

## 26. Laws of Thermodynamics

### Short Answer

#### Answer.1

No, the internal energy of the system necessarily should not increase if heat is added to it.

#### Explanation

1. Internal energy is defined as the sum of the kinetic energy and potential energy of molecules of the system. It includes only the energy associated with the random motion of molecules of the system.

2. Random motion of molecules is associated with the temperature of the system. Thus, any change in temperature will change the internal energy of the system.

3. Change in internal energy is given as

$$\Delta U = C_v \Delta T$$

Where  $\Delta U$  = change in internal energy

$C_v$  = molar specific heat at constant volume

$\Delta T$  = change in temperature.

4. If  $\Delta T = 0$  then,  $\Delta U$  will also be zero.

5. In the isothermal process, where the change in temperature is zero,  $\Delta U$  is also zero.

6. According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

7. Since for an isothermal process  $\Delta T = 0$ , therefore  $\Delta U = 0$ . So, from the first law of thermodynamics  $\Delta Q = \Delta W$ .

8. For an isothermal process, heat supplied to the system is used up entirely in doing work.

### **Answer.2**

No, the internal energy of a system will not necessarily increase if its temperature is increased.

### **Explanation**

1. Internal energy is defined as the sum of the kinetic energy and potential energy of molecules of the system. It includes only the energy associated with the random motion of molecules of the system.
2. The important thing about internal energy is that it depends only on the state of the system, not how that state was achieved.
3. Thus, the internal energy of a given mass of gas depends on its state described by specific values of pressure, volume, and temperature.
4. The internal energy of ideal gas depends on only on temperature.
5. For system other than ideal gas, internal energy depends on pressure, volume and temperature combined.

### **Answer.3**

1. Work done on the gas will be zero.

### **Explanation**

Atmospheric pressure decreases, as we increase the height.

So, when the cylinder is lifted from the first floor to second-floor atmospheric pressure on the cylinder will decrease. Hence, the volume of gas inside the cylinder will increase. In the expansion process, work is always done by the gas. So, in such case, no work will be done on the gas. But gas will do work.

2. Work done by the gas will be  $P\Delta V$ .

### **Explanation**

When the cylinder is lifted from the first floor to second-floor atmospheric pressure on the cylinder will decrease. Hence the volume of gas inside the cylinder

will increase. This means gas expands. In the expansion, process work is done by the gas.

We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area  $\times$  displacement

Therefore,

Work done = pressure  $\times$  volume

Let change in the volume of gas =  $\Delta V$

The pressure at which gas expands = P

Thus, work done by the gas W

$$W = P\Delta V$$

3. Change in internal energy and temperature will depend on the walls of the container, whether they are insulating or conducting. So, we cannot comment about internal energy and temperature unless we know the nature of the wall.

#### **Answer.4**

1. Work done by the force on the block is  $F \times d$ .

#### **Explanation**

Given

Force = F

Displacement due to force = d

We know that

Work done = force  $\times$  displacement  $\times$   $\cos \theta$

$\theta$  in our case is zero as displacement is in the direction of the force.

Work done by the force = F  $\times$  d  $\times$   $\cos 0$

= F  $\times$  d ( $\because \cos 0 = 1$ )

∴ Work done by the force on the block is  $F \times d$ .

2. Internal energy will not change because of work done by the force on the block.

### **Explanation**

Internal energy is defined as the sum of the kinetic energy and potential energy of molecules of the system. It includes only the energy associated with the random motion of molecules of the system.

If the block moves as a whole system with some velocity, the kinetic energy of the box is not to be included in internal energy.

### **Answer.5**

Energy transferred to the gas is heat energy.

### **Explanation**

Since the outer surface of the gas is rubbed and not displaced/shaken, no work will be done on the gas.

We know that,

Work done = force  $\times$  displacement

Force = pressure  $\times$  area

∴ work done = pressure  $\times$  area  $\times$  displacement

So, work will only be done when there is either displacement or the gas is expanded/compressed.

The cylinder and gas become warm because of the heat generated by friction between the polishing machine and surface of the cylinder. This heat will warm up the gas and outer surface of the cylinder.

### **Answer.6**

Our hands become warm when we rub them due to the friction between our hands. Friction between our hands supplies heat to the hands.

### **Answer.7**

Work is done when there is displacement in the body.

Work done = force  $\times$  displacement

As the bottle is shaken vigorously, that means the bottle is displaced from its position. And this displacement is due to external force. Therefore, work is done on the bottle. Due to this work, the temperature of the liquid increases. No external heat is supplied to the bottle and liquid.

### **Answer.8**

Work done by the system is neither necessarily zero nor nonzero when the final volume is equal to the initial volume.

### **Explanation**

We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area  $\times$  displacement

Therefore,

Work done = pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_2 - V_1$

Pressure = P

Thus, work done by the system W at constant pressure

$$W = P\Delta V$$

But if  $V_2 = V_1$  then,  $\Delta V = 0$

Work done by the system  $W = 0$

So, in an isobaric process (where pressure remains constant) work done by the system will be zero if initial and final volume are equal.

But we also know that in the cyclic process the system returns to its initial state. But still work done is not zero. In fact, in the cyclic process since the system returns to its initial state, internal energy becomes zero. This is because internal energy is a state variable. It depends on initial and final state only. And if initial and final state becomes equal, then change in internal energy will be zero.

According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

In cyclic process  $\Delta U = 0$ .

So, from the first law of thermodynamics  $\Delta Q = \Delta W$  i.e. heat supplied to the system is converted entirely into work in a cyclic process.

### **Answer.9**

Yes, work can be done by a system without changing its volume if the process is cyclic.

### **Explanation**

1. In cyclic process, the system returns to its initial state. Therefore, initial volume becomes equal to final volume and hence  $\Delta V = 0$ .

2. Also, since system returns to initial state, the internal energy of the system remains same.

3. This is because internal energy is a state variable. It depends on initial and final state only. And if initial and final state becomes equal, then change in internal energy will be zero.

4. So, for cyclic process  $\Delta V = 0$  and  $\Delta U = 0$ .

5. According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

6. So, from the first law of thermodynamics  $\Delta Q = \Delta W$  i.e. heat supplied to the system is converted entirely into work in a cyclic process.

7. Therefore, work can be done by a system without changing its volume.

### **Answer.10**

Yes, the internal energy will also be doubled.

### **Explanation**

1. Internal energy for an ideal gas is given as

$$U = nC_v T$$

Where  $U$  = internal energy

$N$  = number of moles

$C_v$  = molar specific heat at constant volume

$T$  = temperature

2. Since the ideal gas is continuously pumped into rigid container number of moles are also increasing.

3. When the pressure becomes doubled, the number of moles also gets doubled.

4. It is given that the temperature remains constant. So internal energy depends only on the number of moles.

5. Internal energy will also be doubled as the number of moles is getting doubled.

### **Answer.11**

We know that pressure inside the tyre is greater than the atmospheric pressure of surroundings. So, when a tyre bursts there is an adiabatic expansion of the air (adiabatic because no heat is either supplied or released during expansion). In the expansion process, work is done by the gas. Now work done in an adiabatic process is given as

$$W = \frac{\mu R(T_1 - T_2)}{\gamma - 1} \dots\dots (i)$$

Where  $\mu$ =number of moles

R=gas constant

$T_1$ =initial temperature

$T_2$ =final temperature

$\gamma$  = ratio of specific heat at constant pressure and volume

Since work is done by the gas in adiabatic expansion, therefore,  $W > 0$ . From equation (i), for  $W > 0$ ,  $T_1 > T_2$ . This means that air

coming out of tyre will be at a lower temperature than when it was inside the tyre.

