles. A maximum energy electron combines with an α particles to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination. [Take $h = 4.14 \times 10^{-15}$ eV s] **(IIT-JEE 1999)**

9. When a beam of 1.6 eV photons of intensity 2.0 W m^{-2} falls on a platinum surface of area 1.0×10^{-4} m^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take 1 eV = 1.6×10^{-19} J. **(IIT-JEE 2000)**

10. In a photoelectric experiment set up, photons of energy 5 eV falls on the cathode having work function 3 eV. (a) If the saturation current is $i_A = 4$ μA for intensity 10^{-5} W/m^2, then plot a graph between anode potential and current. (b) Also draw a graph for intensity of incident radiation 2×10^{-5} W/m^2. **(IIT-JEE 2003)**

11. The potential energy of a particle as
$u(x) = E_0$ for $0 \le x \le 1$
$= 0$ for $x > 1$
For $0 \le x \le 1$, de-Broglie wavelength is λ_1 and λ_2 and for $x > 1$ the de-Broglie wavelength is λ_2. Total energy of particle is $2E_0$, find $\frac{\lambda_1}{\lambda_2}$. **(IIT-JEE 2005)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. b. **2.** c. **3.** c. **4.** b. **5.** d.
6. c. **7.** c. **8.** b. **9.** a.

Multiple Correct Answers Type

1. c., d. **2.** a., b., c. **3.** b., d.
4. a., b., c. **5.** a., c. **6.** a., b.
7. a, c.

Linked Comprehension Type

1. a. **2.** b. **3.** d.

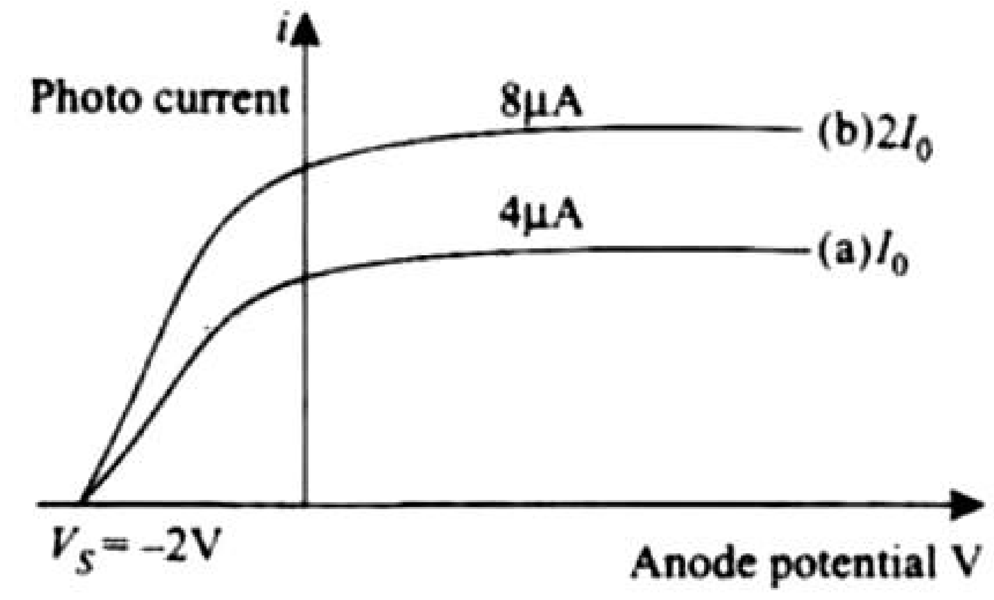
Integer Answer Type

1. (3) **2.** (7) **3.** (7) **4.** (1)

True/False Type

1. False **2.** False

Subjective Type

1. (a) 2eV (b) zero (c) 2 V (d) 3000 Å
2. 1.324×10^{15} Hz
3. 1.32 eV, 3373 Å
4. 1.075×10^{12}
5. (a) $2.87 \times 10^{13}\ s^{-1}m^{-2}$, $2.07 \times 10^{10}\ A$
   (b) $2.06 \times 10^{13}\ s^{-1}\ m^{-2}$, $1.483 \times 10^{-10}\ A$
   (c) 1.06 V in both cases
6. (a) 10^5 per sec (b) 1.1×10^7 (c) 111 s
7. (a) 15.8 eV (b) 0.5 Å
8. During combination 3.4 eV. After combination 2.84 eV, 2.64 eV
9. 6.25×10^{11}, zero, 5.0 eV
10. 

11. $\sqrt{2}$

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. b. Stopping potential is the negative potential applied to stop the electrons having maximum kinetic energy. Therefore, stopping potential will be 4 V.

