Solved Paper–2

Class 9th, Mathematics, SA–2

Time: 3hours **Max. Marks 90**

General Instructions

1. All questions are compulsory.
2. Draw neat labeled diagram wherever necessary to explain your answer.
3. Q.No. 1 to 8 are of objective type questions, carrying 1 mark each.
4. Q.No.9 to 14 are of short answer type questions, carrying 2 marks each.
5. Q. No. 15 to 24 carry 3 marks each. Q. No. 25 to 34 carry 4 marks each.

1. Point (0, –7) lies
   (A) on the x –axis (B) in the second quadrant
   (C) on the y-axis (D) in the fourth quadrant

2. If AB = QR, BC = PR and CA = PQ, then
   (A) Δ ABC $\cong$ Δ PQR (B) Δ CBA $\cong$ Δ PRQ
   (C) Δ BAC $\cong$ Δ RPQ (D) Δ PQR $\cong$ Δ BCA

3. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is:
   (A) 17 cm (B) 15 cm
   (C) 4 cm (D) 8 cm

4. A linear equation in two variables is of the form ax + by + c = 0, where
   (A) $a \neq 0, b \neq 0$ (B) $a = 0, b \neq 0$
   (C) $a \neq 0, b = 0$ (D) $a = 0, c = 0$

5. The class mark of the class 90-120 is :
   (A) 90 (B) 105
   (C) 115 (D) 120

6. The number of planks of dimensions (4 m × 50 cm × 20 cm) that can be stored in a pit which is 16 m long, 12m wide and 4 m deep is
   (A) 1900 (B) 1920
   (C) 1800 (D) 1840

7. In a medical examination of students of a class, the following blood groups are recorded:

Blood group	A	AB	B	O
Number of students	10	13	12	5

A student is selected at random from the class. The probability that he/she has blood group B, is :

(A)1/4 (B)13/40

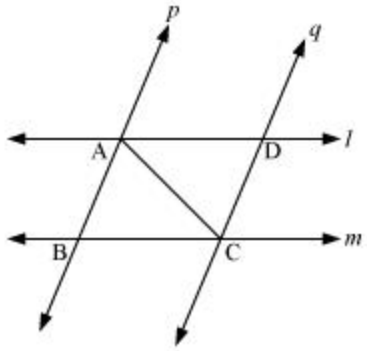
(C)3/10 (D)1/8

8. The lateral surface area of a cube is 256 m^2. The volume of the cube is

(A) 512 m^3 (B) 64 m^3

(C) 216 m^3 (D) 256 m^3

9. *l* and *m* are two parallel lines intersected by another pair of parallel lines *p* and *q* (see the given figure). Show that $\Delta ABC \cong \Delta CDA$.



10. Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm $\left[\text{Assume } \pi = \frac{22}{7}\right]$

11. The distance (in km) of 40 engineers from their residence to their place of work were found as follows:

5	3	10	20	25	11	13	7	12	31
19	10	12	17	18	11	32	17	16	2
7	9	7	8	3	5	12	15	18	3
12	14	2	9	6	15	15	7	6	12

Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0 - 5 (5 not included). What main feature do you observe from this tabular representation?

12.

Concentration of SO_2 (in ppm)	**Number of days (frequency)**
0.00 − 0.04	4
0.04 − 0.08	9
0.08 − 0.12	9
0.12 − 0.16	2
0.16 − 0.20	4
0.20 − 0.24	2
Total	30

The above frequency distribution table represents the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 − 0.16 on any of these days.

13. Prove that line of centres of two intersecting circles subtends equal angles at the two points of intersection.

14. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

0 1 2 2 1 2 3 1 3 0
1 3 1 1 2 2 0 1 2 1
3 0 0 1 1 2 3 2 2 0

Prepare a frequency distribution table for the data given above.

15. Draw the graph of each of the following linear equations in two variables:
(i) $y = 3x$ (ii) $3 = 2x + y$

16. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

17. Construct a triangle PQR in which QR = 6 cm, $\angle Q = 60°$ and PR − PQ = 2 cm

18. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

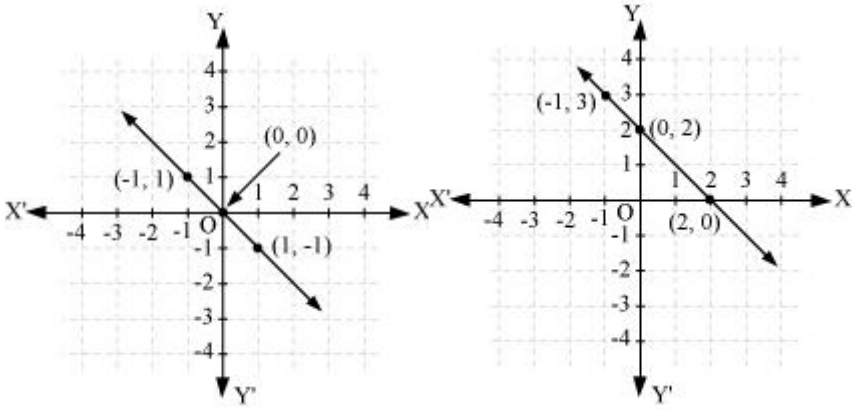
19. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 − 44 (in years) worldwide, found the following figures (in %):

