Sample Paper – 1
Mathematics
Class XI Session 2022-23

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A

(Multiple Choice Questions)
Each question carries 1 mark.

1. The number of relations from an empty set is:
(a) 0 (b) 1
(c) 2 (d) 3 1

2. The value of $\sec^2\theta + \text{cosec}^2\theta$ is equal to:
(a) $\tan^2\theta + \cot^2\theta$ (b) $\sec^2\theta\,\text{cosec}^2\theta$
(c) $\sec\theta\,\text{cosec}\,\theta$ (d) $\sin^2\theta + \cos^2\theta$ 1

3. If $X = \{8^n - 7n - 1 | n \in N\}$ and $Y = \{49n - 49 | n \in N\}$, then:
(a) $X \subset Y$ (b) $Y \subset X$
(c) $X = Y$ (d) $X \cap Y = \phi$ 1

4. $\sqrt{p + iq} = a + ib$ then $\sqrt{p - iq}$ is:
(a) $a^2 + b^2$ (b) $\sqrt{a^2 + b^2}$
(c) $a + ib$ (d) $a - ib$ 1

5. How many three letter words are formed using the letters of the word "TIME"?
(a) 20 (b) 24
(c) 26 (d) 12 1

6. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by : $xRy \Leftrightarrow x$ is factor of y. Then, the domain of R is:
(a) {2, 3, 5} (b) {3, 5}
(c) {2, 3, 4} (d) {2, 3, 4, 5} 1

7. The 4th term from the end of the sequence $16, 8, 4, 2 \ldots \frac{1}{16}$ is:
(a) 1 (b) 2
(c) 3 (d) 4 1

8. The area of a triangle whose vertices are (2, 4), (4, –2) and (–3, 10) is:
(a) 4 (b) 9
(c) 3 (d) 10 1

9. Which of the following set is a finite set?
(a) Set of concentric circles
(b) Set of letters in English alphabets
(c) $x \in R : a < x > 1\}$
(d) None of the above 1

10. The value of 8! is:
(a) 18 (b) 720
(c) 28 (d) 30 1

11. The real value of α for which the expression $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely real is:
(a) $(n+1)\frac{\pi}{2}$ (b) $(2n+1)\frac{\pi}{2}$
(c) $n\pi$ (d) None of these 1

12. If $f(x) = x^2 - 5x + 7$, then $f'(3)$ is:
(a) 11 (b) 1
(c) 4 (d) 0 1

13. Domain of $= \sqrt{a^2 - x^2}$ $(a < 0)$ is:
(a) (a, a) (b) $[-a, a]$
(c) $[0, a]$ (d) $[-a, 0]$ 1

14. The radius of the circle in which a central angle of 30° intercepts an arc of length 22 cm is: (Use $\pi = \frac{22}{7}$)

(a) 43 cm (b) 41 cm
(c) 42 cm (d) 40 cm 1

15. Mean and standard deviation of 100 observation were found to be 40 and 10, respectively. If at the time of calculation two observation were wrongly taken as 30 and 70 in place of 3 and 27 respectively, the correct standard deviation is:
(a) 14.24 (b) 10.24
(c) 19.23 (d) 20 1

16. $3^{\frac{1}{2}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{8}} \times ...$ upto infinite terms is equal to:
(a) 3^2 (b) 3
(c) 3^3 (d) 3^4 1

17. If X = {1, 2, 3}, if n represents any member of x, then all elements of a set, containing element $n + 6$ is given by:
(a) {6, 7, 8} (b) {5, 6, 7}
(c) {7, 8, 9} (d) {8, 9, 10} 1

18. A couple has two children, find the probability that both children are males, if it is known that at least one of the children is male.
(a) 2/3 (b) 1/3
(c) 4/5 (d) 5/3 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): The distance between the points $(1+\sqrt{11}, 0, 0)$ and $(1, -2, 3)$ is $2\sqrt{6}$ units.
Reason (R): Distance between any two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is, $|AB| =$
$$\sqrt{(x_2+x_1)^2+(y_2+y_1)^2+(z_2+z_1)^2}$$
1

20. Assertion (A): Value of sin (– 270)° is 1.
Reason (R): $\sin(180° + \theta) = -\sin\theta$. 1

SECTION - B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. If the vertices of a parallelogram ABCD are A(1, 2, 3), B(–1, –2, –1) and C(2, 3, 2), then find the fourth vertex D.

OR

Find the equation of the set of the points P such that its distances from the points A(2, 0, –5) and B(0, 1, 4) are equal. 2

22. If $f(x) = x^4$ then find the range of the function for $\{x = 1, 2, 3, 4, 5\}$. 2

23. If the 4th and 9th term of a G.P. be 54 and 13122 respectively, find the G.P. 2

24. In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

OR

If the ratio ${}^{2n}C_3 : {}^{n}C_3$ is equal to 11 : 1, find n. 2

25. Find the mean and variance of the frequency distribution given below:

x	$1 \le x < 3$	$3 \le x < 5$	$5 \le x < 7$	$7 \le x < 10$
f	6	4	5	1

2

SECTION - C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. In a town of 10000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find
(A) The number of families which buy newspaper A only.
(B) The number of families which buy none of A, B and C.

OR

110 people play cricket, 160 play tennis, and 70 play both sports, according to a study of 450 people. How many people don't play tennis or cricket? 3

27. Evaluate $\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$. 3

28. Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age (in years)	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number	5	6	12	14	26	12	16	9

Converting the given data in continuous frequency by subtracting 0.5 in upper limit. 3

29. If $a + ib = \frac{(x-i)^2}{2x^2+1}$ prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

OR

Express the given complex number in the form $a + ib$.

