APPENDIX 1

SOLVED EXAMPLE UNIT-1

1. Find the dimensions of a and b in the formula $[P + \frac{a}{V^2}][V - b] = RT$ where P is pressure and V is the volume of the gas

Solution:

By the principle of homogeneity, a/V^2 is of the dimensions of pressure and b is of the dimensions of volume.

$[a] = [\text{pressure}]\,[V^2] = [ML^{-1}T^{-2}]\,[L^6]$
$= [ML^5T^{-2}]$

$[b] = [V] = [L^3]$

2. Show that $(P^{-5/6}\rho^{1/2}E^{1/3})$ is of the dimension of time. Here P is the pressure, ρ is the density and E is the energy of a bubble)

Solution:

Dimension of Pressure $= [ML^{-1}T^{-2}]$

Dimension of density $= [ML^{-3}]$

Dimension of Energy $= [ML^2T^{-2}]$

By substituting in the given equation,

$= [ML^{-1}T^{-2}]^{-5/6}[ML^{-3}]^{1/2}[ML^2T^{-2}]^{1/3}$

$= M^{-5/6+1/2+1/3}L^{5/6-3/2+2/3}T^{5/3-2/3}$

$= M^0L^0T^1 = [T]$

3. Find the dimensions of mass in terms of Energy, length and time

Solution:

Let the dimensions of Energy, Length and Time be $[E],[L],[T]$ respectively.

We know that Force = mass x acceleration

$$\text{Mass} = \frac{Force}{acceleration}$$

$$= \frac{Workdone\ (or)\ Energy}{acceleration \times displacement}$$

$$[m] = \frac{Energy}{[acceleration][displacement]}$$

$$= \frac{[E]}{[LT^{-2}][L]} = \frac{[E]}{L^2T^{-2}} = [EL^{-2}T^2]$$

4. A physical quantity Q is found to depend on quantities x,y,z obeying

relation Q = $\frac{x^2 y^3}{z^1}$. The percentage errors in x, y and z are 2%, 3% and 1% respectively. Find the percentage error in Q.

Solution:

$$\text{Let, } Q = \frac{x^2 y^3}{z}.$$

$$\text{It is given, } \frac{\Delta x}{x} = 2\% \quad \frac{\Delta y}{y} = 3\% \quad \frac{\Delta z}{z} = 1\%$$

$$\frac{\Delta Q}{Q} = 2\left(\frac{\Delta x}{x}\right) + 3\left(\frac{\Delta y}{y}\right) + 1\left(\frac{\Delta z}{z}\right)$$

$$= 2(2\%) + 3\,(3\%) + 1(1\%)$$

$$\frac{\Delta Q}{Q} = 4\% + 9\% + 1\% = 14\%$$

5. The mass and volume of a body are found to be 4±.03 kg and 5±.01 m^3 respectively. Then find the maximum possible percentage error in density.

Solution:

Mass m = 4 ± 0.03 kg (m + Δm)

Volume V = 5 ± .01 m^3 (V + ΔV)

Density = ?

$$\text{Error in mass } = \frac{\Delta m}{m} = \frac{0.03}{4} \times 100$$
$$= 0.75\%$$

$$\text{Error in volume } = \frac{\Delta V}{V} = \frac{0.01}{5} \times 100$$
$$= 0.2\%$$

$$\text{Density} = \frac{mass}{volume}.$$

Error in density = error in mass + error in volume

$$= 0.75\% + 0.2\% = 0.95\%$$

6. Using a Vernier Callipers, the length of a cylinder in different measurements is found to be 2.36 cm, 2.27 cm, 2.26 cm, 2.28 cm, 2.31 cm, 2.28 cm and 2.29 cm. Find the mean value, absolute error, the relative error and the percentage error of the cylinder.

Solution:

The given readings are 2.36 cm, 2.27 cm, 2.26 cm, 2.28 cm, 2.31 cm, 2.28 cm and 2.29 cm

The Mean value $\overline{l}$ =

$$\frac{2.36+2.27+2.26+2.28+2.31+2.28+2.29}{7}$$

$$\frac{16.05}{7} = 2.29 \text{ cm}$$

Absolute errors in the measurement are,

$$|\Delta l_1| = |2.29 - 2.36| = 0.07$$
$$|\Delta l_2| = |2.29 - 2.27| = 0.02$$
$$|\Delta l_3| = |2.29 - 2.26| = 0.03$$
$$|\Delta l_4| = |2.29 - 2.28| = 0.01$$
$$|\Delta l_5| = |2.29 - 2.31| = 0.02$$
$$|\Delta l_6| = |2.29 - 2.28| = 0.01$$
$$|\Delta l_7| = |2.29 - 2.29| = 0.00$$

Mean Absolute error

$$\Delta l_{mean} =$$

$$\frac{0.07+0.02+0.03+0.01+0.02+0.01+0.00}{7}$$

$$= \frac{.16}{7} = .02$$

Relative error

$$= \frac{\Delta l mean}{\bar{l}} = \pm\frac{.02}{2.29} = \pm 8.7 \times 10^{-3}$$

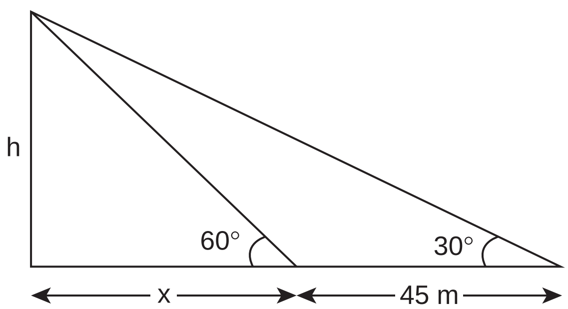
Percentage error $= \pm 8.7 \times 10^{-3} \times 100 =$

$0.87\% \times 100 = \pm(8.7 \times 10^{-1}) = 0.9\%$

7. The shadow of a pole standing on a level ground is found to be 45 m longer when the sun's altitude is 30° than when it was 60°. Determine the height of the pole. [Given $\sqrt{3} = 1.73$]

Solution:

Let the height of the pole be h.



Solution $\frac{x+45}{h} = \cot 30° \Rightarrow h = \frac{x+45}{\cot 30°}$

$$\frac{x}{h} = \cot 60° \Rightarrow x = h\cot 60°$$

Substituting the values of x in the above equation

$$h = \frac{h\cot 60° + 45}{\cot 30°}$$

$$h \cot 30° = h \cot 60° + 45$$

$$h(\cot 30° - \cot 60°) = 45$$

$$h = \frac{45}{\cot 30° - \cot 60°} = \frac{45}{\sqrt{3} - \frac{1}{\sqrt{3}}} = 38.97 \text{ m}$$

8. Calculate the number of times a human heart beats in the life of 100 years old man. Time of one heart beat = 0.8s.

Solution:

Life of the man = 100 years

100 years includes 76 normal years and 24 leap years

Total no of days = 76 × 365 + 24 × 366 = 36524 days

Number of seconds = 36524 × 24 × 3600 = 3.155 x 10^9 second

Number of heart beats =

$$\frac{\textit{Total no of Seconds}}{\textit{Time period of heart beat}}$$

$$= \frac{3.155 \times 10^9}{0.8s} = 3.94 \times 10^9$$

9. The parallax of a heavenly body measured from two points diametrically opposite on equator of earth is 2′. Calculate the distance of the heavenly body. [Given radius of the earth = 6400km] [1″ = 4.85 x 10^{-6} rad]

Solution:

Angle θ = 2^1 = 2 x 60″ = 120″ = 120 × 4.85 × 10^{-6} rad

$$\theta = 5.82 \times 10^{-4} \text{ rad};$$

The distance of heavenly body

$$D = \frac{d}{\theta} = \frac{12800 \times 10^3}{5.82 \times 10^{-4}}$$

$$D = 2.19 \times 10^{10} \text{m}$$

10. Convert a velocity of 72 kmh^{-1} into $m\,s^{-1}$ with the help of dimensional analysis.

Solution:

$n_1 = 72\ kmh^{-1}$ $n_2 = ?\ m\,s^{-1}$

$L_1 = 1Km$ $L_2 = 1m$

$T_1 = 1h$ $T_2 = 1s$

$$n_2 = n_1 \left[\frac{L_1}{L_2}\right]^a \left[\frac{T_1}{T_2}\right]^b$$

The dimensional formula for velocity is $[L\,T^{-1}]$

a = 1 b = -1

$$n_2 = 72\left[\frac{1Km}{1m}\right]^1 \left[\frac{1h}{1s}\right]^{-1}$$

$$n_2 = 72\left[\frac{1000m}{1m}\right]^1 \left[\frac{3600s}{1s}\right]^{-1}$$

$$= 72 \times 1000 \times 1/3600 = 20m\,s^{-1}]$$

$$72\ kmh^{-1} = 20\ m\,s^{-1}$$

11. Check the correctness of the following equation using dimensional analysis. Make a comment on it.
$S = ut + 1/4\ at^2$ where s is the displacement, u is the initial velocity, t is the time and a is the acceleration produced.

Solution:

Dimension for distance s = [L]
Dimension for initial velocity

$$v = [LT^{-1}]$$

Dimension for time t = [T]
Dimension for acceleration

$$a = [LT^{-2}]$$

According to the principle of homogeneity,

Dimensions of LHS = Dimensions of RHS

Substituting the dimensions in the given formula

$S = ut + 1/4\ at^2$, $\frac{1}{4}$ is a number. It has no dimensions

$$[L] = [LT^{-1}]\ [T^1]+[LT^{-2}]\ [T^2]$$
$$[L] = [L] + [L]$$

As the dimensional formula of LHS is same as that of RHS, the equation is dimensionally correct.