2. c. $\lambda_{min} = \dfrac{hc}{\omega} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{4(1.6 \times 10^{-19})} = 310 \times 10^{-9}$ m

3. c. Applying conservation of linear momentum:
Initial momentum = Final momentum

$$0 = m_1v_1 - m_2v_2 \quad \Rightarrow \quad m_1v_1 = m_2v_2$$

Now, $\dfrac{\lambda_1}{\lambda_2} = \dfrac{h\ m_1v_1}{h\ m_2v_2} = 1$

4. b. For photon: $E = h\nu$

or $\quad E = \dfrac{hc}{\lambda}$

$\Rightarrow \quad \lambda_2 = \dfrac{hc}{E} \qquad$ (i)

For proton: $E = \dfrac{1}{2}m_pv_p^2$

$$E = \frac{1}{2}\frac{m_p^2v_p^2}{m} \quad \Rightarrow \quad P = \sqrt{2mE}$$

From de Broglie equation,

$$P = \frac{h}{\lambda_1}$$

$$\Rightarrow \quad \lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \qquad \text{(ii)}$$

$$\frac{h_2}{\lambda_1} = \frac{hc}{E \times \frac{h}{\sqrt{2mE}}} \propto E^{-1/2}$$

5. d. From the graph, it is clear that A and B have the same stopping potential and therefore the same frequency. Also, B and C have the same intensity.

6. c. Since electron shows wave nature, it will show the phenomenon of interference.

For electron: $\lambda = \frac{h}{mv}$

When speed of electron increases, λ will decrease.

The distance between two consecutive fringes

$$= \beta = \frac{\lambda D}{d}$$

As λ decreases, β also decreases.

7. c. $4\,{}_2^4\text{He} \rightarrow {}_8^{16}\text{O}$

$$\text{B.E.} = \Delta m \times 931.5 \text{ MeV}$$
$$= (4 \times 4.0026 - 15.9994) \times 931.5$$
$$= 10.24 \text{ MeV}$$

8. b. $t = 100 \times 10^{-9}$ s, $P = 30 \times 10^{-3}$ W, $C = C \times 10^8$ m s^{-1}

Momentum

$$= \frac{Pt}{C} = \frac{30 \times 10^{-3} \times 1000 \times 10^{-9}}{3 \times 10^8} = 1.0 \times 10^{-17} \text{ kg m s}^{-1}$$

9. a. $\frac{k_1}{k_2} = \left(\frac{u_1}{u_2}\right)^2 = 4$

$$k_1 = \frac{hc}{\lambda} - \phi = \frac{1240}{248} - \phi$$
$$k_1 = (5 - \phi)$$
$$k_2 = \frac{1240}{310} - \phi$$
$$k_2 = (4 - \phi)$$
$$\frac{k_1}{k_2} = \left(\frac{5-\phi}{4-\phi}\right) = 4 \quad \Rightarrow \quad 5 - \phi = 4(4 - \phi)$$

$$\Rightarrow \quad 5 - \phi = 16 - 4\phi$$

$$\Rightarrow \quad 3\phi = 11 \quad \Rightarrow \quad \phi = \frac{11}{3} = 3.7 \text{ eV}$$

Multiple Correct Answers Type

1. c., d.

The threshold wavelength is 5200 Å. For ejection of electrons, the wavelength of the light should be less than 5200 Å so that frequency increases and hence the energy of incident photon increases. UV light has less wavelength than 5200 Å.

2. a., b., c.

Standard result.

3. b., d.

Since the stopping potential depends on the frequency and not on the intensity and the source is same, the stopping potential remains unaffected. The saturation current depends on the intensity of incident light on the cathode of the photocell which in turn depends on the distance of the source from cathode. The intensity of light is inversely proportional to the square of the distance between the light source and photocell.

Intensity, $I \propto 1/r^2$ and saturation current $\propto I$ (Intensity)

$$\Rightarrow \quad \text{Saturation current} \propto \frac{1}{r^2}$$

$$\Rightarrow \quad \frac{(\text{Saturation current})_{\text{final}}}{(\text{Saturation current})_{\text{initial}}} = \frac{r^2_{\text{initial}}}{r^2_{\text{final}}}$$

$$\Rightarrow \quad (\text{Saturation current})_{\text{final}} = \frac{0.2 \times 0.2}{0.6 \times 0.6} \times 18 = 2 \text{ mA}$$

4. a., b., c.