### **Answer.12**

1. We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area  $\times$  displacement

Therefore,

Work done=pressure  $\times$  volume

Let change in the volume of object =  $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the object W

$$W = P\Delta V$$

During expansion volume increases. Hence work done will be positive and equals  $P\Delta V$ .

2. According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$ =heat supplied to the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by the system

Since work done by the gas is not zero, heat supplied to the object will not be equal to internal energy only.

**Answer.13**

No, it is not a reversible process.

**Explanation**

1. When we stir a liquid vigorously, it becomes warm because we do work on the liquid and it increases its temperature.
2. To make it a reversible process, would require it to bring the temperature to its initial value by stirring the liquid in opposite direction. Only then we can call it a reversible process.
3. But we know that this is not possible. The only way we can decrease the temperature is by extracting heat from the liquid and not by stirring it in opposite direction.

**Answer.14**

We know that efficiency  $\eta$  of a Carnot engine is

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \dots (i)$$

Where W=work done by the engine

$Q_1$ =heat absorbed by the engine

$Q_2$ =heat released by the engine

As seen from the formula (i), efficiency will be 1 when:

i)  $W=Q_1$

ii)  $Q_2=0$

**Answer.15**

1. The entropy of the system is the measure of molecular disorder or randomness, of a system.
2. When heat is withdrawn from the system, temperature decreases. This means that there is less randomness in the system. So, entropy will decrease.
3. But the heat withdrawn from the system is supplied to surroundings. Therefore, the entropy of the surroundings will increase.
4. The second law of thermodynamics states that net entropy of the universe always increases.
5. So, if the entropy of the system decreases, the entropy of the surroundings increases. Therefore, there is always a net increase in entropy.
6. Hence second law of thermodynamic is not violated in this case.

## **Objective I**

### **Answer.1**

According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

It is a general law of conservation of energy applicable to any system in which the energy transfer from or to surroundings is considered.

## Answer.2

According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

For an isothermal process  $\Delta T = 0$ , therefore  $\Delta U = 0$ .

Then,

$$\Delta Q = \Delta W$$

When heat is supplied to the system gas expands i.e. the volume of the gas increases.

We know that,

Work done = force  $\times$  displacement

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Volume = area  $\times$  displacement

Therefore,

Work done = pressure  $\times$  volume

Let change in the volume of object =  $\Delta V = V_2 - V_1$

Pressure =  $P$

Thus, work done by the object  $W$

$$W = P\Delta V$$

During expansion volume increases. Hence, work done will be positive and equals  $P\Delta V$ .

### Answer.3

Initial and final points of both processes A and B are same. Therefore, internal energy in both the processes will be the same because internal energy is a state variable, independent of the path taken.

The area under the P-V curve gives the work done on the system. From the graph, it can be seen the area under the curve for process A is more than the area under the curve for process B., therefore, work done on the system in process A  $\Delta W_1$  is more than work done on the system in process B  $\Delta W_2$

$$\Delta W_1 > \Delta W_2 \dots(i)$$

According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$ =heat supplied/extracted to/from the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by/on the system

For process A

$$\Delta Q_1 = \Delta U + \Delta W_1 \dots(ii)$$

For process B

$$\Delta Q_2 = \Delta U + \Delta W_2 \dots(iii)$$

From equation (i) ,(ii) and (iii) it is clear that  $\Delta Q_1 > \Delta Q_2$ .

**Answer.4**

Initial and final points of both processes A and B are same. Therefore, change in internal energy in both the processes will be the same because internal energy is a state variable, independent of the path taken.

Therefore,  $\Delta U_1 = \Delta U_2 = 0$ .

**Answer.5**

We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area  $\times$  displacement

Therefore,

Work done = pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_2 - V_1$

Pressure = P

Thus, work done by the system W at constant pressure

$$W = P\Delta V$$

From the graph we can see that  $V_2 > V_1$  i.e. final volume is greater than the initial volume.

So, work done by the system continuously increases.

### **Answer.6**

Statement A: For an isothermal process (where the temperature remains constant) if heat is added to a system temperature will not increase. So, statement A is wrong.

Statement B: We know that,

Work done = force  $\times$  displacement

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Volume = area  $\times$  displacement

Therefore,

Work done = pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_2 - V_1$

Pressure = P

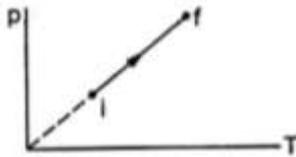
Thus, work done by the system W at constant pressure

$$W = P\Delta V$$

$W > 0$  and positive only when  $V_2 > V_1$ .

Therefore, statement B is correct.

An ideal gas goes from the state i to the state f as shown in the figure. The work done by the gas during the process.



- A. is positive
- B. is negative
- C. is zero
- D. cannot be obtained from this information.

**Answer.7**

Since the graph between P and T is a straight line passing through the origin, therefore  $P \propto T$ .

P can only be proportional to T when the volume is kept constant. This can be easily proved from the ideal gas equation which is

$$PV=RT$$

Since R is already a constant, if volume also becomes constant then  $P \propto T$ .

Constant volume implies  $\Delta V=0$ .

We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area  $\times$  displacement

Therefore,

Work done=pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the system W

$$W=P\Delta V$$

For  $\Delta V=0$ ,  $W=0$ .

**Answer.8**

Given  $\Delta V_1 = \Delta V_2$

We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area  $\times$  displacement

Therefore,

Work done = pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_2 - V_1$

Pressure = P

Thus, work done by the system W

$$\Delta W = P \Delta V$$

For process A  $\Delta W_1 = P_1 \Delta V_1$

For process B  $\Delta W_2 = P_2 \Delta V_2$

Since  $\Delta V_1 = \Delta V_2$  we can write,

$$\frac{\Delta W_1}{P_1} = \frac{\Delta W_2}{P_2}$$

$$\frac{\Delta W_1}{\Delta W_2} = \frac{P_1}{P_2}$$

$P_1 < P_2$  (from graph)

Therefore  $\Delta W_1 < \Delta W_2$ .

### **Answer.9**

1. When the piston is moved to compress the gas, the volume of the gas reduces and pressure and temperature increases.
2. After that piston is maintained at this position. So, volume becomes constant.
3. But during this sudden compression heat is generated. And since the cylinder is metallic this heat can escape out of the cylinder as metals are good conductors of heat.
4. When heat escapes out of the cylinder, the temperature of the gas will decrease which will, in turn, decrease the pressure of the gas.

### **Objective II**

#### **Answer.1**

We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area × displacement

Therefore,

Work done = pressure × volume

Let change in the volume of system =  $\Delta V = V_2 - V_1$

Pressure = P

Thus, work done by the system W

$$\Delta W = P\Delta V$$

It is given that volume is increasing i.e.  $\Delta V > 0$ . So,  $\Delta W > 0$  and positive.