Comment:

But actually it is a ***wrong*** equation. We know that the equation of motion is $s = ut + 1/2\ at^2$

So, a dimensionally correct equation ***need not*** be the true (or) actual equation

But a true equation is ***always*** dimensionally correct.

12. Round - off the following numbers as indicated.
a) 17.234 to 3 digits
b) 3.996×10^5 to 3 digits
c) 3.6925×10^{-3} to 2 digits
d) 124783 to 5 digits.

Solution:

a) 17.2 b) 4.00×10^5
c) 3.7×10^{-3} d) 124780

13. Solve the following with regard to significant figures.
a) $\sqrt{4.5 - 3.31}$
b) $5.9 \times 10^5 - 2.3 \times 10^4$

c) 7.18 + 4.3
d) 6.5 + .0136

Solution:

a) Among the two, the least number of significant after decimal is one

$$\sqrt{4.5-3.31} = \sqrt{1.19} = 1.09$$

b) The number of minimum significant figures is 2

$$5.9\times10^5 - 2.3\times10^4$$
$$= 5.9\times10^5 - 0.23\times10^5$$
$$= 5.67\times10^5 = 5.7\times10^5$$

c) The lowest least number of significant digit after decimal is one

7.18 + 4.3 = 11.48 Rounding off we get 11.5

d) The lowest least number of significant digit after decimal is one

$$6.5 + .0136 = 6.5136 = 6.5$$

14. Arrive at Einstein's mass-energy relation by dimensional method ($E = mc^2$)

Solution:

Let us assume that the Energy E depends on mass m and velocity of light c.

$$E \,\alpha\, m^a c^b$$

$E = km^a c^b$ where K a constant

Dimensions of E = $[ML^2T^{-2}]$

Dimensions of m = [M]

Dimensions of c = $[LT^{-1}]$

Substituting the values in the above equation

$$[ML^2T^{-2}] = K\,[M]^a[LT^{-1}]^b$$

By equating the dimensions,

$$a = 1$$
$$b = 2$$
$$-b = -2$$
$$E = k.mc^2$$

The value of constant k = 1

$E = mc^2$ This is Einstein's mass energy relation

15. The velocity of a body is given by the equation $v = b/t + ct^2 + dt^3$. Find the dimensional formula for b.

Solution:

(b/t) should have the dimensions of velocity
b has the dimensions of (velocity x time)

$$[b] = [LT^{-1}][T] = [L] = [M^0L^1T^0]$$

16. The initial and final temperatures of a liquid in a container are observed to be 75.4 ±0.5°C and 56.8 ±0.2°C. Find the fall in the temperature of the liquid.

Solution:

$t_1 = (75.4 \pm 0.5)$°C
$t_2 = (56.8 \pm 0.2)$°C
Fall in temperature = (75.4 ±0.5°C) – (56.8 ±0.2°C)

$$t = (18.6 \pm 0.7)°C$$

17. Two resistors of resistances R_1 = 150 ± 2 Ohm and $R_2 = 220$ ± 6 Ohm are connected in parallel combination. Calculate the equivalent resistance.

Hint: $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

Solution:

The equivalent resistance of a parallel combination

$$R' = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{150 \times 220}{150 + 220} = \frac{33000}{370} = 89.1\ Ohm$$

We know that, $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{\Delta R'}{(R')^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Delta R' = (R')^2 \frac{\Delta R_1}{R_1^2} + (R')^2 \frac{\Delta R_2}{R_2^2}$$

$$= \left(\frac{R'}{R_1}\right)^2 \Delta R_1 + \left(\frac{R'}{R_2}\right)^2 \Delta R_2$$

Substituting the value,

$$\Delta R' = \left[\frac{89.1}{150}\right]^2 \times 2 + \left[\frac{89.1}{220}\right]^2 \times 6$$

$$= 0.070 + 0.098 = 0.168$$

$R' = 89.1 \pm 0.168$ Ohm.

18. A capacitor of capacitance C = $3.0 \pm 0.1 \mu F$ is charged to a voltage of V = $18 \pm 0.4 Volt$. Calculate the charge Q [Use Q= CV]

Solution:

$(C + \Delta C) = (3.0 \pm 0.1)\ \mu f$

$(V + \Delta V) = (18 \pm 0.4)$ V

Q = CV

$$Q = 3.0 \times 10^{-6} \times 18 = 54 \times 10^{-6} \text{ coulomb}$$

$$\text{Error in C} = \frac{\Delta C}{C} \times 100$$

$$= \frac{0.1}{3} \times 100 = 3.3\%$$

$$\text{Error in V} = \frac{\Delta V}{V} \times 100$$

$$= \frac{0.4}{18} \times 100 = 2.2\%$$

Error in Q = Error in C + Error in V = 3.3% + 2.2% = 5.5%

$\therefore$ Charge $Q = (54 \times 10^{-6} \pm 5.5\%)$ coulomb

SOLVED EXAMPLE UNIT-2

1. The position vector for a particle is represented be $\vec{r} = 3t^2\hat{i} + 5t\hat{j} + 6\hat{k}$, find the velocity and speed of the particle at t = 3 sec?

Solution:

$$\vec{v} = \frac{d\vec{r}}{dt} = 6t\hat{i} + 5\hat{j}.$$

The velocity at any time 't' is given by $\vec{v} = 6t\hat{i} + 5\hat{j}$.

The magnitude of velocity is speed. The speed at any time 't' is then given by

$$\text{Speed} = \sqrt{(6t)^2 + 5^2} = \sqrt{36t^2 + 25}$$

Now the velocity at t = 3sec is given by

$$\vec{v} = 6(3)\hat{i} + 5\hat{j} = 18\hat{i} + 5\hat{j}.$$

and speed at t = 3 sec, is given by

$$\text{speed} = \sqrt{349}\ ms^{-1}$$

2. A gun is fired from a place which is at distance 1.2 km from a hill. The echo of the sound is heard back at the same place of firing after 8 second. Find the speed of sound.

Solution:

The echo will be heard when the sound reaches back at the place of firing. So, the total distance travelled by sound is 2 × 1.2 km = 2.4 km = 2400 m.

$$\text{speed} = \frac{2400\text{ m}}{8s} = 300\text{ m s}^{-1}$$

3. A train 100 m long is moving with a speed of 60 kmh^{-1}. In how many seconds will it cross a bridge of 1 km long?

Solution:

Total distance to be covered = 1 km + 100 m = 1100 m (including both bridge and time)

Then, Speed = 60 km h^{-1}

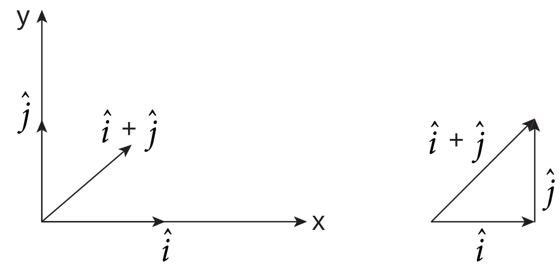
$$= 60 \times \frac{5}{18}\text{ m s}^{-1} = \frac{150}{9}\ ms^{-1}$$

Then, Time taken to cover this

$$\text{distance} = \frac{1100}{150/9}\text{ s} = 66\text{ s}$$

4. Draw the resultant direction of the two unit vectors $\hat{i}$ and $\hat{j}$. Use a 2-dimensional Cartesian co-ordinate system. Is $\hat{i} + \hat{j}$ a unit vector?

By using the triangular law of addition $\hat{i} + \hat{j}$ as shown in the following figure,



The definition of unit vector is $\hat{A}.\hat{A} = 1$

But here,

$$(\hat{i}+\hat{j}).(\hat{i}+\hat{j}) = \hat{i}.\hat{i} + \hat{i}.\hat{j} + \hat{j}.\hat{i} + \hat{j}.\hat{j}$$
$$= 1 + 0 + 0 + 1 = 2$$

So, $\hat{i} + \hat{j}$ is not a unit vector.

To make any vector to a unit vector, must divide the vector by its magnitude,

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

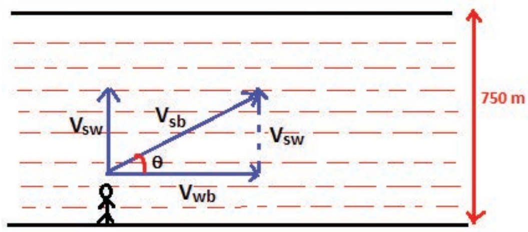
The norm of the vector $\hat{i} + \hat{j} = \sqrt{2}$.

Hence, the unit vector is $\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}$

5. A swimmer moves across the Cauvery river of 750 m wide. The velocity of the swimmer relative to water ($\vec{v}_{sw}$) is 1.5 m s^{-1} and directed perpendicular to the water current. The velocity of water relative to the bank ($\vec{v}_{wb}$) is 1 m s^{-1}. Calculate the
(a) velocity of the swimmer with respect to the bank of the river ($\vec{v}_{sb}$).
(b) time taken by the swimmer to cross the Cauvery river.

Solution:

(a) We can draw the following picture from the given data in the problem.



