WORKSHEET 2

POLYNOMIALS

1. Find the zeros of the polynomial $p(x) = abx^2 + (b^2 - ac)x - bc$ and verify the relationship between the zeros and its coefficients.

2. Form a quadratic polynomial whose zeros are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

3. If α and β are the zeros of polynomial by $p(x) = x^2 - x + k(x + 1)$, such that $\alpha - \beta = 1$, find the value of k.

4. If the sum of squares of the zeros of the quadratic polynomial $p(x) = x^2 - 8x + k$ is is 40, find the value of k.

5. Find the quadratic polynomial each with the given numbers as the sum and the product of its zeros respectively:

(i) $\frac{1}{5}$ and -2 (ii) 0 and 6

6. If α and β, are the zeros of the polynomial $p(x) = x^2 - 5x + 6$, find a polynomial whose zeros are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

7. If the squared difference of the zeros of the quadratic polynomial $p(x) = x^2 + px + 45$ is equal to 144. Find the value of p.

8. If one of the zero of the quadratic polynomial $p(x) = 4x^2 - 8kx - 9$ is negative of the other, find the value of k.

9. Divide the polynomial $30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 - 2x - 4$. Also find the quotient and remainder.

10. Divide the polynomial $p(x) = 5 + x^2 - x^3 + \ldots$ by a polynomial $q(x) = x - 1 - x^2$ and verify division algorithm.

11. On dividing the polynomial $p(x) = x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient $q(x)$ and remainder $r(x)$ where $q(x) = x - 2$ and $r(x) = -2x + 4$ respectively. Find the polynomial $g(x)$.

12. Find the quadratic polynomial such that the sum of its zeros is 23 and the difference between the zeros is 7.

13. If α and β are the zeros of the polynomial $x^2 - 15x + k$ such that $\alpha - \beta = 3$. Find the value of k.

14. If α and β are the zero of the polynomial $x^2 - x - 6$, find a polynomial whose zeros are $\frac{\alpha^2}{\beta^2}$ and $\frac{\beta^2}{\alpha^2}$.

15. If α and β are the zero of quadratic polynomial $p(x) = x^2 - a(x + 1) - b$, then prove that $(\alpha + 1)(\beta + 1) = 1 - b$.

16. If the two zeros of the polynomial $p(x) = x^4 - 4x^3 - 4x^2 + \ldots$ are $2 \pm \sqrt{3}$. Find the other zeros.

17. What should be added to $p(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $g(x) = x^2 + 2x - 3$.

+ 4

18. If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other, then find the value of k.

19. If the polynomial $p(x) = ax^4 + bx - c$ is exactly divisible by the polynomial $g(x) = x^2 + bx + c$, then find the value of ab.

20. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.

Answers

1. $\frac{-a}{b}, \frac{c}{b}$ 2. $(x^2 \ldots)$ 3. $k = 6$ 4. $k = 12$

5. (i) $5x^2 - x - 1$ (ii) $2x^2 - 3x - 12$; 6. $x^2 - 25x + 156$ 7. ±1 8 8. 0

9. $Q = 10x^2$ $R = 24x$ 11. $x^2 - x + 1$ 12. $x^2 - 23x + 120$ 13. 54

14. $36x^2 - 97x + 1$ 16. −1, 1 17. 6 1 x 65 18. $k = 2$

19. 1 20. $-57x + 70$

WORKSHEET 3

POLYNOMIALS

1. Real number α is a zero of the polynomial $f(x)$ if
(a) $f(\alpha)$ (b) $f(\alpha)$ (c) $f(\alpha)$ (d) none

2. The zeroes of a polynomial $f(x)$ are the coordinates of the points where the graph of $y = f(x)$ intersects.
(a) x-axis (b) y-axis (c) origin (d) (x, y)

3. If β is a zero of $f(x)$, then ________ is one of the factors of $f(x)$.
(a) $(x - \beta)$ (b) $(x - 2\beta)$ (c) $(x + \beta)$ (d) $(2x - \beta)$

4. If $(x - c)$ is a zero a factor of $f(x)$, then ________ is a zero of $f(x)$.
(a) x (b) c (c) $2c$ (d) $2x$

5. If 2 is a zero of $f(x) = x^2 - x + k$, then the value of k is
(a) 0 (b) – 2 (c) 1 (d) 3

6. Cubic polynomial $x = f(y)$ cuts y-axis at atmost
(a) one point (b) two point (c) three points (d) four points

7. If 1 is one of the zeroes of the polynomial, $x^2 - ax + b$ then the relationship between a and b is
(a) $a - b = 0$ (b) $a - b - 1 = 0$ (c) $a + 1 = b$ (d) $a + b + 1 = 0$

8. If the degree of a polynomial $f(x)$ is 4, then maximum number of zeroes of $f(x)$ would be
(a) 4 (b) 8 (c) 5 (d) 3

9. If 3 is a zero of both $2x^2 + ax - 12$ and $3x + b$ then the value of $b - 4a$ is
(a) 1 (b) 0 (c) 2 (d) 4

10. The zeroes of the polynomial $f(x) = (x - 1)(x^2 - 5x + 6)$ are
(a) 1, – 2, 3 (b) – 1, 2, 3 (c) 1, 2, – 3 (d) 1, 2, 3

Answers

1.	(b)	2.	(a)	3.	(a)	4.	(b)	5.	(b)
6.	(c)	7.	(b)	8.	(a)	9.	(a)	10.	(d)

ACTIVITY 1

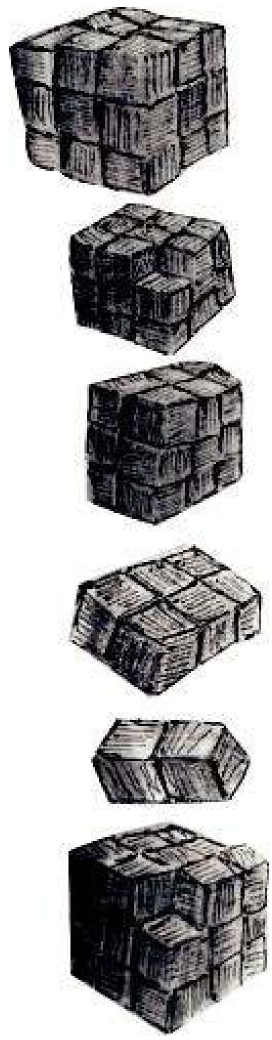
Aim:

To verify the algebraic identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ using unit cubes.

Material Required:

unit cubes

Procedure:



Let $a = 3$ and $b = 1$

Step-1: To represent a^3, make a cube of dimension $a \times a \times a$ ie. $3 \times 3 \times 3$ cubic units.

Step-2: To represent $a^3 - b^3$, extract a cube of dimension $b \times b \times b$ ie. $1 \times 1 \times 1$ from the cube formed in Step-1 of dimension $a \times a \times a$ ie. $3 \times 3 \times 3$ cubic units.

Step-3: To represent $(a - b)a^2$, make a cuboid of dimensions $(a – b) \times a \times a$ ie. $2 \times 3 \times 3$ cubic units.

Step-4: To represent $(a - b)ab$, make a cuboid of dimensions $(a – b) \times a \times b$ ie. $2 \times 3 \times 1$ cubic units.

Step-5: To represent $(a - b)b^2$, make a cuboid of dimensions $(a – b) \times b \times b$ ie. $2 \times 1 \times 1$ cubic units.

Step-6: To represent $(a - b)a^2 + (a - b)ab + (a - b)b^2$ ie. $(a – b)(a^2 + ab + b^2)$. Join all the cuboids formed in the Steps 3, 4 and 5.

Observe the following:

The number of unit cubes in a^3 =

The number of unit cubes in b^3 =

The number of unit cubes in $a^3 - b^3$ =

The number of unit cubes in $(a - b)a^2$ =

The number of unit cubes in $(a - b)ab$ =

The number of unit cubes in $(a - b)b^2$ =

The number of unit cubes in $(a - b)a^2 + (a - b)ab + (a - b)b^2$ =

Observation:

It is observed that the number of unit cubes in $a^3 - b^3$ is equal to the number of unit cubes in $(a - b)a^2 + (a - b)ab + (a - b)b^2$ i.e. $(a - b)(a^2 + ab + b^2)$.

WORKSHEET 4

(RECAPILUTATION)
POLYNOMIALS

Q1. Fill in the blanks

(i) A…………… polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.

(ii) The zeroes of a polynomial $p(x)$ are precisely the …………………… of the points, where the graph of $y = p(x)$ intersects the x -axis.

(iii) A quadratic polynomial can have at most ………….. zeroes and a cubic polynomial can have at most …………… zeroes.

(iv) If a and b are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then $a + b$ = …………… and ab = ………………

(v) The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x)\,q(x) + r(x)$, where $r(x) = 0$ or degree $r(x)$ ……………. degree $g(x)$.

