CBSE Class 10th Mathematics
Standard Sample Paper- 07

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. If $9^{x+2} = 240 + 9^x$, then the value of 'x' is

    a. 0.5

    b. 0.1

    c. 0.3

    d. 0.2

2. The least number n so that 5^n is divisible by 3, where n is:

    a. a whole number

    b. a real number

c. a natural number

d. no natural number

3. If $\sum f_i x_i = 625$ and $\sum f_i = 25$, then the value of $\overline{x}$ is

a. 63

b. 64

c. 25

d. 26

4. A quadratic equation $ax^2 + bx + c = 0$ has real and distinct roots, if

a. $b^2 - 4ac > 0$

b. $b^2 - 4ac < 0$

c. None of these

d. $b^2 - 4ac = 0$

5. In a right ΔPQR , PR is the hypotenuse of length 20 cm and $\angle P = 60^\circ$. The area of the triangle is

a. $50\sqrt{3}cm^2$

b. $100cm^2$

c. $100\sqrt{3}cm^2$

d. $50cm^2$

6. The value of $\sin 45^\circ + \cos 45^\circ$ is

a. $\sqrt{2}$

b. $\frac{1}{\sqrt{2}}$

c. 1

d. $\frac{1}{\sqrt{3}}$

7. The value of $tan(55^\circ - \theta) - cot(35^\circ + \theta)$ is

a. – 1

b. 0

c. $\sqrt{2}$

d. 1

8. A letter is chosen at random from the word ‘ASSASSINATION’. The probability that it is a vowel is

a. $\frac{6}{13}$

b. $\frac{7}{13}$

c. $\frac{6}{31}$

d. $\frac{3}{13}$

9. The base of an equilateral triangle ABC lies on the y-axis. The coordinates of the point C is (0, – 3). If origin is the midpoint of BC, then the coordinates of B are

a. (3, 0)

b. (0, – 3)

c. (– 3, 0)

d. (0, 3)

10. The co – ordinates of the mid – point of the line joining the points (3p, 4) and (– 2, 4) are (5, p). The value of ‘p’ is

a. 1

b. 4

c. 2

d. 3

11. Fill in the blanks:

Surface area of a solid body is the area of all of its surfaces together and it is always measured in ________ units.

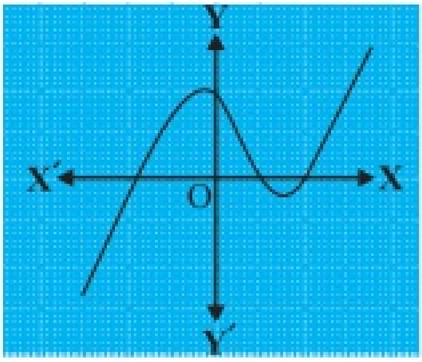
12. Fill in the blanks:

If α, β and γ are zeroes of a cubic polynomial p(x) = $ax^3 + bx^2 + cx + d$, then $\alpha\beta\gamma$ = ________.

OR

Fill in the blanks:

The graph of y = p(x) is given in the figure below, for some polynomial p(x). The number of zeroes of p(x) is ________.



13. Fill in the blanks:

An operation which produces some well-defined outcomes, is called an ________.

14. Fill in the blanks:

The common difference of the AP: $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is ________.

15. Fill in the blanks:

A circle can have ________ parallel tangents at most.

16. Using prime factorisation, find the HCF and LCM of 21, 28, 36, 45.

17. Distance between two parallel lines is 14 cm. Find the radius of the circle which will touch both the lines.

18. If a line intersects a circle in two distinct points, what is it called?

19. If the common difference of an A.P. is 3, then what is the value of $a_{20} - a_{15}$?

OR

Write down the first four terms of the sequences whose general terms are $T_1 = 2$, $T_n = T_{n-1} + 5$, $n \geq 2$.

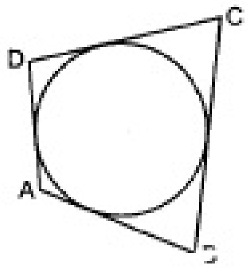
20. Solve the quadratic equation by the method of completing square: $x^2 + 6x - 16 = 0$.

Section B

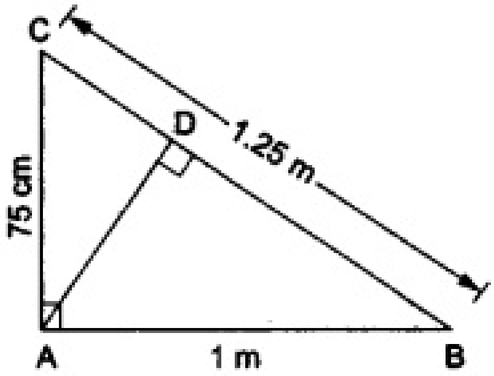
21. All kings, jacks and diamonds have been removed from a pack of cards and the remaining cards are well shuffled. A card is drawn at random. Find the probability that it is

i. a red queen
ii. a face card
iii. a diamond
iv. a black card

22. In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



23. In the given figure, $\angle CAB = 90^\circ$ and $AD \perp BC$. Show that $\triangle BDA \sim \triangle BAC$. If AC = 75 cm, AB = 1 m and BC = 1.25m, Find AD.



OR

In an equilateral triangle of side 24 cm, find the length of the altitude.

24. Two stations due south of a leaning tower which leans towards the north are at distances a and b from its foot. If α, β be the elevations of the top of the tower from these stations, prove that its inclination θ to the horizontal is given by $\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$

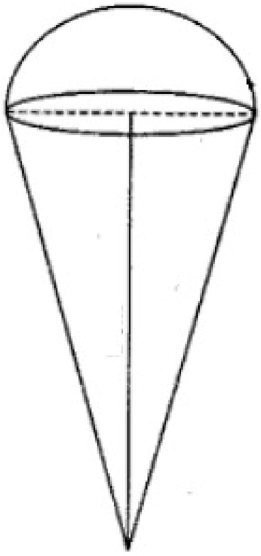
25. Find the values of k for which the given equation has real and equal roots:

$12x^2 + 4kx + 3 = 0$

OR

Find two natural numbers, the sum of whose squares is 25 times their sum and also equal to 50 times their difference.

