Chapter 11. Matrices

Ex 11.1

Answer 2.

$[2a+3b \quad a\text{-}b] = [19 \quad 2]$

$[2a+3b \quad a\text{-}b]$ is 1×2 matrix and $[19 \quad 2]$ is 1×2 matrix

$2a+3b = 19$ ____ (1)

$a\text{-}b = 2$ _____ (2)

$\Rightarrow a = 2+b$

Putting the value of a in equation (1)

$4+2b+3b = 19$ .... (1)

$\Rightarrow 5b = 15$

$\Rightarrow b = 3$

From (2)

$a = 2+3 = 5$

Answer 3.

$$\begin{bmatrix} 3x-y \\ 5 \end{bmatrix}_{2\times1} = \begin{bmatrix} 7 \\ x+y \end{bmatrix}_{2\times1}$$

$$3x-y=7 \quad -(1)$$
$$x+y=5 \quad -(2)$$
$$\Rightarrow x=5-y$$
putting the value of x in (1)
$$3(5-y)-y=7$$
$$\Rightarrow 15-3y-y=7$$
$$\Rightarrow -4y=-8$$
$$\Rightarrow y=2$$
from (2)
$$x+2=5$$
$$\Rightarrow x=3$$

Answer 6.

$A=[4\ 7]_{1\times2}\quad B=[3\ \ 1]_{1\times2}$

(i) A + 2B

$$A+2B=[4\ \ 7]+[6\ \ 2]$$
$$=[4+6\quad 7+2]$$
$$=[10\quad 9]_{1\times2}$$

(ii) A - B

$$A-B = [4-3\quad 7-1]$$
$$= [1\quad 6]_{1\times2}$$

(iii) 2A - 3B

$$2A-3B = [8\quad 14] - [9\quad 3]$$
$$= [-1\quad 11]_{1\times2}$$

Answer 7.

$$P = \begin{bmatrix}2 & 9\\5 & 7\end{bmatrix}_{2\times2},\quad Q=\begin{bmatrix}7 & 3\\4 & 1\end{bmatrix}_{2\times2}$$

(i) $$2P+3Q = \begin{bmatrix}4 & 18\\10 & 14\end{bmatrix}+\begin{bmatrix}21 & 9\\12 & 3\end{bmatrix}$$

$$= \begin{bmatrix}4+21 & 18+9\\10+12 & 14+3\end{bmatrix}=\begin{bmatrix}25 & 27\\22 & 17\end{bmatrix}_{2\times2}$$

(ii) $$2Q-P = \begin{bmatrix}14 & 6\\8 & 2\end{bmatrix}-\begin{bmatrix}2 & 9\\5 & 7\end{bmatrix}$$

$$= \begin{bmatrix}14-2 & 6-9\\8-5 & 2-7\end{bmatrix}=\begin{bmatrix}12 & -3\\3 & -5\end{bmatrix}_{2\times2}$$

(iii) $$3P-2Q = \begin{bmatrix}6 & 27\\15 & 21\end{bmatrix}-\begin{bmatrix}14 & 6\\8 & 2\end{bmatrix}$$

$$= \begin{bmatrix}6-14 & 27-6\\15-8 & 21-2\end{bmatrix}=\begin{bmatrix}-8 & 21\\7 & 19\end{bmatrix}_{2\times2}$$

Answer 8.

$$A=\begin{bmatrix}17 & 5 & 19\\ 11 & 8 & 13\end{bmatrix}_{2\times3},\ B=\begin{bmatrix}9 & 3 & 7\\ 1 & 6 & 5\end{bmatrix}_{2\times3}$$

$$2A-3B=\begin{bmatrix}34 & 10 & 38\\ 22 & 16 & 26\end{bmatrix}-\begin{bmatrix}27 & 9 & 21\\ 3 & 18 & 15\end{bmatrix}$$

$$=\begin{bmatrix}7 & 1 & 17\\ 19 & -2 & 11\end{bmatrix}_{2\times3}$$

Answer 9.

