Chapter – 10

Oscillations

Multiple Choice Questions

Question 1.
In a simple harmonic oscillation, the acceleration against displacement for one complete oscillation will be [model NSEP 2000-01]
(a) an ellipse
(b) a circle
(c) a parabola
(d) a straight line

Answer:
(d) a straight line

Question 2.
A particle executing SHM crosses points A and B with the same velocity. Having taken 3 s in passing from A to B, it returns to B after another 3s. The time period is
(a) 15s
(b) 6s
(c) 12s
(d) 9s

Answer:
(c) 12s
Hint:
Time period of Oscillation = 2 × (time taken to go from A to B + the next time taken to return at B)
$= 2 \times (3 + 3)$
$= 2 \times 6$
Time period = 12s

Question 3.
The length of a second's pendulum on the surface of the Earth is 0.9 m. The

length of the same pendulum on surface of planet X such that the acceleration of the planet X is n times greater than the Earth

(a) 0.9 n

(b) $\frac{0.9}{n}$ m

(c) 0.9 n^2m

(d) $\frac{0.9}{n^2}$

Answer:
(a) 0.9 n
Hint:
Second's pendulum on the surface of planet,
Time period, T = 2 sec

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$2 = 2\pi\sqrt{\frac{l}{ng}} \; ; \; l^2 = \pi^2\left(\frac{l}{ng}\right) \; ; \; l = \frac{9.8n}{(3.14)^2}$$

$$l = \frac{9.8n}{9.8596} \; ; \; l = 0.9n$$

Question 4.
A simple pendulum is suspended from the roof of a school bus which moves in a horizontal direction with an acceleration a, then the time period is .....

(a) $T \propto \frac{1}{g^2 + a^2}$

(b) $T \propto \frac{1}{\sqrt{g^2 + a^2}}$

(c) $T \propto \sqrt{g^2 + a^2}$

(d) $T \propto (g^2 + a^2)$

Answer:
$T \propto \frac{1}{g^2+a^2}$

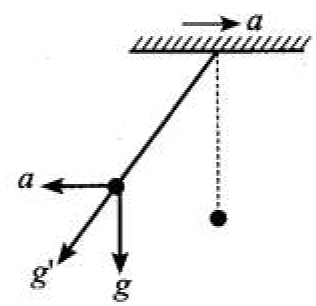
Hint: According to law of acceleration, the time period of simple pendulum

$$T \propto \frac{1}{\sqrt{g'}}$$

From pythagoras theorem, $g' = g^2 + a^2$

$$g'^2 = \sqrt{g^2 + a^2}$$

$$T \propto \frac{1}{\sqrt{\left(\sqrt{g^2 + a^2}\right)}} ; T \propto \frac{1}{g^2 + a^2}$$



Question 5.

Two bodies A and B whose masses are in the ratio 1:2 are suspended from two separate massless springs of force constants k_A and k_B respectively. If the two bodies oscillate vertically such that their maximum velocities are in the ratio 1 : 2, the ratio of the amplitude A to that of B is

(a) $\sqrt{\frac{k_B}{2k_A}}$ (b) $\sqrt{\frac{k_B}{8k_A}}$ (c) $\sqrt{\frac{2k_B}{k_A}}$ (d) $\sqrt{\frac{8k_B}{k_A}}$

Answer:

(b) $\sqrt{\frac{k_B}{8k_A}}$

Hint:

The maximum velocity in the Oscillation is given as $v_{max} = A\omega$

The ratio of maximum velocity, $v_A : v_B = 1 : 2$

$$2v_A = v_B$$

$$2A_A\omega_A = A_B\omega_B$$

$$\frac{A_A}{A_B} = \frac{\omega_B}{2\omega_B} \qquad \text{where, } \omega = \sqrt{\frac{k}{m}}$$

$$\frac{A_A}{A_B} = \frac{1}{2}\sqrt{\frac{k_B}{2m_A} \times \frac{m_B}{k_A}} = \sqrt{\frac{1}{4} \times \frac{k_B}{2k_A}} \qquad \frac{m_A}{m_B} = \frac{1}{2} ; 2m_A = m_B$$

$$\frac{A_A}{A_B} = \sqrt{\frac{K_B}{8K_A}}$$

Question 6.
A spring is connected to a mass m suspended from it and its time period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is