26. An 'ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day a child came to his shop and purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm.



By reading the above-given information, find the following:

i. The volume of the ice-cream without hemispherical end.
ii. The volume of the ice-cream with a hemispherical end.

Section C

27. If d is the HCF of 30 and 72, find the values of x and y satisfying d = 30x + 72y

OR

Explain why $3.\overline{1416}$ is a rational number.

28. Find a point which is equidistant from the points A (-5,4) and B (-1,6). How many such points are there?

29. Represent the following pair of equations graphically and write the coordinates of points where the lines intersect y-axis
x + 3y = 6
2x - 3y =12

OR

Solve for x and y.

$x + 4y = 27xy\,; x + 2y = 21xy$

30. If α and β are the zeroes of the quadratic polynomial f(x) = ax^2 +bx + c, then

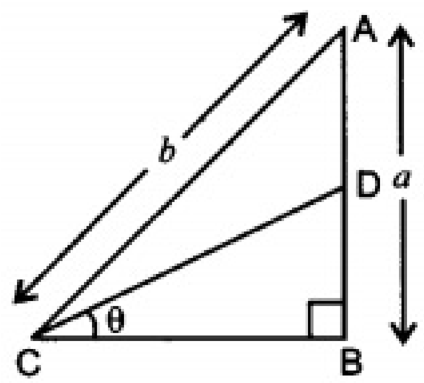
evaluate: $\frac{1}{\alpha} - \frac{1}{\beta}$

31. If 12th term of an AP is 213 and the sum of its four terms is 24, then find the sum of its first 10 terms.

32. Prove that $\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ} = \frac{6-\sqrt{3}}{3}$

OR

In figure AD = BD and $\angle B$ is a right angle. Determine $\sin^2\theta + \cos^2\theta$.



33. The central angles of two sectors of circles of radii 7 cm and 21 cm are respectively 120° and 40°. Find the areas of the two sectors as well as the length of the corresponding arcs. What do you observe?

34. 17 cards numbered 1, 2, 3, 4, ..., 17 are put in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the card drawn bears

i. An odd number
ii. A number divisible by 5.

Section D

35. Draw the incircle of the triangle whose sides are 3 cm, 4 cm and 5 cm and measure its radius. Write the steps of construction also.

OR

Construct a ΔABC in which AB = 4 cm, BC = 5 cm and AC = 6 cm. Now, construct a triangle similar to ΔABC such that each of its sides is two-third of the corresponding sides of ΔABC. Also, prove your assertion.

36. D and E are the points on the sides AB and AC respectively of a ΔABC such that: AD = 8 cm, DB = 12 cm, AE = 6 cm and CE = 9 cm. Prove that BC = 5/2 DE.

37. Ten years ago, father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present ages.

OR

Solve the following pairs of equations by reducing them to a pair of linear equations:
$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ and $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

38. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

OR

A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil, if the specific gravity of the wood is 0.7 gm/cm^3 and that of the graphite is 2.1 gm/ cm ^{3}.

39. A vertical pedestal stands on the ground and is surmounted by a vertical flagstaff of height 5 m. At a point on the ground the angles of elevation of the bottom and the top of the flagstaff are 30° and 60° respectively. Find the height of the pedestal.

40. Draw an ogive by less than method for the following data:

No. of rooms:	1	2	3	4	5	6	7	8	9	10
No. of houses:	4	9	22	28	24	12	8	6	5	2

Solution

Section A

1. (a) 0.5

   Explanation:

   $9^{x+2} = 240 + 9^x$
   $\Rightarrow 9^x.9^2 = 240 + 9^x$
   $\Rightarrow 9^x(81 - 1) = 240$
   $\Rightarrow 9^x = 3$
   $\Rightarrow 9^x = 9^{\frac{1}{2}}$
   $\Rightarrow x = \frac{1}{2} = 0.5$

2. (d) no natural number

   Explanation:

   Since 5 is a prime number so it is not divisible by 3.

   Therefore there is no natural number n

   such that 5^n is divisible by 3.

3. (c) 25

   Explanation:

   $\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$ = $\frac{625}{25}$ = 25

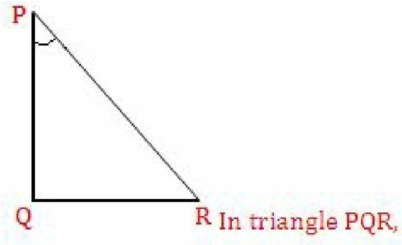
4. (a) $b^2 - 4ac > 0$

   Explanation:

   A quadratic equation $ax^2 + bx + c = 0$ has real and distinct roots, if $b^2 - 4ac > 0$.

5. (a) $50\sqrt{3}cm^2$

Explanation:



In a triangle PQR,

$\cos 60^\circ = \frac{\mathrm{PQ}}{\mathrm{PR}}$

$\Rightarrow \frac{1}{2} = \frac{\mathrm{PQ}}{20}$

$\Rightarrow \mathrm{PQ} = 10 \text{ cm}$

And $\sin 60^\circ = \frac{\mathrm{QR}}{\mathrm{PR}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\mathrm{QR}}{20}$

$\Rightarrow \mathrm{QR} = 10\sqrt{3} \text{ cm}$

$\therefore$ ar $(\Delta \mathrm{PQR})$

= $\frac{1}{2} \times 10\sqrt{3} \times 10$

= $50\sqrt{3} cm^2$

6. (a) $\sqrt{2}$

   Explanation:

   Given: $\sin 45^\circ + \cos 45^\circ$

   = $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

   = $\frac{2}{\sqrt{2}} = \sqrt{2}$

7. (b) 0

   Explanation:

   Given: $\tan(55^\circ - \theta) - \cot(35^\circ + \theta)$

   $=\cot(90^\circ - 55^\circ + \theta) - \cot(35^\circ + \theta)$

   $=\cot(35^\circ + \theta) - \cot(35^\circ + \theta)$ = 0

8. (a) $\frac{6}{13}$

Explanation:

Vowels present in the given word are A, A, I, A, I, O = 6

Number of possible outcomes = {A, A, I, A, I, O} = 6

Number of total outcomes = 13

Required Probability = $\frac{6}{13}$

9. (d) (0, 3)

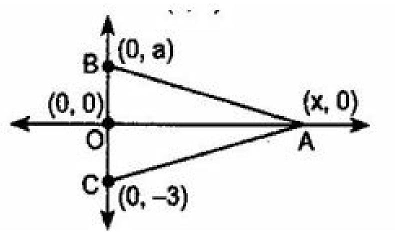
   Explanation:

   Let the coordinate of B be (0, a).

