ગુજરાત શૈક્ષણિક સંશોધન અને તાલીમ પરિષદના પત્ર-ક્રમાંક
જીસીઈઆરટી/સીએન્ડઈ/2018/5808, તા.07/03/2018થી મંજૂર

MATHEMATICS

Textbook for Class VIII

0852

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Gujarat State Board of School Textbooks
'Vidyayan', Sector 10-A, Gandhinagar-382010

Co-ordinator

Shri Ashish H. Borisagar

(Subject Co-ordinator : Maths.)

Preparation and Planning

Shri Haren Shah

(Dy. Director : Academic)

Lay-out and Planning

Shri Haresh S. Limbachiya

(Dy. Director : Production)

PREFACE

With a view to implementing 'Equal Curriculum Policy', Gujarat State Government and GCERT took a decision to implement directly the textbooks of NCERT, New Delhi, in Gujarat according to the proposal no. JSBH/121/Single file-62/N dated : 19-7-2017. Keeping this objective in view, this textbook of **Mathematics**, published by NCERT, is being implemented in **Class 8**. For this, the Gujarati translation of NCERT textbook was prepared first.

During the Gujarati translation process, minor changes have been made in proper nouns, numbers and chapters in accordance with present situation and Gujarat specific with NCERT's prior approval. Now, the changes made in Gujarati version have been mandatorily incorporated in this English medium Mathematics Textbook. For this, expertise and experience Shri Satish Traiya and Dr. Bhavesh Dave have been secured by the Board. The Board is thankful to them for their noble contribution.

The Gujarat State Board of School Textbooks is also obliged to NCERT for their kind co-operation.

Creative suggestions for the enhancement of quality of the textbook are always welcomed by the Board.

P. Bharathi (IAS)

Director | Executive President

Date : 01-01-2020 | Gandhinagar

First Edition : 2019, Re-Print : 2020

Published by : P. Bharathi, Director, on behalf of Gujarat State Board of School Textbooks, 'Vidyayan', Sector 10-A, Gandhinagar

Printed by :

Foreword

The National Curriculum Framework, 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

NCERT appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Dr H.K. Dewan for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

New Delhi
30 November 2007

Preface

This is the final book of the upper primary series. It has been an interesting journey to define mathematics learning in a different way. The attempt has been to retain the nature of mathematics, engage with the question why learn mathematics while making an attempt to create materials that would address the interest of the learners at this stage and provide sufficient and approachable challenge to them. There have been many views on the purpose of school mathematics. These range from the fully utilitarian to the entirely aesthetic perceptions. Both these end up not engaging with the concepts and enriching the apparatus available to the learner for participating in life. The NCF emphasises the need for developing the ability to mathematise ideas and perhaps experiences as well. An ability to explore the ideas and framework given by mathematics in the struggle to find a richer life and a more meaningful relationship with the world around.

This is not even easy to comprehend, far more difficult to operationalise. But NCF adds to this an even more difficult goal. The task is to involve everyone of that age group in the classroom or outside in doing mathematics. This is the aim we have been attempting to make in the series.

We have, therefore, provided space for children to engage in reflection, creating their own rules and definitions based on problems/tasks solved and following their ideas logically. The emphasis is not on remembering algorithms, doing complicated arithmetical problems or remembering proofs, but understanding how mathematics works and being able to identify the way of moving towards solving problems.

The important concern for us has also been to ensure that all students at this stage learn mathematics and begin to feel confident in relating mathematics. We have attempted to help children read the book and to stop and reflect at each step where a new idea has been presented. In order to make the book less formidable we have included illustrations and diagrams. These combined with the text help the child comprehend the idea. Throughout the series and also therefore in this book we have tried to avoid the use of technical words and complex formulations. We have left many things for the student to describe and write in her own words.

We have made an attempt to use child friendly language. To attract attention to some points blurbs have been used. The attempt has been to reduce the weight of long explanations by using these and the diagrams. The illustrations and fillers also attempt to break the monotony and provide contexts.

Class VIII is the bridge to Class IX where children will deal with more formal mathematics. The attempt here has been to introduce some ideas in a way that is moving towards becoming formal. The tasks included expect generalisation from the gradual use of such language by the child.

The team that developed this textbook consisted teachers with experience and appreciation of children learning mathematics. This team also included people with experience of research in mathematics teaching-learning and an experience of producing materials for children. The feedback on the textbooks for Classes VI and VII was kept in mind while developing this textbook. This process of development also included discussions with teachers during review workshop on the manuscript.

In the end, I would like to express the grateful thanks of our team to Professor Krishna Kumar, *Director*, NCERT, Professor G. Ravindra, *Joint Director*, NCERT and Professor Hukum Singh, *Head*, DESM, for giving us an opportunity to work on this task with freedom and with full support. I am also grateful to Professor J.V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics for his suggestions. I am also grateful for the support of the team members from NCERT, Professor S.K. Singh Gautam, Dr V.P. Singh and in particular Dr Ashutosh K. Wazalwar who coordinated this work and made arrangements possible. In the end I must thank the Publication Department of NCERT for its support and advice and those from Vidya Bhawan who helped produce the book.

It need not be said but I cannot help mentioning that all the authors worked as a team and we accepted ideas and advice from each other. We stretched ourselves to the fullest and hope that we have done some justice to the challenge posed before us.

The process of developing materials is, however, a continuous one and we would hope to make this book better. Suggestions and comments on the book are most welcome.

H.K. Dewan
Chief Advisor
Textbook Development Committee

A Note for the Teacher

This is the third and the last book of this series. It is a continuation of the processes initiated to help the learners in abstraction of ideas and principles of mathematics. Our students to be able to deal with mathematical ideas and use them need to have the logical foundations to abstract and use postulates and construct new formulations. The main points reflected in the NCF-2005 suggest relating mathematics to development of wider abilities in children, moving away from complex calculations and algorithm following to understanding and constructing a framework of understanding. As you know, mathematical ideas do not develop by telling them. They also do not reach children by merely giving explanations. Children need their own framework of concepts and a classroom where they are discussing ideas, looking for solutions to problems, setting new problems and finding their own ways of solving problems and their own definitions.

As we have said before, it is important to help children to learn to read the textbook and other books related to mathematics with understanding. The reading of materials is clearly required to help the child learn further mathematics. In Class VIII please take stock of where the students have reached and give them more opportunities to read texts that use language with symbols and have brevity and terseness with no redundancy. For this if you can, please get them to read other texts as well. You could also have them relate the physics they learn and the equations they come across in chemistry to the ideas they have learnt in mathematics. These cross-disciplinary references would help them develop a framework and purpose for mathematics. They need to be able to reconstruct logical arguments and appreciate the need for keeping certain factors and constraints while they relate them to other areas as well. Class VIII children need to have opportunity for all this.

As we have already emphasised, mathematics at the Upper Primary Stage has to be close to the experience and environment of the child and be abstract at the same time. From the comfort of context and/or models linked to their experience they need to move towards working with ideas. Learning to abstract helps formulate and understand arguments. The capacity to see interrelations among concepts helps us deal with ideas in other subjects as well. It also helps us understand and make better patterns, maps, appreciate area and volume and see similarities between shapes and sizes. While this is regarding the relationship of other fields of knowledge to mathematics, its meaning in life and our environment needs to be re-emphasised.

Children should be able to identify the principles to be used in contextual situations, for solving problems sift through and choose the relevant information as the first important step. Once students do that they need to be able to find the way to use the knowledge they have and reach where the problem requires them to go. They need to identify and define a problem, select or design possible solutions and revise or redesign the steps, if required. As they go further there would be more to of this to be done. In Class VIII we have to get them to be conscious of the steps they follow. Helping children to develop the ability to construct appropriate models by breaking up the problems and evolving their own strategies and analysis of problems is extremely important. This is in the place of giving them prescriptive algorithms.

Cooperative learning, learning through conversations, desire and capacity to learn from each other and the recognition that conversation is not noise and consultation not cheating is an important part of change in attitude for you as a teacher and for the students as well. They should be asked to make presentations as a group with the inclusion of examples from the contexts of their own experiences. They should be encouraged to read the book in groups and formulate and express what they understand from it. The assessment pattern has to recognise and appreciate this and the classroom groups should be such that all children enjoy being with each other and are contributing to the learning of the group. As you would have seen different groups use different strategies. Some of these are not as efficient as others as they reflect the modeling done and reflect the thinking used. All these are appropriate and need to be analysed with children. The exposure to a variety of strategies deepens the mathematical understanding. Each group moves from where it is and needs to be given an opportunity for that.

For conciseness we present the key ideas of mathematics learning that we would like you to remember in your classroom.

1. Enquiry to understand is one of the natural ways by which students acquire and construct knowledge. The process can use generation of observations to acquire knowledge. Students need to deal with different forms of questioning and challenging investigations- explorative, open-ended, contextual and even error detection from geometry, arithmetic and generalising it to algebraic relations etc.
2. Children need to learn to provide and follow logical arguments, find loopholes in the arguments presented and understand the requirement of a proof. By now children have entered the formal stage. They need to be encouraged to exercise creativity and imagination and to communicate their mathematical reasoning both verbally and in writing.
3. The mathematics classroom should relate language to learning of mathematics. Children should talk about their ideas using their experiences and language. They should be encouraged to use their own words and language but also gradually shift to formal language and use of symbols.
4. The number system has been taken to the level of generalisation of rational numbers and their properties and developing a framework that includes all previous systems as sub-sets of the generalised rational numbers. Generalisations are to be presented in mathematical language and children have to see that algebra and its language helps us express a lot of text in small symbolic forms.
5. As before children should be required to set and solve a lot of problems. We hope that as the nature of the problems set up by them becomes varied and more complex, they would become confident of the ideas they are dealing with.
6. Class VIII book has attempted to bring together the different aspects of mathematics and emphasise the commonality. Unitary method, Ratio and proportion, Interest and dividends are all part of one common logical framework. The idea of variable and equations is needed wherever we need to find an unknown quantity in any branch of mathematics.

We hope that the book will help children learn to enjoy mathematics and be confident in the concepts introduced. We want to recommend the creation of opportunity for thinking individually and collectively.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching, to be included in the future editions. This can only happen if you would find time to listen carefully to children and identify gaps and on the other hand also find the places where they can be given space to articulate their ideas and verbalise their thoughts.

Textbook Development Committee

CHAIRPERSON, ADVISORY GROUP IN SCIENCE AND MATHEMATICS

J.V. Narlikar, Emeritus Professor, *Chairman,* Advisory Committee, Inter University Centre for Astronomy and Astrophysics (IUCCA), Ganeshkhind, Pune University, Pune

CHIEF ADVISOR

H.K. Dewan, Vidya Bhawan Society, Udaipur, Rajasthan

CHIEF COORDINATOR

Hukum Singh, *Professor and Head*, DESM, NCERT, New Delhi

MEMBERS

Anjali Gupte, *Teacher*, Vidya Bhawan Public School, Udaipur, Rajasthan
Avantika Dam, *TGT*, CIE Experimental Basic School, Department of Education, Delhi

B.C. Basti, *Senior Lecturer*, Regional Institute of Education, Mysore, Karnataka

H.C. Pradhan, *Professor*, Homi Bhabha Centre for Science Education, TIFR, Mumbai Maharashtra

K.A.S.S.V. Kameshwar Rao, *Lecturer*, Regional Institute of Education, Shyamala Hills Bhopal (M.P.)

Mahendra Shankar, *Lecturer (S.G.)* (Retd.), NCERT, New Delhi

Meena Shrimali, *Teacher*, Vidya Bhawan Senior Secondary School, Udaipur, Rajasthan

P. Bhaskar Kumar, *PGT*, Jawahar Navodaya Vidyalaya, Lepakshi, Distt. Anantpur (A.P.)

R. Athmaraman, *Mathematics Education Consultant*, TI Matric Higher Secondary School and AMTI, Chennai, Tamil Nadu

Ram Avtar, *Professor* (Retd.), NCERT, New Delhi

Shailesh Shirali, Rishi Valley School, Rishi Valley, Madanapalle (A.P.)

S.K.S. Gautam, *Professor*, DEME, NCERT, New Delhi

Shradha Agarwal, *Principal*, Florets International School, Panki, Kanpur (U.P.)

Srijata Das, *Senior Lecturer* in Mathematics, SCERT, New Delhi

V.P. Singh, *Reader*, DESM, NCERT, New Delhi

MEMBER-COORDINATOR

Ashutosh K. Wazalwar, *Professor*, DESM, NCERT, New Delhi

ACKNOWLEDGEMENTS

The Council gratefully acknowledges the valuable contributions of the following participants of the Textbook Review Workshop: Shri Pradeep Bhardwaj, *TGT* (Mathematics) Bal Sthali Public Secondary School, Kirari, Nangloi, New Delhi; Shri Sankar Misra, *Teacher* in Mathematics, Demonstration Multipurpose School, Regional Institute of Education, Bhubaneswar (Orissa); Shri Manohar M. Dhok, *Supervisor,* M.P. Deo Smruti Lokanchi Shala, Nagpur (Maharashtra); Shri Manjit Singh Jangra, *Maths teacher,* Government Senior Secondary School, Sector-4/7, Gurgoan (Haryana); Dr. Rajendra Kumar Pooniwala, U.D.T., Government Subhash Excellence School, Burhanpur (M.P.); Shri K. Balaji, TGT (Mathematics), Kendriya Vidyalaya No.1, Tirupati (A.P.); Ms. Mala Mani, Amity International School, Sector-44, Noida; Ms. Omlata Singh, *TGT* (Mathematics), Presentation Convent Senior Secondary School, Delhi; Ms. Manju Dutta, Army Public School, Dhaula Kuan, New Delhi; Ms. Nirupama Sahni, *TGT* (Mathematics), Shri Mahaveer Digambar Jain Senior Secondary School, Jaipur (Rajasthan); Shri Nagesh Shankar Mone, *Head Master,* Kantilal Purshottam Das Shah Prashala, Vishrambag, Sangli (Maharashtra); Shri Anil Bhaskar Joshi, *Senior teacher* (Mathematics), Manutai Kanya Shala, Tilak Road, Akola (Maharashtra); Dr. Sushma Jairath, *Reader,* DWS, NCERT, New Delhi; Shri Ishwar Chandra, *Lecturer (S.G)* (Retd.) NCERT, New Delhi.

The Council is grateful for the suggestions/comments given by the following participants during the workshop of the mathematics Textbook Development Committee – Shri Sanjay Bolia and Shri Deepak Mantri from Vidya Bhawan Basic School, Udaipur; Shri Inder Mohan Singh Chhabra, Vidya Bhawan Educational Resource Centre, Udaipur.

The Council acknowledges the comments/suggestions given by Dr. R.P. Maurya, *Reader,* DESM, NCERT, New Delhi; Dr. Sanjay Mudgal, *Lecturer,* DESM, NCERT, New Delhi; Dr. T.P. Sharma, *Lecturer,* DESM, NCERT, New Delhi for the improvement of the book.

The Council acknowledges the support and facilities provided by Vidya Bhawan Society and its staff, Udaipur, for conducting workshops of the development committee at Udaipur and to the Director, Centre for Science Education and Communication (CSEC), Delhi University for providing library help.

The Council acknowledges the academic and administrative support of Professor Hukum Singh, Head, DESM, NCERT.

The Council also acknowledges the efforts of Sajjad Haider Ansari, Rakesh Kumar, Neelam Walecha, *DTP Operators*; Kanwar Singh, *Copy Editor;* Abhimanu Mohanty, *Proof Reader*, Deepak Kapoor, *Computer Station Incharge*, DESM, NCERT for technical assistance, APC Office and the Administrative Staff, DESM, NCERT and the Publication Department of the NCERT.

Contents

CHAPTER

1

Rational Numbers

0852CH01

1.1 Introduction

In Mathematics, we frequently come across simple equations to be solved. For example, the equation

$$x + 2 = 13 \qquad (1)$$

is solved when $x = 11$, because this value of x satisfies the given equation. The solution 11 is a **natural number**. On the other hand, for the equation

$$x + 5 = 5 \qquad (2)$$

the solution gives the **whole number** 0 (zero). If we consider only natural numbers, equation (2) cannot be solved. To solve equations like (2), we added the number zero to the collection of natural numbers and obtained the whole numbers. Even whole numbers will not be sufficient to solve equations of type

$$x + 18 = 5 \qquad (3)$$

Do you see 'why'? We require the number –13 which is not a whole number. This led us to think of **integers, (positive and negative)**. Note that the positive integers correspond to natural numbers. One may think that we have enough numbers to solve all simple equations with the available list of integers. Now consider the equations

$$2x = 3 \qquad (4)$$

$$5x + 7 = 0 \qquad (5)$$

for which we cannot find a solution from the integers. (Check this)

We need the numbers $\frac{3}{2}$ to solve equation (4) and $\frac{-7}{5}$ to solve equation (5). This leads us to the collection of **rational numbers**.

We have already seen basic operations on rational numbers. We now try to explore some properties of operations on the different types of numbers seen so far.

1.2 Properties of Rational Numbers

1.2.1 Closure

(i) Whole numbers

Let us revisit the closure property for all the operations on whole numbers in brief.

Operation	Numbers	Remarks
Addition	$0 + 5 = 5$, a whole number $4 + 7 = ...$ Is it a whole number? In general, $a + b$ is a whole number for any two whole numbers a and b.	Whole numbers are closed under addition.
Subtraction	$5 - 7 = -2$, which is not a whole number.	Whole numbers are **not** closed under subtraction.
Multiplication	$0 \times 3 = 0$, a whole number $3 \times 7 = ...$ Is it a whole number? In general, if a and b are any two whole numbers, their product ab is a whole number.	Whole numbers are closed under multiplication.
Division	$5 \div 8 = \frac{5}{8}$, which is not a whole number.	Whole numbers are **not** closed under division.

Check for closure property under all the four operations for natural numbers.

(ii) Integers

Let us now recall the operations under which integers are closed.

Operation	Numbers	Remarks
Addition	$-6 + 5 = -1$, an integer Is $-7 + (-5)$ an integer? Is $8 + 5$ an integer? In general, $a + b$ is an integer for any two integers a and b.	Integers are closed under addition.
Subtraction	$7 - 5 = 2$, an integer Is $5 - 7$ an integer? $-6 - 8 = -14$, an integer	Integers are closed under subtraction.

	$-6-(-8)=2$, an integer Is $8-(-6)$ an integer? In general, for any two integers *a* and *b*, $a-b$ is again an integer. Check if $b-a$ is also an integer.	
Multiplication	$5\times 8=40$, an integer Is -5×8 an integer? $-5\times(-8)=40$, an integer In general, for any two integers *a* and *b*, $a\times b$ is also an integer.	Integers are closed under multiplication.
Division	$5\div 8=\frac{5}{8}$, which is not an integer.	Integers are **not** closed under division.

You have seen that whole numbers are closed under addition and multiplication but not under subtraction and division. However, integers are closed under addition, subtraction and multiplication but not under division.

(iii) Rational numbers

Recall that a number which can be written in the form $\frac{p}{q}$, where *p* and *q* are integers and $q\neq 0$ is called a **rational number**. For example, $-\frac{2}{3}, \frac{6}{7}, \frac{9}{-5}$ are all rational numbers. Since the numbers 0, –2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers. (Check it!)

(a) You know how to add two rational numbers. Let us add a few pairs.

$$\frac{3}{8}+\frac{(-5)}{7}=\frac{21+(-40)}{56}=\frac{-19}{56} \quad \text{(a rational number)}$$

$$\frac{-3}{8}+\frac{(-4)}{5}=\frac{-15+(-32)}{40}=\ldots \quad \text{Is it a rational number?}$$

$$\frac{4}{7}+\frac{6}{11}=\ldots \quad \text{Is it a rational number?}$$

We find that sum of two rational numbers is again a rational number. Check it for a few more pairs of rational numbers.

We say that *rational numbers are closed under addition. That is, for any two rational numbers a and b, a + b is also a rational number.*

(b) Will the difference of two rational numbers be again a rational number?

We have,

$$\frac{-5}{7}-\frac{2}{3}=\frac{-5\times 3-2\times 7}{21}=\frac{-29}{21} \quad \text{(a rational number)}$$

$$\frac{5}{8}-\frac{4}{5}=\frac{25-32}{40}=\ldots$$ Is it a rational number?

$$\frac{3}{7}-\left(\frac{-8}{5}\right)=\ldots$$ Is it a rational number?

Try this for some more pairs of rational numbers. We find that *rational numbers are closed under subtraction. That is, for any two rational numbers a and b, a – b is also a rational number*.

(c) Let us now see the product of two rational numbers.

$$\frac{-2}{3}\times\frac{4}{5}=\frac{-8}{15};\ \frac{3}{7}\times\frac{2}{5}=\frac{6}{35}$$ (both the products are rational numbers)

$$-\frac{4}{5}\times\frac{-6}{11}=\ldots$$ Is it a rational number?

Take some more pairs of rational numbers and check that their product is again a rational number.

We say that *rational numbers are closed under multiplication. That is, for any two rational numbers a and b, a × b is also a rational number*.

(d) We note that $\frac{-5}{3}\div\frac{2}{5}=\frac{-25}{6}$ (a rational number)

$\frac{2}{7}\div\frac{5}{3}=\ldots$. Is it a rational number? $\frac{-3}{8}\div\frac{-2}{9}=\ldots$. Is it a rational number?

Can you say that rational numbers are closed under division?

We find that for any rational number a, $a \div 0$ is **not defined**.

So rational numbers are **not closed** under division.

However, if we exclude zero then the collection of, all other rational numbers is closed under division.

TRY THESE

Fill in the blanks in the following table.

Numbers	**Closed under**			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	Yes	Yes	...	No
Integers	...	Yes	...	No
Whole numbers	...	...	Yes	...
Natural numbers	...	No	...	...

1.2.2 Commutativity

(i) Whole numbers

Recall the commutativity of different operations for whole numbers by filling the following table.

Operation	Numbers	Remarks
Addition	$0 + 7 = 7 + 0 = 7$ $2 + 3 = ... + ... = ...$ For any two whole numbers a and b, $a + b = b + a$	Addition is commutative.
Subtraction	....	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

Check whether the commutativity of the operations hold for natural numbers also.

(ii) Integers

Fill in the following table and check the commutativity of different operations for integers:

Operation	Numbers	Remarks
Addition	....	Addition is commutative.
Subtraction	Is $5 - (-3) = - 3 - 5$?	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

(iii) Rational numbers

(a) Addition

You know how to add two rational numbers. Let us add a few pairs here.

$$\frac{-2}{3} + \frac{5}{7} = \frac{1}{21} \text{ and } \frac{5}{7} + \left(\frac{-2}{3}\right) = \frac{1}{21}$$

So, $$\frac{-2}{3} + \frac{5}{7} = \frac{5}{7} + \left(\frac{-2}{3}\right)$$

Also, $$\frac{-6}{5} + \left(\frac{-8}{3}\right) = \ldots \text{ and } \frac{-8}{3} + \left(\frac{-6}{5}\right) = \ldots$$

Is $$\frac{-6}{5} + \left(\frac{-8}{3}\right) = \left(\frac{-8}{3}\right) + \left(\frac{-6}{5}\right)?$$

Is $\frac{-3}{8}+\frac{1}{7}=\frac{1}{7}+\left(\frac{-3}{8}\right)$?

You find that two *rational numbers can be added in any order. We say that addition is commutative for rational numbers. That is, for any two rational numbers a and b, a + b = b + a.*

(b) Subtraction

Is $\frac{2}{3}-\frac{5}{4}=\frac{5}{4}-\frac{2}{3}$?

Is $\frac{1}{2}-\frac{3}{5}=\frac{3}{5}-\frac{1}{2}$?

You will find that subtraction is not commutative for rational numbers.

Note that subtraction is not commutative for integers and integers are also rational numbers. So, subtraction will not be commutative for rational numbers too.

(c) Multiplication

We have, $\frac{-7}{3}\times\frac{6}{5}=\frac{-42}{15}=\frac{6}{5}\times\left(\frac{-7}{3}\right)$

Is $\frac{-8}{9}\times\left(\frac{-4}{7}\right)=\frac{-4}{7}\times\left(\frac{-8}{9}\right)$?

Check for some more such products.

You will find that *multiplication is commutative for rational numbers.*

In general, a × b = b × a for any two rational numbers a and b.

(d) Division

Is $\frac{-5}{4}\div\frac{3}{7}=\frac{3}{7}\div\left(\frac{-5}{4}\right)$?

You will find that expressions on both sides are not equal.

So division is **not commutative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Commutative for			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	Yes	...	...	...
Integers	...	No	...	...
Whole numbers	...	...	Yes	...
Natural numbers	...	...	...	No

1.2.3 Associativity

(i) Whole numbers

Recall the associativity of the four operations for whole numbers through this table:

Operation	Numbers	Remarks
Addition	....	Addition is associative
Subtraction	....	Subtraction is **not** associative
Multiplication	Is $7 \times (2 \times 5) = (7 \times 2) \times 5$? Is $4 \times (6 \times 0) = (4 \times 6) \times 0$? For any three whole numbers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	....	Division is **not** associative

Fill in this table and verify the remarks given in the last column.
Check for yourself the associativity of different operations for natural numbers.

(ii) Integers

Associativity of the four operations for integers can be seen from this table

Operation	Numbers	Remarks
Addition	Is $(-2) + [3 + (-4)]$ $= [(-2) + 3)] + (-4)$? Is $(-6) + [(-4) + (-5)]$ $= [(-6) + (-4)] + (-5)$? For any three integers a, b and c $a + (b + c) = (a + b) + c$	Addition is associative
Subtraction	Is $5 - (7 - 3) = (5 - 7) - 3$?	Subtraction is **not** associative
Multiplication	Is $5 \times [(-7) \times (-8)$ $= [5 \times (-7)] \times (-8)$? Is $(-4) \times [(-8) \times (-5)]$ $= [(-4) \times (-8)] \times (-5)$? For any three integers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	Is $[(-10) \div 2] \div (-5)$ $= (-10) \div [2 \div (-5)]$?	Division is **not** associative

(iii) Rational numbers

(a) Addition

We have $\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\frac{-2}{3}+\left(\frac{-7}{30}\right)=\frac{-27}{30}=\frac{-9}{10}$

$$\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)=\frac{-1}{15}+\left(\frac{-5}{6}\right)=\frac{-27}{30}=\frac{-9}{10}$$

So, $\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)$

Find $\frac{-1}{2}+\left[\frac{3}{7}+\left(\frac{-4}{3}\right)\right]$ and $\left[\frac{-1}{2}+\frac{3}{7}\right]+\left(\frac{-4}{3}\right)$. Are the two sums equal?

Take some more rational numbers, add them as above and see if the two sums are equal. We find that *addition is associative for rational numbers. That is, for any three rational numbers* a, b and c, $a+(b+c)=(a+b)+c$.

(b) Subtraction

You already know that subtraction is not associative for integers, then what about rational numbers.

Is $\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]=\left[\frac{2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$?

Check for yourself.

Subtraction is **not associative** for rational numbers.

(c) Multiplication

Let us check the associativity for multiplication.

$$\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\frac{-7}{3}\times\frac{10}{36}=\frac{-70}{108}=\frac{-35}{54}$$

$$\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}=\ldots$$

We find that $\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}$

Is $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{4}{5}\right)=\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{4}{5}$?

Take some more rational numbers and check for yourself.

We observe that *multiplication is associative for rational numbers. That is for any three rational numbers* a, b and c, $a\times(b\times c)=(a\times b)\times c$.

(d) Division

Recall that division is not associative for integers, then what about rational numbers?

Let us see if $\frac{1}{2}\div\left[\frac{-1}{3}\div\frac{2}{5}\right]=\left[\frac{1}{2}\div\left(\frac{-1}{3}\right)\right]\div\frac{2}{5}$

We have, LHS $=\frac{1}{2}\div\left(\frac{-1}{3}\div\frac{2}{5}\right)=\frac{1}{2}\div\left(\frac{-1}{3}\times\frac{5}{2}\right)$ (reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$)

$=\frac{1}{2}\div\left(-\frac{5}{6}\right)=\ldots$

RHS $=\left[\frac{1}{2}\div\left(\frac{-1}{3}\right)\right]\div\frac{2}{5}$

$=\left(\frac{1}{2}\times\frac{-3}{1}\right)\div\frac{2}{5}=\frac{-3}{2}\div\frac{2}{5}=\ldots$

Is LHS = RHS? Check for yourself. You will find that division is **not associative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Associative for			
	addition	subtraction	multiplication	division
Rational numbers	...	...	...	No
Integers	...	...	Yes	...
Whole numbers	Yes	...	...	...
Natural numbers	...	No	...	...

Example 1: Find $\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\left(\frac{5}{22}\right)$

Solution: $\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\left(\frac{5}{22}\right)$

$=\frac{198}{462}+\left(\frac{-252}{462}\right)+\left(\frac{-176}{462}\right)+\left(\frac{105}{462}\right)$ (Note that 462 is the LCM of 7, 11, 21 and 22)

$=\frac{198-252-176+105}{462}=\frac{-125}{462}$

We can also solve it as.

$$\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\frac{5}{22}$$

$$=\left[\frac{3}{7}+\left(\frac{-8}{21}\right)\right]+\left[\frac{-6}{11}+\frac{5}{22}\right]$$ (by using commutativity and associativity)

$$=\left[\frac{9+(-8)}{21}\right]+\left[\frac{-12+5}{22}\right]$$ (LCM of 7 and 21 is 21; LCM of 11 and 22 is 22)

$$=\frac{1}{21}+\left(\frac{-7}{22}\right)=\frac{22-147}{462}=\frac{-125}{462}$$

Do you think the properties of commutativity and associativity made the calculations easier?

Example 2: Find $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

Solution: We have

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(-\frac{4\times 3}{5\times 7}\right)\times\left(\frac{15\times(-14)}{16\times 9}\right)$$

$$=\frac{-12}{35}\times\left(\frac{-35}{24}\right)=\frac{-12\times(-35)}{35\times 24}=\frac{1}{2}$$

We can also do it as.

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(\frac{-4}{5}\times\frac{15}{16}\right)\times\left[\frac{3}{7}\times\left(\frac{-14}{9}\right)\right]$$ (Using commutativity and associativity)

$$=\frac{-3}{4}\times\left(\frac{-2}{3}\right)=\frac{1}{2}$$

1.2.4 The role of zero (0)

Look at the following.

$2+0=0+2=2$ (Addition of 0 to a whole number)

$-5+0=\ldots+\ldots=-5$ (Addition of 0 to an integer)

$\frac{-2}{7}+\ldots=0+\left(\frac{-2}{7}\right)=\frac{-2}{7}$ (Addition of 0 to a rational number)

You have done such additions earlier also. Do a few more such additions.

What do you observe? You will find that when you add 0 to a whole number, the sum is again that whole number. This happens for integers and rational numbers also.

In general, $a + 0 = 0 + a = a$, where a is a whole number

$b + 0 = 0 + b = b$, where b is an integer

$c + 0 = 0 + c = c$, where c is a rational number

Zero is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers as well.

1.2.5 The role of 1

We have,

$$5 \times 1 = 5 = 1 \times 5 \quad \text{(Multiplication of 1 with a whole number)}$$

$$\frac{-2}{7} \times 1 = \ldots \times \ldots = \frac{-2}{7}$$

$$\frac{3}{8} \times \ldots = 1 \times \frac{3}{8} = \frac{3}{8}$$

What do you find?

You will find that when you multiply any rational number with 1, you get back the same rational number as the product. Check this for a few more rational numbers. You will find that, $a \times 1 = 1 \times a = a$ for any rational number a.

We say *that* 1 *is the multiplicative identity for rational numbers.*

Is 1 the multiplicative identity for integers? For whole numbers?

THINK, DISCUSS AND WRITE

If a property holds for rational numbers, will it also hold for integers? For whole numbers? Which will? Which will not?

1.2.6 Negative of a number

While studying integers you have come across negatives of integers. What is the negative of 1? It is – 1 because $1 + (-1) = (-1) + 1 = 0$

So, what will be the negative of (–1)? It will be 1.

Also, $2 + (-2) = (-2) + 2 = 0$, so we say 2 is the **negative or additive inverse** of –2 and vice-versa. In general, for an integer a, we have, $a + (-a) = (-a) + a = 0$; so, a is the negative of $-a$ and $-a$ is the negative of a.

For the rational number $\frac{2}{3}$, we have,

$$\frac{2}{3} + \left(-\frac{2}{3}\right) = \frac{2 + (-2)}{3} = 0$$

Also, $\left(-\frac{2}{3}\right)+\frac{2}{3}=0$ (How?)

Similarly, $\frac{-8}{9}+\ldots=\ldots+\left(\frac{-8}{9}\right)=0$

$$\ldots+\left(\frac{-11}{7}\right)=\left(\frac{-11}{7}\right)+\ldots=0$$

In general, for a rational number $\frac{a}{b}$, we have, $\frac{a}{b}+\left(-\frac{a}{b}\right)=\left(-\frac{a}{b}\right)+\frac{a}{b}=0$. We say that $-\frac{a}{b}$ *is the additive inverse of* $\frac{a}{b}$ *and* $\frac{a}{b}$ *is the additive inverse of* $\left(-\frac{a}{b}\right)$.

1.2.7 Reciprocal

By which rational number would you multiply $\frac{8}{21}$, to get the product 1? Obviously by $\frac{21}{8}$, since $\frac{8}{21}\times\frac{21}{8}=1$.

Similarly, $\frac{-5}{7}$ must be multiplied by $\frac{7}{-5}$ so as to get the product 1.

We say that $\frac{21}{8}$ is the reciprocal of $\frac{8}{21}$ and $\frac{7}{-5}$ is the reciprocal of $\frac{-5}{7}$.

Can you say what is the reciprocal of 0 (zero)?

Is there a rational number which when multiplied by 0 gives 1? Thus, zero has no reciprocal.

We say that a rational number $\frac{c}{d}$ is called the **reciprocal** or **multiplicative inverse** of another non-zero rational number $\frac{a}{b}$ if $\frac{a}{b}\times\frac{c}{d}=1$.

1.2.8 Distributivity of multiplication over addition for rational numbers

To understand this, consider the rational numbers $\frac{-3}{4}$, $\frac{2}{3}$ and $\frac{-5}{6}$.

$$\frac{-3}{4}\times\left\{\frac{2}{3}+\left(\frac{-5}{6}\right)\right\}=\frac{-3}{4}\times\left\{\frac{(4)+(-5)}{6}\right\}$$

$$=\frac{-3}{4}\times\left(\frac{-1}{6}\right)=\frac{3}{24}=\frac{1}{8}$$

Also $\frac{-3}{4}\times\frac{2}{3}=\frac{-3\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$

And $\frac{-3}{4} \times \frac{-5}{6} = \frac{5}{8}$

Therefore $\left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right) = \frac{-1}{2} + \frac{5}{8} = \frac{1}{8}$

Thus, $\frac{-3}{4} \times \left\{\frac{2}{3} + \frac{-5}{6}\right\} = \left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right)$

> **Distributivity of Multiplication over Addition and Subtraction.**
> For all rational numbers *a*, *b* and *c*,
> $a(b + c) = ab + ac$
> $a(b - c) = ab - ac$

TRY THESE

Find using distributivity. (i) $\left\{\frac{7}{5} \times \left(\frac{-3}{12}\right)\right\} + \left\{\frac{7}{5} \times \frac{5}{12}\right\}$ (ii) $\left\{\frac{9}{16} \times \frac{4}{12}\right\} + \left\{\frac{9}{16} \times \frac{-3}{9}\right\}$

Example 3: Write the additive inverse of the following:

(i) $\frac{-7}{19}$ (ii) $\frac{21}{112}$

> When you use distributivity, you split a product as a sum or difference of two products.