Q2. Find the quadratic polynomial, the sum and product of whose zeroes are

(i) ½, – 2

(ii) – 3, – 7

(iii) $5 + \sqrt{3}$ and $5 - \sqrt{2}$

Q3. Find the zeroes of the quadratic polynomial $p(x) = t^2 - 15$.

Q4. How many maximum zeroes can a polynomial of degree two has?

Q5. What is the zero of the polynomial $ax + b = 0$, $a \neq 0$?

Q6. Find the sum and the product of the zeroes of the polynomial $6x^2 - x - 2$.

Q7. Give examples of polynomials $f(x)$, $g(x)$, $q(x)$ and $r(x)$ which satisfy the division algorithm

(i) deg $r(x) = 0$

(ii) deg $f(x)$ = deg $q(x) = 2$

Q8. Find the quadratic polynomial whose one zero is 5 & product of zeroes is 30.

Q9. The linear polynomial ax + b, a ≠ 0, has exactly one zero, namely, the__________ of the point where the graph of y = ax + b intersects the x-axis.

Q10. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^3 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

Q11. Consider a polynomial $7x^3 + 6x^2 - 5x - 4$

Give your comments on the following statements:

• The coefficient of x is 5

____

• This is a polynomial

____

• Its constant term is – 4

____

• Its degree is 4

____

• It is a cubic polynomial.

____

• The coefficient of x^2 is 6.

____

- If we remove $-5x$ term then it will become a trinomial
- If we remove any of one term, then it will become a trinomial
- If we add one more term having degree 4 then it will become a polynomial of degree 4.

WORKSHEET 5

POLYNOMIALS

1. If $(x-1)$ is a factor of $mx^2 - \sqrt{2}x + 1$, then the value of m is
(a) $\sqrt{2}$ (b) $\sqrt{2}+1$ (c) $\sqrt{2}-1$ (d) 1

2. On factorising $x^2 + (p + 1/p)\,x + 1$, we get
(a) $(x+p)(x-p)$ (b) $(p+1/p)(x-p)$ (c) $(p+1/p)(x+p)$ (d) $(x-1/p)(x-p)$

3. Which of the following is a polynomial?
(a) $x^2 - (1/2x) - 5$ (b) $x^2 + \sqrt{x} - 3$ (c) $(1/\sqrt{x}) + 7$ (d) $x^2 - (1/2)x - 5$

4. The value of polynomial $2x^3 - 3x^2 + 5x - 3$ at $x = 0$ is
(a) 3 (b) 0 (c) -3 (d) 1

5. The degree of $(4x-3)(4x+3)$ is
(a) 4 (b) 16 (c) 2 (d) 1

6. The value of k, for which the polynomial $x^3 - 3x^2 + 3x + k$ has 3 as its zero is
(a) 9 (b) -3 (c) -9 (d) 12

7. Which of the following is a zero of the polynomial $x^3 + 3x^2 - 3x - 1$?
(a) 1 (b) 2 (c) -1 (d) -2

8. On factorizing $-x^2 + 5x - 6$, we get
(a) $(x-2)(x-3)$ (b) $(2+x)(3-x)$ (c) $(2-x)(3-x)$ (d) $-(2-x)(3-x)$

9. The factors of $(2x-3y)^3 + (3y-4z)^3 + 8(2z-x)^3$ are
(a) $2x \times 3y \times 4z$ (b) $2(2x-3y)(3y-4z)(2z-x)$
(c) $6(2x-3y)(3y-4z)(2z-x)$ (d) $3(2x-3y)(3y-4z)(2z-x)$

10. If $P(x) = cx + d$, then zero of polynomial will be
(a) $-d/c$ (b) d/c (c) c/d (d) $-c/d$

Fill in the blanks

1. The value of $P(-1)$ when $P(x)$ is $x^3 - 3x^2 + 5$ is………..
2. On factorizing $8p^2 - 16q^2$, we get …………..
3. The zero of polynomial $P(x) = cx + d$ is ……..

4. The coefficient of x in $-x + t$ is …………..
5. Degree of $(4x - 3)(3x^3 + 4)$ is ………..
6. On expanding $(\sqrt{2}x + 3)^3$, we get ………….
7. The value of k, if $(x+1)$ is a factor of $3x^2 + x + k$ is ………….
8. When $x^{41} + 41$ is divided by $(x + 1)$, the remainder will be ……
9. The value of $f(\sqrt{3})$, when $f(x) = 3x\ 3 + 10$ is …………………
10. The standard form of $6q^6 - 7q^2 + 6q^3$ is …………….

WORKSHEET 6

POLYNOMIALS MIXED ASSIGNMENT

Q1. Define a polynomial.
Q2. Define a quadratic polynomial. Give an example also.
Q3. Define a cubic polynomial. Give an example also.
Q4. Fill in the blanks:

(a) A polynomial of two terms is called as___________.
(b) The degree of cubic polynomial is ____________.
(c) The degree of $\sqrt{5}\,y - 3$ is __________.
(d) If $a + b + c = 0$, then $a^3 + b^3 + c^3$ =________________.
(e) The degree of constant polynomial is _______.
(f) The coefficient of x^2 in $2x^2 + x + 8$ is ________.

Q5. Which of the following expressions are polynomials in one variable and which are not? Give reason for your answer.

(a) $x^2 + x$
(b) $\sqrt{2}y - 8$

Q6. (a) Find the zero of the polynomial $p(x) = 2x + 1$

(b) Check whether – 2 and 2 are the zeroes of polynomial $x + 2$ or not. Q7. Verify that $x^3 + y^3 + z^3 - 3xyz = 1/2(x + y + z)\,[(x - y)^2 + (y - z)^2 + (z - x)^2]$ Q8. (a) Evaluate: $(998)^3$

(b) Without calculating the cube, find the value of $(28)^3 + (-15)^3 + (-13)^3$

Q9. Find the product of $(6z - 3z^2 - 1)(4z + 3)(3z^3 - 1)$.

Q10. For what value of a, $2y^3 + ay^2 + 11y + a + 3$ is exactly divisible by $(2y - 1)$?

Factorization of Polynomials

Q11. Factorize the following:

(i) $25^2x - 10x + 1 - 36z^2$
(ii) $4(x - y)^2 - 12(x + y)(x - y) + 9(x + y)^2$.
(iii) $x^3 - x$.
(iv) $1 - 2ab - (a^2 + b^2)$
(v) $x^2 - 7x - 18$
(vi) $x^2 - 16xy + 60y^2$.
(vii) $9(a - 2b)^2 - 4(a - 2b) - 13$
(viii) $x^4 - 81$
(ix) $2a^5 - 32a$
(x) $x^3 - 5x^2 - 14x$.
(xi) $x^4 - y^4$
(xii) $1 - 6x + 9x^2$
(xiii) $a - b - a^3 + b^3$

Problems based on Remainder theorem and Factor theorem ***Q12. If $x^3 + ax^2 + bx + 6$ has $(x - 2)$ as a factor and leaves a remainder 3 when divided by $x - 3$. Find the values a & b.***

Q13. Factorize the following using factor theorem:

(i) $x^3 + 13x^2 + 32x + 20$
(ii) $2y^3 + y^2 - 2y - 1$

Q14. If $f(x) = 4\,x\,3 - 12x^2 + 14x - 3$. Use the remainder theorem to find the remainder when $f(x)$ is divided by $x - ½$

Q15. The polynomials $kx^3 + 3x^2 - 13$ and $2x^3 - 5x + k$ when divided by $(x + 2)$ leave the same remainder in each case. Find the value of k?

Q16. For what value of 'a' is $x + 3$ is a factor of $3x^2 + ax + 6$?

Q17. Check if $y + 2$ is a factor of $4y^3 - 3y^2 + 2y - 50$ or not using factor theorem.

Q18. If $(x - 2)$ and $(x - ½)$ are the factors of $px^2 + 5x + r$, show that $p = r$.

Q19. If $ax^3 + bx^2 + x - 6$ has $(x + 2)$ as a factor and leaves a remainder 4 when divided by $(x - 2)$, find the values of a and b.

Q20. Show that $(x - 2)$, $(x + 3)$ and $(x - 4)$ are factors of $x^3 - 3x^2 - 10x + 24$.

Q21. Without actual division, show that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

PRACTICE TEST

1 mark questions

1. Which of the following is a polynomial?

(a) $x^2 + (1/2x) - 5$ (b) $x^2 + \sqrt{x} - 3$ (c) $(1/\sqrt{x}) + 7$ (d) $x^2 + (1/2)x - 5$

2. The value of polynomial $2x^3 - 3x^2 + 5x - 3$ at $x = 0$ is

(a) 3 (b) 0 (c) – 3 (d) 1

3. The degree of $(4x - 3)(4x + 3)$ is

(a) 4 (b) 16 (c) 2 (d) 1

2 marks questions

1. Factorise the polynomial $64x^3 - (2x - t)^3$
2. If (x–1) is a factor of a2x3 – 4ax + 4a – 1, find the value of a.

3 marks questions

1. Factorise $12(x^2 + 7x)^2 - 8(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$
2. Show that 2 and –1/3 are the zeroes of the polynomial $3x^3 - 2x^2 - 7x - 2$. Also, find its third zero.