   It is given that (0, 0) is the mid-point of BC.

   Therefore 0 = (0 + 0) /2 , 0 =(a - 3) /2   a - 3 = 0 , a = 3 0 = $\frac{0+0}{2}$, 0 = $\frac{a-3}{2}$, a - 3 = 0, a = 3

   Therefore, the coordinates of B are (0, 3).



10. (b) 4

    Explanation:

    Let the coordinates of midpoint O$(5, p)$ is equidistance from the points A$(3p, 4)$ and B$(-2, 4)$.(because O is the mid-point of AB)

    $\therefore 5 = \frac{3p-2}{2} \Rightarrow 3p - 2 = 10$

    $\Rightarrow 3p = 12 \Rightarrow p = 4$

    Also $p = \frac{4+4}{2} \Rightarrow p = 4$

11. square

12. $\frac{-d}{a}$ OR 3

13. experiment

14. -1

15. two

16. Prime factorization of 28,36,45 is:

21=3 $\times$7

$$28 = 4 \times 7 = 2^2 \times 7$$

$$36 = 4 \times 9 = 2^2 \times 3^2$$

$$45 = 5 \times 9 = 5 \times 3^2$$

Now

HCF = product of the smallest power of each common prime factor in the numbers = 1

LCM = product of the greatest power of each prime factor involved in the numbers = $2^2 \times 3^2 \times 5 \times 7 = 1260$

17. Circle touches both the parallel lines

Given, Distance between the parallel lines = 14 cm

We know that, Diameter of circle = Distance between the parallel lines

$\therefore$Radius = $\frac{14}{2}$ = 7 cm

18. The line which intersects a circle in two distinct points is called secant.

19. Let the first term of the AP be a.

Given, common difference (d) = 3

$a_n = a + (n - 1)d$

Now,

$a_{20} – a_{15} = [a + (20 – 1) d] – [a + (15 – 1)d]$

= 19d – 14d

= 5d

= 5 × 3

= 15.

OR

$T_1 = 2, T_n = T_{n-1} + 5, n > 2$

$\Rightarrow T_2 = T_{2-1} + 5 = T_1 + 5 = 2 + 5 = 7$

$T_3 = T_{3-1} + 5 = T_2 + 5 = 7 + 5 = 12$ and $T_4 = T_{4-1} + 5 = T_3 + 5 = 12 + 5 = 17$

$\therefore$ 1st four terms are 2, 7,12 and 17.

20. $x^2 + 6x - 16 = 0$
$\Rightarrow x^2 + 6x = 16$
$\Rightarrow x^2 + 6x + 9 = 16 + 9$ [Adding on both sides square of coefficient of x, i.e. $(\frac{6}{2})^2$]
$\Rightarrow (x+3)^2 = 25$
$\Rightarrow$ x + 3 = $\pm\sqrt{25}$
$\Rightarrow$ x + 3 =5 or x + 3 = -5
$\Rightarrow$ x = 2 or x = -8

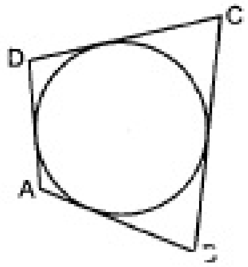
Section B

21. When all kings, jacks and diamonds have been removed, number of cards remaining
= 52 - (4+4+11) = 52 - 19 = 33
Total no. of outcomes = 33

   i. Let A be the event of getting a red queen.
   Thus, favorable outcomes = 1
   P(A) = $\frac{1}{33}$
   ii. Let B be the event of getting a face card
   Thus, favorable outcomes = 3
   P(B) = $\frac{3}{33} = \frac{1}{11}$
   iii. Let C be the event of getting a diamond.
   Thus, favorable outcomes = 0 (all diamonds are removed)
   P(C) = 0
   iv. Let D be the event of getting a black card
   Black cards left are : 11 clubs+11 spades=22
   Thus, favorable outcomes = 22
   P(D) = $\frac{22}{33} = \frac{2}{3}$

22. Given, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB =

6 cm, BC = 9 cm and CD = 8 cm.



If a circle touches all the four sides of quadrilateral ABCD, then

AB + CD = AD + BC

$\therefore$ 6 + 8 = AD + 9

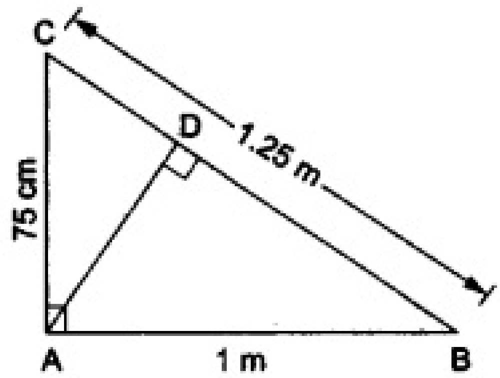
$\Rightarrow$ 14 = AD + 9

$\Rightarrow$ 14-9 = AD

$\Rightarrow$ AD = 5 cm

23. Given, $\angle CAB = 90^\circ$ and $AD \perp BC$.

Also given, AC = 75 cm = 0.75 m , AB = 1 m and BC = 1.25 m.