The velocity of the swimmer relative to the bank $\vec{v}_{sb} = \vec{v}_{sw} + \vec{v}_{wb}$

Since the swimmer travels in the perpendicular direction against the water current

The magnitude is given by

$$|\vec{v}_{sb}| = \sqrt{v_{sw}^2 + v_{wb}^2} =$$

$$\sqrt{1.5^2 + 1^2} = \sqrt{3.25}\, m s^{-1} \cong 1.802\, m s^{-1}$$

The direction of the swimmer relative to the bank is given by

$$tan\theta = \frac{v_{sw}}{v_{wb}} = \frac{1.5}{1} = 1.5$$

$$\theta = tan^{-1}(1.5) \approx 56°$$

(c) The time taken by the swimmer to cross the river is equal to the total distance covered by the swimmer with velocity 1.802 ms^{-1}.

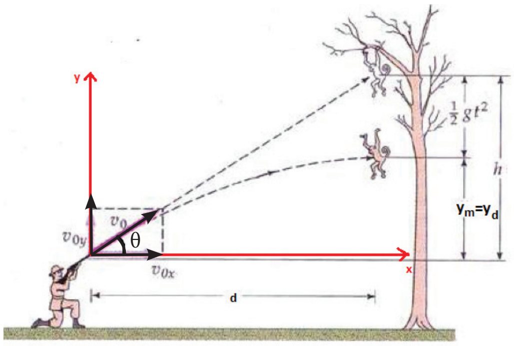
The total distance covered by him,

$$d = \frac{width\ of\ the\ river}{\sin 56^{\circ}} = \frac{750}{0.829} = 904.7\, m$$

The time taken by the swimmer,

$$T = \frac{d}{v_{sb}} = \frac{904.7}{1.802} \approx 502\, s$$

6. A monkey hangs on a tree. A hunter aims a gun at the monkey and fires the bullet with velocity v_0 which makes angle θ with horizontal direction. At the instant gun fires, monkey leaves the branch and falls straight down to escape from the bullet as shown in the figure. Will bullet hit the monkey or will the monkey escape the bullet? (ignore air resistance)



$$y_m = y_d = v_{oy}t - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$$

As soon as the monkey begins to fall, it will have downward vertical motion with acceleration due to gravity g.

Its equation of motion at any time t is

given by

$$y_m = h - \frac{1}{2}gt^2 \qquad (1)$$

When the bullet comes out of the gun, it has both vertical and horizontal components of velocity given by

$$v_{0x} = v_0\, cos\theta ; v_{0y} = v_0\, sin\theta \qquad (2)$$

Let us assume the horizontal distance between the monkey and hunter is 'd'.

At time t, the horizontal distance travelled by the bullet $x = v_0\, cos\theta\, t$.

When the horizontal position of bullet, $x = d$, the time $d = v_{0x}T$. It implies that $T = d / v_{0x}$

At this time T, the vertical distance covered by the bullet is

$$y_b = v_{0y}T - \frac{1}{2}gT^2 = \frac{v_{0y}d}{v_{0x}} - \frac{1}{2}gT^2.$$

$$y_b = \frac{d\,v_0\,sin\theta}{v_0\,cos\theta} - \frac{1}{2}gT^2$$

$$= d\,tan\theta - \frac{1}{2}gT^2 \qquad (3)$$

But from the figure we can write, $tan\theta = \frac{h}{d}$.

$$h = d\,tan\theta\,.$$

By substituting this in the equation (3), we get,

$$y_b = h - \frac{1}{2}gT^2 \qquad (4)$$

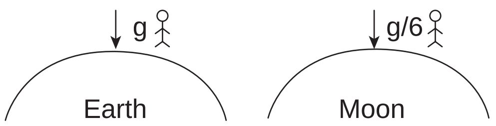
At this same time T, the vertical position of the monkey can be calculated from the equation (1)

$$y_m = h - \frac{1}{2}gT^2 \qquad (5)$$

Note that at the time T, the y coordinate of both monkey and bullet is same. It implies that the bullet will hit the monkey.

7. A three storey building of height 100 m is located on Earth and a similar building is also located on Moon. If two persons jump from the top of these buildings on Earth and Moon simultaneously, when they reach the ground, what will be their speed? ($g = 10$ m s^{-2})

Solution:



For both persons, the Kinematic equations are the same, with u = 0, a_e = g and $a_{moon} = \frac{g}{6}$. Then

$$a_e = g \text{ and } a_m = \frac{g}{6}$$

For a person on earth, $V_{earth} = \sqrt{2gh}$ $= \sqrt{2g100} = \sqrt{2\times10\times100}$

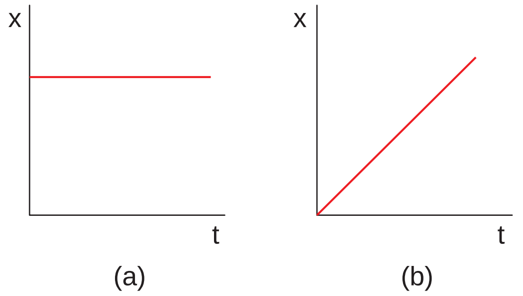
$Hence, V_{earth} = \sqrt{2000}\ m\ s^{-1}$ gives the velocity at the ground, on earth.

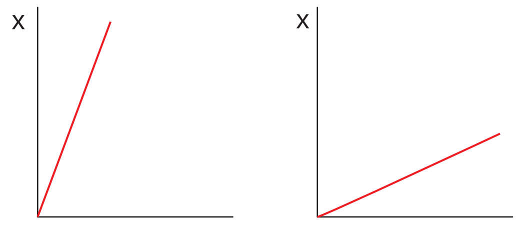
Similarly, for a person on the moon,

$$V_{moon} = \sqrt{\frac{2gh}{6}} = \frac{\sqrt{2000}}{\sqrt{6}}\ m\ s^{-1}$$

The person on earth reaches ground with greater velocity than the person on the moon

8. The following graphs represent position – time graphs. Arrange the graphs in ascending order of increasing speed.





(c) (d)

The slope in the position – time graph will give the speed of the particle.

In the graph (a) slope is zero. Graph (c) has higher slope than graphs (b) and (d). So we can arrange the speeds in ascending order as

$$v_a < v_d < v_b < v_c$$

SOLVED EXAMPLE UNIT-3

1. A body of mass 100 kg is moving with an acceleration of 50 cm s^{-2}. Calculate the force experienced by it.

Solution:

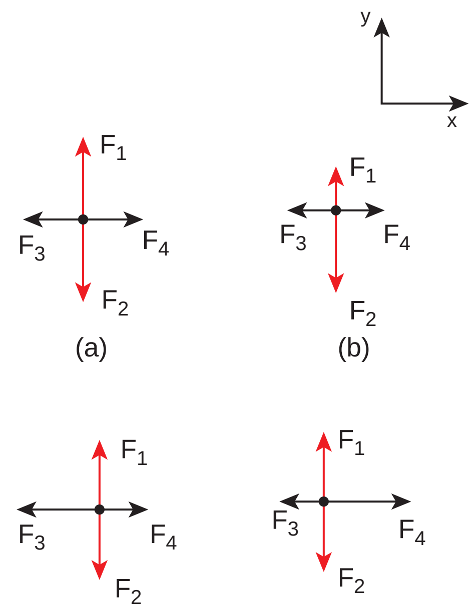
Mass m = 100 kg
Acceleration a = 50 cm s^{-2} = 0.5 m s^{-2}
Using Newton's second law,

$$F = ma$$

$$F = 100 \text{ kg x } 0.5 \text{ m s}^{-2} = 50 \text{ N}$$

2. Identify the free body diagram that represents the particle accelerating in positive *x* direction in the following.

The relative magnitude of forces should be indicated when the free body diagram for mass m is drawn.



Case (a):
The forces F_1 and F_2 have equal length but opposite direction. So net force along y-direction is zero. Since the force is zero, acceleration is also zero along Y-direction (Newton's second law). Similarly in the x direction, F_3 and F_4 have equal length and opposite in direction. So net force is zero in the x direction. So there is no acceleration in x direction.

Case (b):
The forces F_1 and F_2 are not equal in length and act opposite to each other. The figure (b) shows that there are unbalanced forces along the y-direction. So the particle has acceleration in the -y direction. The forces F_3 and F_4 are having equal length and act in opposite directions. So there is no net force along the x direction. So the particle has no acceleration in the x direction.

Case (c):
The forces F_1 and F_2 are equal in magnitude and act opposite to each

other. The net force is zero in y direction. So in y-direction there is no acceleration. The forces F_3 and F_4 are not equal in magnitude and F_3 is greater than F_4. So there is a net acceleration in negative x direction

Case (d):

The forces F_1 and F_2 are equal in magnitude and act opposite to each other. The net force is zero in y direction. So there is no acceleration in y-direction. The forces F_3 and F_4 are not equal in magnitude. The force F_4 is greater than the force F_3. So there is a net acceleration in the positive x direction.

3. A gun weighing 25 kg fires a bullet weighing 30 g with the speed of 200 ms^{-1}. What is the speed of recoil of the gun.

Solution:

Mass of the gun M = 25 kg
Mass of the bullet m = 30 g = 30 × 10^{-3} kg
Speed of bullet v = 200 m s^{-1}
Speed of gun V = ?
The motion is in one dimension.
As per law of conservation of momentum,

$$MV + mv = 0$$

$$V = \frac{-mv}{M}$$

$$V = \frac{-30\times10^{-3}\times200}{25} = -240\times10^{-3}\,ms^{-1}$$

The negative sign shows that the gun moves in the opposite direction of the bullet. Further the magnitude of the recoil speed is very small compared to the bullet's speed.