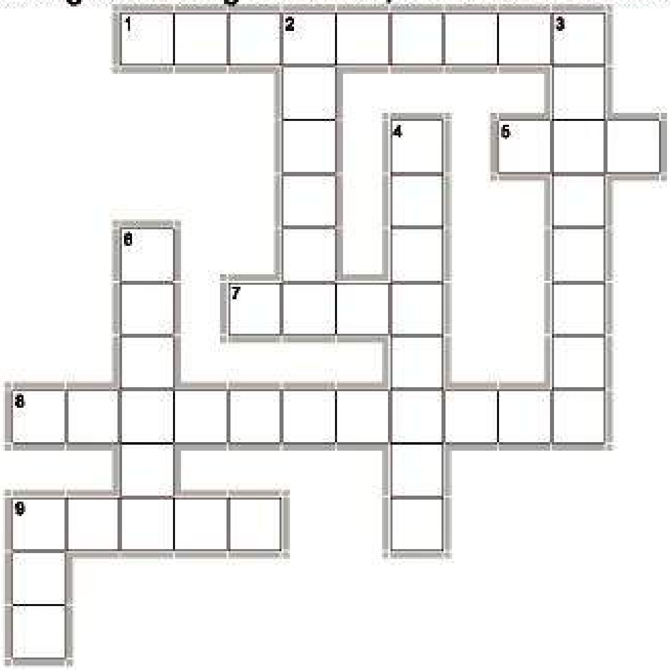
4 marks questions

1. If $x3 + mx^2 - x + 6$ has $(9x - 2)$ as a factor and leaves a remainder n when divided by $(x - 3)$, find the values of m and n.
2. Prove that $(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x) = 2(x^3 + y^3 + z^3 - 3xyz)$
3. When the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $(x + 1)$ and $(x - 2)$ respectively, leaves remainder A and B and $2A + B = 6$, find the value of a.

WORKSHEET 7

Polynomials

Using the clues given below, solve the crossword.



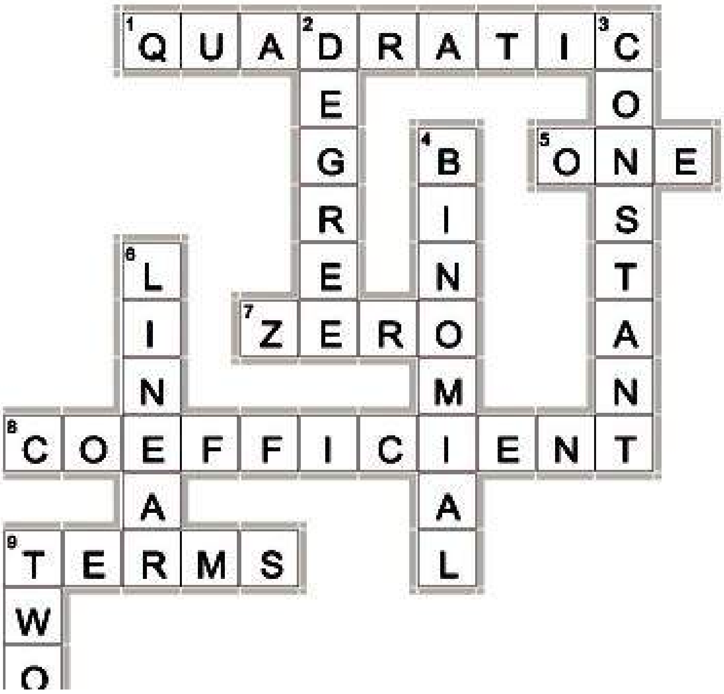
Across

1. polynomial of degree two
5. degree of 7m-7
7. degree of non zero constant polynomial
8. 4 in 4x+3
9. x and 8 in x-8

Down

2. highest power of variable
3. 5 is .... polynomial
4. polynomial having two terms
6. polynomial of degree one
9. degree of $5t^2$-7t+1

Solution



MISCONCEPTION/COMMON ERRORS

Common Errors

1. At $x = 3$, the value of $2x^2$ is written as 23^2 in place of 2×3^2.
2. zero of a polynomial is understood as 0 while zero of a polynomial can be any real number.
3. Negative sign of a team is not included in the numerical coefficient of the team. For example, in the polynomial $x^2 - 5x + 4$, coefficient of the team $-5x$ is taken as 5 instead of -5.
4. $(x - 2)(x - 3) = 6 \Rightarrow x - 2 = 6,\ x - 3 = 6$ as the students do in case of $(x - 2)(x - 3) = 0$ $\Rightarrow x - 2 = 0,\ x - 3 = 0$
4. $x \times x = 2x$ as the students do in case of $x \times x = x^2$

"Mathematics is the art of saying things in different ways." – Maxwell

LINEAR EQUATIONS IN TWO VARIABLES

Introduction

Two real numbers or two algebraic expressions related by the sign of equality (=) form an equation.

An algebraic equation is a statement (involving one or more than one variables/literals in which one expression equals to another expression. For example

2 + 3 = 5, 13 – (–3) = 16, (–9) + (– 2) etc. are numeric statements of equality. Now, lets take a statement "The sum of sonali's marks in English and Science is 168". In the above statement, find

(i) How many variables are required if she had scored equal marks in both subjects.

(ii) If the marks in the two subjects are different, how many variables will be required?

(iii) Express the two above situations in the form of an equation.

(iv) How many pairs of marks in the two subjects, she can score, if the possible answer to the above situation can be

(a) 1 variable (x, y, z etc.)

(b) 2 variable (x and y or p and q etc.)

(c) $x + x = 168$ or $2x = 168$ [for (i)]
$x + y = 168$ or $y + z = 168$ [for (ii)]

(d) If marks in Science are 90, then marks in English can be 78 and so on many more

Key Concepts

An equation of the form $ax+b= 0$, where a and b are real numbers and $a\neq 0$ is a linear equation in one variable.

An equation of the form $ax + by + c = 0$, where a, b and c are real numbers such that $a \neq 0$, $b \neq 0$, is called a linear equation in two variables.

Note: If $a = 0$ or $b = 0$, then the equation is not a linear equation in two variables.

The process of finding solution(s) is called solving an equation.

The solution of a linear equation is not affected when

(i) the same number is added to (subtracted from) both sides of the equation.

(ii) both sides of the equation are multiplied by or divided by the same non-zero real numbers.

A linear equation in two variables has infinitely many solutions.

A linear equation in two variables can be solved in two ways

(i) Algebraic Method

(ii) Geometric Method or Graphic Method

The graph of every linear equation in two variables is a straight line and every point on the graph of the straight line represents a solution of the linear equation.

Every solution of the linear equation can be represented by a unique point on the graph of the equation.

The graph of $x = a$ and $y = b$ are the lines parallel to y-axis and x-axis respectively.

The graph of the line $y = mn$ ($m \neq 0$), is a line that passes through the origin always.

The graph of the line $az + by + c = 0$ intersects x-axis and y-axis at points $\left(-\frac{c}{a}, 0\right)$ and $\left(0, \frac{-c}{b}\right)$ respectively.

Pair of Linear equation in two variables

Two linear equations in the same two variables are said to form a pair of linear equations in two variables.

General form of a pair of linear equations is

$a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, where a_1, b_1, c_1 and a_2, b_2, c_2 are real numbers such that $a_1^2 + b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$.

A pair of linear equations is said to be consistent if it has either a unique solution or infinitely many solutions.

When the linear equations in two variables has infinitely many solution, then the system of linear equations is dependent and consistent.

A pair of linear equations is said to be inconsistent if it has no solution.

In the pair of linear equation in two variables

$a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

(i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then

(a) the pair of linear equations is consistent with unique solution.

(b) the graph will be a pair of lines intersecting at a unique point. The point of intersection is the solution of the pair of equations.

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then

(a) the pair of linear equations is inconsistent.

(b) the graph will be a pair of parallel lines and the pair of equations will have no solution.

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then

(a) the pair of linear equations is dependent and consistent.

(b) the graph will be a pair of coincident lines. Each point on the lines will be a solution and so the pair of equations will have infinitely many solution.

A pair of linear equations can be solved by any of the following three methods:

(i) Substitution Method
(ii) Elimination Method
(iii) Cross-multiplication Method

The pair of linear equations can also be solved geometrically/graphically.

WORKSHEET 1

State whether the statement is true or false

(a) $2x+y = 3(x+y) + 1$ is a linear equation in two variable.

(b) A linear equation in two variables can represent a line || to x-axis.

(c) A linear equation in two variables has only finite solution.