Solution:

(i) $\frac{7}{19}$ is the additive inverse of $\frac{-7}{19}$ because $\frac{-7}{19} + \frac{7}{19} = \frac{-7+7}{19} = \frac{0}{19} = 0$

(ii) The additive inverse of $\frac{21}{112}$ is $\frac{-21}{112}$ (Check!)

Example 4: Verify that $-(-x)$ is the same as x for

(i) $x = \frac{13}{17}$ (ii) $x = \frac{-21}{31}$

Solution: (i) We have, $x = \frac{13}{17}$

The additive inverse of $x = \frac{13}{17}$ is $-x = \frac{-13}{17}$ since $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$.

The same equality $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$, shows that the additive inverse of $\frac{-13}{17}$ is $\frac{13}{17}$

or $-\left(\frac{-13}{17}\right) = \frac{13}{17}$, i.e., $-(-x) = x$.

(ii) Additive inverse of $x = \frac{-21}{31}$ is $-x = \frac{21}{31}$ since $\frac{-21}{31} + \frac{21}{31} = 0$.

The same equality $\frac{-21}{31} + \frac{21}{31} = 0$, shows that the additive inverse of $\frac{21}{31}$ is $\frac{-21}{31}$,
i.e., $-(-x) = x$.

Example 5: Find $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}$

Solution: $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5} = \frac{2}{5}\times\frac{-3}{7}-\frac{3}{7}\times\frac{3}{5}-\frac{1}{14}$ (by commutativity)

$$= \frac{2}{5}\times\frac{-3}{7}+\left(\frac{-3}{7}\right)\times\frac{3}{5}-\frac{1}{14}$$

$$= \frac{-3}{7}\left(\frac{2}{5}+\frac{3}{5}\right)-\frac{1}{14} \quad \text{(by distributivity)}$$

$$= \frac{-3}{7}\times 1-\frac{1}{14} = \frac{-6-1}{14} = \frac{-1}{2}$$

EXERCISE 1.1

1. Using appropriate properties find.

   (i) $-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$ (ii) $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

2. Write the additive inverse of each of the following.

   (i) $\frac{2}{8}$ (ii) $\frac{-5}{9}$ (iii) $\frac{-6}{-5}$ (iv) $\frac{2}{-9}$ (v) $\frac{19}{-6}$

3. Verify that $-(-x) = x$ for.

   (i) $x = \frac{11}{15}$ (ii) $x = -\frac{13}{17}$

4. Find the multiplicative inverse of the following.

   (i) -13 (ii) $\frac{-13}{19}$ (iii) $\frac{1}{5}$ (iv) $\frac{-5}{8}\times\frac{-3}{7}$

   (v) $-1\times\frac{-2}{5}$ (vi) -1

5. Name the property under multiplication used in each of the following.

   (i) $\frac{-4}{5}\times 1 = 1\times\frac{-4}{5} = -\frac{4}{5}$ (ii) $-\frac{13}{17}\times\frac{-2}{7} = \frac{-2}{7}\times\frac{-13}{17}$

   (iii) $\frac{-19}{29}\times\frac{29}{-19} = 1$

6. Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.

7. Tell what property allows you to compute $\frac{1}{3}\times\left(6\times\frac{4}{3}\right)$ as $\left(\frac{1}{3}\times 6\right)\times\frac{4}{3}$.

8. Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

9. Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

10. Write.

(i) The rational number that does not have a reciprocal.

(ii) The rational numbers that are equal to their reciprocals.

(iii) The rational number that is equal to its negative.

11. Fill in the blanks.

(i) Zero has ________ reciprocal.

(ii) The numbers ________ and ________ are their own reciprocals

(iii) The reciprocal of – 5 is ________.

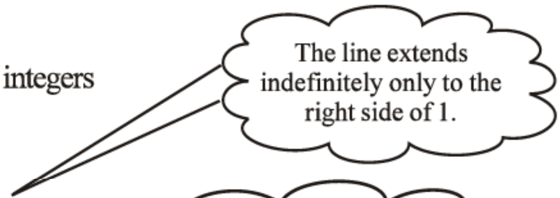
(iv) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is ________.

(v) The product of two rational numbers is always a _______.

(vi) The reciprocal of a positive rational number is ________.

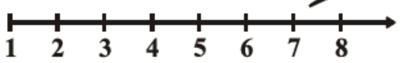
1.3 Representation of Rational Numbers on the Number Line

You have learnt to represent natural numbers, whole numbers, integers and rational numbers on a number line. Let us revise them.



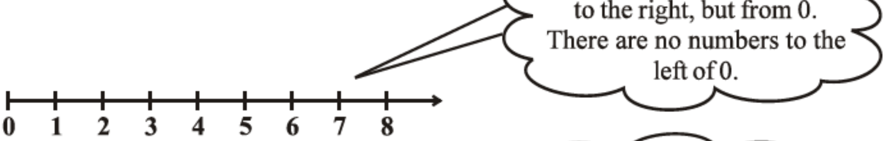
Natural numbers

(i)



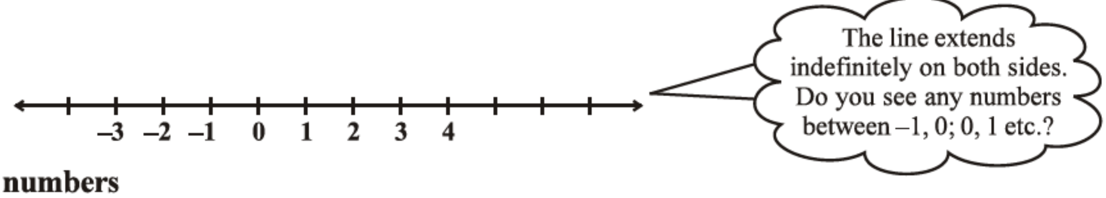
Whole numbers

(ii)



Integers

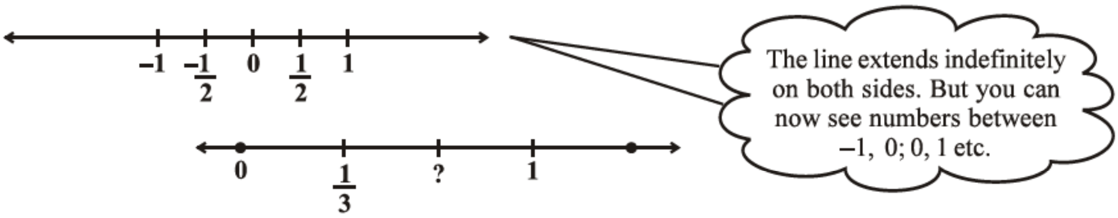
(iii)



Rational numbers

(iv)

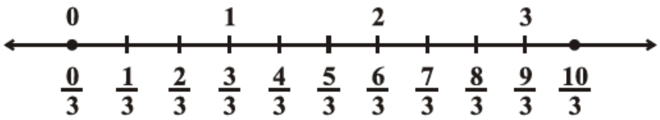
(v)

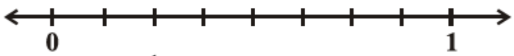


The point on the number line (iv) which is half way between 0 and 1 has been labelled $\frac{1}{2}$. Also, the first of the equally spaced points that divides the distance between 0 and 1 into three equal parts can be labelled $\frac{1}{3}$, as on number line (v). How would you label the second of these division points on number line (v)?

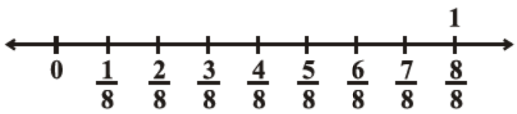
The point to be labelled is twice as far from and to the right of 0 as the point labelled $\frac{1}{3}$. So it is two times $\frac{1}{3}$, i.e., $\frac{2}{3}$. You can continue to label equally-spaced points on the number line in the same way. In this continuation, the next marking is 1. You can see that 1 is the same as $\frac{3}{3}$.

Then comes $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}$ (or 2), $\frac{7}{3}$ and so on as shown on the number line (vi)

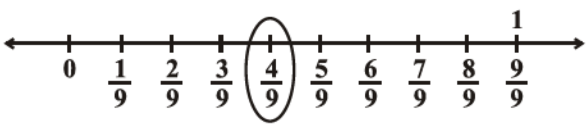
(vi) 

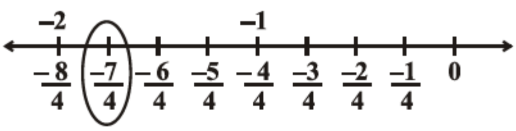
Similarly, to represent $\frac{1}{8}$, the number line may be divided into eight equal parts as shown: 

We use the number $\frac{1}{8}$ to name the first point of this division. The second point of division will be labelled $\frac{2}{8}$, the third point $\frac{3}{8}$, and so on as shown on number line (vii)

(vii) 

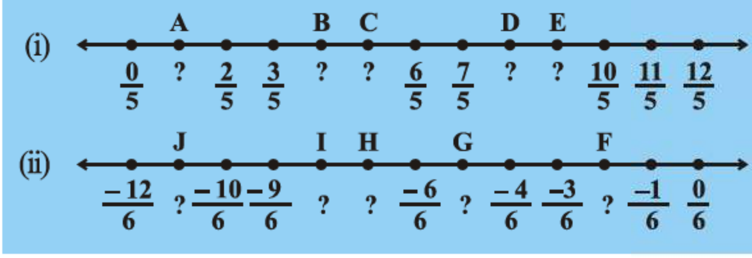
Any rational number can be represented on the number line in this way. In a rational number, the numeral below the bar, i.e., the denominator, tells the number of equal parts into which the first unit has been divided. The numeral above the bar i.e., the numerator, tells 'how many' of these parts are considered. So, a rational number such as $\frac{4}{9}$ means four of nine equal parts on the right of 0 (number line viii) and for $\frac{-7}{4}$, we make 7 markings of distance $\frac{1}{4}$ each on the *left* of zero and starting from 0. The seventh marking is $\frac{-7}{4}$ [number line (ix)].

(viii) 

(ix) 

TRY THESE

Write the rational number for each point labelled with a letter.



1.4 Rational Numbers between Two Rational Numbers

Can you tell the natural numbers between 1 and 5? They are 2, 3 and 4.
How many natural numbers are there between 7 and 9? There is one and it is 8.
How many natural numbers are there between 10 and 11? Obviously none.
List the integers that lie between –5 and 4. They are – 4, – 3, –2, –1, 0, 1, 2, 3.
How many integers are there between –1 and 1?
How many integers are there between –9 and –10?

You will find a definite number of natural numbers (integers) between two natural numbers (integers).

How many rational numbers are there between $\frac{3}{10}$ and $\frac{7}{10}$?

You may have thought that they are only $\frac{4}{10}, \frac{5}{10}$ and $\frac{6}{10}$.

But you can also write $\frac{3}{10}$ as $\frac{30}{100}$ and $\frac{7}{10}$ as $\frac{70}{100}$. Now the numbers, $\frac{31}{100}, \frac{32}{100}, \frac{33}{100}$, ... $\frac{68}{100}, \frac{69}{100}$, are all between $\frac{3}{10}$ and $\frac{7}{10}$. The number of these rational numbers is 39.

Also $\frac{3}{10}$ can be expressed as $\frac{3000}{10000}$ and $\frac{7}{10}$ as $\frac{7000}{10000}$. Now, we see that the rational numbers $\frac{3001}{10000}, \frac{3002}{10000}, ..., \frac{6998}{10000}, \frac{6999}{10000}$ are between $\frac{3}{10}$ and $\frac{7}{10}$. These are 3999 numbers in all.

In this way, we can go on inserting more and more rational numbers between $\frac{3}{10}$ and $\frac{7}{10}$. So unlike natural numbers and integers, the number of rational numbers between two rational numbers is not definite. Here is one more example.

How many rational numbers are there between $\frac{-1}{10}$ and $\frac{3}{10}$?

Obviously $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}$ are rational numbers between the given numbers.

If we write $\frac{-1}{10}$ as $\frac{-10000}{100000}$ and $\frac{3}{10}$ as $\frac{30000}{100000}$, we get the rational numbers $\frac{-9999}{100000}, \frac{-9998}{100000}, ..., \frac{-29998}{100000}, \frac{29999}{100000}$, between $\frac{-1}{10}$ and $\frac{3}{10}$.

You will find that *you get countless rational numbers between any two given rational numbers*.

Example 6: Write any 3 rational numbers between –2 and 0.

Solution: –2 can be written as $\frac{-20}{10}$ and 0 as $\frac{0}{10}$.

Thus we have $\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \ldots, \frac{-1}{10}$ between –2 and 0.

You can take any three of these.

Example 7: Find any ten rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.

Solution: We first convert $\frac{-5}{6}$ and $\frac{5}{8}$ to rational numbers with the same denominators.

$$\frac{-5 \times 4}{6 \times 4} = \frac{-20}{24} \quad \text{and} \quad \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

Thus we have $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, ..., \frac{14}{24}$ as the rational numbers between $\frac{-20}{24}$ and $\frac{15}{24}$.

You can take any ten of these.

Another Method

Let us find rational numbers between 1 and 2. One of them is 1.5 or $1\frac{1}{2}$ or $\frac{3}{2}$. This is the **mean** of 1 and 2. You have studied mean in Class VII.

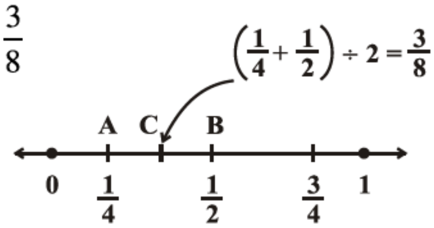
We find that *between any two given numbers, we need not necessarily get an integer but there will always lie a rational number*.

We can use the idea of mean also to find rational numbers between any two given rational numbers.

Example 8: Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$.

Solution: We find the mean of the given rational numbers.

$$\left(\frac{1}{4} + \frac{1}{2}\right) \div 2 = \left(\frac{1+2}{4}\right) \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$



$\frac{3}{8}$ lies between $\frac{1}{4}$ and $\frac{1}{2}$.

This can be seen on the number line also.

We find the mid point of AB which is C, represented by $\left(\frac{1}{4}+\frac{1}{2}\right) \div 2 = \frac{3}{8}$.

We find that $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.

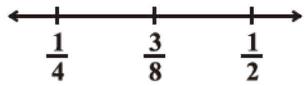
If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

This again shows that there are countless number of rational numbers between any two given rational numbers.

Example 9: Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

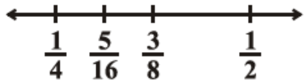
Solution: We find the mean of the given rational numbers.

As given in the above example, the mean is $\frac{3}{8}$ and $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.



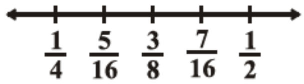
We now find another rational number between $\frac{1}{4}$ and $\frac{3}{8}$. For this, we again find the mean of $\frac{1}{4}$ and $\frac{3}{8}$. That is, $\left(\frac{1}{4}+\frac{3}{8}\right) \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$

$$\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{1}{2}$$



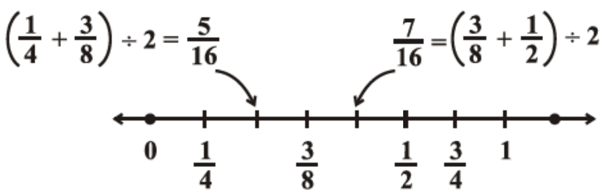
Now find the mean of $\frac{3}{8}$ and $\frac{1}{2}$. We have, $\left(\frac{3}{8}+\frac{1}{2}\right) \div 2 = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$

Thus we get $\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{7}{16} < \frac{1}{2}$.



Thus, $\frac{5}{16}, \frac{3}{8}, \frac{7}{16}$ are the three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

This can clearly be shown on the number line as follows:



In the same way we can obtain as many rational numbers as we want between two given rational numbers. You have noticed that there are countless rational numbers between any two given rational numbers.

EXERCISE 1.2

1. Represent these numbers on the number line. (i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$
2. Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ on the number line.
3. Write five rational numbers which are smaller than 2.
4. Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.
5. Find five rational numbers between.

   (i) $\frac{2}{3}$ and $\frac{4}{5}$ (ii) $\frac{-3}{2}$ and $\frac{5}{3}$ (iii) $\frac{1}{4}$ and $\frac{1}{2}$
6. Write five rational numbers greater than –2.
7. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

WHAT HAVE WE DISCUSSED?

1. Rational numbers are **closed** under the operations of addition, subtraction and multiplication.
2. The operations addition and multiplication are
   (i) **commutative** for rational numbers.
   (ii) **associative** for rational numbers.
3. The rational number 0 is the **additive identity** for rational numbers.
4. The rational number 1 is the **multiplicative identity** for rational numbers.
5. The **additive inverse** of the rational number $\frac{a}{b}$ is $-\frac{a}{b}$ and vice-versa.
6. The **reciprocal** or **multiplicative inverse** of the rational number $\frac{a}{b}$ is $\frac{c}{d}$ if $\frac{a}{b} \times \frac{c}{d} = 1$.
7. **Distributivity** of rational numbers: For all rational numbers a, b and c,
   $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
8. Rational numbers can be represented on a number line.
9. Between any two given rational numbers there are countless rational numbers. The idea of **mean** helps us to find rational numbers between two rational numbers.

CHAPTER

2

Linear Equations in One Variable

2.1 Introduction

In the earlier classes, you have come across several **algebraic expressions** and **equations**. Some examples of expressions we have so far worked with are:

$$5x,\ 2x - 3,\ 3x + y,\ 2xy + 5,\ xyz + x + y + z,\ x^2 + 1,\ y + y^2$$

Some examples of equations are: $5x = 25$, $2x - 3 = 9$, $2y + \frac{5}{2} - \frac{37}{2}$, $6z + 10 - -2$

You would remember that equations use the *equality* (=) sign; it is missing in expressions.

Of these given expressions, many have more than one variable. For example, $2xy + 5$ has two variables. We however, restrict to expressions with only one variable when we form equations. Moreover, the expressions we use to form equations are linear. This means that the highest power of the variable appearing in the expression is 1.

These are linear expressions:

$$2x,\ 2x + 1,\ 3y - 7,\ 12 - 5z,\ \frac{5}{4}(x - 4) + 10$$

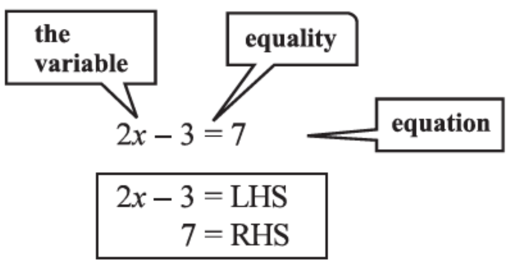
These are **not** linear expressions:

$x^2 + 1,\ y + y^2,\ 1 + z + z^2 + z^3$ (since highest power of variable > 1)

Here we will deal with equations with linear expressions in one variable only. Such equations are known as **linear equations in one variable**. The simple equations which you studied in the earlier classes were all of this type.

Let us briefly revise what we know:

(a) *An algebraic equation is an equality involving variables*. It has an *equality sign*. The expression on the left of the equality sign is the *Left Hand Side* (LHS). The expression on the right of the equality sign is the *Right Hand Side* (RHS).



(b) In an equation the *values of the expressions on the LHS and RHS are equal*. This happens to be *true* only for certain values of the variable. These values are the **solutions** of the equation.

$x = 5$ is the solution of the equation $2x - 3 = 7$. For $x = 5$,
LHS $= 2 \times 5 - 3 = 7 =$ RHS
On the other hand $x = 10$ is not a solution of the equation. For $x = 10$, LHS $= 2 \times 10 - 3 = 17$. This is not equal to the RHS

(c) *How to find the solution of an equation?*

We assume that the two sides of the equation are balanced. We perform the same mathematical operations on both sides of the equation, so that the balance is not disturbed. A few such steps give the solution.

2.2 Solving Equations which have Linear Expressions on one Side and Numbers on the other Side

Let us recall the technique of solving equations with some examples. Observe the solutions; they can be any rational number.

Example 1: Find the solution of $2x - 3 = 7$

Solution:

Step 1 Add 3 to both sides.

$$2x - 3 + 3 = 7 + 3 \quad \text{(The balance is not disturbed)}$$

or $$2x = 10$$

Step 2 Next divide both sides by 2.

$$\frac{2x}{2} = \frac{10}{2}$$

or $$x = 5 \quad \text{(required solution)}$$

Example 2: Solve $2y + 9 = 4$

Solution: Transposing 9 to RHS

$$2y = 4 - 9$$

or $$2y = -5$$

Dividing both sides by 2, $$y = \frac{-5}{2} \quad \text{(solution)}$$

To check the answer: LHS $= 2\left(\frac{-5}{2}\right) + 9 = -5 + 9 = 4 =$ RHS (as required)

Do you notice that the solution $\left(\frac{-5}{2}\right)$ is a rational number? In Class VII, the equations we solved did not have such solutions.

Example 3: Solve $\frac{x}{3}+\frac{5}{2}=-\frac{3}{2}$

Solution: Transposing $\frac{5}{2}$ to the RHS, we get $\frac{x}{3}=\frac{-3}{2}-\frac{5}{2}=-\frac{8}{2}$

or $\frac{x}{3}=-4$

Multiply both sides by 3, $x=-4\times 3$

or $x=-12$ (solution)

***Check*:** LHS $=-\frac{12}{3}+\frac{5}{2}=-4+\frac{5}{2}=\frac{-8+5}{2}=\frac{-3}{2}=$ RHS (as required)

Do you now see that the coefficient of a variable in an equation need not be an integer?

Example 4: Solve $\frac{15}{4}-7x=9$

Solution: We have $\frac{15}{4}-7x=9$

or $-7x=9-\frac{15}{4}$ (transposing $\frac{15}{4}$ to R H S)

or $-7x=\frac{21}{4}$

or $x=\frac{21}{4\times(-7)}$ (dividing both sides by -7)

or $x=-\frac{3\times 7}{4\times 7}$

or $x=-\frac{3}{4}$ (solution)

***Check*:** LHS $=\frac{15}{4}-7\left(\frac{-3}{4}\right)=\frac{15}{4}+\frac{21}{4}=\frac{36}{4}=9=$ RHS (as required)

EXERCISE 2.1

Solve the following equations.

1. $x-2=7$
2. $y+3=10$
3. $6=z+2$
4. $\frac{3}{7}+x=\frac{17}{7}$
5. $6x=12$
6. $\frac{t}{5}=10$
7. $\frac{2x}{3}=18$
8. $1.6=\frac{y}{1.5}$
9. $7x-9=16$

10. $14y - 8 = 13$ **11.** $17 + 6p = 9$ **12.** $\frac{x}{3} + 1 - \frac{7}{15}$

2.3 Some Applications

We begin with a simple example.

Sum of two numbers is 74. One of the numbers is 10 more than the other. What are the numbers?

We have a puzzle here. We do not know either of the two numbers, and we have to find them. We are given two conditions.

(i) One of the numbers is 10 more than the other.

(ii) Their sum is 74.

We already know from Class VII how to proceed. If the smaller number is taken to be x, the larger number is 10 more than x, i.e., $x + 10$. The other condition says that the sum of these two numbers x and $x + 10$ is 74.

This means that $x + (x + 10) = 74$.

or $2x + 10 = 74$

Transposing 10 to RHS, $2x = 74 - 10$

or $2x = 64$

Dividing both sides by 2, $x = 32$. This is one number.

The other number is $x + 10 = 32 + 10 = 42$

The desired numbers are 32 and 42. (Their sum is indeed 74 as given and also one number is 10 more than the other.)

We shall now consider several examples to show how useful this method is.

Example 5: What should be added to twice the rational number $\frac{-7}{3}$ to get $\frac{3}{7}$?

Solution: Twice the rational number $\frac{-7}{3}$ is $2 \times \left(\frac{-7}{3}\right) = \frac{-14}{3}$. Suppose x added to this number gives $\frac{3}{7}$; i.e.,

$$x + \left(\frac{-14}{3}\right) = \frac{3}{7}$$

or $$x - \frac{14}{3} = \frac{3}{7}$$

or $$x = \frac{3}{7} + \frac{14}{3} \qquad \text{(transposing } \frac{14}{3} \text{ to RHS)}$$

$$= \frac{(3 \times 3) + (14 \times 7)}{21} = \frac{9 + 98}{21} - \frac{107}{21}.$$

Thus $\frac{107}{21}$ should be added to $2 \times \left(\frac{-7}{3}\right)$ to give $\frac{3}{7}$.

Example 6: The perimeter of a rectangle is 13 cm and its width is $2\frac{3}{4}$ cm. Find its length.

Solution: Assume the length of the rectangle to be x cm.

The perimeter of the rectangle = 2 × (length + width)

$$= 2 \times (x + 2\frac{3}{4})$$

$$= 2\left(x + \frac{11}{4}\right)$$

The perimeter is given to be 13 cm. Therefore,

$$2\left(x + \frac{11}{4}\right) = 13$$

or $\quad x + \frac{11}{4} = \frac{13}{2}$ (dividing both sides by 2)

or $\quad x = \frac{13}{2} - \frac{11}{4}$

$$= \frac{26}{4} - \frac{11}{4} = \frac{15}{4} = 3\frac{3}{4}$$

The length of the rectangle is $3\frac{3}{4}$ cm.

Example 7: The present age of Sahil's mother is three times the present age of Sahil. After 5 years their ages will add to 66 years. Find their present ages.

Solution: Let Sahil's present age be x years.

We could also choose Sahil's age 5 years later to be x and proceed. Why don't you try it that way?

	Sahil	Mother	Sum
Present age	x	$3x$	
Age 5 years later	$x + 5$	$3x + 5$	$4x + 10$

It is given that this sum is 66 years.

Therefore, $\quad 4x + 10 = 66$

This equation determines Sahil's present age which is x years. To solve the equation, we transpose 10 to RHS,

$$4x = 66 - 10$$

or $$4x = 56$$

or $$x = \frac{56}{4} = 14 \qquad \text{(solution)}$$

Thus, Sahil's present age is 14 years and his mother's age is 42 years. (You may easily check that 5 years from now the sum of their ages will be 66 years.)

Example 8: Bansi has 3 times as many two-rupee coins as he has five-rupee coins. If he has in all a sum of ₹ 77, how many coins of each denomination does he have?

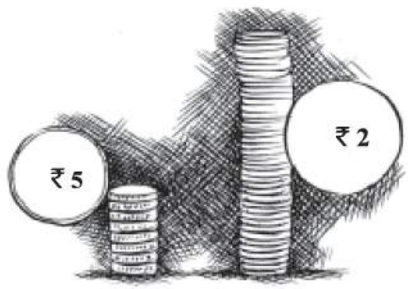
Solution: Let the number of five-rupee coins that Bansi has be x. Then the number of two-rupee coins he has is 3 times x or $3x$.

The amount Bansi has:

(i) from 5 rupee coins, ₹ $5 \times x$ = ₹ $5x$

(ii) from 2 rupee coins, ₹ $2 \times 3x$ = ₹ $6x$

Hence the total money he has = ₹ $11x$



But this is given to be ₹ 77; therefore,

$$11x = 77$$

or $$x = \frac{77}{11} = 7$$

Thus, number of five-rupee coins = $x = 7$

and number of two-rupee coins = $3x = 21$ (solution)

(You can check that the total money with Bansi is ₹ 77.)

Example 9: The sum of three consecutive multiples of 11 is 363. Find these multiples.

Solution: If x is a multiple of 11, the next multiple is $x + 11$. The next to this is $x + 11 + 11$ or $x + 22$. So we can take three consecutive multiples of 11 as x, $x + 11$ and $x + 22$.

It is given that the sum of these consecutive multiples of 11 is 363. This will give the following equation:

$$x + (x + 11) + (x + 22) = 363$$

or $$x + x + 11 + x + 22 = 363$$

or $$3x + 33 = 363$$

or $$3x = 363 - 33$$

or $$3x = 330$$

> Alternatively, we may think of the multiple of 11 immediately before x. This is $(x - 11)$. Therefore, we may take three consecutive multiples of 11 as $x - 11$, x, $x + 11$.
>
> In this case we arrive at the equation
>
> $$(x - 11) + x + (x + 11) = 363$$
>
> or $$3x = 363$$

or $x = \frac{330}{3}$

$= 110$

Hence, the three consecutive multiples are 110, 121, 132 (answer).

or $x = \frac{363}{3} = 121$. Therefore,

$x = 121, x - 11 = 110, x + 11 = 132$

Hence, the three consecutive multiples are 110, 121, 132.

We can see that we can adopt different ways to find a solution for the problem.

Example 10: The difference between two whole numbers is 66. The ratio of the two numbers is 2 : 5. What are the two numbers?

Solution: Since the ratio of the two numbers is 2 : 5, we may take one number to be $2x$ and the other to be $5x$. (Note that $2x : 5x$ is same as 2 : 5.)

The difference between the two numbers is $(5x - 2x)$. It is given that the difference is 66. Therefore,

$$5x - 2x = 66$$

or $3x = 66$

or $x = 22$

Since the numbers are $2x$ and $5x$, they are 2×22 or 44 and 5×22 or 110, respectively.

The difference between the two numbers is $110 - 44 = 66$ as desired.

Example 11: Deveshi has a total of ₹ 590 as currency notes in the denominations of ₹ 50, ₹ 20 and ₹ 10. The ratio of the number of ₹ 50 notes and ₹ 20 notes is 3:5. If she has a total of 25 notes, how many notes of each denomination she has?

Solution: Let the number of ₹ 50 notes and ₹ 20 notes be $3x$ and $5x$, respectively.

But she has 25 notes in total.

Therefore, the number of ₹ 10 notes $= 25 - (3x + 5x) = 25 - 8x$

The amount she has

from ₹ 50 notes : $3x \times 50 =$ ₹ $150x$

from ₹ 20 notes : $5x \times 20 =$ ₹ $100x$

from ₹ 10 notes : $(25 - 8x) \times 10 =$ ₹ $(250 - 80x)$

Hence the total money she has $= 150x + 100x + (250 - 80x) =$ ₹ $(170x + 250)$

But she has ₹ 590. Therefore, $170x + 250 = 590$

or $170x = 590 - 250 = 340$

or $x = \frac{340}{170} = 2$

The number of ₹ 50 notes she has $= 3x$

$= 3 \times 2 = 6$

The number of ₹ 20 notes she has $= 5x = 5 \times 2 = 10$

The number of ₹ 10 notes she has $= 25 - 8x$

$= 25 - (8 \times 2) = 25 - 16 = 9$

EXERCISE 2.2

1. If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?
2. The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?
3. The base of an isosceles triangle is $\frac{4}{3}$ cm. The perimeter of the triangle is $4\frac{2}{15}$ cm. What is the length of either of the remaining equal sides?
4. Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.
5. Two numbers are in the ratio 5:3. If they differ by 18, what are the numbers?
6. Three consecutive integers add up to 51. What are these integers?
7. The sum of three consecutive multiples of 8 is 888. Find the multiples.
8. Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add up to 74. Find these numbers.
9. The ages of Rahul and Haroon are in the ratio 5:7. Four years later the sum of their ages will be 56 years. What are their present ages?
10. The number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. What is the total class strength?
11. Bharat's father is 26 years younger than Bharat's grandfather and 29 years older than Bharat. The sum of the ages of all the three is 135 years. What is the age of each one of them?
12. Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?
13. A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

14. Lakshmi is a cashier in a bank. She has currency notes of denominations ₹ 100, ₹ 50 and ₹ 10, respectively. The ratio of the number of these notes is 2:3:5. The total cash with Lakshmi is ₹ 4,00,000. How many notes of each denomination does she have?
15. I have a total of ₹ 300 in coins of denomination ₹ 1, ₹ 2 and ₹ 5. The number of ₹ 2 coins is 3 times the number of ₹ 5 coins. The total number of coins is 160. How many coins of each denomination are with me?
16. The organisers of an essay competition decide that a winner in the competition gets a prize of ₹ 100 and a participant who does not win gets a prize of ₹ 25. The total prize money distributed is ₹ 3,000. Find the number of winners, if the total number of participants is 63.

2.4 Solving Equations having the Variable on both Sides

An equation is the equality of the values of two expressions. In the equation $2x - 3 = 7$, the two expressions are $2x - 3$ and 7. In most examples that we have come across so far, the RHS is just a number. But this need not always be so; both sides could have expressions with variables. For example, the equation $2x - 3 = x + 2$ has expressions with a variable on both sides; the expression on the LHS is $(2x - 3)$ and the expression on the RHS is $(x + 2)$.

- We now discuss how to solve such equations which have expressions with the variable on both sides.

Example 12: Solve $2x - 3 = x + 2$

Solution: We have

$$2x = x + 2 + 3$$

or $$2x = x + 5$$

or $$2x - x = x + 5 - x \quad \text{(subtracting } x \text{ from both sides)}$$

or $$x = 5 \quad \text{(solution)}$$

Here we subtracted from both sides of the equation, not a number (constant), but a term involving the variable. We can do this as variables are also numbers. Also, note that subtracting x from both sides amounts to transposing x to LHS.

Example 13: Solve $5x + \frac{7}{2} = \frac{3}{2}x - 14$

Solution: Multiply both sides of the equation by 2. We get

$$2 \times \left(5x + \frac{7}{2}\right) = 2 \times \left(\frac{3}{2}x - 14\right)$$

$$(2 \times 5x) + \left(2 \times \frac{7}{2}\right) = \left(2 \times \frac{3}{2}x\right) - (2 \times 14)$$

or $$10x + 7 = 3x - 28$$

or $$10x - 3x + 7 = -28 \quad \text{(transposing } 3x \text{ to LHS)}$$

or $$7x + 7 = -28$$

or $$7x = -28 - 7$$

or $$7x = -35$$

or $x = \frac{-35}{7}$ or $x = -5$ (solution)

EXERCISE 2.3

Solve the following equations and check your results.

1. $3x = 2x + 18$
2. $5t - 3 = 3t - 5$
3. $5x + 9 = 5 + 3x$
4. $4z + 3 = 6 + 2z$
5. $2x - 1 = 14 - x$
6. $8x + 4 = 3\,(x - 1) + 7$
7. $x = \frac{4}{5}\,(x + 10)$
8. $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$
9. $2y + \frac{5}{3} = \frac{26}{3} - y$
10. $3m = 5\,m - \frac{8}{5}$

2.5 Some More Applications

Example 14: The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number?

Solution: Take, for example, a two-digit number, say, 56. It can be written as $56 = (10 \times 5) + 6$.

If the digits in 56 are interchanged, we get 65, which can be written as $(10 \times 6) + 5$.

Let us take the two digit number such that the digit in the units place is b. The digit in the tens place differs from b by 3. Let us take it as $b + 3$. So the two-digit number is $10\,(b + 3) + b = 10b + 30 + b = 11b + 30$.