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$ 3

30. Evaluate the given limit:

$$\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}.$$ 3

31. If the angle between two lines is $\frac{5\pi}{4}$ and the slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

OR

Find the equation of the line which passes through $(2, 2\sqrt{3})$ and inclined with the x-axis at an angle of 75°. 3

SECTION - D

(This section comprises of long answer-type questions (LA) of 5 marks each.)

32. Prove that:

$$\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x.$$ 5

33. Three coins are tossed once. Find the probability of getting:

(A) 3 tails (B) exactly two tails

(C) no tails (D) at most two tails

(E) no head 5

34. Find the equation of the hyperbola whose conjugate axis is 6 and distance between the focii is 12.

OR

Find the coordinates of focus, axis of parabola, the equation of directrix, coordinate of vertex of the parabola $x^2 - 2x - 4y - 11 = 0$. 5

35. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second?

[Hint: If x is the length of the shortest board, then x, $(x + 3)$ and $2x$ are the lengths of the second and third piece, respectively. Thus, $x + (x + 3) + 2x \geq 91$ and $2x \geq (x + 3) + 5$]

OR

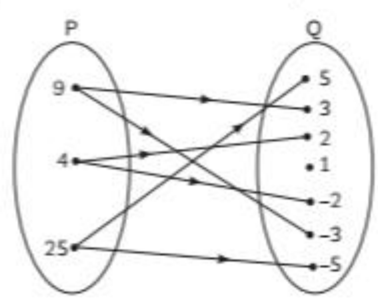
A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460L of the 9% solution, how many litres of 3% solution will have to be added? 5

SECTION - E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (A), (B), (C) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1:**

The figure shows a relation between the sets P and Q.



(A) Write this relation in set builder form. 1

(B) Find the domain of relation. 1

(C) Find the range of relation and also find the total number of relation from set P.

OR

If $R = \{(x, y): x, y \in z, x^2 + y^2 \leq 16\}$ is a relation on z, then find the domain of R. 2

37. **Case-Study 2:**

Riya and her 5 friends went for a trip to Shimla. They stayed in a hotel. There were 4 vacant rooms A, B, C and D. Out of these 4 vacant rooms, two rooms A and B were double share rooms and two rooms C and D can contain one person each.

(A) Find the number of in which room A can be filled. 1

(B) If room A and B are already filled each, then find the number of ways in which room C and be filled. 1

(C) Determine n, if ${}^{2n}C_4 : {}^{n}C_3 = 12 : 1$

OR

If room A is filled with 2 persons, then find the number in which rooms C and D can be filled. 2

38. **Case-Study 3:**

A class teacher Shinu Sharma of Class XI writes Let $f : R \to [0, \infty)$ be a function defined as $f(x) = |x|$ and $g(x) = f(x + 1) + f(x - 1)$, $\forall \in R$. and asked students to answer the following question based on the above information.

(A) Find the value of $g(x)$ and if $\lim_{x \to -1} g(x) = a$, then find the value of a. 2

(B) Evaluate the given limit:

$$\lim_{x \to +2} \left[\frac{x^2 - 4}{x^3 - 4x^2 + 4x} \right]$$ 2

SOLUTION

SECTION - A

1. *(b) 1*

Explanation: Empty set contains no elements means number of relations from empty set is

$$2^0 = 1$$

2. *(b) $sec^2 \theta \, cosec^2 \theta$*

Explanation:

$$\sec^2 \theta + \text{cosec}^2 \theta = \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$= \frac{1}{\sin^2 \theta \cos^2 \theta} = \text{cosec}^2 \theta \sec^2 \theta$$

3. *(a) $X \subset Y$*

Explanation:

$X = \{8^n - 7n - 1 | n \in N\} = \{0, 49, 490, ...\}$

$Y = \{49n - 49 | n \in N\} = \{0, 49, 98, 147, ... 490, ...\}$

Clearly, every element of X is in Y but every element of Y is not in X.

$\therefore \quad (X \subset Y)$

4. *(d) $a - ib$*

Explanation: If $\sqrt{p + iq} = a + ib$

Then, $\sqrt{p - iq} = a - ib$

5. *(b) 24*

Explanation: The number of letters in the given word is 4.

The number of 3 letter words that can be formed using these four letters is $^4P_3$

$$= 4 \times 3 \times 2$$
$$= 24$$

6. *(a) {2, 3, 5}*

Explanation: Relations x is a factor of y. Here in set {2, 3, 4, 5} the factors of y are {2, 3, 5}.

So, domain of R = {2, 3, 5}.

7. *(b) 2*

Explanation: Here, $a = 16$

$$r = \frac{1}{2}$$

So, 4th term, $a_4 = ar^3$

$$= 16 \times \left(\frac{1}{2}\right)^3$$

$$= 16 \times \frac{1}{8} = 2$$

8. *(b) 9*

Explanation: Area of Triangle (2, 4), (4, −2), and (−3, 10)

$$= \frac{1}{2}\ \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$= \frac{1}{2}|2(-2-10)+4(10-4)+(-3)(4+2)|$$

$$= \left|\frac{-24+24-18}{2}\right|$$

$$= \left|\frac{-18}{2}\right| = 9$$

9. *(b) Set of letters in English alphabets.*

Explanation: From the above options, only option b contains countable elements, as we can count the elements of a set of letters in English alphabets, hence it is a finite set.