(a) $T' = \sqrt{2}T$ (b) $T' = \frac{T}{\sqrt{2}}$ (c) $T' = \sqrt{2T}$ (d) $T' = \sqrt{\frac{T}{2}}$

Answer:

(b) $T' = \frac{T}{\sqrt{2}}$

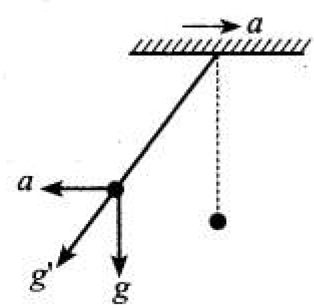
Hint:
Spring constant of spring depends upon number of coil in the spring! When you cut the spring then the number of coil remain half of the original, so
k' = 2k
Hint:
According to law of accelaration, the time period of simple pendulum

$$T \propto \frac{1}{\sqrt{g'}}$$

From pythagoras theorem, $g' = g^2 + a^2$

$$g'^2 = \sqrt{g^2 + a^2}$$

$$T \propto \frac{1}{\sqrt{\left(\sqrt{g^2 + a^2}\right)}} ; T \propto \frac{1}{g^2 + a^2}$$



Question 5.
Two bodies A and B whose masses are in the ratio 1 : 2 are suspended from two separate massless springs of force constants k_A and k_B respectively. If the two bodies oscillate vertically such that their maximum velocities are in the ratio 1 : 2, the ratio of the amplitude A to that of B is

(a) $\sqrt{\frac{k_B}{2k_A}}$ (b) $\sqrt{\frac{k_B}{8k_A}}$ (c) $\sqrt{\frac{2k_B}{k_A}}$ (d) $\sqrt{\frac{8k_B}{k_A}}$

Answer:

(b) $\sqrt{\frac{k_B}{8k_A}}$

Hint:
The maximum velocity in the Oscillation is given as $v_{max} = A\omega$
The ratio of maximum velocity, $v_A : v_B = 1 : 2$

Hint: The maximum velocity in the Oscillation is given as $v_{max} = A\omega$
The ratio of maximum velocity, $v_A : v_B = 1 : 2$

$$2v_A = v_B$$

$$2A_A\omega_A = A_B\omega_B$$

$$\frac{A_A}{A_B} = \frac{\omega_B}{2\omega_B} \qquad \text{where, } \omega = \sqrt{\frac{k}{m}}$$

$$\frac{A_A}{A_B} = \frac{1}{2}\sqrt{\frac{k_B}{2m_A} \times \frac{m_B}{k_A}} = \sqrt{\frac{1}{4} \times \frac{k_B}{2k_A}} \qquad \frac{m_A}{m_B} = \frac{1}{2};\ 2m_A = m_B$$

$$\frac{A_A}{A_B} = \sqrt{\frac{K_B}{8K_A}}$$

Question 6.
A spring is connected to a mass m suspended from it and its time period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is

(a) $T' = \sqrt{2}T$ (b) $T' = \frac{T}{\sqrt{2}}$ (c) $T' = \sqrt{2T}$ (d) $T' = \sqrt{\frac{T}{2}}$

Answer:

(b) $T' = \frac{T}{\sqrt{2}}$

Hint:
Spring constant of spring depends upon number of coil in the spring! When you cut the spring then the number of coil remain half of the original, so

k' = 2k

Time period of the spring, $T = 2\pi\sqrt{\frac{m}{k}}$

If the spring is halves, So the time period

$$T' = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

$$T' = \frac{1}{\sqrt{2}} \times 2\pi\sqrt{\frac{m}{k}} \Rightarrow T' = \frac{T}{\sqrt{2}}$$

Question 7.