(d) The point (1, 2) lies on the line $x + 2y = 5$.

(e) $x = 2$ and y = 1 is the solution of the linear equation $2x - 2y = 0$.

(f) Any point on y-axis is of the form $(0, y)$

(g) A linear equation in two variables is of the form $ax + by + c = 0$ if a and b are any two real numbers.

(h) The point (a, a) lines on the line $x + y = 0$.

(i) The line $x = 5$ in two variable can be written as $1x + 1y = 5$.

(j) The graph of the line $3x + y = 0$ cuts x-axis at the point (0, 3).

WORKSHEET 2

1. The equation of the x-axis is

(a) $x = 0$ (b) $y = 0$ (c) $x + y = 0$ (d) $x = y$

2. Graph of the equation $2x + 3y = 6$ cuts the axes at points

(a) (2, 0) and (0, 3) (b) (2, 3) and (3, 2)

(c) (3, 0) and (0, 2) (d) (0, 3) and (2, 0)

3. The point (0, 3) lies on the line

(a) $x + y = 1$ (b) $2x – 3y = 0$ (c) $2x + y = 3$ (d) $2x – y = 3$

4. The point (1, –1) lies on the line

(a) $x + y = 2$ (b) $x – y = 2$ (c) $y – x = 2$ (d) $y + x = 2$

5. How many linear equations in x and y can be satisfied by $x = 1$ and $y = 3$

(a) only 1 (b) 2

(c) 3 (d) Infinitely many

6. The graph of $x = 3$ is a line

(a) || to x-axis (b) || to y-axis

(c) passes through origin (d) All of these

7. If (3, 4) lies on the line $2y = 3x + k$, then value of k is

(a) 2 (b) 3 (c) 4 (d) 10

8. The equation of the line || to y-axis and 3 units left of origin is

(a) $x – 3 = 0$ (b) $x + 3 = 0$ (c) $y – 3 = 0$ (d) $y + 3 = 0$

9. The value of k, for which the line $x + y = k$ passes through the origin is

(a) 0 (b) 1 (c) –1 (d) 2

10. Negative solutions of the equation $ax + by + c = 0$ lies in

(a) 1st quadrant (b) 2nd quadrant (c) 3rd quadrant (d) 4th quadrant

WORKSHEET 3

1. Graphically the pair of lines $6x - 3y + 10 = 0$ and $2x - y + 2 = 0$ represents
(a) Intersecting lines (b) Parallel lines (c) Coincident lines (d) None of these
2. One equation of a pair of dependent linear equation is $2x - 3y = 5$. The second equation can be

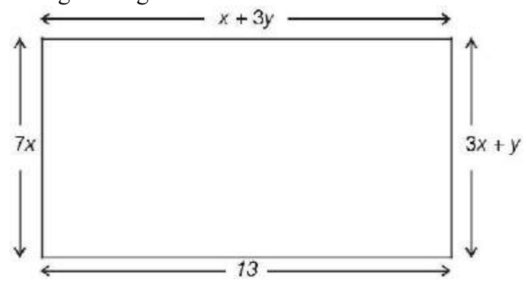
(a) $6x - 9y = 10$ (b) $\frac{x}{2} - \frac{4}{3} = \frac{y5}{3}$ (c) $\frac{x}{2} - \frac{4}{3} = \frac{y-5}{4}$ (d) $2x + 3y = 5$

3. The value of k for which the pair of equations $kx - y = 2$ and $6x - 2y = 3$ will have infinitely many solution is
(a) 3 (b) – 3 (c) – 12 (d) No value
4. If a pair of lines is consistent, then the lines will be

(a) parallel (b) always coincident

(c) intersecting or coincident (d) always intersecting

5. The value of x and y in the following rectangle



(a) 4 and 1 (b) 1 and 3 (c) 3 and 2 (d) 1 and 4
6. The angles of a are x, y and 40°. The difference between the two angles x and y is 30°. The values of x and y are
(a) 45°, 75° (b) 50°, 80° (c) 55°, 85° (d) 55°, 95°

7. Area of the triangle formed by the vertices cut off by the line $3x + 4y = 12$ and the axis is

(a) 6 sq. yards (b) 8 sq. yards (c) 12 sq. yards (d) 24 sq. yards

8. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present age (in years) of the son and the father are respectively

(a) 4 and 24 (b) 5 and 30 (c) 6 and 36 (d) 8 and 48

WORKSHEET 4

1. Draw the graph of the following

(a) $2x + y = 4$ (b) $3x – 2y = 6$

2. Express x in terms of y in the expression $5x – 4y = 12$. Check whether (4, 2) lies on the line $5x – 4y = 12$ or not.
3. Draw the graph of the line $2x + 3y = 12$. Find the points where the line $2x + 3y = 12$ cuts x-axis and y-axis. Also, find the area of the triangle formed by the origin and the line and the axis.
4. The taxi fare in a city consists of some fixed Rs. 20 for the first kilometer and Rs. x per Km for the subsequent distance covered (in Kms). Taking the total fare as Rs. y. Find the amount spent for 10 Kms. Form a linear equation and also draw its graph.

5. The linear equation that converts Fahrenheit (°F) to Celsius (°C) is given by $C =$

(i) If the temperature is 86°F, what is the temperature in °C.

(ii) At what numerical value that both scales represent the same temperature.

WORKSHEET 5

$\frac{5F - 160}{9}$

1. For what values of p and q, will the following pair of lines have infinitely many solutions? $4x + 5y = 2$

$(2p + 7q)x + (p + 8q)y = 2q - p + 1$

2. Draw the graph of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines drawn and x-axis.

3. If $x + 1$ is a factor of $x^4 + ax^3 + 2bx^2 + 3x = 4$, then find the values of a and b given that $a + 4b = 12$.

4. Solve the following pairs of equations:

(i) $x + y$ 3.3 $\frac{0.6}{3x - 2y} = -1, 3x - 2y \neq 0$

(ii) $\frac{1}{2x} - \frac{1}{y} = -1$, $\frac{1}{x} + \frac{1}{2y} = 8, x, y \neq 0$

(iii) $\frac{x}{a} + \frac{y}{b} = a + b$, $\frac{x}{a^2} + \frac{y}{b^2} = 2$, $a, b \neq 0$

(iv) $43x + 67y = -2y$, $67x + 43y = 2y$

5. Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} = 1$ and $\frac{x}{8} + \frac{y}{6} - 15 = 0$. Hence, find λ if $y = \lambda x + 5$.

6. The angles of a cyclic quadrilateral ABCD are
$\angle A = (6x + 10)°, \angle B = 5x°, \angle C = (x + y)°, \angle D = (3x - 10)°$
Find x and y and hence the values of the four angles.

7. In a competitive examination, one mark is awarded for each correct answer while ½ mark is deducted for every wrong answer. Vandana answered 120 questions and got 90 marks. How many questions did she answer correctly?

8. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of ages of his children. Find the age of father.

9. A person, rowing at the rate of 5 Km/hr in still water, takes thrice as much time in going 40 Km upstream as in going 40 Km downstream. Find the speed of the stream.

10. A two digit number is obtained by either multiplying the sum of the digits by 8 and subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

ANSWERS

Woksheet 1

1. (a) T	(b) T	(c) F	(d) T	(e) T
(f) T	(g) F	(h) F	(i) F	(j) F

Woksheet 2

1. (b)	2. (c)	3. (c)	4. (b)	5. (d)
6. (b)	7. (a)	8. (b)	9. (a)	10. (c)

Woksheet 3

1. (b)	2. (c)	3. (a)	4. (c)	5. (b)
6. (c)	7. (a)	8. (c)		

Woksheet 4

2. Yes, (4, 2) lies on the line $5x - 4y = 12$
3. $2x + 3y = 12$ cuts x-axis at (6, 0) and y-axis at (0, 4)
Area( ) =12 sq. yards.
4. $y = 9x +$ 11
5. (i) 20°C 40 (ii) –
$p = -1, q = 2$

Woksheet 5

x = – 340, y = – 165, λ 1

1.

2. 6 sq. yards

3. $a = 0$, $b = 3$

4. (i) $x = 1.2$, $y = 2.1$

(ii) $x = \frac{1}{6}$, $y = \frac{1}{4}$

(iii) $x = a^2$, $y = b^2$

(iv) $x = 1$, $y = -1$

5. - –

2

6. $x = 20$, $y = 30$

$A = 130°$, $B = 100°$, $C = 50°$, $D = 80°$

7. 100

8. 40 years

9. 2.5 Km/hr

10. 83

Errors committed by students

1. Axis are not represented properly.
2. Units of various terms are not mentioned like speed is written 2.5 which should be written as 2.5 Km/hr.
3. While calculating the area of whose base vertices suppose (– 1, 0) and (4, 0), the base is taken as 3 instead of 5.