For metal A: $4.25 = W_A + T_A$ (i)

$$\text{Also, } T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}\frac{m^2v_A^2}{m} = \frac{p_A^2}{2m}\frac{h^2}{2m\lambda_A^2} \qquad \text{(ii)}$$

$$\left[\because \lambda = \frac{h}{p}\right]$$

For metal B: $4.7 = (T_A - 1.5) + W_B$ (iii)

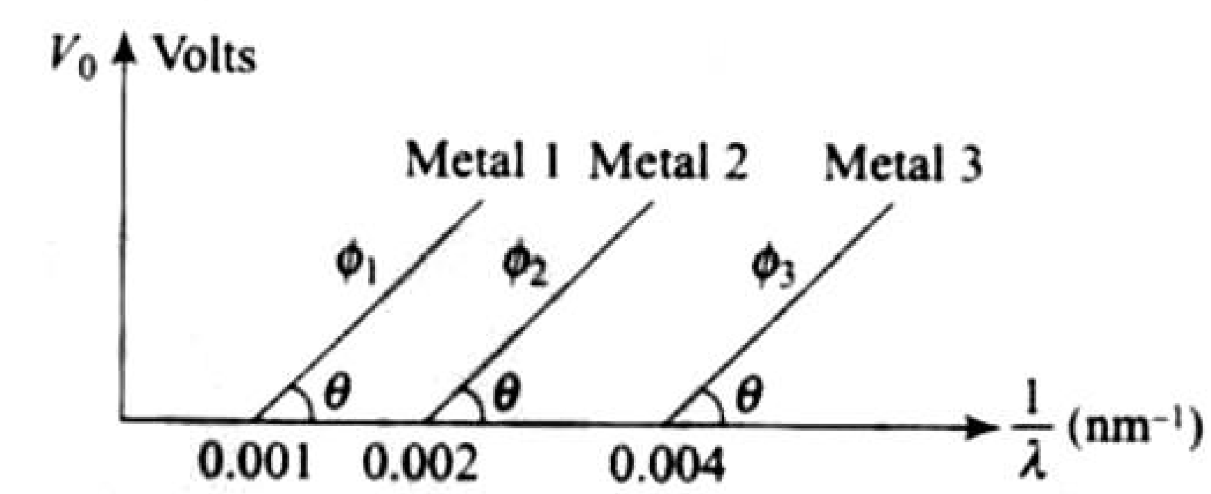
$$\text{Also, } T_B = \frac{h^2}{2m\lambda_B^2} \times \frac{2m\lambda_A^2}{h^2} = \frac{\lambda_A^2}{\lambda_B^2}$$

$$\Rightarrow \quad \frac{T_A - 1.5}{T_A} = \frac{\lambda_A^2}{2\lambda_A^2} = \frac{\lambda_A^2}{4\lambda_A^2} = \frac{1}{4} \qquad [\because \lambda_B = 2\lambda_A \text{ given}]$$

$$\Rightarrow \quad 4T_A - 6 = T_A \quad \Rightarrow \quad T_A = 2 \text{ eV}$$

5. a., c.

$$\phi_1 : \phi_2 : \phi_3 = eV_{0_1} : eV_{0_2} : eV_{0_3}$$



$$V_{0_1} : V_{0_2} : V_{0_3} = 0.001 : 0.002 : 0.004 = 1:2:4$$

Therefore, option (a) is correct.

By Einstein's photoelectric equation, $\frac{hc}{\lambda} - \phi = eV$

$$\Rightarrow \quad V = \frac{hc}{e\lambda} - \frac{\phi}{e} \qquad \text{(i)}$$

Comparing Eq. (i) by $y = mx + c$, we get the slope of the line

$$m = \frac{hc}{e} = \tan\theta$$

⇒ Option (c) is correct.

From the graph it is clear that,

$$\frac{1}{\lambda_{0_1}} = 0.001 \text{ nm}^{-1}$$

$$\Rightarrow \quad \lambda_{0_1} = \frac{1}{0.001} = 1000 \text{ nm}$$

Also, $\frac{1}{\lambda_{0_2}} = 0.002\ \text{nm}^{-1} \Rightarrow \lambda_{0_2} = 500$ nm

and $\lambda_{0_3} = 250$ nm

Violet color light will have wavelength less than 400 nm. Therefore, this light will be unable to show photoelectric effect on plate 3 ⇒ Option (d) is wrong.

6. a., b.

For $d = \lambda$, there will be only one central maxima.

For $\lambda < d < 2\lambda$, there will be three maxima on the screen corresponding to path difference,

$\Delta x = 0$ and $\Delta x = \pm \lambda$

∴ Correct options are (a) and (b).

7. a., c.