From ideal gas equation, we know that

$$PV = nRT$$

Where P = pressure

V = volume

n = number of moles

R = gas constant

T = temperature

So, if both pressure and volume are increasing, then

the temperature must also increase as n and R are constant.

## **Answer.2**

From ideal gas equation, we know that

$$PV = nRT$$

Where  $P$ =pressure

$V$ =volume

$n$ =number of moles

$R$ =gas constant

$T$ =temperature

If  $P_i V_i = P_f V_f$

Then  $nRT_i = nRT_f$

$\therefore T_i = T_f$

Since the initial state of the system ( $P_i, V_i, T_i$ ) is equal to final state of the system ( $P_f, V_f, T_f$ ), initial and final internal energy will also be equal as internal energy is a state variable.

### **Answer.3**

According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$ =heat supplied/extracted to/from the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by/on the system

$$\therefore \Delta U = \Delta Q - \Delta W$$

It is given that initial state ( $p_1, V_1$ ) and final state ( $p_2, V_2$ ) is same for both the method.

So, change in internal energy will be the same for both the methods as internal energy is a state variable and is independent of the path taken to achieve a state.

**Answer.4**

As seen from the figure initial state of A is the final state of B and vice-versa.

So, the change in internal energy of process A and B will be equal in magnitude but with a negative sign.

$$\text{So } \Delta U_1 = -\Delta U_2$$

$$\therefore \Delta U_1 + \Delta U_2 = 0.$$

**Answer.5**

1. The internal energy of an ideal gas is proportional to the temperature. Since internal energy is decreasing, therefore, the temperature must decrease.

2. According to the question, change in internal energy is equal to change in work done.

3. According to First law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied/extracted to/from the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by/on the system

4. We know for adiabatic process  $\Delta Q = 0$ . So, from first law  $\Delta U = \Delta W$ .

## Exercises

### Answer.1

Given

Initial temperature of water  $T_1 = 15^\circ\text{C} = 288\text{K}$

Final temperature of water  $T_2 = 17^\circ\text{C} = 290\text{K}$

Specific heat capacity of copper  $c_c = 420 \text{ J kg}^{-1} \text{ K}^{-1}$

Specific heat capacity of water  $c_w = 4200 \text{ J kg}^{-1} \text{ K}^{-1}$

Mass of copper vessel  $m_c = 100\text{g} = 100 \times 10^{-3} \text{kg}$

Mass of water  $m_w = 200\text{g} = 200 \times 10^{-3} \text{kg}$

a) It is given that copper vessel is thermally insulated. Therefore, no heat from the surroundings can be transferred to the liquid-vessel system.

b) Work done on this system will be

$$\Delta W = m_w c_w \Delta T + m_c c_c \Delta T$$

$$\Delta T = T_2 - T_1 = 290 - 288 = 2\text{K}$$

So,

$$\Delta W = 200 \times 10^{-3} \times 4200 \times 2 + 100 \times 10^{-3} \times 420 \times 2$$

$$\Delta W = 1680 + 84 = 1764 \text{ J}$$

$\therefore$  work done on the system is 1764J.

c) From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

From part (a) we have concluded that  $\Delta Q = 0$

So, first law becomes

$$\Delta U = -\Delta W$$

Work done on the system  $\Delta W = 1764 \text{ J}$

So, work done by the system  $\Delta W = -1764 \text{ J}$

$$\Delta U = -(-1764) = 1764 \text{ J}$$

$\therefore$  Increase in internal energy of the system = 1764J.

## **Answer.2**

Given

Mass attaches to the pulley  $m = 12\text{kg}$

Heat capacity of liquid  $s = 4200 \text{ JK}^{-1}$

Height through which mass fall =  $70\text{cm} = 0.7\text{m}$

a) Paddle immersed in liquid is kept in an adiabatic container. So, no heat can be either supplied or extracted to the liquid. Therefore, heat given to liquid is zero.

b) As no heat is supplied to liquid and pulley is frictionless, work done on the liquid will be equal to the potential energy of the mass attached to the pulley.

Work done on the liquid = potential energy of mass

Potential energy of mass =  $mgh$

Where  $g$  = acceleration due to gravity =  $10\text{ms}^{-2}$

Potential energy of mass =  $12 \times 10 \times 0.7 = 84\text{J}$

$\therefore$  work done on liquid =  $84\text{J}$ .

c) The mechanical work calculated in the second part will be converted into heat. This heat will be supplied to liquid and due to which temperature of the liquid will rise.

We know that,

$$\text{Heat capacity } s = \frac{\Delta Q}{\Delta T}$$

Where  $\Delta Q$  = heat supplied

$\Delta T$  = rise in temperature

$$\Delta T = \frac{\Delta Q}{s}$$

Since work done is equal to heat supplied, therefore

$$\Delta T = \frac{84}{4200} = \frac{1}{50} = 0.02\text{K}$$

$\therefore$  the rise in temperature of the liquid will be  $0.02\text{K}$ .

### Answer.3

Given

Mass of block  $m=100\text{kg}$

Initial velocity  $u=2\text{ m/s}$

Coefficient of kinetic friction  $\mu = 0.20$

a) Since the block comes to stop final velocity will be zero.

So, final velocity  $v=0\text{ m/s}$

When the block is moving over belt there is kinetic friction between the lower surface of the block and upper surface of the belt. And we know that heat is produced due to friction between two surfaces. Now because of this heat, the internal energy of block will change.

So,

Kinetic energy lost in heat due to friction = change in the internal energy

Kinetic energy lost = initial kinetic energy- final kinetic energy

$$K.E \text{ lost} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 100 \times 2 \times 2 - 0 = 200J$$

∴ change in internal energy is 200J.

b) Given

The velocity of the frame of reference  $u_0= 2\text{m/s}$

So, in this frame of reference initial and final velocity of the block will change.

New initial velocity  $u'=u-u_0=2-2=0\text{m/s}$

New final velocity  $v' = v-u_0=0-2=-2\text{m/s}$

So, increase in kinetic energy = (final – initial) kinetic energy

$$= \frac{1}{2}mv'^2 - \frac{1}{2}mu'^2$$

$$= \frac{1}{2} \times 100 \times (-2)^2 - 0 = 200J$$

Increase in kinetic energy in com frame of reference is 200J.

c) Total work done in com frame of reference will be work done due to friction plus work done to give final velocity.

We know that force of friction  $f = \mu N$

Where  $\mu$  = coefficient of friction = 0.02

$N$  = normal reaction =  $mg$

$$f = 0.02 \times 100 \times 10 = 200N$$

from newton's second law of motion

force = mass  $\times$  acceleration

so,

$$200 = 100 \times \text{acceleration}$$

$$\text{acceleration 'a'} = \frac{200}{100} = 2ms^{-2}$$

Using the third equation of motion,

$$v'^2 - u'^2 = 2as$$

where  $s$  = displacement as seen in com frame of reference

$$(-2)^2 - 0 = 2 \times 2 \times s$$

$$s = 1m$$

work done due to friction  $W_f$  = force  $\times$  displacement

$$W_f = 200 \times 1 = 200J$$

Now to calculate work done to give final velocity, we will work-energy theorem.