4. A wooden box is lying on an inclined plane. What is the coefficient of friction if the box starts sliding when the angle of inclination is 45°.

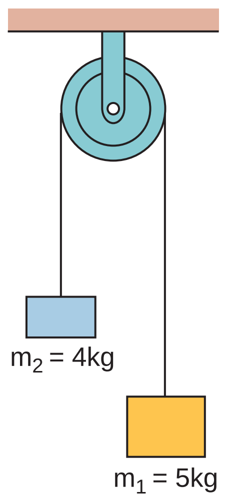
Solution:

Angle of inclination $\Theta = 45°$
∴ Coefficient of friction $\mu = \tan\Theta = \tan 45° = 1$

5. Two masses m_1 = 5 kg and m_2 = 4 kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of each mass when left free to move? (g = 10 m s^{-2})

$$a = \frac{m_1 - m_2}{m_1 + m_2}\times g$$

$$= \frac{5-4}{5+4}\times10 = \frac{1}{9}\times10 = 1.1\,ms^{-2}$$



6. A block of mass m is pushed momentarily along a horizontal surface with an initial velocity u. If μ_k is the coefficient of kinetic friction between the object and surface, find the time at which the block comes to rest.

Solution:

When the block slides, the force acting on the block is kinetic friction which is equal to $f_k = \mu_k mg$

From Newton's second law $ma = -\mu_k mg$

The negative sign implies that force acts on the opposite direction of motion.

The acceleration of the block while sliding $a = -\mu_k g$.

The negative sign implies that the acceleration is in opposite direction of the velocity.

Note that the acceleration depends only on g and the coefficient of kinetic friction μ_k

We can apply the following kinematic equation

$$v = u + at$$

The final velocity is zero.

$$0 = u - \mu_k gt$$
$$t = \frac{u}{\mu_k g}$$

7. Three blocks of masses 10 kg, 7 kg and 2 kg are placed in contact with each other on a frictionless table. A force of 50 N is applied on the heaviest mass. What is the acceleration of the system?

Solution:

We know that

$$a = \left[\frac{F}{m_1 + m_2 + m_3}\right] = \frac{50N}{10\,kg + 7kg + 2\,kg}$$
$$= \frac{50}{19} = 2.63\,m\,s^{-2}$$

8. The coefficient of friction between a block and plane is $\frac{1}{\sqrt{3}}$. If the inclination of the plane gradually increases, at what angle will the object begin to slide?

Since the coefficient of friction is $\frac{1}{\sqrt{3}}$

$$\text{Tan } \Theta = \frac{1}{\sqrt{3}} \Rightarrow \Theta = 30°$$

9. Find the maximum speed at which a car can turn round a curve of 36 m radius on a level road. Given the coefficient of friction between the tyre and the road is 0.53.

Radius of the curve r = 36 m

Coefficient of friction $\mu = 0.53$

Acceleration due to gravity $g = 10\ ms^{-2}$

$$v_{max} = \sqrt{\mu rg} = \sqrt{0.53 \times 36 \times 10}$$
$$= 13.81\,m\,s^{-1}$$

10. Calculate the centripetal acceleration of the Earth which orbits around the

Sun. The Sun to Earth distance is apprriximately 150 million km. (Assume the orbit of Earth to be circular)

The centripetal acceleration $a_c = \dfrac{v^2}{r}$

V - velocity of Earth around the orbit

r - radius of orbit or distance of Earth to Sun

Velocity of Earth is written in terms of angular velocity (ω) as

$$v = \omega r$$

By substituting in the centripetal acceleration formula, $a_c = \dfrac{\omega^2 r^2}{r} = \omega^2 r$

But $\omega = \dfrac{2\pi}{T}$ where T is time for the Earth to orbit around the sun, which is one year.

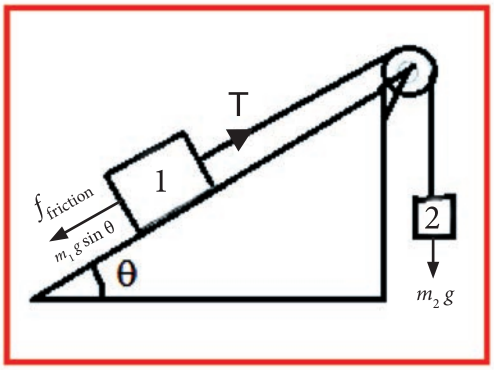
$$\text{T} = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s} = 3.1 \times 10^7 \, s$$

$$\omega = 2.02 \times 10^{-7} \, rad \ per \ sec$$

$$a_c = \left(2.02 \times 10^{-7}\right)^2 \times \left(150 \times 10^9\right)$$

$$a_c = 6.12 \times 10^{-3} \, m s^{-2}$$

11. A block 1 of mass m_1, constrained to move along a plane inclined at an angle θ to the horizontal, is connected via a frictionless, massless and inextensible string that passes over a massless pulley, to a second block 2 of mass m_2. Assume the coefficient of static friction between the block and the inclined plane is μ_s and the coefficient of kinetic friction is μ_k



What is the relation between the masses of block 1 and block 2 such that the system just starts to slip?

Solution:

For all parts of this problem, it will be convenient to use different coordinate systems for the two different blocks. For block 1, take the positive x -direction to be up the incline, parallel to the plane, and the positive y -direction to be perpendicular to the plane, directed with a positive upward component. Take the positive direction of the position of block 2 to be downward.

The normal component N of the contact force between block 1 and the ramp will be

$$N = m_1 g \cos\theta \,. \qquad (1)$$

The net x -component of the force on block 1 is then

$$F_{1x} = T - f_{\text{friction}} - m_1 g \sin\theta \qquad (2)$$

where T is the tension in the string

For the just-slipping condition, the frictional force has magnitude

$$f_{\text{friction}} = \mu_s N = \mu_s m_1 g \cos\theta. \qquad (3)$$

The tension in the string is the gravitational force of the suspended mass,

$$T = m_2 g. \quad (4)$$

For the just-slipping condition, the net force on block 1 must be zero. Equations (2), (3) and (4) gives

$0 = m_2 g - \mu_s m_1 g \cos\theta - m_1 g \sin\theta$

$m_2 = m_1(\mu_s \cos\theta + \sin\theta)$

12. Consider two objects of masses 5 kg and 20 kg which are initially at rest. A force 100 N is applied on the two objects for 5 second.

a) What is the momentum gained by each object after 5 s.
b) What is the speed gained by each object after 5 s.

Final momentum on each object $\Delta P = F\Delta t = 100X\ 5 = 500\ \text{kgms}^{-1}$

Final speed on the object of mass 5 kg = 500 /5 =100 m s^{-1}

Final speed on the object of mass 20 kg =500/20 =25 m s^{-1}

Note that momentum on each object is the same after 5 seconds but speed is not the same after 5 seconds. The heavier mass acquires lesser speed than the one with lower mass.

13. An object of mass 5 kg is initially at rest on the surface. The surface has coefficient of kinetic friction $\mu_k = 0.6$. What initial velocity must be given to the object so that it travels 10 m before coming to rest?

When the object moves on the surface it will experience three forces.

a) Downward gravitational force (mg)
b) Upward normal force (N)
c) Frictional force opposite to the motion of the object.

Since there is no motion along the vertical direction, magnitude of normal force is equal to the magnitude of gravitational force.

$$N = mg$$

Applying Newton's second law along the x direction

$$m\vec{a} = -\mu_k mg\hat{i}$$

The acceleration is $\vec{a} = -\mu_k mg\hat{i}$

Note that the acceleration is along the x direction since the frictional force acts along the negative x direction.

$$\text{Or} \quad a = -\mu_k g$$

Note that the acceleration is uniform during the entire motion. We can use Newton's kinematic equation to find the final velocity.

Along the x direction $v^2 = u^2 + 2as$

Here v = final velocity and u = initial velocity to be given to travel a distance s.

In this problem s = 10 m

Since the particle comes to rest, the final velocity v = 0

$$0 = u^2 - 2\mu_k gs$$

$$u = \sqrt{2\mu_k gs}$$

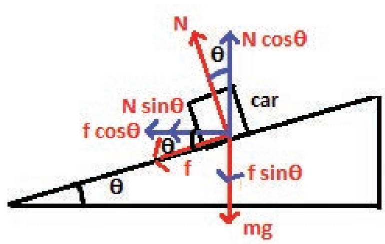
$$u = \sqrt{2 \times 0.6 \times 9.8 \times 10} = 10.8\ ms^{-1}$$

14. In the section 3.7.3 (Banking of road) we have not included the friction exerted by the road on the car. Suppose the coefficient of static friction between the car tyre and the surface of the road is μ_s, calculate the minimum speed with which the car can take safe turn?

When the car takes turn in the banked road, the following three forces act on the car.

(1) The gravitational force mg acting downwards
(2) The normal force N acting perpendicular to the surface of the road
(3) The static frictional force f acting on the car along the surface.

The following figure shows the forces acting on the horizontal and vertical direction.