S.No.	Causes	Female fatality rate (%)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the information given above graphically.
(ii) Which condition is the major cause of women's ill health and death worldwide?
(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

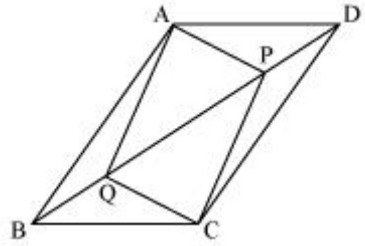
20. From the choices given below, choose the equation whose graphs are given in the given figures.

	For the first figure		**For the second figure**
(i)	$y = x$	(i)	$y = x + 2$
(ii)	$x + y = 0$	(ii)	$y = x - 2$
(iii)	$y = 2x$	(iii)	$y = -x + 2$
(iv)	$2 + 3y = 7x$	(iv)	$x + 2y = 6$



21. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

22. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that:



(i) $\Delta APD \cong \Delta CQB$
(ii) AP = CQ
(iii) $\Delta AQB \cong \Delta CPD$
(iv) AQ = CP
(v) APCQ is a parallelogram

23. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:
(i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

24. The following table gives the life times of neon lamps:

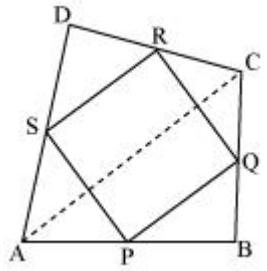
Length (in hours)	**Number of lamps**
300 − 400	14
400 − 500	56
500 − 600	60

600 − 700	86
700 − 800	74
800 − 900	62
900 − 1000	48

(i) Represent the given information with the help of a histogram.

(ii) How many lamps have a lifetime of more than 700 hours?

25. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:



(i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

26. Construct an equilateral triangle, given its side and justify the construction

27. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for *x*-axis and Fahrenheit for *y*-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

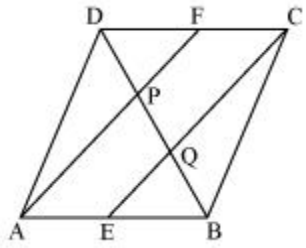
(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

28. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

29. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

30. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



31. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

Number of letters	**Number of surnames**
1 − 4	6
4 − 6	30
6 − 8	44
8 − 12	16
12 − 20	4

(i) Draw a histogram to depict the given information.
(ii) Write the class interval in which the maximum number of surname lie.

32. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}A, 90° - \frac{1}{2}B$ and $90° - \frac{1}{2}C$.

33. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs 498.96. If the cost of white-washing is Rs 2.00 per square meter, find the

(i) inside surface area of the dome,

(ii) volume of the air inside the dome. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

34. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

Solutions

1. C
2. B
3. D
4. B
5. B
6. B
7. C
8. A

9. In ΔABC and ΔCDA,
$\angle BAC = \angle DCA$ (Alternate interior angles, as $p \parallel q$)
$AC = CA$ (Common)
$\angle BCA = \angle DAC$ (Alternate interior angles, as $l \parallel m$)
$\therefore \Delta ABC \cong \Delta CDA$ (By ASA congruence rule)

10. (i) Radius (r) of cone = 6 cm
Height (h) of cone = 7 cm
Volume of cone $= \frac{1}{3}\pi r^2 h$
$= \left[\frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7\right] \text{ cm}^3$
$= (12 \times 22) \text{ cm}^3$
$= 264 \text{ cm}^3$
Therefore, the volume of the cone is 264 cm^3.

(ii) Radius (r) of cone = 3.5 cm
Height (h) of cone = 12 cm
Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\right] \text{ cm}^3$$

$$= \left(\frac{1}{3} \times 22 \times \frac{1}{2} \times 3.5 \times 12\right) \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

Therefore, the volume of the cone is 154 cm^3.

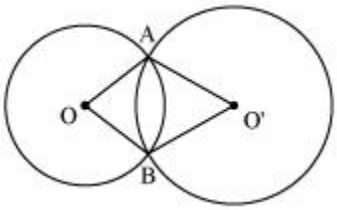
11. It is given that a grouped frequency distribution table of class size 5 has to be constructed. Therefore, the class intervals will be 0 − 5, 5 − 10, 10 − 15, 15 −20…
By observing the data given as above, a grouped frequency distribution table can be constructed as follows.

