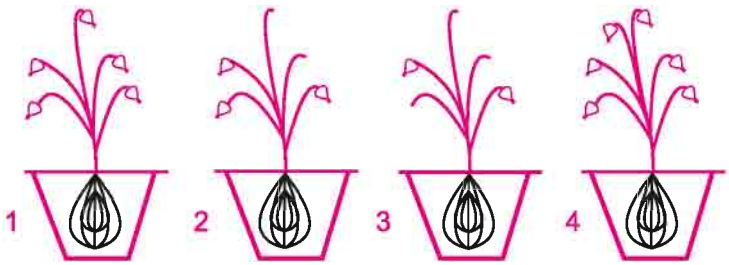
Chapter 12

Algebra

12.1 Our study so far has been with numbers made from the digits 0 to 9. We also learnt different operations on these numbers. We applied our knowledge of numbers to various problems in our day to day life. The branch of mathematics in which we studied numbers is arithmetic. Sometimes, arithmetic alone is not effective enough to solve complicated problems of numbers. Hence, we begin the study of another branch of mathematics to handle problems that we can't solve using just arithmetic. It is called algebra. In algebra, we often use letters like (a,b,c .....) to represent numbers. Letters are used as symbols for generalizing numbers. These letters are called literals.

By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities.

12.2 The Idea of a Variable See the figure below:



In the above figure, some seeds are sowed. Number of fruits obtained from each seed is as follows:

Seed 1 Number of fruits = 5
Seed 2 Number of fruits = 3
Seed 3 Number of fruits = 2
Seed 4 Number of fruits = 6

For convenience, let us write the letter 'x' for the number of fruits obtained from a seed. Then,

For seed 1 $x = 5$
For seed 2 $x = 3$
For seed 3 $x = 2$
For seed 4 $x = 6$

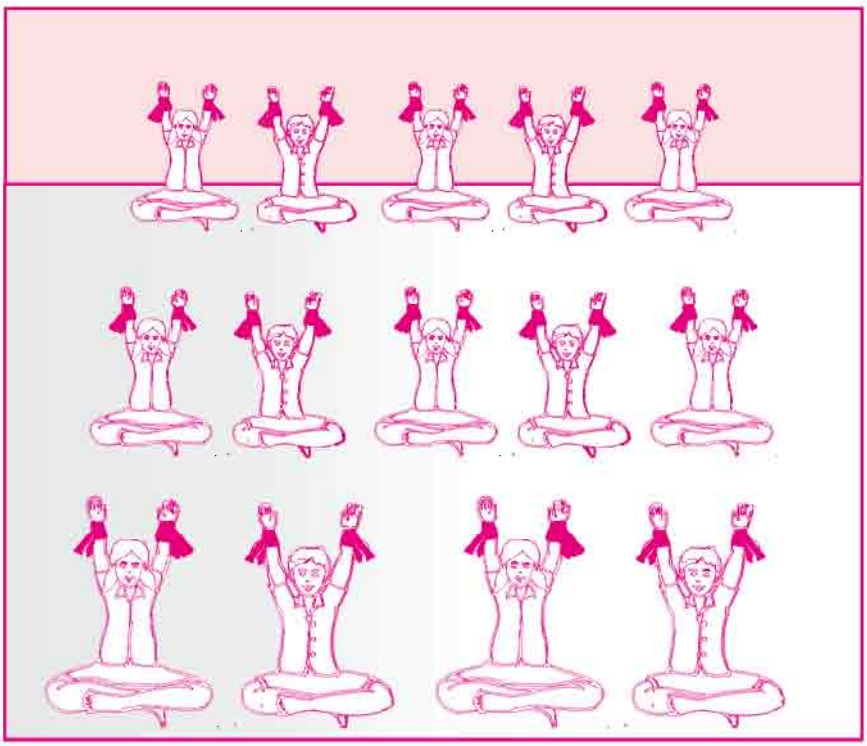
The value of x (number of fruits obtained) goes on changing for different seeds. x is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4, ... The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values. Thus, a Variable is a number which can take different values in different situations. Instead, the arithmetic numbers which have fixed values are called Constants.

Do and Learn.