When the car takes turn with the speed v, the centripetal force is exerted by horizontal component of normal force and static frictional force. It is given by

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \qquad (1)$$

In the vertical direction, there is no acceleration. It implies that the vertical component of normal force is balanced by downward gravitational force and downward vertical component of frictional force. This can be expressed as

$$N\cos\theta = mg + f\sin\theta$$

Or $$N\cos\theta - f\sin\theta = mg \qquad (2)$$

Diving the equation (1) by equation (2), we get

$$\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} = \frac{v^2}{rg} \qquad (3)$$

To calculate the maximum speed for the safe turn, we can use the maximum static friction is given by $f = \mu_s N$. By substituting this relation in equation (3), we get

$$\frac{N\sin\theta + \mu_s N\cos\theta}{N\cos\theta - \mu_s N\sin\theta} = \frac{v_{\max}{}^2}{rg}$$

By simplifying this equation,

$$\frac{N\cos\theta\left\{\left(\frac{N\sin\theta}{N\cos\theta}\right) + \mu_s\right\}}{N\cos\theta\left(1 - \mu_s\frac{N\sin\theta}{N\cos\theta}\right)} = \frac{v_{\max}{}^2}{rg}$$

$$\frac{(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta} = \frac{v_{\max}{}^2}{rg}$$

The Maximum speed for safe turn is given by $v_{\max} = \sqrt{rg\frac{(\tan\theta + \mu_s)}{(1 - \mu_s\tan\theta)}}$ (4)

Suppose we neglect the effect of friction ($\mu_s = 0$), then safe speed

$$v_{safe} = \sqrt{rg\tan\theta} \qquad (5)$$

Note that the maximum speed with which the car takes safe turn is increased by friction (equation (4)). Suppose the car turns with speed $v < v_{safe}$, then the static friction acts up in the slope to prevent from inward skidding.

If the car turns with the speed little greater than, then the static friction acts down the slope to prevent outward skidding. But if the car turns with the speed much greater than v_{safe} then static friction cannot prevent from outward skidding.

SOLVED EXAMPLE UNIT-4

1. A force $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ N acts on a particle and displaces it through a distance $\vec{S} = 4\hat{i} + 6\hat{j}$ m. Calculate the work done.

Solution:

Force $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ N

Distance $\vec{S} = 4\hat{i} + 6\hat{j}$ m

Work done = $\vec{F} \cdot \vec{S} = (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (4\hat{i} + 6\hat{j})$

$= 4+12+0 = 16$ J

2. A particle moves along X- axis from x=0 to x=8 under the influence of a force given by F= $3x^2 - 4x + 5$. Find the work done in the process.

Solution:

Work done in moving a particle from x=0 to x=8 will be

$$W = \int_0^8 F dx = \int_0^8 (3x^2 - 4x + 5) dx$$

$$= \left[\frac{3x^3}{3} - \frac{4x^2}{2} + 5x \right]_0^8$$

$$W = \left[3\frac{(8)^3}{3} - 4\left(\frac{8^2}{2}\right) + 40 \right]$$

$$= [512 - 128 + 40] = 424 J$$

3. A body of mass 10kg at rest is subjected to a force of 16N. Find the kinetic energy at the end of 10 s.

Solution:

Mass m = 10 kg

Force F = 16 N

time t = 10 s

$$a = F/m = \frac{16}{10} = 1.6 m s^{-2}$$

we know that, $v = u + at$

$= 0 + 1.6 \times 10 = 16\ m\ s^{-1}$

Kinetic energy K.E $= \frac{1}{2} mv^2$

$= \frac{1}{2} \times 10 \times 16 \times 16$

$= 1280 J$

4. A body of mass 5kg is thrown up vertically with a kinetic energy of 1000 J. If acceleration due to gravity is $10\ m\ s^{-2}$, find the height at which the kinetic energy becomes half of the original value.

Solution:

Mass m = 5 kg

K.E E = 1000 J

g = $10\ m\ s^{-2}$

$$\text{At a height 'h', mgh} = \frac{E}{2}$$

$$5 \times 10 \times h = \frac{1000}{2}$$

$$h = \frac{500}{50} = 10 m$$

5. Two bodies of masses 60 kg and 30 kg moving in the same direction along straight line with velocity 40 cm s^{-1} and 30 cm s^{-1}

respectively suffer one dimensional elastic collision. Find their velocities after collision.

Solution:

Mass $m_1 = 60$ kg

Mass $m_2 = 30$ kg

$$V_1 = 40\ cms^{-1}$$
$$V_2 = 30\ cms^{-1}$$

Solution:

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$
$$v_2 = \frac{(m_2 - m_1)}{m_1 + m_2}u_2 + \frac{2m_1}{m_1 + m_2}u_1$$

Substituting the values, we get,

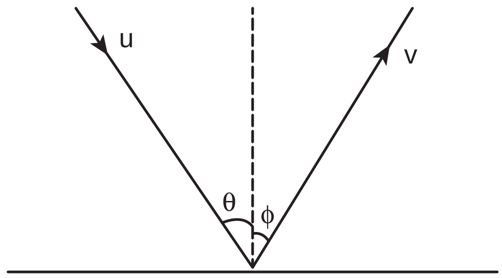
$$v_1 = \frac{(60-30)}{90}\times 40 + \frac{2\times 30}{90}\times 30$$

$$v_1 = \frac{1}{90}\,[\,1200+1800\,]$$
$$= \frac{3000}{90} = 33.3 cms^{-1}$$

Likewise,

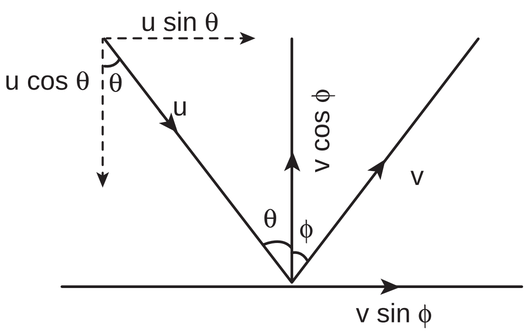
$$v_2 = \frac{(30-60)}{90}\times 30 + \frac{2\times 60}{90}\times 40$$
$$v_2 = \frac{1}{90}[-900+4800]$$
$$= \frac{3900}{90} = 43.3 cms^{-1}$$

6. A particle strikes a horizontal frictionless floor with a speed u at an angle θ with the vertical and rebounds with the speed v at an angle ϕ with the vertical. The coefficient of restitution between the particle and floor is e. What is the magnitude of v?



Solution:

Applying component of velocities,



The x - component of velocity is

$$u\sin\theta = v\sin\phi \qquad (1)$$

The magnitude of y – component of velocity is not same, therefore, using coefficient of restitution,

$$e = \frac{v\cos\varphi}{u\cos\theta} \qquad (2)$$

Squaring (1) and (2) and adding we get

$$v^2\sin^2\varphi = u^2\sin^2\theta$$
$$v^2\cos^2\varphi = e^2u^2\cos^2\theta$$
adding
$$v^2 = u^2\sin^2\theta + e^2u^2\cos^2\theta$$
$$\therefore v^2 = u^2\left[\sin^2\theta + e^2\cos^2\theta\right]$$
$$v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

7. A particle of mass m is fixed to one end of a light spring of force constant k and un-stretched length *l*. It is rotated with an angular velocity ω in horizontal circle. What will be the length increase in the spring?

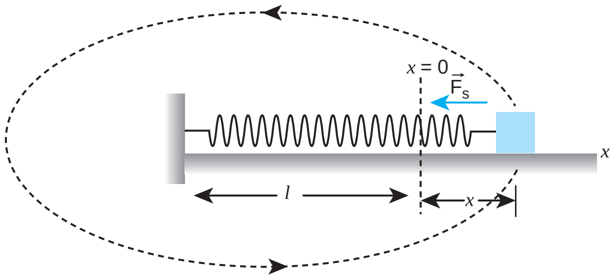
Solution:

Mass of particle = m

Force constant = k

Un-stretched length = *l*

Angular velocity = ω



Let 'x' be the increase in the length of the spring.

The new length = (*l*+x) = r

When the spring is rotated in a horizontal circle,

Spring force = centripetal force.

$$kx = m\omega^2(l+x)$$

$$x = \frac{m\omega^2 l}{k - m\omega^2}$$

8. A gun fires 8 bullets per second into a target X. If the mass of each bullet is 3 g and its speed 600 ms^{-1}, then calculate the power delivered by the bullets.

Solution:

Power = work done per second = total kinetic energy of 8 bullets per second

$P = 8 \times$ (kinetic energy of each bullet per second)

$$= 8 \times \frac{1}{2} \times (3 \times 10^{-3}) \times (600)^2$$

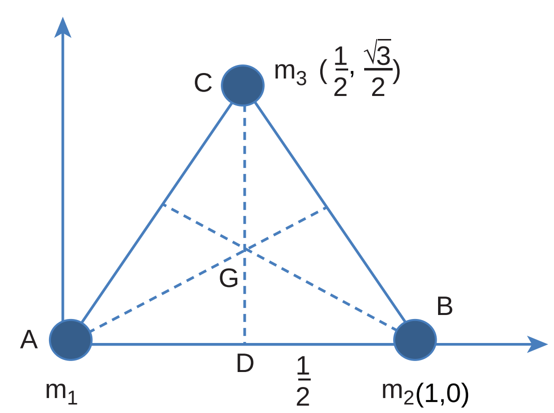
P = 4320 W

P = 4.320 kW

SOLVED EXAMPLE UNIT-5

1. Three particles of masses m_1 = 1 kg, $m_2 = 2$ kg and $m_3 = 3$ kg are placed at the corners of an equilateral triangle of side 1m as shown in Figure. Find the position of centre of mass.

Solution:



The centre of mass of an equilateral triangle lies at its geometrical centre G.

The positions of the mass m_1, m_2 and m_3 are at positions A, B and C as shown in the Figure.

From the given position of the masses, the coordinates of the masses m_1 and m_2 are easily marked as (0,0) and (1,0) respectively.