QUADRATIC EQUATIONS

Introduction

We are familiar with linear equations in the variable and their solutions. In this section, we will learn the solutions of the equations of the type $p(x)$ is a quadratic (of degree 2) polynomial in one variable. We intend to study some applications of quadratic equations in day-in-day life situations.

Key Concepts

Quadratic Equation

An equation with the variable (or unknown), in which the highest power of the variable is two, is called a quadratic equation.

The standard form of the quadratic equation is:

$ax^2 + bx + c = 0$ where a, b, c are real numbers and $a \neq 0$

OR

An equation of the form $p(x) = 0$, where $p(x)$ is a polynomial of degree 2, is called a quadratic equation. In $p(x) = 0$, when we arrange the terms of $p(x)$ in descending order of their degrees, we get the standard form, i.e., $ax^2 + bx + c = 0$, $a \neq 0$, therefore, $ax^2 + bx + c = 0$, $a \neq 0$ is called the standard form of a quadratic equation.

Solution of a quadratic equation /Roots of a quadratic equation:

If two real numbers say α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + b + c$, $a \neq 0$, then α and β are known as the roots/solutions of the quadratic equation $p(x) = 0$.

Note: **A quadratic equation cannot have there then two roots may be distinct or equal.**

Quadratic equation may be solved by:

(i) Factorisation
(ii) Quadratic Formula
(iii) Completing the Squares

(i) Roots of a quadratic equation by Factorisation: Resolve the quadratic equation $ax^2 + bx + c = 0$ into the product of two linear factors say $(px + q)$ and $(rx + s)$, where p, q, r and s are real numbers and $p, r \neq 0$, put each linear factor equal to zero and we get the possible value of x:

i.e. $px + q = 0$,

$x = \frac{-q}{p}$,

Thus, $x = \frac{-q}{p}$ and

$rx + s = 0$

$-s =$

$x = \frac{-s}{r}$ are the two roots of quadratic equation.

(ii) Quadratic Formula (Sridharacharya Formula)

In the previous section, we have learnt about factorization method of solving quadratic equations. In some cases, it is not convenient to solve quadratic equations by factorization method. For example, consider the equations $x^2 + 6x + 3 = 0$. In order to solve this equation by factorization method we will have to split the coefficient of the middle term 6 into two integers, where sum is 6 and product is 3. Clearly, this not possible in integers, therefore, this equation cannot be solved by any factorization method. In this section, we shall discuss a method to solve such quadratic equations. The method which we will discuss below in popularly known as *Sridharacharya's Formula* as it was first given by an ancient Indian mathematician Sridharacharya around 1025 AD.

Now, by quadratic formula (Sridharacharya Formula)

Find $D = b^2 - 4ac$ and if $b^2 - 4ac \geq 0$ then the roots of quadratic equation $ax^2 + b + c = 0$, $a \neq 0$ are given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

DISCRIMINANT: If $ax^2 + bx + c = 0$, $b^2 - 4ac$ is known as the discriminant and $a \neq 0$ is a quadratic equation, then the expression is generally denoted by D.

(iii) Completing of the Square

In this section, we shall learn about the method of completing squares. We may use the following algorithm to obtain the roots of a quadratic equation by using the method of completing Square.

ALGORITHM

STEP I: Obtain the quadratic equation, Let the quadratic equation be $ax^2 + b + c = 0$, $a \neq 0$.

STEP II: Make the coefficient of x^2 unity by dividing throughout by it, it is not unity

i.e., obtain $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

STEP III: Shift the constant term $\frac{c}{a}$ on *RHS* to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$

STEP IV: Add square of half of the coefficient of x i.e. $\left(\frac{b}{2a}\right)^2$ on both sides to obain

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

STEP V: Write LHS as the perfect square of a binomial expression and simplify RHS to get

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

STEP VI: Take square root of both sides to get

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

STEP VII: Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on *RHS*

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{D}}{2a} \quad \text{where } D = b^2 - 4ac$$

or $\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Product of roots = $\alpha\beta = \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

Factorised form of $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

If S be the sum and P be the product of roots, then quadratic equation is:

$x^2 - (S)x + (P) = 0$

Nature of the roots:

Nature of roots of a quadratic equation $ax^2 + bx + c = 0$ means whether the roots are real on complex by analysing it expression $b^2 - 4ac$ (called as discriminant, D), we can set an idea about the nature of the roots as follows:

1. (a) if $D < 0$ ($b^2 - 4ac < 0$), then the roots of the quadratic equation are non-real.

   (b) if $D = 0$ ($b^2 - 4ac = 0$), then the roots are real and equal.

Equal roots = $\dfrac{-b}{2a}$

   (b) if $D > 0$ ($b^2 - 4ac > 0$), then the roots are real and unequal.

WORKSHEET 1

Multiple Choice Questions

1. The discriminant of quadratic equation $3\sqrt{2}x^2 - \sqrt{3}x - \sqrt{18} = 0$ is
(a) 50 (b) 75 (c) 60 (d) 25

2. If the discriminant of quadratic equation $ax^2 + bx + c = 0$ is equal to zero, then two equal roots are:

(a) $\dfrac{-b}{2a}$ (b) $\dfrac{b}{2a}$ (c) $-\dfrac{b}{a}$ (d) $\dfrac{-a}{2b}$

3. For any real value of p, the root of the quadratic equation $x^2 + px - 4 = 0$ are

(a)	real and equal	(b)	real and unequal
(c)	non-real	(d)	none of these

4. For what value of p, the equation $px^2 - 18x + 1 = 0$ is a perfect square?

(a) 18 (b) – 18 (c) 9 (d) 81

5. If the root of $x^2 + mx + 12 = 0$ are in the ratio 1 : 3, then m =

(a) ± 6 (b) ± 8 (c) ± 9 (d) ± 7

6. If p, q are the roots of equation $x^2 - 7x + 10 = 0$, then equations whose roots are $p + 2$, $q + 2$ is

(a) $x^2 + 9x - 12 = 0$ (b) $x^2 + 11x + 28 = 0$

(c) $x^2 - 11x + 28 = 0$ (d) $x^2 = 9x + 12 = 0$

7. If $x = \sqrt{5 + \sqrt{5 + \sqrt{5 + \ldots}}}$ to α and x is a rational number, then we have

(a) $x^2 + x - 5 = 0$ (b) $x^2 - x - 5 = 0$

(c) $x^2 + 5x + 5 = 0$ (d) $x^2 + x - 5 = 0$

8. If the sides of a right angled triangle are x, $x + 2$, $x + 4$ (where $x > 0$), then x =
(a) 6 (b) 10 (c) 12 (d) 8

9. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are

(a) $2, -\frac{3}{2}$ (b) $-2, \frac{3}{2}$ (c) $2, \frac{3}{2}$ (d) $-2, \frac{3}{2}$

10. The product of two consecutive integers is 306 the quadratic representation of this situation is:

(a) $x(x + 1) = 306$ (b) $x + (x + 1)^2 = 306$

(c) $x + (x + 1) = 306$ (d) $x^2 + (x + 1) = 306$

WORKSHEET 2

Multiple Choice Questions

11. If 1 is a root of the equation $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$ then ab equals:

(a) $-\frac{7}{2}$ (b) – 3 (c) 3 (d) 8

12. The quadratic equation where roots are real and equal is:

(a) $x^2 - 4x + 4 = 0$ (b) $x^2 - 2\sqrt{2}x - 6 = 0$

(c) $2x^2 - 4x + 3 = 0$ (d) $3x^2 - 5x + 2 = 0$

13. Which of the following equation has two distinct real roots?

(a) $x^2 + x + 5 = 0$ (b) $5x^2 - 3x + 1 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

14. For the quadratic equation $x^2 - 2x + 1 = 0$ the value of $x + \frac{1}{x}$ is:

(a) – 1 (b) 1 (c) 2 (d) – 2

15. One of the roots of the quadratic equation $6x^2 - x - 1 = 0$ is:

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) – 1 (d) $-\frac{2}{3}$

16. If 8 is a root of the equation $x^2 - 10x + k = 0$, then the value of k is:

(a) 2 (b) – 8 (c) 8 (d) 16

17. If $x^2 + 2kx + 4 = 0$ has a root $x = 2$, then the value of k is:

(a) – 2 (b) 2 (c) – 4 (d) – 1

18. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is:

(a) – 2 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 2

19. If $r = 3$ is a root of quadratic equation $kr^2 - kr - 3 = 0$, then the value of k is:

(a) 2 (b) $\frac{1}{2}$ (c) – 2 (d) $-\frac{1}{2}$

20. The values of k for which the equation $2x^2 - (k - 1)x + 8 = 0$ will have real and equal roots are:

(a) 9 and – 7 (b) only – 7 (c) only 9 (d) – 9 and – 7

TRUE/FALSE

Write 'T' for True statement and 'F' for false statement

21. $(x - 2)(x + 1) = (x - 1)(x + 3)$ represent a quadratic equation

22. The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ was first given by an Indian Mathematician Sridharacharya.

Fill in the blanks

23. If the given quadratic equation $2x^2 + 2x + p = 0$ has equal roots, then $p =$ ………..
24. One year ago, a man was eight times as old as old as his son. Now, his age is equal to the **square** of his son's age. Representative of above situation is given by……………..
25. **Match the Columns:**

Column-I

(i) Value of a, for which $x^2 + 4x + 9 = 0$ is a perfect square.