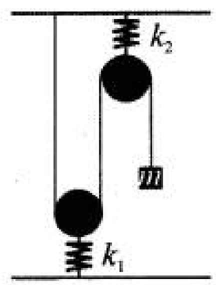
The time period for small vertical oscillations of block of mass m when the masses of the pulleys are negligible and spring constant k_1 and k_2 is ....

(a) $T = 4\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$

(b) $T = 2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$

(c) $T = 4\pi\sqrt{m(k_1 + k_2)}$

(d) $T = 2\pi\sqrt{m(k_1 + k_2)}$



Answer:

(a) $T = 4\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$

Question 8.

A simple pendulum has a time period T_1. When its point of suspension is moved vertically upwards according as $y = kt^2$, where y is vertical distance covered and $k = 1\ ms^{-2}$, its time period becomes T_2.

Then, $\frac{T_1^2}{T_2^2}$ is ($g = 10\ ms^{-2}$)

Then [IIT 2005]

(a) $\frac{5}{6}$ (b) $\frac{11}{10}$ (c) $\frac{6}{5}$ (d) $\frac{4}{5}$

Answer:

(c) 6/5

Hint:

$$y = kt^2$$

$$\frac{dy}{dt} = 2kt, \frac{d^2y}{dt^2} = 2k$$

Here $k = 1\ ms^{-2}$

$$\frac{d^2y}{dt^2} = 2 \times 1 = 2\ ms^{-2}\ ;\ g_2 = g_1 + 2 = 10 + 2 = 12\ ms^{-2}$$

For a pendulum, Time period $T = 2\pi\sqrt{\frac{l}{g}}$

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{g_2}{g_1} = \frac{12}{10} = \frac{6}{5}$$

Question 9.
An ideal spring of spring constant k, is suspended from the ceiling of a room and a block of mass M is fastened to its lower end. If the block is released when the spring is un-stretched, then the maximum extension in the spring is : [IIT 2002]

(a) $4\frac{Mg}{k}$ (b) $\frac{Mg}{k}$ (c) $2\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

Answer:

(c) $2\frac{Mg}{k}$

Hint:
Work by gravity + Work by spring = Change in kinetic energy

$$Mgx + \left(-\frac{1}{2}kx^2\right) = 0$$

$$Mgx = \frac{1}{2}kx^2$$

$$Mg = \frac{kx}{2} \Rightarrow \therefore\ x = \frac{2Mg}{k}$$

Question 10.
A pendulum is hung in a very high building oscillates to and fro motion freely like a simple harmonic oscillator. If the acceleration of the bob is 16 ms-2 at a

distance of 4 m from the mean position, then the time period is [NEET 2018 model]
(a) 2s
(b) 1s
(c) 2πs
(d) πs

Answer:
(d) πs
Hint:
Acceleration, a = 16 m/s^2
displacement, y = 4m
According to SHM,

$$a = \omega^2 y$$

$$\omega^2 = \frac{16}{4} = 4$$

$$\therefore \omega = 2 \text{ rad/s}$$

Time period, $$T = \frac{2\pi}{\omega} = \frac{2\pi}{2}$$

$$T = \pi \text{ sec}$$

Question 11.
A hollow sphere is filled with water. It is hung by a long thread. As the water flows out of a hole at the bottom, the period of oscillation will
(a) first increase and then decrease
(b) first decrease and then increase
(c) increase continuously
(d) decrease continuously

Answer:
(a) first increase and then decrease

Question 12.
The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are ....
(a) kg ms^{-1} (b) kg ms^{-2} (c) kg s^{-1} (d) kg s

Answer:
(c) kgs^{-1}

Hint: Damping force $\propto$ Velocity

$$F \propto v \; ; \; F = kv$$

$$k = \frac{f}{v} \quad k \text{ is constant of proportionality}$$

$$= \frac{kg\,ms^{-2}}{ms^{-1}}$$

$$k = kg\,s^{-1}$$

Question 13.
When a damped harmonic oscillator completes 100 oscillations, its amplitude is reduced to 1/3 of its initial value. What will be its amplitude when it completes 200 oscillations?