$$M=\begin{bmatrix}8 & 3\\ 9 & 7\\ 4 & 3\end{bmatrix}_{3\times2},\quad N=\begin{bmatrix}4 & 7\\ 5 & 3\\ 10 & 1\end{bmatrix}_{3\times2}$$

$$\text{(i)}\quad M+N=\begin{bmatrix}8+4 & 3+7\\ 9+5 & 7+3\\ 4+10 & 3+1\end{bmatrix}=\begin{bmatrix}12 & 10\\ 14 & 10\\ 14 & 4\end{bmatrix}_{3\times2}$$

$$\text{(ii)}\quad M-N=\begin{bmatrix}8-4 & 3-7\\ 9-5 & 7-3\\ 4-10 & 3-1\end{bmatrix}=\begin{bmatrix}4 & -4\\ 4 & 4\\ -6 & 2\end{bmatrix}$$

Answer 10.

$$A=\begin{bmatrix}1 & 9 & 4\\ 5 & 0 & 3\end{bmatrix}_{2\times3}$$

$$\text{(i)}\ \text{Negative}\,A=\begin{bmatrix}-1 & -9 & -4\\ -5 & 0 & -3\end{bmatrix}_{2\times3}$$

$$\text{(ii)}\ A^t=\begin{bmatrix}1 & 5\\ 9 & 0\\ 4 & 3\end{bmatrix}_{3\times2}$$

Answer 11.

$$P = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix}_{2\times2}$$

$$\text{(i)}\quad P^t = \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix}_{2\times2}$$

$$\text{(ii)}\quad P + P^t = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 12 \\ 12 & 4 \end{bmatrix}_{2\times2}$$

$$\text{(iii)}\quad P + p^t = \begin{bmatrix} 8 & 5 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 8 & 7 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}_{2\times2}$$

Answer 12.

$$B = \begin{bmatrix} 15 & 13 \\ 11 & 12 \\ 10 & 17 \end{bmatrix}_{3\times2}$$

$$B^t = \begin{bmatrix} 15 & 11 & 10 \\ 13 & 12 & 17 \end{bmatrix}_{2\times3}$$

To add two matrixes their order i.e. their corresponding number of rows and number of columns should be same, whereas in this case order of B and B^t are not same hence, we cannot add them.

Answer 13.

$$A=\begin{bmatrix}5 & r\\ p & 7\end{bmatrix}, B=\begin{bmatrix}q & 4\\ 3 & s\end{bmatrix}$$

$$A+B=\begin{bmatrix}s & r\\ p & 7\end{bmatrix}+\begin{bmatrix}q & 4\\ 3 & s\end{bmatrix}$$

$$A+B=\begin{bmatrix}s+q & r+4\\ p+3 & 7+s\end{bmatrix}_{2\times 2} \qquad -(1)$$

But, given

$$A+B=\begin{bmatrix}9 & 7\\ 5 & 8\end{bmatrix}_{2\times 2} \qquad -(2)$$

from (1) & (2)

$$\begin{bmatrix}5+q & r+4\\ p+3 & 7+s\end{bmatrix}=\begin{bmatrix}9 & 7\\ 5 & 8\end{bmatrix}$$

$5+q=9$

$\Rightarrow q=4$

$r+4=7$

$\Rightarrow r=3$

$p+3=5$

$\Rightarrow p=2$

$7+s=8$

$\Rightarrow s=1$

Answer 14.

$$A=\begin{bmatrix} p & q \\ 8 & 5 \end{bmatrix}_{2\times2}, \quad B=\begin{bmatrix} 3p & 5q \\ 2q & 7 \end{bmatrix}_{2\times2}$$

$$A+B=\begin{bmatrix} p+3q & q+5q \\ 8+2q & 5+7 \end{bmatrix}$$

$$A+B=\begin{bmatrix} 4p & 6q \\ 8+2q & 12 \end{bmatrix}_{2\times2} \quad - \quad (1)$$

$$\text{But given, } A+B=\begin{bmatrix} 12 & 6 \\ 2r & 3s \end{bmatrix}_{2\times2} \quad -(2)$$

from (1) and (2)

$$\begin{bmatrix} 4p & 6q \\ 8+2q & 12 \end{bmatrix}=\begin{bmatrix} 12 & 6 \\ 2r & 3s \end{bmatrix}$$

$4p=12$

$\Rightarrow p=3$

$6q=6$

$\Rightarrow q=1$

$8+2q=2r$

$8+2=2r$

$\Rightarrow r=5$

$12=3s$

$\Rightarrow s=4$

Answer 15.