Distance (in km)	Tally mark	Number of engineers
0 − 5	𝍸	5
5 − 10	𝍸 𝍸 |	11
10 −15	𝍸 𝍸 |	11
15 − 20	𝍸 ||||	9
20 − 25	|	1
25 − 30	|	1
30 − 35	||	2
Total		40

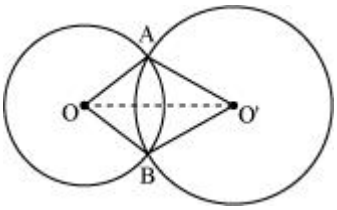
It can be observed that there are very few engineers whose homes are at more than or equal to 20 km distance from their work place. Most of the engineers have their workplace up to 15 km distance from their homes.

12. Number days for which the concentration of sulphur dioxide was in the interval of 0.12 − 0.16 = 2
Total number of days = 30
Hence, required probability, $P = \frac{2}{30} = \frac{1}{15}$

13.



Let two circles having their centres as O and O' intersect each other at point A and B respectively. Let us join OO'.



In $\Delta AOO'$ and BOO',

OA = OB (Radius of circle 1)

$O'A = O'B$ (Radius of circle 2)

$OO' = OO'$ (Common)

$\Delta AOO' \cong \Delta BOO'$ (By SSS congruence rule)

$\angle OAO' = \angle OBO'$ (By CPCT)

Therefore, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

14. By observing the data given above, the required frequency distribution table can be constructed as follows.

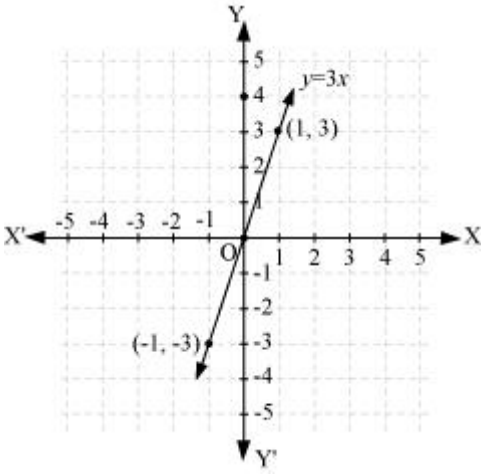
Number of heads	**Number of times (frequency)**
0	6
1	10
2	9
3	5
Total	30

15. (i) $y = 3x$

It can be observed that $x = -1$, $y = -3$ and $x = 1$, $y = 3$ are solutions of the above equation. Therefore, the solution table is as follows.

x	-1	1
y	-3	3

The graph of the above equation is constructed as follows.

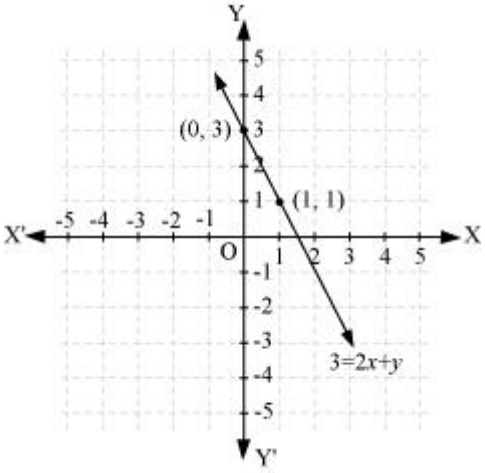


(ii) $3 = 2x + y$

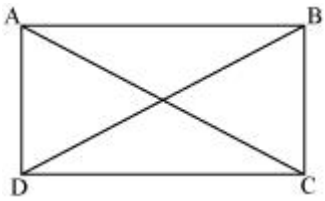
It can be observed that $x = 0$, $y = 3$ and $x = 1$, $y = 1$ are solutions of the above equation. Therefore, the solution table is as follows.

x	0	1
y	3	1

The graph of this equation is constructed as follows.

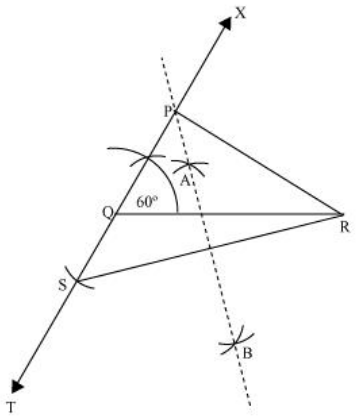


16. Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is 90º.



In ΔABC and ΔDCB,
AB = DC (Opposite sides of a parallelogram are equal)
BC = BC (Common)
AC = DB (Given)
∴ ΔABC ≅ ΔDCB (By SSS Congruence rule)
⇒ ∠ABC = ∠DCB
It is known that the sum of the measures of angles on the same side of transversal is 180º.
∠ABC + ∠DCB = 180º (AB || CD)
⇒ ∠ABC + ∠ABC = 180º
⇒ 2∠ABC = 180º
⇒ ∠ABC = 90º
Since ABCD is a parallelogram and one of its interior angles is 90º, ABCD is a rectangle.