According to work-energy theorem,

Work done = change in kinetic energy

So,

$$W' = \frac{1}{2}mv'^2 - \frac{1}{2}mu'^2 = \frac{1}{2} \times 100 \times (-2)^2 - 0 = 200J$$

Total work done  $W=W' + W_f = 200+200=400J$ .

$\therefore$  work done in com frame of reference= 400J.

#### **Answer.4**

Given

Heat supplied  $\Delta Q = 100J$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$ =heat supplied to the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by the system

We know that,

Work done = force  $\times$  displacement

$$pressure = \frac{force}{area}$$

Volume = area  $\times$  displacement

Therefore,

Work done=pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_2 - V_1$

Pressure =P

Thus, work done by the gas

$$\Delta W = P\Delta V$$

Since the gas is kept in a rigid container, therefore  $\Delta V=0$  in this case.

So,  $\Delta W=0$

Thus, first law will become

$$\Delta Q = \Delta U = 100J$$

$\therefore$  change in internal energy will be 100J.

### Answer.5

Given

Initial pressure  $P_1=10\text{kPa}=10\times 10^3\text{Pa}$

Final pressure  $P_2=50\text{kPa}=50\times 10^3\text{Pa}$

Initial volume  $V_1=200\text{cc}=200\times 10^{-6}\text{m}^3$

Final volume  $V_2=50\text{cc}= 50\times 10^{-6}\text{m}^3$

a) We know that,

Work done = force  $\times$  displacement

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Volume = area  $\times$  displacement

Therefore,

Work done=pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_2-V_1$

Pressure =P

Thus, work done by the gas

$$\Delta W=P\Delta V$$

Here we have given two values of pressure. So, we will take the average value of pressure

Average pressure P

$$P = \frac{P_1 + P_2}{2}$$

$$P = \frac{10 \times 10^3 + 50 \times 10^3}{2} = 30 \times 10^3 \text{Pa}$$

Therefore,  $\Delta W = 30 \times 10^3 \times (50 - 200) \times 10^{-6}$

$$\Delta W = -4.5 \text{ J}$$

$\therefore$  Work done by the gas is -4.5J.

b) Given that no heat is supplied or extracted from the gas.

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

Since  $\Delta Q = 0$

$$\text{Therefore } \Delta U = -\Delta W = -(-4.5) \text{ J} = 4.5 \text{ J}$$

$\therefore$  the change in internal energy of the gas is 4.5J.

### **Answer.6**

Given

Initial pressure  $P_i$

Final pressure  $P_f$

Initial temperature  $T_i$

Final temperature  $T_f$

It is given that

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \dots \dots (i)$$

From ideal gas equation, we know

$$PV = nRT$$

Where  $V$  = volume of gas

$R$  = gas constant

n= number of moles

applying ideal gas equation for both processes, we get

$$\therefore \frac{P_i}{T_i} = \frac{nR}{V_i} \dots\dots (ii)$$

And

$$\frac{P_f}{T_f} = \frac{nR}{V_f} \dots\dots (iii)$$

From equation (i), (ii) and (iii), we get

$$\frac{nR}{V_i} = \frac{nR}{V_f}$$

$$\therefore V_i = V_f$$

We know that,

Work done = force  $\times$  displacement

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Volume = area  $\times$  displacement

Therefore,

Work done = pressure  $\times$  volume

Let change in the volume of system =  $\Delta V = V_f - V_i$

Pressure = P

Thus, work done by the gas

$$\Delta W = P \Delta V = P (V_f - V_i) = 0$$

$\therefore$  work done by the gas is zero.

### Answer.7

We know that work done by the gas is given as

$$\Delta W = P\Delta V$$

From graph we can write

$$V_A = V_C = 10\text{cc} = 10 \times 10^{-6} \text{m}^3$$

$$V_D = V_B = 25\text{cc} = 25 \times 10^{-6} \text{m}^3$$

$$P_B = P_C = 30\text{kPa} = 30 \times 10^3 \text{Pa}$$

$$P_A = P_D = 10\text{kPa} = 10 \times 10^3 \text{Pa}$$

Work done in path ADB  $W_{ADB} = W_{AD} + W_{DB}$

$$W_{ADB} = P_A (V_D - V_A) + 0 \quad (\because W_{DB} = 0 \text{ because } V_D = V_B)$$

$$= 10 \times 10^3 \times (25 - 10) \times 10^{-6}$$

$$= 0.15\text{J}$$

Work done in path AB  $W_{AB} = P_{\text{avg}}(V_B - V_A)$

$$P_{\text{avg}} = \frac{P_B + P_A}{2} = \frac{10 \times 10^3 + 30 \times 10^3}{2} = 20 \times 10^3 \text{Pa}$$

$$W_{AB} = 20 \times 10^3 \times (25 - 10) \times 10^{-6}$$

$$= 0.30\text{J}$$

Work done in path ACB  $W_{ACB} = W_{AC} + W_{BC}$

$$W_{ACB} = 0 + P_B(V_B - V_C) \quad (\because W_{AC} = 0 \text{ because } V_A = V_C)$$

$$= 30 \times 10^3 \times (25 - 10) \times 10^{-6}$$

$$= 0.45\text{J}$$

## Answer.8

Given

Heat absorbed in process abc  $\Delta Q_1=80\text{J}$

Work done by the system in process abc  $\Delta W_1=30\text{J}$

Work done by the system in process adc  $\Delta W_2=10\text{J}$

Let heat absorbed into the system during process adc  $=\Delta Q_2$

Now initial point a and final point c is the same for both the processes is the same. So, change in internal energy will be the same for both the process, as internal energy is a state function independent of the path taken.

Therefore,

$$\Delta U_1=\Delta U_2=\Delta U \dots\dots(i)$$

From first law of thermodynamics, we know that,

$$\Delta Q=\Delta U+\Delta W$$

Where  $\Delta Q$ =heat supplied to the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by the system

Using first law of thermodynamics for process abc

$$\Delta Q_1=\Delta U_1+\Delta W_1$$

$$\Delta U_1=\Delta Q_1-\Delta W_1=80-30=50\text{J}$$

Using first law of thermodynamics for process adc

$$\Delta Q_2=\Delta U_2+\Delta W_2$$

$$=\Delta U_1+\Delta W_2 \text{ (from (i))}$$

$$= 50+10=60\text{J}$$

$\therefore$  heat absorbed into the system during process adc= 60J.