In $\triangle \mathrm{BDA}$ and $\triangle \mathrm{BAC}$, we have:

$\angle BDA = \angle BAC = 90^0$

$\angle DBA = \angle CBA$ (common)

$\therefore \triangle \mathrm{BDA} \sim \triangle \mathrm{BAC}$ [By AA similarity theorem]

$\Rightarrow \frac{AD}{AC} = \frac{AB}{BC}$ [By proportionality theorem]

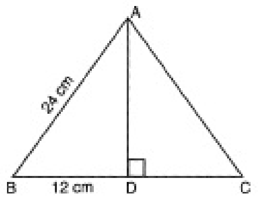
$\Rightarrow \frac{AD}{0.75} = \frac{1}{1.25}$

$\Rightarrow \mathrm{AD} = \frac{0.75}{1.25}$

= 0.6 m or 60 cm

$\therefore$AD = 60 cm.

OR



Let $\triangle$ABC be an equilateral triangle of side 24 cm and AD is altitude

In an equilateral triangle, altitude is also a perpendicular bisector.

$\therefore$ AD is perpendicular bisector of side BC

$\therefore$ BD = $\frac{BC}{2} = \frac{24}{2}$ = 12 cm

AB = 24 cm (Given)

$\triangle$ABD is a right angled triangle, using pythagoras theorem,

AD = $\sqrt{AB^2 - BD^2}$

$= \sqrt{(24)^2 - (12)^2}$

$= \sqrt{576 - 144}$

$= \sqrt{432}$

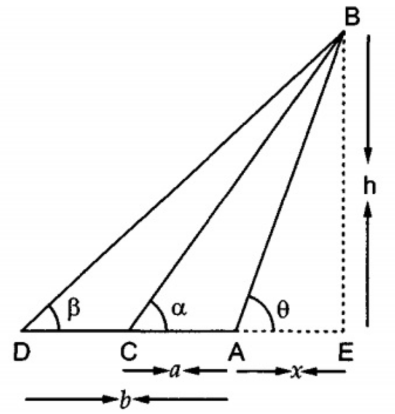
AD = 12$\sqrt{3}$ cm

$\therefore$ the length of the altitude is 12$\sqrt{3}$ cm.

24. Let AB be the leaning tower and let C and D be two given stations at distances a and b respectively from the foot A of the tower.

Let AE = x and BE = h

In $\triangle AEB$, we have



$\tan\theta = \frac{BE}{AE}$

$\Rightarrow \quad \tan\theta = \frac{h}{x}$

$\Rightarrow \quad x = h\cot\theta$ ....(i)

In ΔCEB, we have

$\tan\alpha = \frac{BE}{CE}$

$\Rightarrow \quad \tan\alpha = \frac{h}{a+x}$

$\Rightarrow \quad a + x = h\cot\alpha$

$\Rightarrow \quad x = h\cot\alpha - a$.........(ii)

In ΔDEB, we have

$\tan\beta = \frac{BE}{DE}$

$\Rightarrow \quad \tan\beta = \frac{h}{b+x}$

$\Rightarrow \quad b + x = h\cot\beta$

$\Rightarrow \quad x = h\cot\beta - b$ ....(iii)

On equating the values of x obtained from equations (i) and (ii), we have

$h\cot\theta = h\cot\alpha - a$

$\Rightarrow \quad h(\cot\alpha - \cot\theta) = a$

$\Rightarrow \quad h = \frac{a}{\cot\alpha - \cot\theta}$ ....(iv)

On equating the values of x obtained from equations (i) and (iii), we get

$h\cot\theta = h\cot\beta - b$

$\Rightarrow \quad h(\cot\beta - \cot\theta) = b$

$\Rightarrow \quad h = \frac{b}{\cot\beta - \cot\theta}$ ....(v)

Equating the values of h from equations (iv) and (v), we get

$\frac{a}{\cot\alpha - \cot\theta} = \frac{b}{\cot\beta - \cot\theta}$

$\Rightarrow \quad a(\cot\beta - a\cot\theta) = b(\cot\alpha - \cot\theta)$

$\Rightarrow \quad (b - a)\cot\theta = b\cot\alpha - a\cot\beta$

$\Rightarrow \quad \cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$

25. We have to find the values of k for which the given equation has real and equal roots.

The given equation is $12x^2 + 4kx + 3 = 0$. Here, a = 12, b = 4k and c = 3

$\therefore D = b^2 - 4ac = (4k)^2 - 4 \times 12 \times 3 = 16k^2 - 144$

The given equation will have real and equal roots, if

$D = 0 \Rightarrow 16k^2 - 144 = 0 \Rightarrow 16k^2 = 144 \Rightarrow k^2 = 9 \Rightarrow k = \pm 3$

OR

Let the required numbers be x and y, then

$x^2 + y^2 = 25(x + y)$ ....(1)

$x^2 + y^2 = 50(x - y)$ ....(2)

$\Rightarrow 25(x+y) = 50(x-y)$

$\Rightarrow x+y = 2(x-y)$

$\Rightarrow$ x = 3y

putting x = 3y in (1), we get

$9y^2 + y^2 = 100y$

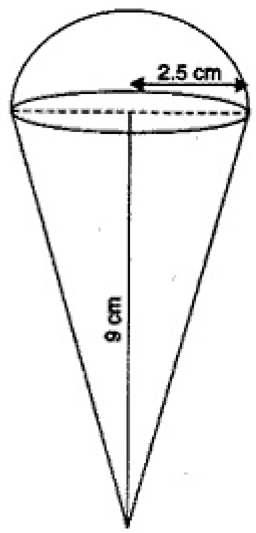
$\Rightarrow 10y^2 - 100y = 0$

$\Rightarrow$ 10y(y - 10) = 0

$\Rightarrow$ y = 10

Hence, x = 30 and y = 10

26. For cone, Radius of the base (r)
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$
Height (h) = 9 cm
$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$
$= \frac{825}{14}\text{cm}^3$



For hemisphere,
Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$
$\therefore$ Volume = $\frac{2}{3}\pi r^3$
$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

i. The volume of the ice-cream without hemispherical end = Volume of the cone
$= \frac{825}{14}\text{cm}^3$

ii. Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere
$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$
$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

Section C

27. Using Euclid's algorithm, the HCF(30, 72)

72 = 30 $\times$ 2 + 12

30 = 12 $\times$ 2 + 6.