Given,

$$\begin{bmatrix} 2a+b & c \\ d & 3a-b \end{bmatrix}_{2\times2} = \begin{bmatrix} 4 & 3a \\ 7 & 6 \end{bmatrix}_{2\times2}$$

$$2a+b=4 \quad -(1)$$
$$3a-b=6 \quad -(2)$$
Adding (1) and (2), we get
$$5a=10$$
$$\Rightarrow a=2$$
from (1)
$$2(2)+b=4$$
$$\Rightarrow b=0$$
$$c=3a$$
$$\Rightarrow c=3\times2$$
$$\Rightarrow c=6$$
$$\Rightarrow d=7$$

Answer 17.

$$A=\begin{bmatrix} 15 & 7 \\ 13 & 8 \end{bmatrix}_{2\times2}, B=\begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix}_{2\times2}$$

(1) $A+X=B$

$X=B-A$

$$\therefore X=\begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix}-\begin{bmatrix} 15 & 7 \\ 13 & 8 \end{bmatrix}=\begin{bmatrix} 1 & 5 \\ 14 & 3 \end{bmatrix}_{2\times2}$$

(ii) $2A-X=B$

$X=2A-B$

$$X=\begin{bmatrix} 30 & 14 \\ 26 & 16 \end{bmatrix}-\begin{bmatrix} 16 & 12 \\ 27 & 11 \end{bmatrix}$$
$$=\begin{bmatrix} 14 & 2 \\ -1 & 5 \end{bmatrix}_{2\times2}$$

Answer 18.

$$P = \begin{bmatrix} 14 & 17 \\ 13 & 1 \end{bmatrix}_{2\times2}, \quad Q = \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}_{2\times2}$$

$$P - M = 3Q$$

$$M = P - 3Q$$

$$M = \begin{bmatrix} 14 & 17 \\ 13 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ 9 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 14-6 & 17-3 \\ 13-9 & 1+9 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 14 \\ 4 & 10 \end{bmatrix}_{2\times2}$$

Ex 11.2

Answer 2.

$$A=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}_{2\times 2},\quad B=\begin{bmatrix}-2 & 3\\ -4 & 1\end{bmatrix}_{2\times 2}$$

(a) $$AB=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}\begin{bmatrix}-2 & 3\\ -4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-2-12 & 3+3\\ -6-8 & 9+2\end{bmatrix}=\begin{bmatrix}-14 & 6\\ -14 & 11\end{bmatrix}_{2\times 2}$$

(b) $$BA=\begin{bmatrix}-2 & 3\\ -4 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-2+9 & -6+6\\ -4+3 & -12+2\end{bmatrix}$$

$$=\begin{bmatrix}7 & 0\\ -1 & -10\end{bmatrix}_{2\times 2}$$

Answer 3.

Sol: Let $A=\begin{bmatrix}p & q\\ r & s\end{bmatrix}_{2\times 2}$

$$AI=\begin{bmatrix}p & q\\ r & s\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}p+0 & 0+q\\ r+0 & 0+s\end{bmatrix}$$

$$=\begin{bmatrix}p & q\\ r & s\end{bmatrix}=A$$

$$IA=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}p & q\\ r & s\end{bmatrix}$$

$$=\begin{bmatrix}p+0 & q+0\\ 0+r & 0+s\end{bmatrix}$$

$$=\begin{bmatrix}p & q\\ r & s\end{bmatrix}=a$$

Hence proved $AI=IA=A$.

Answer 4.

$$P = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2\times 2}, \quad Q = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}_{2\times 2}$$

$$\begin{aligned} QP &= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}_{2\times 2} \end{aligned}$$

$$\begin{aligned} P(QP) &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} \\ &= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}_{2\times 2} \end{aligned}$$