1. Let y denote age in years. Write down the value of y for your 10 friends.

12.2.1 The Yoga Class

Let, the Yoga class is going on in your school. Students are doing Yoga with their hands up. Ribbons are tied in each hand of each student. Fill in the table below with the help of the figure given below.



Number of students	1	2	3	4	5						
Number of ribbons	2	4	6	8	10						

Table 12.1

How many ribbons are tied to 10 students?
As per the table, the answer is 2 x 10.
Hence, number of ribbons = 2 x number of students
Let n denotes number of students. Then,
Number of ribbons = 2 x n
n = 1, 2, 3, 4,.....
As per the table, number of ribbons keeps on increasing with increasing value of n.

12.2.2 Generalising with Matchstick Patterns

Chinu and Chotu are making patterns with matchsticks. They decide to make triangular patterns. They make 2 triangles. Their friend Ramu comes in. He looks at the pattern and forms one more triangle. Ramu always asks questions. He asks Chinu, "How many matchsticks will be required to make further triangles"? They go on forming the patterns and prepare a table.

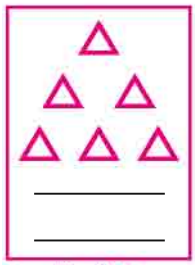


Fig. 12.1

Number of triangles formed	1	2	3	4	5	6
Number of matchsticks required	3	6	9	12	15	18

Table 12.2

How many matchsticks will be required to make 8 triangles?

Number of matchsticks required = 3 × Number of triangles formed

If T denotes number of triangles formed, then,

number of matchsticks required = 3 × T

Here, T is an example of variable whose value is not fixed. T = 1,2,3,4, ...

12.3 Algebraic Expressions

Game of matchsticks – Raju and his friends are making patterns with matchsticks. Raju puts a matchstick on a table. Pappu takes two matchsticks and forms an open container. Then Kavita also picks two sticks, forms second open container adjacent to the one made by Pappu. Then Sanju also forms third open container adjacent to the one made by Kavita. Following the same pattern, how many matchsticks are required to form 8 containers? Let's make a table.

Number of containers formed	1	2	3	4	.....	n
Number of matchsticks required	3	5	7	.....	.....	.....

Table 12.3

While writing the table, Raju realises that 2 matchsticks are required to add one container.

Number of matchsticks required for 1 container = 1 + 2 × 1

Number of matchsticks required for 2 containers = 1 + 2 × 2

Number of matchsticks required for 3 containers = 1 + 2 × 3

Number of matchsticks required for n containers = 1 + 2 × n = 1 + 2n, here n is a variable.

(1 + 2n) is an example of an algebraic expression. An algebraic expression is an expression built up from constants, variables, and the algebraic operations like addition, subtraction, multiplication, division, etc. For example, in the algebraic expression (1 + 2n), n is multiplied by 2 and then 1 is added to the product. Algebraic expression can be monomial (1 term), binomial (2 terms), or polynomial (more than 2 terms).

Now, we will learn to form more algebraic expressions.

	Algebraic expression	How it is formed
(i)	$2+7$	adding 7 in 2
(ii)	$x+9$	adding 9 in x
(iii)	$p+q$	adding q in p
(iv)	$a-3$	subtracting 3 from a
(v)	$3-b$	subtracting b from 3
(vi)	$x-y$	subtracting y from x
(vii)	$3 \times x$	x is multiplied by 3
(viii)	$\frac{13}{a}$	13 is divided by a
(ix)	$\frac{x}{y}$	x is divided by y

Write 10 other such simple expressions and tell how they have been formed.

Do and Learn.

1. Write algebraic expressions through given instructions about how to form it.
   (i) Sum of 5 and a variable----------
   (ii) Difference between 7 and a variable----------
   (iii) 3 times of a variable----------
   (iv) 12 less than 6 times of a variable----------
   (v) Half of a variable----------
   (vi) 200 less than one third of a variable----------
2. Shweta secured 75 marks in Mathematics. Her score in Science is not known. Let her Science score be x. What is her total score?
3. Sakshi has some candies with her. Ashu has 4 times as many candies as Sakshi. How many candies are there in total?.....

Forming Algebraic expressions with given statements.

