Sample Paper 12

Class- X Exam - 2022-23

Mathematics - Standard

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section A consists of 20 questions of 1 mark each.)

1. The prime factorisation of 96 is:

(a) $2^5 \times 3$ (b) 2^6

(c) $2^4 \times 3$ (d) $2^4 \times 32$ 1

2. The zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$ is:

(a) $\frac{1}{2}, 0$ (b) $0, \frac{1}{2}$

(c) $\frac{5}{6}, \frac{1}{2}$ (d) $\frac{3}{2}, \frac{3}{2}$ 1

3. If $x = a$, $y = b$ is the solution of the pair of equations $x - y = 2$ and $x + y = 4$, then the values of a and b is:

(a) $a = 3, b = 1$ (b) $a = 5, b = 2$

(c) $a = 1, b = 3$ (d) $a = 4, b = 2$ 1

4. Discriminant of the quadratic equation $2x^2 + 4x - 7 = 0$ is:

(a) 28 (b) 72

(c) 36 (d) 48 1

5. The roots of quadratic equation $x^2 - 4x + 2 = 0$ is:

(a) $2 \pm \sqrt{2}$ (b) $\sqrt{2}, 2$

(c) 3, 2 (d) $3 \pm 2\sqrt{2}$ 1

6. If $S_n = 5n^2 + 3n$, then its n^{th} term is:

(a) $5n - 1$ (b) $10n^2$

(c) $10n - 2$ (d) $8n - 3$ 1

7. If the common difference of an A.P. is 5, then find $a_{18} - a_{13}$.

(a) 38 (b) 40

(c) 18 (d) 25 1

8. In an A.P., a = –6 and d = 2. The sum of its first 20 terms is:

(a) 270 (b) 200

(c) 195 (d) 260 1

9. Write the relationship between the coefficients, if the following pair of equations is inconsistent. $ax + by + c = 0$; $a'x + b'y + c' = 0$.

(a) $\frac{a}{a'} \neq \frac{b}{b'} = \frac{c}{c'}$ (b) $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$

(c) $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$ (d) 0 1

10. In a ΔABC, right-angled at B, if AB : AC = 1 : 2, then the value of $\frac{2\tan A}{1+\tan^2 A}$ is:

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$ 1

11. If a tower 6 metres high casts a shadow on the ground that is $2\sqrt{3}$ metres long, then the elevation of the sun is:

(a) 60° (b) 45°
(c) 30° (d) 90° 1

12. The area of a circle, whose circumference is 22 cm, is:

(a) 38 cm^2 (b) 36 cm^2
(c) 38.5 cm^2 (d) 40 cm^2 1

13. If the ratio between the volumes of two spheres is 8 : 27, then the ratio between their surface areas is:

(a) 2 : 3 (b) 1 : 2
(c) 25 : 16 (d) 4 : 9 1

14. The class-mark of the class interval 10-25 is:

(a) 17.5 (b) 16
(c) 14 (d) 18 1

15. One card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is either red or a queen.

(a) $\frac{5}{14}$ (b) $\frac{1}{14}$
(c) $\frac{5}{13}$ (d) $\frac{7}{13}$ 1

16. How many face cards are there in a pack of 52 cards?

(a) 12 (b) 10
(c) 14 (d) 16 1

17. Determine the upper limit of the modal class of the following frequency distribution:

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

(a) 16 (b) 19.5
(c) 18 (d) 17.5 1

18. The empirical relationship among the three measures of central tendency mean, mode and median.

(a) 3 Median = Mode + 2 Mean
(b) Mode = 2 Median – Mean
(c) Mean = 3 Mode + 2 Median
(d) Median = Mode – Mean 1

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).

Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

19. **Statement A (Assertion):** If the circumference of a circle is 176 cm, then its radius is 28 cm.

Statement R (Reason): Circumference = $2\pi \times$ radius. 1

20. **Statement A (Assertion):** If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.

Statement R (Reason): Median = mode + 2 mean 1

SECTION - B

10 marks

(Section B consists of 5 questions of 2 marks each.)

21. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, –5) is the mid-point of PQ, find the coordinates of P and Q.

OR

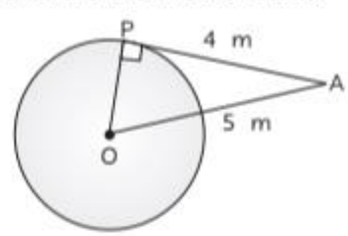
Find the third vertex of a triangle, if two of its vertices are at (–3, 1) and (0, –2) and the centroid is at the origin. 2

22. Explain why $3 \times 5 \times 7 \times 9 \times 11 + 11$ is a composite number.

OR

If $n = 2^3 \times 3^4 \times 5^4 \times 7$, where n is a natural number, then find the number of consecutive zeros in n. 2

23. A tangent from point A, which is placed 5 cm away from the circle's centre, has a length of 4 cm. Find the circle's radius.



2

24. Find the missing frequency for the given data if mean of distribution is 52.

Wages in Rs	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Workers	5	3	4	f	2	6	13

2

25. If a number x is chosen at random from the numbers $-2, -1, 0, 1, 2$, then what is the probability that $x^2 < 2$? 2

SECTION - C 18 marks

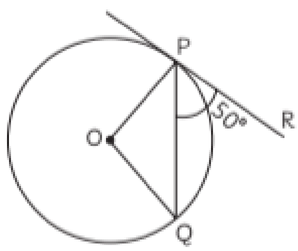
(Section C consists of 6 questions of 3 marks each.)

26. If p is a prime number, then prove that $\sqrt{p}$ is irrational. 3

27. Find the zeros of the polynomial $2x^2 - (1 + 2\sqrt{2})x + \sqrt{2}$. 3

28. Solve for x: $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0, 1, 2$ 3

29. In the figure, if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with the chord PQ, then determine $\angle POQ$.