12 = 6 $\times$ 2 + 0.

HCF(30, 72) = 6

Now it given that

HCF=30x+72y

So 6 = 30x + 72y

or 1 = 5x + 12y

$x = \frac{1-12y}{5}$ .... (1)

if y = -2 then $x = \frac{1+24}{5} = 5$

if y = -12 then $x = \frac{1+144}{5} = 29$

if y = -22 then $x = \frac{1+264}{5} = 53$

So the value of (x, y) possible are (5, -2), (29, -12) (53, -22)

Hence infinite no. of solutions are possible.

OR

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers.

Let $x = 3.\overline{1416}$

$\Rightarrow x = 3.141614161416$ ....(i)

Since there are four repeating digits, we multiply by 1000.

$\Rightarrow 1000x = 31416.14161416$ ....(ii)

Subtracting (i) from (ii), we get

$1000x - x = 31416.14161416 - 3.14161416$

$999x = 31413$

$\Rightarrow x = \frac{31413}{999}$

which is of the form $\frac{p}{q}$ and $q \neq 0$.

So , $3.\overline{1416}$ is a rational number.

28. Let P(x, y) be equidistant from the points A(-5, 4) and B(-1, 6).

Now,

AP = BP

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x + 5)^2 + (y - 4)^2 = (x + 1)^2 + (y - 6)^2$

$\Rightarrow x^2 + 25 + 10x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$

$\Rightarrow 10x + 41 - 8y = 2x + 37 - 12y$

$\Rightarrow 8x + 4y + 4 = 0$

$\Rightarrow 2x + y + 1 = 0$

Thus, all the points which lie on line 2x + y + 1 =0 are equidistant from A and B.

29. The given systems of equations are:

$x + 3y = 6 \text{ and } 2x - 3y = 12$

Now, $x + 3y = 6$

$y = \frac{6-x}{3}$

Table for equation $x + 3y = 6$

x	0	3
y	2	1

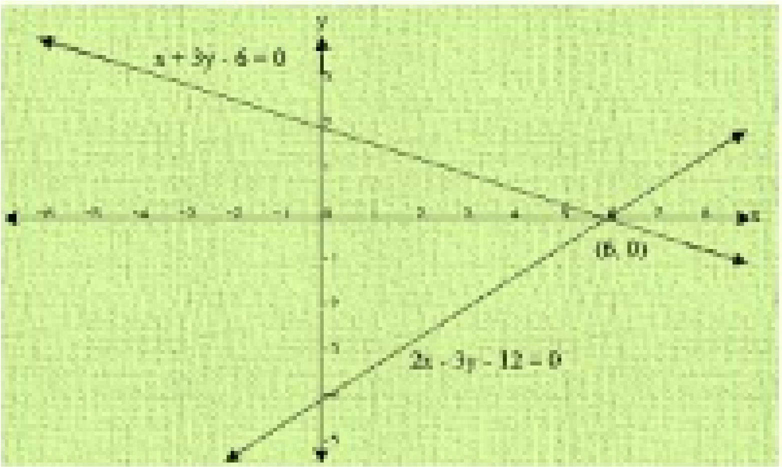
Now, $2x - 3y = 12$

$y = \frac{2x-12}{3}$

Table for equation $2x - 3y = 12$

x	**0**	**6**
y	-4	0

Graph of the given system of equations are :



Clearly the two lines meet y-axis at B(0, 2) and C(0, -4) respectively.

Hence the required coordinates are (0,2) and (0, -4)

OR

Given equations are $x+4y=27xy$ and $x+2y=21xy$

On dividing both sides of the above equations by xy, we get

$\frac{1}{y}+\frac{4}{x}=27$ and $\frac{1}{y}+\frac{2}{x}=21$

On putting $\frac{1}{y}=u$ and $\frac{1}{x}=v$, we get

$u+4v=27$ and $u+2v=21$

On Subtracting equations, we get

$2v=6 \Rightarrow v=3$

On putting value of v in any equation, we get u= 15

Now, $v=3 \Rightarrow \frac{1}{x}=3 \Rightarrow x=\frac{1}{3}$

and $u=15 \Rightarrow \frac{1}{y}=15 \Rightarrow y=\frac{1}{15}$

Hence, $x=\frac{1}{3}$ and $y=\frac{1}{15}$ is the required solution.

30. The quadratic polynomial $ax^2 + bx + c = f(x)$

α and β are the zeroes of an equation.

$\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$

$\frac{1}{\alpha}-\frac{1}{\beta}=\frac{\beta-\alpha}{\alpha\beta}=\frac{-(\alpha-\beta)}{\alpha\beta}\ldots\ldots$ (i)

consider,

$(\alpha-\beta)^2=(\alpha+\beta)^2+4\alpha\beta$

$(\alpha-\beta)^2=\left(\frac{-b}{a}\right)^2+\frac{4c}{a}$

$\alpha-\beta=\sqrt{\frac{b^2}{a^2}+\frac{4c}{a}}=\sqrt{\frac{b^2+4ac}{a^2}}=\frac{\sqrt{b^2+4ac}}{a}$

$\frac{1}{\alpha}-\frac{1}{\beta}=\frac{-(\alpha-\beta)}{\alpha\beta}=\frac{\frac{\sqrt{b^2+4ac}}{a}}{\frac{c}{a}}=\frac{\sqrt{b^2+4ac}}{c}$

31. Given, 12^{th} term=213,

i.e. $a_{12}=213$

$a_n=a+(n-1)d$

$\Rightarrow a+(12-1)d=213$

$\Rightarrow a+11d=213 \qquad \ldots.(1)$

and $S_4=24$

$\Rightarrow \frac{4}{2}[2a+(4-1)d]=24$

$\Rightarrow 2a+3d=12 \qquad \ldots..(2)$

On solving Eqs. (1) and (2), we get

$S_{10}=\frac{10}{2}\left[2\times\left(\frac{-507}{19}\right)+(10-1)\times\frac{414}{19}\right]$