17. The below given steps will be followed to construct the required triangle.
Step I: Draw line segment QR of 6 cm. At point Q, draw an angle of 60°, say ∠XQR.
Step II: Cut a line segment QS of 2 cm from the line segment QT extended in the opposite side of line segment XQ. (As PR > PQ and PR − PQ = 2 cm). Join SR.
Step III: Draw perpendicular bisector AB of line segment SR. Let it intersect QX at point P. Join PQ, PR.
ΔPQR is the required triangle.



18. Radius (r) of pit $= \left(\frac{3.5}{2}\right)$ m $= 1.75$ m

Height (h) of pit = Depth of pit = 12 m

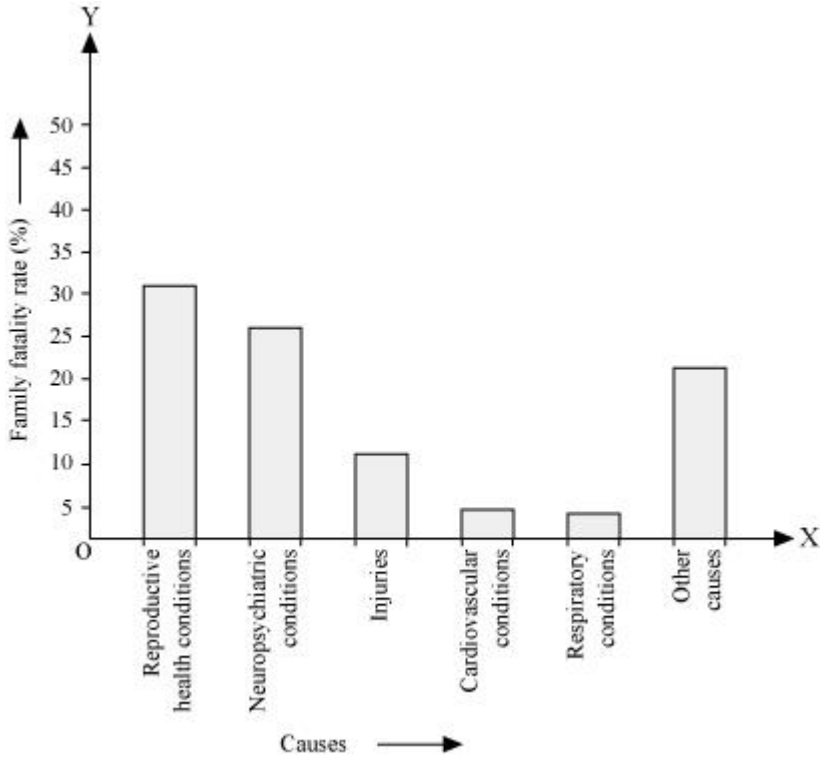
Volume of pit $= \frac{1}{3}\pi r^2 h$

$= \left[\frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12\right]$ cm^3

= 38.5 m^3

Thus, capacity of the pit = (38.5 × 1) kilolitres = 38.5 kilolitres

19. (i) By representing causes on x-axis and family fatality rate on y-axis and choosing an appropriate scale (1 unit = 5% for y axis), the graph of the information given above can be constructed as follows.



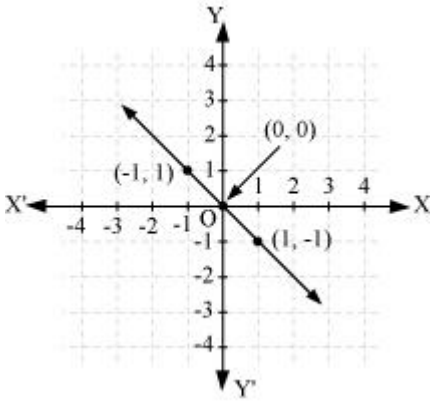
All the rectangle bars are of the same width and have equal spacing between them.

(ii) Reproductive health condition is the major cause of women's ill health and death worldwide as 31.8% of women are affected by it.

(iii) The factors are as follows.

1. Lack of medical facilities
2. Lack of correct knowledge of treatment

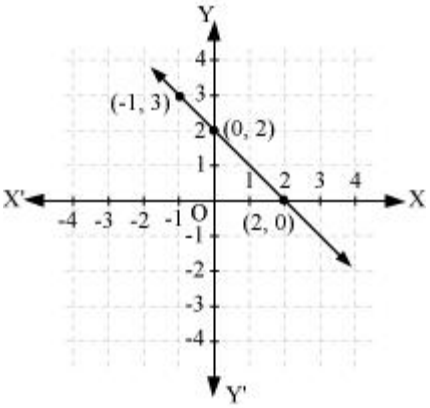
20.



Points on the given line are (−1, 1), (0, 0), and (1, −1).

It can be observed that the coordinates of the points of the graph satisfy the equation $x + y = 0$. Therefore, $x + y = 0$ is the equation corresponding to the graph as shown in the first figure.