OR

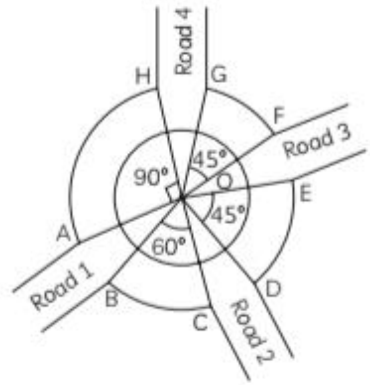
O is point inside a triangle ABC. The bisectors of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA at point D, E and F respectively, show that

$AD \times BE \times CF = DB \times EC \times FA$ 3

30. The diagram shows a round about at the junction of four roads (of equal width).

The central park is in the form of a circle with centre O and radius 14 m.

The curbs BC, DE, FG and HA are in the form of arcs that lie on a circle with centre O and radius 21 m. The angles subtended by these curbs at O are 60°, 45°, 45°, 90°.



(A) Find the total lengths of the curbs; $1\frac{1}{2}$

(B) Find the area of the circular road surrounded the central park. $1\frac{1}{2}$

31. Find the median of the following data:

Marks (out of 90)	No. of Students
0-10	2
10-20	2
20-30	4
30-40	6
40-50	6
50-60	5
60-70	2
70-80	4
80-90	4
Total	35

OR

Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another. Find the probability that both will visit the shop:

(A) the same day

(B) different days

(C) consecutive days. 3

SECTION - D

20 marks

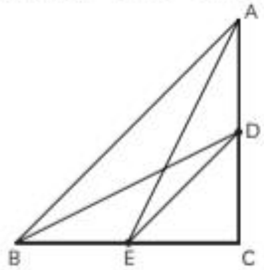
(Section D consists of 4 questions of 5 marks each.)

32. State and prove basic proportionality theorem.

OR

D and E are points on the sides CA and CB respectively of a $\triangle ABC$, right-angled at C.

Prove that: $AE^2 + BD^2 = AB^2 + DE^2$

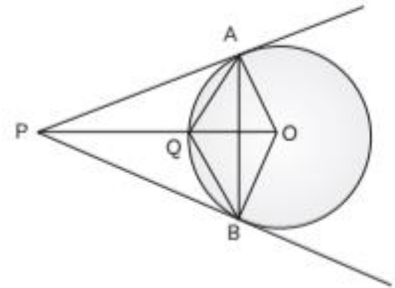


5

33. Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot A at the rate of ₹ 2 for 3 bananas and the second lot B at the rate of ₹ 1 per banana, and got a total collection of ₹ 400. If he had sold the first lot A at the rate of ₹ 1 per banana, and the second lot B at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460.

Determine the total number of bananas he had. 5

34. Two tangents are drawn from a point P to a circle with a centre of O. Prove that $\triangle APB$ is equilateral, if OP = diameter of the circle.



5

35. Trigonometric ratios sin A, sec A, and tan A should be expressed in the form of cotA.

OR

The angles of depression of the top and bottom of building 50 metres high as observed from the top of a tower are 30° and 60°, respectively. Find the height of the tower and also the horizontal distances between the building and the tower. 5

SECTION - E

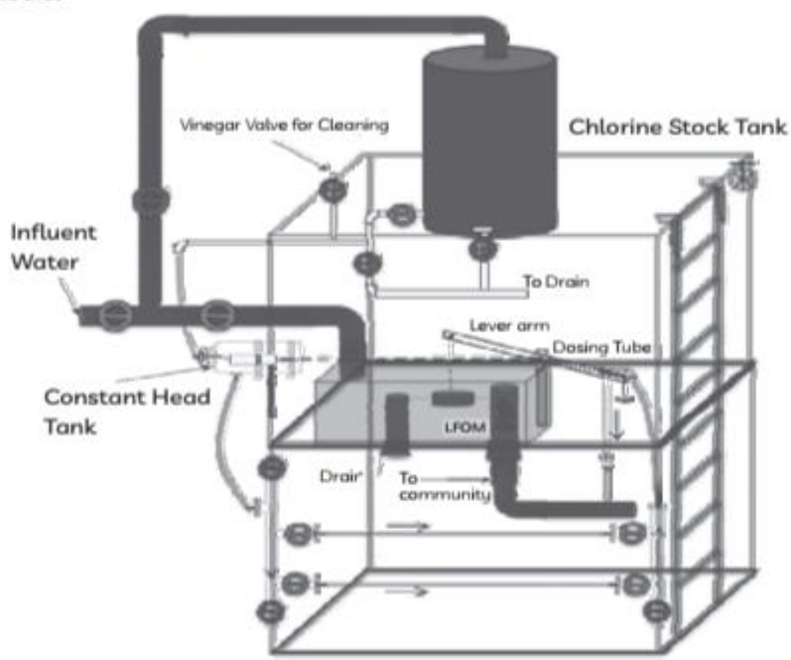
12 marks

(Case Study Based Questions)

(Section E consists of 3 questions. All are compulsory.)

36. *Selvi is setting up a water purifier system in her house which includes setting up an overhead tank in the shape of a right circular cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid.*

The underground water tank (sump) is a sturdy single moulded piece built to with stand underground pressure and is available in the storage capacity of 2000 L.



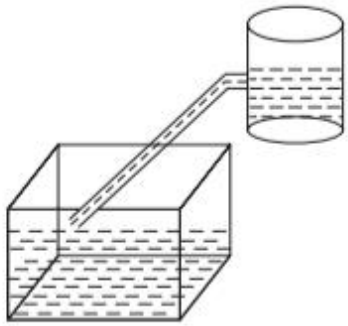
These, along with hassle-free installation and minimum maintenance needs make it the ideal water storage solution.