With interchange of digits, the resulting two-digit number will be

$$10b + (b + 3) = 11b + 3$$

> Could we take the tens place digit to be $(b - 3)$? Try it and see what solution you get.

If we add these two two-digit numbers, their sum is

$$(11b + 30) + (11b + 3) = 11b + 11b + 30 + 3 = 22b + 33$$

It is given that the sum is 143. Therefore, $22b + 33 = 143$

or $\quad 22b = 143 - 33$

or $\quad 22b = 110$

or $\quad b = \frac{110}{22}$

or $\quad b = 5$

> Remember, this is the solution when we choose the tens digits to be 3 more than the unit's digits. What happens if we take the tens digit to be $(b - 3)$?

The units digit is 5 and therefore the tens digit is $5 + 3$ which is 8. The number is 85.

Check: On interchange of digits the number we get is 58. The sum of 85 and 58 is 143 as given.

> The statement of the example is valid for both 58 and 85 and both are correct answers.

Example 15: Arjun is twice as old as Shriya. Five years ago his age was three times Shriya's age. Find their present ages.

Solution: Let us take Shriya's present age to be x years.

Then Arjun's present age would be $2x$ years.

Shriya's age five years ago was $(x - 5)$ years.

Arjun's age five years ago was $(2x - 5)$ years.

It is given that Arjun's age five years ago was three times Shriya's age.

Thus, $2x - 5 = 3(x - 5)$

or $2x - 5 = 3x - 15$

or $15 - 5 = 3x - 2x$

or $10 = x$

So, Shriya's present age $= x = 10$ years.

Therefore, Arjun's present age $= 2x = 2 \times 10 = 20$ years.

EXERCISE 2.4

1. Amina thinks of a number and subtracts $\frac{5}{2}$ from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?
2. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?
3. Sum of the digits of a two-digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?
4. One of the two digits of a two digit number is three times the other digit. If you interchange the digits of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?
5. Saroj's mother's present age is six times Saroj's present age. Saroj's age five years from now will be one third of his mother's present age. What are their present ages?
6. There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11:4. At the rate ₹100 per metre it will cost the village panchayat ₹ 75,000 to fence the plot. What are the dimensions of the plot?
7. Hasan buys two kinds of cloth materials for school uniforms, shirt material that costs him ₹ 50 per metre and trouser material that costs him ₹ 90 per metre.

For every 3 meters of the shirt material he buys 2 metres of the trouser material. He sells the materials at 12% and 10% profit respectively. His total sale is ₹ 36,600. How much trouser material did he buy?

8. Half of a herd of deer are grazing in the field and three fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.

9. A grandfather is ten times older than his granddaughter. He is also 54 years older than her. Find their present ages.

10. Aman's age is three times his son's age. Ten years ago he was five times his son's age. Find their present ages.

2.6 Reducing Equations to Simpler Form

Example 16: Solve $\frac{6x+1}{3}+1-\frac{x-3}{6}$

Solution: Multiplying both sides of the equation by 6,

> Why 6? Because it is the smallest multiple (or LCM) of the given denominators.

$$\frac{6\,(6x+1)}{3}+6\times 1 = \frac{6(x-3)}{6}$$

or $\quad 2\,(6x + 1) + 6 = x - 3$

or $\quad 12x + 2 + 6 = x - 3$ (opening the brackets)

or $\quad 12x + 8 = x - 3$

or $\quad 12x - x + 8 = -3$

or $\quad 11x + 8 = -3$

or $\quad 11x = -3 - 8$

or $\quad 11x = -11$

or $\quad x = -1$ (required solution)

Check: LHS $=\frac{6(-1)+1}{3}+1=\frac{-6+1}{3}+1 = \frac{-5}{3}+\frac{3}{3}=\frac{-5+3}{3}=\frac{-2}{3}$

RHS $=\frac{(-1)-3}{6}-\frac{-4}{6}-\frac{-2}{3}$

LHS = RHS. (as required)

Example 17: Solve $5x - 2\,(2x - 7) = 2\,(3x - 1) + \frac{7}{2}$

Solution: Let us open the brackets,

LHS $= 5x - 4x + 14 = x + 14$

$$\text{RHS} = 6x - 2 + \frac{7}{2} = 6x - \frac{4}{2} + \frac{7}{2} - 6x + \frac{3}{2}$$

The equation is $x + 14 = 6x + \frac{3}{2}$

or $\quad 14 = 6x - x + \frac{3}{2}$

or $\quad 14 = 5x + \frac{3}{2}$

or $\quad 14 - \frac{3}{2} = 5x \qquad$ (transposing $\frac{3}{2}$)

or $\quad \frac{28-3}{2} = 5x$

or $\quad \frac{25}{2} = 5x$

or $\quad x = \frac{25}{2} \times \frac{1}{5} - \frac{5 \times 5}{2 \times 5} - \frac{5}{2}$

Did you observe how we simplified the form of the given equation? Here, we had to multiply both sides of the equation by the LCM of the denominators of the terms in the expressions of the equation.

Therefore, required solution is $x = \frac{5}{2}$.

Check: $\text{LHS} = 5 \times \frac{5}{2} - 2\left(\frac{5}{2} \times 2 - 7\right)$

$$= \frac{25}{2} - 2(5-7) = \frac{25}{2} - 2(-2) = \frac{25}{2} + 4 = \frac{25+8}{2} = \frac{33}{2}$$

$$\text{RHS} = 2\left(\frac{5}{2} \times 3 - 1\right) + \frac{7}{2} = 2\left(\frac{15}{2} - \frac{2}{2}\right) + \frac{7}{2} = \frac{2 \times 13}{2} + \frac{7}{2}$$

$$= \frac{26+7}{2} - \frac{33}{2} = \text{LHS.} \quad \text{(as required)}$$

Note, in this example we brought the equation to a simpler form by opening brackets and combining like terms on both sides of the equation.

EXERCISE 2.5

Solve the following linear equations.

1. $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$ **2.** $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$ **3.** $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{2}$

4. $\frac{x-5}{3} - \frac{x-3}{5}$ **5.** $\frac{3t-2}{4} - \frac{2t+3}{3} - \frac{2}{3} - t$ **6.** $m - \frac{m-1}{2} - 1 - \frac{m-2}{3}$

Simplify and solve the following linear equations.

7. $3(t-3) = 5(2t+1)$ **8.** $15(y-4) - 2(y-9) + 5(y+6) = 0$

9. $3(5z-7) - 2(9z-11) = 4(8z-13) - 17$

10. $0.25(4f-3) = 0.05(10f-9)$

2.7 Equations Reducible to the Linear Form

Example 18: Solve $\frac{x+1}{2x+3} = \frac{3}{8}$

Solution: Observe that the equation is not a linear equation, since the expression on its LHS is not linear. But we can put it into the form of a linear equation. We multiply both sides of the equation by $(2x+3)$,

$$\left(\frac{x+1}{2x+3}\right) \times (2x+3) = \frac{3}{8} \times (2x+3)$$

Note that $2x + 3 \neq 0$ (Why?)

Notice that $(2x+3)$ gets cancelled on the LHS We have then,

$$x + 1 = \frac{3\,(2x+3)}{8}$$

We have now a linear equation which we know how to solve.
Multiplying both sides by 8

$$8\,(x+1) = 3\,(2x+3)$$

This step can be directly obtained by 'cross-multiplication'

$$\frac{x+1}{2x+3} \nearrow\!\!\!\!\nwarrow \frac{3}{8}$$

or $8x + 8 = 6x + 9$

or $8x = 6x + 9 - 8$

or $8x = 6x + 1$

or $8x - 6x = 1$

or $2x = 1$

or $x = \frac{1}{2}$

The solution is $x = \frac{1}{2}$.

Check : Numerator of LHS $= \frac{1}{2} + 1 = \frac{1+2}{2} = \frac{3}{2}$

Denominator of LHS $= 2x + 3 = 2 \times \frac{1}{2} + 3 = 1 + 3 = 4$

LHS = numerator ÷ denominator = $\frac{3}{2} \div 4 = \frac{3}{2}\times\frac{1}{4} = \frac{3}{8}$

LHS = RHS.

Example 19: Present ages of Anu and Raj are in the ratio 4:5. Eight years from now the ratio of their ages will be 5:6. Find their present ages.

Solution: Let the present ages of Anu and Raj be $4x$ years and $5x$ years respectively.

After eight years. Anu's age = $(4x + 8)$ years;

After eight years, Raj's age = $(5x + 8)$ years.

Therefore, the ratio of their ages after eight years = $\frac{4x+8}{5x+8}$

This is given to be 5 : 6

Therefore, $\frac{4x+8}{5x+8} = \frac{5}{6}$

Cross-multiplication gives $6\,(4x + 8) = 5\,(5x + 8)$

or $24x + 48 = 25x + 40$

or $24x + 48 - 40 = 25x$

or $24x + 8 = 25x$

or $8 = 25x - 24x$

or $8 = x$

Therefore, Anu's present age = $4x = 4 \times 8 = 32$ years

Raj's present age = $5x = 5 \times 8 = 40$ years

EXERCISE 2.6

Solve the following equations.

1. $\frac{8x-3}{3x} = 2$ **2.** $\frac{9x}{7-6x} = 15$ **3.** $\frac{z}{z+15} = \frac{4}{9}$

4. $\frac{3y+4}{2-6y} = \frac{-2}{5}$ **5.** $\frac{7y+4}{y+2} = \frac{-4}{3}$

6. The ages of Hari and Harry are in the ratio 5:7. Four years from now the ratio of their ages will be 3:4. Find their present ages.

7. The denominator of a rational number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

WHAT HAVE WE DISCUSSED?

1. An algebraic equation is an equality involving variables. It says that the value of the expression on one side of the equality sign is equal to the value of the expression on the other side.
2. The equations we study in Classes VI, VII and VIII are linear equations in one variable. In such equations, the expressions which form the equation contain only one variable. Further, the equations are linear, i.e., the highest power of the variable appearing in the equation is 1.
3. A linear equation may have for its solution any rational number.
4. An equation may have linear expressions on both sides. Equations that we studied in Classes VI and VII had just a number on one side of the equation.
5. Just as numbers, variables can, also, be transposed from one side of the equation to the other.
6. Occasionally, the expressions forming equations have to be simplified before we can solve them by usual methods. Some equations may not even be linear to begin with, but they can be brought to a linear form by multiplying both sides of the equation by a suitable expression.
7. The utility of linear equations is in their diverse applications; different problems on numbers, ages, perimeters, combination of currency notes, and so on can be solved using linear equations.

CHAPTER 3

Understanding Quadrilaterals

0852CH03

3.1 Introduction

You know that the paper is a model for a **plane surface**. When you join a number of points without lifting a pencil from the paper (and without retracing any portion of the drawing other than single points), you get a **plane curve**.

Try to recall different varieties of curves you have seen in the earlier classes.

Match the following: (Caution! A figure may match to more than one type).

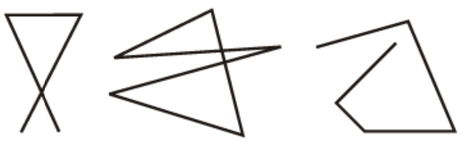
Figure	Type
(1) 	(a) Simple closed curve
(2) 	(b) A closed curve that is not simple
(3) 	(c) Simple curve that is not closed
(4) 	(d) Not a simple curve

Compare your matchings with those of your friends. Do they agree?

3.2 Polygons

A simple closed curve made up of only line segments is called a **polygon**.

Curves that are polygons

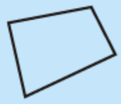
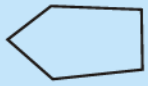
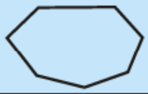
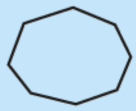


Curves that are not polygons

Try to give a few more examples and non-examples for a polygon.
Draw a rough figure of a polygon and identify its sides and vertices.

3.2.1 Classification of polygons

We classify polygons according to the number of sides (or vertices) they have.

Number of sides or vertices	Classification	Sample figure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	
⋮	⋮	⋮
n	n-gon	

3.2.2 Diagonals

A diagonal is a line segment connecting two non-consecutive vertices of a polygon (Fig 3.1).

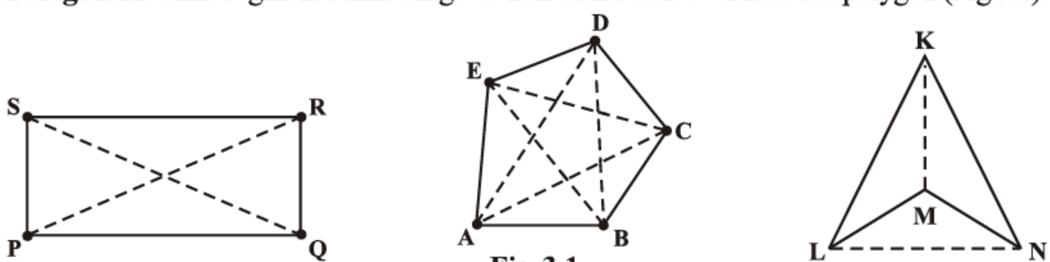
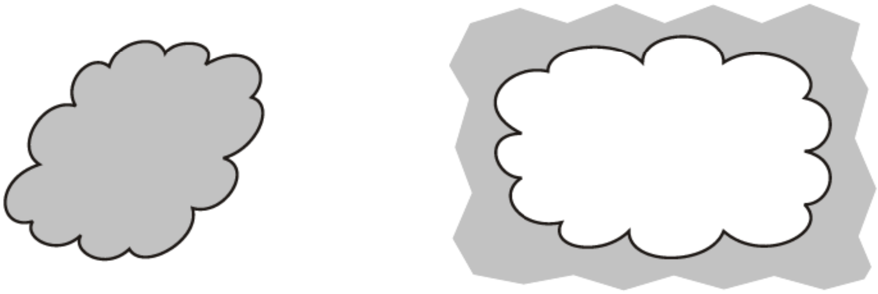


Fig 3.1

Can you name the diagonals in each of the above figures? (Fig 3.1)

Is $\overline{PQ}$ a diagonal? What about $\overline{LN}$?

You already know what we mean by **interior** and **exterior** of a closed curve (Fig 3.2).

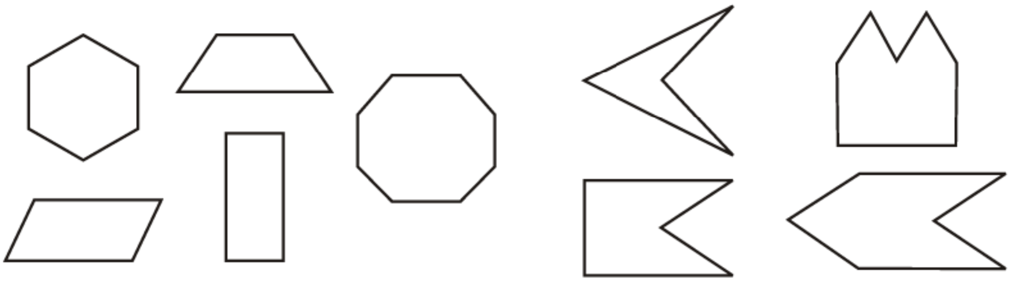


Interior **Fig 3.2** **Exterior**

The interior has a boundary. Does the exterior have a boundary? Discuss with your friends.

3.2.3 Convex and concave polygons

Here are some convex polygons and some concave polygons. (Fig 3.3)



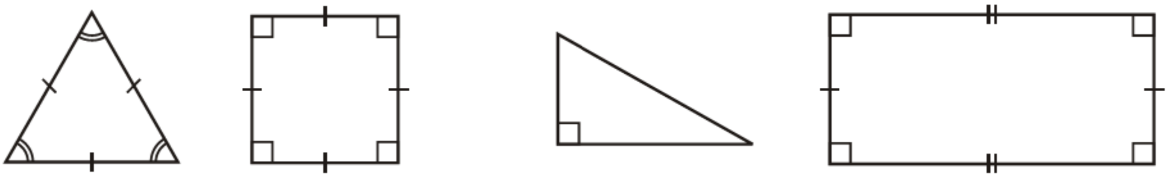
Convex polygons **Fig 3.3** **Concave polygons**

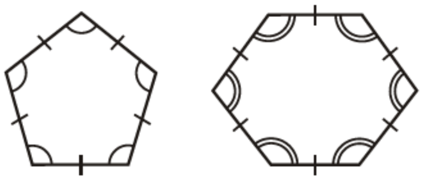
Can you find how these types of polygons differ from one another? Polygons that are convex have no portions of their diagonals in their exteriors or any line segment joining any two different points, in the interior of the polygon, lies wholly in the interior of it . Is this true with concave polygons? Study the figures given. Then try to describe in your own words what we mean by a convex polygon and what we mean by a concave polygon. Give two rough sketches of each kind.

In our work in this class, we will be dealing with convex polygons only.

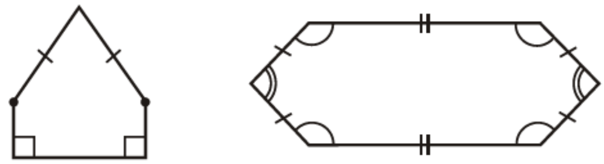
3.2.4 Regular and irregular polygons

A regular polygon is both 'equiangular' and 'equilateral'. For example, a square has sides of equal length and angles of equal measure. Hence it is a regular polygon. A rectangle is equiangular but not equilateral. Is a rectangle a regular polygon? Is an equilateral triangle a regular polygon? Why?





Regular polygons



Polygons that are not regular

[**Note:** Use of  or  indicates segments of equal length].

In the previous classes, have you come across any quadrilateral that is equilateral but not equiangular? Recall the quadrilateral shapes you saw in earlier classes – Rectangle, Square, Rhombus etc.

Is there a triangle that is equilateral but not equiangular?

3.2.5 Angle sum property

Do you remember the angle-sum property of a triangle? The sum of the measures of the three angles of a triangle is 180°. Recall the methods by which we tried to visualise this fact. We now extend these ideas to a quadrilateral.

DO THIS

1. Take any quadrilateral, say ABCD (Fig 3.4). Divide it into two triangles, by drawing a diagonal. You get six angles 1, 2, 3, 4, 5 and 6.

   Use the angle-sum property of a triangle and argue how the sum of the measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ amounts to $180° + 180° = 360°$.

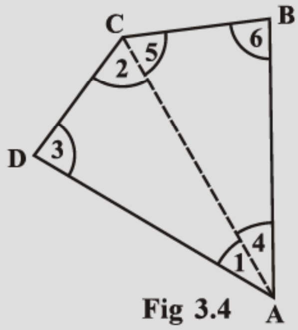


Fig 3.4

2. Take four congruent card-board copies of any quadrilateral ABCD, with angles as shown [Fig 3.5 (i)]. Arrange the copies as shown in the figure, where angles $\angle 1, \angle 2, \angle 3, \angle 4$ meet at a point [Fig 3.5 (ii)].

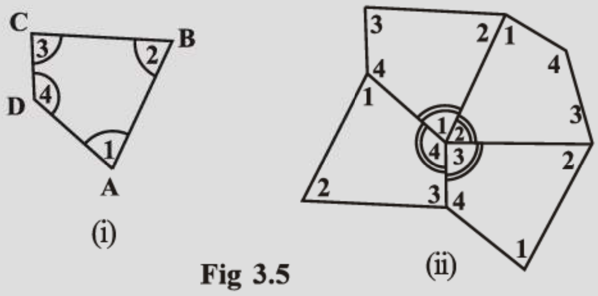
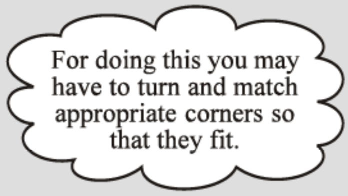


Fig 3.5



What can you say about the sum of the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$?

[**Note:** We denote the angles by $\angle 1, \angle 2, \angle 3$, etc., and their respective measures by $m\angle 1$, $m\angle 2$, $m\angle 3$, etc.]

The sum of the measures of the four angles of a quadrilateral is __________.

You may arrive at this result in several other ways also.

3. As before consider quadrilateral ABCD (Fig 3.6). Let P be any point in its interior. Join P to vertices A, B, C and D. In the figure, consider ΔPAB. From this we see $x = 180° - m\angle 2 - m\angle 3$; similarly from ΔPBC, $y = 180° - m\angle 4 - m\angle 5$, from ΔPCD, $z = 180° - m\angle 6 - m\angle 7$ and from ΔPDA, $w = 180° - m\angle 8 - m\angle 1$. Use this to find the total measure $m\angle 1 + m\angle 2 + ... + m\angle 8$, does it help you to arrive at the result? Remember $\angle x + \angle y + \angle z + \angle w = 360°$.

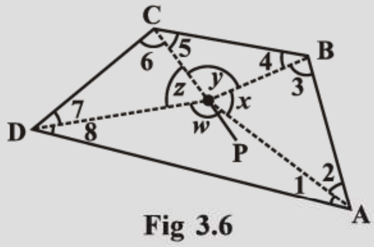


Fig 3.6

4. These quadrilaterals were convex. What would happen if the quadrilateral is not convex? Consider quadrilateral ABCD. Split it into two triangles and find the sum of the interior angles (Fig 3.7).

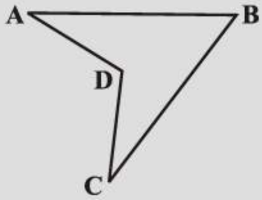


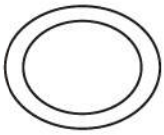
Fig 3.7

EXERCISE 3.1

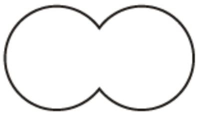
1. Given here are some figures.


(1)


(2)


(3)


(4)


(5)


(6)

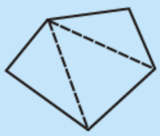
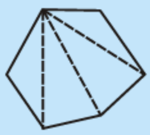

(7)


(8)

Classify each of them on the basis of the following.

(a) Simple curve (b) Simple closed curve (c) Polygon
(d) Convex polygon (e) Concave polygon

2. How many diagonals does each of the following have?
   (a) A convex quadrilateral (b) A regular hexagon (c) A triangle
3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)
4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	180°	$2 \times 180°$ $= (4 - 2) \times 180°$	$3 \times 180°$ $= (5 - 2) \times 180°$	$4 \times 180°$ $= (6 - 2) \times 180°$

What can you say about the angle sum of a convex polygon with number of sides?

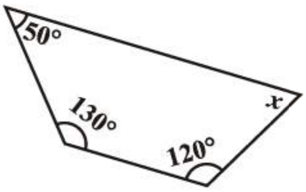
(a) 7 (b) 8 (c) 10 (d) n

5. What is a regular polygon?

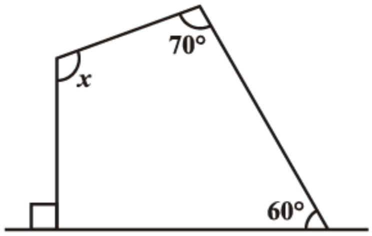
State the name of a regular polygon of

(i) 3 sides (ii) 4 sides (iii) 6 sides

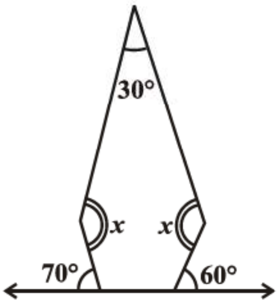
6. Find the angle measure x in the following figures.



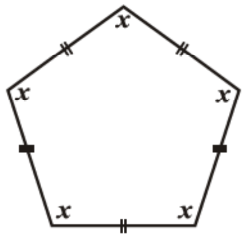
(a)



(b)

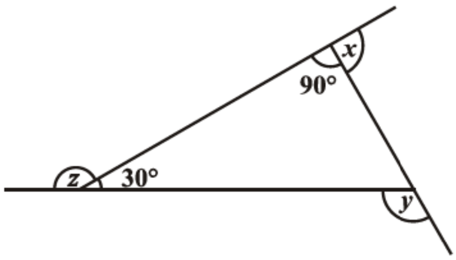


(c)

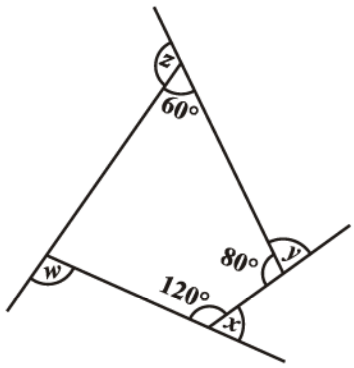


(d)

7.



(a) Find $x + y + z$



(b) Find $x + y + z + w$

3.3 Sum of the Measures of the Exterior Angles of a Polygon

On many occasions a knowledge of exterior angles may throw light on the nature of interior angles and sides.

DO THIS

Draw a polygon on the floor, using a piece of chalk. (In the figure, a pentagon ABCDE is shown) (Fig 3.8).

We want to know the total measure of angles, i.e, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5$. Start at A. Walk along $\overline{AB}$. On reaching B, you need to turn through an angle of $m\angle 1$, to walk along $\overline{BC}$. When you reach at C, you need to turn through an angle of $m\angle 2$ to walk along $\overline{CD}$. You continue to move in this manner, until you return to side AB. You would have in fact made one complete turn.

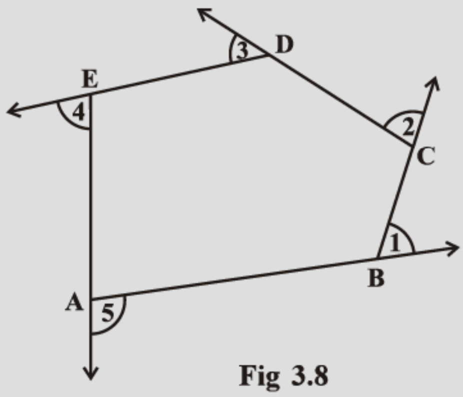


Fig 3.8

Therefore, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

This is true whatever be the number of sides of the polygon.

Therefore, *the sum of the measures of the external angles of any polygon is 360°.*

Example 1: Find measure x in Fig 3.9.

Solution: $x + 90° + 50° + 110° = 360°$ (Why?)

$x + 250° = 360°$

$x = 110°$

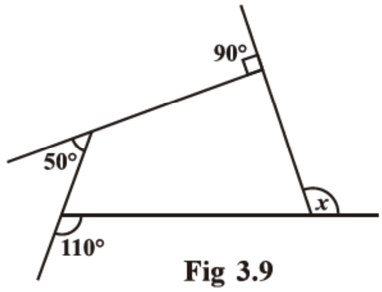


Fig 3.9

TRY THESE

Take a regular hexagon Fig 3.10.

1. What is the sum of the measures of its exterior angles x, y, z, p, q, r?
2. Is $x = y = z = p = q = r$? Why?
3. What is the measure of each?
   (i) exterior angle (ii) interior angle
4. Repeat this activity for the cases of
   (i) a regular octagon (ii) a regular 20-gon

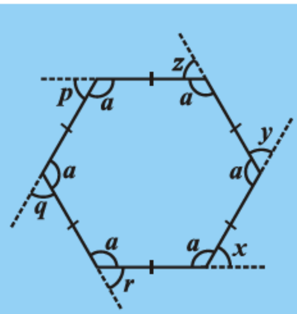


Fig 3.10

Example 2: Find the number of sides of a regular polygon whose each exterior angle has a measure of 45°.

Solution: Total measure of all exterior angles = 360°

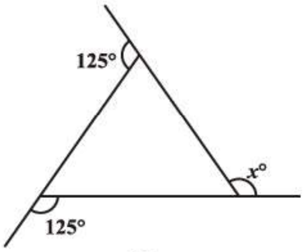
Measure of each exterior angle = 45°

Therefore, the number of exterior angles $= \frac{360}{45} = 8$

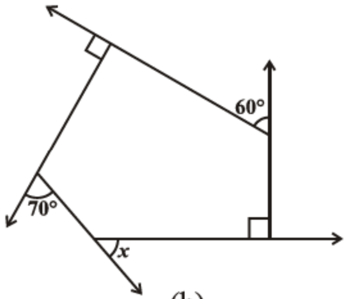
The polygon has 8 sides.

EXERCISE 3.2

1. Find x in the following figures.



(a)



(b)

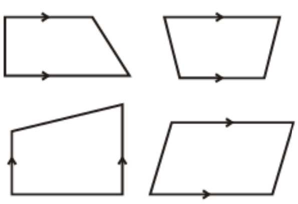
2. Find the measure of each exterior angle of a regular polygon of
   (i) 9 sides (ii) 15 sides
3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?
4. How many sides does a regular polygon have if each of its interior angles is 165°?
5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?
   (b) Can it be an interior angle of a regular polygon? Why?
6. (a) What is the minimum interior angle possible for a regular polygon? Why?
   (b) What is the maximum exterior angle possible for a regular polygon?

3.4 Kinds of Quadrilaterals

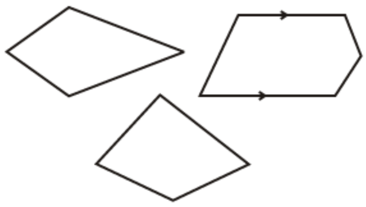
Based on the nature of the sides or angles of a quadrilateral, it gets special names.

3.4.1 Trapezium

Trapezium is a quadrilateral with a pair of parallel sides.



These are trapeziums



These are not trapeziums

Study the above figures and discuss with your friends why some of them are trapeziums while some are not. (**Note:** *The arrow marks indicate parallel lines*).

DO THIS

1. Take identical cut-outs of congruent triangles of sides 3 cm, 4 cm, 5 cm. Arrange them as shown (Fig 3.11).

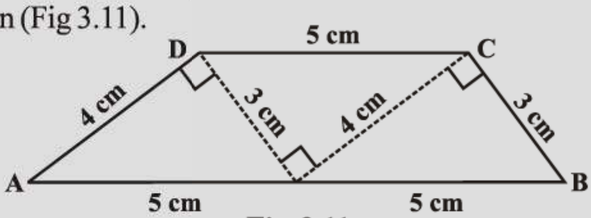


Fig 3.11

You get a trapezium. (Check it!) Which are the parallel sides here? Should the non-parallel sides be equal?

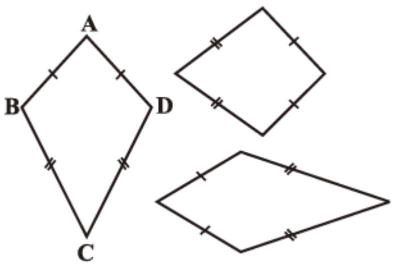
You can get two more trapeziums using the same set of triangles. Find them out and discuss their shapes.

2. Take four set-squares from your and your friend's instrument boxes. Use different numbers of them to place side-by-side and obtain different trapeziums.

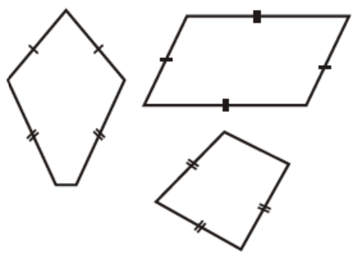
   If the non-parallel sides of a trapezium are of equal length, we call it an *isosceles trapezium*. Did you get an isoceles trapezium in any of your investigations given above?

3.4.2 Kite

Kite is a special type of a quadrilateral. The sides with the same markings in each figure are equal. For example AB = AD and BC = CD.



These are kites



These are not kites

Study these figures and try to describe what a kite is. Observe that

(i) A kite has 4 sides (It is a quadrilateral).

(ii) There are exactly two **distinct consecutive pairs** of sides of equal length.

Check whether a square is a kite.

DO THIS

Take a thick white sheet.

Fold the paper once.

Draw two line segments of different lengths as shown in Fig 3.12.

Cut along the line segments and open up.

You have the shape of a kite (Fig 3.13).

Has the kite any line symmetry?

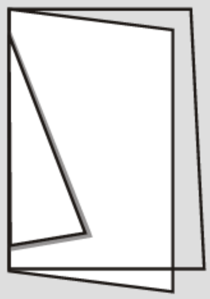


Fig 3.12

Show that ΔABC and ΔADC are congruent. What do we infer from this?

Fold both the diagonals of the kite. Use the set-square to check if they cut at right angles. Are the diagonals equal in length?

Verify (by paper-folding or measurement) if the diagonals bisect each other.

By folding an angle of the kite on its opposite, check for angles of equal measure.

Observe the diagonal folds; do they indicate any diagonal being an angle bisector?

Share your findings with others and list them. A summary of these results are given elsewhere in the chapter for your reference.

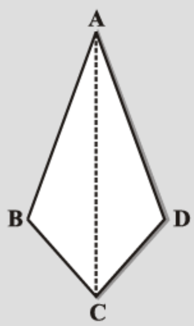
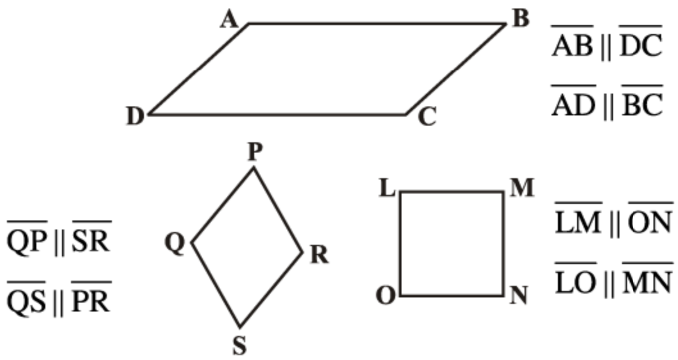


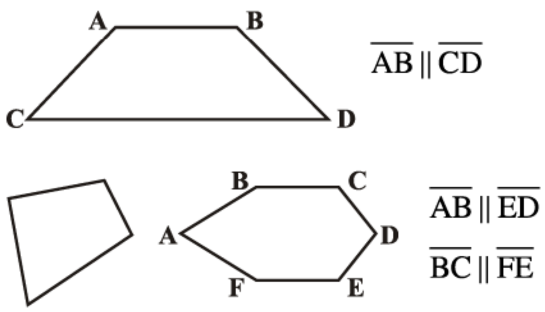
Fig 3.13

3.4.3 Parallelogram

A parallelogram is a quadrilateral. As the name suggests, it has something to do with parallel lines.



These are parallelograms



These are not parallelograms

Study these figures and try to describe in your own words what we mean by a parallelogram. Share your observations with your friends.

Check whether a rectangle is also a parallelogram.

DO THIS

Take two different rectangular cardboard strips of different widths (Fig 3.14).

Strip 1 **Fig 3.14** **Strip 2**

Place one strip horizontally and draw lines along its edge as drawn in the figure (Fig 3.15).

Now place the other strip in a slant position over the lines drawn and use this to draw two more lines as shown (Fig 3.16).

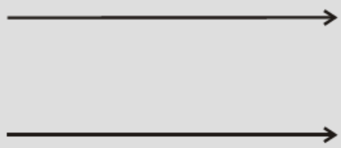


Fig 3.15

These four lines enclose a quadrilateral. This is made up of two pairs of parallel lines (Fig 3.17).

Fig 3.16

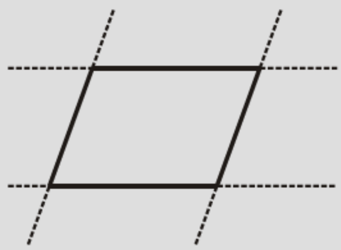


Fig 3.17

It is a parallelogram.