10. *(b) 720*

Explanation: $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$$= 720$$

11. *(c) nπ*

Explanation: Rationalizing the denominator

$$\Rightarrow \quad \frac{(1 - i\sin\alpha)(1 - 2i\sin\alpha)}{(1 + 2i\sin\alpha)(1 - 2i\sin\alpha)}$$

$$= \frac{1 - i\sin\alpha - 2i\sin\alpha + 2i^2\sin^2\alpha}{1 - 4i^2\sin^2\alpha}$$

$$= \frac{1 - 3i\sin\alpha - 2\sin^2\alpha}{1 + 4\sin^2\alpha}$$

$$= \frac{1 - 2\sin^2\alpha}{1 + 4\sin^2\alpha} - \frac{3i\sin\alpha}{1 + 4\sin^2\alpha}$$

It is given that z is purely real.

$$\Rightarrow \quad \frac{-3i\sin\alpha}{1 + 4\sin^2\alpha} = 0$$

$$\Rightarrow \quad -3\sin\alpha = 0$$

$$\Rightarrow \quad \sin\alpha = 0$$

$$\alpha = n\pi, n \in 1$$

12. *(b) 1*

Explanation: $f(x) = x^2 - 5x + 7$

$$f'(x) = 2x - 5$$
$$f'(3) = 2 \times 3 - 5$$
$$= 6 - 5$$
$$= 1$$

13. *(b) [−a, a]*

Explanation: Let, $f(x) = \sqrt{a^2 - x^2}$

For $f(x)$ to be defined $a^2 - x^2 \geq 0$

$$x^2 \leq a^2$$
$$x \leq [-a, a]$$

14. *(c) 42 cm*

Explanation: Here $l = 22$ cm and $\theta = 30° = \frac{\pi}{6}$

Hence, by $r = \frac{l}{\theta}$, we have

$$r = \frac{22 \times 6}{\pi} = \frac{22 \times 6 \times 7}{22} = 42 \text{ cm}$$

15. *(b) 10.24*

Explanation: Given mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation, two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively.

Now, we have to find the correct standard deviation.

As per given criteria,

Number of observations, $n = 100$

Mean of the given observations before correction, $\bar{x} = 40$

But we know,

$$\bar{x} = \frac{\Sigma x_i}{100}$$

$\Rightarrow$ $\Sigma x_i = 40 \times 100 = 4000$

It is said two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively,

So, $\Sigma x_i = 4000 - 30 - 70 + 3 + 27 = 3930$

So, the correction mean after correction is

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{3930}{100} = 39.3$$

Also given the standard deviation of the 100 observations is 10 before correction *i.e.*, $\sigma = 10$

But we know,

$$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

Substituting the corresponding values, we get

$$10 = \sqrt{\frac{\Sigma x_i^2}{100} - \left(\frac{4000}{100}\right)^2}$$

Now taking square on both sides, we get

$$10^2 = \frac{\Sigma x_i^2}{100} - (40)^2$$

$$\Rightarrow \quad 100 = \frac{\Sigma x_i^2}{100} - 1600$$

$$\Rightarrow \quad 100 + 1600 = \frac{\Sigma x_i^2}{100}$$

$$\Rightarrow \quad \frac{\Sigma x_i^2}{100} = 1700$$

$$\Rightarrow \quad \Sigma x_i^2 = 170000$$

It is said two observation were wrongly taken as 30 and 70 in place of 3 and 27 respectively, so correction is

$\Rightarrow \Sigma x_i^2 = 170000 - (30)^2 - (70)^2 + 3^2 + (27)^2$

$\Rightarrow \Sigma x_i^2 = 170000 - 900 - 4900 + 9 + 729$

$= 164938$

$\Rightarrow$ $\Sigma x_i^2 = 164938$

So the correct standard deviation after correction is

$$\sigma = \sqrt{\frac{164938}{100} - \left(\frac{3930}{100}\right)^2}$$

$$\sigma = \sqrt{1649.38 - (39.3)^2}$$

$$\sigma = \sqrt{1649.38 - 1544.49} = \sqrt{104.89}$$

$$\sigma = 10.24$$

Thus, the correct standard deviation is 10.24.

16. *(b) 3*

Explanation: Given

$$3^{\frac{1}{2}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{8}} \times \ldots$$

$$= 3^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots}$$

$$= 3^{\frac{1}{2}\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots\right)}$$

$$= 3^{\frac{1}{2} \times \left(\frac{1}{1 - \frac{1}{2}}\right)} \qquad [\because\ a = 1 \text{ and } r = \frac{1}{2}]$$

$$= 3^{\frac{1}{2} \times \frac{2}{1}} = 3$$

17. *(c) {7, 8, 9}*

Explanation: The elements in a set containing $n + 6$ elements where $n \in x$ will

$1 + 6, 2 + 6, 3 + 6 = \{7, 8\ 9\}$.

18. *(b) 1/3*

Explanation: S = {MM, MF, FM, FF}

Total sample space is 4.

E : Both children are males.

F : atleast one child is male

E = {(M, M)}

F = {(M, F) (F, M) (M, M)}

$$P(E) = \frac{1}{4}$$

$$P(F') = \frac{3}{4}$$

$$P(E \cap F) = \frac{1}{4}$$

$$P(E/F) = \frac{1/4}{3/4} = \frac{1}{3}$$

19. *(c) A is true but R is false.*

Explanation: Let A = $(1+\sqrt{11}, 0, 0)$ and B = (1, –2, 3)

$$\therefore\ AB = \sqrt{(1 - 1 - \sqrt{11})^2 + (-2 - 0)^2 + (3 - 0)^2}$$

$$= \sqrt{11 + 4 + 9} = \sqrt{24} = 2\sqrt{6} \text{ units}$$

20. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: $\sin(-270°) = -\sin(180° + 90°)$

$= \sin 90° = 1$

$\sin(180° + \theta) = -\sin\theta$ as it goes in 3[rd] quadrant.