### Answer.9

From graph we can write

$$V_A = V_D = 200\text{cc} = 200 \times 10^{-6} \text{m}^3$$

$$V_B = V_C = 400\text{cc} = 400 \times 10^{-6} \text{m}^3$$

$$P_B = P_D = 155\text{kPa} = 155 \times 10^3 \text{Pa}$$

$$P_A = P_C = 50\text{kPa} = 50 \times 10^3 \text{Pa}$$

Given

Heat absorbed in process ABC  $\Delta Q_1 = 50\text{cal} = 50 \times 4.2 \text{J} = 210\text{J}$

Let heat absorbed into the system during process ADC  $= \Delta Q_2$

We know that work done by the gas is given as

$$\Delta W = P \Delta V$$

Work done in path ACB  $W_{ACB} = \Delta W_1 = W_{AC} + W_{BC}$

$$\Delta W_1 = P_A (V_C - V_A) + 0 \quad (W_{BC} = 0 \text{ because } V_B = V_C)$$

$$= 50 \times 10^3 \times (400 - 200) \times 10^{-6}$$

$$= 10\text{J}$$

Work done in path ADB  $W_{ADB} = \Delta W_2 = W_{AD} + W_{DB}$

$$\Delta W_2 = P_B (V_D - V_B) + 0 \quad (\because W_{AD} = 0 \text{ because } V_A = V_D)$$

$$= 155 \times 10^3 \times (400 - 200) \times 10^{-6}$$

$$= 31\text{J}$$

Now initial point A and final point C is the same for both the processes is the same. So, change in internal energy will be the same for both the process, as internal energy is a state function independent of the path taken.

Therefore,

$$\Delta U_1 = \Delta U_2 = \Delta U \dots\dots(i)$$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

Using first law of thermodynamics for process ABC

$$\Delta Q_1 = \Delta U_1 + \Delta W_1$$

$$\Delta U_1 = \Delta Q_1 - \Delta W_1 = 210 - 10 = 200 \text{ J}$$

Using first law of thermodynamics for process adc

$$\Delta Q_2 = \Delta U_2 + \Delta W_2$$

$$= \Delta U_1 + \Delta W_2 \text{ (from (i))}$$

$$= 200 + 31 = 231 \text{ J.}$$

$\therefore$  heat supplied to the system during process ADC = 231 J.

### **Answer.10**

We know that in the cyclic process the system returns to its initial state. So, change internal energy in the cyclic process will be zero as internal energy is a state function.

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

Since  $\Delta U = 0$ , first law becomes

$$\Delta Q = \Delta W$$

In a PV graph work done is equal to the area under the curve.

$$\Delta W = \text{area of a circle}$$

$$\Delta Q = \Delta W = \text{area of circle}$$

$$\text{Diameter of the circle} = 300 - 100 = 200$$

$$\text{radius} = \frac{\text{diameter}}{2} = \frac{200}{2} = 100$$

$$\text{Area of the circle} = \pi \times (\text{radius})^2$$

$$= \pi \times 100 \times 100 \times 10^{-6} \times 10^3$$

( $10^{-6} \times 10^3$  is because volume and pressure are given in cc and kPa respectively)

$$\text{Area of circle} = 3.14 \times 10 = 31.4$$

∴ Heat absorbed by a system = 31.4J.

### Answer.11

'J' is mechanical equivalent of heat a conversion factor between two different units of energy: calorie to joule

From the graph we can write

$$V_A = V_B = 500 \text{cc} = 500 \times 10^{-6} \text{m}^3$$

$$V_C = 700 \text{cc} = 700 \times 10^{-6} \text{m}^3$$

$$P_A = P_C = 100 \text{kPa} = 100 \times 10^3 \text{ Pa}$$

$$P_B = 200 \text{kPa} = 200 \times 10^3 \text{ Pa}$$

We know that work done by the gas is given as

$$\Delta W = P \Delta V$$

Work done in path AB=0 as  $V_A=V_B$ .

Work done in path CA  $\Delta W_1=P_A(V_A-V_C)$

$$=100 \times 10^3 \times (500-700) \times 10^{-6}$$

$$=-20\text{J}$$

Work done in path BC  $\Delta W_2= P_{\text{avg}}(V_C-V_B)$

$$P_{\text{avg}} = \frac{P_B + P_C}{2} = \frac{100 \times 10^3 + 200 \times 10^3}{2} = 150 \times 10^3 \text{ Pa}$$

$$\Delta W_2=150 \times 10^3 \times (700-500) \times 10^{-6}$$

$$=30\text{J}$$

Total work done in process ABCA= $\Delta W=\Delta W_1+\Delta W_2$

$$=30-20=10\text{J}$$

We know that in the cyclic process the system returns to its initial state. So, change internal energy in the cyclic process will be zero as internal energy is a state function.

From first law of thermodynamics, we know that,

$$\Delta Q=\Delta U+\Delta W$$

Where  $\Delta Q$ =heat supplied to the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by the system

Since  $\Delta U=0$ , first law becomes

$$\Delta Q=\Delta W=10\text{J}$$

But it is given in question that  $\Delta Q=2.4\text{cal}$

So,  $2.4 \times J=10\text{Joule}$

$$J = \frac{10}{2.4} = 4.17\text{Joule/cal}$$

$\therefore$  value of 'J' is 4.17J/cal.

## Answer.12

'J' is mechanical equivalent of heat a conversion factor between two different units of energy: calorie to the joule.

Given

Heat given to system = 2625 cal = 2625 × J

Change in internal energy = 5000 J

From graph

$$V_a = 0.02 \text{ m}^3$$

$$V_b = V_c = 0.05 \text{ m}^3$$

$$P_a = P_b = 200 \text{ kPa} = 200 \times 10^3 \text{ Pa}$$

$$P_c = 300 \text{ kPa} = 300 \times 10^3 \text{ Pa}$$

We know that work done by the gas is given as

$$\Delta W = P \Delta V$$

Work done in process abc =  $\Delta W = W_{ab} + W_{bc}$

$$\Delta W = P_a(V_b - V_a) + 0 \quad (\because W_{bc} = 0 \text{ because } V_b = V_c)$$

$$\Delta W = 200 \times 10^3 \times (0.05 - 0.02) = 6000 \text{ J}$$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

$$\Delta Q = 5000 + 6000 = 11000 \text{ J}$$

But  $\Delta Q = 2625 \text{ cal} = 2625 \times J$

Therefore,

$$2625 \times J = 11000$$

$$J = \frac{11000}{2625} = 4.19 \text{ joule/cal}$$

∴ value of 'J' is 4.19 joule/cal.

### Answer.13

Given

$$\text{Heat extracted from the system } \Delta Q = -70 \text{ cal} = -70 \times 4.2 = -294 \text{ J}$$

The negative sign is because heat is extracted from the system.

From graph

$$V_A = 250 \text{ cc} = 250 \times 10^{-6} \text{ m}^3$$

$$V_B = 100 \text{ cc} = 100 \times 10^{-6} \text{ m}^3$$

$$P_A = 200 \text{ kPa} = 200 \times 10^3 \text{ Pa}$$

$$P_B = 500 \text{ kPa} = 500 \times 10^3 \text{ Pa}$$

We know that work done by the gas is given as

$$\Delta W = P \Delta V$$

Here since we have two values of pressure we will take average pressure.

$$P_{avg} = \frac{P_A + P_B}{2} = \frac{200 \times 10^3 + 500 \times 10^3}{2} = 350 \times 10^3 \text{ Pa}$$

$$\Delta W = P_{avg} (V_A - V_B)$$

$$= 350 \times 10^3 \times (250 - 100) \times 10^{-6}$$

$$= -52.5 \text{ J}$$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

Therefore,

$$\Delta U = -294 - (-52.5) = -241.5\text{J}$$

$\therefore$  change in internal energy is -241.5J.