$=\frac{10}{2}[\frac{-1014}{19}+9\times\frac{414}{19}]$

$=\frac{10}{2}(\frac{-1014+3726}{19})=\frac{27120}{38}$

32. L.H.S = $\frac{\cos 58^\circ}{\sin 32^\circ}+\frac{\sin 22^\circ}{\cos 68^\circ}-\frac{\cos 38^\circ \text{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$

$=\frac{\cos(90^\circ-32^\circ)}{\sin 32^\circ}+\frac{\sin(90^\circ-68^\circ)}{\cos 68^\circ}-\frac{\cos(90^\circ-52^\circ)\cos ec 52^\circ}{\tan(90^\circ-72^\circ)\tan(90^\circ-55^\circ)\times\sqrt{3}\times\tan 72^\circ\tan 55^\circ}$

$=\frac{\sin 32^\circ}{\sin 32^\circ}+\frac{\cos 68^\circ}{\cos 68^\circ}-\frac{\sin 52^\circ\times\frac{1}{\sin 52^\circ}}{\cot 72^\circ\times\cot 55^\circ\times\sqrt{3}\times\frac{1}{\cot 72^\circ}\times\frac{1}{\cot 55^\circ}}$

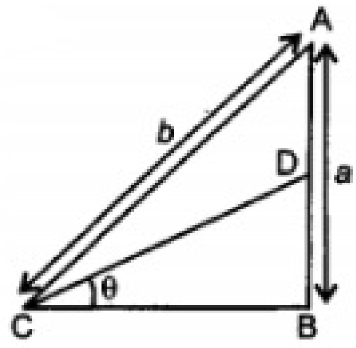
$=1+1-\frac{1}{\sqrt{3}}$

$=2-\frac{1}{\sqrt{3}}$

$= 2 - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= 2 - \frac{\sqrt{3}}{3}$
= $\frac{6-\sqrt{3}}{3}$

= R.H.S. Proved.

OR



Given, AD = BD.......(1)

According to given figure , AB =a, AC = b, $\angle BCD = \theta$

In $\triangle$ABC, use Pythagoras theorem;

$AB^2 + BC^2 = AC^2 \Rightarrow a^2 + BC^2 = b^2$
$\therefore$BC = $\sqrt{b^2 - a^2}$......(2)
Now, in $\triangle$DBC, DB = $\frac{1}{2}$ a [since, AD = BD = $\frac{1}{2}$ AB]
and BC = $\sqrt{b^2 - a^2}$

Again using pythagoras theorem in Δ BCD;

$CD^2 = BD^2 + BC^2 \Rightarrow CD^2 = \frac{a^2}{4} + (b^2 - a^2)$
$\therefore$ CD = $\frac{\sqrt{4b^2-3a^2}}{2}$
Now,in ΔBCD,

$\sin\theta = \frac{BD}{CD} = \frac{a}{\sqrt{4b^2-3a^2}}$
$\cos\theta = \frac{BC}{CD} = \frac{2\sqrt{b^2-a^2}}{\sqrt{4b^2-3a^2}}$

NOW,

$\sin^2\theta + \cos^2\theta = \frac{a^2}{4b^2-3a^2} + \frac{4(b^2-a^2)}{4b^2-3a^2} = 1$

33.

	Sector I	Sector II
Radius	r_1 = 7 cm	r_2 = 21 cm
Sector angle	θ_1 =120°	θ_2 = 40°
Sector areas	$A_1 = \frac{\theta_1}{360} \times \pi r_1^2$	$A_2 = \frac{\theta_2}{360} \times \pi r_2^2$
Sector arc	$l_1 = \frac{\theta_1}{360} \times 2\pi r_1$	$l_2 = \frac{\theta_2}{360} \times 2\pi r_2$

We find that

$A_1 = \frac{\theta_1}{360} \times \pi r_1^2 = \frac{120}{360} \times \frac{22}{7} \times 7^2 \text{cm}^2 = \frac{154}{3} \text{cm}^2$

$A_2 = \frac{\theta_2}{360} \times \pi r_2^2 = \frac{40}{360} \times \frac{22}{7} \times 21^2 \text{cm}^2 = 154 \text{ cm}^2$

$l_1 = \frac{\theta_1}{360} \times 2\pi r_1 = \frac{120}{360} \times 2 \times \frac{22}{7} \times 7\text{cm}^2 = \frac{44}{3}\text{cm}$

$l_2 = \frac{\theta_2}{360} \times 2\pi r_2 = \frac{40}{360} \times 2 \times \frac{22}{7} \times 21\text{cm} = \frac{44}{3}\text{cm}$

We observe that the arc lengths of two circles of different radii may be same but areas need not be equal.

34. Given, 17 cards numbered 1, 2, 3, 4, ..., 17 are put in a box and mixed thoroughly. A card is drawn at random from the box.

Therefore, total number of cards = 17.

i. Let E_l be the event of choosing an odd number.

These numbers are 1,3,5,..., 17.

Let their number be n. Then,

$T_n = 17 \Rightarrow 1 + (n-1) \times 2 = 17 \Rightarrow n = 9$

$\therefore P(E_1) = \frac{9}{17}$

ii. Let E_2 be the event of choosing a number divisible by 5. Numbers divisible by 5 are 5,10,15. Their number is 3.