So, reason is the correct explanation of assertion.

SECTION - B

21. Let the fourth vertex of the parallelogram ABCD is D(x, y, z). Then, the mid-point of AC is P.

$$P\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right), \text{ i.e., } P\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$$

Mid-point of BD is also P (as diagonal bisect each other)

$$\frac{3}{2} = \frac{-1+x}{2}$$

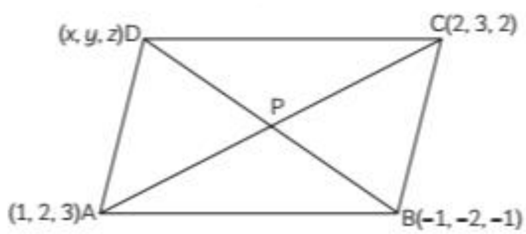
$$\Rightarrow \quad x = 4$$

$$\frac{5}{2} = \frac{-2+y}{2}$$

$$\Rightarrow \quad y = 7$$

$$\frac{5}{2} = \frac{-1+z}{2}$$

$$\Rightarrow \quad z = 6$$



∴ The coordinates of fourth vertex is (4, 7, 6).

OR

Given points are A(2, 0, –5) and B(0, 1, 4).

Let P(x, y, z) be any point such that its distance from the points A and B are equal.

$$\therefore \quad AP = BP$$

$$\Rightarrow \quad \sqrt{(x-3)+(y-0)^2+(z+5)^2}$$

$$= \sqrt{(x+0)^2+(y-1)^2+(z-4)^2}$$

On squaring both sides, we get

$(x-3)^2 + (y)^2 + (z+5)^2 = (x)^2 + (y-1)^2 + (z-4)^2$

$\Rightarrow -6x + 2y + 18z + 17 = 0$

Which is the required equation.

22. Here $f(x) = x^4$

For $x = 1$

$f(x) = 1^4 = 1$

For $x = 2$

$f(x) = 2^4 = 16$

For $x = 3$

$f(x) = 3^4 = 81$

For $x = 4$

$f(x) = 4^4 = 256$

For $x = 5$

$f(x) = 5^4 = 625$

Hence, Range of $f(x) = x^4$ for $x = 1, 2, 3, 4, 5$ is $= \{1, 16, 81, 256, 625\}$

23. Let a be the first term and r the common ratio of the given G.P., then

$$a_4 = 54 \text{ and } a_9 = 13122$$

$$\Rightarrow \quad ar^3 = 54 \text{ and } ar^8 = 13122$$

$$\Rightarrow \quad \frac{ar^8}{ar^3} = \frac{13122}{54}$$

$$\Rightarrow \quad r^5 = 243$$

$$\Rightarrow \quad r^5 = 3^5$$

$$\Rightarrow \quad r = 3$$

Putting $r = 3$ in $ar^3 = 54$, we get:

$$a(3)^3 = 54$$

$$\Rightarrow \quad a = 2$$

Thus, the given G.P. is $a, ar, ar^2, ar^3, \ldots$ i.e. 2, 6, 18, 54.

24. Since, boys are to be separated. Therefore, let us first seat 5 girls. This can be done in 5! ways. For each such arrangement, three boys can be seated only at the cross-marked places.

× G × G × G × G × G ×

There are 6 crossed marked places and three boys can be seated in ${}^6C_3 \times 3!$ ways. Hence, by the fundamental principles of counting, the total number of ways is $5! \times {}^6C_3 \times 3!$

$= 14400.$

OR

We have,

$${}^{2n}C_3 : {}^nC_3 = 11 : 1$$

$$\Rightarrow \quad \frac{{}^{2n}C_3}{{}^nC_3} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{\frac{(2n)!}{(2n-3)!3!}}{\frac{n!}{(n-3)!(3!)}} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{(2n)!}{(2n-3)!} \times \frac{(n-3)!}{n!} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$$

$$\Rightarrow \quad \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$$

$$\frac{4(2n-1)}{n-2}=\frac{11}{1}$$

$\Rightarrow \quad 8n - 4 = 11n - 22$

$\Rightarrow \quad 3n = 18$

$\Rightarrow \quad n = 6$

25. Given the frequency distribution.

Now, we have to find the mean and variance.

Converting the ranges of x to groups, the given table can be rewritten as shown below,

X (class)	f_i	x_i	$f_i x_i$	$f_i x_i^2$
1 – 3	6	2	12	24
3 – 5	4	4	16	64
5 – 7	5	6	30	180
7–10	1	8.5	8.5	72.25
Total	**16**		**66.5**	**340.25**

And we know variance can be written as

$$\sigma = \frac{\Sigma f_i x_i^2}{n} - \left(\frac{\Sigma f_i x_i^2}{n}\right)$$

Substituting values from above table, we get

$$\sigma^2 = \frac{340.25}{16} - \left(\frac{66.5}{16}\right)^2$$

On simplifying, we get

$\sigma^2 = 21.265 - (4.16)^2$

$\sigma^2 = 21.265 - 17.305 = 3.96$

We also know that mean can be written as

$$\bar{x} = \frac{\Sigma f_i x_i}{n}$$

Substituting values from above table, we get

$$\bar{x} = \frac{66.5}{16} = 4.16$$

Hence, the mean and variance of the given frequency distribution are 4.16 and 3.96 respectively.