(a) $\frac{1}{5}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

Answer:
(d) 1/9

Hint:
In damped vibration, amplitude at any instant t is

$$a = a_0 e^{-bt}$$

If, $t = 100\,T$ and $a = \frac{a_0}{3}$

$$\therefore \frac{a_0}{3} = a_0 e^{-100bT}$$

If, $t = 200\,T$

$$a = a_0 e^{-bt} = a_0 e^{-b(200\,T)}$$

$$= a_0 (e^{-100\,6T})^2$$

$$a = a_0 \left(\frac{1}{3}\right)^2$$

$$a = \frac{a_0}{9}$$

Question 14.
Which of the following differential equations represents a damped harmonic oscillator?

(a) $\frac{d^2y}{dt^2} + y = 0$

(b) $\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + y = 0$

(c) $\frac{d^2y}{dt^2} + k^2y = 0$

(d) $\frac{dy}{dt} + y = 0$

Answer:
(b) $\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + y = 0$

Question 15.
If the inertial mass and gravitational mass of the simple pendulum of length / are not equal, then the time period of the simple pendulum is .....

(a) $T = 2\pi\sqrt{\frac{m_i l}{m_g g}}$ (b) $T = 2\pi\sqrt{\frac{m_g l}{m_i g}}$ (c) $T = 2\pi\frac{m_g}{m_i}\sqrt{\frac{l}{g}}$ (d) $T = 2\pi\frac{m_i}{m_g}\sqrt{\frac{l}{g}}$

Answer:
(a) $T = 2\pi\sqrt{\frac{m_i l}{m_g g}}$

Short Answer Questions

Question 1.
What is meant by periodic and non-periodic motion? Give any two examples, for each motion.

Answer:
Periodic motion: Any motion which repeats itself in a fixed time interval is known as periodic motion. Examples: Hands in pendulum clock, swing of a cradle.

Non-Periodic motion: Any motion which does not repeat itself after a regular interval of time is known as non-periodic motion. Example: Occurrence of Earth quake, eruption of volcano.

Question 2.
What is meant by force constant of a spring?

Answer:
The force constant (or) spring factor is defined as the restoring force produced per unit displacement.

Question 3.
Define time period of simple harmonic motion.

Answer:
Time period: The time period is defined as the time taken by a particle to complete one oscillation. It is usually denoted by T. For one complete revolution, the time taken is t = T, therefore,

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$$

Question 4.
Define frequency of simple harmonic motion.

Answer:
The number of oscillations produced by the particle per second is called frequency. It is denoted by f. SI unit for frequency is s^{-1} or hertz (Hz). Mathematically, frequency is related to time period by f = 1/T

Question 5.
What is an epoch?

Answer:
The phase of a vibrating particle corresponding to time t = 0 is called initial phase or epoch. At, $t = \varphi$, $\varphi = \varphi_0$.

The constant φ_0 is called initial phase or epoch. It tells about the initial state of motion of the vibrating particle.

Question 6.
Write short notes on two springs connected in series.

Answer:
Phase: The phase of a vibrating particle at any instant completely specifies the

state of the particle. It expresses the position and direction of motion of the particle at that instant with respect to its mean position the phase of the vibrating particle. At time t = 0 s (initial time), the phase $\varphi = \varphi_0$ is called epoch (initial phase) where φ_0 is called the angle of epoch.

Question 7.
Write short notes on two springs connected in parallel.

Answer:
When two or more springs are connected in parallel, we can replace (by removing) all these springs with an equivalent spring (effective spring) whose net effect is same as if all the springs are in parallel connection.