Dimensions (sump):

1.57 m × 1.44 m × 95 cm.

Dimensions (overhead tank):

Radius is 60 cm and Height is 95 cm



Water flow conditions at the required overload capacity should be checked for critical pressure drop to ensure that valves are adequately sized.

On the basis of the above information, answer the following questions:

(A) Find the ratio of the capacity of the sump to the capacity of the overhead tank. 1

(B) If overhead tank need to be painted to save it from corrosion, how much area need to be painted? 1

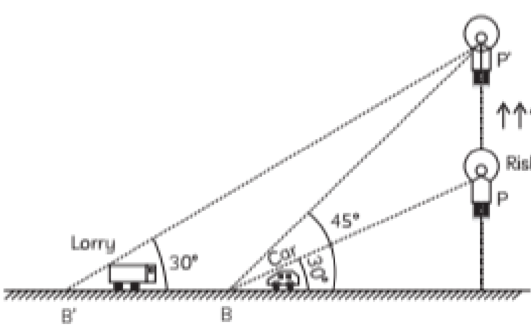
(C) If water is filled in the overhead tank at the rate of 20 litre per minute, the tank will be completely filled in how much time?

OR

If the amount of water in the sump, at an instant, is 1500 litres , then find the water level in the sump at that instant? 2

37. *Rishu is riding in a hot air balloon. After reaching a point P, he spots a car parked at B on the ground at an angle of depression of 30º. The balloon rises further by 50 metres and now he spots the same car at an angle of depression of 45º and a lorry parked at B' at an angle of depression of 30º.*

(Use $\sqrt{3}$ = 1.73)



The measurement of Rishu facing vertically is the height. Distance is defined as the measurement of car/lorry from a point in a horizontal direction. If an imaginary line is drawn from the observation point to the top edge of the car/lorry, a triangle is formed by the vertical, horizontal and imaginary line.

On the basis of the above information, answer the following questions:

(A) If the height of the balloon at point P is 'h' m and distance AB is 'x' m, then find the relation between 'x' and 'h'. 1

(B) Find the relation between the height of the balloon at point P' and distance AB. 1

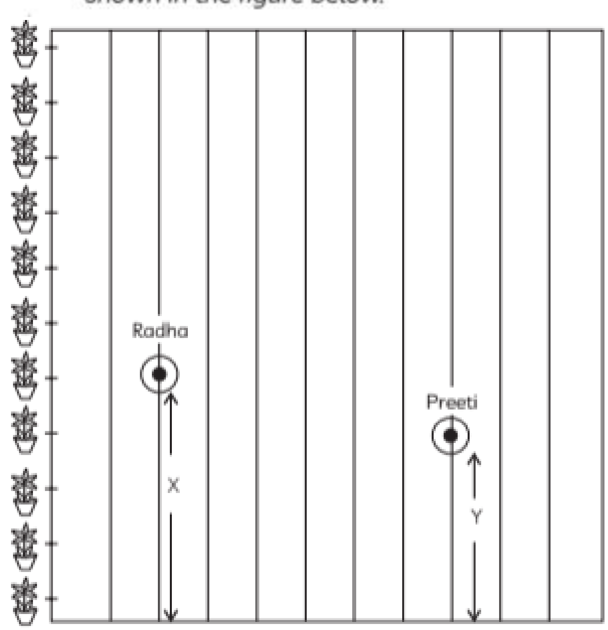
(C) Find the height of the balloon at point P, and the distance AB on the ground.

OR

Find the distance B'B on the ground. 2

38. *To conduct sport day activities in the rectangular school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each along DB.*

100 flower pots have been placed at a distance of 1 m from each other along DA as shown in the figure below.



Radha runs $\frac{1}{4}$th of the distance DA on 2nd line and post a green flag at X . Preeti runs $\frac{1}{5}$th of the distance DA on other line post a red flag at Y.

On the basis of the above information, answer the following questions:

(A) Treating DB as x-axis and DA as y-axis, then find the position of green flag. 1

(B) Treating DB as x-axis and DA as y–axis, the find the position of red flag. 1

(C) Find the distance (in complete metres) between the two flags.

OR

Find the perimeter (in complete metres) of the triangular region OXY. 2

SOLUTION

SECTION - A

1. (a) $2^5 \times 3$

Explanation: The prime factorisation of 96 is:

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

2. (d) $\frac{3}{2}, \frac{3}{2}$

Explanation: $p(x) = 4x^2 - 12x + 9 = (2x - 3)^2 = 0$

Thus, $x = \frac{3}{2}$ and $\frac{3}{2}$ are the two zeroes of $p(x)$.

3. (a) $a = 3, b = 1$

Explanation: As $x = a, y = b$ is the solution of:

$x - y = 2$ and $x + y = 4$

we have,

$a - b = 2$ and $a + b = 4$

$\Rightarrow$ $a = 3$ and $b = 1$

Caution

Derive the value of either x or y, but do which is more convenient and don't mess up the process.

4. (b) 72

Explanation: The discriminant of $2x^2 + 4x - 7 = 0$ is $[(4)^2 - 4(2)(-7)] = 16 + 56 = 72$

5. (a) $2 \pm \sqrt{2}$

Explanation: Here, quadratic equation is:

$$p(x) = x^2 - 4x + 2$$

On comparing it with

$ax^2 + bx + c = 0$

Then, $a = 1, b = -4, c = 2$

$$D = b^2 - 4ac$$
$$= (-4)^2 - 4 \times 1 \times 2$$
$$= 16 - 8 = 8$$

Then, roots, $x = \frac{-b + \sqrt{D}}{2a}$

$$= \frac{-(-4) \pm \sqrt{8}}{2}$$
$$= \frac{4 \pm 2\sqrt{2}}{2}$$
$$= 2 \pm \sqrt{2}$$

Hence, roots of the equation are $2 + \sqrt{2}$ and $2 - \sqrt{2}$.