To find the position of m_3 the Pythagoras theorem is applied. As the ΔDBC is a right angle triangle,

$$BC^2 = CD^2 + DB^2$$
$$CD^2 = BC^2 - DB^2$$

$$CD^2 = 1^2 - \left(\frac{1}{2}\right)^2 = 1 - \left(\frac{1}{4}\right) = \frac{3}{4}$$
$$CD = \frac{\sqrt{3}}{2}$$

The position of mass m_3 is

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \text{ or } \left(0.5, 0.5\sqrt{3}\right)$$

X Coordinate of centre of mass,

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$x_{CM} = \frac{(1\times 0) + (2\times 1) + (3\times 0.5)}{1+2+3} = \frac{3.5}{6}$$
$$x_{CM} = \frac{7}{12}\text{ m}$$

Y Coordinate of centre of mass,

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{(1\times 0) + (2\times 0) + (3\times 0.5\times\sqrt{3})}{1+2+3} = \frac{1.5\sqrt{3}}{6}$$
$$y_{CM} = \frac{\sqrt{3}}{4}\text{ m.}$$

∴ The coordinates of centre of mass G

$$\left(x_{CM}, y_{CM}\right) \text{ is } \left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$$

2. An electron of mass $9\times 10^{-31}\,kg$ revolves around a nucleus in a circular orbit of radius 0.53 Å. What is the angular momentum of the electron? (Velocity of electron is, $v = 2.2\times 10^{6}\,m\,s^{-1}$)

Solution:

Mass of the electron, $m = 9\times 10^{-31}\,kg$

Radius of the electron, r = 0.53 Å = $0.53\times 10^{-10}\,m$

Velocity of the electron, v = $2.2\times 10^{6}\,m\,s^{-1}$

Angular momentum of electron is, $L = I\,\omega$

Electron is considered as a point mass. Hence, its moment of inertia is, $I = m\,r^2$

The relation, $\omega = \frac{v}{r}$ could be used.

Angular momentum, $L = mr^2\times\frac{v}{r}$

$= mvr$

$$= 9.1\times 10^{-31}\times 2.2\times 10^{6}\times 0.53\times 10^{-10}$$
$$L = 1.06\times 10^{-34}\,kg\,m^2\,s^{-1}$$

3. A solid sphere of mass 20 kg and radius 0.25 m rotates about an axis passing through the centre. What is the angular momentum if the angular velocity is 5 rad s^{-1}

Solution:

Mass of the sphere, m = 20 kg

Radius r = 0.25 m

Angular velocity $\omega = 5 \text{ rad s}^{-1}$

Solution:

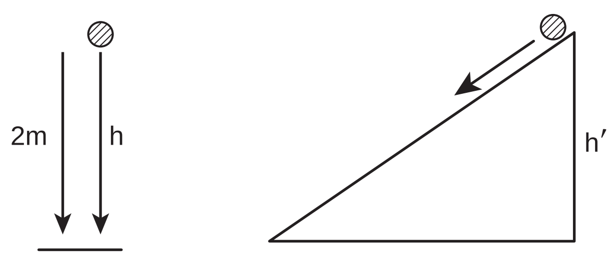
Angular momentum $L = I\omega = \frac{2}{5}mr^2\omega$

$$= \frac{2}{5} \times 20 \times (0.25)^2 \times 5 = 40 \times (0.0625) = 2.5$$

$$L = 2.5 \text{ kg m}^2\text{s}^{-1}$$

4. A solid cylinder when dropped from a height of 2 m acquires a velocity while reaching the ground. If the same cylinder is rolled down from the top of an inclined plane to reach the ground with same velocity, what must be the height of the inclined plane? Also compute the velocity.

Solution:



In the first case,

potential energy = kinetic energy

$$mgh = \frac{1}{2}mv^2$$

$$mg \times 2 = \frac{1}{2}mv^2 \qquad (1)$$

In second case,

potential energy = translational kinetic energy + rotational kinetic energy

$$mgh' = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh' = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{v^2}{r^2}\right)$$

$$\therefore mgh' = \frac{3}{4}mv^2 \qquad (2)$$

Dividing (2) by (1),

$$\frac{mgh'}{mg \times 2} = \frac{\frac{3}{4}mv^2}{\frac{1}{2}mv^2} = \frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$$

$$h' = 3\,m$$

From equation (1), $2mg = \frac{1}{2}mv^2$

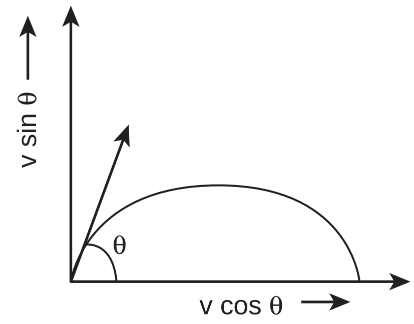
$$v = \sqrt{4g} = 2\sqrt{g}$$

$$v = 2 \times \sqrt{9.81}$$

$$v = 6.3\,m\,s^{-1}$$

5. A small particle of mass m is projected with an initial velocity v at an angle θ with x axis in X-Y plane as shown in Figure. Find the angular momentum of the particle.

Solution:



Let the particle of mass m cross a horizontal distance x in time t.

$$\text{Angular momentum } \vec{L} = \int \vec{\tau}\, dt$$

$$\text{But } \vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{r} = x\hat{i} + y\hat{j} \text{ and } \vec{F} = -mg\hat{j}$$

$$\therefore \vec{\tau} = \left(x\hat{i} + y\hat{j}\right) \times \left(-mg\hat{j}\right)$$

$$\vec{\tau} = -mgx\left(\hat{i} \times \hat{j}\right) = -mgx\hat{k}$$

$$\vec{L} = -mg\int \left(xdt\right)\hat{k} = -mgv\cos\theta\left(\int tdt\right)\hat{k}$$

Let initial time t = 0 and final time $t = t_f$

$$\therefore \vec{L} = -mgv\cos\theta\left(\int_0^{t_f} tdt\right)\hat{k} = -\frac{1}{2}mgv\cos\theta t_f^2\ \hat{k}$$

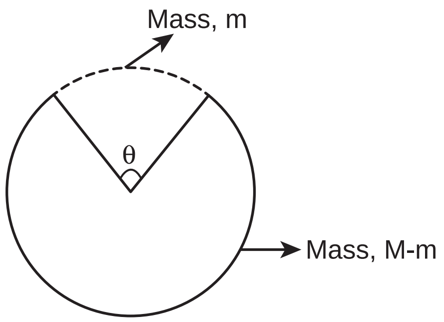
Negative sign indicates, $\vec{L}$ point inwards

6. From a complete ring of mass M and radius R, a sector angle θ is removed. What is the moment of inertia of the incomplete ring about axis passing through the centre of the ring and perpendicular to the plane of the ring?

Solution:

Let R be the radius of the ring and M be the total mass of the complete ring.

Let m be the mass of the section removed from the ring then, mass of the incomplete ring is M-m



Let us introduce a positive integer (n), such that, $n\theta = 360^\circ$, or $n = \frac{360^\circ}{\theta}$

$$mass\ of\ incomplete\ ring = M - m$$

$$m = \frac{M}{360} \times \theta$$

$$\therefore mass\ of\ incomplete\ ring = M - \frac{M}{360} \times \theta$$

$$mass\ of\ incomplete\ ring = M - \frac{M}{n} = M\frac{(n-1)}{n}$$

For example, $a)\ when\ \theta = 60^\circ;\ n = \frac{360^\circ}{60^\circ} = 6$

$$\therefore n - 1 = 5$$

$$\text{mass of incomplete ring} = \frac{5}{6}M$$

$$b)\ when\ \theta = 30^\circ,\ n = \frac{360^\circ}{30^\circ} = 12$$

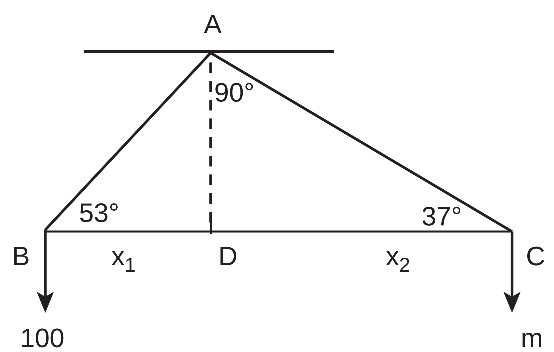
$$n - 1 = 11$$

$$\text{mass of incomplete ring} = \frac{11}{12}M$$

The moment of inertia of the incomplete ring is, $I = M\frac{(n-1)}{n}R^2$

7. A massless right angled triangle is suspended with its right angle corner. A mass of 100 kg is suspended from another corner B which subtends an angle 53°. Find the mass m that should be suspended from other corner C so that BC (hypotenuse) remains horizontal.

Solution:



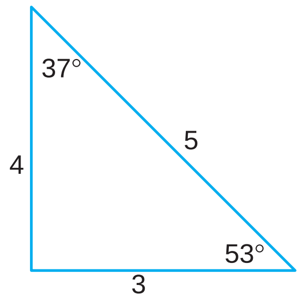
From the principle of moments,

$$100 \times g \times x_1 = m \times g \times x_2$$

$$100 \times \cos 53° = m \times \cos 37° \qquad (1)$$

Where, x_1 and x_2 are the arm lengths.

The right angle triangle with angles 37°, 53° and 90° is a special triangle which has the respective sides in the ratio, 3:4:5 as shown in the diagram.