Question 8.
Write down the time period of simple pendulum.

Answer:
The angular frequency of this oscillator (natural frequency of this system) is

$$\omega^2 = \frac{g}{l}$$

$$\therefore \quad \omega = \sqrt{\frac{g}{l}} \text{ in rad s}^{-1}$$

The frequency of oscillations is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \text{ in Hz}$$

and time period of oscillations is

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ in second}$$

Question 9.
State the laws of simple pendulum?

Answer:
Law of length: For a given value of acceleration due to gravity, the time period of a simple pendulum is directly proportional to the square root of length of the pendulum.

$T \propto \sqrt{l}$

Law of acceleration: For a fixed length, the time period of a simple pendulum is inversely proportional to square root of acceleration due to gravity.

$T \propto \frac{1}{\sqrt{g}}$

Question 10.
Write down the equation of time period for linear harmonic oscillator.

Answer:
From Newton's second law, we can write the equation for the particle executing simple harmonic motion

$$m\frac{d^2x}{dt^2} = -kx \quad ...(1)$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \quad ...(2)$$

Comparing the equation with simple harmonic motion equation, we get

$$\omega^2 = \frac{k}{m} \quad ...(3)$$

which means the angular frequency or natural frequency of the oscillator is

$$\omega = \sqrt{\frac{k}{m}} \text{ rad s}^{-1} \quad ...(4)$$

The frequency of the oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \text{ Hertz} \quad ...(5)$$

and the time period of the oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \text{ seconds} \quad ...(6)$$

Question 11.
What is meant by free oscillation?

Answer:
When the oscillator is allowed to oscillate by displacing its position from

equilibrium position, it oscillates with a frequency which is equal to the natural frequency of the oscillator. Such an oscillation or vibration is known as free oscillation or free vibration.

Question 12.
Explain damped oscillation. Give an example.

Answer:
The oscillations in which the amplitude decreases gradually with the passage of time are called damped Oscillations.

Example:

1. The oscillations of a pendulum or pendulum oscillating inside an oil filled container.
2. Electromagnetic oscillations in a tank circuit.
3. Oscillations in a dead beat and ballistic galvanometers.

Question 13.
Define forced oscillation. Give an example.

Answer:
The body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations.

Example: Sound boards of stringed instruments.

Question 14.
What is meant by maintained oscillation? Give an example.

Answer:
While playing in swing, the oscillations will stop after a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations, are known as maintained vibrations.

Example: The vibration of a tuning fork getting energy from a battery or from external power supply.

Question 15.
Explain resonance. Give an example.

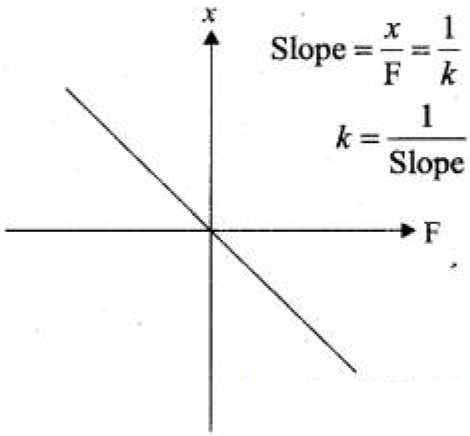
Answer:
The frequency of external periodic force (or driving force) matches with the natural frequency of the vibrating body (driven). As a result the oscillating body begins to vibrate such that its amplitude increases at each step and ultimately it has a large amplitude. Such a phenomenon is known as resonance and the corresponding vibrations are known as resonance vibrations.
Example: The breaking of glass due to sound

Long Answer Questions

Question 1.
What is meant by simple harmonic oscillation? Give examples and explain why every simple harmonic motion is a periodic motion whereas the converse need not be true.