6. (c) $10n - 2$

Explanation: $a_n = S_n - S_{n-1}$

$= 5n^2 + 3n - [5(n-1)^2 + 3(n-1)]$

$= 5n^2 + 3n - [5n^2 + 5 - 10n + 3n - 3]$

$= 10n - 2$

Caution

It is very important to remember that the difference of sum of first (n −1) terms and first n terms of an A.P. gives the value of n^{th} terms. i.e., $a_n = S_n - S_{n-1}$

7. (d) 25

Explanation: $a_{18} - a_{13} = a + 17d - a - 12d$

$= 5d = 5 \times 5$

$= 25$

8. (d) 260

Explanation: $S_n = \frac{n}{2}[2a + (n-1)d]$

So, $S_{20} = \frac{20}{2}[2(-6)+(20-1)(2)]$

$= 10[-12 + 38]$

$= 10 \times 26 = 260$

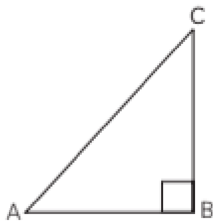
9. *(b)* $\frac{a}{a'}=\frac{b}{b'}\neq\frac{c}{c'}$

Explanation: The required relationship is:

$$\frac{a}{a'} = \frac{b}{b'}\neq\frac{c}{c'}$$

10. *(c)* $\frac{\sqrt{3}}{2}$

Explanation:



Here, $\frac{AB}{AC} = \frac{1}{2}$

Then, $\cos A = \frac{AB}{AC}$

$\cos A = \frac{1}{2}$

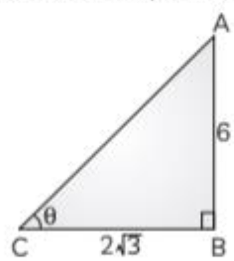
$\therefore \quad A = 60°$

Now, $\frac{2\tan A}{1+\tan^2 A} = \frac{2\tan 60°}{1+\tan^2 60°} = \frac{2\sqrt{3}}{1+(\sqrt{3})^2}$

$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

11. *(a)* *60°*

Explanation: As per the given question:



Hence,

$\tan\theta = \frac{6}{2\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \tan 60°$

$\theta = 60°$

12. *(c)* *38.5 cm²*

Explanation: Let r be the radius of the circle. Then,

$$2\pi r = 22 \text{ cm}$$

$\Rightarrow \quad r = \frac{22}{\left(2 \times \frac{22}{7}\right)}$

$\Rightarrow \quad r = \frac{7}{2}$ cm or 3.5 cm

Thus,

Area $= \pi r^2 = \frac{22}{7} \times 3.5 \times 3.5 \text{ cm}^2 = 38.5 \text{ cm}^2$

13. *(d)* *4 : 9*

Explanation: Let, the radius of 2 sphere be r and R

$$\frac{V_1}{V_2} = \frac{8}{27}$$

Then, $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27}$

$\Rightarrow \quad \frac{r}{R} = \frac{2}{3}$ ...(i)

Then, $\frac{S_1}{S_2} = \frac{4\pi r^2}{4\pi R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Hence, the required ratio is 4 : 9.

14. *(a)* *17.5*

Explanation: The class mark of 10 – 25 is $\frac{10+25}{2} = 17.5$.

15. *(d)* $\frac{7}{13}$

Explanation: In a pack of 52 cards, there are 26 red cards and 2 black queens.

So, total possible outcomes = 26 + 2 = 28

P(a red or a queen) $= \frac{28}{52} = \frac{7}{13}$

16. *(a)* *12*

Explanation: Face cards are jacks, queen and kings

$\therefore$ Total face cards = 4 + 4 + 4 = 12

17. *(d)* *17.5*

Explanation: The given frequency distribution in the exclusive form is:

Class	0.5 – 5.5	5.5 – 11.5	11.5- 17.5	17.5 – 23.5	23.5 – 29.5
Frequency	13	10	15	8	11

Here, the modal class is 11.5 – 17.5

So, the upper limit of the modal class is 17.5.

18. (a) *3 Median = Mode + 2 Mean*

Explanation: The required relationship is: 3Median = Mode + 2Mean

19. (a) *Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: Circumference (C) = 176 cm

$$2\pi r = 176$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 176$$

$$\Rightarrow \quad \frac{44}{7} \times r = 176$$

$$\Rightarrow \quad r = 176 \times \frac{7}{44} = 28 \text{ cm}$$

$\therefore$ The radius of the circle is 28 cm.

20. (c) *Assertion (A) is true but reason(R) is false.*

Explanation: $\text{Median} = \frac{1}{3}(\text{Mode} + 2 \text{ mean})$

$$= \frac{1}{3}(60 + 2 \times 66) = 64$$

SECTION - B

21. Let the coordinates of P and Q be $(0, y)$ and $(x, 0)$ respectively.

$\therefore$ Mid-point of PQ $= \left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

i.e. $\left(\frac{x}{2}, \frac{y}{2}\right)$

Equating it with (2, –5), we have,

$$\frac{x}{2} = 2; \ \frac{y}{2} = -5$$

$$\Rightarrow \quad x = 4, \quad y = -10$$

Thus, the coordinates of P and Q are (0, –10) and (4, 0) respectively.

OR

Given that two points are (– 3, 1) and (0, –2) and its centroid is (0, 0).

Let the third vertex be (x, y). Then,

$$\left(\frac{x-3+0}{3}, \frac{y+1-2}{3}\right) = (0, 0)$$

$$\Rightarrow \quad \frac{x-3}{3} = 0 \quad \text{and} \quad \frac{y-1}{3} = 0$$

$$\Rightarrow \quad x = 3 \quad \text{and} \quad y = 1$$

Thus, the third vertex is (3, 1).