Hence, (ii) is the correct answer.



Points on the given line are (−1, 3), (0, 2), and (2, 0). It can be observed that the coordinates of the points of the graph satisfy the equation $y = -x + 2$.

Therefore, $y = -x + 2$ is the equation corresponding to the graph shown in the second figure.

Hence, (iii) is the correct answer.

21. Radius (r) of heap $= \left(\frac{10.5}{2}\right)$ m $= 5.25$ m

Height (h) of heap = 3 m

Volume of heap $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3 \right) \text{ m}^3$$

$$= 86.625 \text{ m}^3$$

Therefore, the volume of the heap of wheat is 86.625 m^3.

Area of canvas required = CSA of cone

$$= \pi r l = \pi r \sqrt{r^2 + h^2}$$

$$= \left[\frac{22}{7} \times 5.25 \times \sqrt{(5.25)^2 + (3)^2} \right] \text{ m}^2$$

$$= \left(\frac{22}{7} \times 5.25 \times 6.05 \right) \text{ m}^2$$

$$= 99.825 \text{ m}^2$$

Therefore, 99.825 m^2 canvas will be required to protect the heap from rain.

22. (i) In ΔAPD and ΔCQB,
∠ADP = ∠CBQ (Alternate interior angles for BC || AD)
AD = CB (Opposite sides of parallelogram ABCD)
DP = BQ (Given)
∴ ΔAPD ≅ ΔCQB (Using SAS congruence rule)

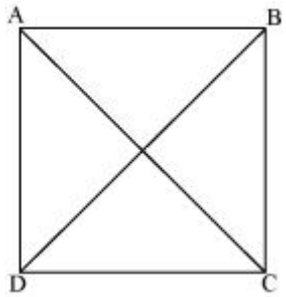
(ii) As we had observed that ΔAPD ≅ ΔCQB,
∴ AP = CQ (CPCT)

(iii) In ΔAQB and ΔCPD,
∠ABQ = ∠CDP (Alternate interior angles for AB || CD)
AB = CD (Opposite sides of parallelogram ABCD)
BQ = DP (Given)
∴ ΔAQB ≅ ΔCPD (Using SAS congruence rule)

(iv) As we had observed that ΔAQB ≅ ΔCPD,
∴ AQ = CP (CPCT)

(v) From the result obtained in (ii) and (iv),
AQ = CP and
AP = CQ
Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

23.



(i) It is given that ABCD is a rectangle.

$\therefore \angle A = \angle C$

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \angle DAC = \angle DCA$ $\quad (AC \text{ bisects } \angle A \text{ and } \angle C)$

CD = DA (Sides opposite to equal angles are also equal)

However, DA = BC and AB = CD (Opposite sides of a rectangle are equal)

∴ AB = BC = CD = DA

ABCD is a rectangle and all of its sides are equal.

Hence, ABCD is a square.

(ii) Let us join BD.

In ΔBCD,

BC = CD (Sides of a square are equal to each other)

∠CDB = ∠CBD (Angles opposite to equal sides are equal)

However, ∠CDB = ∠ABD (Alternate interior angles for AB || CD)

∴ ∠CBD = ∠ABD

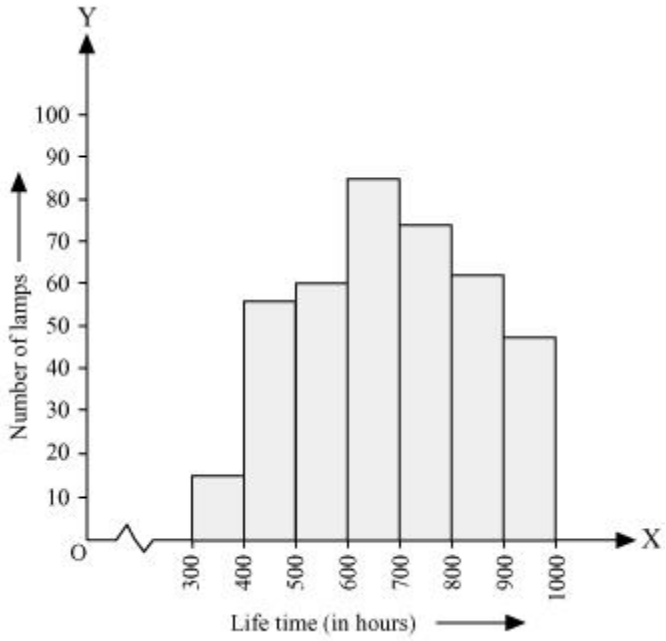
⇒ BD bisects ∠B.

Also, ∠CBD = ∠ADB (Alternate interior angles for BC || AD)

⇒ ∠CDB = ∠ABD

∴ BD bisects ∠D.

24. (i) By taking life time (in hours) of neon lamps on *x*-axis and the number of lamps on *y*-axis, the histogram of the given information can be drawn as follows.