Answer:
Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point. In one dimensional case, let x be the .displacement of the particle and a_x be the acceleration of the particle, then

$$a_x \propto x \qquad \text{...(1)}$$
$$a_x = -b\,x \qquad \text{...(2)}$$



where b is a constant which measures acceleration per unit displacement and dimensionally it is equal to T^{-2}.

By multiplying by mass of the particle on both sides of equation (1) and from Newton's second law, the force is
$F_x = -kx$ ...(3)

where k is a force constant which is defined as force per unit length. The negative sign indicates that displacement and force (or acceleration) are in opposite directions. This means that when the displacement of the particle is taken towards right of equilibrium position (x takes positive value), the force (or acceleration) will point towards equilibrium (towards left) and similarly, when the displacement of the particle is taken towards left of equilibrium position (x takes negative value), the force (or acceleration) will point towards equilibrium (towards right).

This type of force is known as restoring force because it always directs the particle executing simple harmonic motion to restore to its original (equilibrium or mean) position. This force (restoring force) is central and attractive whose center of attraction is the equilibrium position.

In order to represent in two or three dimensions, we can write using vector notation
$\vec{F} = -k\vec{r}$ ...(4)
where $\vec{r}$ is the displacement of the particle from the chosen origin. Note that the force and displacement have a linear relationship. This means that the exponent of force $\vec{F}$ and the exponent of displacement $\vec{r}$ are unity. The

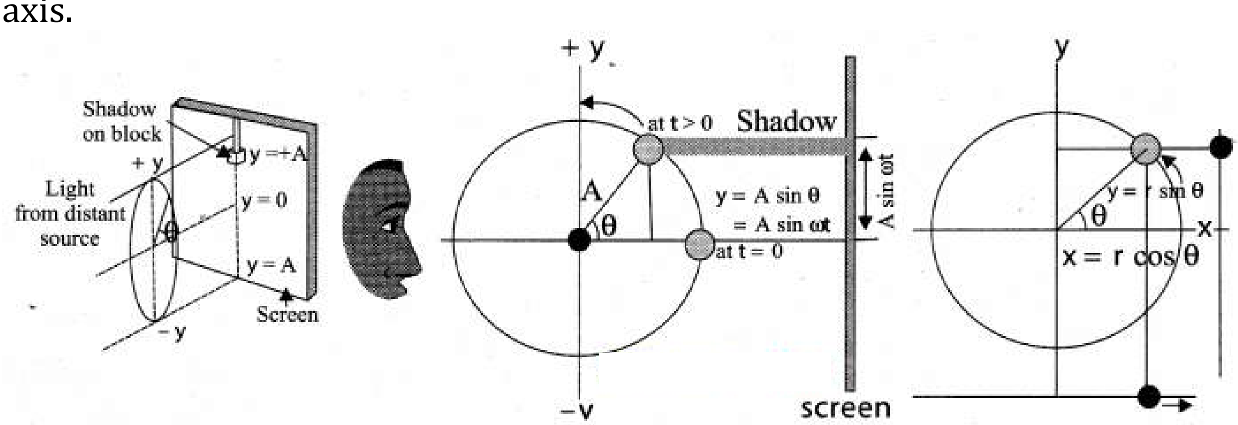
sketch between cause (magnitude of force $|\vec{\mathbf{F}}|$) and effect (magnitude of displacement $|\vec{r}|$) is a straight line passing through second and fourth quadrant.

By measuring slope 1/k, one can find the numerical value of force constant k.

Question 2.
Describe Simple Harmonic Motion as a projection of uniform circular motion.

Answer:
The projection of uniform circular motion on a diameter of SHM
Consider a particle of mass m moving with uniform speed v along the circumference of a circle whose radius is r in anti-clockwise direction (as shown in figure). Let us assume that the origin of the coordinate system coincides with the center O of the circle. If ω is the angular velocity of the particle and θ the angular displacement of the particle at any instant of time t, then $\theta = \omega t$. By projecting the uniform circular motion on its diameter gives a simple harmonic motion. This means that we can associate a map (or a relationship) between uniform circular (or revolution) motion to vibratory motion. Conversely, any vibratory motion or revolution can be mapped to uniform circular motion. In other words, these two motions are similar in nature.