(ii) If roots of $ax^2 + bx + c = 0$ are equal, the value of c.

(iii) One root of a quadratic equation is $3 - \sqrt{2}$, then theother.

Column-II

(a) $3 + \sqrt{2}$

(b) $(x-1)(x-1)$

(c) 4

(iv) Value of m, for which the quadratic equation 2

(d) b^2

$mx + 8x + 2 = 0$ by real roots.

(v) Factors of $x^2 - 2x + 1$

4

a

(d) $m \le 8$

ANSWERS

1.	(b)	9.	(b)	17.	(a)	25. (i)	(c) $\frac{b^2}{4a}$
2.	(a)	10.	(a)	18.	(d)	(ii)	(d) $3 + \sqrt{2}$
3.	(b)	11.	(c)	19.	(b)	(iii)	(a) 2
4.	(d)	12.	(a)	20.	(a)	(iv)	(e) $m \le 8$
5.	(b)	13.	(a)	21.	F	(v)	(b) $(x-1)(x-1)$
6.	(c)	14.	(c)	22.	T		
7.	(b)	15.	(b)	23.	$\frac{1}{2}$		
8.	(a)	16.	(d)	24.	$x^2 - 8x + 7 = 0$ and $y^2 = x$		

WORKSHEET 3

Quadratic Equations

1. Find the roots of the following equation:

$(x+3)(x-1) = 3x - \frac{1}{3}$ ($x = 2, -1$)

2. Find the nature of roots of the quadratic equation:

$2x^2 - 3x + 5 = 0$ (No real roots)

3. Find the roots of quadratic equation $x^2 - \frac{3}{2}x + \frac{1}{2} = 0$ by completing of the square. ($x = 1, \frac{1}{2}$)

4. Find value of p such that the quadratic equation $(p - 12)x^2 - 2(p - 12)x + 2 = 0$ has equal roots. ($p = 14$)

5. One root of the equation $2x^2 - 8x - m = 0$ is $\frac{5}{2}$. Find the other root and the value of m.

$m = \frac{-15}{2}$, other root is $\frac{3}{2}$

6. The sum of two numbers is 9 and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers (3, 6)

7. Solve for x: $\frac{1}{x-3} - \frac{2}{x-2} = \frac{8}{x}$; $x \neq 0, 2, 3$ ($x = \frac{12}{5}, 4$)

8. Solve for x: $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ ($x = -a, -b$)

9. The altitude of a right triangle is 7 cm more than its base. If the hypotenuse is 13 cm. find the other two sites. (5, 12)

10. A Piece of cloth costs Rs. 200. If the piece was 5 m longer and each meter of cloth costs Rs. 2 less. The cost of the piece would have remain unchanged. How long is the piece and what is the original rate per meter. **[20 m, ` 10 / m]**

11. Find two consecutive positive integers, sum of when square is 421. **[14, 15]**

12. Find the nature of roots of the quadrate equation $2x^2 - 2\sqrt{6}x + 3 = 0$, if the real roots exist, find them, **[$\frac{\sqrt{3}}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}$]**

13. Solve for x. $(a+b)^2x^2 - 4abx - (a-b)^2 = 0$

14. If (– 5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$, find p and k. **[$p = 7, k = -140$]**

15. Find roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ by Quadratic Formula. **[$-\frac{\sqrt{3}}{2}, -+\frac{\sqrt{3}}{2}$]**

ACTIVITY 1

Quadratic Equations

Objective

Solving a quadratic equation by making it a perfect square.

Pre-requisite knowledge

Area of square and rectangle.

Required materials

Coloured papers/(Graph paper and pencil colours for shading) A pair of scissors Eraser Geometry box

Procedure

Suppose we have to solve $x^2 + 6x = 55$

1. Cut a square of x units (say 5 cm) for a coloured paper.

2. Divide the square into small square of 1 × 1 dimension

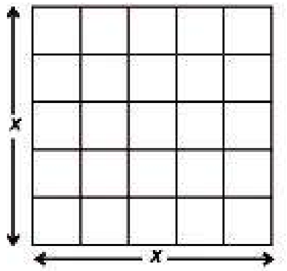


Fig 1

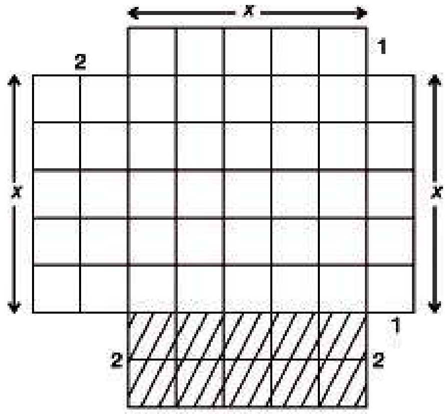


Fig 2

3. Now cut 6 strips of $x \times 1$ dimensions of different colour and arrange then around the previous square as shown in fig.
4. Now cut the square of 1 × 1 dimension for another colour and arrange them around the previous square to complete it as a squares.

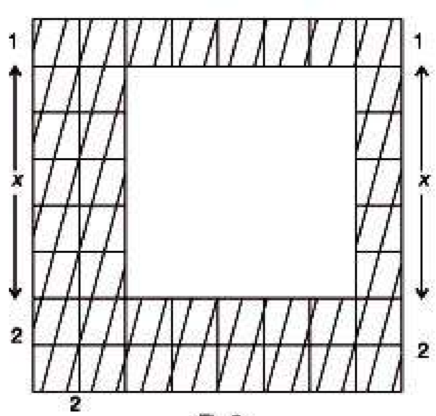


Fig 3

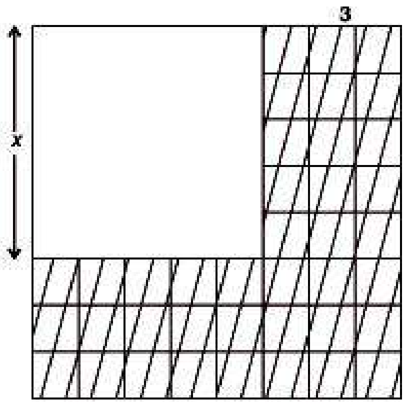


Fig 4

5. Now cut a square of $(x + 2)$ unit for a graph sheet and arrange the cutout of Fig. 3 as shown in Fig. 4.

Observation:

1. Fig 3 represents the area are $x^2 + 6x + 9$

2. Fig 4 has 64 small square i.e. $(x + 3)^2$

$$x^2 + 6x + 4 = 64 \quad x^2 + 6x + 9 = 8^2 \quad x + 3 = \pm 8$$

$x = 5, -11$ Since x is a side of square, so, $x = 5$

Found the solution of quadratic equation by making it a perfect square.

ACTIVITY 2

Cross Number Puzzle

			6
	10		
		9	
		8	

Down

1. Cube of 9.
2. Missing number to make 12, __, 37, a Pythagorean triplet
3. Smallest number by which 248 be multiplied to make the represent a prefect cube number.
5. Square of 75
6. Smaller square number that is divisible by each of 5 and 11
9. Without adding, find the sum of 1 + 3 + 5 + 7 + 5 + 11
10. Smaller number which added to 7669 makes the resultant a perfect square.

Across

2. Square of 19
3. By just looking at given number, find the number which cannot be a perfect square such 81, 100, 144, 25000
7. Square root of 4489
8. Smaller natural number other then /which is a perfect a square as will as perfect cube number.
10. Cube root of 357911
11. Smaller number which when subtracts from 374695 make the smaller perfect square number.

Down		***Across***	
1.	$9^3 = 725$	2.	$19^2 = 361$
2.	35	3.	25000
4.	961	7.	67
5.	5625	8.	64
6.	3025	10.	71
9.	36	11.	151
10.	75		

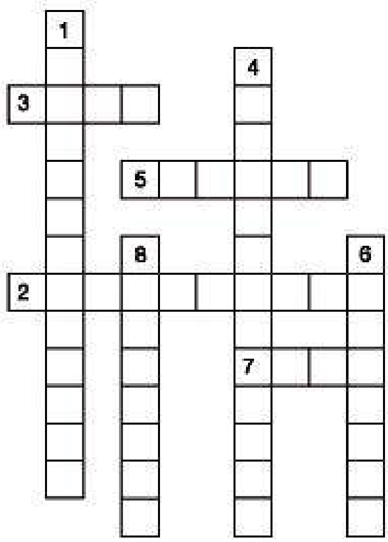
ACTIVITY 3

[1] 7	[2] 3	6	1	[6] 3
[3] 2	5	0	0	0
[4] 9	[7] 6	[10] 7	1	2
6	2	5	[9] 3	5
[11] 1	4	1	[8] 6	4

Mathematics Puzzle

Cross word Puzzle 1

1. $2\pi R$ = ___________ of a circle of radius R.
2. $2(\ell + b)$ = ___________ of a rectangle.
3. πr^2 = ___________ of a circle of radius r.
4. Base × height = Area of a ___________.
5. Side × side = Area of a ___________.