A parallelogram is a quadrilateral whose opposite sides are parallel.

3.4.4 Elements of a parallelogram

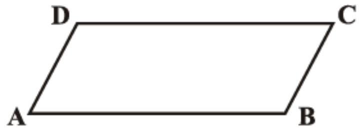


Fig 3.18

There are four sides and four angles in a parallelogram. Some of these are equal. There are some terms associated with these elements that you need to remember.

Given a parallelogram ABCD (Fig 3.18).

$\overline{AB}$ and $\overline{DC}$, are **opposite sides**. $\overline{AD}$ and $\overline{BC}$ form another pair of opposite sides.

$\angle$A and $\angle$C are a pair of **opposite angles**; another pair of opposite angles would be $\angle$B and $\angle$D.

$\overline{AB}$ and $\overline{BC}$ are **adjacent sides**. This means, one of the sides starts where the other ends. Are $\overline{BC}$ and $\overline{CD}$ adjacent sides too? Try to find two more pairs of adjacent sides.

$\angle$A and $\angle$B are **adjacent angles**. They are at the ends of the same side. $\angle$B and $\angle$C are also adjacent. Identify other pairs of adjacent angles of the parallelogram.

DO THIS

Take cut-outs of two identical parallelograms, say ABCD and A′B′C′D′ (Fig 3.19).

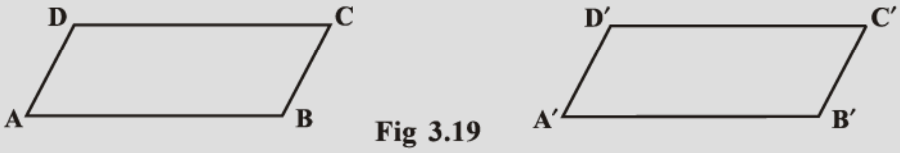


Fig 3.19

Here $\overline{AB}$ is same as $\overline{A'B'}$ except for the name. Similarly the other corresponding sides are equal too.

Place $\overline{A'B'}$ over $\overline{DC}$. Do they coincide? What can you now say about the lengths $\overline{AB}$ and $\overline{DC}$?

Similarly examine the lengths $\overline{AD}$ and $\overline{BC}$. What do you find?

You may also arrive at this result by measuring $\overline{AB}$ and $\overline{DC}$.

Property: *The opposite sides of a parallelogram are of equal length.*

TRY THESE

Take two identical set squares with angles 30° – 60° – 90° and place them adjacently to form a parallelogram as shown in Fig 3.20. Does this help you to verify the above property?

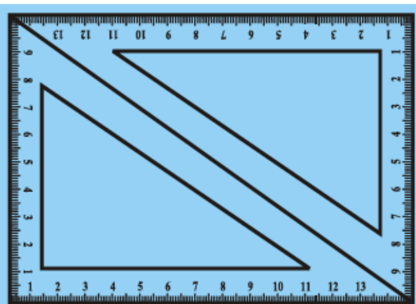


Fig 3.20

You can further strengthen this idea through a logical argument also.

Consider a parallelogram ABCD (Fig 3.21). Draw any one diagonal, say $\overline{AC}$.

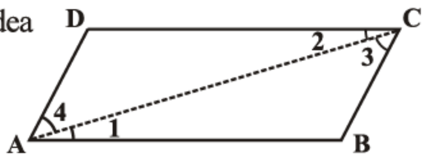


Fig 3.21

Looking at the angles,

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4 \quad \text{(Why?)}$$

Since in triangles ABC and ADC, $\angle 1 = \angle 2$, $\angle 3 = \angle 4$ and $\overline{AC}$ is common, so, by ASA congruency condition,

$$\Delta ABC \cong \Delta CDA \qquad \text{(How is ASA used here?)}$$

This gives $\quad AB = DC \quad$ and $\quad BC = AD$.

Example 3: Find the perimeter of the parallelogram PQRS (Fig 3.22).

Solution: In a parallelogram, the opposite sides have same length.

Therefore, PQ = SR = 12 cm and QR = PS = 7 cm

So, Perimeter = PQ + QR + RS + SP

= 12 cm + 7 cm + 12 cm + 7 cm = 38 cm

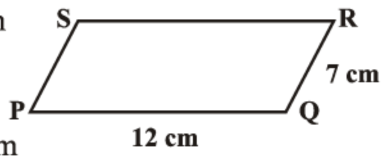


Fig 3.22

3.4.5 Angles of a parallelogram

We studied a property of parallelograms concerning the (opposite) sides. What can we say about the angles?

DO THIS

Let ABCD be a parallelogram (Fig 3.23). Copy it on a tracing sheet. Name this copy as A′B′C′D′. Place A′B′C′D′ on ABCD. Pin them together at the point where the diagonals meet. Rotate the transparent sheet by 180°. The parallelograms still concide; but you now find A′ lying exactly on C and vice-versa; similarly B′ lies on D and vice-versa.

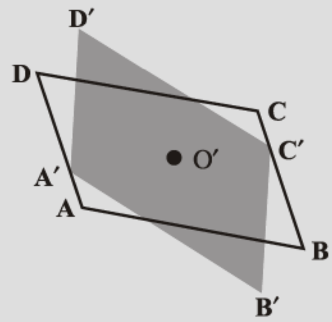


Fig 3.23

Does this tell you anything about the measures of the angles A and C? Examine the same for angles B and D. State your findings.

Property: *The opposite angles of a parallelogram are of equal measure.*

TRY THESE

Take two identical 30° – 60° – 90° set-squares and form a parallelogram as before. Does the figure obtained help you to confirm the above property?

You can further justify this idea through logical arguments.

If $\overline{AC}$ and $\overline{BD}$ are the diagonals of the parallelogram, (Fig 3.24) you find that

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4 \quad \text{(Why?)}$$

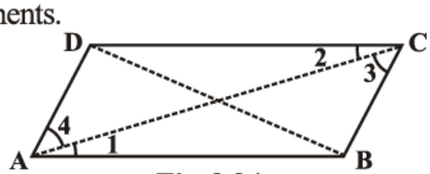


Fig 3.24

Studying ΔABC and ΔADC (Fig 3.25) separately, will help you to see that by ASA congruency condition,

$$\Delta\,ABC \cong \Delta\,CDA \qquad \text{(How?)}$$

Fig 3.25

This shows that $\angle$B and $\angle$D have same measure. In the same way you can get $m\angle A = m\,\angle C$.

Alternatively, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, we have, $m\angle A = \angle 1 + \angle 4 = \angle 2 + \angle C \;\; m\angle C$

Example 4: In Fig 3.26, BEST is a parallelogram. Find the values x, y and z.

Solution: S is opposite to B.

So, $x = 100°$ (opposite angles property)

$y = 100°$ (measure of angle corresponding to $\angle x$)

$z = 80°$ (since $\angle y$, $\angle z$ is a linear pair)

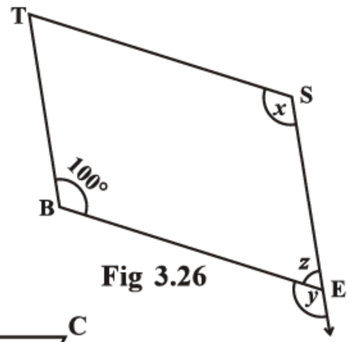


Fig 3.26

We now turn our attention to adjacent angles of a parallelogram.

In parallelogram ABCD, (Fig 3.27).

$\angle$A and $\angle$D are supplementary since $\overline{DC} \parallel \overline{AB}$ and with transversal $\overline{DA}$, these two angles are interior opposite.

$\angle$A and $\angle$B are also supplementary. Can you say 'why'?

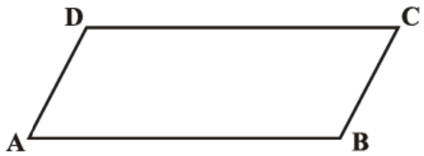


Fig 3.27

$\overline{AD} \parallel \overline{BC}$ and $\overline{BA}$ is a transversal, making $\angle$A and $\angle$B interior opposite.

Identify two more pairs of supplementary angles from the figure.

Property: *The adjacent angles in a parallelogram are supplementary.*

Example 5: In a parallelogram RING, (Fig 3.28) if $m\angle R = 70°$, find all the other angles.

Solution: Given $m\angle R = 70°$

Then $m\angle N = 70°$

because $\angle$R and $\angle$N are opposite angles of a parallelogram.

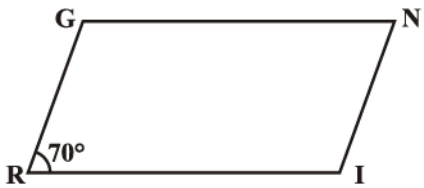


Fig 3.28

Since $\angle$R and $\angle$I are supplementary,

$$m\angle I = 180° - 70° = 110°$$

Also, $m\angle G = 110°$ since $\angle$G is opposite to $\angle$I

Thus, $m\angle R = m\angle N = 70°$ and $m\angle I = m\angle G = 110°$

THINK, DISCUSS AND WRITE

After showing $m\angle R = m\angle N = 70°$, can you find $m\angle I$ and $m\angle G$ by any other method?

3.4.6 Diagonals of a parallelogram

The diagonals of a parallelogram, in general, are not of equal length. (Did you check this in your earlier activity?) However, the diagonals of a parallelogram have an interesting property.

DO THIS

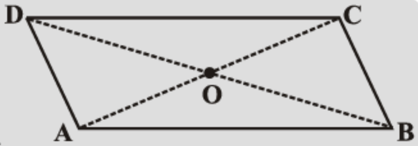


Fig 3.29

Take a cut-out of a parallelogram, say, ABCD (Fig 3.29). Let its diagonals $\overline{AC}$ and $\overline{DB}$ meet at O.

Find the mid point of $\overline{AC}$ by a fold, placing C on A. Is the mid-point same as O?

Does this show that diagonal $\overline{DB}$ bisects the diagonal $\overline{AC}$ at the point O? Discuss it with your friends. Repeat the activity to find where the mid point of $\overline{DB}$ could lie.

Property: *The diagonals of a parallelogram bisect each other (at the point of their intersection, of course!)*

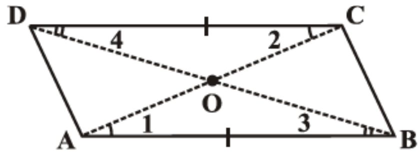


Fig 3.30

To argue and justify this property is not very difficult. From Fig 3.30, applying ASA criterion, it is easy to see that

Δ AOB $\cong$ Δ COD (How is ASA used here?)

This gives AO = CO and BO = DO

Example 6: In Fig 3.31 HELP is a parallelogram. (Lengths are in cms). Given that OE = 4 and HL is 5 more than PE? Find OH.

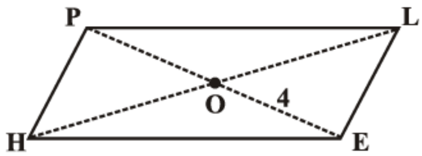


Fig 3.31

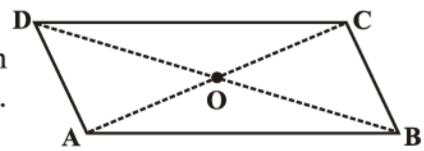
Solution : If OE = 4 then OP also is 4 (Why?)

So PE = 8, (Why?)

Therefore HL = 8 + 5 = 13

Hence $OH = \frac{1}{2} \times 13 = 6.5$ (cms)

EXERCISE 3.3

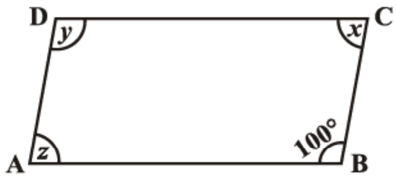


1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.

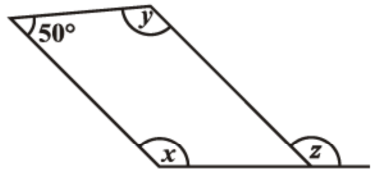
   (i) AD = ... (ii) $\angle$ DCB = ...

   (iii) OC = ... (iv) $m\angle DAB + m\angle CDA$ = ...

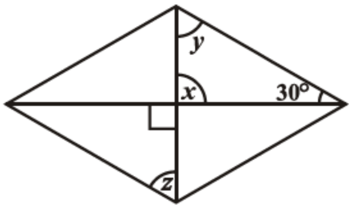
2. Consider the following parallelograms. Find the values of the unknowns x, y, z.



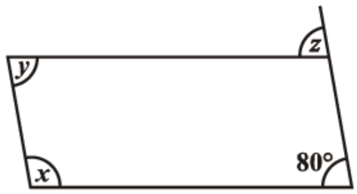
(i)



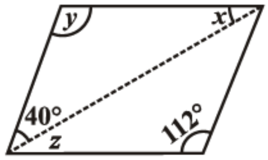
(ii)



(iii)



(iv)



(v)

3. Can a quadrilateral ABCD be a parallelogram if

(i) $\angle D + \angle B = 180°$? (ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

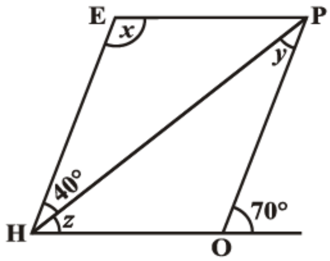
(iii) $\angle A = 70°$ and $\angle C = 65°$?

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

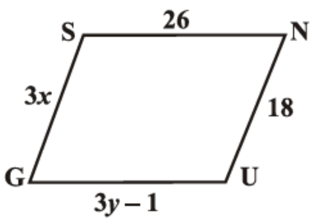
6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

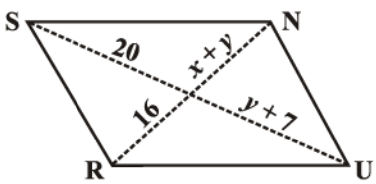


8. The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)

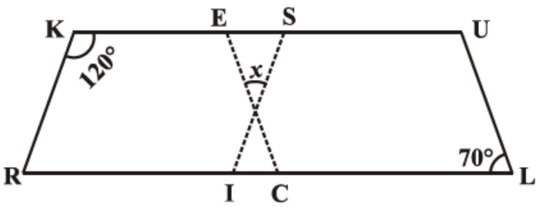
(i)



(ii)



9.



In the above figure both RISK and CLUE are parallelograms. Find the value of x.

10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 3.32)

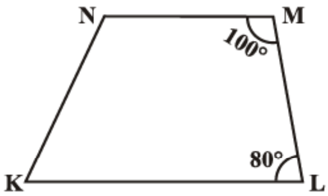


Fig 3.32

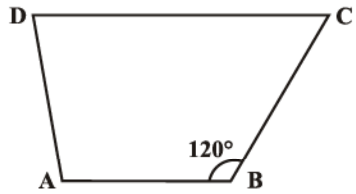


Fig 3.33

11. Find $m\angle C$ in Fig 3.33 if $\overline{AB} \parallel \overline{DC}$.
12. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in Fig 3.34.
(If you find $m\angle R$, is there more than one method to find $m\angle P$?)

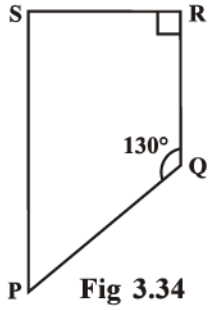


Fig 3.34

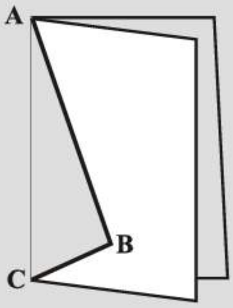
3.5 Some Special Parallelograms

3.5.1 Rhombus

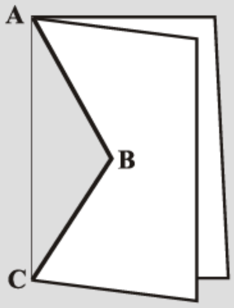
We obtain a Rhombus (which, you will see, is a parallelogram) as a special case of kite (which is not a a parallelogram).

DO THIS

Recall the paper-cut kite you made earlier.



Kite-cut



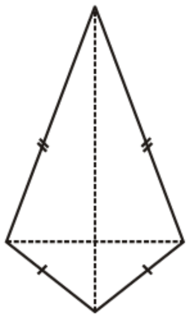
Rhombus-cut

When you cut along ABC and opened up, you got a kite. Here lengths AB and BC were different. If you draw AB = BC, then the kite you obtain is called a **rhombus**.

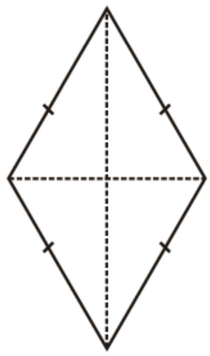
Note that the sides of rhombus are all of same length; this is not the case with the kite.

A rhombus is a quadrilateral with sides of equal length.

Since the opposite sides of a rhombus have the same length, it is also a parallelogram. So, *a rhombus has all the properties of a parallelogram and also that of a kite*. Try to list them out. You can then verify your list with the check list summarised in the book elsewhere.



Kite



Rhombus

The most useful property of a rhombus is that of its diagonals.

Property: *The diagonals of a rhombus are perpendicular bisectors of one another.*

DO THIS

Take a copy of rhombus. By paper-folding verify if the point of intersection is the mid-point of each diagonal. You may also check if they intersect at right angles, using the corner of a set-square.

Here is an outline justifying this property using logical steps.

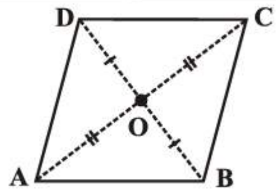


Fig 3.35

ABCD is a rhombus (Fig 3.35). Therefore it is a parallelogram too.

Since diagonals bisect each other, OA = OC and OB = OD.

We have to show that $m\angle AOD = m\angle COD = 90°$

It can be seen that by SSS congruency criterion

$$\Delta AOD \cong \Delta COD$$

Since AO = CO (Why?)
AD = CD (Why?)
OD = OD

Therefore, $m\angle AOD = m\angle COD$

Since $\angle AOD$ and $\angle COD$ are a linear pair,

$$m\angle AOD = m\angle COD = 90°$$

Example 7:

RICE is a rhombus (Fig 3.36). Find x, y, z. Justify your findings.

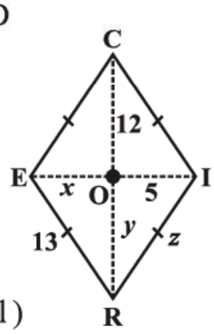


Fig 3.36

Solution:

x = OE	y = OR	z = side of the rhombus
= OI (diagonals bisect)	= OC (diagonals bisect)	= 13 (all sides are equal)
= 5	= 12	

3.5.2 A rectangle

A rectangle is a parallelogram with equal angles (Fig 3.37).

Fig 3.37

What is the full meaning of this definition? Discuss with your friends.

If the rectangle is to be equiangular, what could be the measure of each angle?

Let the measure of each angle be $x°$.

Then $4x° = 360°$ (Why)?

Therefore, $x° = 90°$

Thus each angle of a rectangle is a right angle.

So, a rectangle is a parallelogram in which every angle is a right angle.

Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other.

In a parallelogram, the diagonals can be of different lengths. (Check this); but surprisingly the rectangle (being a special case) has diagonals of equal length.

Property: *The diagonals of a rectangle are of equal length.*

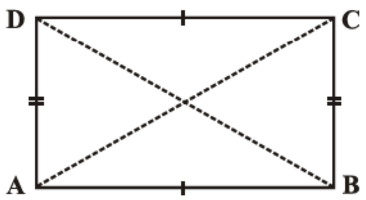


Fig 3.38

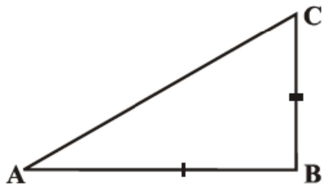


Fig 3.39

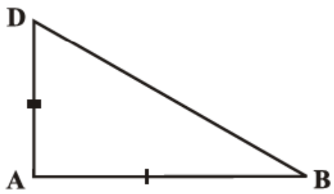


Fig 3.40

This is easy to justify. If ABCD is a rectangle (Fig 3.38), then looking at triangles ABC and ABD separately [(Fig 3.39) and (Fig 3.40) respectively], we have

$$\Delta ABC \cong \Delta ABD$$

This is because

	AB = AB	(Common)
	BC = AD	(Why?)
	$m\angle A = m\angle B = 90°$	(Why?)

The congruency follows by SAS criterion.

Thus AC = BD

and *in a rectangle the diagonals, besides being equal in length bisect each other* (Why?)

Example 8: RENT is a rectangle (Fig 3.41). Its diagonals meet at O. Find x, if OR = $2x + 4$ and OT = $3x + 1$.

Solution: $\overline{OT}$ is half of the diagonal $\overline{TE}$,

$\overline{OR}$ is half of the diagonal $\overline{RN}$.

Diagonals are equal here. (Why?)

So, their halves are also equal.

Therefore $3x + 1 = 2x + 4$

or $x = 3$

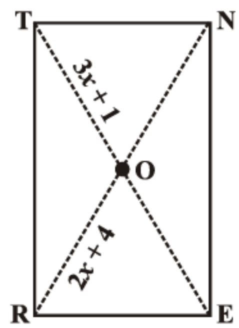


Fig 3.41

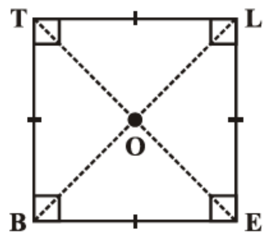
3.5.3 A square

A square is a rectangle with equal sides.

This means a square has all the properties of a rectangle with an additional requirement that all the sides have equal length.

The square, like the rectangle, has diagonals of equal length.

In a rectangle, there is no requirement for the diagonals to be perpendicular to one another, (Check this).



BELT is a square, BE = EL = LT = TB

$\angle B$, $\angle E$, $\angle L$, $\angle T$ are right angles.

BL = ET and $\overline{BL} \perp \overline{ET}$.

OB = OL and OE = OT.

In a square the diagonals.

(i) bisect one another (square being a parallelogram)
(ii) are of equal length (square being a rectangle) and
(iii) are perpendicular to one another.

Hence, we get the following property.

Property: *The diagonals of a square are perpendicular bisectors of each other.*

DO THIS

Take a square sheet, say PQRS (Fig 3.42).

Fold along both the diagonals. Are their mid-points the same?

Check if the angle at O is 90° by using a set-square.

This verifies the property stated above.

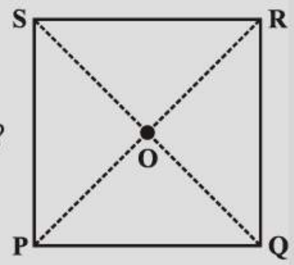


Fig 3.42

We can justify this also by arguing logically:

ABCD is a square whose diagonals meet at O (Fig 3.43).

OA = OC (Since the square is a parallelogram)

By SSS congruency condition, we now see that

Δ AOD $\cong$ Δ COD (How?)

Therefore, $m\angle AOD = m\angle COD$

These angles being a linear pair, each is right angle.

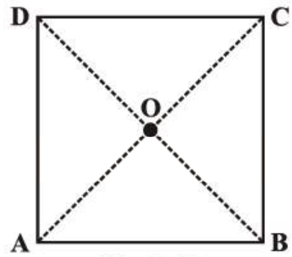
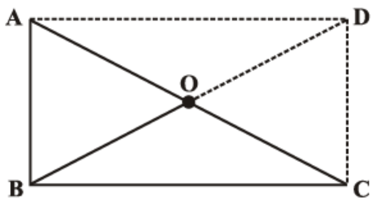


Fig 3.43

EXERCISE 3.4

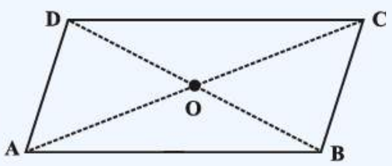
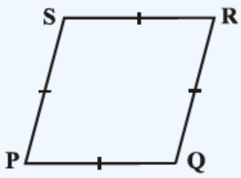
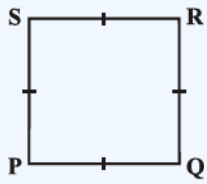
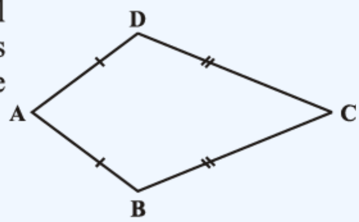
1. State whether True or False.
   (a) All rectangles are squares
   (b) All rhombuses are parallelograms
   (c) All squares are rhombuses and also rectangles
   (d) All squares are not parallelograms.
   (e) All kites are rhombuses.
   (f) All rhombuses are kites.
   (g) All parallelograms are trapeziums.
   (h) All squares are trapeziums.
2. Identify all the quadrilaterals that have.
   (a) four sides of equal length
   (b) four right angles
3. Explain how a square is.
   (i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle
4. Name the quadrilaterals whose diagonals.
   (i) bisect each other (ii) are perpendicular bisectors of each other (iii) are equal
5. Explain why a rectangle is a convex quadrilateral.
6. ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



THINK, DISCUSS AND WRITE

1. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?
2. A square was defined as a rectangle with all sides equal. Can we define it as rhombus with equal angles? Explore this idea.
3. Can a trapezium have all angles equal? Can it have all sides equal? Explain.

WHAT HAVE WE DISCUSSED?

Quadrilateral	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	(1) Opposite sides are equal. (2) Opposite angles are equal. (3) Diagonals bisect one another.
Rhombus: A parallelogram with sides of equal length. 	(1) All the properties of a parallelogram. (2) Diagonals are perpendicular to each other.
Rectangle: A parallelogram with a right angle. 	(1) All the properties of a parallelogram. (2) Each of the angles is a right angle. (3) Diagonals are equal.
Square: A rectangle with sides of equal length. 	All the properties of a parallelogram, rhombus and a rectangle.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides 	(1) The diagonals are perpendicular to one another (2) One of the diagonals bisects the other. (3) In the figure $m\angle B = m\angle D$ but $m\angle A \neq m\angle C$.

CHAPTER

4

Practical Geometry

4.1 Introduction

You have learnt how to draw triangles in Class VII. We require three measurements (of sides and angles) to draw a unique triangle.

Since three measurements were enough to draw a triangle, a natural question arises whether four measurements would be sufficient to draw a unique four sided closed figure, namely, a quadrilateral.

DO THIS

Take a pair of sticks of equal lengths, say 10 cm. Take another pair of sticks of equal lengths, say, 8 cm. Hinge them up suitably to get a rectangle of length 10 cm and breadth 8 cm.

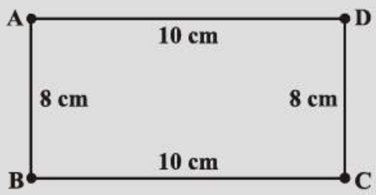


Fig 4.1

This rectangle has been created with the 4 available measurements.

Now just push along the breadth of the rectangle. Is the new shape obtained, still a rectangle (Fig 4.2)? Observe that the rectangle has now become a parallelogram. Have you altered the lengths of the sticks? No! The measurements of sides remain the same.

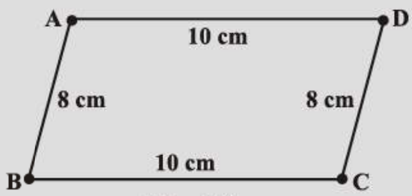


Fig 4.2

Give another push to the newly obtained shape in a different direction; what do you get? You again get a parallelogram, which is altogether different (Fig 4.3), yet the four measurements remain the same.

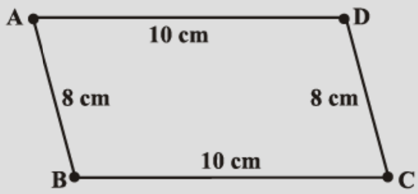


Fig 4.3

This shows that 4 measurements of a quadrilateral cannot determine it uniquely. Can 5 measurements determine a quadrilateral uniquely? Let us go back to the activity!

You have constructed a rectangle with two sticks each of length 10 cm and other two sticks each of length 8 cm. Now introduce another stick of length equal to BD and tie it along BD (Fig 4.4). If you push the breadth now, does the shape change? No! It cannot, without making the figure open. The introduction of the fifth stick has fixed the rectangle uniquely, i.e., there is no other quadrilateral (with the given lengths of sides) possible now.

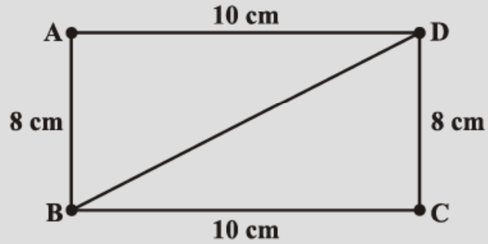


Fig 4.4

Thus, we observe that five measurements can determine a quadrilateral uniquely. But will any five measurements (of sides and angles) be sufficient to draw a unique quadrilateral?

THINK, DISCUSS AND WRITE

Arshad has five measurements of a quadrilateral ABCD. These are AB = 5 cm, $\angle A = 50°$, AC = 4 cm, BD = 5 cm and AD = 6 cm. Can he construct a unique quadrilateral? Give reasons for your answer.

4.2 Constructing a Quadrilateral

We shall learn how to construct a unique quadrilateral given the following measurements:

- When four sides and one diagonal are given.
- When two diagonals and three sides are given.
- When two adjacent sides and three angles are given.
- When three sides and two included angles are given.
- When other special properties are known.

Let us take up these constructions one-by-one.

4.2.1 When the lengths of four sides and a diagonal are given

We shall explain this construction through an example.

Example 1: Construct a quadrilateral PQRS where PQ = 4 cm, QR = 6 cm, RS = 5 cm, PS = 5.5 cm and PR = 7 cm.

Solution: [A rough sketch will help us in visualising the quadrilateral. We draw this first and mark the measurements.] (Fig 4.5)

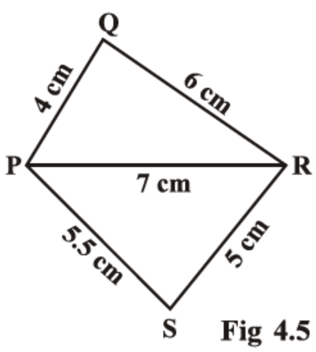


Fig 4.5

Step 1 From the rough sketch, it is easy to see that ΔPQR can be constructed using SSS construction condition. Draw ΔPQR (Fig 4.6).

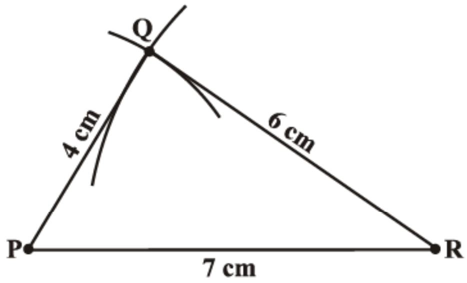


Fig 4.6

Step 2 Now, we have to locate the fourth point S. This 'S' would be on the side opposite to Q with reference to PR. For that, we have two measurements.

S is 5.5 cm away from P. So, with P as centre, draw an arc of radius 5.5 cm. (The point S is somewhere on this arc!) (Fig 4.7).

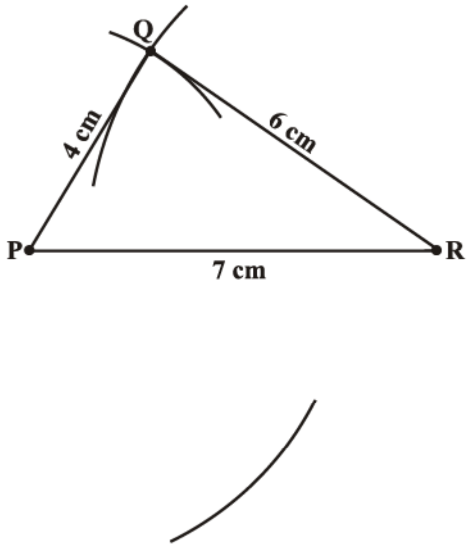


Fig 4.7

Step 3 S is 5 cm away from R. So with R as centre, draw an arc of radius 5 cm (The point S is somewhere on this arc also!) (Fig 4.8).

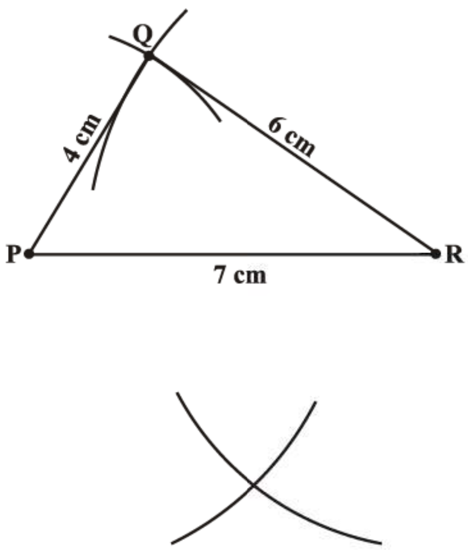


Fig 4.8

Step 4 S should lie on both the arcs drawn. So it is the point of intersection of the two arcs. Mark S and complete PQRS. PQRS is the required quadrilateral (Fig 4.9).

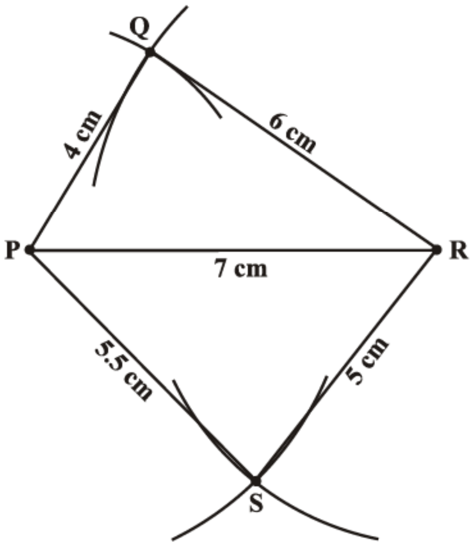


Fig 4.9

THINK, DISCUSS AND WRITE

(i) We saw that 5 measurements of a quadrilateral can determine a quadrilateral uniquely. Do you think any five measurements of the quadrilateral can do this?

(ii) Can you draw a parallelogram BATS where BA = 5 cm, AT = 6 cm and AS = 6.5 cm? Why?

(iii) Can you draw a rhombus ZEAL where ZE = 3.5 cm, diagonal EL = 5 cm? Why?

(iv) A student attempted to draw a quadrilateral PLAY where PL = 3 cm, LA = 4 cm, AY = 4.5 cm, PY = 2 cm and LY = 6 cm, but could not draw it. What is the reason?
[**Hint:** Discuss it using a rough sketch].

EXERCISE 4.1

1. Construct the following quadrilaterals.

(i) Quadrilateral ABCD.
AB = 4.5 cm
BC = 5.5 cm
CD = 4 cm
AD = 6 cm
AC = 7 cm

(ii) Quadrilateral JUMP
JU = 3.5 cm
UM = 4 cm
MP = 5 cm
PJ = 4.5 cm
PU = 6.5 cm

(iii) Parallelogram MORE
OR = 6 cm
RE = 4.5 cm
EO = 7.5 cm

(iv) Rhombus BEST
BE = 4.5 cm
ET = 6 cm

4.2.2 When two diagonals and three sides are given

When four sides and a diagonal were given, we first drew a triangle with the available data and then tried to locate the fourth point. The same technique is used here.