#### **Answer.14**

Given

Constant pressure  $p = 1.0 \times 10^5$  Pa

Change in volume  $\Delta V = (200 - 100) \times 10^{-6} \text{m}^3 = 10^{-4} \text{m}^3$

Internal energy  $U = 1.5pV$

So, change in internal energy  $\Delta U = 1.5p\Delta V$

$$= 1.5 \times 1.0 \times 10^5 \times 10^{-4}$$

$$= 15\text{J}$$

We know that work done by the gas is given as

$$\Delta W = p\Delta V = 1.0 \times 10^5 \times 10^{-4} = 10\text{J}$$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

Therefore,

$$\Delta Q = 15 + 10 = 25\text{J}$$

$\therefore$  heat absorbed by the system = 25J.

### Answer.15

Given

Heat supplied to system  $\Delta Q = 10\text{J}$

Atmospheric pressure  $P = 100\text{kPa} = 100 \times 10^3\text{Pa}$

Displacement of the piston  $d = 10\text{cm} = 0.1\text{m}$

Area of cross section of cylinder  $A = 4\text{cm}^2 = 4 \times 10^{-4}\text{m}^2$

Gas will expand when is heat to the system. Therefore, the volume of gas expanded  $\Delta V$

$\Delta V = \text{area} \times \text{displacement}$

$= A \times d$

$\Delta V = 4 \times 0.1 \times 10^{-4} = 40 \times 10^{-6}\text{m}^3$

We know that work done by the gas is given as

$\Delta W = P\Delta V$

$= 100 \times 10^3 \times 40 \times 10^{-6}$

$\Delta W = 4\text{J}$

From first law of thermodynamics, we know that,

$\Delta Q = \Delta U + \Delta W$

Where  $\Delta Q =$ heat supplied to the system

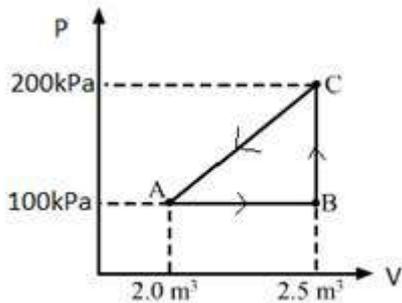
$\Delta U =$ change in internal energy

$\Delta W =$ work done by the system

$\Delta U = \Delta Q - \Delta W = 10 - 4 = 6\text{J}$

Thus, the increase in internal energy is 6J.

### Answer.16



From graph

$$V_a = 2\text{m}^3$$

$$V_b = V_c = 2.5\text{m}^3$$

$$P_a = P_b = 100\text{kPa} = 100 \times 10^3\text{Pa}$$

$$P_c = 200\text{kPa} = 200 \times 10^3\text{Pa}$$

Work done in process ABCA = area enclosed by the triangle ABC

$$\Delta W = 0.5 \times BC \times AB$$

Where BC = height of the triangle

AB = base of triangle

So,

$$\Delta W = 0.5 \times (200 - 100) \times 10^3 \times (2.5 - 2)$$

$$\Delta W = 0.5 \times 100 \times 0.5 \times 10^3 = 25000\text{J}$$

a) Process ABCA is a cyclic process. The system is brought back to its initial state. Since internal energy is a state function, change in internal energy will be zero.

So,  $\Delta U = 0$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

$$\therefore \Delta Q = \Delta W = 25000\text{J}$$

Since work done is positive, work is done by the gas. When work is done by the gas heat is supplied to the system.

b) Amount of heat supplied = work done by the gas in the cyclic process

$$\therefore \Delta Q = \Delta W = 25000\text{J}.$$

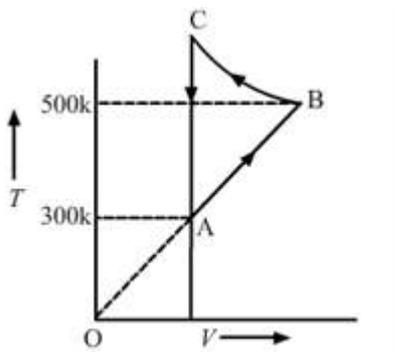
### Answer.17

Given

Heat extracted from the system  $\Delta Q = -1200\text{J}$

(negative sign is because heat is extracted from the system)

Number of moles of the gas  $n = 2.0$



From the graph we can write

$$T_A=300K$$

$$T_B=500K$$

$$V_A=V_C$$

We know that work done by the gas is given as

$$\Delta W=P\Delta V$$

Where  $\Delta V$  =change in volume

P =pressure

So, work done along line CA will be zero, as  $V_A=V_C$ .

Thus, total work done will be

$$\Delta W=W_{AB}+W_{BC}$$

$$W_{AB} = P(V_B-V_A)$$

But we know that ideal gas equation is

$$PV=nRT$$

Where n= number of moles

R=gas constant

T=temperature

$$\therefore P\Delta V=nR\Delta T$$

Thus, we can write  $W_{AB} = P(V_B-V_A) = nR(T_B-T_A)$

$$\therefore \Delta W= nR(T_B-T_A) + W_{BC}$$

From first law of thermodynamics, we know that,

$$\Delta Q=\Delta U+\Delta W$$

Where  $\Delta Q$ =heat supplied to the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by the system

Process ABCA is a cyclic process. The system is brought back to its initial state. Since internal energy is a state function, change in internal energy will be zero.

So,  $\Delta U=0$ .

So, first law becomes

$$\Delta Q = \Delta W$$

$$\Delta Q = nR(T_B - T_A) + W_{BC}$$

We know that  $R = 8.31 \text{ J/K mol}$

$$W_{BC} = \Delta Q - nR(T_B - T_A)$$

$$= -1200 - 2 \times 8.31 \times (500 - 300)$$

$$= -1200 - 3324$$

$$= -4524 \text{ J}$$

Thus, work done along path BC = -4524J

### **Answer.18**

Given

Number of moles  $n = 2$

Temperature at b  $T_b = 500 \text{ K}$

Temperature at c  $T_c = 300 \text{ K}$

From the graph it is clear that

$$T_b = T_a \text{ and } T_d = T_c$$

Thus, path ab and cd are isothermal paths.

We know that work done by the gas is given as

$$\Delta W = P \Delta V$$

Where  $\Delta V$  = change in volume

P = pressure

Again, from the graph we can see that  $\Delta V=0$  for path bc and da.

Therefore, work done along path bc and da are zero.