Here, 1 unit on *y*-axis represents 10 lamps.

(ii) It can be concluded that the number of neon lamps having their lifetime more than 700 is the sum of the number of neon lamps having their lifetime as 700 − 800, 800 − 900, and 900 − 1000.

Therefore, the number of neon lamps having their lifetime more than 700 hours is 184. (74 + 62 + 48 = 184)

25. (i) In ΔADC, S and R are the mid-points of sides AD and CD respectively.

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

$\therefore$ SR || AC and SR = $\frac{1}{2}$ AC ... (1)

(ii) In ΔABC, P and Q are mid-points of sides AB and BC respectively. Therefore, by using mid-point theorem,

PQ || AC and PQ = $\frac{1}{2}$ AC ... (2)

Using equations (1) and (2), we obtain

PQ || SR and PQ = SR ... (3)

$\Rightarrow$ PQ = SR

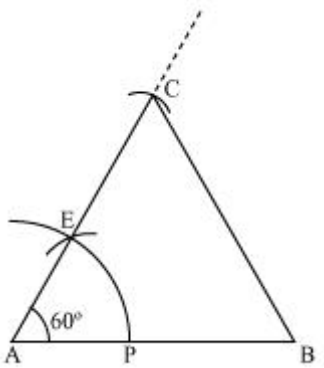
(iii) From equation (3), we obtained

PQ || SR and PQ = SR

Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

Hence, PQRS is a parallelogram.

26. Let us draw an equilateral triangle of side 5 cm. We know that all sides of an equilateral triangle are equal. Therefore, all sides of the equilateral triangle will be 5 cm. We also know that each angle of an equilateral triangle is 60º.
The below given steps will be followed to draw an equilateral triangle of 5 cm side.
Step I: Draw a line segment AB of 5 cm length. Draw an arc of some radius, while taking A as its centre. Let it intersect AB at P.
Step II: Taking P as centre, draw an arc to intersect the previous arc at E. Join AE.
Step III: Taking A as centre, draw an arc of 5 cm radius, which intersects extended line segment AE at C. Join AC and BC. ΔABC is the required equilateral triangle of side 5 cm.



Justification of Construction:
We can justify the construction by showing ABC as an equilateral triangle i.e., AB = BC = AC = 5 cm and ∠A = ∠B = ∠C = 60°.
In ΔABC, we have AC = AB = 5 cm and ∠A = 60°.
Since AC = AB,
∠B = ∠C (Angles opposite to equal sides of a triangle)
In ΔABC,
∠A + ∠B + ∠C = 180° (Angle sum property of a triangle)
⇒ 60° + ∠C + ∠C = 180°
⇒ 60° + 2 ∠C = 180°
⇒ 2 ∠C = 180° − 60° = 120°
⇒ ∠C = 60°
∴ ∠B = ∠C = 60°
We have, ∠A = ∠B = ∠C = 60° ... (1)
⇒ ∠A = ∠B and ∠A = ∠C
⇒ BC = AC and BC = AB (Sides opposite to equal angles of a triangle)

$\Rightarrow$ AB = BC = AC = 5 cm ... (2)

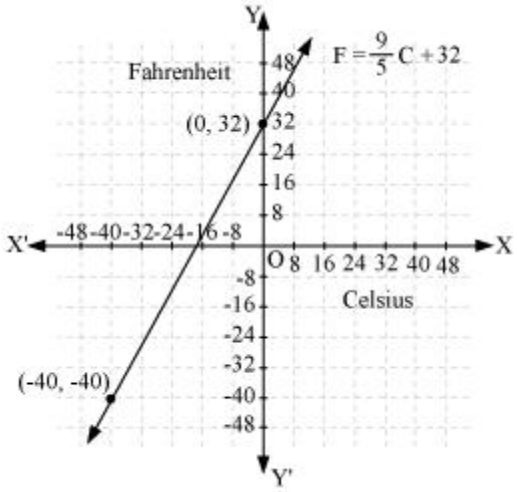
From equations (1) and (2), ΔABC is an equilateral triangle.

27. (i) $F = \left(\frac{9}{5}\right)C + 32$

It can be observed that points (0, 32) and (−40, −40) satisfy the given equation.

Therefore, these points are the solutions of this equation.

The graph of the above equation is constructed as follows.



(ii) Temperature = 30°C

$$F = \left(\frac{9}{5}\right)C + 32$$

$$F = \left(\frac{9}{5}\right)30 + 32 = 54 + 32 = 86$$

Therefore, the temperature in Fahrenheit is 86°F.

(iii) Temperature = 95°F

$$F = \left(\frac{9}{5}\right)C + 32$$

$$95 = \left(\frac{9}{5}\right)C + 32$$

$$63 = \left(\frac{9}{5}\right)C$$

$$C = 35$$

Therefore, the temperature in Celsius is 35°C.