	Statement	Algebraic expression
(i)	Subtracting 7 from z	$z-7$
(ii)	Subtracting 4 from p	$p-4$
(iii)	Subtracting 16 from a	$a-16$
(iv)	y is divided by 3	$\frac{y}{3}$

(v)	Multiplying m by 7	7m
(vi)	Multiplying x by 3	$3x$
(vii)	Multiplying x by 5	$5x$

Do and Learn.

Match the algebraic expressions with appropriate situations in the following:

(i)	$x+4$	(a) Prashant has 4 times as many wealth as Kamli.
(ii)	$x-4$	(b) Malti has Rs.4/- more than Seema.
(iii)	$4-x$	(c) My weight is 4 kgs less than Nancy.
(iv)	$4y$	(d) I had Rs.4/- from which I spent some money. How much I am left with?
(v)	$\frac{y}{4}$	(e) Banshi had some marbles. He distributed them between his 4 friends equally. How many marbles each friend get?

Exercise 12.1

1. Make the matchstick patterns of the letters given below. Draw the figures of the patterns in your notebook. Create rules to find out number of matchsticks required for each pattern. (You can use literals like a, b, x, y, etc to create rules.)
   - (i) Pattern of T — T, TT, TTT,……
   - (ii) Pattern of N — N, NN, NNN,…
   - (iii) Pattern of W — W, WW, WWW,…
2. Tree Plantation Program was held in a school. 4 Trees were planted in each row. Write the number of trees planted in terms of the number of rows.
3. Ranu is 5 years younger than Leela.
   - (i) Let Leela's age be x years. Write the age of Ranu in terms of x.
   - (ii) Let Ranu's age be P years. Write the age of Leela in terms of P.
4. Cost of a pen is Rs.5/-. Madan has some money with him. He pays all of that money to buy those pens. Write the number of pens purchased in terms of the money he had.

5. Complete the table given below:

x	1	2	3	4	5	---	---
$2x+3$	5	7	---	---	--	15	---

12.4 Rules for Algebraic Expressions

12.4.1 Commutativity

(i) For Addition – We know that interchanging the order of numbers in addition does not change the sum. For example, $2+5=5+2$.

This property of numbers is known as the commutativity of addition of numbers. Commuting means interchanging. So it means that if the order of numbers is commuted in addition, the sum will remain same. Similarly, if x and y are two variables, then, we can say that, $\boxed{x+y=y+x}$

(ii) For Multiplication – Similarly, for the product of two numbers, $5\times2 = 2\times5=10$

That is, for multiplication of two numbers, the order of the two numbers being multiplied does not matter. This property of numbers is known as commutativity of multiplication of numbers. $\boxed{x \times y = y \times x}$

12.4.2 Distributivity - Suppose we are asked to calculate 8×35. We obviously do not know the table of 35. So, we do the following-

$$8\times35=8\times(30+5)$$
$$= 8\times30+8\times5$$
$$= 240+40$$
$$= 280$$

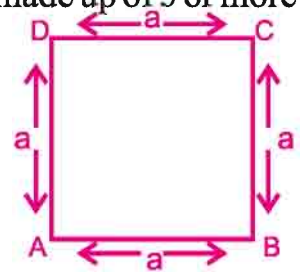
By using variables, we can write this property of numbers also in a general and concise way. Let x, y and z be three variables, each of which can take any number. Then

$$\boxed{x \times (y+z) = x \times y + x \times z}$$

This property is known as distributivity of multiplication over addition of numbers.

12.4.3 Rules from Geometry (in the form of algebraic expressions) We can write the perimeters of rectangle and square in the form of algebraic expressions. We know that perimeter of any polygon (a closed figure made up of 3 or more line segments) is the sum of the lengths of its sides

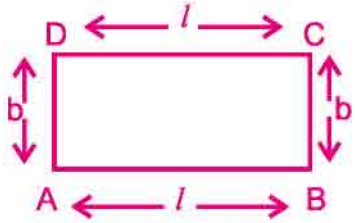
(i) Perimeter of a Square



Perimeter of the square ABCD = AB + BC + CD + DA

$$= a+a+a+a$$
$$= 4a = 4\times \text{side}$$

(ii) Perimeter of a Rectangle- Opposite sides of a rectangle are equal. Thus, in the rectangle ABCD, let us denote by l, the length of the sides AB or CD and, by b, the breadth of the sides AD or BC.