So, total work done  $\Delta W=W_{ab}+W_{cd}$

We know that work done in an isothermal process is given as

$$W = nRT \ln \frac{V_f}{V_i}$$

Where n=number of moles

R=gas constant =8.31J/Kmol

T=temperature

$V_f$ =final volume

$V_i$ =initial volume

$$W_{ab} = nRT_a \ln \frac{V_b}{V_a}$$

$$W_{ab} = 2 \times 8.31 \times 500 \times \ln \frac{2V_o}{V_o}$$

$$W_{ab} = 8310 \times \ln 2$$

Similarly,

$$W_{cd} = nRT_c \ln \frac{V_d}{V_c}$$

$$W_{cd} = 2 \times 8.31 \times 300 \times \ln \frac{V_o}{2V_o}$$

$$W_{cd} = -2 \times 8.31 \times 300 \times \ln \frac{2V_o}{V_o}$$

$$W_{cd} = -4986 \times \ln 2$$

So, total work done  $\Delta W=W_{ab}+W_{cd}$

$$=8310 \times \ln 2 - 4986 \times \ln 2$$

$$=3324 \times \ln 2$$

$$\Delta W = 3324 \times 0.693 = 2304.02 \text{ J}$$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$ =heat supplied to the system

$\Delta U$ =change in internal energy

$\Delta W$ =work done by the system

Process ABCA is a cyclic process. The system is brought back to its initial state. Since internal energy is a state function, change in internal energy will be zero.

So,  $\Delta U=0$ .

So, first law becomes

$$\Delta Q=\Delta W=2304.02\text{J}$$

Thus, heat absorbed by the system is 2304.02J.

### **Answer.19**

Given

Mass of water  $m = 2\text{kg}$

Change in temperature  $\Delta T = 4^\circ\text{C} - 0^\circ\text{C} = 4^\circ\text{C}$

Specific heat capacity of water  $c = 4200 \text{ J kg}^{-1} \text{ K}^{-1}$

Density of water at  $0^\circ\text{C} = 999.9 \text{ kg m}^{-3}$

Density of water at  $4^\circ\text{C} = 1000 \text{ kg m}^{-3}$

Atmospheric pressure  $P = 10^5 \text{ Pa}$ .

We know that specific heat capacity is given by

$$c = \frac{\Delta Q}{m\Delta T}$$

Where  $\Delta Q$  = heat supplied to the system

Therefore,  $\Delta Q = cm\Delta T$

$$= 4200 \times 2 \times 4 = 33600\text{J}$$

We know that work done by the gas is given as

$$\Delta W = P\Delta V$$

Where  $\Delta V$  =change in volume

P =pressure

Also,

$$volume = \frac{mass}{density}$$

Volume at 0°C  $V_1$

$$V_o = \frac{m}{density\ at\ 0^\circ C} = \frac{2}{999.9} m^3$$

Similarly, volume at 4°C  $V_2$

$$V_2 = \frac{m}{density\ at\ 4^\circ C} = \frac{2}{1000} m^3$$

$$\therefore \Delta W = P(V_2 - V_1)$$

$$\Delta W = 10^5 \times \left( \frac{2}{1000} - \frac{2}{999.9} \right) = -0.02J$$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  =heat supplied to the system

$\Delta U$  =change in internal energy

$\Delta W$  =work done by the system

$$\Delta U = \Delta Q - \Delta W$$

$$= 33600 - (-0.02)$$

$$= 33599.98J$$

Thus, change in internal energy is 33599.98J.

**Answer.20**

Given

The density of steam  $\rho' = 0.6 \text{ kg m}^{-3}$

Mass of water  $m = 10 \text{ g} = 0.010 \text{ kg}$

Specific heat capacity of water  $c = 4200 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$

latent heat of vaporization of water  $L = 2.25 \times 10^6 \text{ J kg}^{-1}$ .

Pressure  $P = 100 \text{ kPa} = 100 \times 10^5 \text{ Pa}$

Change in temperature  $\Delta T = (100 - 0) \text{ }^\circ\text{C} = 100 \text{ }^\circ\text{C}$

Density of water  $\rho = 1000 \text{ kg m}^{-3}$

We know that specific heat capacity is given by

$$c = \frac{\Delta Q}{m\Delta T}$$

Where  $\Delta Q$  = heat supplied to the system

Therefore,  $\Delta Q = cm\Delta T$

Also,  $\Delta Q = mL$

Where  $m$  = mass of the substance

$L$  = latent heat

Therefore,  $\Delta Q = mL + cm\Delta T$

$$= 0.010 \times 2.25 \times 10^6 + 4200 \times 0.01 \times 100$$

$$= 22500 + 4200$$

$$= 26700 \text{ J}$$

We know that work done by the gas is given as

$$\Delta W = P\Delta V$$

Where  $\Delta V$  = change in volume

$P$  = pressure

Also,

$$volume = \frac{mass}{density}$$

$$\Delta W = P \times \left( \frac{m}{\rho'} - \frac{m}{\rho} \right)$$

$$\Delta W = 10^5 \times \left( \frac{0.01}{0.06} - \frac{0.01}{1000} \right)$$

$$\Delta W = 10^5 \times 0.01699 = 1699 \text{ J}$$

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

$$\Delta U = \Delta Q - \Delta W$$

$$= 26700 - 1699$$

$$= 25001 \text{ J}$$

Thus, change in internal energy is 25001J.

### **Answer.21**

a) According to the question, the diathermic separator between both the part is fixed. So, no change in volume will be observed. And as we know work done on gas is  $P\Delta V$ . Therefore, no work will be done on the left part during the process as the volume is not changing.

First, we will calculate the final temperature and then final pressure.

(c) Given

Pressure of left chamber =  $P_1$

Pressure of right chamber =  $P_2$

Temperature of left chamber =  $T_1$

Temperature of right chamber =  $T_2$

Let the number of moles in the left chamber be  $n_1$

Number of moles in the right chamber be  $n_2$

Diathermic wall has divided the tube in two equal part. So, the volume of the left and the right chamber will be  $V/2$ .

Applying ideal gas equation in the left chamber,

$$\frac{P_1 V}{2} = n_1 R T_1$$

$$n_1 = \frac{P_1 V}{2 R T_1}$$

Similarly applying ideal gas equation in the right chamber,

$$\frac{P_2 V}{2} = n_2 R T_2$$

$$n_2 = \frac{P_2 V}{2 R T_2}$$

Total number of moles  $n = n_1 + n_2$

$$n = \frac{P_1 V}{2 R T_1} + \frac{P_2 V}{2 R T_2}$$

$$n = \frac{V}{2 R} \left( \frac{P_1}{T_1} + \frac{P_2}{T_2} \right)$$

$$n = \frac{V}{2 R} \left( \frac{T_2 P_1 + P_2 T_1}{T_1 T_2} \right) \dots\dots (i)$$

The internal energy of ideal gas is given as

$$U = n C_v T$$

Where  $C_v$  = molar specific heat at constant volume

$T$  = temperature.

According to question,

$$U=1.5nRT$$

$$\therefore nC_vT=1.5nRT$$

$$\text{So, } C_v=1.5R$$

The internal energy of the left chamber  $U_1=n_1C_vT_1$

The internal energy of right chamber  $U_2= n_2C_vT_2$

Total internal energy  $U= U_1+U_2$

$$1.5nRT= n_1C_vT_1+ n_2C_vT_2$$

$$1.5nRT=C_v(n_1T_1+n_2T_2)$$

Substituting the value of  $C_v$

$$1.5nRT=1.5R(n_1T_1+n_2T_2)$$

$$nT=n_1T_1+n_2T_2$$

substituting the value of  $n_1$  and  $n_2$  in above equation

$$nT = \frac{P_1V}{2RT_1} \times T_1 + \frac{P_2V}{2RT_2} \times T_2$$

$$nT = \frac{V(P_1 + P_2)}{2R}$$

$$T = \frac{V(P_1 + P_2)}{2Rn}$$

Substituting the value of  $n$  in the above equation,

$$T = \frac{V(P_1 + P_2)}{2R} \times \frac{T_1T_2}{T_2P_1 + P_2T_1} \times \frac{2R}{V}$$

$$T = \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$$

Thus, final equilibrium temperature  $T = \frac{(T_1T_2) \times (P_1 + P_2)}{T_2P_1 + P_2T_1}$ .