Example 2: Construct a quadrilateral ABCD, given that BC = 4.5 cm, AD = 5.5 cm, CD = 5 cm the diagonal AC = 5.5 cm and diagonal BD = 7 cm.

Solution:

Here is the rough sketch of the quadrilateral ABCD (Fig 4.10). Studying this sketch, we can easily see that it is possible to draw Δ ACD first (How?).

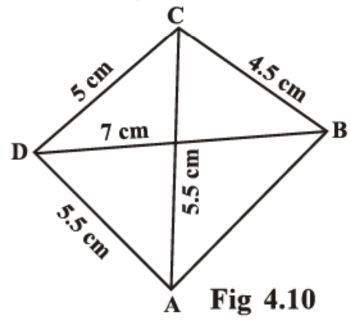


Fig 4.10

Step 1 Draw Δ ACD using SSS construction (Fig 4.11). (We now need to find B at a distance of 4.5 cm from C and 7 cm from D).

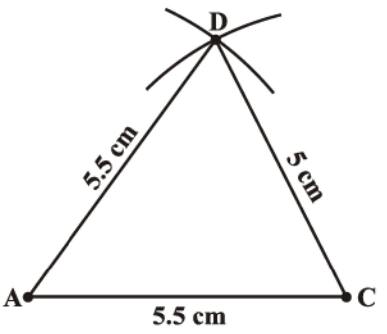


Fig 4.11

Step 2 With D as centre, draw an arc of radius 7 cm. (B is somewhere on this arc) (Fig 4.12).

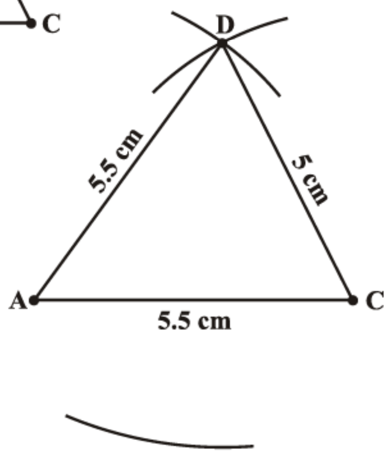


Fig 4.12

Step 3 With C as centre, draw an arc of radius 4.5 cm (B is somewhere on this arc also) (Fig 4.13).

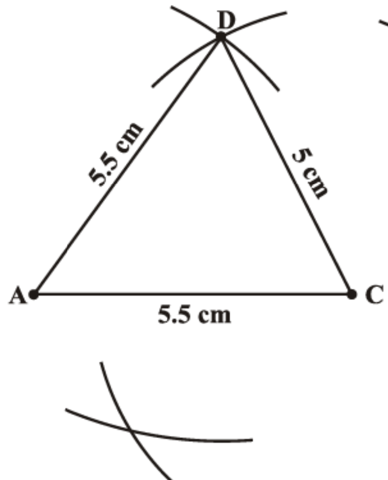


Fig 4.13

Step 4 Since B lies on both the arcs, B is the point intersection of the two arcs. Join BC and AB. Then, complete ABCD. ABCD is the required quadrilateral (Fig 4.14).

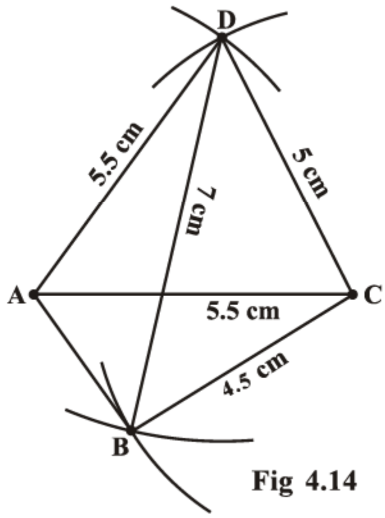


Fig 4.14

THINK, DISCUSS AND WRITE

1. In the above example, can we draw the quadrilateral by drawing ΔABD first and then find the fourth point C?
2. Can you construct a quadrilateral PQRS with PQ = 3 cm, RS = 3 cm, PS = 7.5 cm, PR = 8 cm and SQ = 4 cm? Justify your answer.

EXERCISE 4.2

1. Construct the following quadrilaterals.

(i) quadrilateral LIFT

LI = 4 cm

IF = 3 cm

TL = 2.5 cm

LF = 4.5 cm

IT = 4 cm

(ii) Quadrilateral GOLD

OL = 7.5 cm

GL = 6 cm

GD = 6 cm

LD = 5 cm

OD = 10 cm

(iii) Rhombus BEND

BN = 5.6 cm

DE = 6.5 cm

4.2.3 When two adjacent sides and three angles are known

As before, we start with constructing a triangle and then look for the fourth point to complete the quadrilateral.

Example 3: Construct a quadrilateral MIST where MI = 3.5 cm, IS = 6.5 cm, $\angle$M = 75°, $\angle$I = 105° and $\angle$S = 120°.

Solution:

Here is a rough sketch that would help us in deciding our steps of construction. We give only hints for various steps (Fig 4.15).

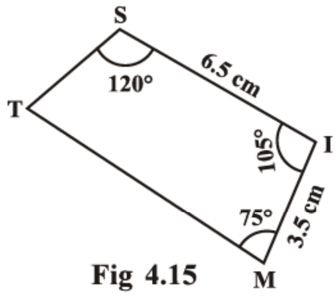


Fig 4.15

Step 1 How do you locate the points? What choice do you make for the base and what is the first step? (Fig 4.16)

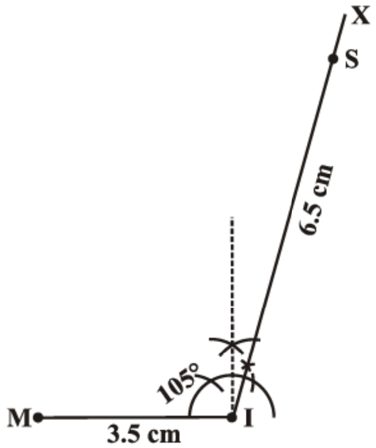


Fig 4.16

Step 2 Make ∠ISY = 120° at S (Fig 4.17).

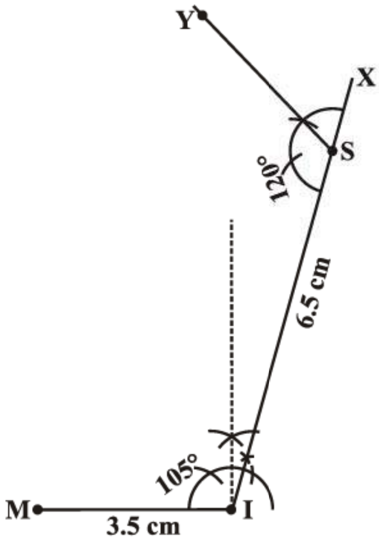


Fig 4.17

Step 3 Make ∠IMZ = 75° at M. (where will SY and MZ meet?) Mark that point as T. We get the required quadrilateral MIST (Fig 4.18).

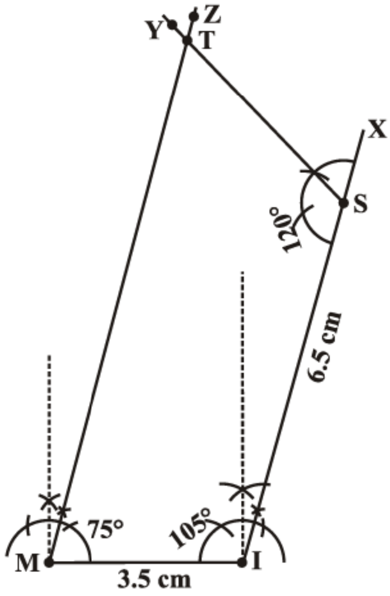


Fig 4.18

THINK, DISCUSS AND WRITE

1. Can you construct the above quadrilateral MIST if we have 100° at M instead of 75°?
2. Can you construct the quadrilateral PLAN if PL = 6 cm, LA = 9.5 cm, ∠P = 75°, ∠L =150° and ∠A = 140°? (**Hint:** Recall angle-sum property).
3. In a parallelogram, the lengths of adjacent sides are known. Do we still need measures of the angles to construct as in the example above?

EXERCISE 4.3

1. Construct the following quadrilaterals.

(i) Quadrilateral MORE
MO = 6 cm
OR = 4.5 cm
∠M = 60°
∠O = 105°
∠R = 105°

(ii) Quadrilateral PLAN
PL = 4 cm
LA = 6.5 cm
∠P = 90°
∠A = 110°
∠N = 85°

(iii) Parallelogram HEAR
HE = 5 cm
EA = 6 cm
∠R = 85°

(iv) Rectangle OKAY
OK = 7 cm
KA = 5 cm

4.2.4 When three sides and two included angles are given

Under this type, when you draw a rough sketch, note carefully the "included" angles in particular.

Example 4: Construct a quadrilateral ABCD, where AB = 4 cm, BC = 5 cm, CD = 6.5 cm and ∠B = 105° and ∠C = 80°.

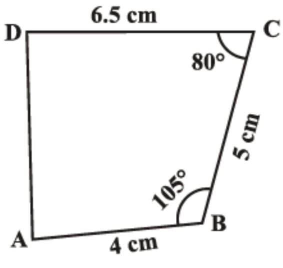


Fig 4.19

Solution:

We draw a rough sketch, as usual, to get an idea of how we can start off. Then we can devise a plan to locate the four points (Fig 4.19).

Step 1 Start with taking BC = 5 cm on B. Draw an angle of 105° along BX. Locate A 4 cm away on this. We now have B, C and A (Fig 4.20).

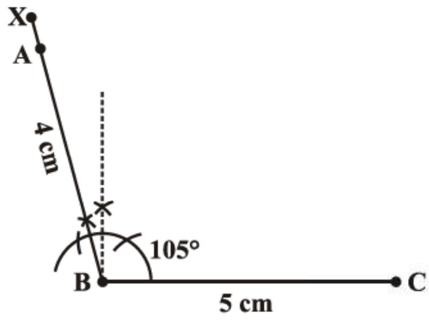


Fig 4.20

Step 2 The fourth point D is on CY which is inclined at 80° to BC. So make ∠BCY = 80° at C on BC (Fig 4.21).

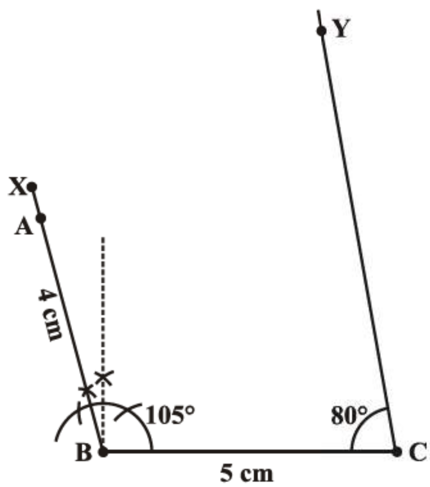


Fig 4.21

Step 3 D is at a distance of 6.5 cm on CY. With C as centre, draw an arc of length 6.5 cm. It cuts CY at D (Fig 4.22).

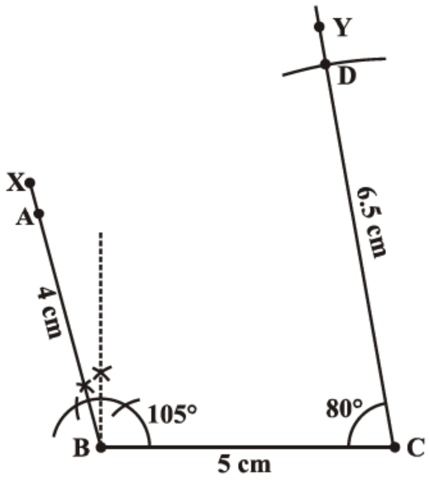


Fig 4.22

Step 4 Complete the quadrilateral ABCD. ABCD is the required quadrilateral (Fig 4.23).

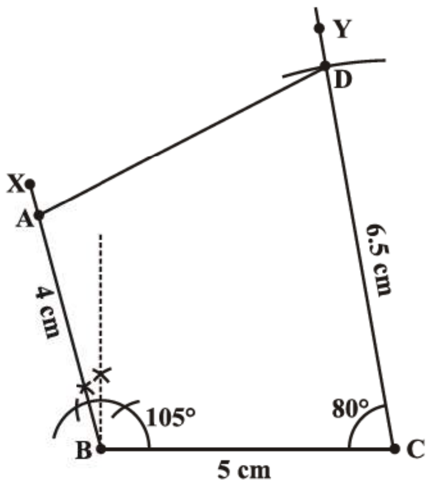


Fig 4.23

THINK, DISCUSS AND WRITE

1. In the above example, we first drew BC. Instead, what could have been be the other starting points?
2. We used some five measurements to draw quadrilaterals so far. Can there be different sets of five measurements (other than seen so far) to draw a quadrilateral? The following problems may help you in answering the question.
   (i) Quadrilateral ABCD with AB = 5 cm, BC = 5.5 cm, CD = 4 cm, AD = 6 cm and $\angle B = 80°$.
   (ii) Quadrilateral PQRS with PQ = 4.5 cm, $\angle P = 70°$, $\angle Q = 100°$, $\angle R = 80°$ and $\angle S = 110°$.

   Construct a few more examples of your own to find sufficiency/insufficiency of the data for construction of a quadrilateral.

EXERCISE 4.4

1. Construct the following quadrilaterals.

   (i) Quadrilateral DEAR
   DE = 4 cm
   EA = 5 cm
   AR = 4.5 cm
   ∠E = 60°
   ∠A = 90°

   (ii) Quadrilateral TRUE
   TR = 3.5 cm
   RU = 3 cm
   UE = 4 cm
   ∠R = 75°
   ∠U = 120°

4.3 Some Special Cases

To draw a quadrilateral, we used 5 measurements in our work. Is there any quadrilateral which can be drawn with less number of available measurements? The following examples examine such special cases.

Example 5: Draw a square of side 4.5 cm.

Solution: Initially it appears that only one measurement has been given. Actually we have many more details with us, because the figure is a special quadrilateral, namely a square. We now know that each of its angles is a right angle. (See the rough figure) (Fig 4.24)

This enables us to draw ΔABC using SAS condition. Then D can be easily located. Try yourself now to draw the square with the given measurements.

Rough figure

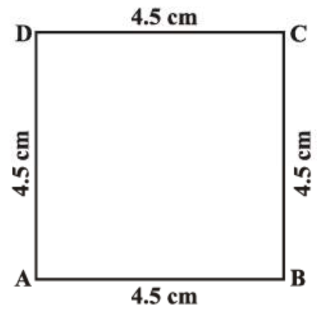


Fig 4.24

Example 6: Is it possible to construct a rhombus ABCD where AC = 7 cm and BD = 6 cm? Justify your answer.

Solution: Only two (diagonal) measurements of the rhombus are given. However, since it is a rhombus, we can find more help from its properties.

The diagonals of a rhombus are perpendicular bisectors of one another.

So, first draw AC = 7 cm and then construct its perpendicular bisector. Let them meet at 0. Cut off 3 cm lengths on either side of the drawn bisector. You now get B and D.

Draw the rhombus now, based on the method described above (Fig 4.25).

Rough figure

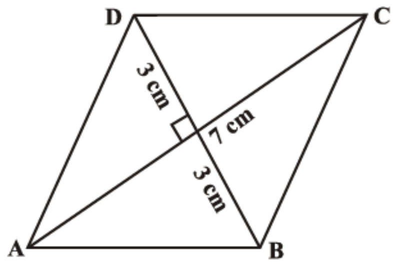


Fig 4.25

TRY THESE

1. How will you construct a rectangle PQRS if you know only the lengths PQ and QR?
2. Construct the kite EASY if AY = 8 cm, EY = 4 cm and SY = 6 cm (Fig 4.26). Which properties of the kite did you use in the process?

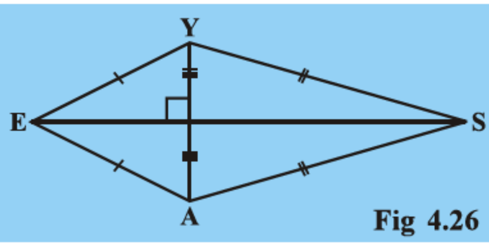


Fig 4.26

EXERCISE 4.5

Draw the following.

1. The square READ with RE = 5.1 cm.
2. A rhombus whose diagonals are 5.2 cm and 6.4 cm long.
3. A rectangle with adjacent sides of lengths 5 cm and 4 cm.
4. A parallelogram OKAY where OK = 5.5 cm and KA = 4.2 cm. Is it unique?

WHAT HAVE WE DISCUSSED?

1. Five measurements can determine a quadrilateral uniquely.
2. A quadrilateral can be constructed uniquely if the lengths of its four sides and a diagonal is given.
3. A quadrilateral can be constructed uniquely if its two diagonals and three sides are known.
4. A quadrilateral can be constructed uniquely if its two adjacent sides and three angles are known.
5. A quadrilateral can be constructed uniquely if its three sides and two included angles are given.

CHAPTER 5

Data Handling

0852CH05

5.1 Looking for Information

In your day-to-day life, you might have come across information, such as:

(a) Runs made by a batsman in the last 10 test matches.

(b) Number of wickets taken by a bowler in the last 10 ODIs.

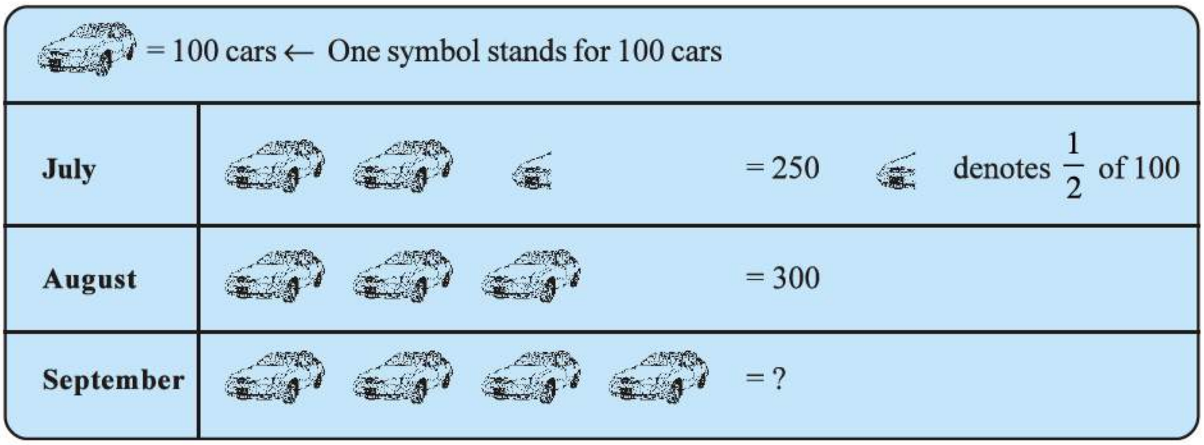
(c) Marks scored by the students of your class in the Mathematics unit test.

(d) Number of story books read by each of your friends etc.

The information collected in all such cases is called **data**. Data is usually collected in the context of a situation that we want to study. For example, a teacher may like to know the average height of students in her class. To find this, she will write the heights of all the students in her class, organise the data in a systematic manner and then interpret it accordingly.

Sometimes, data is represented **graphically** to give a clear idea of what it represents. Do you remember the different types of graphs which we have learnt in earlier classes?

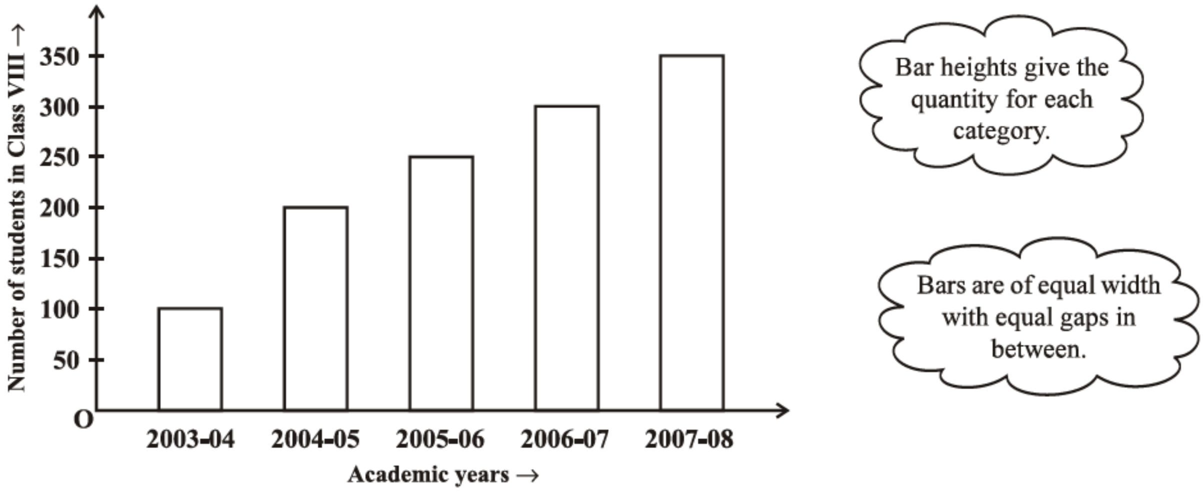
1. **A Pictograph:** Pictorial representation of data using symbols.



(i) How many cars were produced in the month of July?

(ii) In which month were maximum number of cars produced?

2. **A bar graph:** A display of information using bars of uniform width, their heights being proportional to the respective values.



(i) What is the information given by the bar graph?

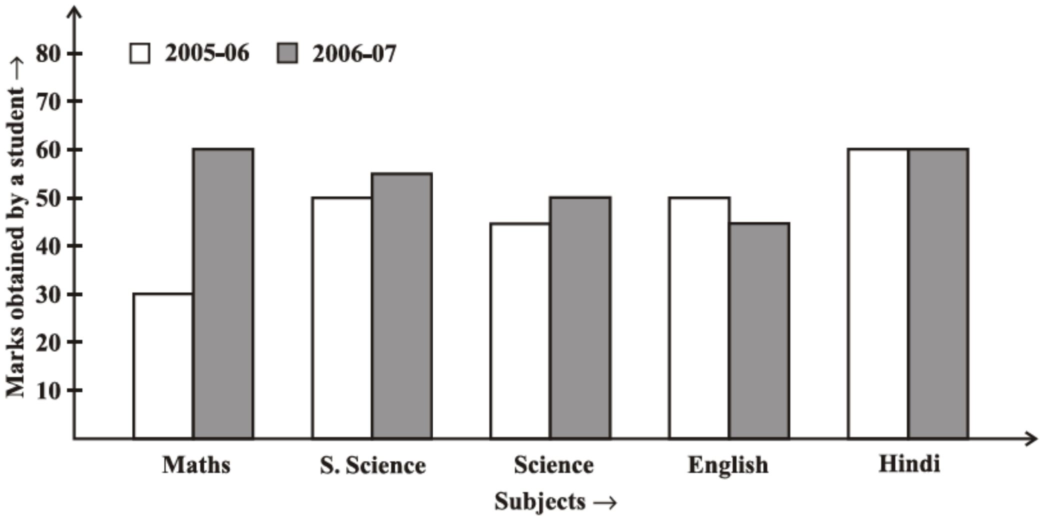
(ii) In which year is the increase in the number of students maximum?

(iii) In which year is the number of students maximum?

(iv) State whether true or false:

'The number of students during 2005-06 is twice that of 2003-04.'

3. **Double Bar Graph:** A bar graph showing two sets of data simultaneously. It is useful for the comparison of the data.



(i) What is the information given by the double bar graph?

(ii) In which subject has the performance improved the most?

(iii) In which subject has the performance deteriorated?

(iv) In which subject is the performance at par?

THINK, DISCUSS AND WRITE

If we change the position of any of the bars of a bar graph, would it change the information being conveyed? Why?

TRY THESE

Draw an appropriate graph to represent the given information.

1.

Month	July	August	September	October	November	December
Number of watches sold	1000	1500	1500	2000	2500	1500

2.

Children who prefer	School A	School B	School C
Walking	40	55	15
Cycling	45	25	35

3. Percentage wins in ODI by 8 top cricket teams.

Teams	From Champions Trophy to World Cup-06	Last 10 ODI in 07
South Africa	75%	78%
Australia	61%	40%
Sri Lanka	54%	38%
New Zealand	47%	50%
England	46%	50%
Pakistan	45%	44%
West Indies	44%	30%
India	43%	56%

5.2 Organising Data

Usually, data available to us is in an unorganised form called **raw data**. To draw meaningful inferences, we need to organise the data systematically. For example, a group of students was asked for their favourite subject. The results were as listed below:

Art, Mathematics, Science, English, Mathematics, Art, English, Mathematics, English, Art, Science, Art, Science, Science, Mathematics, Art, English, Art, Science, Mathematics, Science, Art.

Which is the most liked subject and the one least liked?

It is not easy to answer the question looking at the choices written haphazardly. We arrange the data in Table 5.1 using tally marks.

Table 5.1

Subject	Tally Marks	Number of Students
Art	卌 ||	7
Mathematics	卌	5
Science	卌 |	6
English	||||	4

The number of tallies before each subject gives the number of students who like that particular subject.

This is known as the **frequency** of that subject.

Frequency gives the number of times that a particular entry occurs.

From Table 5.1, Frequency of students who like English is 4
Frequency of students who like Mathematics is 5

The table made is known as **frequency distribution table** as it gives the number of times an entry occurs.

TRY THESE

1. A group of students were asked to say which animal they would like most to have as a pet. The results are given below:
   dog, cat, cat, fish, cat, rabbit, dog, cat, rabbit, dog, cat, dog, dog, dog, cat, cow, fish, rabbit, dog, cat, dog, cat, cat, dog, rabbit, cat, fish, dog.
   Make a frequency distribution table for the same.

5.3 Grouping Data

The data regarding choice of subjects showed the occurrence of each of the entries several times. For example, Art is liked by 7 students, Mathematics is liked by 5 students and so on (Table 5.1). This information can be displayed graphically using a pictograph or a bargraph. Sometimes, however, we have to deal with a large data. For example, consider the following marks (out of 50) obtained in Mathematics by 60 students of Class VIII:

21, 10, 30, 22, 33, 5, 37, 12, 25, 42, 15, 39, 26, 32, 18, 27, 28, 19, 29, 35, 31, 24, 36, 18, 20, 38, 22, 44, 16, 24, 10, 27, 39, 28, 49, 29, 32, 23, 31, 21, 34, 22, 23, 36, 24, 36, 33, 47, 48, 50, 39, 20, 7, 16, 36, 45, 47, 30, 22, 17.

If we make a frequency distribution table for each observation, then the table would be too long, so, for convenience, we make groups of observations say, 0-10, 10-20 and so on, and obtain a frequency distribution of the number of observations falling in each group. Thus, the frequency distribution table for the above data can be.

Table 5.2

Groups	Tally Marks	Frequency
0-10	𝍷𝍷	2
10-20	𝍸 𝍸	10
20-30	𝍸 𝍸 𝍸 𝍸 𝍷	21
30-40	𝍸 𝍸 𝍸 𝍷𝍷𝍷𝍷	19
40-50	𝍸 𝍷𝍷	7
50-60	𝍷	1
	Total	**60**

Data presented in this manner is said to be **grouped** and the distribution obtained is called **grouped frequency distribution**. It helps us to draw meaningful inferences like–

(1) Most of the students have scored between 20 and 40.

(2) Eight students have scored more than 40 marks out of 50 and so on.

Each of the groups 0-10, 10-20, 20-30, etc., is called a **Class Interval** (or briefly a class).

Observe that 10 occurs in both the classes, i.e., 0-10 as well as 10-20. Similarly, 20 occurs in classes 10-20 and 20-30. But it is not possible that an observation (say 10 or 20) can belong simultaneously to two classes. To avoid this, we adopt the convention that the common observation will belong to the higher class, i.e., 10 belongs to the class interval 10-20 (and not to 0-10). Similarly, 20 belongs to 20-30 (and not to 10-20). In the class interval, 10-20, 10 is called the **lower class limit** and 20 is called the **upper class limit**. Similarly, in the class interval 20-30, 20 is the lower class limit and 30 is the upper class limit. Observe that the difference between the upper class limit and lower class limit for each of the class intervals 0-10, 10-20, 20-30 etc., is equal, (10 in this case). This difference between the upper class limit and lower class limit is called the **width** or **size** of the class interval.

TRY THESE

1. Study the following frequency distribution table and answer the questions given below.

Frequency Distribution of Daily Income of 550 workers of a factory

Table 5.3

Class Interval (Daily Income in `)	Frequency (Number of workers)
100-125	45
125-150	25

150-175	55
175-200	125
200-225	140
225-250	55
250-275	35
275-300	50
300-325	20
Total	**550**

(i) What is the size of the class intervals?
(ii) Which class has the highest frequency?
(iii) Which class has the lowest frequency?
(iv) What is the upper limit of the class interval 250-275?
(v) Which two classes have the same frequency?

2. Construct a frequency distribution table for the data on weights (in kg) of 20 students of a class using intervals 30-35, 35-40 and so on.

   40, 38, 33, 48, 60, 53, 31, 46, 34, 36, 49, 41, 55, 49, 65, 42, 44, 47, 38, 39.

5.3.1 Bars with a difference

Let us again consider the grouped frequency distribution of the marks obtained by 60 students in Mathematics test. (Table 5.4)

Table 5.4

Class Interval	Frequency
0-10	2
10-20	10
20-30	21
30-40	19
40-50	7
50-60	1
Total	**60**

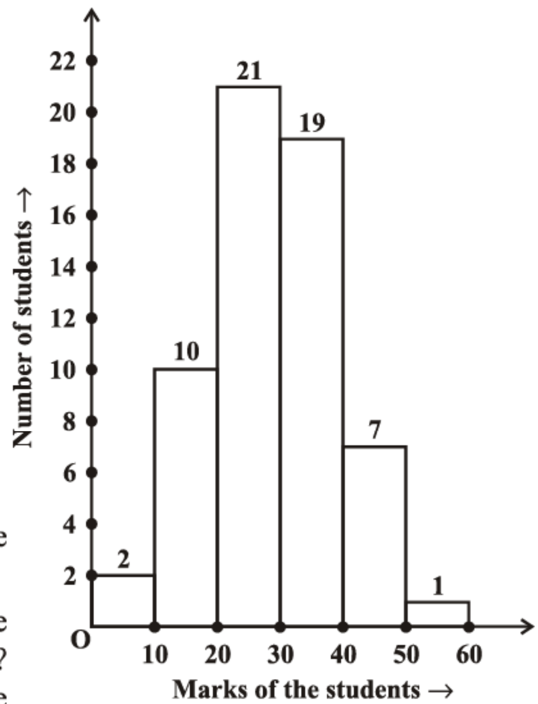


Fig 5.1

This is displayed graphically as in the adjoining graph (Fig 5.1).

Is this graph in any way different from the bar graphs which you have drawn in Class VII? Observe that, here we have represented the groups of observations (i.e., class intervals) on the horizontal axis. The **height** of the bars show the **frequency** of the class-interval. Also, there is no gap between the bars as there is no gap between the class-intervals.

The graphical representation of data in this manner is called a **histogram**.

The following graph is another histogram (Fig 5.2).

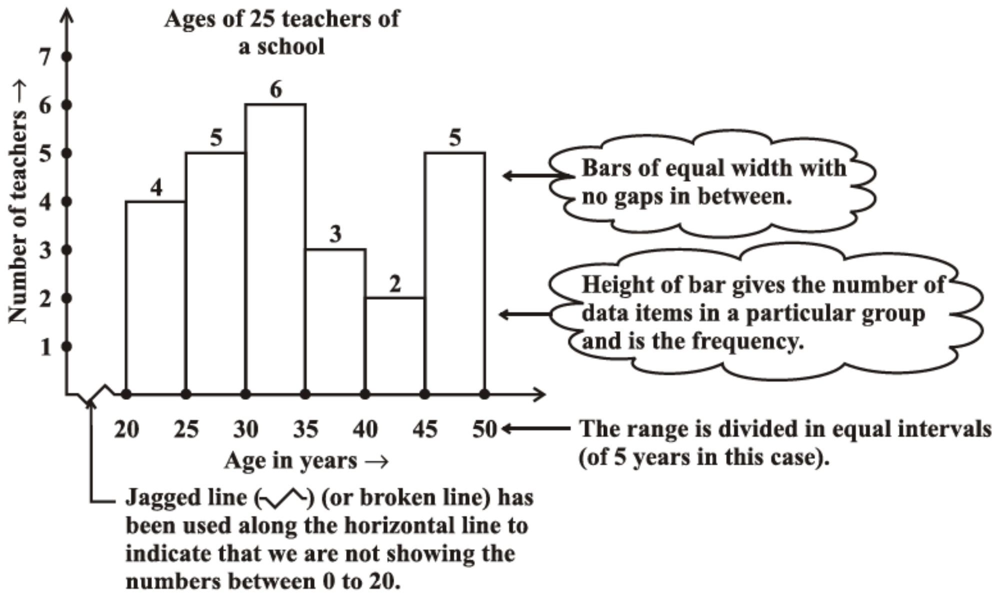


Fig 5.2

From the bars of this histogram, we can answer the following questions:

(i) How many teachers are of age 45 years or more but less than 50 years?

(ii) How many teachers are of age less than 35 years?

TRY THESE

1. Observe the histogram (Fig 5.3) and answer the questions given below.

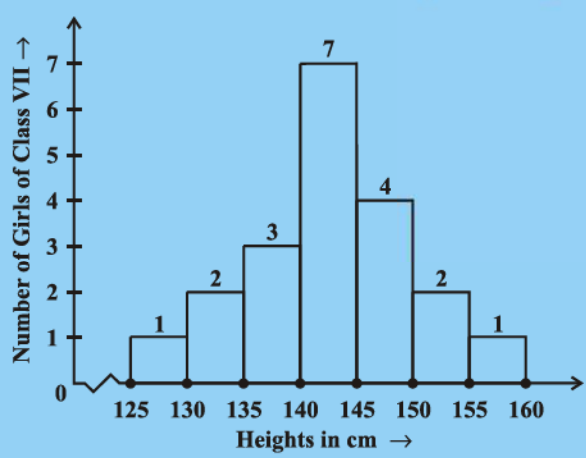


Fig 5.3

(i) What information is being given by the histogram?

(ii) Which group contains maximum girls?

(iii) How many girls have a height of 145 cms and more?

(iv) If we divide the girls into the following three categories, how many would there be in each?

150 cm and more — Group A

140 cm to less than 150 cm — Group B

Less than 140 cm — Group C

EXERCISE 5.1

1. For which of these would you use a histogram to show the data?

(a) The number of letters for different areas in a postman's bag.

(b) The height of competitors in an athletics meet.

(c) The number of cassettes produced by 5 companies.

(d) The number of passengers boarding trains from 7:00 a.m. to 7:00 p.m. at a station.

