CHAPTER 14

SYSTEMS IN MOTION

§14.01 *Introduction*

Except in chapter 13 we have hitherto tacitly assumed that each system considered was at rest or that its kinetic energy was negligible. In this chapter we shall briefly describe what happens when this restriction is removed. We shall use the formulae of special relativity since these are more revealing and not much more difficult than their prerelativistic approximations.

§14.02 *Mechanics and hydrodynamics*

We begin by quoting without derivation some important relativistic formulae in the field of mechanics and hydrodynamics. We denote by $\boldsymbol{u}$ the velocity of the system relative to a chosen frame and by c the constant speed of light in a vacuum. We define the auxiliary parameter γ by

$$\gamma = (1 - \boldsymbol{u}^2/c^2)^{\frac{1}{2}} < 1. \qquad 14.02.1$$

When several parts of an isolated system interact, the energy E of the whole system remains unchanged and the linear momentum $\boldsymbol{L}$ of the whole system remains unchanged. We use the subscript $_0$ to denote the value taken when $\boldsymbol{u} = 0$. We have the standard relation

$$E = \gamma^{-1} E_0 = \gamma^{-1} m_0 c^2 \qquad 14.02.2$$

where m_0 denotes the rest-mass of the system. $E_0 = m_0 c^2$ differs from U of previous chapters only in having an absolute value whereas the zero of U is arbitrary. We also have the standard relation

$$\boldsymbol{L} = \gamma^{-1} m_0 \boldsymbol{u}. \qquad 14.02.3$$

When we differentiate (2) we obtain

$$\mathrm{d}E = \gamma^{-3} m_0 \boldsymbol{u}\, \mathrm{d}\boldsymbol{u} + \gamma^{-1} \mathrm{d}E_0 \qquad 14.02.4$$

and when we differentiate (3) we obtain

$$\boldsymbol{u}\, \mathrm{d}\boldsymbol{L} = \gamma^{-3} m_0 \boldsymbol{u}\, \mathrm{d}\boldsymbol{u} + \gamma^{-1} \boldsymbol{u}^2\, \mathrm{d}m_0 = \gamma^{-3} m_0 \boldsymbol{u}\, \mathrm{d}\boldsymbol{u} + \gamma^{-1} (\boldsymbol{u}^2/c^2)\, \mathrm{d}E_0. \qquad 14.02.5$$

Eliminating d$\boldsymbol{u}$ from (4) and (5) we obtain

$$dE = \gamma dE_0 + \boldsymbol{u} dL. \qquad 14.02.6$$

We also have the relations

$$dV = \gamma dV_0 \qquad 14.02.7$$

$$P = P_0. \qquad 14.02.8$$

§14.03 *Entropy*

Since E_0 differs from U only by an arbitrary constant, we have for a closed system at rest

$$\begin{aligned} dE_0 &= (\partial E_0/\partial S_0) dS_0 + (\partial E_0/\partial V_0) dV_0 \\ &= (\partial E_0/\partial S_0) dS_0 - P_0 dV_0. \end{aligned} \qquad 14.03.1$$

Substituting (1) into (14.02.6) we obtain

$$dE = \gamma(\partial E_0/\partial S_0) - \gamma P_0 dV_0 + \boldsymbol{u} dL. \qquad 14.03.2$$

From the statistical mechanical interpretation of entropy it follows that S is independent of $\boldsymbol{u}$. Consequently we have

$$S = S_0 \qquad 14.03.3$$

and (2) may be rewritten

$$dE = \gamma(\partial E_0/\partial S) dS - \gamma P_0 dV_0 + \boldsymbol{u} dL. \qquad 14.03.4$$

§14.04 *Thermal equilibrium*

We now consider two identical systems α and β moving relative to each other with different but constant values of $\boldsymbol{L}$. Then by repeating the argument of §1.17 we obtain as the condition for thermal equilibrium

$$[\gamma(\partial E_0/\partial S)]^{\alpha} = [\gamma(\partial E_0/\partial S)]^{\beta}. \qquad 14.04.1$$

§14.05 *Temperature*

Formula (14.04.1) is a complete and unambiguous statement of the condition for thermal equilibrium between two identical systems in relative motion. There is no need to mention temperature and indeed the property of temperature will depend on its precise definition. We may define temperature T by

$$T = \gamma(\partial E_0/\partial S) \qquad 14.05.1$$

and the condition for thermal equilibrium then becomes

$$T^{\alpha}=T^{\beta}. \tag{14.05.2}$$

§ 14.06 *Fundamental equations*

If we substitute (14.05.1) into (14.03.4) we obtain

$$\mathrm{d}E=T\mathrm{d}S-\gamma P_0\mathrm{d}V_0+\boldsymbol{u}\mathrm{d}\boldsymbol{L}. \tag{14.06.1}$$

Using (14.02.7) and (14.02.8) we obtain the fundamental equation

$$\mathrm{d}E=T\mathrm{d}S-P\mathrm{d}V+\boldsymbol{u}\mathrm{d}\boldsymbol{L}. \tag{14.06.2}$$

If we define the Helmholtz function $\mathcal{F}$ by

$$\mathcal{F}=E-TS \tag{14.06.3}$$

and use this with (2) we obtain the second fundamental equation

$$\mathrm{d}\mathcal{F}=-S\mathrm{d}T-P\mathrm{d}V+\boldsymbol{u}\mathrm{d}\boldsymbol{L}. \tag{14.06.4}$$