Therefore, Perimeter of a rectangle

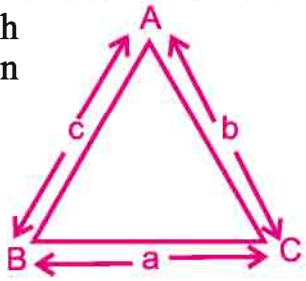
= length of AB + length of BC + length of CD + length of AD

= 2 × length of CD + 2 × length of BC

$= 2l + 2b$

$= 2(l + b)$

Here, both l and b are variables. They take on different values for different rectangles. Also, they take on values independent of each other. i.e. the value one variable takes does not depend on what value the other variable has taken.



(iii) Perimeter of a triangle-

Perimeter of a triangle = sum of the lengths of its sides

= BC + CA + AB

= a + b + c

Exercise 12.2

1. With the numbers 3, 7 and 4, form arithmetic expressions using-
   (i) Only addition and subtraction operations.
   (ii) Only multiplication and addtion operations.
2. For each of the below expressions, mention whether it is an arithmetic expression or algebraic expression?
   (i) $3x + 5$ (ii) $5 \times 4 + 7$ (iii) $3 + 4 \times 3 + 5$
   (iv) $2x + 1$ (v) $\frac{x}{2} + 5 - x$ (vi) $3x$
3. Observe the expressions in the table carefully. Mention, what all operations are used to form that expression by putting the sign of Right/Wrong in the table.

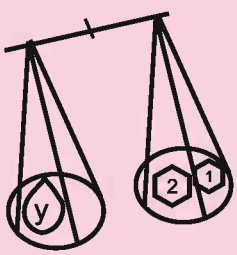
S.No.	Expression	Addition	Subtraction	Multiplication	Division
1	$x + 5$				
2	$7m + 3$				
3	$y - 3x$				
4	$x - y - z$				
5	$3x - 10 - \frac{z}{5}$				
6	$\frac{y}{17}$				

4. Write algebraic expressions for the following situations.
   (i) 7 added to a (ii) 10 subtracted from b (iii) x multiplied by 4
   (iv) x divided by 4 (v) x subtracted from 7 (vi) 10 divided by q
5. Give expressions for the following cases.
   (i) 15 added to 2n (ii) 15 subtracted from $2x$
   (iii) 3 added to twice of p (iv) 3 subtracted from twice of q
   (v) 11 subtracted from the product of y and 5
   (vi) 11 added to the product of z and -3
6. Form algebraic expressions using q, 5 and -3.
7. Nathu has Rs x/- with him. Then,
   (i) How much money do Bina have if she owns twice as much as Nathu does?
   (ii) How much money is Nathu left with after buying books worth Rs. 150/-?
   (iii) How much money do Seema have if she owns half as much as Nathu had initially?
   (iv) How much money do Milli have if she owns thrice as much as Nathu does?
8. The height of a triangle is 5 more than twice of its base. What is its height if base is b?
9. The present age of Vimal is p years.
   (i) How old was he 10 years ago?
   (ii) How old will Vimal be 5 years from now?
   (iii) Vimal's aunt is thrice as old as Vimal. How old is Vimal's aunt?
   (iv) Age of Vimal's mother is 5 years less than twice of Vimal's age. How old is his mother?

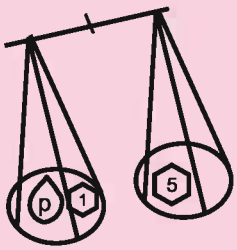
12.5 Introduction to Equations

Seema bought a papaya from a fruit seller. He measured the weight of the papaya in the weighing scale. He put the papaya on one pan. The scale was balanced when he put a 2 kgs weight on another pan of the scale. Denoting the weight of the papaya with a variable x, we can write it in a mathematical sentence as, $x = 2$.

Observe the mathematical sentence in the following conditions.



Y= 2+1



P+1=5

These mathematical sentences are called Equations.

12.5.1 Forming Equations from given Statements

Observe the equations formed from the statement given.