SECTION - C

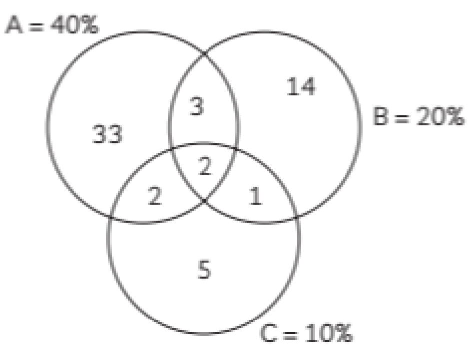
26. Here,

$n(A) = 40\%$

$n(U) = 20\%, n(C) = 10\%, n(A \cap U) = 5\%,$

$n(U \cap C) = 3\%, n(C \cap A) = 4\%, n[A \cap B \cap C] = 2\%$

We can draw a Venn-diagram using the above values.



(A) $n(A)$ only $= n(A) - [n(A \cap B) + n(A \cap C)] + n(A \cap B \cap C)$

$= 40\% - [5\% + 4\%] + 2\%$

$= 40\% - [9\%] + 2\%$

$= 33\%$

Thus, the number of families who buy newspaper A only

$= 33\%$ of 10000

$= 3300$

(B) The number of families which buy none of of A, B and C is

Total families - those who buy either A, B or C

$= 100\% - [n(A) + n(B) + n(C) - n(A \cap B)$

$- n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)]$

$= 100\% - [40\% + 20\% + 10\% - 5\% - 3\% - 4\% - 2\%]$

$= 100\% - 60\% = 40\%$

$\therefore$ Number of families, who buy none of A, B and C newspapers out of 10000 families are

$$= 10000 \times \frac{40}{100}$$

$= 4000$ families

OR

Hint: U–set of people surveyed

A–set of people who play cricket

B–set of people who play tennis

Number of people who play neither cricket nor tennis

$= n[(A \cup B)'] = n(U) - n(A \cup B)$

$= 450 - 200$

$= 250$

27. Firstly, the expression $(a + b)^6 - (a - b)^6$ is simplified by using Binomial Theorem. This can be done as

$(a + b)^6 = {}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5a^1b^5 + {}^6C_6b^6$

$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

$(a - b)^6 = {}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5a^1b^5 + {}^6C_6b^6$

$= a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$

$\therefore [(a + b)^6 - (a - b)^6 = 2[6a^5b + 20a^3b^3 + 6ab^5]$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$$

$$= 2\left[6\left(\sqrt{3}\right)^5\left(\sqrt{2}\right) + 20\left(\sqrt{3}\right)^3\left(\sqrt{2}\right)^3 + 6\left(\sqrt{3}\right)\left(\sqrt{2}\right)^5\right]$$

$$= 2[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6}]$$

$$= 2 \times 198\sqrt{6}$$

$$= 396\sqrt{6}$$

28.

Age	Number	Cumulative Frequency (c.f.)	Mid Point (x_i)	$\|x - M\|$	$f_i \|x_i - M\|$
15.5 – 20.5	5	5	18	$\|18 - 38\| = 20$	$5 \times 20 = 100$
20.5 – 25.5	6	$5 + 6 = 11$	23	$\|23 - 38\| = 20$	$6 \times 15 = 90$
25.5 – 30.5	12	$11 + 12 = 23$	28	$\|28 - 38\| = 20$	$12 \times 10 = 120$
30.5 – 35.5	14	$23 + 14 = 37$	33	$\|33 - 38\| = 20$	$14 \times 5 = 70$
35.5 – 40.5	26	$37 + 26 = 63$	38	$\|38 - 38\| = 20$	$26 \times 0 = 0$
40.5 – 45.5	12	$63 + 12 = 75$	43	$\|43 - 38\| = 20$	$12 \times 5 = 60$
45.5 – 50.5	16	$75 + 16 = 91$	48	$\|48 - 38\| = 20$	$16 \times 10 = 160$
50.5 – 55.5	9	$91 + 9 = 100$	53	$\|53 - 38\| = 20$	$9 \times 15 = 135$
$\Sigma f_i = 100$					$\Sigma f_i \|x_i - M\| = 735$

$$N = \Sigma f_i = 100$$

$$\text{Median class} = \left(\frac{N}{2}\right)^{th} \text{term}$$

$$= \left(\frac{100}{2}\right)^{th} \text{term}$$

$$= 50^{th} \text{term}$$

In the above data, cumulative frequency of class 35.5 – 40.5 is 63 which is greater than 50.

$\therefore$ Median class = 35.5 – 40.5

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Where,

l = lower limits of median class

N = sum of frequencies

f = frequency of median class

C = cumulative frequency of class before median class.