Given data

$$4.5 a_0 = a_0 \frac{n^2}{Z} \quad \text{(i)}$$

$$\frac{nh}{2\pi} = \frac{3h}{2\pi} \quad \text{(ii)}$$

So $n = 3$ and $z = 2$

So possible wavelength are

$$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_1 = \frac{9}{32R}$$

$$\frac{1}{\lambda_2} = RZ^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \Rightarrow \lambda_2 = \frac{1}{3R}$$

$$\frac{1}{\lambda_3} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_3 = \frac{9}{5R}$$

Linked Comprehension Type

1. a. $a = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2a}{n}$

$$\lambda_{\text{de Broglie}} = \frac{h}{p}$$

$$\frac{2a}{n} = \frac{h}{p} \Rightarrow p = \frac{nh}{2a}$$

$$E = \frac{p^2}{2m} = \frac{n^2h^2}{8a^2m}$$

⇒ $E \propto 1/a^2$

2. b. $E = \frac{h^2}{8a^2m} = \frac{(6.6\times10^{-34})^2}{8\times(6.6\times10^{-9})^2\times10^{-30}\times1.6\times10^{-19}}$

$= 8$ meV

3. d. $mv = \frac{nh}{2a}$

or $v = \frac{nh}{2am} \Rightarrow v \propto n$

Integer Answer Type

1. (3) de Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$

$$\lambda = \frac{h}{\sqrt{2mqV}} \quad [\because \text{Kinetic energy, } K = qV]$$

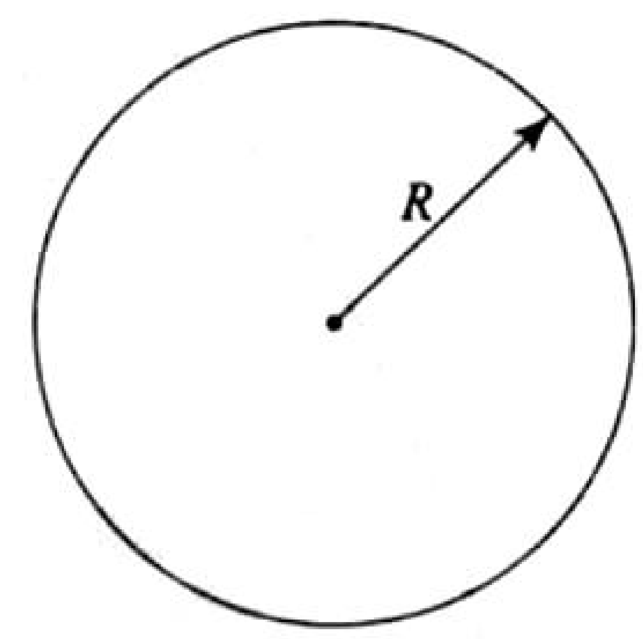
$$\lambda_p = \frac{h}{\sqrt{2m_pq_pV}}, \lambda_\alpha = \frac{h}{\sqrt{2m_\alpha q_\alpha V}}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \frac{h}{\sqrt{2m_pq_pV}} \times \frac{\sqrt{2m_\alpha q_\alpha V}}{h}$$

$$\frac{\sqrt{m_\alpha q_\alpha}}{\sqrt{m_pq_p}} = \frac{\sqrt{(4m_p)(2q_p)}}{\sqrt{(m_pq_p)}} = \sqrt{8} = 2\sqrt{2} \approx 3$$

2. (7) $R = 1$ cm

$f = 4.7$ cm



$$\frac{hc}{\lambda} = \phi + eV$$

$$\frac{1240(ev)(nm)}{200(nm)} = 4.7\ (eV) + eV$$

$$\frac{1240}{200}e = 4.7e + eV$$

$6.2 - 4.2 = V$

∴ $V = 1.5$ volt

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 1.5$$

$$(9\times10^9)\frac{Ne}{\frac{1}{100}} = 1.5$$

$9\times10^{11}Ne = 15$

$$N = \frac{1.5}{9\times10^{11}\times1.6\times10^{-19}} = \frac{15}{16}\times\frac{1}{9}\times10^8$$

$$= \frac{5}{3\times16}\times10^8 = \frac{50}{48}\times10^7$$

∴ $Z = 7$

3. (7)

+120 e —— r = 10 fm —— +e

$$\frac{(9\times10^9)(120e)(e)}{10\times10^{-15}} = \frac{p^2}{2m}$$

$$\lambda = \frac{h}{p}$$

$$\lambda^2 = \frac{h^2}{p^2} = \frac{h^2\times10\times10^{-15}}{2m\times9\times10^{-9}\times120e^2}$$

$$= \frac{4.2\times4.2\times10^{-30}\times10\times10^{-15}}{2(5/3)\times10^{-27}\times9\times10^{-9}\times120}$$

$$= \frac{42\times42}{36}\times10^{-30}$$

⇒ $\lambda = 7\times10^{-15}$ m $= 7$ fm

4. (1) According to Einstein's photoelectric equation

$$K_{max} = h\nu - \phi_0$$

where the symbols have their usual meaning.