Twice of a number is 10.	$2n = 10$
5 more than thrice of a number is 17.	$3n + 5 = 17$
3 less than half of a number is 6.	$\frac{x}{2} - 3 = 6$
Adding 15 in twice of a number results in 51.	----------

12.5.2 Deriving Statements from given Equations

$3x = 21$	Thrice of a number is 21.
$2x - 7 = 19$	7 less than twice of a number is 19.
$23 = 4x + 3$	23 is 3 more than 4 times a number.
$3x - 7 = 11$	.....

Thus, a relation between some constants and variables in which mathematical operations are used along with the 'equal to' sign "=" is called an Equation. An equation has two sides: left hand side (L.H.S.) and right hand side (R.H.S) which are separated with an 'equal to' sign.

12.5.3 Solution of an Equation

Let us take the equation: $x + 1 = 5$

L.H.S. R.H.S.

Let's try and find out the value of x in above equation which satisfies the equation.

Value of x	L.H.S.	R.H.S	Result
0	0 + 1 = 1	5	Not equal
1	1 + 1 = 2	5	Not equal
2	2 + 1 = 3	5	Not equal
3	3 + 1 = 4	5	Not equal
4	4 + 1 = 5	5	Equal

Keeping the value of x as 4, L.H.S. of the equation becomes equal to R.H.S. of the equation. Hence, the equation is satisfied when x takes the value 4.

Let us take one more equation: $3x - 2 = 2x + 1$

L.H.S. ↗ ↖ R.H.S.

Value of x	L.H.S.	R.H.S.	Result
0	$3\times0-2=-2$	$2\times0+1=1$	Not equal
1	$3\times1-2=3-2=1$	$2\times1+1=2+1=3$	Not equal
2	$3\times2-2=6-2=4$	$2\times2+1=4+1=5$	Not equal
3	$3\times3-2=9-2=7$	$2\times3+1=6+1=7$	Equal

The value of the variable in an equation which satisfies the equation is called a solution to the equation. The method we used to find the solution is a trial and error method.

Exercise 12.3

1. State which of the following are equations (with a variable). Identify the variable from the equations with a variable.

(i) $5x = 0$ (ii) $t - 7 > 5$ (iii) $4 \div 2 = 2$

(iv) $2x - 1 < 5$ (v) $7 = 14 \times 2 + q$ (vi) $15000 = 2t + 3500$

2. For the equation, $10y = 50$, pick out the solution which satisfies the equation from the values $y = 10$, $y = 8$ and $y = 5$.
3. A possible solution is given with each of the equations given below. Put the value of the variable in the equation and show that the value satisfy / do not satisfy the equation.

(i) $3x - 7 = 5$ $x = 5$ (ii) $3p + 2 = 8$ $p = 2$

4. Complete the table and by inspection of the table find the solution to the equation :

(i) $3x = 15$

x	0	1	2	3	4	5	6	7
$3x$	0	3						

(ii) $\frac{p}{3} = 4$

P	1	2	3	4	5	6	7	8	9	10	11	12
$\frac{P}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$									

(iii) $x - 3 = 5$

x	1	2	3	4	5	6	7	8	9	10	11
$x-3$	-2	-1	0								

we learnt

1. We learnt making patterns using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape.

2. A variable takes on different values, its value is not fixed. For example, the radius of a circle can have any value. It is a variable. But the sum of the interior angles of a quadrilateral has a fixed value. It is not a variable.

3. We may use any letter x, y, z, p, q etc. to show a variable.

4. Using different operations we formed expressions with variables and constants like:
   $x+4, x-3, 3p, 4q$, etc.

5. Variables allow us to express many common rules in both algebra and arithmetic in a general way. For example, The commutative law of addition and the commutative law of multiplication can be expressed as
   $a+b=b+a$ and $a \times b = b \times a$ respectively.

6. An equation is a condition on a variable. It is expressed by saying that an expression with a variable is equal to a fixed number, e.g. $x+4=8$.

7. We learnt trial and error method to get the solution of an equation. In this method, we give some value to the variable and check whether it satisfies the equation. We go on giving this way different values to the variable, until we find the right value which satisfies the equation.