Here, $l = 35.5$, N = 100, $h = 5$, $f = 26$

$$\text{Median} = 35.5 + \frac{\frac{100}{2} - 37}{26} \times 5$$

$$= 35.5 + \frac{50 - 37}{26} \times 5$$

$$= 35.5 + \frac{13}{26} \times 5$$

$$= 35.5 + 2.5 = 38$$

Now,

$$\Sigma f_i = 100$$

$$\Sigma f_i \,|x_i - M| = 735$$

$$\therefore \text{Mean deviation (M)} = \frac{\sum f_i |x_i - M|}{f_i}$$

$$= \frac{735}{100}$$

$$= 7.35$$

29. Given that

$$a + ib = \frac{(x - i)^2}{2x^2 + 1}$$

$$\Rightarrow \quad |a + ib| = \left|\frac{(x - i)^2}{2x^2 + 1}\right|$$

[On taking modulus on both sides]

$$\Rightarrow \quad |a + ib| = \left|\frac{(x - i)\;(x - i)}{2x^2 + 1}\right|$$

$$\Rightarrow \quad |a + ib| = \frac{|(x - i)(x - i)|}{|2x^2 + 1|}$$

$$\Rightarrow \quad |a + ib| = \frac{|x - i||x - i|}{2x^2 + 1}$$

$$\Rightarrow \quad \sqrt{a^2 + b^2} = \frac{\sqrt{x^2 + 1}\,\sqrt{x^2 + 1}}{2x^2 + 1}$$

$$\Rightarrow \quad \sqrt{a^2 + b^2} = \frac{x^2 + 1}{2x^2 + 1}$$

$$= a^2 + b^2 = a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

[on squaring both sides]

Hence, proved.

OR

$$\text{Let } z = \left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$$

$$= \frac{1}{5} + i\frac{2}{5} - 4 - i\frac{5}{2}$$

$$= \frac{1}{5} - 4 + i\frac{2}{5} - i\frac{5}{2}$$

$$= \left(\frac{1}{5} - 4\right) + \left(i\frac{2}{5} - i\frac{5}{2}\right)$$

$$= \left(\frac{1 - 4 \times 5}{5}\right) + i\left(\frac{2 \times 2 - 5 \times 5}{10}\right)$$

$$= \left(\frac{1-20}{5}\right) + i\left(\frac{4-25}{10}\right)$$

$$= \left(\frac{-19}{5}\right) + \left(\frac{-21}{10}i\right)$$

30. $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

Putting $x = 3$

$$= \frac{(3)^4 - 81}{2(3)^2 - 5(3) - 3}$$

$$= \frac{81 - 81}{18 - 15 - 3}$$

$$= \frac{0}{0}$$

Since, it is of the $\frac{0}{0}$ form.

We simplify as

$$\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} = \lim_{x \to 3} \frac{(x^2)^2 - (9)^2}{2x^2 - 6x + x - 3}$$

$[\because\ a^2 - b^2 = (a - b)(a + b)]$

$$= \lim_{x \to 3} \frac{(x^2 - 9)(x^2 + 9)}{2x(x-3) + 1(x-3)}$$

$$= \lim_{x \to 3} \frac{(x^2 - 9)(x^2 + 9)}{2x(x-3) + 1(x-3)}$$

$$= \lim_{x \to 3} \frac{(x-3)(x+3)(x^2+9)}{(2x+1)(x-3)}$$

$$= \lim_{x \to 3} \frac{(x+3)(x^2+9)}{2x+1}$$

Putting $x = 3$

$$= \frac{(3+3)[(3)^2 + 9]}{2 \times 3 + 1}$$

$$= \frac{6(9+9)}{6+1}$$

$$= \frac{6(18)}{7}$$

$$= \frac{(108)}{7}$$

31. Given that slope of one line is $m_1 = \frac{1}{2}$.

Let, m_2 be the slope of the other line.

Also, given that the angle between the two lines is $\theta = \frac{\pi}{4}$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow$ $\tan\frac{\pi}{4} = \left|\frac{\frac{1}{2} - m_2}{1 + \left(\frac{1}{2}\right)m_2}\right|$

$\Rightarrow$ $1 = \left|\frac{1 - 2m_2}{2 + m_2}\right|$

$\Rightarrow$ $\pm 1 = \frac{1 - 2m_2}{2 + m_2}$

$\therefore$ $\frac{1 - 2m_2}{2 + m_2} = 1$

$\Rightarrow$ $1 - 2m_2 = 2 + m_2$

$\Rightarrow$ $m_2 = -\frac{1}{3}$

$\therefore$ $\frac{1 - 2m_2}{2 + m_2} = -1$

$\Rightarrow$ $1 - 2m_2 = -2 - m_2$

$\Rightarrow$ $m_2 = 3$

Hence, the slope of the other line is $-\frac{1`}{3}$ or 3.

OR

We know that

Equation of line passing through point (x_0, y_0) with slope m is

$$y - y_0 = m(x - x_0)$$

Here, point $(x_0, y_0) = (2, 2\sqrt{3})$

Hence, $x_0 = 2$, $y_0 = 2\sqrt{3}$

And slope $= m = \tan\theta$

Given $\theta = 75°$

$\therefore$ $m = \tan(75°)$

$= \tan(45 + 30)°$

$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$\left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\therefore \quad m = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Putting value of m in $(y - y_0) = m(x - x_0)$