But $K_{max} = eV_s$ where V_s is the stopping potential $V_s = \frac{h}{e}v - \frac{\phi_0}{e}$

Thus, the graph between $V_s - v$ is a straight line.
Compare the above relation with $y = mx + C$

$\therefore$ Slope of $V_s - v$ graph = $\frac{h}{e}$

It is same for both the metals.
$\therefore$ Ratio of the slopes = 1

Fill in the Blanks Type

1. According to laws of photoelectric effect,

$$\frac{1}{2}mv^2 = hv = hv_0$$

i.e., the maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the frequency of incident radiation.

True/False Type

1. False.
For photoelectric effect:
$hv - hv_0 = (\text{KE})_{max}$
where h = Planck's constant,
v = frequency of incident radiation, and
v_0 = threshold frequency.
$\Rightarrow$ $(\text{K.E.})_{max} \propto n$
KE does not depend on the intensity of incident radiation.
2. False. $(\text{KE.})_{max} = hv - hv_0 \Rightarrow (\text{KE})_{max} \propto v$
Thus, maximum kinetic energy is proportional to frequency and not intensity.

Subjective Type

1. Energy corresponding to incident photon,

$$hv = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}}$$

$$= 9.9 \times 10^{-19}\ \text{J} = \frac{9.9 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV} = 6.2\ \text{eV}$$

a. The kinetic energy of fastest electrons.
$E_k = hv - W$
or $E_k = 6.2\ \text{eV} - 4.2\ \text{eV} = 2\ \text{eV}$

b. The kinetic energy of slowest electrons is zero, since the emitted electrons have all possible energies from 0 to certain maximum value E_k.

c. If V_s is the stopping potential, then
$E_k = eV_s$
or $V_s = \frac{E_k}{e} = \frac{2\ \text{eV}}{e} = 2\ \text{V}$

d. If λ_0 is the cut-off wavelength for aluminium, then $W = (hc/\lambda_0)$
or $\lambda_0 = (hc/W)$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.2 \times 1.6 \times 10^{-19}} = 3000 \times 10^{-10}\ \text{m}$$

$$= 3000\ \text{Å}$$

2. According to Einstein's photoelectric equation,
$E_k = hv - W$

If V_s is retarding or stopping potential and v_0, the threshold frequency, then above equation becomes
$eV_s = hv - hv_0$
or $hv = eV_s + hv_0$
or $v = \frac{eV_s}{h} + v_0$

Hence, $e = 1.6 \times 10^{-10}$ coulomb, $V_s = 3$ V, and $v_0 = 6 \times 10^{-14}$ Hz.
Therefore, required frequency

$$v = \frac{1.6 \times 10^{-10} \times 3}{6.63 \times 10^{-34}} + 6 \times 10^{14}$$

$$= 7.24 \times 10^{14} + 6 \times 10^{14} = 13.24 \times 10^{14}\ \text{Hz}$$
$$= 1.324 \times 10^{15}\ \text{Hz}$$

3. Energy of each photon,

$$\varepsilon = \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2480 \times 10^{-10}}\ \text{J} = 8.0 \times 10^{-19}\ \text{J}$$

Number of photons emitted per second,

$$N = \frac{P}{\varepsilon} = \frac{40}{8.0 \times 10^{-19}} = 5.0 \times 1019\ \text{s}^{-1}$$

These photons spread in all directions over surface area $4\pi r^2$, therefore, the number of photons incident per unit area per second

$$N_s = \frac{N}{4\pi r^2} = \frac{5.0 \times 10^{19}}{4 \times 3.14 \times (2)^2}$$

(As $r = 2m$) $= 9.95 \times 10^{13}\ \text{s}^{-1}$

From Einstein's photoelectric equation,
$E_k = hv - W$

$$\varepsilon = hv = \frac{hc}{\lambda} = 8.0 \times 10^{-19}\text{J}$$

$$= \frac{8.0 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV} = 5.0\ \text{eV}$$

$\therefore$ $E_k = 5.0\ \text{eV} - 3.68\ \text{eV} = 1.32\ \text{eV}$

Threshold (or maximum) wavelength for photoelectrons emission,

$$\lambda_0 = \frac{hc}{W} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3.68 \times 1.6 \times 10^{-19}}\ \text{m}$$

$$= 3.373 \times 10^{-7} = 3373\ \text{Å}$$

4. We know that threshold wavelength $(\lambda_0) = \frac{hc}{\phi}$.

$$\lambda_0 = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{2.3 \times (1.6 \times 10^{-19})} = 5.404 \times 10^{-7}\ \text{m}$$

$$= 5404\ \text{Å}$$

Thus, wavelengths 4144 Å and 4972 Å will emit electrons from the metal surface.
Energy incident on the surface per unit time for each wavelength

$= \text{Intensity of each wavelength} \times \text{Area of the surface}$

$$= \frac{3.6 \times 10^{-3}}{3} \times (1.0\,\text{cm}^2) = 1.2 \times 10^{-7}\,\text{W}$$