$\therefore P(E_2) = \frac{3}{17}$

Section D

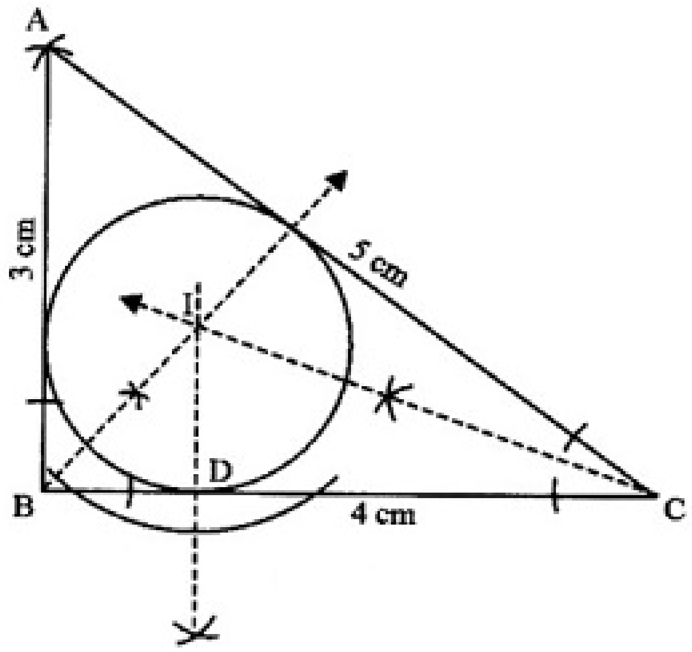
35. Given: A $\triangle ABC$ in which AB = 3 cm, BC = 4 cm and CA = 5 cm.

Required: To construct the incircle of $\triangle ABC$

Steps of construction:

i. Draw a line segment BC = 4cm.
ii. With B as centre and radius = 3 cm, draw an arc.
iii. With C as centre and radius = 5 cm, draw another arc intersecting the first arc at A.
iv. Join B to A and C to A.
v. Draw the bisector of angles. ABC and ACB intersecting at I. Then I is the incentre of $\triangle ABC$.
vi. Draw $ID \perp BC$.
vii. Draw the circle with I as centre and radius equal to ID.

Then this circle is the required of $\triangle ABC$.



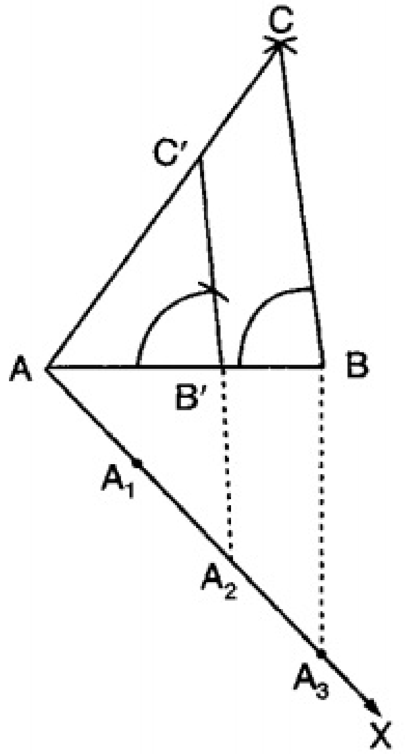
OR

Steps of construction

STEP I Draw a line segment AB = 4 cm.

STEP II With A as centre and radius = AC = 6 cm, draw an arc.

STEP III With B as centre and radius = BC = 5 cm, draw another arc, intersecting the arc drawn in step II at C.



STEP IV Join AC and BC to obtainΔABC.

STEP V Below AB, make an acute angle $\angle BAX$.

STEP VI Along AX, mark off three points (greater of 2 and 3 in $\frac{2}{3}$) A_1 A_2, A_3 such that $AA_1 = A_1 A_2 = A_2A_3$.

STEP VII Join A3B.

STEP VIII Since we have to construct a triangle each of whose sides is two-third of the corresponding sides ofΔABC. So, take two parts out of three equal parts on AX i.e. from point A_2, draw A_2B' || A_3B, meeting AB at B'.

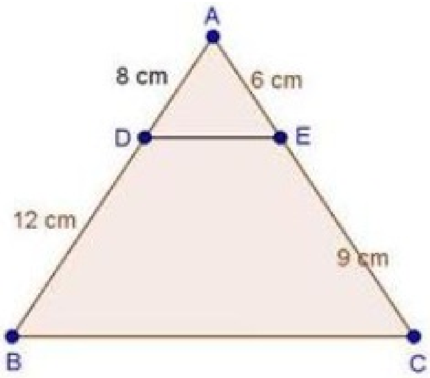
STEP IX From B', draw B'C' || BC, meeting AC at C. AB'C' is the required triangle, each of the whose sides is two third of the corresponding sides of ΔABC.

Justification: Since B'C' || BC. So, $\Delta ABC \sim \Delta AB'C'$

$\therefore \quad \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{AB'}{AB} = \frac{2}{3}$

Let ABC be the given triangle and we want to construct a triangle similarΔABC such that each of its sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding sides ofΔABC such that m < n. We follow the following steps to construct the same.

36. We have,



$\frac{AD}{DB} = \frac{8}{12} = \frac{2}{3}$

And, $\frac{AE}{EC} = \frac{6}{9} = \frac{2}{3}$

Since, $\frac{AD}{DB} = \frac{AE}{EC}$

Therefore, according to the converse of basic proportionality theorem,we have

DE||BC

In ΔADE and ΔABC

$\angle A = \angle A$ [Common]

$\angle ADE = \angle B$ [Corresponding angles]

Then, ΔADE ~ ΔABC [By AA similarity]

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{8}{20} = \frac{DE}{BC}$

$\Rightarrow \frac{2}{5} = \frac{DE}{BC}$

$\Rightarrow BC = \frac{5}{2}DE$

37. Suppose, the present ages of father and son be x years and y years respectively.

According to the question,

Ten years ago,

Father's age = $(x - 10)years$

Son's age = $(y - 10)years$

$\therefore x - 10 = 12(y - 10)$

$\Rightarrow x - 12y + 110 = 0$ ....(i)

Ten years later,

Father's age = $(x + 10)years$.

Son's age = $(y + 10)years$

$\therefore x+10=2(y+10)$

$\Rightarrow x-2y-10=0$ ....(ii)

Subtracting (ii) from (i), we get

-10y + 120 = 0

$\Rightarrow$ 10y = 120

$\Rightarrow$ y = 12

Putting y = 12 in (i), we get

x - 144 + 110 = 0 $\Rightarrow$ x = 34

Thus, present age of father is 34 years and the present age of son is 12 years.