(b) Now we will find final pressures on both sides.

Let final pressure of left chamber  $P_1'$

Final pressure of right chamber  $P_2'$

Applying ideal gas equation in the left chamber before and after equilibrium

$$\frac{P_1 V}{2} = nRT_1 \dots (i)$$

$$\frac{P'_1 V}{2} = nRT \dots (ii)$$

From equation (i) and (ii),

$$\frac{P_1}{T_1} = \frac{P'_1}{T}$$

$$P'_1 = \frac{P_1}{T_1} T$$

Substituting the value of T,

$$P'_1 = \frac{P_1}{T_1} \times \frac{(T_1 T_2) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1}$$

Applying ideal gas equation in the right chamber before and after equilibrium

$$\frac{P_2 V}{2} = nRT_2 \dots (iii)$$

$$\frac{P'_2 V}{2} = nRT \dots (iv)$$

From equation (iii) and (iv),

$$\frac{P_2}{T_2} = \frac{P'_2}{T}$$

$$P'_2 = \frac{P_2}{T_2} T$$

Substituting the value of T,

$$P'_2 = \frac{P_2}{T_2} \times \frac{(T_1 T_2) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1}$$

(d) The internal energy of ideal gas is given as

$$U = nC_v T$$

Where  $C_v$  = molar specific heat at constant volume

T = temperature.

As stated in part (a) no work will be done on either chamber of the vessel as the diathermic separator is fixed.

So,  $\Delta W = 0$  for the right chamber of the tube.

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

$$\Delta Q = \Delta U$$

Change in internal energy of the right chamber after equilibrium has reached will be

$$\Delta U = n_2 C_v T_2 - n_2 C_v T$$

Substituting the value of  $n_2$ ,  $C_v$  and  $T$  in the above equation

$$\Delta U = \frac{P_2 V}{2RT_2} \times 1.5 \times R \times T_2 - \frac{P_2 V}{2RT_2} \times 1.5 \times R \times \frac{(T_1 T_2) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1}$$

$$\Delta U = \frac{P_2 V}{2RT_2} \times 1.5 \times R \times T_2 \left( 1 - \frac{(T_1) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1} \right)$$

$$\Delta Q = \Delta U = \frac{P_2 V}{2} \times 1.5 \left( 1 - \frac{(T_1) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1} \right)$$

$$\Delta Q = \frac{3P_2 V}{4} \times \left( \frac{T_2 P_1 + P_2 T_1 - (T_1) \times (P_1 + P_2)}{T_2 P_1 + P_2 T_1} \right)$$

$$\Delta Q = \frac{3P_2 V}{4} \times \left( \frac{T_2 P_1 - T_1 P_1}{T_2 P_1 + P_2 T_1} \right)$$

$$\Delta Q = \frac{3P_1 P_2 V}{4} \times \left( \frac{T_2 - T_1}{T_2 P_1 + P_2 T_1} \right)$$

Thus, heat flown from left to right chamber is  $\Delta Q = \frac{3P_1 P_2 V}{4} \times \left( \frac{T_2 - T_1}{T_2 P_1 + P_2 T_1} \right)$ .

## Answer.22

a) According to the question, conducting separator between both the part is fixed. So, no change in volume will be observed. And as we know work done a gas is  $P\Delta V$ . Therefore, no work will be done on the left part during the process as the volume is not changing.

b) Given

The vessel is divided into two equal parts.

So, the volume of left part  $V_1$  and volume of right part  $V_2$  will half of the total volume.

$$V_1 = V_2 = \frac{V}{2}$$

Initial pressure on each side is  $p$

Number of moles on left side  $n_1 = 1$

Number of moles on right side  $n_2 = 2$

Let initial temperature for the left and right part be  $T_1$  and  $T_2$  respectively.

For the left part, applying the ideal gas equation

$$\frac{pV}{2} = 1 \times R \times T_1$$

$$T_1 = \frac{pV}{2R}$$

Similarly, for the right part

$$\frac{pV}{2} = 2 \times R \times T_2$$

$$T_2 = \frac{pV}{4R}$$

c) It is given that the internal energy of the gas is

$$U = 1.5nRT$$

Where  $n$  = number of moles

$R$  = gas constant

$T$  = final equilibrium temperature of both parts

The internal energy of ideal gas is given as

$$U = nC_v T$$

Where  $C_v$  = molar specific heat at constant volume

$T$  = temperature.

Total moles  $n = n_1 + n_2 = 1 + 2 = 3$

$$U = 3C_v T$$

Let  $U_1$  and  $U_2$  be the internal energy of the left and right part respectively.

$$\text{So, } U_1 = n_1 C_v T_1 = C_v T_1$$

$$\text{And } U_2 = n_2 C_v T_2 = 2C_v T_2$$

(gas is same on both the part so  $C_v$  will be same)

Now

$$U = U_1 + U_2$$

$$3C_v T = C_v T_1 + 2C_v T_2$$

$$3T = T_1 + 2T_2$$

$$3T = \frac{pV}{2R} + \frac{2pV}{4R}$$

$$T = \frac{pV}{3R}$$

d) As stated in part (a) no work will be done on either part of the vessel as conducting separator is fixed.

So,  $\Delta W = 0$  for the right part of the vessel.

From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Where  $\Delta Q$  = heat supplied to the system

$\Delta U$  = change in internal energy

$\Delta W$  = work done by the system

$$\therefore \Delta Q = \Delta U$$

It is given that the internal energy of the gas is

$$U = 1.5nRT$$

Where  $n$ =number of moles

$R$ =gas constant

$T$ =final equilibrium temperature of both parts

$$\Delta U = 1.5nR\Delta T$$

For the right part, change in internal energy after equilibrium has reached will be due to the change in temperature from  $T_2$  to  $T$ .

$$\Delta U = 1.5n_2R(T - T_2)$$

$$\Delta U = 1.5 \times 2 \times R \times \left( \frac{pV}{3R} - \frac{pV}{4R} \right)$$

$$\Delta U = 3 \times R \times \frac{pV}{3 \times 4 \times R}$$

$$\Delta U = \frac{pV}{4}$$

Thus, heat given to the right part is  $\Delta Q = \frac{pV}{4}$ .

e) Since  $\Delta W$  is zero as the volume is fixed, therefore, the first law of thermodynamics.

But since heat is given to the right part that means heat is extracted from the left part. So, the internal energy of the left part will decrease.

Therefore, heat given to the right part will be equal to the negative internal energy of the left part.

$$\Delta Q = -\Delta U \text{ (left part)}$$

$$\therefore \Delta U = -\frac{pV}{4}$$