$$(y - 2\sqrt{3}) = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$

$$(y - 2\sqrt{3})(\sqrt{3}-1) = (\sqrt{3}+1)(x-2)$$

$$y(\sqrt{3}-1) - 2\sqrt{3}(\sqrt{3}-1)$$

$$= x(\sqrt{3}+1) - 2(\sqrt{3}+1)$$

$$y(\sqrt{3}-1) - 2\sqrt{3} \times \sqrt{3} + 2\sqrt{3}$$

$$= x(\sqrt{3}+1) - 2\sqrt{3} - 2$$

$$y(\sqrt{3}-1) - 6 + 2\sqrt{3} = x(\sqrt{3}+1) - 2\sqrt{3} - 2$$

$$y(\sqrt{3}-1) = x(\sqrt{3}+1) - 2\sqrt{3} - 2 + 6 - 2\sqrt{3}$$

$$y(\sqrt{3}-1) = x(\sqrt{3}+1) - 4\sqrt{3} + 4$$

$$y(\sqrt{3}-1) - x(\sqrt{3}+1) = -4\sqrt{3} + 4$$

$$x(\sqrt{3}+1) - y(\sqrt{3}+1) = 4\sqrt{3} - 4$$

$$x(\sqrt{3}+1) - y(\sqrt{3}-1) = 4(\sqrt{3}-1)$$

SECTION - D

32. $$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\frac{4x+2x}{2}\cos\frac{4x-2x}{2} + \cos 3x}{2\sin\frac{4x+2x}{2}\cos\frac{4x-2x}{2} + \sin 3x}$$

$$\left[\because \begin{array}{l} \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2} \text{ and} \\ \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2} \end{array}\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x\,(2\cos x + 1)}{\sin 3x(2\cos x + 1)} = \cot 3x = \text{R.H.S.}$$

Hence proved.

33. When three coin is tossed, so the possible sample space contains,

S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Where S is a sample space and here $n(S) = 8$

(A) 3 tails

Let F be the event of getting 3 tails.

F = {TTT}

$n(F) = 1$

Probability of getting 3 tails = P(F)

$$= \frac{n(F)}{n(S)}$$

$$= \frac{1}{8}$$

(B) Exactly two tails

Let G be the event of getting exactly 2 tails

G = {HTT, THT, TTH}

$n(G) = 3$

Probability of getting exactly two tails = P(G)

$$= \frac{n(G)}{n(S)}$$

$$= \frac{3}{8}$$

(C) no tails

Let H be the event of getting no tail

H = {HHH}

$n(H) = 1$

Probability of getting no tails = P(H)

$$= \frac{n(H)}{n(S)}$$

$$= \frac{1}{8}$$

(D) At most two tails

Let I be the event of getting at most 2 tails.

i.e., getting 0 tails, 1 tail or 2 tail

I = {HHH, HHT, HTH, THH, HTT, THT, TTH}

$$n(I) = 7$$

Probability of getting at most tails = P(I)

$$= \frac{n(I)}{n(S)}$$

$$= \frac{7}{8}$$

$$= P(J)$$

(E) no heads

Let us assume 'J' be the event of getting no heads. J = {TTT}

$$n(J) = 1$$

Probability of getting no heads = P(E)

$$P(J) = \frac{n(J)}{n(S)}$$

$$= \frac{1}{8}$$

34. Let the equation of the ellipse be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{...(i) } [a > b]$$

Length of conjugate axis = 6

$\Rightarrow \quad 2b = 6$

$\Rightarrow \quad b = 3$

And distance between the focii = 12

$\Rightarrow \quad 2c = 12$

$\Rightarrow \quad c = 6$

Since, $\quad c^2 = a^2 + b^2$

$\Rightarrow \quad a^2 = c^2 - b^2 = 36 - 9 = 27$

$\Rightarrow \quad a^2 = 27$

$\therefore \quad \frac{x^2}{27} - \frac{y^2}{9} = 1$

$\frac{x^2}{27} - \frac{y^2}{9} = 1$, is the required equation of the Hyperbola.

OR

Given, equation of the parabola is

$$x^2 - 2x - 4y - 11 = 0$$

$\Rightarrow \quad (x^2 - 6x + 4) - 4 - 4y - 11 = 0$

$\Rightarrow \quad (x - 2)^2 = 4y + 20$

$\Rightarrow \quad (x - 2)^2 = 4(y + 5) = 4 \times 1 \times (y + 5)$

For vertex, $\quad x - 2 = 0$

$\Rightarrow \quad x = 2$

$y + 5 = 0$

$\Rightarrow \quad y = -5$

Axis of parabola $x - 2 = 0$

$\Rightarrow \quad x = 2$

For directrix, $y + 5 = -1$

$\Rightarrow \quad y + 6 = 0$

And for focus, $y + 5 = 1$ and $x - 2 = 0$,

$\Rightarrow \quad y = -4$

$\Rightarrow \quad x = 2$

$\therefore$ Focus $= (2, -4)$, axis of parabola is $x = 2$, equation of directrix is $y + 6 = 0$ and co-ordinates of vertex are $(2, -5)$.

35. Let the length of the shortest board be x cm.

Length of second board 3 cm longer than the shortest side = $x + 3$ cm

and length of the third board = Twice the shortest board = $2x$ cm

Given that maximum sum of length of the boards can be 91 cm.

i.e., sum of length of boards ≤ 91

$$x + (x + 3) + 2x \leq 91$$

$$4x + 3 \leq 91$$

$$4x \leq 91 - 3$$

$$4x \leq 88$$

$$x \leq \frac{88}{4}$$

$$x \leq 22$$

Also, the third piece is at least 5cm longer than the second piece.

$$2x \geq (x + 3) + 5$$

$$2x \geq x + 8$$

$$2x - x \geq 8$$

$$x \geq 8$$

Hence, $x \leq 22$ and $x \geq 8$

i.e., $\quad 8 \leq x \leq 22$

Thus, the possible length of the shortest board is greater than or equal to 8 cm but less than or equal to 22 cm.

OR

Let x L of 3% solution be added to 460 L of 9% solution of acid.