22. $11(3 \times 5 \times 7 \times 9 + 1) = 11\ (945 + 1)$

$$= 11 \times 946$$

$$= 11 \times 2 \times 11 \times 43$$

Since, it has more than 2 factors.

Therefore, it is a composite number.

Caution

Understand the clear difference between prime and composite number.

OR

According to questions,

$$n = 2^3 \times 3^4 \times 5^4 \times 7$$

$$= 2^3 \times 3^4 \times 5^3 \times 5 \times 7$$

$$= (2 \times 5)^3 \times 3^4 \times 5 \times 7$$

$$= (10)^3 \times 3^4 \times 5 \times 7$$

$$= 3^4 \times 5 \times 7 \times 1000$$

Thus, in the given natural number '*n*' there are 3 zeros.

23. We know that the tangent at any point of a circle is $\perp$ to the radius through the point of contact.

$\therefore \quad \angle OPA = 90°$

$\therefore \quad OA^2 = OP^2 = AP^2$

(By Pythagoras theorem)

$\Rightarrow \quad (5)^2 = (OP)^2 + (4)^2$

$\Rightarrow \quad 25 = (OP)^2 + 16$

$\Rightarrow \quad OP^2 = 9$

$\Rightarrow \quad OP = 3$ cm

(OP = – 3 cm is moved)

24. Given data is

Find the missing frequency for the given data is mean of distribution is 52.

Wages in Rs	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Workers	5	3	4	f	2	6	13

Given data is

Wages	f_i	x_i	$f_i x_i$
10-20	5	15	75
20-30	3	25	75
30-40	4	35	140
40-50	f	45	$45f$
50-60	2	55	110
60-70	6	65	390
70-80	13	75	975
Total	$33 + f$		$1765 + 45f$

Then, $\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$$52 = \frac{1765 + 45f}{33 + f}$$

$$\Rightarrow \quad 7f = 1765 - 1716$$
$$= 49$$
$$\Rightarrow \quad f = 7$$

Hence, missing frequency is 7.

Caution

Understand the meaning of terms in the formula, so the correct placing of values can be done.

25. Total outcomes = 5

Favourable outcomes (i.e., $x^2 < 2$) are

4, 1, 0, 1, 4 i.e. 3

$$\therefore \quad P(x^2 < 2) = \frac{3}{5}$$

Hence, the required probability is $\frac{3}{5}$.

SECTION - C

26. Let us assume, to the contrary that $\sqrt{p}$ is rational.

So, we can find co-prime integers 'a' and 'b' ($b \neq 0$), such that $\sqrt{p} = \frac{a}{b}$

$$\Rightarrow \quad \sqrt{p}\, b = a$$
$$\Rightarrow \quad pb^2 = a^2 \qquad \text{...(i)}$$

$\Rightarrow a^2$ is divisible by p

$\Rightarrow a$ is divisible by p.

So, we can write $a = pc$ for some integer c.

Therefore, $a^2 = p^2c^2$ (squaring both side)

$$\Rightarrow \quad pb^2 = p^2c^2 \qquad \text{(from (i))}$$
$$\Rightarrow \quad b^2 = pc^2$$

$\Rightarrow b^2$ is divisible by p

$\Rightarrow b$ is divisible by p

$\Rightarrow p$ divides both a and b.

$\Rightarrow$ 'a' and 'b' have at least 'p' as a common factor

But this contradicts the fact that 'a' and 'b' are coprime.

This contradiction arises because we have assumed that $\sqrt{p}$ is rational.

Therefore, $\sqrt{p}$ is irrational.

27. $p(x) = 2x^2 - (1 + 2\sqrt{2})\,x + \sqrt{2} = 0$

$$= 2x^2 - x - 2\sqrt{2}\,x + \sqrt{2} = 0$$
$$= x(2x - 1) - \sqrt{2}\,(2x - 1) = 0$$
$$= (2x - 1)\,(x - \sqrt{2}) = 0$$

Either, $2x - 1 = 0$ or $x - \sqrt{2} = 0$

$$\Rightarrow \quad x = \frac{1}{2} \quad \text{or} \quad x = \sqrt{2}$$

Thus, zeros of the given polynomial are $\frac{1}{2}$ and $\sqrt{2}$.

28. Here, $\frac{1}{x - 2} + \frac{2}{x - 1} = \frac{6}{x}, x \neq 0, 1, 2$

$$\Rightarrow \quad \frac{(x - 1) + 2(x - 2)}{(x - 2)(x - 1)} = \frac{6}{x}$$
$$\Rightarrow \quad \frac{x - 1 + 2x - 4}{x^2 - 2x - x + 2} = \frac{6}{x}$$
$$\Rightarrow \quad (3x - 5)x = 6(x^2 - 3x + 2)$$
$$\Rightarrow \quad 3x^2 - 5x = 6x^2 - 18x + 12$$
$$\Rightarrow \quad 3x^2 - 13x + 12 = 0$$
$$\Rightarrow \quad 3x^2 - 9x - 4x + 12 = 0$$
$$\Rightarrow \quad 3x(x - 3) - 4(x - 3) = 0$$
$$\Rightarrow \quad (3x - 4)\,(x - 3) = 0$$
$$\Rightarrow \quad x = \frac{4}{3}, 3$$

Hence, the values of x are $\frac{4}{3}$ and 3.