Let us first project the position of a particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter as shown in figure. Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.



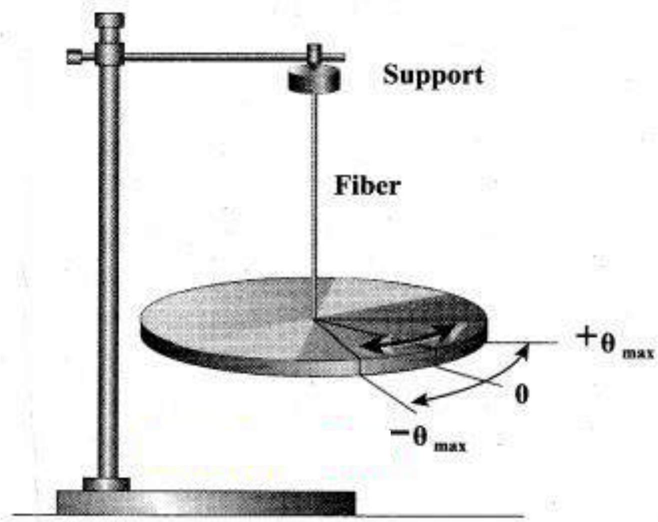
Projection of moving particle on a circle on a diameter

As a specific example, consider a spring mass system (or oscillation of pendulum). When the spring moves up and down (or pendulum moves to and fro), the motion of the mass or bob is mapped to points on the circular motion. Thus, if a particle undergoes uniform circular motion then the projection of the particle on the diameter of the circle (or on a line parallel to the diameter) traces straight line motion which is simple harmonic in nature. The circle is known as reference circle of the simple harmonic motion. The simple harmonic motion can also be defined as the motion of the projection of a particle on any diameter of a circle of reference.

Question 3.
What is meant by angular harmonic oscillation? Compute the time period of angular harmonic oscillation.

Answer:

Time period and frequency of angular SHM:
When a body is allowed to rotate freely about a given axis then the oscillation is known as the angular oscillation. The point at which the resultant torque acting on the body is taken to be zero is called mean position. If the body is displaced from the mean position, then the resultant torque acts such that it is proportional to the angular displacement and this torque has a tendency to bring the body towards the mean position.



A body (disc) allowed to rotate freely about an axis

Let $\vec{\theta}$ be the angular displacement of the body and the resultant torque $\vec{\tau}$ acting on the body is

$$\vec{\tau} \propto \vec{\theta} \quad ...(1)$$

$$\vec{\tau} = -\kappa\vec{\theta} \quad ...(2)$$

κ is the restoring torsion constant, which is torque per unit angular displacement. If I is the moment of inertia of the body and a is the angular acceleration then

$$\vec{\tau} = I\vec{\alpha} = -\kappa\vec{\theta} \quad ...(3)$$

But $\vec{\alpha} = \frac{d^2\vec{\theta}}{dt^2}$ and therefore,

$$\frac{d^2\vec{\theta}}{dt^2} = -\frac{\kappa}{I}\vec{\theta} \quad ...(4)$$

This differential equation resembles simple harmonic differential equation. So, comparing equation (4) with simple harmonic motion given in equation $a = \frac{d^2y}{dt^2} = -\omega^2 y$, we have

$$\omega = \sqrt{\frac{\kappa}{I}} \text{ rad s}^{-1} \quad ...(5)$$

The frequency of the angular harmonic motion is

$$f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}} \text{ Hz} \quad ...(6)$$

The time period is $T = 2\pi\sqrt{\frac{\kappa}{I}}$ seconds ...(7)

Question 4.