Then, the total quantity of mixture = $(460 + x)$ L

Total acid content in the $(460 + x)$ L of mixture

$$= \left(460 \times \frac{9}{100} + x \times \frac{3}{100}\right)$$

It is given that acid content in the resulting mixture must be more than 5% but less than 7% acid.

Therefore,

$$5\% \text{ of } (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < 7\% \text{ of } (460 + x)$$

$$\Rightarrow \frac{5}{100} \times (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < \frac{7}{100} \times (460 + x)$$

$$\Rightarrow 5 \times (460 + x) < 460 \times 9 + 3x < 7 \times (460 + x)$$

(multiplying by 100)

$$\Rightarrow 2300 + 5x < 4140 + 3x < 3220 + 7x$$

Taking first two inequalities,

$$2300 + 5x < 4140 + 3x$$

$$\Rightarrow 5x - 3x < 4140 - 2300$$

$$\Rightarrow 2x < 1840$$

$$\Rightarrow x < \frac{1840}{2}$$

$$\Rightarrow x < 920$$

Taking last two inequalities,

$$4140 + 3x < 3220 + 7x$$

$$\Rightarrow 3x - 7x < 3220 - 4140$$

$$\Rightarrow -4x < -920$$

$$\Rightarrow 4x > 920$$

$$\Rightarrow x > \frac{920}{4}$$

$$\Rightarrow x > 230 \quad ...(ii)$$

Hence, the number of litres of the 3% solution of acid must be more than 230 L and less than 920 L.

SECTION - E

36. (A) Relation R is "x is the square of y".

$\therefore$ In set builder form, R = {(x, y) : x is the square of y, $x \in$ P, $y \in$ Q}.

(B) The domain of relation is an element of set P, *i.e.*, {4, 9, 25}.

(C) The range of relation is {–2, 2, –3, 3, –5, 5}.

Total number of ordered pair in R = 6 (note that total no. of ordered pairs possible are $3 \times 7 = 21$)

$\therefore$ Total number of relation = $2^6 = 64$

OR

$R = \{(x, y) : x, y \in z, x^2 + y^2 \leq 16\}$

then domain,

$$y^2 = 16 - x^2$$

$$y = \sqrt{16 - x^2}$$

$$16 - x^2 \geq 0$$

$$16 \geq x^2 - 4 \leq 0\ x \leq 4$$

So, the domain is {–4, –3, –2, –1, 0, 1, 2, 3, 4}

37. (A) Total members = 6

$\because$ Room A is a double shared room.

$\therefore$ The number of ways in which room A can be filled = ${}^6C_2 = 15$

(B) Now, rooms A and B can be filled with 2 members each and room C can be filled with 1 person.

$\therefore$ Required number of ways = ${}^2C_1 = 2$

(C) We have ${}^{2n}C_4 : {}^nC_3 = 12 : 1$

$$\Rightarrow \frac{{}^{2n}C_4}{{}^nC_3} = \frac{12}{1}$$

$$\Rightarrow {}^{2n}C_4 = 12\ {}^nC_3$$

$$\Rightarrow \frac{(2n)!}{4!(2n-4)!} = 12 \times \frac{(2n)!}{3!(2n-3)!}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)!}{4 \times 3 \times 2!(2n-4)} = 12 \times \frac{0(n-1)(n-2)(n-3)!}{2!(n-3)!}$$

$$\Rightarrow 4(2n-1)(2n-3) = 6(n-1)(n-2) \quad [\because n \neq 0]$$

$$2n(5n-7) = 0$$

$$n = \frac{7}{5} \text{ or } 0$$

OR

As, room A is filled with 2 persons

Now, the remaining persons = 4

Given that room C and D can occupy 1 person each.

$\therefore$ The number of ways in which rooms C and D can be filled = ${}^4C_1 \times {}^3C_1 = 12$

38. (A) Given $f(x) = |x|$ and $g(x) = f(x + 1) + f(x - 1)$

So,

$g(x) = |x + 1| + |x - 1|$

$$g(x) = \begin{cases} -(x+1)-(x-1), & \text{if } x < -1 \\ (x+1)-(x-1), & \text{if } -1 \leq x < 1 \\ (x+1)+(x-1), & \text{if } x \geq 1 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} -2x & \text{if } x < -1 \\ 2 & \text{if } -1 \le x < 1 \\ 2x & \text{if } x \ge 1 \end{cases}$$

For $\lim_{x \to -1} g(x)$ to exist

$$\lim_{x \to -1^-} g(x) = \lim_{x \to -1^+} g(x)$$

$$\Rightarrow \lim_{x \to -1^-} (-2x) = \lim_{x \to -1^+} 2$$

$$\Rightarrow \quad 2 = 2$$

$\lim_{x \to -1} g(x)$ exists and equal to $2 \Rightarrow a = 2$

(B) Consider $f(x) = \dfrac{x^2 - 4}{x^3 - 4x^2 + 4x}$

On putting $x = 2$, we get

$f(2) = \dfrac{4-4}{8-16+8} = \dfrac{0}{0}$, *i.e*, it is the form $\dfrac{0}{0}$.

So, let us first factorise it.

Consider, $\lim_{x \to 2} \dfrac{x^2 - 4}{x^3 - 4x^2 + 4x}$

$$\lim_{x \to 2} \frac{(x+2)(x-2)}{x^3 - 4x^2 + 4x}$$

$$= \lim_{x \to 2} \frac{(x+2)}{x(x-2)}$$

$$= \frac{2+2}{2(2-2)} = \frac{4}{0}$$

Which is not difined.