Give reasons for each.

2. The shoppers who come to a departmental store are marked as: man (M), woman (W), boy (B) or girl (G). The following list gives the shoppers who came during the first hour in the morning:

W W W G B W W M G G M M W W W W G B M W B G G M W W M M W W
W M W B W G M W W W W G W M M W W M W G W M G W M M B G G W

Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

3. The weekly wages (in `) of 30 workers in a factory are.

830, 835, 890, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840

Using tally marks make a frequency table with intervals as 800–810, 810–820 and so on.

4. Draw a histogram for the frequency table made for the data in Question 3, and answer the following questions.

(i) Which group has the maximum number of workers?

(ii) How many workers earn ` 850 and more?

(iii) How many workers earn less than ` 850?

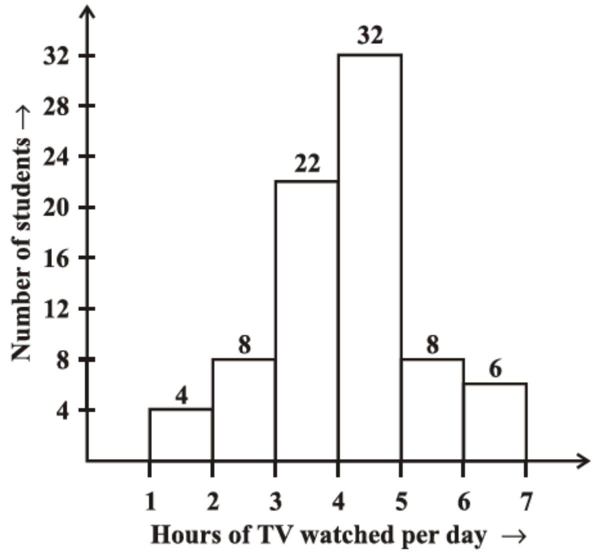
5. The number of hours for which students of a particular class watched television during holidays is shown through the given graph.

Answer the following.

(i) For how many hours did the maximum number of students watch TV?

(ii) How many students watched TV for less than 4 hours?

(iii) How many students spent more than 5 hours in watching TV?



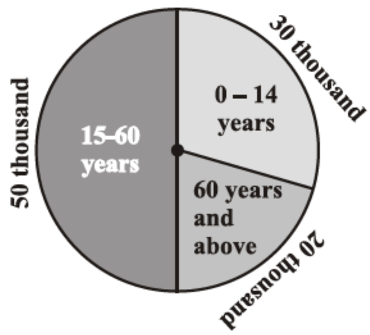
5.4 Circle Graph or Pie Chart

Have you ever come across data represented in circular form as shown (Fig 5.4)?

The time spent by a child during a day

(i)

Age groups of people in a town



(ii)

Fig 5.4

These are called **circle graphs**. A circle graph shows the relationship between a whole and its parts. Here, the whole circle is divided into sectors. The size of each sector is proportional to the activity or information it represents.

For example, in the above graph, the proportion of the sector for hours spent in sleeping

$$= \frac{\text{number of sleeping hours}}{\text{whole day}} = \frac{8 \text{ hours}}{24 \text{ hours}} = \frac{1}{3}$$

So, this sector is drawn as $\frac{1}{3}$rd part of the circle. Similarly, the proportion of the sector for hours spent in school $= \frac{\text{number of school hours}}{\text{whole day}} = \frac{6 \text{ hours}}{24 \text{ hours}} = \frac{1}{4}$

So this sector is drawn $\frac{1}{4}$th of the circle. Similarly, the size of other sectors can be found.

Add up the fractions for all the activities. Do you get the total as one?

A circle graph is also called a **pie chart**.

TRY THESE

1. Each of the following pie charts (Fig 5.5) gives you a different piece of information about your class. Find the fraction of the circle representing each of these information.

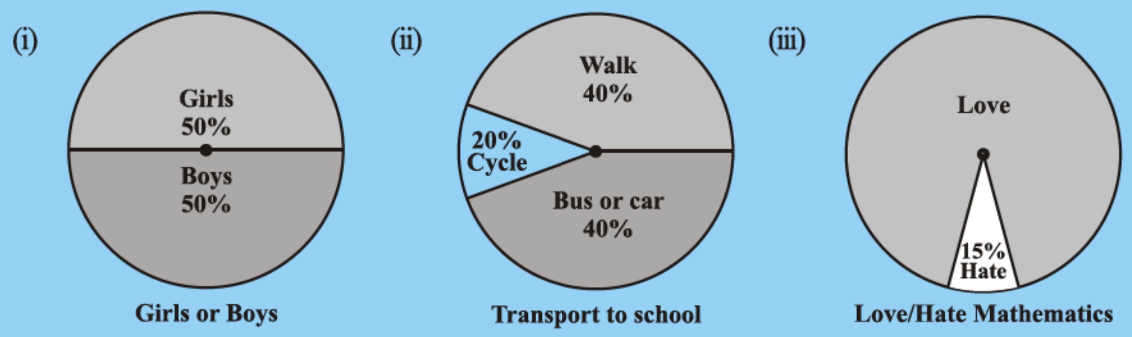
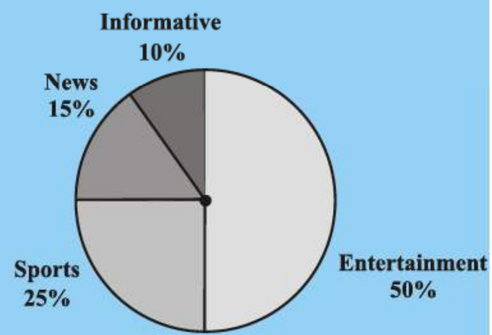


Fig 5.5

2. Answer the following questions based on the pie chart given (Fig 5.6).
   (i) Which type of programmes are viewed the most?
   (ii) Which two types of programmes have number of viewers equal to those watching sports channels?



Viewers watching different types of channels on T.V.

Fig 5.6

5.4.1 Drawing pie charts

The favourite flavours of ice-creams for students of a school is given in percentages as follows.

Flavours	Percentage of students Preferring the flavours
Chocolate	50%
Vanilla	25%
Other flavours	25%

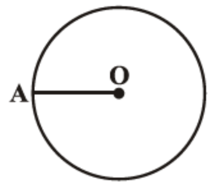
Let us represent this data in a pie chart.

The total angle at the centre of a circle is 360°. The central angle of the sectors will be a fraction of 360°. We make a table to find the central angle of the sectors (Table 5.5).

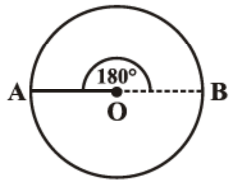
Table 5.5

Flavours	Students in per cent preferring the flavours	In fractions	Fraction of 360°
Chocolate	50%	$\frac{50}{100}=\frac{1}{2}$	$\frac{1}{2}$ of 360° = 180°
Vanilla	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°
Other flavours	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°

1. Draw a circle with any convenient radius. Mark its centre (O) and a radius (OA).



2. The angle of the sector for chocolate is 180°. Use the protractor to draw ∠AOB = 180°.



3. Continue marking the remaining sectors.

Example 1: Adjoining pie chart (Fig 5.7) gives the expenditure (in percentage) on various items and savings of a family during a month.

(i) On which item, the expenditure was maximum?

(ii) Expenditure on which item is equal to the total savings of the family?

(iii) If the monthly savings of the family is ` 3000, what is the monthly expenditure on clothes?

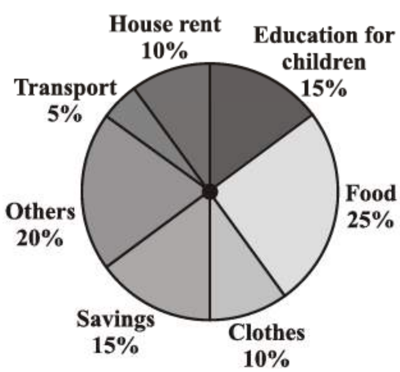


Fig 5.7

Solution:

(i) Expenditure is maximum on food.

(ii) Expenditure on Education of children is the same (i.e., 15%) as the savings of the family.

(iii) 15% represents ` 3000

Therefore, 10% represents ` $\frac{3000}{15} \times 10$ = ` 2000

Example 2: On a particular day, the sales (in rupees) of different items of a baker's shop are given below.

ordinary bread	: 320
fruit bread	: 80
cakes and pastries	: 160
biscuits	: 120
others	: 40
Total	**: 720**

Draw a pie chart for this data.

Solution: We find the central angle of each sector. Here the total sale = ` 720. We thus have this table.

Item	Sales (in `)	In Fraction	Central Angle
Ordinary Bread	320	$\frac{320}{720} = \frac{4}{9}$	$\frac{4}{9} \times 360° = 160°$
Biscuits	120	$\frac{120}{720} = \frac{1}{6}$	$\frac{1}{6} \times 360° = 60°$
Cakes and pastries	160	$\frac{160}{720} = \frac{2}{9}$	$\frac{2}{9} \times 360° = 80°$
Fruit Bread	80	$\frac{80}{720} = \frac{1}{9}$	$\frac{1}{9} \times 360° = 40°$
Others	40	$\frac{40}{720} = \frac{1}{18}$	$\frac{1}{18} \times 360° = 20°$

Now, we make the pie chart (Fig 5.8):

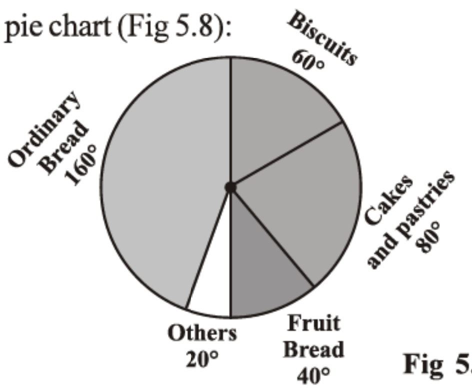


Fig 5.8

TRY THESE

Draw a pie chart of the data given below.

The time spent by a child during a day.

Sleep — 8 hours
School — 6 hours
Home work — 4 hours
Play — 4 hours
Others — 2 hours

THINK, DISCUSS AND WRITE

Which form of graph would be appropriate to display the following data.

1. Production of food grains of a state.

Year	2001	2002	2003	2004	2005	2006
Production (in lakh tons)	60	50	70	55	80	85

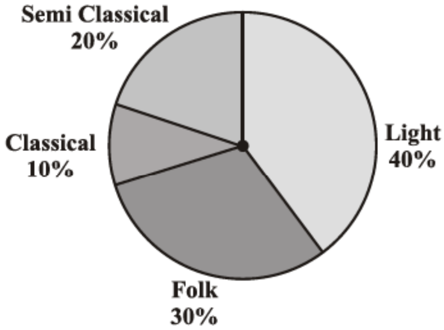
2. Choice of food for a group of people.

Favourite food	**Number of people**
North Indian	30
South Indian	40
Gujarati	25
Others	25
Total	**120**

3. The daily income of a group of a factory workers.

Daily Income (in Rupees)	**Number of workers (in a factory)**
75-100	45
100-125	35
125-150	55
150-175	30
175-200	50
200-225	125
225-250	140
Total	**480**

EXERCISE 5.2



1. A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the findings of this survey.
   From this pie chart answer the following:
   (i) If 20 people liked classical music, how many young people were surveyed?
   (ii) Which type of music is liked by the maximum number of people?
   (iii) If a cassette company were to make 1000 CD's, how many of each type would they make?

2. A group of 360 people were asked to vote for their favourite season from the three seasons rainy, winter and summer.
   (i) Which season got the most votes?
   (ii) Find the central angle of each sector.
   (iii) Draw a pie chart to show this information.

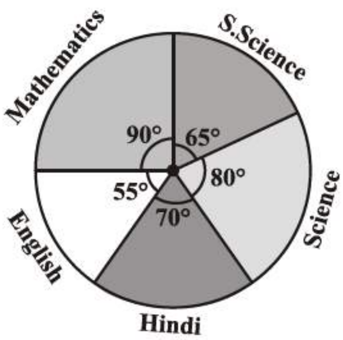
Season	No. of votes
Summer 	90
Rainy 	120
Winter 	150

3. Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.

Colours	Number of people
Blue	18
Green	9
Red	6
Yellow	3
Total	**36**

> Find the proportion of each sector. For example, Blue is $\frac{18}{36}=\frac{1}{2}$; Green is $\frac{9}{36}=\frac{1}{4}$ and so on. Use this to find the corresponding angles.

4. The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions.
   (i) In which subject did the student score 105 marks?
   (**Hint:** for 540 marks, the central angle = 360°. So, for 105 marks, what is the central angle?)
   (ii) How many more marks were obtained by the student in Mathematics than in Hindi?
   (iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.
   (**Hint:** Just study the central angles).



5. The number of students in a hostel, speaking different languages is given below. Display the data in a pie chart.

Language	Gujarati	English	Urdu	Hindi	Sindhi	Total
Number of students	40	12	9	7	4	72

5.5 Chance and Probability

Sometimes it happens that during rainy season, you carry a raincoat every day and it does not rain for many days. However, by chance, one day you forget to take the raincoat and it rains heavily on that day.

Sometimes it so happens that a student prepares 4 chapters out of 5, very well for a test. But a major question is asked from the chapter that she left unprepared.

Everyone knows that a particular train runs in time but the day you reach well in time it is late!

You face a lot of situations such as these where you take a chance and it does not go the way you want it to. Can you give some more examples? These are examples where the chances of a certain thing happening or not happening are not equal. The chances of the train being in time or being late are not the same. When you buy a ticket which is wait listed, you do take a chance. You hope that it might get confirmed by the time you travel.

We however, consider here certain experiments whose results have an equal chance of occurring.

5.5.1 Getting a result

You might have seen that before a cricket match starts, captains of the two teams go out to toss a coin to decide which team will bat first.

What are the possible results you get when a coin is tossed? Of course, Head or Tail.

Imagine that you are the captain of one team and your friend is the captain of the other team. You toss a coin and ask your friend to make the call. Can you control the result of the toss? Can you get a head if you want one? Or a tail if you want that? No, that is not possible. Such an experiment is called a **random experiment**. Head or Tail are the two **outcomes** of this experiment.

TRY THESE

1. If you try to start a scooter, what are the possible outcomes?
2. When a die is thrown, what are the six possible outcomes?

3. When you spin the wheel shown, what are the possible outcomes? (Fig 5.9) List them.
   (Outcome here means the sector at which the pointer stops).

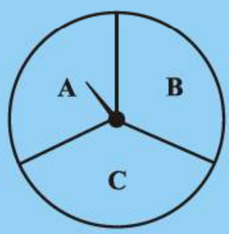


Fig 5.9

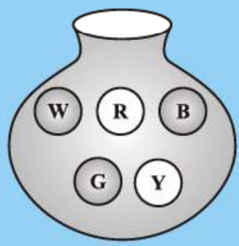


Fig 5.10

4. You have a pot with five identical balls of different colours and you are to pull out (draw) a ball without looking at it; list the outcomes you would get (Fig 5.10).

THINK, DISCUSS AND WRITE

In throwing a die:

- Does the first player have a greater chance of getting a six?
- Would the player who played after him have a lesser chance of getting a six?
- Suppose the second player got a six. Does it mean that the third player would not have a chance of getting a six?

5.5.2 Equally likely outcomes:

A coin is tossed several times and the number of times we get head or tail is noted. Let us look at the result sheet where we keep on increasing the tosses:

Number of tosses	Tally marks (H)	Number of heads	Tally mark (T)	Number of tails
50	卌 卌 卌 卌 卌 ||	27	卌 卌 卌 卌 |||	23
60	卌 卌 卌 卌 卌 |||	28	卌 卌 卌 卌 卌 卌 ||	32
70	...	33	...	37
80	...	38	...	42
90	...	44	...	46
100	...	48	...	52

Observe that as you increase the number of tosses more and more, the number of heads and the number of tails come closer and closer to each other.

This could also be done with a die, when tossed a large number of times. Number of each of the six outcomes become almost equal to each other.

In such cases, we may say that the different outcomes of the experiment are equally likely. This means that each of the outcomes has the same chance of occurring.

5.5.3 Linking chances to probability

Consider the experiment of tossing a coin once. What are the outcomes? There are only two outcomes – Head or Tail. Both the outcomes are equally likely. Likelihood of getting a head is one out of two outcomes, i.e., $\frac{1}{2}$. In other words, we say that the probability of getting a head = $\frac{1}{2}$. What is the probability of getting a tail?

Now take the example of throwing a die marked with 1, 2, 3, 4, 5, 6 on its faces (one number on one face). If you throw it once, what are the outcomes?

The outcomes are: 1, 2, 3, 4, 5, 6. Thus, there are six equally likely outcomes.

What is the probability of getting the outcome '2'?

It is $\frac{1}{6}$ $\begin{array}{l}\leftarrow \text{Number of outcomes giving 2}\\ \leftarrow \text{Number of equally likely outcomes.}\end{array}$

What is the probability of getting the number 5? What is the probability of getting the number 7? What is the probability of getting a number 1 through 6?

5.5.4 Outcomes as events

Each outcome of an experiment or a collection of outcomes make an **event**.

For example in the experiment of tossing a coin, getting a Head is an event and getting a Tail is also an event.

In case of throwing a die, getting each of the outcomes 1, 2, 3, 4, 5 or 6 is an event.

Is getting an even number an event? Since an even number could be 2, 4 or 6, getting an even number is also an event. What will be the probability of getting an even number?

It is $\frac{3}{6}$ ← Number of outcomes that make the event
← Total number of outcomes of the experiment.

Example 3: A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is probability of getting a red ball? Is it more or less than getting a yellow ball?

Solution: There are in all (4 + 2 =) 6 outcomes of the event. Getting a red ball consists of 4 outcomes. (Why?)

Therefore, the probability of getting a red ball is $\frac{4}{6} = \frac{2}{3}$. In the same way the probability of getting a yellow ball $= \frac{2}{6} = \frac{1}{3}$ (Why?). Therefore, the probability of getting a red ball is more than that of getting a yellow ball.

TRY THESE

Suppose you spin the wheel

1. (i) List the number of outcomes of getting a green sector and not getting a green sector on this wheel (Fig 5.11).
   (ii) Find the probability of getting a green sector.
   (iii) Find the probability of not getting a green sector.

Fig 5.11

5.5.5 Chance and probability related to real life

We talked about the chance that it rains just on the day when we do not carry a rain coat.

What could you say about the chance in terms of probability? Could it be one in 10 days during a rainy season? The probability that it rains is then $\frac{1}{10}$. The probability that it does not rain $= \frac{9}{10}$. (Assuming raining or not raining on a day are equally likely)

The use of probability is made in various cases in real life.

1. To find characteristics of a large group by using a small part of the group.

   For example, during elections 'an exit poll' is taken. This involves asking the people whom they have voted for, when they come out after voting at the centres which are chosen off hand and distributed over the whole area. This gives an idea of chance of winning of each candidate and predictions are made based on it accordingly.

2. Metrological Department predicts weather by observing trends from the data over many years in the past.

EXERCISE 5.3

1. List the outcomes you can see in these experiments.

(a) Spinning a wheel

(b) Tossing two coins together

2. When a die is thrown, list the outcomes of an event of getting

(i) (a) a prime number (b) not a prime number.

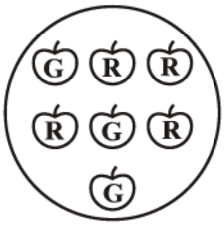
(ii) (a) a number greater than 5 (b) a number not greater than 5.

3. Find the.

(a) Probability of the pointer stopping on D in (Question 1-(a))?

(b) Probability of getting an ace from a well shuffled deck of 52 playing cards?

(c) Probability of getting a red apple. (See figure below)



4. Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of .

(i) getting a number 6?

(ii) getting a number less than 6?

(iii) getting a number greater than 6?

(iv) getting a 1-digit number?

5. If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non blue sector?

6. Find the probabilities of the events given in Question 2.

WHAT HAVE WE DISCUSSED?

1. Data mostly available to us in an unorganised form is called **raw data**.

2. In order to draw meaningful inferences from any data, we need to organise the data systematically.

3. **Frequency** gives the number of times that a particular entry occurs.
4. Raw data can be 'grouped' and presented systematically through 'grouped frequency distribution'.
5. Grouped data can be presented using **histogram**. Histogram is a type of bar diagram, where the class intervals are shown on the horizontal axis and the heights of the bars show the frequency of the class interval. Also, there is no gap between the bars as there is no gap between the class intervals.
6. Data can also presented using **circle graph** or **pie chart**. A circle graph shows the relationship between a whole and its part.
7. There are certain experiments whose outcomes have an equal chance of occurring.
8. A **random experiment** is one whose outcome cannot be predicted exactly in advance.
9. Outcomes of an experiment are **equally likely** if each has the same chance of occurring.
10. **Probability of an event** $= \dfrac{\text{Number of outcomes that make an event}}{\text{Total number of outcomes of the experiment}}$, when the outcomes are equally likely.
11. One or more outcomes of an experiment make an **event**.
12. Chances and probability are related to real life.

CHAPTER

6

Squares and Square Roots

0852CH06

6.1 Introduction

You know that the area of a square = side × side (where 'side' means 'the length of a side'). Study the following table.

Side of a square (in cm)	Area of the square (in cm^2)
1	$1 \times 1 = 1 = 1^2$
2	$2 \times 2 = 4 = 2^2$
3	$3 \times 3 = 9 = 3^2$
5	$5 \times 5 = 25 = 5^2$
8	$8 \times 8 = 64 = 8^2$
a	$a \times a = a^2$

What is special about the numbers 4, 9, 25, 64 and other such numbers?

Since, 4 can be expressed as $2 \times 2 = 2^2$, 9 can be expressed as $3 \times 3 = 3^2$, all such numbers can be expressed as the product of the number with itself.

Such numbers like 1, 4, 9, 16, 25, ... are known as **square numbers**.

In general, if a natural number m can be expressed as n^2, where n is also a natural number, then m is a **square number**. Is 32 a square number?

We know that $5^2 = 25$ and $6^2 = 36$. If 32 is a square number, it must be the square of a natural number between 5 and 6. But there is no natural number between 5 and 6.

Therefore 32 is not a square number.

Consider the following numbers and their squares.

Number	Square
1	$1 \times 1 = 1$
2	$2 \times 2 = 4$

3	$3 \times 3 = 9$
4	$4 \times 4 = 16$
5	$5 \times 5 = 25$
6	-----------
7	-----------
8	-----------
9	-----------
10	-----------

Can you complete it?

From the above table, can we enlist the square numbers between 1 and 100? Are there any natural square numbers upto 100 left out?

You will find that the rest of the numbers are not square numbers.

The numbers 1, 4, 9, 16 ... are square numbers. These numbers are also called **perfect squares**.

TRY THESE

1. Find the perfect square numbers between (i) 30 and 40 (ii) 50 and 60

6.2 Properties of Square Numbers

Following table shows the squares of numbers from 1 to 20.

Number	Square	Number	Square
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Study the square numbers in the above table. What are the ending digits (that is, digits in the units place) of the square numbers? All these numbers end with 0, 1, 4, 5, 6 or 9 at units place. None of these end with 2, 3, 7 or 8 at unit's place.

Can we say that if a number ends in 0, 1, 4, 5, 6 or 9, then it must be a square number? Think about it.

TRY THESE

1. Can we say whether the following numbers are perfect squares? How do we know?
   (i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
   (v) 1069 (vi) 2061

Write five numbers which you can decide by looking at their units digit that they are not square numbers.

2. Write five numbers which you cannot decide just by looking at their units digit (or units place) whether they are square numbers or not.

- Study the following table of some numbers and their squares and observe the one's place in both.

Table 1

Number	Square	Number	Square	Number	Square
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	30	900
7	49	17	289	35	1225
8	64	18	324	40	1600
9	81	19	361	45	2025
10	100	20	400	50	2500

The following square numbers end with digit 1.

Square	Number
1	1
81	9
121	11
361	19
441	21

TRY THESE

Which of 123^2, 77^2, 82^2, 161^2, 109^2 would end with digit 1?

Write the next two square numbers which end in 1 and their corresponding numbers.
You will see that if a number has 1 or 9 in the units place, then it's square ends in 1.

- Let us consider square numbers ending in 6.

Square	Number
16	4
36	6
196	14
256	16

TRY THESE

Which of the following numbers would have digit 6 at unit place.

(i) 19^2 (ii) 24^2 (iii) 26^2
(iv) 36^2 (v) 34^2

We can see that *when a square number ends in 6, the number whose square it is, will have either 4 or 6 in unit's place.*

Can you find more such rules by observing the numbers and their squares (Table 1)?

TRY THESE

What will be the "one's digit" in the square of the following numbers?

(i) 1234 (ii) 26387 (iii) 52698 (iv) 99880
(v) 21222 (vi) 9106

- Consider the following numbers and their squares.

We have one zero

$10^2 = 100$
$20^2 = 400$
$80^2 = 6400$

But we have two zeros

We have two zeros

$100^2 = 10000$
$200^2 = 40000$
$700^2 = 490000$
$900^2 = 810000$

But we have four zeros

If a number contains 3 zeros at the end, how many zeros will its square have ?
What do you notice about the number of zeros at the end of the number and the number of zeros at the end of its square?
Can we say that square numbers can only have even number of zeros at the end?

- See Table 1 with numbers and their squares.
What can you say about the squares of even numbers and squares of odd numbers?

TRY THESE

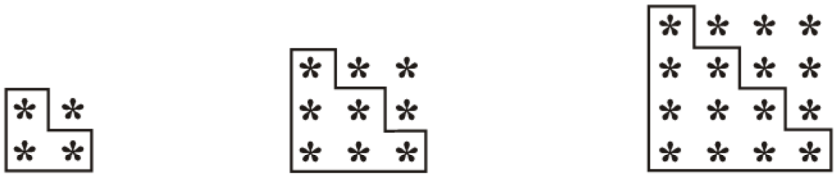
1. The square of which of the following numbers would be an odd number/an even number? Why?
(i) 727 (ii) 158 (iii) 269 (iv) 1980
2. What will be the number of zeros in the square of the following numbers?
(i) 60 (ii) 400

6.3 Some More Interesting Patterns

1. **Adding triangular numbers.**
Do you remember triangular numbers (numbers whose dot patterns can be arranged as triangles)?

```
                              *
                    *         * *
            *       * *       * **
     *      * *     * * *     * ***
*    * *    * * *   * * * *   * ****
1    3      6       10        15
```

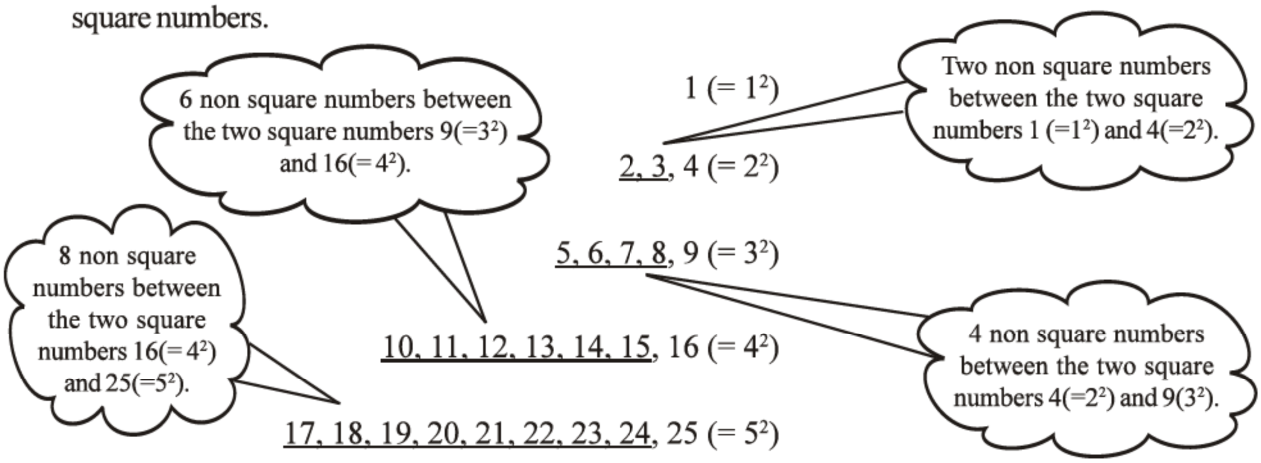
If we combine two consecutive triangular numbers, we get a square number, like



$1 + 3 = 4 = 2^2$ $\quad$ $3 + 6 = 9 = 3^2$ $\quad$ $6 + 10 = 16 = 4^2$

2. Numbers between square numbers

Let us now see if we can find some interesting pattern between two consecutive square numbers.



Between $1^2(=1)$ and $2^2(=4)$ there are two (i.e., 2×1) non square numbers 2, 3.

Between $2^2(=4)$ and $3^2(=9)$ there are four (i.e., 2×2) non square numbers 5, 6, 7, 8.

Now, $\quad 3^2 = 9, \quad 4^2 = 16$

Therefore, $\quad 4^2 - 3^2 = 16 - 9 = 7$

Between $9(=3^2)$ and $16(=4^2)$ the numbers are 10, 11, 12, 13, 14, 15 that is, six non-square numbers which is 1 less than the difference of two squares.

We have $\quad 4^2 = 16 \quad$ and $\quad 5^2 = 25$

Therefore, $\quad 5^2 - 4^2 = 9$

Between $16(=4^2)$ and $25(=5^2)$ the numbers are 17, 18, ... , 24 that is, eight non square numbers which is 1 less than the difference of two squares.

Consider 7^2 and 6^2. Can you say how many numbers are there between 6^2 and 7^2? If we think of any natural number n and $(n + 1)$, then,

$$(n + 1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1.$$

We find that between n^2 and $(n + 1)^2$ there are $2n$ numbers which is 1 less than the difference of two squares.

Thus, in general we can say that *there are 2n non perfect square numbers between the squares of the numbers n and* $(n + 1)$. Check for $n = 5$, $n = 6$ etc., and verify.

TRY THESE

1. How many natural numbers lie between 9^2 and 10^2 ? Between 11^2 and 12^2?
2. How many non square numbers lie between the following pairs of numbers
   (i) 100^2 and 101^2 (ii) 90^2 and 91^2 (iii) 1000^2 and 1001^2

3. Adding odd numbers

Consider the following

1 [one odd number]	$= 1 = 1^2$
1 + 3 [sum of first two odd numbers]	$= 4 = 2^2$
1 + 3 + 5 [sum of first three odd numbers]	$= 9 = 3^2$
1 + 3 + 5 + 7 [...]	$= 16 = 4^2$
1 + 3 + 5 + 7 + 9 [...]	$= 25 = 5^2$
1 + 3 + 5 + 7 + 9 + 11 [...]	$= 36 = 6^2$

So we can say that the *sum of first n odd natural numbers is* n^2.

Looking at it in a different way, we can say: 'If the number is a square number, it has to be the sum of successive **odd** numbers starting from 1.

Consider those numbers which are not perfect squares, say 2, 3, 5, 6, ... Can you express these numbers as a sum of successive odd natural numbers beginning from 1?

You will find that these numbers cannot be expressed in this form.

Consider the number 25. Successively subtract 1, 3, 5, 7, 9, ... from it

(i) $25 - 1 = 24$ (ii) $24 - 3 = 21$ (iii) $21 - 5 = 16$ (iv) $16 - 7 = 9$
(v) $9 - 9 = 0$

This means, $25 = 1 + 3 + 5 + 7 + 9$. Also, 25 is a perfect square.

Now consider another number 38, and again do as above.

(i) $38 - 1 = 37$ (ii) $37 - 3 = 34$ (iii) $34 - 5 = 29$ (iv) $29 - 7 = 22$
(v) $22 - 9 = 13$ (vi) $13 - 11 = 2$ (vii) $2 - 13 = -11$

This shows that we are not able to express 38 as the sum of consecutive odd numbers starting with 1. Also, 38 is not a perfect square.

So we can also say that *if a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.*

We can use this result to find whether a number is a perfect square or not.

TRY THESE

Find whether each of the following numbers is a perfect square or not?

(i) 121 (ii) 55 (iii) 81
(iv) 49 (v) 69

4. A sum of consecutive natural numbers

Consider the following

$3^2 = 9 = 4 + 5$

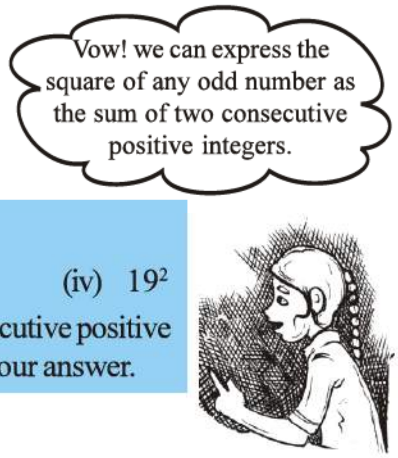
$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

First Number $= \dfrac{3^2 - 1}{2}$

Second Number $= \dfrac{3^2 + 1}{2}$

$9^2 = 81 = 40 + 41$
$11^2 = 121 = 60 + 61$
$15^2 = 225 = 112 + 113$



TRY THESE

1. Express the following as the sum of two consecutive integers.
   (i) 21^2 (ii) 13^2 (iii) 11^2 (iv) 19^2
2. Do you think the reverse is also true, i.e., is the sum of any two consecutive positive integers is perfect square of a number? Give example to support your answer.

5. Product of two consecutive even or odd natural numbers

$11 \times 13 = 143 = 12^2 - 1$

Also $11 \times 13 = (12 - 1) \times (12 + 1)$

Therefore, $11 \times 13 = (12 - 1) \times (12 + 1) = 12^2 - 1$

Similarly, $13 \times 15 = (14 - 1) \times (14 + 1) = 14^2 - 1$

$29 \times 31 = (30 - 1) \times (30 + 1) = 30^2 - 1$

$44 \times 46 = (45 - 1) \times (45 + 1) = 45^2 - 1$

So in general we can say that $(a + 1) \times (a - 1) = a^2 - 1$.

6. Some more patterns in square numbers

Observe the squares of numbers; 1, 11, 111 ... etc. They give a beautiful pattern:

$1^2 =$								1							
$11^2 =$							1	2	1						
$111^2 =$						1	2	3	2	1					
$1111^2 =$					1	2	3	4	3	2	1				
$11111^2 =$				1	2	3	4	5	4	3	2	1			
$11111111^2 =$	1	2	3	4	5	6	7	8	7	6	5	4	3	2	1

Another interesting pattern.

$7^2 = 49$
$67^2 = 4489$
$667^2 = 444889$
$6667^2 = 44448889$
$66667^2 = 4444488889$
$666667^2 = 444444888889$

The fun is in being able to find out why this happens. May be it would be interesting for you to explore and think about such questions even if the answers come some years later.