Energy incident on the surface for each wavelength in 2 s,

$$E = (1.2 \times 10^{-7}) \times (2) = 2.4 \times 10^{-7}\,\text{J}$$

Number of photons n_1 due to wavelength 4144 Å (= 4144 × 10^{-10} m),

$$n_1 = \frac{(2.4 \times 10^{-7})(4144 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)} = 0.5 \times 10^{12}$$

Number of photons n_2 due to wavelength 4972 Å,

$$n_2 = \frac{(2.4 \times 10^{-7})(4972 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)} = 0.575 \times 10^{12}$$

$$N = n_1 + n_2 = 0.5 \times 10^{12} + 0.575 \times 10^{12}$$
$$= 1.075 \times 10^{12}$$

5. a. Energy of each photon $E = \dfrac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10}} = 3.315 \times 10^{-19}\,\text{joules}$$

Number of photons emitted per sec

$$N = \frac{\text{Power } (P)}{\text{Energy of each photon } (e)}$$

$$= \frac{2}{3.315 \times 10^{-19}} = 6.033 \times 10^{18}\,\text{sec}^{-1}$$

Solid angle subtended by aperture on source

$$\omega_1 = \frac{A_1}{r_1^2} = \frac{\pi \times (0.05)^2}{(0.6)^2} = \frac{1}{36} \times 0.785\,\text{steradian}$$

Solid angle subtended by detector at source

$$\omega_2 = \frac{A_2}{r_2^2} = \frac{0.5 \times 10^{-4}}{(6)^2} = \frac{1}{36} \times 5 \times 10^{-5}\,\text{steradian}$$

The solid angle subtended by detector is less than that subtended by aperture on the source, this indicates that all photons reaching the detector are allowed by aperture. Hence the number of photons reaching the detector per m² per second or

$$\text{Photon flux} = \frac{N}{4\pi^2} = \frac{6.033 \times 10^{18}}{4 \times 3.14 \times (6.0)^2}$$
$$= 1.334 \times 10^{16}\,\text{m}^2\,\text{sec}$$

Photo current $I = ne = [0.9 \times \text{photon flux} \times (\text{area of detector})]e$

$$= 0.9 \times 0.1334 \times 10^{16} \times 0.5 \times 10^{-4} \times 1.6 \times 10^{-19}$$
$$= 0.096 \times 10^{-6}\,\text{amp} = 0.096\,\mu\text{A}$$

As the source is at the focus of concave lens, the concave lens forms its image at a distance $\dfrac{f}{2} = 0.3$ m to the left of lens

$$\left[\text{since } \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \text{ or } \frac{1}{v} = \frac{1}{f} + \frac{1}{u}\right.$$
$$\left. = \frac{1}{-0.6} + \frac{1}{-0.6} = -\frac{1}{0.3} \therefore v = -0.3\,\text{m}\right]$$

Therefore the photon incident on lens may be supposed to be concentrated at a distance 0.3 m from the lens or 5.7 m from the detector.

Number of photons reaching aperture per sec

$$= N\frac{\pi r^2}{4\pi R^2} = N\frac{r^2}{4R^2}$$

$$= \frac{6.033 \times 10^{18} \times (0.05)^2}{4 \times (0.6)^2} = 1.047 \times 10^{16}$$

b. The detector transmits 80% photons, therefore the number of photons reaching the detector

$$= \frac{80}{100} \times (1.047 \times 10^{16})$$
$$\times \frac{\text{Solid angle subtended by detector}}{\text{Solid angle subtended by aperture}}$$

$$= \frac{80}{100} \times (1.047 \times 10^{16}) \times \frac{0.5 \times 10^{-4} \times (0.3)^2}{(5.7)^2 \times 3.14 \times (0.05)^2}$$

$$= 1.478 \times 10^{11}\,\text{per sec}$$

Photon flux reaching at detector

$$\frac{1.478 \times 10^{11}}{\text{Area of detector}} = \frac{1.478 \times 10^{11}}{0.5 \times 10^{-4}} = 2.956 \times 10^{15}/\text{m}^2\text{sec}$$

∴ Photon current

$$= 0.9 \times 1.478 \times 10^{11} \times 1.6 \times 10^{-19}$$
$$= 2.13 \times 10^{-8}\,\text{amp} = 0.0213\,\mu\text{A}$$

c. The stopping potential is independent of photocurrent and according to Einstein's photoelectric equation, in both cases, is given by