29. Here, $\angle QPR = 50°$

and $\angle OPR = 90°$

[$\because$ Radius is perpendicular to tangent]

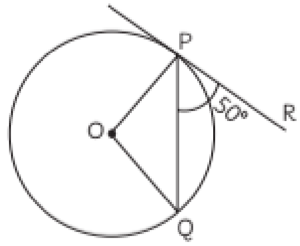
$\therefore \quad \angle OPQ + \angle QPR = 90°$

$\Rightarrow \quad \angle OPQ + 50° = 90°$

$\Rightarrow \quad \angle OPQ = 90° - 50° = 40°$

Now, $\quad OP = OQ$

[Radii of same circle]



$\therefore \quad \angle OPQ = \angle OQP = 40°$

In ΔOPQ,

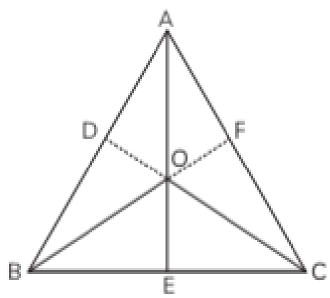
$\angle POQ + \angle OPQ + \angle OQP = 180°$

$\Rightarrow \quad \angle POQ + 40° + 40° = 180°$

$\Rightarrow \quad \angle POQ = 180° - 80° = 100°$

OR

Given : A ΔABC, in which O is a point inside it. The bisector of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CD at points D, E and F respectively.



To Prove : $AD \times BE \times CF = DB \times EC \times FA$

Proof : In ΔAOB, OD is the bisector of $\angle AOB$.

$$\therefore \quad \frac{OA}{OB} = \frac{AD}{BD} \qquad ...(i)$$

In ΔBOC, OE is the bisector of $\angle BOC$.

$$\therefore \quad \frac{OB}{OC} = \frac{BE}{EC} \qquad ...(ii)$$

In ΔCOA, OF is the bisector of $\angle COA$.

$$\therefore \quad \frac{OC}{OA} = \frac{CF}{FA} \qquad ...(iii)$$

Multiplying the corresponding sides of (i), (ii) and (iii), we get

$$\frac{OA}{OB} \times \frac{OB}{OC} \times \frac{OC}{OA} = \frac{AD}{BD} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

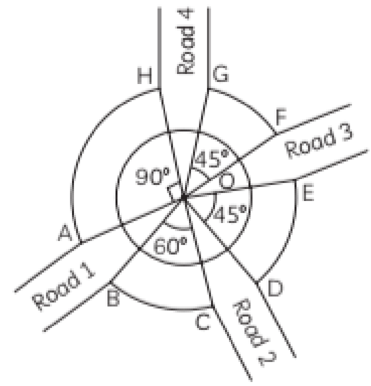
$$\Rightarrow \quad 1 = \frac{AD}{BD} \times \frac{BE}{EC} \times \frac{CF}{FA}$$

$\Rightarrow DB \times EC \times FA = AD \times BF \times CF$

or $AD \times BE \times CF = DB \times EC \times FA$

Hence, proved

30. (A) Total length of curbs: $\overset{\frown}{BC} + \overset{\frown}{DE} + \overset{\frown}{FG} + \overset{\frown}{HA}$



$$= 2\pi(21)\ \frac{60°}{360°} + 2\pi(21)\ \frac{45°}{360°} + 2\pi(21)\ \frac{45°}{360°} + 2\pi(21)\ \frac{90°}{360°}$$

$$= \frac{42\pi}{360}\ (60 + 45 + 45 + 90)$$

$$= \frac{42\neq \times 240}{360} = 88 \text{ m} \ \left(\text{Taking } \pi = \frac{22}{7}\right)$$

(B) Area of the circular road surrounding the central park

$= [\pi(21)^2 - \pi(14)^2]\ m^2$

$= \frac{22}{7} \times (21 + 14)\ (21 - 14)\ m^2$

$= 770\ m^2$.

31.

Marks	No. of Students	c.f.	
0-10	2	2	
10-20	2	4	
20-30	4	8	
30-40	6	14	
40-50	6	20	Median Class
50-60	5	25	
60-70	2	27	
70-80	4	31	
80-90	4	35	
Total	35		

Then, $\frac{N}{2} = \frac{35}{2} = 17.5$

Then, median class is 40 – 50

$l = 40$ $f = 6$, $c.f. = 14$, $h = 10$

Then, $\quad Me = l + \frac{(N/2 - cf)}{f} \times h$

$= 40 + \frac{(17.5 - 14)}{6} \times 10$

$= 40 + \frac{3.5}{6} \times 10 = 40 + 5.83$

$= 45.83$

Thus, median of the given data is 45.83

OR

Two customer can visit the shop in 6 × 6 ways = 36 ways.

Total no. of events = 36

(A) Two customer can visit the shop on the same day in one of the following ways, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday

$\therefore$ P(of visiting on same day) = $\frac{6}{36} = \frac{1}{6}$

(B) P(of visiting on different day) = $1 - \frac{1}{6}$

$= \frac{5}{6}$

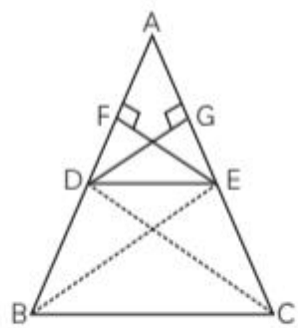
(C) Two customers can visit the shop on 2 consecutive days in the following ways : (Mon, Tue), (Tue, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Sat) = 5 ways

$\therefore$ P (of visiting on consecutive days) = $\frac{5}{36}$

SECTION - D

32. Statement "If a side is parallel to one side of a triangle and it intersects the other two sides in 2 distinct points, then it divides the other 2 sides in same proportion."

Proof :



Given : A ΔABC, a line DE parallel to BC intersect AB and D and AC at E.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw EF ⊥ AB and DG ⊥ AC Join BE and CE

Proof : Since, EF ⊥ AB

EF is the height of triangle ADE and DBE.