Substituting the values in equation (1),

$$100 \times \cos 53° = m \times \cos 37°$$

$$100 \times \frac{3}{5} = m \times \frac{4}{5}$$

$$m = 100 \times \frac{3}{4}$$

$$m = 75\,kg$$

8. If energy of 1000 J is spent in increasing the speed of a flywheel from 30 rpm to 720 rpm, find the moment of inertia of the wheel.

Solution:

$$\omega_1 = 30rpm = 2\pi \times \frac{30}{60} rads^{-1} = \pi\ rads$$

$$\omega_2 = 720rpm$$

$$= 2\ \pi \times \frac{720}{60} rads^{-1} = 24\pi\ rads$$

Change in kinetic energy,

$$\Delta KE = \frac{1}{2} I(\omega_2^2 - \omega_1^2)$$

$$I = \frac{2 \times \Delta KE}{(\omega_2^2 - \omega_1^2)} = \frac{2 \times 1000}{(24\pi)^2 - (\pi)^2}$$

$$I = \frac{2000}{25\pi \times 23\pi}$$

Remember:

$a^2 - b^2 = (a + b)(a - b)$

$I \approx 0.35\ kg\ m^2$ as $\pi^2 \approx 10$

9. Consider two cylinders with same radius and same mass. Let one of the cylinders be solid and another one be hollow. When subjected to same torque, which one among them gets more angular acceleration than the other?

Solution:

Moment of inertia of a solid cylinder about its axis $I_s = \frac{1}{2}MR^2$

Moment of inertia of a hollow cylinder about its axis $I_h = MR^2$

$$I_s = \frac{1}{2}I_h \text{ or } I_h = 2I_s$$

$$torque\ \tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$\alpha_s = \frac{\tau}{I_s} \text{ and } \alpha_h = \frac{\tau}{I_h}$$

$$\alpha_s I_s = \alpha_h I_h \Rightarrow \alpha_s = \alpha_h \frac{I_h}{I_s}$$

Since, $$I_h > I_s \Rightarrow \frac{I_h}{I_s} > 1$$

$$\therefore \alpha_s > \alpha_h$$

For the same torque, a solid cylinder gets more acceleration than a hollow cylinder.

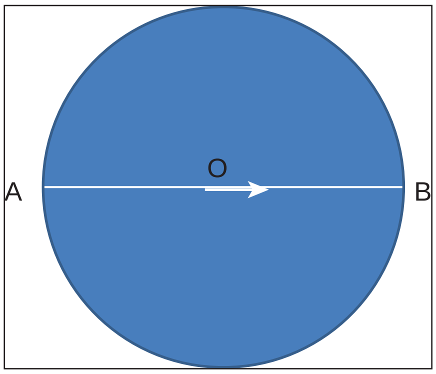
Note: The above two cylinders must be made up of materials of different density. (Say why?)

10. **A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect goes from A to point B along its diameter as shown in Figure. Discuss how the angular speed of the circular disc changes?**

Solution:

As the disc is freely rotating, with the insect on it, the angular momentum of the system is conserved.

$$L = I\omega = \text{constant}$$



When the insect moves towards the centre (from A to O), the moment of inertia (I) decreases. Thus, the angular velocity (ω) increases. When it moves away from centre (from O to B), the moment of inertia (I) increases. Thus, the angular velocity (ω) decreases.

11. (i) What is the shape of the graph between $\sqrt{E_{kr}}$ and L? (E_{kr} is the rotational kinetic energy and L is angular momentum)

(ii) What information can you get from the slope of the graph?

(iii) You are given the graph of $\sqrt{E_{kr}}$ and L for two bodies A and B. Which one has more moment of inertia?

Solution:

i) We know that, Rotational kinetic Energy

$$E_{kr} = \frac{1}{2}I\omega^2$$

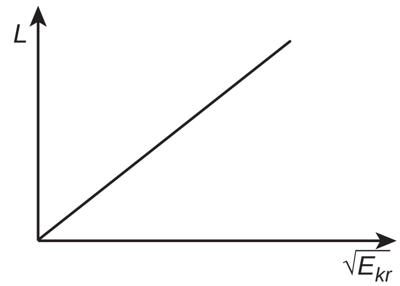
$$= \frac{1}{2}I\omega \times \omega = \frac{1}{2}.L.\omega = \frac{1}{2}\frac{L^2}{I} \qquad \because L = I\omega,\ \omega = L/I$$

$$E_{kr} = \frac{L^2}{2I}$$

$$L^2 = 2IE_{kr}$$

$$L = \sqrt{2IE_{kr}} = \sqrt{2I}.\sqrt{E_{kr}}$$

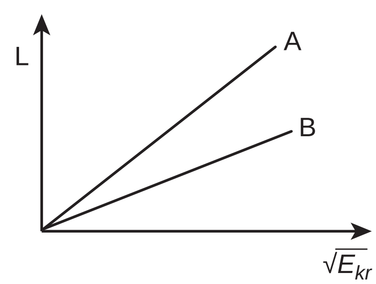
Graph of $\sqrt{E_{kr}}$ and L:



The shape of the graph is a straight line

ii) The slope of the graph gives the value of moment of inertia I.

iii) We know that the slope gives the value of moment of Inertia. The line A has higher slope and hence more moment of Inertia.



12. **Consider a thin uniform circular ring rolling down in an inclined plane without slipping. Compute the linear acceleration along the inclined plane if the angle of inclination is 45°.**

Solution:

The linear acceleration along the inclined plane can be computed by

$$a = \frac{g \sin\theta}{1 + \frac{K^2}{R^2}}$$

For a thin uniform circular ring, axis passing through its centre is $I = MR^2$

$$\therefore K^2 = R^2 \Rightarrow \frac{K^2}{R^2} = 1.$$

And the angle of inclination, $\theta = 45^\circ$

$\Rightarrow (\sin 45^\circ = \frac{1}{\sqrt{2}})$

Hence,

$$a = \frac{\frac{g}{\sqrt{2}}}{1+1}$$

$$a = \frac{g}{2\sqrt{2}} ms^{-2}$$

COMPETITIVE EXAM CORNER

APPENDIX 2

AWET7

Table A1.1 Systematic developments in Physics over centuries

Observation of motion of Sun, Stars, Planets and Moon	Around 3000 BC (Early Greeks)
In the universe everything is changing. Nothing remains in the same state indefinitely. The idea of 'time' started with this understanding.	Around 500 BC (Heraclitus).
Everything is composed of entirely small indivisible elements called 'atoms'. Development of theory of 'atomism'. But that time it is a hypothesis. No experimental proof.	End of 500 BC (Democritus and Leucippus)
▪ Idea of natural laws behind every day phenomena ▪ Idea of motion with gravity ▪ Theory of four elements(Everything is made up of Earth, water, fire, air able to inter-transform into each other) ▪ Earth is centre of universe(Hypothesis) ▪ Force is required to move the object ▪ Heavier object falls faster towards Earth than lighter object	Around 3rd century BC (Aristotle)
▪ Earth is sphere ▪ Measurement of radius of the Earth almost accurately	Around 240 BC (Eratosthenes)

Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ Sun is centre of the solar system (Hypothesis. Could not be proven by experimental evidence) ▪ Idea of Earth rotation around its own axis.	Around 2nd century BC (Aristarchus of Samos) Around 2nd century BC (Seleucia)
▪ Foundation of hydrostatics ▪ Idea of lever ▪ Mechanical work using pulleys ▪ Law of Buoyancy known as a Archimedes principle ▪ First accurate value of the number 'pi'	Around 3rd century BC (Archimedes)
▪ Focused on motion of planets and stars ▪ Prediction of solar eclipses ▪ Calculation of distance of Earth to Moon, Earth to Sun ▪ Astronomical observation were recorded	End of 2nd century BC (Hippachrus)
▪ Geo centric model (Not hypothesis. Explained a lot of naked eye observation) ▪ Explanation of Planets 'retrograde motion' ▪ "Almagest"- First book on astronomy	Around 100 AD(CE) (Ptolemy)
▪ Idea of Earth's rotation about its own axis ▪ Idea of zero ▪ Contribution to mathematics	5th Century AD (Aryabhatta-India)
▪ Understanding of early optics ▪ Boon on 'Treasury of astronomy'- Accurate astronomical table than Ptolemy's data	9th century AD (Ibn al-Hayatham- Arabia) 12th Century AD (Nasir al-Din- Persian astronomer)
▪ From 7th century to 14th century major development in science happened in muslim countries (Arabia, Persia, Iran etc.)	

Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ Copernicun Revolution ▪ Heliocentric model (Not hypothesis. It provides simplest explanation than Ptolemy model for motion of stars and Planets) ▪ Accurate astronomical datas ▪ Laws of Planetary motion	15^{th} Century AD AD 1543 (Copernicus) Tycho Brahe Kepler
▪ Law of inertia ▪ Telescope observation(Founder of modern observational astronomy) ▪ Calculation of time period of the Moons of the Jupiter ▪ Earth is not flat ▪ All object fall to Earth at the same rate (Disproved Aristotle's argument) ▪ Law of inertia(Force need not required to maintain the motion- disproved Aristotle's argument) ▪ Pendulum, inclined plane experiments ▪ Study of projectile motion ▪ Introduction of 'controlled experiments'	Gallieo Galilei (1564- 1642)
▪ Introduction of Cartesian coordinate system ▪ Idea of analytical geometry	Rane Descarte (1596-1650)
▪ 17^{th} and 18^{th}century Development	
▪ Laws of motion ▪ Quantitative idea of motion ▪ Law of gravitation ▪ Development of calculus independent of Leibnitz ▪ Founder of modern optics (reflection, dispersion, prism) ▪ Light consists of minute particles 'corpuscules' ▪ Derivation of Kepler laws ▪ Greatest book 'The principia mathematica'(1687) ▪ Wave theory of light	Isaac Newton (1642-1727) Christian Huygens; Father of modern optics

Table A1.1 Systematic developments in physics over centuries (*Cont*)

■ Work on magnetism	William Gilbert (Around 1600)
■ Behavior of gases	Robert Boyle (1627- 1691)& Robert Hooke
■ Fluid dynamics - Bernoulli's theorem (1734)	Daniel Bernoulli(1700- 1782)
■ Early ideas of kinetic theory of gases	Daniel Bernoulli (1700 - 1782)
■ Study of spring motion(Hooke's law)	Robert Hooke
■ Derivation of frequency of vibration in strings(1714)	Taylor
■ Reformulation of Newtonian mechanics using Energy approach (Lagrangian mechanics)	De Alembert, Lagrange
■ Invention of steam engine(1781)-	James Watt
■ Force between charges (Coulomb's law)	Coulomb
■ 19^{th} century development	
■ Early ideas of Thermodynamics(1840s)	James Joule, Carnot
■ Caloric theory	
■ Laws of thermodynamics(1850s)	Kelvin, Clausius
■ Behavior of gases, velocity and speed(1860s)	James Clark Maxwell
■ Foundation of statistical mechanics and entropy formula (around 1870s)	Boltzmann
■ Wave nature of light- Experiments	Young and Fresnel
■ Behavior of electric charges	
■ Magnetic effect of electric current(1820s)	Oersted
■ Force between two parallel currents	Ampere
■ Principle of Least action, Hamilton mechanics (1821)	William Hamilton
■ Electric powered motor, electricity demonstration	Micheal Faraday
■ Theory of electromagnetism(1873)- Bridge between electricity and magnetism	James Clerk Maxwell
■ Maxwell equations –Paved way to modern technology	
■ Alternating current	Tesla

Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ 20^{th} century developments	
▪ Study of black body radiation ▪ Discovery of an electron ▪ Rutherford atomic model ▪ Study on radioactivity	Max planck(around 1900) J.J.Thomson Rutherford(1910s) Marie Curie(1920s and 30s)
▪ Special theory of relativity, Photo electric effect, Existence of atom(1905), $E=mc^2$ ▪ Revolution of physics after Newton ▪ New Idea of space and time ▪ The General theory of relativity(Greatest theory of 20^{th} century)- 1915 ▪ Study of specific heat capacities	Albert Einstein
▪ Study of atoms ▪ Bohr atom model (1912) ▪ Behavior of electron, proton – Schrodinger equation ▪ Uncertainty principle ▪ Formulation of Quantum Mechanics ▪ Formulation of quantum field theory ▪ Particle physics, standard model ▪ X-ray diffraction(1930s)- Paved way to understanding the materials ▪ Raman effect ▪ Study of stars and black holes ▪ Invention of transistor(1947) ▪ Classification of stars using temperature (Astro thermodynamics), Saha ionization formula ▪ Field of Cosmology (1920s) ▪ Expanding universe model(1922) ▪ Discovery of redshift	 Niels Bohr Schrodinger Heisenberg Paul Dirac Dirac, Feynman, Schwinger Gellman, Weinberg, Abdus salam C.V. Raman(India) Chandrasekhar(Indian origin) John Bardeen, Walter Brattain,William Schokley Megnad Saha(India) Eddington, Schwarchild Thomas Friedmann Edwin Hubble

Table A1.1 Systematic developments in physics over centuries (*Cont*)

▪ Birth of materials science ▪ Nanotechnology, Condensed matter physics ▪ Gravitational waves, Dark energy, Dark matter, String theory	

APPENDIX A1.2

vi) Error involving the division of two quantities (using the method of differentiation)

Consider $Z = \frac{A^n}{B^m}$

Taking logarithms of both the sides,

$$\log Z = \log A^n - \log B^m$$
$$\log Z = n\log A - m\log B$$

Differentiating both sides, we get,

$$\frac{dZ}{Z} = n\frac{dA}{A} - m\frac{dB}{B}$$

In terms of fractional error, this may be written as,

$$\pm\frac{\Delta Z}{Z} = \pm n\frac{\Delta A}{A} \mp m\frac{\Delta B}{B}$$

Maximum possible relative error in z is given by

$$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A} + m\frac{\Delta B}{B}$$

vii) Error involving the product of three quantities (using calculus method)

let $u = x^m y^n Z^p$

Taking logarithms on both sides, we get

$$\log u = \log x^m + \log y^n + \log Z^p$$
$$\log u = m\log x + n\log y + p\log Z$$

Differentiating both sides, we get

$$\frac{du}{u} = m\frac{dx}{x} + n\frac{dy}{y} + p\frac{dz}{z}$$

In terms of fractional error, the equation maybe rewritten as,

$$\pm\frac{\Delta u}{u} = \pm m\frac{\Delta x}{x} \pm n\frac{\Delta y}{y} \pm p\frac{\Delta z}{z}$$

APPENDIX A2.1

Parallelogram Law of Vector addition:

If two non-zero vectors $\vec{A}$ and $\vec{B}$ are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

Consider the figure 2.19 below. Here the two vectors $\vec{A}$ and $\vec{B}$ are connected by a common tail at an angle θ. The parallelogram OACB is next constructed.

Then the diagonal OC is the resultant $(\vec{R})$ passing through the common tail O.

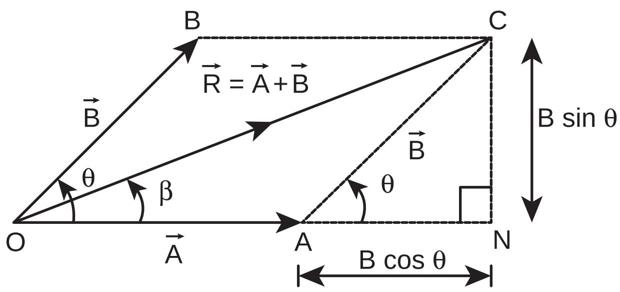


Figure 2.19 Magnitude of resultant vector by parallelogram method

We next find the magnitude and direction of this resultant vector.

(i) Magnitude:

First extend OA to the point N, so that we get ON. Then CN is drawn perpendicular to this ON, from C. Then ONC is a right angled triangle. We can write $R^2 = ON^2 + CN^2$

$$R^2 = (OA + AN)^2 + CN^2$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\therefore R = |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Special cases:

When $\theta = 0°$, then R = A+B

When $\theta = 180°$, then R = A-B

When $\theta = 90°$, then $R = \sqrt{A^2 + B^2}$

(ii) Direction

Let β be the angle between the vectors $\vec{A}$ and $\vec{R}$. Then

$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

APPENDIX 3

Some important formulae for Integral

(1) $\int dx = x$; $\frac{d}{dx}(x) = 1$

(2) $\int x^n dx = \left(\frac{x^{n+1}}{n+1}\right)$; $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$

(3) $\int cudx = c\int udx$ where c is a constant

(4) $\int (u \pm v \pm w)dx = \int udx \pm \int vdx \pm \int wdx$

(5) $\int \frac{1}{x}dx = \ln x + \text{C}$

(6) $\int e^x dx = e^x + \text{C}$

(7) $\int \cos\theta d\theta = \sin\theta + \text{C}$

(8) $\int \sin\theta d\theta = -\cos\theta + \text{C}$

Some important formulae in Differential calculus

1. $\frac{d}{dx}(c) = 0,$ if c is a constant

2. If y = cu, where c is a constant

   and u is a function of x then
   $$\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$$

3. If y = u ±v ±w where u,v and w are functions of x then
   $$\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w) = \frac{du}{ds} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$

4. If $y = x^n$, where n is the real number then
   $$\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$$

5. If y = uv, where u and v are functions of x then
   $$\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

6. If y is a function of x, then $dy = \frac{dy}{dx}.dx$

7. $\frac{d}{dx}(e^x) = e^x$

8. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

9. $\frac{d}{d\theta}(\sin\theta) = \cos\theta$

10. $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$

11. If y is a trigonometric function of θ and θ is the function of t, then
    $$\frac{d}{dt}(\sin\theta) = \cos\theta\frac{d\theta}{dt}$$
    $$\frac{d}{dt}(\cos\theta) = -\sin\theta\frac{d\theta}{dt}$$

APPENDIX 4

(THE GREEK ALPHABET)

(கிரேக்க எழுத்துகள்)

The Greek Alphabet	Upper Case	Lower Case
Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	Ε	ε
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Iota	Ι	ι
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	€	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ
Upsilon	Υ	υ
Phi	Φ	ϕ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω

SOME IMPORTANT CONSTANTS IN PHYSICS

Name	Symbols	Value
Speed of light in vacuum	c	2.9979×10^{8} m s^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2} kg^{-2}
Acceleration due to gravity (sea level, at 45° latitude)	g	9.8 m s^{-2}
Planck constant	h	6.626×10^{-34} J s
Boltzmann constant	k	1.38×10^{-23} J K^{-1}
Avogadro number	N_A	6.023×10^{23} mol^{-1}
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Stefan – Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Wien's constant	b	2.898×10^{-3} m K
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Standard atmospheric pressure	1 atm	1.013×10^{5} Pa