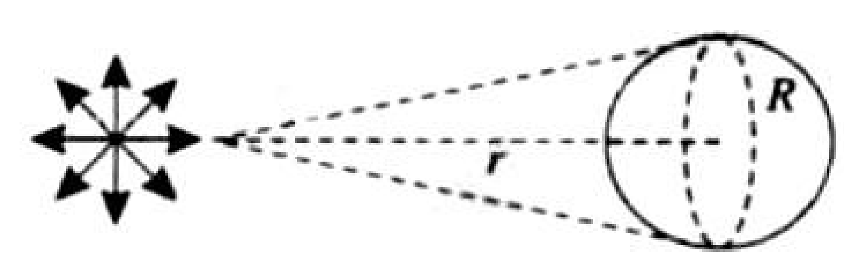
$$e = W + eV_s$$
$$3.315 \times 10^{-9} = 1\,eV + eV_s$$
$$\frac{3.315 \times 10^{-19}}{1.6 \times 10^{-19}} eV = 1\,eV + eV_s$$
$$2.07\,eV = 1\,eV + eV_s$$
$$\therefore \quad V_s = (2.07 - 1) = 1.07\,\text{volt}$$

6. a. If P is power of a point source, then intensity I at distance r is given by

$$I = \frac{P}{4\pi r^2}$$

If R is the radius of metallic sphere, then energy reaching the metallic sphere per second is

$$I \cdot \pi R^2 = \frac{P}{4\pi r^2}\pi R^2 \frac{PR^2}{4r^2}$$



If ε is the energy of a photon, then the number of photons incident on sphere per second

$$= \frac{PR^2/4r^2}{\varepsilon}$$

As η is efficiency of photoelectron emission, then the number of electrons emitted per second

$$n_e = \eta\frac{PR^2/4r^2}{\varepsilon} = \eta\frac{PR^2}{4r^2\varepsilon}$$

Given $\eta = \dfrac{1}{10^6} = 10^{-6}$, $P = 3.2 \times 10^{-3}$ W

$$r = 0.8,\ R = 8.0 \times 10^{-3}\,\text{m}$$
$$\varepsilon = 5.0\,\text{eV} = 5.0 \times 1.6 \times 10^{-19}\,\text{J}$$

$\therefore$ Number of electrons emitted per second

$$n_e = \frac{10^{-6} \times (3.2 \times 10^{-3})(8 \times 10^{-3})^2}{4 \times (0.8)^2 \times 5.0 \times 1.6 \times 10^{-19}} = 10^5 \text{ per sec}$$

b. Due to emission of photoelectrons, the sphere will become positively charged and it will oppose the removal of electrons due to attractive force between electron and positive sphere. Hence the photoelectric emission will stop when sphere has acquired positive potential equal to the stopping potential

i.e. $V = V_S$

$\Rightarrow$ $eV = eV_S$

But $E_k = eV_s$

$\therefore$ $eV = E_k$

Here $E_k = \varepsilon - W = 5.0 - 3.0 = 2.0$ eV

$\therefore$ $eV = 2.0$ eV $\Rightarrow$ $V = 2$ volt

For a sphere of radius R, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

But $Q = Ne$, where N is the total number of electrons emitted.

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Ne}{R}$$

$$N = \frac{4\pi\varepsilon_0 VR}{e}$$

$$= \frac{\left(\frac{1}{9 \times 10^9}\right) \times (2) \times (8 \times 10^{-3})}{1.6 \times 10^{-19}} = 1.1 \times 10^7$$

c. As metallic sphere is emitting electrons at the rate $n_e = 10^5$ electrons/sec; therefore time t taken by sphere at attain required potential.

$$n_e t = 1.11 \times 10^7$$

$$t = \frac{1.11 \times 10^7}{n_e} = \frac{1.11 \times 10^7}{10^5} = 111 \text{ s}$$

7. For standing wave, the separation between consecutive nodes (or antinodes) is $\lambda/2$.

$$\therefore \quad d = \frac{n\lambda}{2}, \text{ where } n \text{ is integer.} \qquad \text{(i)}$$

For next node,

$$D' = (n + 1)\frac{\lambda}{2}$$

$$\therefore \quad d' - d = \frac{\lambda}{2}$$

$\Rightarrow$ $\lambda = 2(d' - d)$

Here, $d = 2$ Å, $d' = 2.5$ Å

$\therefore$ $\lambda = 2(2.5 \text{ Å} - 2 \text{ Å}) = 1$ Å

Therefore, energy of electrons,

$$E = \frac{p^2}{2m}$$

$$\Rightarrow \quad = \frac{h^2}{2m\lambda^2}$$

Here $h = 6.63 \times 10^{-34}$ J-s

$m = 9.1 \times 10^{-31}$ kg

$\lambda = 1$ Å $= 10^{-10}$ m

$$\therefore \quad E = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10^{-10})^2} \text{ J}$$

$$= \frac{(6.63)^2}{2 \times 9.1} \times 10^{-17} \text{ joule} = 2.415 \times 10^{-17} \text{ J}$$

$$= \frac{2.415 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} \approx 151 \text{ eV}$$