TRY THESE

Write the square, making use of the above pattern.
(i) 111111^2 (ii) 1111111^2

TRY THESE

Can you find the square of the following numbers using the above pattern?
(i) 6666667^2 (ii) 66666667^2

EXERCISE 6.1

1. What will be the unit digit of the squares of the following numbers?
   (i) 81 (ii) 272 (iii) 799 (iv) 3853
   (v) 1234 (vi) 26387 (vii) 52698 (viii) 99880
   (ix) 12796 (x) 55555
2. The following numbers are obviously not perfect squares. Give reason.
   (i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
   (v) 64000 (vi) 89722 (vii) 222000 (viii) 505050
3. The squares of which of the following would be odd numbers?
   (i) 431 (ii) 2826 (iii) 7779 (iv) 82004
4. Observe the following pattern and find the missing digits.

$$11^2 = 121$$
$$101^2 = 10201$$
$$1001^2 = 1002001$$
$$100001^2 = 1 \text{ .... } 2 \text{ .... } 1$$
$$10000001^2 = \text{..........................}$$

5. Observe the following pattern and supply the missing numbers.

$$11^2 = 1\ 2\ 1$$
$$101^2 = 1\ 0\ 2\ 0\ 1$$
$$10101^2 = 102030201$$
$$1010101^2 = \text{..........................}$$
$$\text{............}^2 = 10203040504030201$$

6. Using the given pattern, find the missing numbers.

$$1^2 + 2^2 + 2^2 = 3^2$$
$$2^2 + 3^2 + 6^2 = 7^2$$
$$3^2 + 4^2 + 12^2 = 13^2$$
$$4^2 + 5^2 + \_^2 = 21^2$$
$$5^2 + \_^2 + 30^2 = 31^2$$
$$6^2 + 7^2 + \_^2 = \_\_^2$$

To find pattern
Third number is related to first and second number. How?
Fourth number is related to third number. How?

7. Without adding, find the sum.
   (i) 1 + 3 + 5 + 7 + 9
   (ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 +19
   (iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23
8. (i) Express 49 as the sum of 7 odd numbers.
   (ii) Express 121 as the sum of 11 odd numbers.
9. How many numbers lie between squares of the following numbers?
   (i) 12 and 13 (ii) 25 and 26 (iii) 99 and 100

6.4 Finding the Square of a Number

Squares of small numbers like 3, 4, 5, 6, 7, ... etc. are easy to find. But can we find the square of 23 so quickly?

The answer is not so easy and we may need to multiply 23 by 23.

There is a way to find this without having to multiply 23 × 23.

We know $23 = 20 + 3$

Therefore $23^2 = (20 + 3)^2 = 20(20 + 3) + 3(20 + 3)$

$= 20^2 + 20 \times 3 + 3 \times 20 + 3^2$

$= 400 + 60 + 60 + 9 = 529$

Example 1: Find the square of the following numbers without actual multiplication.

(i) 39 (ii) 42

Solution: (i) $39^2 = (30 + 9)^2 = 30(30 + 9) + 9(30 + 9)$

$= 30^2 + 30 \times 9 + 9 \times 30 + 9^2$

$= 900 + 270 + 270 + 81 = 1521$

(ii) $42^2 = (40 + 2)^2 = 40(40 + 2) + 2(40 + 2)$

$= 40^2 + 40 \times 2 + 2 \times 40 + 2^2$

$= 1600 + 80 + 80 + 4 = 1764$

6.4.1 Other patterns in squares

Consider the following pattern:

$25^2 = 625 = (2 \times 3)$ hundreds + 25

$35^2 = 1225 = (3 \times 4)$ hundreds + 25

$75^2 = 5625 = (7 \times 8)$ hundreds + 25

$125^2 = 15625 = (12 \times 13)$ hundreds + 25

Now can you find the square of 95?

Consider a number with unit digit 5, i.e., $a5$

$(a5)^2 = (10a + 5)^2$

$= 10a(10a + 5) + 5(10a + 5)$

$= 100a^2 + 50a + 50a + 25$

$= 100a(a + 1) + 25$

$= a(a + 1)$ hundred + 25

TRY THESE

Find the squares of the following numbers containing 5 in unit's place.

(i) 15 (ii) 95 (iii) 105 (iv) 205

6.4.2 Pythagorean triplets

Consider the following

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

The collection of numbers 3, 4 and 5 is known as **Pythagorean triplet**. 6, 8, 10 is also a Pythagorean triplet, since

$$6^2 + 8^2 = 36 + 64 = 100 = 10^2$$

Again, observe that

$5^2 + 12^2 = 25 + 144 = 169 = 13^2$. The numbers 5, 12, 13 form another such triplet.

Can you find more such triplets?

For any natural number $m > 1$, we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$. So, $2m$, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet.

Try to find some more Pythagorean triplets using this form.

Example 2: Write a Pythagorean triplet whose smallest member is 8.

Solution: We can get Pythagorean triplets by using general form $2m$, $m^2 - 1$, $m^2 + 1$.

Let us first take $m^2 - 1 = 8$

So, $m^2 = 8 + 1 = 9$

which gives $m = 3$

Therefore, $2m = 6$ and $m^2 + 1 = 10$

The triplet is thus 6, 8, 10. But 8 is not the smallest member of this.

So, let us try $2m = 8$

then $m = 4$

We get $m^2 - 1 = 16 - 1 = 15$

and $m^2 + 1 = 16 + 1 = 17$

The triplet is 8, 15, 17 with 8 as the smallest member.

Example 3: Find a Pythagorean triplet in which one member is 12.

Solution: If we take $m^2 - 1 = 12$

Then, $m^2 = 12 + 1 = 13$

Then the value of m will not be an integer.

So, we try to take $m^2 + 1 = 12$. Again $m^2 = 11$ will not give an integer value for m.

So, let us take $2m = 12$

then $m = 6$

Thus, $m^2 - 1 = 36 - 1 = 35$ and $m^2 + 1 = 36 + 1 = 37$

Therefore, the required triplet is 12, 35, 37.

Note: All Pythagorean triplets may not be obtained using this form. For example another triplet 5, 12, 13 also has 12 as a member.

EXERCISE 6.2

1. Find the square of the following numbers.
   (i) 32 (ii) 35 (iii) 86 (iv) 93
   (v) 71 (vi) 46
2. Write a Pythagorean triplet whose one member is.
   (i) 6 (ii) 14 (iii) 16 (iv) 18

6.5 Square Roots

Study the following situations.

(a) Area of a square is 144 cm². What could be the side of the square?

We know that the area of a square = side^2

If we assume the length of the side to be 'a', then $144 = a^2$

To find the length of side it is necessary to find a number whose square is 144.

(b) What is the length of a diagonal of a square of side 8 cm (Fig 6.1)?

Can we use Pythagoras theorem to solve this ?

We have, $AB^2 + BC^2 = AC^2$

i.e., $8^2 + 8^2 = AC^2$

or $64 + 64 = AC^2$

or $128 = AC^2$

Again to get AC we need to think of a number whose square is 128.

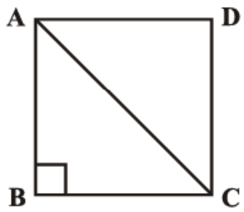


Fig 6.1

(c) In a right triangle the length of the hypotenuse and a side are respectively 5 cm and 3 cm (Fig 6.2).

Can you find the third side?

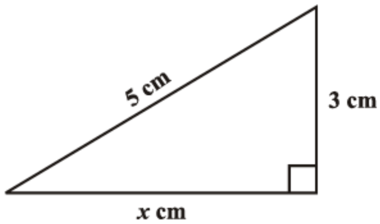


Fig 6.2

Let x cm be the length of the third side.

Using Pythagoras theorem $5^2 = x^2 + 3^2$

$25 - 9 = x^2$

$16 = x^2$

Again, to find x we need a number whose square is 16.

In all the above cases, we need to find a number whose square is known. Finding the number with the known square is known as finding the square root.

6.5.1 Finding square roots

The inverse (opposite) operation of addition is subtraction and the inverse operation of multiplication is division. Similarly, finding the square root is the inverse operation of squaring.

We have, $1^2 = 1$, therefore square root of 1 is 1

$2^2 = 4$, therefore square root of 4 is 2

$3^2 = 9$, therefore square root of 9 is 3

> Since $9^2 = 81$,
> and $(-9)^2 = 81$
> We say that square roots of 81 are 9 and -9.

TRY THESE

(i) $11^2 = 121$. What is the square root of 121?

(ii) $14^2 = 196$. What is the square root of 196?

THINK, DISCUSS AND WRITE

$(-1)^2 = 1$. Is -1, a square root of 1? $(-2)^2 = 4$. Is -2, a square root of 4?

$(-9)^2 = 81$. Is -9 a square root of 81?

From the above, you may say that there are two integral square roots of a perfect square number. In this chapter, we shall take up only positive square root of a natural number. Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

For example: $\sqrt{4} = 2$ (not -2); $\sqrt{9} = 3$ (not -3) etc.

Statement	Inference
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$

Statement	Inference
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$

6.5.2 Finding square root through repeated subtraction

Do you remember that the sum of the first n odd natural numbers is n^2? That is, every square number can be expressed as a sum of successive odd natural numbers starting from 1.

Consider $\sqrt{81}$. Then,

(i) $81 - 1 = 80$ (ii) $80 - 3 = 77$ (iii) $77 - 5 = 72$ (iv) $72 - 7 = 65$
(v) $65 - 9 = 56$ (vi) $56 - 11 = 45$ (vii) $45 - 13 = 32$ (viii) $32 - 15 = 17$
(ix) $17 - 17 = 0$

From 81 we have subtracted successive odd numbers starting from 1 and obtained 0 at 9^{th} step.

Therefore $\sqrt{81} = 9$.

Can you find the square root of 729 using this method? Yes, but it will be time consuming. Let us try to find it in a simpler way.

TRY THESE

By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root.

(i) 121
(ii) 55
(iii) 36
(iv) 49
(v) 90

6.5.3 Finding square root through prime factorisation

Consider the prime factorisation of the following numbers and their squares.

Prime factorisation of a Number	Prime factorisation of its Square
$6 = 2 \times 3$	$36 = 2 \times 2 \times 3 \times 3$
$8 = 2 \times 2 \times 2$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$	$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$15 = 3 \times 5$	$225 = 3 \times 3 \times 5 \times 5$

How many times does 2 occur in the prime factorisation of 6? Once. How many times does 2 occur in the prime factorisation of 36? Twice. Similarly, observe the occurrence of 3 in 6 and 36 of 2 in 8 and 64 etc.

You will find that each prime factor in the prime factorisation of the square of a number, occurs twice the number of times it occurs in the prime factorisation of the number itself. Let us use this to find the square root of a given square number, say 324.

We know that the prime factorisation of 324 is

2	324
2	162
3	81
3	27
3	9
3	3
	1

$$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

By pairing the prime factors, we get

$$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} = 2^2 \times 3^2 \times 3^2 = (2 \times 3 \times 3)^2$$

So, $\sqrt{324} = 2 \times 3 \times 3 = 18$

Similarly can you find the square root of 256? Prime factorisation of 256 is

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

By pairing the prime factors we get,

$$256 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} = (2 \times 2 \times 2 \times 2)^2$$

Therefore, $\sqrt{256} = 2 \times 2 \times 2 \times 2 = 16$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Is 48 a perfect square?

We know $48 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3$

Since all the factors are not in pairs so 48 is not a perfect square.

Suppose we want to find the smallest multiple of 48 that is a perfect square, how should we proceed? Making pairs of the prime factors of 48 we see that 3 is the only factor that does not have a pair. So we need to multiply by 3 to complete the pair.

Hence $48 \times 3 = 144$ is a perfect square.

Can you tell by which number should we divide 48 to get a perfect square?

The factor 3 is not in pair, so if we divide 48 by 3 we get $48 \div 3 = 16 = \underline{2 \times 2} \times \underline{2 \times 2}$ and this number 16 is a perfect square too.

Example 4: Find the square root of 6400.

Solution: Write $6400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

Therefore $\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5 = 80$

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

Example 5: Is 90 a perfect square?

Solution: We have $90 = 2 \times 3 \times 3 \times 5$

2	90
3	45
3	15
5	5
	1

The prime factors 2 and 5 do not occur in pairs. Therefore, 90 is not a perfect square. That 90 is not a perfect square can also be seen from the fact that it has only one zero.

Example 6: Is 2352 a perfect square? If not, find the smallest multiple of 2352 which is a perfect square. Find the square root of the new number.

Solution: We have $2352 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

As the prime factor 3 has no pair, 2352 is not a perfect square.

If 3 gets a pair then the number will become perfect square. So, we multiply 2352 by 3 to get,

$$2352 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Now each prime factor is in a pair. Therefore, $2352 \times 3 = 7056$ is a perfect square. Thus the required smallest multiple of 2352 is 7056 which is a perfect square.

And, $\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$

2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

Example 7: Find the smallest number by which 9408 must be divided so that the quotient is a perfect square. Find the square root of the quotient.

Solution: We have, $9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

If we divide 9408 by the factor 3, then

$9408 \div 3 = 3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$ which is a perfect square. (Why?)

Therefore, the required smallest number is 3.

And, $\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56.$

Example 8: Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.

2	6, 9, 15
3	3, 9, 15
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

Solution: This has to be done in two steps. First find the smallest common multiple and then find the square number needed. The least number divisible by each one of 6, 9 and 15 is their LCM. The LCM of 6, 9 and 15 is $2 \times 3 \times 3 \times 5 = 90$.

Prime factorisation of 90 is $90 = 2 \times \underline{3 \times 3} \times 5$.

We see that prime factors 2 and 5 are not in pairs. Therefore 90 is not a perfect square.

In order to get a perfect square, each factor of 90 must be paired. So we need to make pairs of 2 and 5. Therefore, 90 should be multiplied by 2×5, i.e., 10.

Hence, the required square number is $90 \times 10 = 900$.

EXERCISE 6.3

1. What could be the possible 'one's' digits of the square root of each of the following numbers?
   (i) 9801 (ii) 99856 (iii) 998001 (iv) 657666025
2. Without doing any calculation, find the numbers which are surely not perfect squares.
   (i) 153 (ii) 257 (iii) 408 (iv) 441
3. Find the square roots of 100 and 169 by the method of repeated subtraction.
4. Find the square roots of the following numbers by the Prime Factorisation Method.
   (i) 729 (ii) 400 (iii) 1764 (iv) 4096
   (v) 7744 (vi) 9604 (vii) 5929 (viii) 9216
   (ix) 529 (x) 8100
5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.
   (i) 252 (ii) 180 (iii) 1008 (iv) 2028
   (v) 1458 (vi) 768
6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.
   (i) 252 (ii) 2925 (iii) 396 (iv) 2645
   (v) 2800 (vi) 1620
7. The students of Class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.
9. Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.
10. Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

6.5.4 Finding square root by division method

When the numbers are large, even the method of finding square root by prime factorisation becomes lengthy and difficult. To overcome this problem we use Long Division Method.

For this we need to determine the number of digits in the square root.
See the following table:

Number	Square	
10	100	which is the smallest 3-digit perfect square
31	961	which is the greatest 3-digit perfect square
32	1024	which is the smallest 4-digit perfect square
99	9801	which is the greatest 4-digit perfect square

So, what can we say about the number of digits in the square root if a perfect square is a 3-digit or a 4-digit number? We can say that, if a perfect square is a 3-digit or a 4-digit number, then its square root will have 2-digits.

Can you tell the number of digits in the square root of a 5-digit or a 6-digit perfect square?

The smallest 3-digit perfect square number is 100 which is the square of 10 and the greatest 3-digit perfect square number is 961 which is the square of 31. The smallest 4-digit square number is 1024 which is the square of 32 and the greatest 4-digit number is 9801 which is the square of 99.

THINK, DISCUSS AND WRITE

Can we say that if a perfect square is of n-digits, then its square root will have $\frac{n}{2}$ digits if n is even or $\frac{(n+1)}{2}$ if n is odd?

The use of the number of digits in square root of a number is useful in the following method:

- Consider the following steps to find the square root of 529.

  Can you estimate the number of digits in the square root of this number?

Step 1 Place a bar over every pair of digits starting from the digit at one's place. If the number of digits in it is odd, then the left-most single digit too will have a bar.

Thus we have, $\overline{5}\,\overline{29}$.

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($2^2 < 5 < 3^2$). Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend (here 5). Divide and get the remainder (1 in this case).

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} & \quad 2 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline & 129 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 29 in this case) to the right of the remainder. So the new dividend is 129.

Step 4 Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & \quad 2 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline 4\_ & 129 \end{array}$$

Step 5 Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

In this case $42 \times 2 = 84$.

As $43 \times 3 = 129$ so we choose the new digit as 3. Get the remainder.

$$\begin{array}{r|l} & \quad 23 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline 43 & 129 \\ & -129 \\ \hline & \quad 0 \end{array}$$

Step 6 Since the remainder is 0 and no digits are left in the given number, therefore, $\sqrt{529} = 23$.

- Now consider $\sqrt{4096}$

Step 1 Place a bar over every pair of digits starting from the one's digit. ($\overline{40}\ \overline{96}$).

$$\begin{array}{r|l} & \quad 6 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline & \quad 4 \end{array}$$

Step 2 Find the largest number whose square is less than or equal to the number under the left-most bar ($6^2 < 40 < 7^2$). Take this number as the divisor and the number under the left-most bar as the dividend. Divide and get the remainder i.e., 4 in this case.

$$\begin{array}{r|l} & \quad 6 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline & 496 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 96) to the right of the remainder. The new dividend is 496.

Step 4 Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & \quad 6 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline 12\_ & 496 \end{array}$$

Step 5 Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new quotient the product is less than or equal to the dividend. In this case we see that $124 \times 4 = 496$.

So the new digit in the quotient is 4. Get the remainder.

$$\begin{array}{r|l} & \quad 64 \\ \hline 6 & \overline{40}\,\overline{96} \\ & -36 \\ \hline 124 & 496 \\ & -496 \\ \hline & \quad 0 \end{array}$$

Step 6 Since the remainder is 0 and no bar left, therefore, $\sqrt{4096} = 64$.

Estimating the number

We use bars to find the number of digits in the square root of a perfect square number.

$$\sqrt{\overline{5}\,\overline{29}} = 23 \qquad \text{and} \qquad \sqrt{\overline{40}\,\overline{96}} = 64$$

In both the numbers 529 and 4096 there are two bars and the number of digits in their square root is 2. Can you tell the number of digits in the square root of 14400?

By placing bars we get $\overline{1}\,\overline{44}\,\overline{00}$. Since there are 3 bars, the square root will be of 3 digit.

TRY THESE

Without calculating square roots, find the number of digits in the square root of the following numbers.

(i) 25600 (ii) 100000000 (iii) 36864

Example 9: Find the square root of : (i) 729 (ii) 1296

Solution:

(i)

	27
2	$\overline{7}\,\overline{29}$
	– 4
47	329
	329
	0

Therefore $\sqrt{729} = 27$

(ii)

	36
3	$\overline{12}\,\overline{96}$
	– 9
66	396
	396
	0

Therefore $\sqrt{1296} = 36$

Example 10: Find the least number that must be subtracted from 5607 so as to get a perfect square. Also find the square root of the perfect square.

Solution: Let us try to find $\sqrt{5607}$ by long division method. We get the remainder 131. It shows that 74^2 is less than 5607 by 131.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 5607 – 131 = 5476. And, $\sqrt{5476} = 74$.

	74
7	$\overline{56}\,\overline{07}$
	– 49
144	707
	–576
	131

Example 11: Find the greatest 4-digit number which is a perfect square.

Solution: Greatest number of 4-digits = 9999. We find $\sqrt{9999}$ by long division method. The remainder is 198. This shows 99^2 is less than 9999 by 198.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 9999 – 198 = 9801.

And, $\sqrt{9801} = 99$

	99
9	$\overline{99}\,\overline{99}$
	– 81
189	1899
	– 1701
	198

Example 12: Find the least number that must be added to 1300 so as to get a perfect square. Also find the square root of the perfect square.

Solution: We find $\sqrt{1300}$ by long division method. The remainder is 4.

This shows that $36^2 < 1300$.

Next perfect square number is $37^2 = 1369$.

Hence, the number to be added is $37^2 – 1300 = 1369 – 1300 = 69$.

	36
3	$\overline{13}\,\overline{00}$
	– 9
66	400
	– 396
	4

6.6 Square Roots of Decimals

Consider $\sqrt{17.64}$

Step 1 To find the square root of a decimal number we put bars on the integral part (i.e., 17) of the number in the usual manner. And place bars on the decimal part

$$\begin{array}{r|l} & \quad 4 \\ \hline 4 & \overline{17}.\overline{64} \\ & -16 \\ \hline & \quad 1 \end{array}$$

(i.e., 64) on every pair of digits beginning with the first decimal place. Proceed as usual. We get $\overline{17}.\overline{64}$.

Step 2 Now proceed in a similar manner. The left most bar is on 17 and $4^2 < 17 < 5^2$. Take this number as the divisor and the number under the left-most bar as the dividend, i.e., 17. Divide and get the remainder.

$$\begin{array}{r|l} & \quad 4 \\ \hline 4 & \overline{17}.\overline{64} \\ & -16 \\ \hline 8\_ & \;\; 1\,64 \end{array}$$

Step 3 The remainder is 1. Write the number under the next bar (i.e., 64) to the right of this remainder, to get 164.

$$\begin{array}{r|l} & \quad 4. \\ \hline 4 & 17.64 \\ & -16 \\ \hline 82 & \;\; 164 \end{array}$$

Step 4 Double the divisor and enter it with a blank on its right. Since 64 is the decimal part so put a decimal point in the quotient.

Step 5 We know $82 \times 2 = 164$, therefore, the new digit is 2. Divide and get the remainder.

$$\begin{array}{r|l} & \quad 4.2 \\ \hline 4 & 17.64 \\ & -16 \\ \hline 82 & \;\; 164 \\ & -164 \\ \hline & \quad 0 \end{array}$$

Step 6 Since the remainder is 0 and no bar left, therefore $\sqrt{17.64} = 4.2$.

Example 13: Find the square root of 12.25.

Solution:

$$\begin{array}{r|l} & \quad 3.5 \\ \hline 3 & \overline{12}.\overline{25} \\ & -9 \\ \hline 65 & \;\; 325 \\ & \;\; 325 \\ \hline & \quad 0 \end{array}$$

Therefore, $\sqrt{12.25} = 3.5$

Which way to move

Consider a number 176.341. Put bars on both integral part and decimal part. In what way is putting bars on decimal part different from integral part? Notice for 176 we start from the unit's place close to the decimal and move towards left. The first bar is over 76 and the second bar over 1. For .341, we start from the decimal and move towards right. First bar is over 34 and for the second bar we put 0 after 1 and make $.\overline{34}\overline{10}$.

$$\begin{array}{r|l} & \quad 48 \\ \hline 4 & \overline{23}\overline{04} \\ & -16 \\ \hline 88 & \;\; 704 \\ & \;\; 704 \\ \hline & \quad 0 \end{array}$$

Example 14: Area of a square plot is 2304 m^2. Find the side of the square plot.

Solution: Area of square plot = 2304 m^2

Therefore, side of the square plot = $\sqrt{2304}$ m

We find that, $\sqrt{2304} = 48$

Thus, the side of the square plot is 48 m.

Example 15: There are 2401 students in a school. P.T. teacher wants them to stand in rows and columns such that the number of rows is equal to the number of columns. Find the number of rows.

Solution: Let the number of rows be x

So, the number of columns = x

Therefore, number of students = $x \times x = x^2$

Thus, $x^2 = 2401$ gives $x = \sqrt{2401} = 49$

The number of rows = 49.

	49
4	$\overline{24}\,\overline{01}$
	–16
89	801
	801
	0

6.7 Estimating Square Root

Consider the following situations:

1. Deveshi has a square piece of cloth of area 125 cm^2. She wants to know whether she can make a handkerchief of side 15 cm. If that is not possible she wants to know what is the maximum length of the side of a handkerchief that can be made from this piece.
2. Meena and Shobha played a game. One told a number and other gave its square root. Meena started first. She said 25 and Shobha answered quickly as 5. Then Shobha said 81 and Meena answered 9. It went on, till at one point Meena gave the number 250. And Shobha could not answer. Then Meena asked Shobha if she could atleast tell a number whose square is closer to 250.

In all such cases we need to *estimate* the square root.

We know that $100 < 250 < 400$ and $\sqrt{100} = 10$ and $\sqrt{400} = 20$.

So $10 < \sqrt{250} < 20$

But still we are not very close to the square number.

We know that $15^2 = 225$ and $16^2 = 256$

Therefore, $15 < \sqrt{250} < 16$ and 256 is much closer to 250 than 225.

So, $\sqrt{250}$ is approximately 16.

TRY THESE

Estimate the value of the following to the nearest whole number.

(i) $\sqrt{80}$ (ii) $\sqrt{1000}$ (iii) $\sqrt{350}$ (iv) $\sqrt{500}$

EXERCISE 6.4

1. Find the square root of each of the following numbers by Division method.

(i) 2304 (ii) 4489 (iii) 3481 (iv) 529
(v) 3249 (vi) 1369 (vii) 5776 (viii) 7921
(ix) 576 (x) 1024 (xi) 3136 (xii) 900

2. Find the number of digits in the square root of each of the following numbers (without any calculation).

(i) 64 (ii) 144 (iii) 4489 (iv) 27225
(v) 390625

3. Find the square root of the following decimal numbers.
   (i) 2.56 (ii) 7.29 (iii) 51.84 (iv) 42.25
   (v) 31.36
4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.
   (i) 402 (ii) 1989 (iii) 3250 (iv) 825
   (v) 4000
5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.
   (i) 525 (ii) 1750 (iii) 252 (iv) 1825
   (v) 6412
6. Find the length of the side of a square whose area is 441 m^2.
7. In a right triangle ABC, $\angle B = 90°$.
   (a) If AB = 6 cm, BC = 8 cm, find AC (b) If AC = 13 cm, BC = 5 cm, find AB
8. A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.
9. There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement.

WHAT HAVE WE DISCUSSED?

1. If a natural number m can be expressed as n^2, where n is also a natural number, then m is a **square number**.
2. All square numbers end with 0, 1, 4, 5, 6 or 9 at units place.
3. Square numbers can only have even number of zeros at the end.
4. **Square root** is the inverse operation of square.
5. There are two integral square roots of a perfect square number.

   Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

   For example, $3^2 = 9$ gives $\sqrt{9} = 3$

CHAPTER 7

Cubes and Cube Roots

7.1 Introduction

This is a story about one of India's great mathematical geniuses, S. Ramanujan. Once another famous mathematician Prof. G.H. Hardy came to visit him in a taxi whose number was 1729. While talking to Ramanujan, Hardy described this number "a dull number". Ramanujan quickly pointed out that 1729 was indeed interesting. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways:

$$1729 = 1728 + 1 = 12^3 + 1^3$$

$$1729 = 1000 + 729 = 10^3 + 9^3$$

1729 has since been known as the Hardy – Ramanujan Number, even though this feature of 1729 was known more than 300 years before Ramanujan.

Hardy – Ramanujan Number

1729 is the smallest Hardy–Ramanujan Number. There are an infinitely many such numbers. Few are 4104 (2, 16; 9, 15), 13832 (18, 20; 2, 24), Check it with the numbers given in the brackets.

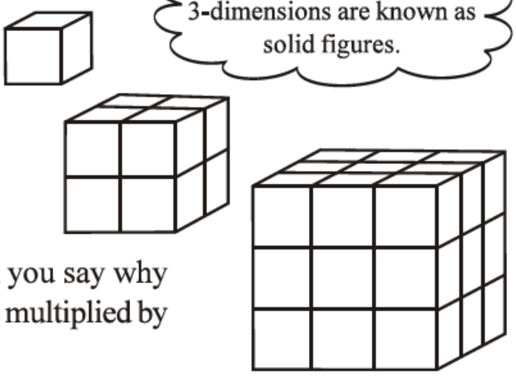
How did Ramanujan know this? Well, he loved numbers. All through his life, he experimented with numbers. He probably found numbers that were expressed as the sum of two squares and sum of two cubes also.

There are many other interesting patterns of cubes. Let us learn about cubes, cube roots and many other interesting facts related to them.

7.2 Cubes

Figures which have 3-dimensions are known as solid figures.

You know that the word 'cube' is used in geometry. A cube is a solid figure which has all its sides equal. How many cubes of side 1 cm will make a cube of side 2 cm?



How many cubes of side 1 cm will make a cube of side 3 cm?

Consider the numbers 1, 8, 27, ...

These are called **perfect cubes or cube numbers**. Can you say why they are named so? Each of them is obtained when a number is multiplied by taking it three times.

We note that $1 = 1 \times 1 \times 1 = 1^3$; $8 = 2 \times 2 \times 2 = 2^3$; $27 = 3 \times 3 \times 3 = 3^3$.
Since $5^3 = 5 \times 5 \times 5 = 125$, therefore 125 is a cube number.

Is 9 a cube number? No, as $9 = 3 \times 3$ and there is no natural number which multiplied by taking three times gives 9. We can see also that $2 \times 2 \times 2 = 8$ and $3 \times 3 \times 3 = 27$. This shows that 9 is not a perfect cube.

The following are the cubes of numbers from 1 to 10.

Table 1

Number	Cube
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
4	$4^3 = 64$
5	$5^3 =$ ____
6	$6^3 =$ ____
7	$7^3 =$ ____
8	$8^3 =$ ____
9	$9^3 =$ ____
10	$10^3 =$ ____

The numbers 729, 1000, 1728 are also perfect cubes.

Complete it.

There are only ten perfect cubes from 1 to 1000. (Check this). How many perfect cubes are there from 1 to 100?

Observe the cubes of even numbers. Are they all even? What can you say about the cubes of odd numbers?

Following are the cubes of the numbers from 11 to 20.

Table 2

Number	Cube
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

We are even, so are our cubes

We are odd so are our cubes

Consider a few numbers having 1 as the one's digit (or unit's). Find the cube of each of them. What can you say about the one's digit of the cube of a number having 1 as the one's digit?

Similarly, explore the one's digit of cubes of numbers ending in 2, 3, 4, ... , etc.

TRY THESE

Find the one's digit of the cube of each of the following numbers.

(i) 3331 (ii) 8888 (iii) 149 (iv) 1005

(v) 1024 (vi) 77 (vii) 5022 (viii) 53

7.2.1 Some interesting patterns

1. Adding consecutive odd numbers

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$

Is it not interesting? How many consecutive odd numbers will be needed to obtain the sum as 10^3?

TRY THESE

Express the following numbers as the sum of odd numbers using the above pattern?

(a) 6^3 (b) 8^3 (c) 7^3

Consider the following pattern.

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$

Using the above pattern, find the value of the following.

(i) $7^3 - 6^3$ (ii) $12^3 - 11^3$ (iii) $20^3 - 19^3$ (iv) $51^3 - 50^3$

2. Cubes and their prime factors

Consider the following prime factorisation of the numbers and their cubes.

each prime factor appears three times in its cubes

Prime factorisation of a number	Prime factorisation of its cube
$4 = 2 \times 2$	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
$6 = 2 \times 3$	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
$15 = 3 \times 5$	$15^3 = 3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$
$12 = 2 \times 2 \times 3$	$12^3 = 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3 \times 2^3 \times 3^3$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Observe that each prime factor of a number appears three times in the prime factorisation of its cube.

Do you remember that $a^m \times b^m = (a \times b)^m$

In the prime factorisation of any number, if each factor appears three times, then, is the number a perfect cube? Think about it. Is 216 a perfect cube?

By prime factorisation, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Each factor appears 3 times. $216 = 2^3 \times 3^3 = (2 \times 3)^3$

$= 6^3$ which is a perfect cube!

Is 729 a perfect cube? $729 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

factors can be grouped in triples

Yes, 729 is a perfect cube.

Now let us check for 500.

Prime factorisation of 500 is $2 \times 2 \times \underline{5 \times 5 \times 5}$.

So, 500 is not a perfect cube.

There are three 5's in the product but only two 2's.

Example 1: Is 243 a perfect cube?

Solution: $243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

In the above factorisation 3×3 remains after grouping the 3's in triplets. Therefore, 243 is not a perfect cube.

TRY THESE

Which of the following are perfect cubes?

1. 400
2. 3375
3. 8000
4. 15625
5. 9000
6. 6859
7. 2025
8. 10648

7.2.2 Smallest multiple that is a perfect cube

Raj made a cuboid of plasticine. Length, breadth and height of the cuboid are 15 cm, 30 cm, 15 cm respectively.

Anu asks how many such cuboids will she need to make a perfect cube? Can you tell?

Raj said, Volume of cuboid is $15 \times 30 \times 15 = 3 \times 5 \times 2 \times 3 \times 5 \times 3 \times 5$

$= 2 \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Since there is only one 2 in the prime factorisation. So we need 2×2, i.e., 4 to make it a perfect cube. Therefore, we need 4 such cuboids to make a cube.

Example 2: Is 392 a perfect cube? If not, find the smallest natural number by which 392 must be multiplied so that the product is a perfect cube.

Solution: $392 = \underline{2 \times 2 \times 2} \times 7 \times 7$

The prime factor 7 does not appear in a group of three. Therefore, 392 is not a perfect cube. To make its a cube, we need one more 7. In that case

$392 \times 7 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7} = 2744$ which is a perfect cube.

Hence the smallest natural number by which 392 should be multiplied to make a perfect cube (2744) is 7.

Example 3: Is 53240 a perfect cube? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube?

Solution: $53240 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11} \times 5$

The prime factor 5 does not appear in a group of three. So, 53240 is not a perfect cube. In the factorisation 5 appears only one time. If we divide the number by 5, then the prime factorisation of the quotient will not contain 5.

So, $53240 \div 5 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

Hence the smallest number by which 53240 should be divided to make it a perfect cube is 5.

The perfect cube in that case is = 10648.

Example 4: Is 1188 a perfect cube? If not, by which smallest natural number should 1188 be divided so that the quotient is a perfect cube?

Solution: $1188 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times 11$

The primes 2 and 11 do not appear in groups of three. So, 1188 is not a perfect cube. In the factorisation of 1188 the prime 2 appears only two times and the prime 11 appears once. So, if we divide 1188 by $2 \times 2 \times 11 = 44$, then the prime factorisation of the quotient will not contain 2 and 11.

Hence the smallest natural number by which 1188 should be divided to make it a perfect cube is 44.

And the resulting perfect cube is $1188 \div 44 = 27$ $(=3^3)$.

Example 5: Is 68600 a perfect cube? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

Solution: We have, $68600 = 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7 \times 7$. In this factorisation, we find that there is no triplet of 5.

So, 68600 is not a perfect cube. To make it a perfect cube we multiply it by 5.

Thus, $68600 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$

$= 343000$, which is a perfect cube.

Observe that 343 is a perfect cube. From Example 5 we know that 343000 is also perfect cube.

THINK, DISCUSS AND WRITE

Check which of the following are perfect cubes. (i) 2700 (ii) 16000 (iii) 64000 (iv) 900 (v) 125000 (vi) 36000 (vii) 21600 (viii) 10,000 (ix) 27000000 (x) 1000. What pattern do you observe in these perfect cubes?

EXERCISE 7.1

1. Which of the following numbers are not perfect cubes?
   (i) 216 (ii) 128 (iii) 1000 (iv) 100
   (v) 46656
2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.
   (i) 243 (ii) 256 (iii) 72 (iv) 675
   (v) 100
3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.
   (i) 81 (ii) 128 (iii) 135 (iv) 192
   (v) 704
4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

7.3 Cube Roots

If the volume of a cube is 125 cm^3, what would be the length of its side? To get the length of the side of the cube, we need to know a number whose cube is 125.