Area of $\Delta ADE = \frac{1}{2} \times b \times h = \frac{1}{2} \times AD \times EF$

Area of $\Delta DBE = \frac{1}{2} \times DB \times EF$

$\therefore \quad \frac{ar\,(\Delta ADE)}{ar\,(\Delta DBE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF} = \frac{AD}{DB} \qquad ...(i)$

Similarly,

$\frac{ar\,(\Delta ADE)}{ar\,(\Delta DCE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC} \qquad ...(ii)$

But ΔDBE and ΔDCE are on the same base DE and between the same parallel straight lines BC and DE.

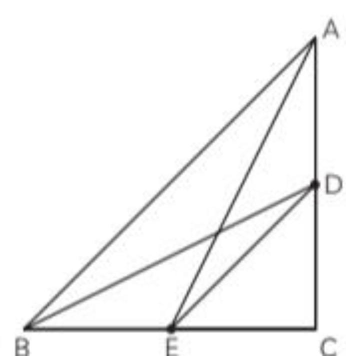
Area of ΔDBE = Area of ΔDCE ...(iii)

From, (i), (ii) and (iii), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, proved.

OR



In the figure, ACB is a right angled triangle, right-angled at C. D and E are points on sides CA and BC respectively.

We Join DE, BD and AE.

In right triangle ACE, we have:

$$AE^2 = AC^2 + EC^2 \quad ...(i)$$

In right triangle BCD, we have:

$$BD^2 = BC^2 + DC^2 \quad ...(ii)$$

Adding (i) and (ii), we get:

$$\begin{aligned} AE^2 + BD^2 &= (AC^2 + EC^2) + (BC^2 + DC^2) \\ &= (BC^2 + CA^2) + (CE^2 + CD^2) \\ &= AB^2 + DE^2 \end{aligned}$$

Hence proved

33. Let lot A contains 'x' bananas; and lot B contains 'y' bananas. Then, according to the question,

$$\frac{2}{3}x + y = 400 \; ; \quad x + \frac{4}{5}y = 460$$

$\Rightarrow \quad 2x + 3y = 1200 \; ; \; 5x + 4y = 2300$

$\Rightarrow \quad 10x + 15y = 6000 \quad ...(i)$

and $\quad 10x + 8y = 4600 \quad ...(ii)$

Subtracting (ii) from (i), we get

$\Rightarrow \quad 7y = 1400 \quad \Rightarrow \quad y = 200.$

From (i), we get

$$10x + 15 \times 300 = 6000$$

$\Rightarrow \quad 10x = 1500$

$\Rightarrow \quad x = 300$

Thus, lot A contains 300 bananas and lot B contains 200 bananas.

$\therefore$ Vijay had 500 bananas in all.

34. Join OP.

Suppose OP meets the circle at Q. Join AQ.

We have

i.e., $\quad OP = \text{diameter}$

$\therefore \quad OQ + PQ = \text{diameter}$

$\therefore \quad OQ + PQ = \text{diameter}$

$\quad PQ = \text{Diameter} - \text{radius} \quad [\because OQ = r]$

$\therefore \quad PQ = \text{radius}$

Thus, $\quad OQ = PQ = \text{radius}$

Thus, OP is the hypotenuse of right triangle OAP and Q is the mid-point of OP

$\therefore \quad OA = AQ = OQ$

[$\because$ mid-point of hypotenuse of a right triangle is equidistant from the vertices]

$\Rightarrow \Delta OAQ$ is equilateral

$\Rightarrow \quad \angle AOQ = 60°$

So, $\quad \angle APO = 30°$

$\therefore \quad \angle APB = 2\angle APO = 60°$

Also $\quad PA = PB$

$\Rightarrow \quad \angle PAB = \angle PBA$

But $\quad \angle APB = 60°$

$\therefore \quad \angle PAB = \angle PBA = 60°$

Hence, ΔAPB is equilateral triangle.

35. For sin A,

Bu using identity, $\text{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \quad \text{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow \quad \dfrac{1}{\sin^2 A} = 1 + \cot^2 A$

$\Rightarrow \quad \sin^2 A = \dfrac{1}{1+\cot^2 A}$

$\Rightarrow \quad \sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

For sec A,

By using identity, $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \quad \sec^2 A = 1 + \tan^2 A$

$\Rightarrow \quad \sec^2 A = 1 + \dfrac{1}{\cot^2 A} = \dfrac{\cot^2 A + 1}{\cot^2 A}$

$\Rightarrow \quad \sec^2 A = \dfrac{1+\cot^2 A}{\cot^2 A}$

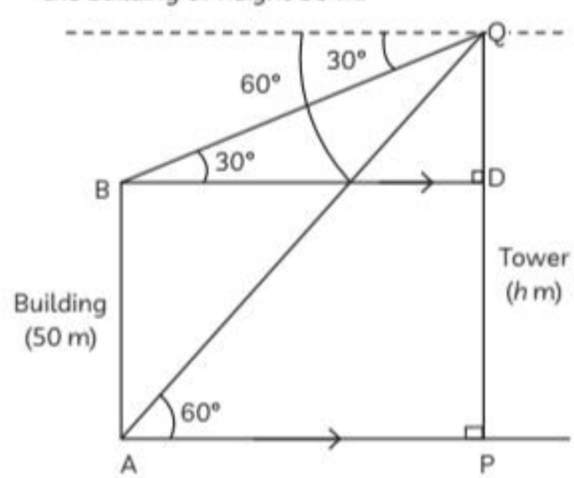
$\Rightarrow \quad \sec A = \dfrac{\sqrt{1+\cot^2 A}}{\cot A}$

For tan A,

$\Rightarrow \quad \tan A = \dfrac{1}{\cot A}$

OR

Let PQ be the tower of height h m and AB be the building of height 50 m.