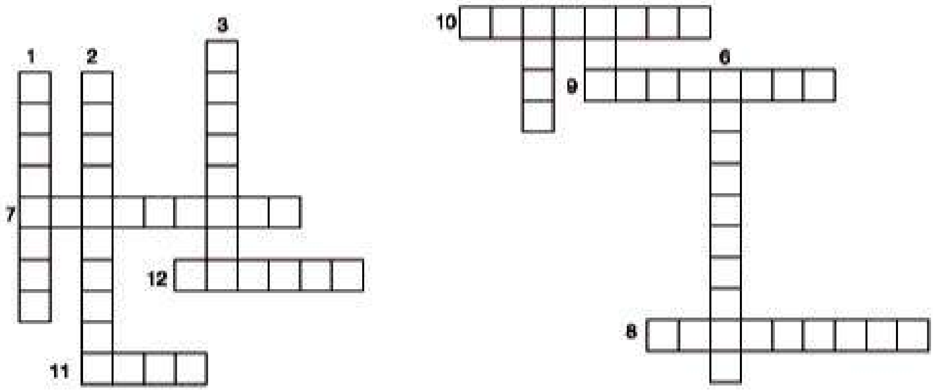


6. Area of __________ $= \frac{1}{2}$ × base × height.
7. 10000 cm^2 = ________ hectare.
8. _________ = 2 × radius.

Answer

1. CIRCUMFERENCE
2. PERIMETER
3. AREA
4. PARALLELOGRAM
5. SQUARE
6. TRIANGLE
7. ONE
8. DIAMETER

Cross Word Number Puzzle 2



Down

1. A polynomial with two terms.
2. An expression containing one or more terms with non zero coefficient.
3. To find the value of a mathematical expression.
4. A ________ is formal by the product of variable and constants.
5. The abbreviation of the greatest no. (or expression).
6. A polynomial with three terms.

Across

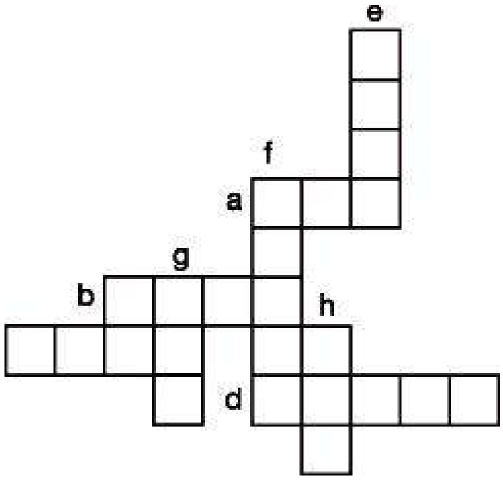
7. A polynomial with only one term.
8. An expression of the second degree.
9. Terms can be written as product of its___
10. The number _____ –3, –2, –1, 0, 1, 2, 3 are known as_____
11. ______ terms are formed from the same variable and the powers of this variable are the some terms.
12. The highest power of a polynomial is called the ______ of the polynomial.

Answer

1.	Binomial	2.	Polynomial	3.	EVALUATE	4.	TERM
5.	GCF	6.	Trinomial	7.	Monomial	8.	Quadratic
9.	Factors	10.	INTEGERS	11.	Like	12.	Degree

Mathematics Word Puzzle 3

If $x = 0, y = 1, z = 2$



Cross:

(a) $xy + yz + zx$

(b) $x^2 y^2 + z^2 - 2xyz$

(c) $8 - (x+y)$

(d) $x^2 y^3 + y^3 z^3 + z^2 x^3$

Down:

C

(a) $x^2 - 2xy\,(y - 2)\,x^3 + y^3 + z^3$

(b)

3

(c) $x^3 + y^3 + z^3 - 2yz^2$

(d) $2x + 2y + 2z$

Answer

(a)	TWO	(b)	FOUR	(c)	SEVEN	(d)	EIGHT
(e)	ZERO	(f)	THREE	(g)	ONE	(h)	SIX

WORKSHEET 4

1. A two digit number is such that the product of digit is 35, when 18 is added to the number, the digits interchange their places. Find the number.
2. Find two number whose sum is 27 and product is 182.
3. A motor boat whose speed is 9 Km/hr in still water goes 12 Km downstream and comes back in a total time 3 hrs. Find the speed of the stream.
4. A train travels 360 Km at uniform speed. If the speed has been 5 Km/hr more it could have taken the less for the same journey. Find the speed of the train.
5. The hypotenuse of right angled triangle is 6 cm more than twice the shortest side. If the third side is 2 cm less than the hypotenuse, find the sides of the triangle.

6. By a reduction of Rs. 2 per kg in the price of sugar. Anita can purchase 8 kg sugar more for Rs. 224 . Find the original price of sugar per kg.
7. Rs 9000 were divided equally among a certain number of students. Had there been 20 more students, each would have got Rs. 160 less. Find the original no. of students.
8. An aeroplane takes an hour less for a journey of 1200 Km, if the speed is increased by 100 Km/hr from its usual speed. Find the usual speed of the aeroplane.
9. 7 years ago age of Aditi was 5 times the square of the age of Sarthak. After the 3years, age of Sarthak will be 2/5 of the age of Aditi. Find their present ages.
10. Two years ago a man's age was three times the square of his son's age. Three year hence his age will be four times of his son's age. Find their present age.
11. In a cricket match against Sri Lanka, Sehwag took one wicket less than twice the number of wickets taken by Amit Mishra. If the product of the number of wickets taken by these two is 15. Find the number of wickets taken by each.
12. A peacock is sitting on the top of pillar, which is meter high from a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of pillar seeing the snake, the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught.
13. The numerators of a fraction is 1 less than its denominator. If three is added to each of the numerator and denominator the fraction is increased by 3/28. Find the fraction.
14. (a) $x^2 - 8x + 16 = 0$

 (b) $a^2x^2 + (a^2 + b^2)x - b^2 = 0$

 (c) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

 (d) $abx^2 + (b^2 - ac)x - bc = 0$

 (e) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}$, $x \neq 2, x \neq 4$

 (f) $\frac{1}{x+4} - \frac{1}{x-3} = \frac{11}{30}$ $x \neq -4; x \neq 7$

 (g) $3x^2 + 2 \quad 5x - 5 = 0$

 (h) $\frac{1}{a} + \frac{1}{b} + \frac{1}{x} = \frac{1}{a+b+x}$, $a \neq 0,\ b \neq 0,\ x \neq 0, x \neq -(a+b)$

WORKSHEET 5

QUADRATIC EQUATIONS

Find the discriminant of each of the following quadratic equations. Solve using quadratic formula.

1. $x^2 + 3x - 4 = 0$
2. $7x^2 + 21x - 28 = 0$
3. $-x^2 + 4x - 7 = 0$
4. $x^2 + (-14x) + 45 = 0$
5. $x^2 - \frac{7}{6}x + \frac{1}{3} = 0$

Solve each equation by completing the square.

1. $p^2 + 14p - 38 = 0$
2. $v^2 + 6v - 59 = 0$
3. $a^2 + 14a - 51 = 0$
4. $3x^2 - 12x + 11 = 0$
5. $2x^2 + 6x + 5 = 0$

Solve the following

√‾‾

1. One side of a right triangle is 12 cm while the hypotenuse is $4\sqrt{10}$ cm. Find the length of the other side.
2. Suppose that one side of a right triangle is 1 more than the other side and the hypotenuse is 1 less than 2 times the shorter side. Find the lengths of all the sides.
3. Find two positive consecutive odd integers whose product is 195.
4. The width of a rectangle is 16 cm less than 3 times the length. If the area is 35 square cm, find the dimensions of the rectangle.
5. A ladder is resting against a wall. The top of the ladder touches the wall at a height of 15 feet. Find the distance from the wall to the bottom of the ladder if the length of the ladder is one feet more than twice its distance from the wall.

ARITHMETIC PROGRESSION

Introduction

In your daily life you must have observed that in nature many things follow patterns such as petals of flowers, the hole of honey-comb, the spiral of a pineapple etc. Patterns are important to study because on the basis of patterns we can make generalisations which leads to algebra. We need to understand what rule they follow and make generalisations. In this topic you will study one special type of pattern called arithmetic progression (A.P.).

Initially, a sequence is an ordered list of objects i.e.