(iv) $F = \left(\frac{9}{5}\right)C + 32$

If C = 0°C, then

$$F = \left(\frac{9}{5}\right)0 + 32 = 32$$

Therefore, if C = 0°C, then F = 32°F

If F = 0°F, then

$$0 = \left(\frac{9}{5}\right)C + 32$$

$$\left(\frac{9}{5}\right)C = -32$$

$$C = \frac{-160}{9} = -17.77$$

Therefore, if F = 0°F, then C = −17.8°C

(v) $$F = \left(\frac{9}{5}\right)C + 32$$

Here, F = C

$$F = \left(\frac{9}{5}\right)F + 32$$

$$\left(\frac{9}{5} - 1\right)F + 32 = 0$$

$$\left(\frac{4}{5}\right)F = -32$$

$$F = -40$$

Yes, there is a temperature, −40°, which is numerically the same in both Fahrenheit and Celsius.

28. Let the diameter of earth be *d*. Therefore, the radius of earth will be $\frac{d}{2}$.

Diameter of moon will be $\frac{d}{4}$ and the radius of moon will be $\frac{d}{8}$.

Volume of moon = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{8}\right)^3 = \frac{1}{512} \times \frac{4}{3}\pi d^3$

Volume of earth = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{1}{8} \times \frac{4}{3}\pi d^3$

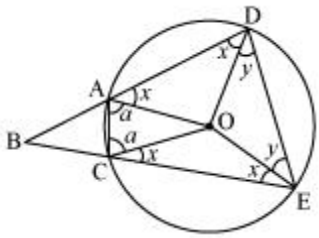
$$\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{1}{512}\times\frac{4}{3}\pi d^3}{\frac{1}{8}\times\frac{4}{3}\pi d^3}$$

$$= \frac{1}{64}$$

$$\Rightarrow \text{Volume of moon} = \frac{1}{64}\text{Volume of earth}$$

Therefore, the volume of moon is $\frac{1}{64}$ of the volume of earth.

29.



In ΔAOD and ΔCOE,
OA = OC (Radii of the same circle)
OD = OE (Radii of the same circle)
AD = CE (Given)
∴ ΔAOD ≅ ΔCOE (SSS congruence rule)
∠OAD = ∠OCE (By CPCT) ... (1)
∠ODA = ∠OEC (By CPCT) ... (2)
Also,
∠OAD = ∠ODA (As OA = OD) ... (3)
From equations (1), (2), and (3), we obtain
∠OAD = ∠OCE = ∠ODA = ∠OEC
Let ∠OAD = ∠OCE = ∠ODA = ∠OEC = *x*
In Δ OAC,
OA = OC
∴ ∠OCA = ∠OAC (Let *a*)
In Δ ODE,
OD = OE
∠OED = ∠ODE (Let *y*)
ADEC is a cyclic quadrilateral.
∴ ∠CAD + ∠DEC = 180° (Opposite angles are supplementary)

$x + a + x + y = 180°$

$2x + a + y = 180°$

$y = 180^o - 2x - a$ ... (4)

However, $\angle DOE = 180^o - 2y$

And, $\angle AOC = 180^o - 2a$

$\angle DOE - \angle AOC = 2a - 2y = 2a - 2\ (180^o - 2x - a)$

$= 4a + 4x - 360°$ ... (5)

$\angle BAC + \angle CAD = 180^o$ (Linear pair)

$\Rightarrow \angle BAC = 180^o - \angle CAD = 180^o - (a + x)$

Similarly, $\angle ACB = 180^o - (a + x)$

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180^o$ (Angle sum property of a triangle)

$\angle ABC = 180^o - \angle BAC - \angle ACB$

$= 180^o - (180^o - a - x) - (180^o - a - x)$

$= 2a + 2x - 180^o$

$= \frac{1}{2}[4a + 4x - 360°]$

$\angle ABC = \frac{1}{2}[\angle DOE - \angle AOC]$ [Using equation (5)]

30. ABCD is a parallelogram.

$\therefore AB \parallel CD$

And hence, AE || FC

Again, AB = CD (Opposite sides of parallelogram ABCD)

$\frac{1}{2}AB = \frac{1}{2}CD$

AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other. Therefore, AECF is a parallelogram.

$\Rightarrow$ AF || EC (Opposite sides of a parallelogram)

In ΔDQC, F is the mid-point of side DC and FP || CQ (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

$\Rightarrow$ DP = PQ ... (1)

Similarly, in ΔAPB, E is the mid-point of side AB and EQ || AP (as AF || EC).