For least value of d, $n = 1$. Therefore, from Eq. (i)

$$d_{min} = \frac{\lambda}{2} = \frac{1}{2} \text{ Å} = 0.5 \text{ Å}$$

8. From Einstein's photoelectric equation, maximum kinetic energy of emitted electrons

$$E_k = \frac{hc}{\lambda} - W$$

$$\text{Now, } E = \frac{hc}{\lambda} = \frac{6.624 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}$$

$$= 4.968 \times 10^{-19} \text{ J-s} = \frac{4.968 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.1 \text{ eV}$$

$E_k = 3.1$ eV $- 1.9$ eV $= 1.2$ eV

$e + \alpha \rightarrow He^+ +$ Photon

Energy of He atom in their fourth excited state ($n = 5$) is

$$E_n = -\frac{Z^2 Rhc}{n^2} = -\frac{(2)^2 \times 13.6}{(5)^2} = -2.176 \text{ eV}$$

From conservation of energy, 1.2 eV + 0 = −2.176 eV + E_γ

Energy of photon during combination,

$E_\gamma = 1.2 + 2.176 = 3.376$ eV

$$\text{Energy of helium ion, } E_n = -\frac{Z^2 Rhc}{n^2} = -\frac{4 \times 13.6}{n^2}$$

$$= -\frac{54.4}{n^2} \text{eV}, \ n = 1, 2, 3, \ldots$$

$= -54.4$ eV, -13.6eV, -6.04 eV, -3.4 eV, -2.176 eV, -1.51 eV

Difference of energies lying between 2 and 4 eV is

$-3.4 + 6.04 = 2.64$ eV

$-2.176 + 6.04 = 3.86$ eV

Energies of photons emitted are 2.64 eV and 3.86 eV.

9. Energy incident on surface per second

$P = IA$

$= 2.0 \times 1.0 \times 10^{-4} = 2 \times 10^{-4}$ J

Energy of each photon

$= 10.6$ eV $= 10.6 \times 1.6 \times 10^{-19}$ J

Number of photons incident on the surface

$$= \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

Number of photoelectrons emitted

$$= \frac{0.53}{100} \times \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}} = 6.25 \times 10^{11}$$

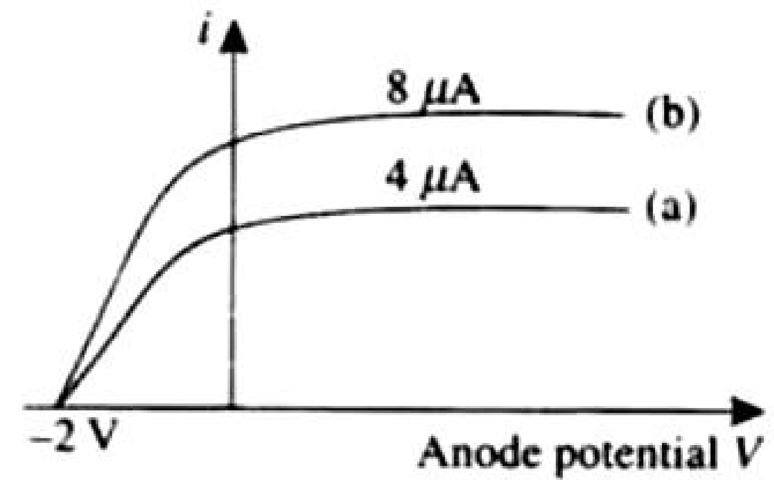
According to Einstein's photoelectric equation, maximum KE of photoelectrons

$E_k = \varepsilon - W = 10.6$ eV $- 5.6$ eV $= 5$ eV

Minimum kinetic energy of photoelectrons = zero.

10. Maximum kinetic energy of the photoelectrons would be

$$K_{max} = E - W = (5 - 3)\text{ eV} = 2\text{ eV}$$



Therefore, the stopping potential is $2V$. Saturation current depends on the intensity of light incident. When the intensity is doubled the saturation current will also become twofold. The corresponding graphs are shown in the above figure.

11. For $0 \le x \le 1$, $PE = E_0$

$\therefore$ Kinetic energy K_1 = total energy $- PE = 2E_0 - E_0 = E_0$

$$\lambda_1 = \frac{h}{\sqrt{2mE_0}} \qquad \text{(i)}$$

For $x > 1$, $PE = 0$

$\therefore$ Kinetic energy K_2 = Total energy $= 3E_0$

$$\lambda_2 = \frac{h}{\sqrt{2m(E_0)}} \qquad \text{(ii)}$$

From equations (i) and (ii), we have $\dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$