A. 2, 0, – 2, 2, 0, – 2, 2, 0,………………   B. 0 000 00000 0000000………

C. 1, 2, 4, 8, 16,……………………………   D. I II III IIII………….

You can observe that there is a pattern in each of these sequences. This pattern may be easy to observe and but in same cases it may be difficult also. But with the help of generalisations we can tell the numbers at any positions. It is obvious there are no limits to the kinds of patterns we can form and thus infinite kind of sequences can be generated. Students can write the general terms for these sequences. Also they can be given the general terms and asked to write the first few terms of the sequence. They shall also realise that if general term is given they can write the specified terms without writing all terms in continuation.

Key Concepts

Sequence

A sequence is a list or an ordered arrangement of numbers, figures or objects. The members, which are also elements, are called the terms of the sequence. A general sequence can be written as a_1, a_2, a_3, a_4, a_5, a_6,………………………..

where a_1 *is* the first term, a_2 is the second term, and so on. The nth term is denoted as a_n.

Arithmetic Sequence

An arithmetic sequence is a list of numbers in which the difference between two consecutive terms is constant.

The common difference is called *d.*

If $d > 0$, then the terms of the sequence are increasing, and if $d < 0$, then the terms are decreasing.

If *a* is the first term and *d,* the common difference of an AP, then the AP can be written as $a, a + d, a + 2d, a + 3d$, ……..

Note that in case of an AP, it is obvious that

Common difference = second term – first term

= third term – second term

= fourth term – third term and so on.

General Terms of an A. P.

The recursive formula (or rule) for nth term of an arithmetic sequence is $a_n = a_{n-1} + d$ where a_1 is given.

The explicit formula (or rule) for an arithmetic sequence is $a_n = a_1 (n — 1)d$.

Sam of first n terms of an A.P.

$S_n = (n / 2)[2a + (n - 1)d]$

$S_n = (n / 2)[a + l]$ where l is the last term Important Facts

if a, b, c are in A.P. then $b – a = c – b$ or $b = (a + c)/2$

if a constant is added to each term of an A.P., then the resulting sequence is also an A.P..

if a constant is subtracted to each term of an A.P., then the resulting sequence is also an A.P..

if a constant is multiplied to each term of an A.P., then the resulting sequence is also an A.P..

if each term of an A.P. is divided by a non-zero constant, then the resulting sequence is also an A.P..

LEARNING TEACHING STRATEGIES

ACTIVITY 1

Let us consider an example:

Rita deposits Rs.1000 in a bank at the simple interest of 10% per annum. The amount at the end of first, second, third and fourth years, in rupees will be respectively: 1100, 1200, 1300, 1400

Do you observe any pattern? You can see that amount increases every year by a fixed amount of Rs.100.

Students can be asked to observe the similarities between the following sequences:

2, 4, 6, 8……….

4, 7, 10………

11, 16, 21

Let them come out with the observation that in each case the difference between the consecutive terms is same. For such sequence there is special name i.e. Arithmetic sequence. Emphasis is to be laid on building the vocabulary as all the terms are new for the students — first term, common difference, arithmetic sequence, arithmetic progression etc.

Teacher should take care that learner is able to use the vocabulary and symbols carefully as this concept also laid the foundation for functions.

Moreover nth terms or any other term of A.P. can be calculated if the sum of n terms of A.P. is expressed as linear relationship in term of n. Formula can be derived as $S_{n+1} - S_n = a_{n+1}$

$a_{n+1} - a_n = d$ Teacher can link the topic with daily life activities and make generalisations, such as: You all have seen a calendar in your home, school etc. for watching dates, days for planning your holidays or work.

SUN	MON	TUE	WED	THU	FRI	SAT
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Do you observe any calendar for any mathematical purpose?

Do you observe any type of pattern in rows or columns or diagonally of above calendar?

Have a look again and think!

1, 2, 3, 4, 5, 6, 7

Or

8, 9, 10, 11, 12, 13, 14

Or

1, 8, 15, 22, 29

Or

5, 11, 17, 23, 29

If yes, all are arithmetic progressions.

After observing the pattern, can you find a pattern in sum of elements of rows, columns or diagonals?

Addition of 1, 2, 3, 4, 5, 6, 7 = 28 (No. of terms are odd) It is 7 times of middle term? How?

1 + 7 is 8 i.e. sum of first and last term.

2 + 6 is 8 i.e. sum of second term and second last term. 3 + 5 is 8 i.e. sum of third term and third last term.

4 is the middle term.

Now if you observe the sum, you will din 28 = (7) × (8/2)

= (total term/2) × (sum of first term and last term)

= (n/2) × (a + ℓ) where n is number of terms and a & ℓ are first and last terms respectively.

Hence you can drive a formula for that A.P. in which number of terms is odd.

Reflection:

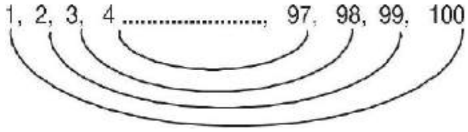
How would you develop a formula for that A.P. in which number of terms is even?

Similarly, you can generalise the formula for any column, diagonal or any other type of A.P..

Let an A.P. is given as under.

1, 2, 3, ………………….., 100

Sum of first term and last term or sum of second term and second last term is 101.



Now, Sum of A.P. = (100/2) × 101

= 50 × 101

= 5050

The above method is similar to the one used by the great German Mathematician Carl Friedrich Gauss (1777-1855) when he was elementary school. His teacher asked the class to find the sum of first 100 natural numbers. Gauss found the sum as follows: 1 + 2 + 3 + 4+ ………+ 98 + 99 + 100

100 + 99 + 98 + 97 + ………+ 3 + 2 + 1

+ 101 + 101 +

101 ………… + 101 + 101 + 101

101, 100 times

i.e., sum = (101 × 100)/2 = 5050

Applications of Arithmetic Progression

A.P. has number of applications in daily life situations. We need explain them through some examples such as: **Problem:** A ball, starting from rest rolls down a uniform slope. It passes over distances 10cm, 30cm, 50cm, etc in successive seconds. How long will it take for the ball to roll a distance of 10 meters? How much distance will it roll in 12th second?

Solution: In 1st, 2nd, 3rd, seconds, etc the ball rolls down the distances 10 cm, 30 cm, 50 cm etc. Thus, we have an A.P.

10, 30, 50,……………….with a = 10, d = 20.

Let n be the number of seconds in which the ball covers a distance of 10 m.

Using the formula,

S_n = [2a + (n – l)d],

we get, 10 × 100 = (n/2)[2 × l0 + (n – l)(20)] [l m = 100 cm]

$1000 = 10n^2$

$n^2 = 100$

n = +10 or –10

Since n must be positive, so n = 10.

Hence, the ball will cover the distance the distance of 10m in 10 seconds.

In order to find the distance covered in 12th second, we have to find a_{12}.

$a_{12} = 10 + (n – l)(20)$

= 10 +(12 – 1) (20)

= 230

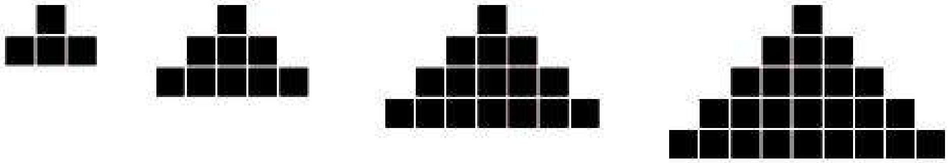
So, the distance travelled in 12th second is 230 cm.

ACTIVITY 2

DESIGNS AND NUMBERS

Objective:

To draw the next shape by observing the pattern.



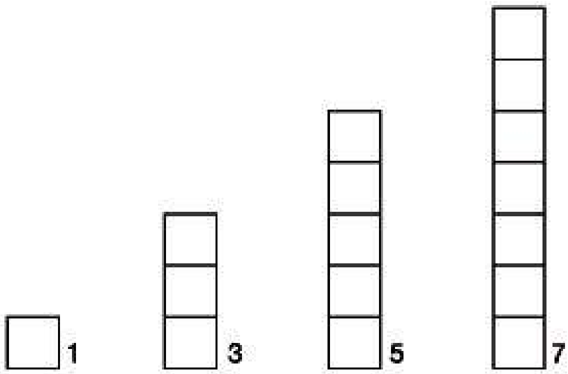
In this activity students will observe and able to observe pattern in shapes (if any), generate next in turn using pattern and associate patterns and numbers. Students make generalisations for predicting any term in the sequence by using that rules.

ACTIVITY 3

TERMS OF AN A.P.

Objective:

To explore that the nth term of an A.P. is a linear expression and common difference of an A.P is free from n.



Through this activity students will explore various other facts regarding the general term.

In this activity students will observe and able to write the specified term when nth term of an A.P. is given, understand that linear expressions forms an A.P. and find that common difference of an A.P is always a constant