Finding the square root, as you know, is the inverse operation of squaring. Similarly, finding the cube root is the inverse operation of finding cube.

We know that $2^3 = 8$; so we say that the cube root of 8 is 2.

We write $\sqrt[3]{8} = 2$. **The symbol $\sqrt[3]{}$ denotes 'cube-root.'**

Consider the following:

Statement	Inference
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$
$3^3 = 27$	$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$

Statement	Inference
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

7.3.1 Cube root through prime factorisation method

Consider 3375. We find its cube root by prime factorisation:

$$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} = 3^3 \times 5^3 = (3 \times 5)^3$$

Therefore, cube root of 3375 = $\sqrt[3]{3375} = 3 \times 5 = 15$

Similarly, to find $\sqrt[3]{74088}$, we have,

$$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7} = 2^3 \times 3^3 \times 7^3 = (2 \times 3 \times 7)^3$$

Therefore, $\sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

Example 6: Find the cube root of 8000.

Solution: Prime factorisation of 8000 is $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

So, $\sqrt[3]{8000} = 2 \times 2 \times 5 = 20$

Example 7: Find the cube root of 13824 by prime factorisation method.

Solution:

$$13824 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 2^3 \times 2^3 \times 2^3 \times 3^3.$$

Therefore, $\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

THINK, DISCUSS AND WRITE

State true or false: for any integer m, $m^2 < m^3$. Why?

7.3.2 Cube root of a cube number

If you know that the given number is a cube number then following method can be used.

Step 1 Take any cube number say 857375 and start making groups of three digits starting from the right most digit of the number.

857	375
↓	↓
second group	first group

We can estimate the cube root of a given cube number through a step by step process.

We get 375 and 857 as two groups of three digits each.

Step 2 First group, i.e., 375 will give you the one's (or unit's) digit of the required cube root.

The number 375 ends with 5. We know that 5 comes at the unit's place of a number only when it's cube root ends in 5.

So, we get 5 at the unit's place of the cube root.

Step 3 Now take another group, i.e., 857.

We know that $9^3 = 729$ and $10^3 = 1000$. Also, $729 < 857 < 1000$. We take the one's place, of the smaller number 729 as the ten's place of the required cube root. So, we get $\sqrt[3]{857375} = 95$.

Example 8: Find the cube root of 17576 through estimation.

Solution: The given number is 17576.

Step 1 Form groups of three starting from the rightmost digit of 17576.

17 576. In this case one group i.e., 576 has three digits whereas 17 has only two digits.

Step 2 Take 576.

The digit 6 is at its one's place.

We take the one's place of the required cube root as 6.

Step 3 Take the other group, i.e., 17.
Cube of 2 is 8 and cube of 3 is 27. 17 lies between 8 and 27.
The smaller number among 2 and 3 is 2.

The one's place of 2 is 2 itself. Take 2 as ten's place of the cube root of 17576.

Thus, $\sqrt[3]{17576} = 26$ (Check it!)

EXERCISE 7.2

1. Find the cube root of each of the following numbers by prime factorisation method.
   (i) 64 (ii) 512 (iii) 10648 (iv) 27000
   (v) 15625 (vi) 13824 (vii) 110592 (viii) 46656
   (ix) 175616 (x) 91125
2. State true or false.
   (i) Cube of any odd number is even.
   (ii) A perfect cube does not end with two zeros.
   (iii) If square of a number ends with 5, then its cube ends with 25.
   (iv) There is no perfect cube which ends with 8.
   (v) The cube of a two digit number may be a three digit number.
   (vi) The cube of a two digit number may have seven or more digits.
   (vii) The cube of a single digit number may be a single digit number.
3. You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768.

WHAT HAVE WE DISCUSSED?

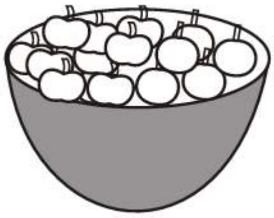
1. Numbers like 1729, 4104, 13832, are known as Hardy – Ramanujan Numbers. They can be expressed as sum of two cubes in two different ways.
2. Numbers obtained when a number is multiplied by itself three times are known as **cube numbers**. For example 1, 8, 27, ... etc.
3. If in the prime factorisation of any number each factor appears three times, then the number is a perfect cube.
4. The symbol $\sqrt[3]{\ }$ denotes cube root. For example $\sqrt[3]{27} = 3$.

Comparing Quantities

CHAPTER
8

0852CH08

8.1 Recalling Ratios and Percentages



We know, ratio means comparing two quantities.

A basket has two types of fruits, say, 20 apples and 5 oranges.

Then, the ratio of the number of oranges to the number of apples = 5 : 20.

The comparison can be done by using fractions as, $\frac{5}{20} = \frac{1}{4}$

The number of oranges is $\frac{1}{4}$th the number of apples. In terms of ratio, this is 1 : 4, read as, "1 is to 4"

OR

Number of apples to number of oranges = $\frac{20}{5} = \frac{4}{1}$ which means, the number of apples is 4 times the number of oranges. This comparison can also be done using percentages.

There are 5 oranges out of 25 fruits. So percentage of oranges is $\frac{5}{25} \times \frac{4}{4} = \frac{20}{100} = 20\%$ [Denominator made 100].	OR	By unitary method: Out of 25 fruits, number of oranges are 5. So out of 100 fruits, number of oranges $= \frac{5}{25} \times 100 = 20$.

Since  contains only apples and oranges,

So, percentage of apples + percentage of oranges = 100

or percentage of apples + 20 = 100

or percentage of apples = 100 – 20 = 80

Thus the basket has 20% oranges and 80% apples.

Example 1: A picnic is being planned in a school for Class VII. Girls are 60% of the total number of students and are 18 in number.

The picnic site is 55 km from the school and the transport company is charging at the rate of ₹ 12 per km. The total cost of refreshments will be ₹ 4280.

Can you tell.

1. The ratio of the number of girls to the number of boys in the class?
2. The cost per head if two teachers are also going with the class?
3. If their first stop is at a place 22 km from the school, what per cent of the total distance of 55 km is this? What per cent of the distance is left to be covered?

Solution:

1. To find the ratio of girls to boys.

   Ashima and John came up with the following answers.

   They needed to know the number of boys and also the total number of students.

Ashima did this

Let the total number of students be x. 60% of x is girls.

Therefore, 60% of x = 18

$$\frac{60}{100} \times x = 18$$

or, $x = \frac{18 \times 100}{60} = 30$

Number of students = 30.

OR

John used the unitary method

There are 60 girls out of 100 students.

There is one girl out of $\frac{100}{60}$ students.

So, 18 girls are out of how many students?

Number of students = $\frac{100}{60} \times 18$

$= 30$

So, the number of boys = 30 – 18 = 12.

Hence, ratio of the number of girls to the number of boys is 18 : 12 or $\frac{18}{12} = \frac{3}{2}$.

$\frac{3}{2}$ is written as 3 : 2 and read as 3 is to 2.

2. To find the cost per person.

Transportation charge = Distance both ways × Rate

= ₹ (55 × 2) × 12

= ₹ 110 × 12 = ₹ 1320

Total expenses = Refreshment charge + Transportation charge

= ₹ 4280 + ₹ 1320

= ₹ 5600

Total number of persons = 18 girls + 12 boys + 2 teachers

= 32 persons

Ashima and John then used unitary method to find the cost per head.

For 32 persons, amount spent would be ₹ 5600.

The amount spent for 1 person = ₹ $\frac{5600}{32}$ = ₹ 175.

3. The distance of the place where first stop was made = 22 km.

To find the percentage of distance:

Ashima used this method:

$$\frac{22}{55} = \frac{22}{55} \times \frac{100}{100} = 40\%$$

[She is multiplying the ratio by $\frac{100}{100} = 1$ and converting to percentage.]

OR

John used the unitary method:

Out of 55 km, 22 km are travelled.

Out of 1 km, $\frac{22}{55}$ km are travelled.

Out of 100 km, $\frac{22}{55} \times 100$ km are travelled.

That is 40% of the total distance is travelled.

Both came out with the same answer that the distance from their school of the place where they stopped at was 40% of the total distance they had to travel.

Therefore, the percent distance left to be travelled = 100% – 40% = 60%.

TRY THESE

In a primary school, the parents were asked about the number of hours they spend per day in helping their children to do homework. There were 90 parents who helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours. The distribution of parents according to the time for which, they said they helped is given in the adjoining figure ; 20% helped for more than $1\frac{1}{2}$ hours per day;

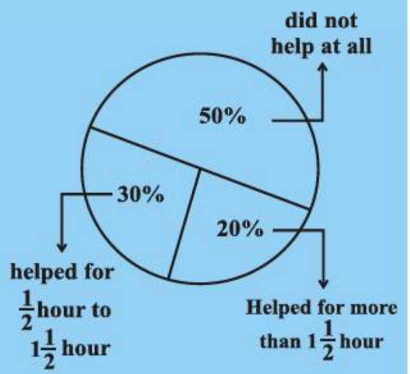
30% helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours; 50% did not help at all.

Using this, answer the following:

(i) How many parents were surveyed?

(ii) How many said that they did not help?

(iii) How many said that they helped for more than $1\frac{1}{2}$ hours?



EXERCISE 8.1

1. Find the ratio of the following.
   (a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.
   (b) 5 m to 10 km (c) 50 paise to ₹ 5
2. Convert the following ratios to percentages.
   (a) 3 : 4 (b) 2 : 3
3. 72% of 25 students are interested in mathematics. How many are not interested in mathematics?
4. A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?
5. If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?

6. If 60% people in a city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people is 50 lakh, find the exact number who like each type of game.

8.2 Finding the Increase or Decrease Per cent

We often come across such information in our daily life as.

(i) 25% off on marked prices (ii) 10% hike in the price of petrol

Let us consider a few such examples.

Example 2: The price of a scooter was ₹ 34,000 last year. It has increased by 20% this year. What is the price now?

Solution:

Amita said that she would first find the increase in the price, which is 20% of ₹ 34,000, and then find the new price.

20% of ₹ 34000 = ₹ $\frac{20}{100} \times 34000$

= ₹ 6800

New price = Old price + Increase

= ₹ 34,000 + ₹ 6,800

= ₹ 40,800

OR

Sunita used the unitary method.
20% increase means,
₹ 100 increased to ₹ 120.
So, ₹ 34,000 will increase to?

Increased price = ₹ $\frac{120}{100} \times 34000$

= ₹ 40,800

Similarly, a percentage decrease in price would imply finding the actual decrease followed by its subtraction the from original price.

Suppose in order to increase its sale, the price of scooter was decreased by 5%. Then let us find the price of scooter.

Price of scooter = ₹ 34000

Reduction = 5% of ₹ 34000

= ₹ $\frac{5}{100} \times 34000$ = ₹ 1700

New price = Old price – Reduction

= ₹ 34000 – ₹ 1700 = ₹ 32300

We will also use this in the next section of the chapter.

8.3 Finding Discounts

Discount is a reduction given on the Marked Price (MP) of the article.

This is generally given to attract customers to buy goods or to promote sales of the goods. You can find the discount by subtracting its sale price from its marked price.

So, Discount = Marked price – Sale price

Example 3: An item marked at ₹ 840 is sold for ₹ 714. What is the discount and discount %?

Solution: Discount = Marked Price – Sale Price

= ₹ 840 – ₹ 714

= ₹ 126

Since discount is on marked price, we will have to use marked price as the base.

On marked price of ₹ 840, the discount is ₹ 126.
On MP of ₹ 100, how much will the discount be?

$$\text{Discount} = \frac{126}{840} \times 100\% = 15\%$$

You can also find discount when discount % is given.

Example 4: The list price of a frock is ₹ 220. A discount of 20% is announced on sales. What is the amount of discount on it and its sale price.

Solution: Marked price is same as the list price.
20% discount means that on ₹ 100 (MP), the discount is ₹ 20.

By unitary method, on ₹1 the discount will be ₹ $\frac{20}{100}$.

On ₹ 220, discount = ₹ $\frac{20}{100} \times 220$ = ₹ 44

The sale price = (₹ 220 – ₹ 44) or ₹ 176

Rehana found the sale price like this —

A discount of 20% means for a MP of ₹ 100, discount is ₹ 20. Hence the sale price is ₹ 80. Using unitary method, when MP is ₹ 100, sale price is ₹ 80;

When MP is ₹ 1, sale price is ₹ $\frac{80}{100}$.

Hence when MP is ₹ 220, sale price = ₹ $\frac{80}{100} \times 220$ = ₹ 176.

Even though the discount was not found, I could find the sale price directly.

TRY THESE

1. A shop gives 20% discount. What would the sale price of each of these be?
   (a) A dress marked at ₹ 120 (b) A pair of shoes marked at ₹ 750
   (c) A bag marked at ₹ 250
2. A table marked at ₹ 15,000 is available for ₹ 14,400. Find the discount given and the discount per cent.
3. An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

8.3.1 Estimation in percentages

Your bill in a shop is ₹ 577.80 and the shopkeeper gives a discount of 15%. How would you estimate the amount to be paid?

(i) Round off the bill to the nearest tens of ₹ 577.80, i.e., to ₹ 580.

(ii) Find 10% of this, i.e., ₹ $\frac{10}{100} \times 580 =$ ₹ 58 .

(iii) Take half of this, i.e., $\frac{1}{2} \times 58 =$ ₹ 29 .

(iv) Add the amounts in (ii) and (iii) to get ₹ 87.

You could therefore reduce your bill amount by ₹ 87 or by about ₹ 85, which will be ₹ 495 approximately.

1. Try estimating 20% of the same bill amount. 2. Try finding 15% of ₹ 375.

8.4 Prices Related to Buying and Selling (Profit and Loss)

For the school fair (mela) I am going to put a stall of lucky dips. I will charge ₹ 10 for one lucky dip but I will buy items which are worth ₹ 5.

So you are making a profit of 100%.

No, I will spend ₹ 3 on paper to wrap the gift and tape. So my expenditure is ₹ 8.

This gives me a profit of ₹ 2, which is, $\frac{2}{8} \times 100\% = 25\%$ only.

Sometimes when an article is bought, some additional expenses are made while buying or before selling it. These expenses have to be included in the cost price.

These expenses are sometimes referred to as **overhead charges**. These may include expenses like amount spent on repairs, labour charges, transportation etc.

8.4.1 Finding cost price/selling price, profit %/loss%

Example 5: Sohan bought a second hand refrigerator for ₹ 2,500, then spent ₹ 500 on its repairs and sold it for ₹ 3,300. Find his loss or gain per cent.

Solution: Cost Price (CP) = ₹ 2500 + ₹ 500 (overhead expenses are added to give CP)

= ₹ 3000

Sale Price (SP) = ₹ 3300

As SP > CP, he made a profit = ₹ 3300 – ₹ 3000 = ₹ 300

His profit on ₹ 3,000, is ₹ 300. How much would be his profit on ₹ 100?

Profit $= \frac{300}{3000} \times 100\% = \frac{30}{3}\% = 10\%$

$$P\% = \frac{P}{CP} \times 100$$

TRY THESE

1. Find selling price (SP) if a profit of 5% is made on
   (a) a cycle of ₹ 700 with ₹ 50 as overhead charges.
   (b) a lawn mower bought at ₹ 1150 with ₹ 50 as transportation charges.
   (c) a fan bought for ₹ 560 and expenses of ₹ 40 made on its repairs.

Example 6: A shopkeeper purchased 200 bulbs for ₹ 10 each. However 5 bulbs were fused and had to be thrown away. The remaining were sold at ₹ 12 each. Find the gain or loss %.

Solution: Cost price of 200 bulbs = ₹ 200 × 10 = ₹ 2000

5 bulbs were fused. Hence, number of bulbs left = 200 – 5 = 195

These were sold at ₹ 12 each.

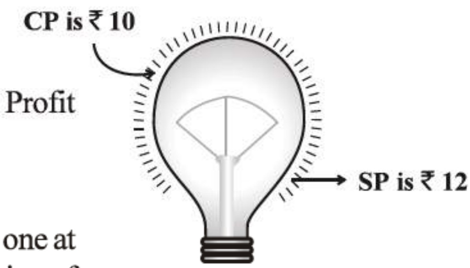
The SP of 195 bulbs = ₹ 195 × 12 = ₹ 2340

He obviously made a profit (as SP > CP).

Profit = ₹ 2340 – ₹ 2000 = ₹ 340

On ₹ 2000, the profit is ₹ 340. How much profit is made on ₹ 100? Profit

$$= \frac{340}{2000} \times 100\% = 17\%.$$



Example 7: Meenu bought two fans for ₹ 1200 each. She sold one at a loss of 5% and the other at a profit of 10%. Find the selling price of each. Also find out the total profit or loss.

Solution: Overall CP of each fan = ₹ 1200. One is sold at a loss of 5%.

This means if CP is ₹ 100, SP is ₹ 95.

Therefore, when CP is ₹ 1200, then SP = ₹ $\frac{95}{100} \times 1200$ = ₹ 1140

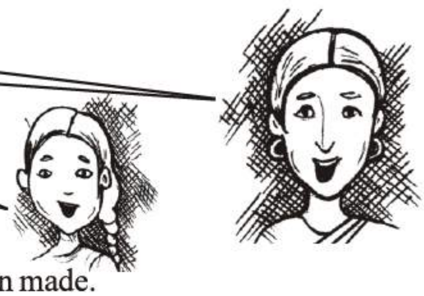
Also second fan is sold at a profit of 10%.

It means, if CP is ₹ 100, SP is ₹ 110.

Therefore, when CP is ₹ 1200, then SP = ₹ $\frac{110}{100} \times 1200$ = ₹ 1320

Was there an overall loss or gain?

We need to find the combined CP and SP to say whether there was an overall profit or loss.



Total CP = ₹ 1200 + ₹ 1200 = ₹ 2400

Total SP = ₹ 1140 + ₹ 1320 = ₹ 2460

Since total SP > total CP, a profit of ₹ (2460 – 2400) or ₹ 60 has been made.

TRY THESE

1. A shopkeeper bought two TV sets at ₹ 10,000 each. He sold one at a profit 10% and the other at a loss of 10%. Find whether he made an overall profit or loss.

8.5 GST(Goods and Services Tax) based Questions

The teacher showed the class a bill in which the following heads were written.

Bill No.			Date	
Menu				
S.No.	Item	Quantity	Rate	Amount
		Bill amount + GST (5%)		
	Total			

Sales tax (ST) is charged by the government on the sale of an item. It is collected by the shopkeeper from the customer and given to the government. This is, therefore, always on the selling price of an item and is added to the value of the bill. There is another type of tax which is included in the prices known as **Value Added Tax (VAT)**.

From July 1, 2017, Government of India introduced GST which stands for Goods and Services Tax which is levied on supply of goods or services or both.

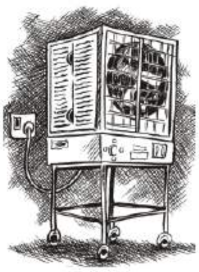
Example 8: (**Finding Sales Tax**) The cost of a pair of roller skates at a shop was ₹ 450. The GST charged was 5%. Find the bill amount.

Solution: On ₹ 100, the GST paid was ₹ 5.

On ₹ 450, the GST paid would be = ₹ $\frac{5}{100} \times 450$

= ₹ 22.50

Bill amount = Cost of item + GST = ₹ 450 + ₹ 22.50 = ₹ 472.50.



Example 9: Waheeda bought an air cooler for ₹ 2240 including GST of 12%. Find the price of the air cooler before GST was added.

Solution: The price includes the GST, i.e., Goods and Service Tax. Thus, a 12% GST means if the price without GST is ₹ 100 then price including GST is ₹ 112.

Now, when price including GST is ₹ 112, original price is ₹ 100.

Hence when price including GST is ₹ 2240, the original price = ₹ $\frac{100}{112} \times 2240$ = ₹ 2000.

Example 10: Salim bought an article for ₹ 784 which included GST of 12% . What is the price of the article before GST was added?

Solution: Let original price of the article be ₹ 100. GST = 12%.

Price after GST is included = ₹ (100+12) = ₹ 112

When the selling price is ₹ 112 then original price = ₹ 100.

When the selling price is ₹ 784, then original price = ₹ $\frac{100}{12} \times 784$ = ₹ 700

THINK, DISCUSS AND WRITE

1. Two times a number is a 100% increase in the number. If we take half the number what would be the decrease in per cent?
2. By what per cent is ₹ 2,000 less than ₹ 2,400? Is it the same as the per cent by which ₹ 2,400 is more than ₹ 2,000?

EXERCISE 8.2

1. A man got a 10% increase in his salary. If his new salary is ₹ 1,54,000, find his original salary.

2. On Sunday 845 people went to the Zoo. On Monday only 169 people went. What is the per cent decrease in the people visiting the Zoo on Monday?

3. A shopkeeper buys 80 articles for ₹ 2,400 and sells them for a profit of 16%. Find the selling price of one article.

4. The cost of an article was ₹ 15,500. ₹ 450 were spent on its repairs. If it is sold for a profit of 15%, find the selling price of the article.

5. A VCR and TV were bought for ₹ 8,000 each. The shopkeeper made a loss of 4% on the VCR and a profit of 8% on the TV. Find the gain or loss percent on the whole transaction.

6. During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1450 and two shirts marked at ₹ 850 each?

7. A milkman sold two of his buffaloes for ₹ 20,000 each. On one he made a gain of 5% and on the other a loss of 10%. Find his overall gain or loss. (**Hint:** Find CP of each)

8. The price of a TV is ₹ 13,000. The GST charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it.

9. Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is ₹ 1,600, find the marked price.

10. I purchased a hair-dryer for ₹ 5,400 including 18% GST. Find the price before GST was added.

11. An article was purchased for ₹ 1239 including GST of 18%. Find the price of the article before GST was added?

8.6 Compound Interest

You might have come across statements like "one year interest for FD (fixed deposit) in the bank @ 9% per annum" or 'Savings account with interest @ 5% per annum'.

Interest is the extra money paid by institutions like banks or post offices on money deposited (kept) with them. Interest is also paid by people when they borrow money. We already know how to calculate **Simple Interest**.

Example 10: A sum of ₹ 10,000 is borrowed at a rate of interest 15% per annum for 2 years. Find the simple interest on this sum and the amount to be paid at the end of 2 years.

Solution: On ₹ 100, interest charged for 1 year is ₹ 15.

So, on ₹ 10,000, interest charged $= \frac{15}{100} \times 10000 =$ ₹ 1500

Interest for 2 years = ₹ 1500 × 2 = ₹ 3000

Amount to be paid at the end of 2 years = Principal + Interest

= ₹ 10000 + ₹ 3000 = ₹ 13000

TRY THESE

Find interest and amount to be paid on ₹ 15000 at 5% per annum after 2 years.

My father has kept some money in the post office for 3 years. Every year the money increases as more than the previous year.

We have some money in the bank. Every year some interest is added to it, which is shown in the passbook. This interest is not the same, each year it increases.

Normally, the interest paid or charged is never simple. The interest is calculated on the amount of the previous year. This is known as interest compounded or **Compound Interest** (C.I.).

Let us take an example and find the interest year by year. Each year our sum or principal changes.

Calculating Compound Interest

A sum of ₹ 20,000 is borrowed by Heena for 2 years at an interest of 8% compounded annually. Find the Compound Interest (C.I.) and the amount she has to pay at the end of 2 years.

Aslam asked the teacher whether this means that they should find the interest year by year. The teacher said 'yes', and asked him to use the following steps :

1. Find the Simple Interest (S.I.) for one year.
   Let the principal for the first year be P_1. Here, P_1 = ₹ 20,000

   SI_1 = SI at 8% p.a. for 1st year = ₹ $\frac{20000 \times 8}{100}$ = ₹ 1600

2. Then find the amount which will be paid or received. This becomes principal for the next year.

   Amount at the end of 1st year = $P_1 + SI_1$ = ₹ 20000 + ₹ 1600

   = ₹ 21600 = P_2 (Principal for 2nd year)

3. Again find the interest on this sum for another year.

$$SI_2 = \text{SI at 8\% p.a. for 2nd year} = ₹\ \frac{21600 \times 8}{100}$$

$$= ₹\ 1728$$

4. Find the amount which has to be paid or received at the end of second year.

$$\text{Amount at the end of 2nd year} = P_2 + SI_2$$

$$= ₹\ 21600 + ₹\ 1728$$

$$= ₹\ 23328$$

$$\text{Total interest given} = ₹\ 1600 + ₹\ 1728$$

$$= ₹\ 3328$$

Reeta asked whether the amount would be different for simple interest. The teacher told her to find the interest for two years and see for herself.

$$\text{SI for 2 years} = ₹\ \frac{20000 \times 8 \times 2}{100} = ₹\ 3200$$

Reeta said that when compound interest was used Heena would pay ₹ 128 more.

Let us look at the difference between simple interest and compound interest. We start with ₹ 100. Try completing the chart.

		Under Simple Interest	Under Compound Interest
First year	Principal	₹ 100.00	₹ 100.00
	Interest at 10%	₹ 10.00	₹ 10.00
	Year-end amount	₹ 110.00	₹ 110.00
Second year	Principal	₹ 100.00	₹ 110.00
	Interest at 10%	₹ 10.00	₹ 11.00
	Year-end amount	₹(110 + 10) = ₹ 120	₹ 121.00
Third year	Principal	₹ 100.00	₹ 121.00
	Interest at 10%	₹ 10.00	₹ 12.10
	Year-end amount	₹(120 + 10) = ₹ 130	₹ 133.10

Which means you pay interest on the interest accumulated till then!

Note that in 3 years,

Interest earned by Simple Interest = ₹ (130 – 100) = ₹ 30, whereas,

Interest earned by Compound Interest = ₹ (133.10 – 100) = ₹ 33.10

Note also that the Principal remains the same under Simple Interest, while it changes year after year under compound interest.

8.7 Deducing a Formula for Compound Interest

Zubeda asked her teacher, 'Is there an easier way to find compound interest?' The teacher said 'There is a shorter way of finding compound interest. Let us try to find it.'

Suppose P_1 is the sum on which interest is compounded annually at a rate of R% per annum.

Let P_1 = ₹ 5000 and R = 5%. Then by the steps mentioned above

1. $SI_1 = ₹\ \frac{5000 \times 5 \times 1}{100}$ or $SI_1 = ₹\ \frac{P_1 \times R \times 1}{100}$

so, $A_1 = ₹\ 5000 + \frac{5000 \times 5 \times 1}{100}$ or $A_1 = P_1 + SI_1 = P_1 + \frac{P_1 R}{100}$

$= ₹\ 5000\left(1+\frac{5}{100}\right) = P_2$ $\qquad$ $= P_1\left(1+\frac{R}{100}\right) = P_2$

2. $SI_2 = ₹\ 5000\left(1+\frac{5}{100}\right) \times \frac{5 \times 1}{100}$ or $SI_2 = \frac{P_2 \times R \times 1}{100}$

$= ₹\ \frac{5000 \times 5}{100}\left(1+\frac{5}{100}\right)$ $\qquad$ $= P_1\left(1+\frac{R}{100}\right) \times \frac{R}{100}$

$$= \frac{P_1 R}{100}\left(1+\frac{R}{100}\right)$$

$A_2 = ₹\ 5000\left(1+\frac{5}{100}\right) + ₹\ \frac{5000 \times 5}{100}\left(1+\frac{5}{100}\right)$ $\qquad$ $A_2 = P_2 + SI_2$

$= ₹\ 5000\left(1+\frac{5}{100}\right)\left(1+\frac{5}{100}\right)$ $\qquad$ $= P_1\left(1+\frac{R}{100}\right) + P_1\frac{R}{100}\left(1+\frac{R}{100}\right)$

$= ₹\ 5000\left(1+\frac{5}{100}\right)^2 = P_3$ $\qquad$ $= P_1\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$

$$= P_1\left(1+\frac{R}{100}\right)^2 = P_3$$

Proceeding in this way the amount at the end of *n* years will be

$$A_n = P_1\left(1+\frac{R}{100}\right)^n$$

Or, we can say $\qquad A = P\left(1+\frac{R}{100}\right)^n$

So, Zubeda said, but using this we get only the formula for the amount to be paid at the end of n years, and not the formula for compound interest.

Aruna at once said that we know CI = A – P, so we can easily find the compound interest too.

Example 11: Find CI on ₹ 12600 for 2 years at 10% per annum compounded annually.

Solution: We have, $A = P\left(1+\frac{R}{100}\right)^n$, where Principal (P) = ₹ 12600, Rate (R) = 10, Number of years (n) = 2

$$= ₹\ 12600\left(1+\frac{10}{100}\right)^2 = ₹\ 12600\left(\frac{11}{10}\right)^2$$

$$= ₹\ 12600 \times \frac{11}{10} \times \frac{11}{10} = ₹\ 15246$$

CI = A – P = ₹ 15246 – ₹ 12600 = ₹ 2646

TRY THESE

1. Find CI on a sum of ₹ 8000 for 2 years at 5% per annum compounded annually.

8.8 Rate Compounded Annually or Half Yearly (Semi Annually)

You may want to know why 'compounded annually' was mentioned after 'rate'. Does it mean anything?

It does, because we can also have interest rates compounded half yearly or quarterly. Let us see what happens to ₹ 100 over a period of one year if an interest is compounded annually or half yearly.

Time period and rate when interest not compounded annually

The time period after which the interest is added each time to form a new principal is called the **conversion period**. When the interest is compounded half yearly, there are two conversion periods in a year each after 6 months. In such situations, the half yearly rate will be half of the annual rate. What will happen if interest is compounded quarterly? In this case, there are 4 conversion periods in a year and the quarterly rate will be one-fourth of the annual rate.

P = ₹ 100 at 10% per annum compounded annually	P = ₹ 100 at 10% per annum compounded half yearly
The time period taken is 1 year	The time period is 6 months or $\frac{1}{2}$ year
$I = ₹\ \frac{100 \times 10 \times 1}{100}$ = Rs 10	$I = ₹\ \frac{100 \times \boxed{10 \times \frac{1}{2}}}{100} = ₹\ 5$ (Rate becomes half)
A = ₹ 100 + ₹ 10 = ₹ 110	A = ₹ 100 + ₹ 5 = ₹ 105 Now for next 6 months the P = ₹ 105
	So, $I = ₹\ \frac{105 \times 10 \times \frac{1}{2}}{100} = ₹\ 5.25$ and A = ₹ 105 + ₹ 5.25 = ₹ 110.25

Do you see that, if interest is compounded half yearly, we compute the interest two times. So time period becomes twice and rate is taken half.

TRY THESE

Find the time period and rate for each .

1. A sum taken for $1\frac{1}{2}$ years at 8% per annum is compounded half yearly.
2. A sum taken for 2 years at 4% per annum compounded half yearly.

THINK, DISCUSS AND WRITE

A sum is taken for one year at 16% p.a. If interest is compounded after every three months, how many times will interest be charged in one year?

Example 12: What amount is to be repaid on a loan of ₹ 12000 for $1\frac{1}{2}$ years at 10% per annum compounded half yearly.

Solution:

Principal for first 6 months = ₹ 12,000	**Principal for first 6 months = ₹ 12,000**
There are 3 half years in $1\frac{1}{2}$ years. Therefore, compounding has to be done 3 times. Rate of interest = half of 10% = 5% half yearly $A = P\left(1+\frac{R}{100}\right)^n$ $= ₹\ 12000\left(1+\frac{5}{100}\right)^3$ $= ₹\ 12000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$ = ₹ 13,891.50	Time = 6 months = $\frac{6}{12}$ year = $\frac{1}{2}$ year Rate = 10% $I = ₹\ \frac{12000 \times 10 \times \frac{1}{2}}{100} = ₹\ 600$ A = P + I = ₹ 12000 + ₹ 600 = ₹12600. It is principal for next 6 months. $I = ₹\ \frac{12600 \times 10 \times \frac{1}{2}}{100} = ₹\ 630$ Principal for third period = ₹ 12600 + ₹ 630 = ₹ 13,230. $I = ₹\ \frac{13230 \times 10 \times \frac{1}{2}}{100} = ₹\ 661.50$ A = P + I = ₹ 13230 + ₹ 661.50 = ₹ 13,891.50

TRY THESE

Find the amount to be paid

1. At the end of 2 years on ₹ 2,400 at 5% per annum compounded annually.
2. At the end of 1 year on ₹ 1,800 at 8% per annum compounded quarterly.

Example 13: Find CI paid when a sum of ₹ 10,000 is invested for 1 year and 3 months at $8\frac{1}{2}$% per annum compounded annually.

Solution: Mayuri first converted the time in years.

$$1 \text{ year } 3 \text{ months} = 1\frac{3}{12} \text{ year} = 1\frac{1}{4} \text{ years}$$

Mayuri tried putting the values in the known formula and came up with:

$$A = ₹\ 10000 \left(1 + \frac{17}{200}\right)^{1\frac{1}{4}}$$

Now she was stuck. She asked her teacher how would she find a power which is fractional? The teacher then gave her a hint:

Find the amount for the whole part, i.e., 1 year in this case. Then use this as principal to get simple interest for $\frac{1}{4}$ year more. Thus,

$$A = ₹\ 10000 \left(1 + \frac{17}{200}\right)$$

$$= ₹\ 10000 \times \frac{217}{200} = ₹\ 10,850$$

Now this would act as principal for the next $\frac{1}{4}$ year. We find the SI on ₹ 10,850 for $\frac{1}{4}$ year.

$$SI = ₹\ \frac{10850 \times \frac{1}{4} \times 17}{100 \times 2}$$

$$= ₹\ \frac{10850 \times 1 \times 17}{800} = ₹\ 230.56$$

Interest for first year = ₹ 10850 – ₹ 10000 = ₹ 850

And, interest for the next $\frac{1}{4}$ year = ₹ 230.56

Therefore, total compound Interest = 850 + 230.56 = ₹ 1080.56.

8.9 Applications of Compound Interest Formula

There are some situations where we could use the formula for calculation of amount in CI. Here are a few.

(i) Increase (or decrease) in population.

(ii) The growth of a bacteria if the rate of growth is known.

(iii) The value of an item, if its price increases or decreases in the intermediate years.

Example 14: The population of a city was 20,000 in the year 1997. It increased at the rate of 5% p.a. Find the population at the end of the year 2000.

Solution: There is 5% increase in population every year, so every new year has new population. Thus, we can say it is increasing in compounded form.

Population in the beginning of 1998 = 20000 (we treat this as the principal for the 1st year)

$$\text{Increase at } 5\% = \frac{5}{100} \times 20000 = 1000$$

$$\text{Population in } 1999 = 20000 + 1000 = 21000$$

Treat as the Principal for the 2nd year.

$$\text{Increase at } 5\% = \frac{5}{100} \times 21000 = 1050$$

$$\text{Population in } 2000 = 21000 + 1050$$

$$= 22050$$

Treat as the Principal for the 3rd year.

$$\text{Increase at } 5\% = \frac{5}{100} \times 22050$$

$$= 1102.5$$

At the end of 2000 the population = 22050 + 1102.5 = 23152.5

or, $$\text{Population at the end of } 2000 = 20000 \left(1 + \frac{5}{100}\right)^3$$

$$= 20000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= 23152.5$$

So, the estimated population = 23153.