Therefore, by using the converse of mid-point theorem, it can be said that

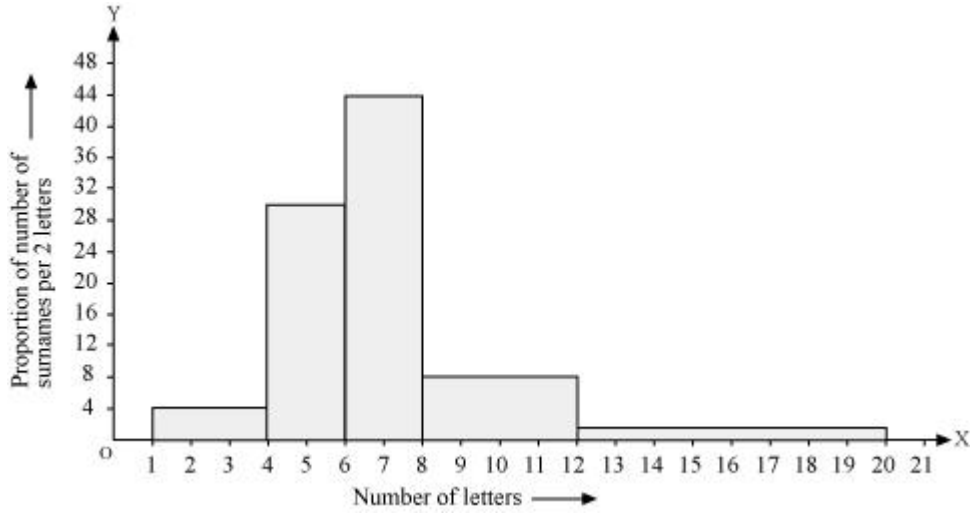
Q is the mid-point of PB.

⇒ PQ = QB ... (2)
From equations (1) and (2),
DP = PQ = BQ
Hence, the line segments AF and EC trisect the diagonal BD.

31. (i) Here, it can be observed that the data has class intervals of varying width. The proportion of the number of surnames per 2 letters interval can be calculated as follows.

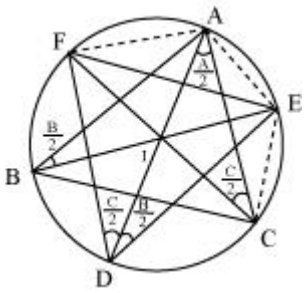
Number of letters	**Frequency (Number of surnames)**	**Width of class**	**Length of rectangle**
1 − 4	6	3	$\frac{6\times 2}{3}=4$
4 − 6	30	2	$\frac{30\times 2}{2}=30$
6 − 8	44	2	$\frac{44\times 2}{2}=44$
8 − 12	16	4	$\frac{16\times 2}{4}=8$
12 − 20	4	8	$\frac{4\times 2}{8}=1$

By taking the number of letters on *x*-axis and the proportion of the number of surnames per 2 letters interval on *y*-axis and choosing an appropriate scale (1 unit = 4 students for *y* axis), the histogram can be constructed as follows.



(ii) The class interval in which the maximum number of surnames lies is 6 − 8 as it has 44 surnames in it i.e., the maximum for this data.

32.



It is given that BE is the bisector of ∠B.

$\therefore \angle ABE = \frac{\angle B}{2}$

However, ∠ADE = ∠ABE (Angles in the same segment for chord AE)

$\Rightarrow \angle ADE = \frac{\angle B}{2}$

Similarly, $\angle ACF = \angle ADF = \frac{\angle C}{2}$ (Angle in the same segment for chord AF)

∠D = ∠ADE + ∠ADF

$$= \frac{\angle B}{2} + \frac{\angle C}{2}$$

$$= \frac{1}{2}(\angle B + \angle C)$$

$$= \frac{1}{2}(180° - \angle A)$$

$$= 90° - \frac{1}{2}\angle A$$

Similarly, it can be proved that

$$\angle E = 90° - \frac{1}{2}\angle B$$

$$\angle F = 90° - \frac{1}{2}\angle C$$

33. (i) Cost of white-washing the dome from inside = Rs 498.96

Cost of white-washing 1 m^2 area = Rs 2

Therefore, CSA of the inner side of dome = $\left(\frac{498.96}{2}\right)$ m^2

= 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be r.

CSA of inner side of dome = 249.48 m^2

$2\pi r^2 = 249.48$ m^2

$\Rightarrow 2\times\frac{22}{7}\times r^2 = 249.48 \text{ m}^2$

$\Rightarrow r^2 = \left(\frac{249.48\times 7}{2\times 22}\right) \text{m}^2 = 39.69 \text{ m}^2$

$\Rightarrow r = 6.3$ m

Volume of air inside the dome = Volume of hemispherical dome

$= \frac{2}{3}\pi r^3$

$= \left[\frac{2}{3}\times\frac{22}{7}\times(6.3)^3\right] \text{m}^3$

= 523.908 m^3

= 523.9 m^3 (approximately)

Therefore, the volume of air inside the dome is 523.9 m^3.

34. Arranging the data in an ascending order,

14, 14, 14, 14, 17, 18, 18, 18, 22, 23, 25, 28

It can be observed that 14 has the highest frequency, i.e. 4, in the given data. Therefore, mode of the given data is 14.