From right triangle BDQ,

$$\frac{DQ}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BD = \sqrt{3}\,DQ \quad .....(i)$$

Also, from right triangle APQ,

$$\frac{PQ}{AP} = \tan 60° = \sqrt{3}$$

or $$\frac{PQ}{BD} = \sqrt{3} \quad [\because AP = BD]$$

or $$BD = \frac{PQ}{\sqrt{3}} \quad .....(ii)$$

From (i) and (ii), we have $DQ = \frac{PQ}{3}$.

Further,

$$PQ = PD + DQ$$

$$PQ = 50 + \frac{PQ}{3}$$

$[\because PD = AB]$

$$\Rightarrow \quad \frac{2}{3}PQ = 50$$

$$\Rightarrow \quad PQ = 50 \times \frac{3}{2} = 75 \text{ m}$$

Thus, the height of the tower is 75 m.

From Eq (i), we have:

$$AP = BD$$

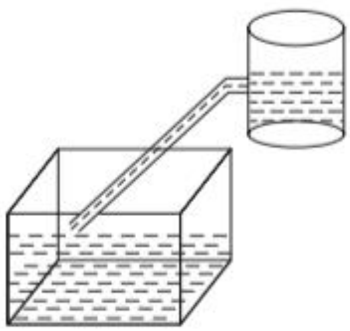
$$= \frac{PQ}{\sqrt{3}} = \frac{75}{\sqrt{3}}$$

$$= 25\sqrt{3} \text{ i.e. } 43.25 \text{ m.}$$

Thus, the horizontal distances between the tower and the building is 43.25 m.

SECTION - E

36.



(A) $$\frac{\text{Capacity of tank}}{\text{Capacity of sump}} = \frac{\pi r^2 H}{lbh}$$

$$= \frac{3.14 \times 0.6 \times 0.6 \times 0.95}{1.57 \times 1.44 \times 0.95}$$

$$= \frac{1}{2}$$

(B) C.S.A. of cylindrical tank $= 2\pi rH$

$= 2 \times 3.14 \times 60 \times 95$

$= 35{,}796 \text{ cm}^2$

$= 3.5796 \text{ m}^2$

$= 3.6 \text{ m}^2$

(C) Volume of water in cylindrical tank

$= \pi r^2 h$

$= 3.14 \times 60 \times 60 \times 95$

$= 1073880 \text{ cm}^3$

Now, $1l = 1000 \text{ cm}^3$

$\therefore$ Volume of tank $= 1073.88\ l$

20l tank is filled in 1 minute

$\therefore$ 1073.88l tank is filled in $\frac{1073.88l}{20}$

$= 53.69$

$= 54$ minutes

OR

Volume of water in sump

$= 1500$ litres

$= 1.5 \text{ m}^3$

Then, $V = lbh$

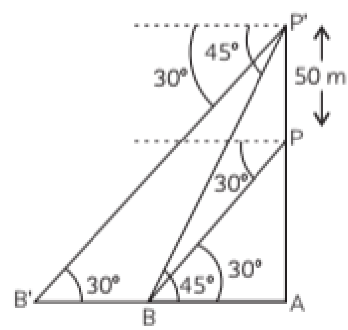
$1.57 \times 1.44 \times h = 1.5$

$$h = \frac{1.5}{1.57 \times 1.44}$$

$= 0.663$ m

$= 66.3$ cm

37.



(A) In ΔAPB,

$$\tan 30° = \frac{AP}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}h$$

(B) In $\Delta AP'B$

$$\tan 45° = \frac{AP'}{AB}$$

$$\Rightarrow \quad AB = AP'$$

$$\Rightarrow \quad x = h + 50$$

(C) On solving equation obtained in (A) and (B), we get

$$\sqrt{3}h = h + 50$$

$$\Rightarrow \quad h(\sqrt{3}-1) = 50$$

$$\Rightarrow \quad h = \frac{50}{0.732} = 68.25$$

In ΔAPB,

$$\tan 30° = \frac{AP}{AB}$$

$$\Rightarrow \quad AB = \frac{AP}{\tan 30°} = \frac{68.25}{1/\sqrt{3}}$$

$$= 68.25 \times 1.732$$

$$= 118 \text{ m}$$

Caution

→ *In solving word problems, drawing of correct figure is very important, otherwise the answer obtained will be wrong.*

OR

In $\Delta AP'B'$

$$\tan 30° = \frac{AP'}{AB'}$$

$$\frac{1}{\sqrt{3}} = \frac{68.25 + 50}{AB'}$$

$$\Rightarrow \quad AB' = 118.25 \times 1.732$$

$$= 204.809$$

$$BB' = AB' - AB$$

$$= 204.809 - 118$$

$$= 86.80 = 87 \text{ m}$$

38. (A) Radha's distance on x-axis is 2 and on y-axis, she is at $\frac{1}{4} \times 100 = 25°$

Green flag coordinates are (2, 25)

(B) X-coordinate = 8

Y-coordinate $= \frac{1}{5} \times 100 = 20$

$\therefore$ Coordinates of red flag (8, 20)

(C) coordinates of green flag is (2, 25)

$\therefore$ coordinates of red flag is (8, 20)

$\therefore$ By distance formula

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{36 + 25}$$

$$= \sqrt{61} = 7.8 \text{ cm}$$

$$= 8 \text{ cm (approx)}$$

OR

$$OX = \sqrt{(2-0)^2 + (25-0)^2}$$

$$= \sqrt{4 + 625} = \sqrt{629} = 25.07$$

$$OY = \sqrt{(8-0)^2 + (20-0)^2}$$

$$= \sqrt{64 + 400} = \sqrt{464} = 21.54$$

$$XY = \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{36 + 25} = \sqrt{61} = 7.81$$

$$\text{Perimeter} = OX + OY + YY$$

$$= 25.07 + 21.54 + 7.81$$

$$= 54.42$$

$$= 55 \text{ m}$$