OR

The given pair of equations is

$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ ....(1)

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$ ....(2)

Put $\frac{1}{\sqrt{x}}=u$ ....(3)

and $\frac{1}{\sqrt{y}}=v$ ....(4)

Then equation (1) and (2) can be written as

2u + 3v = 2 ....(5)

4u - 9v = -1 ....(6)

Multiplying equation (5) by 3, we get

6u + 9v = 6 ....(7)

Adding equation (6) and (7), we get 10u = 5

$\Rightarrow u=\frac{5}{10}=\frac{1}{2}$ ....(8)

Substituting the value of u in equation (5), we get $2\left(\frac{1}{2}\right)+3v=2$

$\Rightarrow$ 1 + 3v = 2

$\Rightarrow$ 3v = 2 - 1 = 1

$\Rightarrow\ v=\frac{1}{3}$ .....(9)

From equation (3) and equation (8), we get $\frac{1}{\sqrt{x}}=\frac{1}{2}$

$\Rightarrow\ \sqrt{x}=2$

$\Rightarrow$ x = 4 ....squaring

From equation (4) and equation (9), we get $\frac{1}{\sqrt{y}}=\frac{1}{3}$

$\Rightarrow \sqrt{y}=3$

$\Rightarrow$ y = 9 .... squaring

Hence, the solution of the given pair of equations is

x = 4, y = 9

Verification. Substituting x = 4, y = 9,

We find that both the equations (1) and (2) are satisfied as shown below:

$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=\frac{2}{\sqrt{4}}+\frac{3}{\sqrt{9}}=\frac{2}{2}+\frac{3}{3}=1+1=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=\frac{4}{\sqrt{4}}-\frac{9}{\sqrt{9}}=\frac{4}{2}-\frac{9}{3}=2-3=-1$

Hence, the solution we have got is correct.

38. By the question,Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s

Volume of water flowing through pipe in 1 sec

$=\pi R^2 H$

$=\pi\times(1)^2\times 0.4\times 100\text{cm}^3$

Volume of water flowing in 30 min (30 $\times$ 60 sec)

$=\pi\times(1)^2\times 0.4\times 100\times 30\times 60$

Volume of water in cylindrical tank in 30 min

$=\pi r^2 h$

$=\pi\times(40)^2\times h$

$\pi\times(40)^2\times h=\pi\times(1)^2\times 0.4\times 100\times 30\times 60$

Rise in water level

$h=\frac{\pi\times(1)^2\times 0.4\times 100\times 30\times 60}{\pi\times 40\times 40}$

= 45 cm

$\therefore$ Level of water in the tank is 45 cm.

OR

We have, Diameter of the graphite cylinder = 1 mm = $\frac{1}{10}$ cm

$\therefore$ Radius of graphite (r) = $\frac{1}{20}\text{cm}$ = 0.05 cm

Length of the graphite cylinder =10 cm

Volume of the graphite cylinder = $\frac{22}{7} \times (0.05)^2 \times 10$

= 0.0785 cm^3

Weight of graphite = Volume $\times$ Specific gravity

= 0.0785 11px}{\times2.1}

= 0.164 gm

Diameter of pencil = $7\text{mm} = \frac{7}{10}\text{cm}$ = 0.7 cm

$\therefore$ Radius of pencil = $\frac{7}{20}$ cm = 0.35 cm

and, Length of pencil = 10 cm

$\therefore$ Volume of pencil = $\pi r^2 h$

= $\frac{22}{7} \times (0.35)^2 \times 10\ \text{cm}^3$ = 3.85 cm^3

Volume of wood = volume of the pencil - volume of graphite

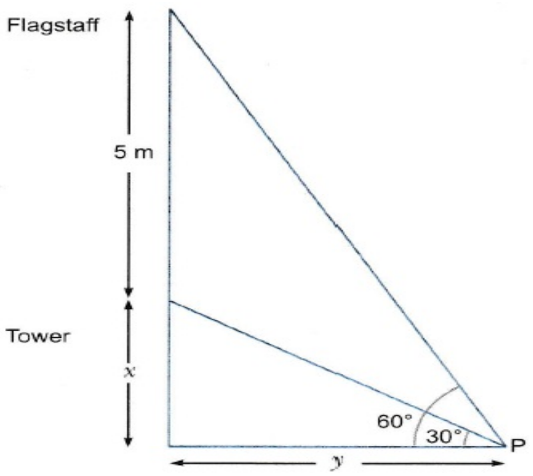
= (3 .85 - 0.164) cm^3= 3. 686 gm

$\therefore$ Weight of wood = volume density

= 3. 686 × 0.7 = 3.73

Hence, Total weight = (3.73+ 0.164) gm = 3.894gm.

39.



Let us suppose that the height of tower be x m and let us suppose that the distance of point from tower be y m.

i. From the fig. $\frac{x}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}}$
$\Rightarrow y = \sqrt{3}x$

ii. $\frac{x+5}{y} = \tan 60^\circ = \sqrt{3}$
$\Rightarrow \frac{x+5}{\sqrt{3}x} = \sqrt{3}[\because y = \sqrt{3}x]$
$\Rightarrow \ x + 5 = 3x$
$\Rightarrow x = \frac{5}{2} = 2.5$
Therefore, height of tower is 2.5 m
Hence, Distance of point from tower = y = $\sqrt{3}$ x
= (2.5 $\times$ 1.732) or 4.33 m

40.

$No.\ of\ rooms$	$No.\ of\ houses$	$Cumulative\ Frequency$
Less than or equal to 1	4	4
Less than or equal to 2	9	13
Less than or equal to 3	22	35
Less than or equal to 4	28	63
Less than or equal to 5	24	87
Less than or equal to 6	12	99
Less than or equal to 7	8	107

Less than or equal to 8	6	113
Less than or equal to 9	5	118
Less than or equal to 10	2	120

We need to plot the points $(1,4),(2,13),(3,35),(4,63),(5,87),(6,99),(7,107),(8,113),(9,118),(10,120)$ o cumulative frequency is plotted along y-axis and number of rooms is plotted along x-axis.