Write down the difference between simple harmonic motion and angular simple harmonic motion.

Answer:

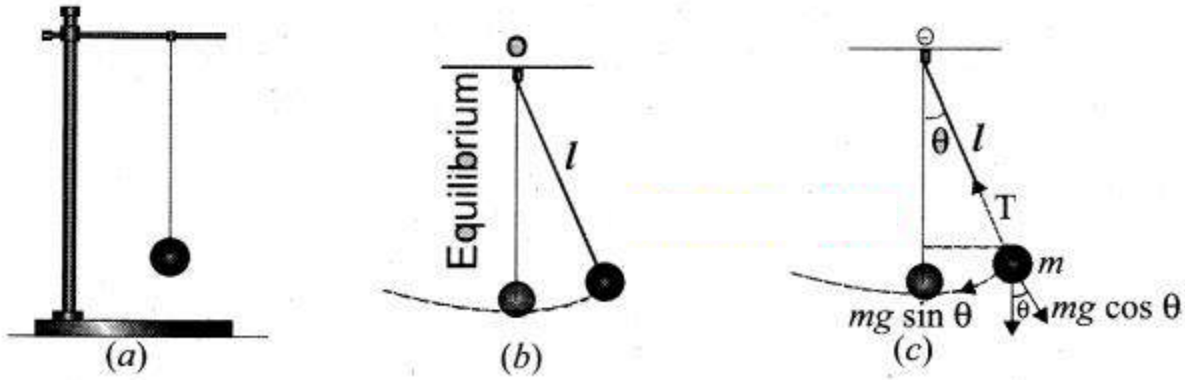
Comparision of simple harmonic motion and angular harmonic motion

S.No.	Simple Harmonic Motion	Angular Harmonic Motion
1.	The displacement of the particle is measured in terms of linear displacement $\vec{r}$.	The displacement of the particle is measured in terms of angular displacement $\vec{\theta}$ (also known as angle of twist).
2.	Acceleration of the particle is $\vec{a} = -\omega^2\vec{r}$	Angular acceleration of the particle is $\vec{\alpha} = -\omega^2\vec{\theta}$
3.	Force, $\vec{F} = m\vec{a}$, where m is called mass of the particle.	Torque, $\vec{\tau} = I\vec{\alpha}$, where I is called moment of inertia of a body.
4.	The restoring force $\vec{F} = -k\vec{r}$, where k is restoring force constant.	The restoring torque $\vec{\tau} = -\kappa\vec{\theta}$, where the symbol κ (Greek alphabet is pronounced as 'kappa') is called restoring torsion constant. It depends on the property of a particular torsion fiber.
5.	Angular frequency, $\omega = \sqrt{\frac{k}{m}}$ rad s^{-1}	Angular frequency, $\omega = \sqrt{\frac{\kappa}{I}}$ rad s^{-1}

Question 5.
Discuss the simple pendulum in detail.

Answer:
Simple pendulum: A pendulum is a mechanical system which exhibits periodic motion. It has a bob with mass m suspended by a long string (assumed to be massless and inextensible string) and the other end is fixed on a stand. At equilibrium, the pendulum does not oscillate and hangs vertically downward. Such a position is known as mean position or equilibrium position.

When a pendulum is displaced through a small displacement from its equilibrium position and released, the bob of the pendulum executes to and fro motion. Let l be the length of the pendulum which is taken as the distance between the point of suspension and the centre of gravity of the bob. Two forces act on the bob of the pendulum at any displaced position.



(i) The gravitational force acting on the body ($\vec{F} = m\vec{g}$) which acts vertically downwards.

(ii) The tension in the string $\vec{T}$ which acts along the string to the point of suspension. Resolving the gravitational force into its components:

(a) Normal component: The component along the string but in opposition to the direction of tension, F_{as} = mg cos θ

(b) Tangential component: The component perpendicular to the string i.e., along tangential direction of arc of swing, F_{ps} = mg sin θ

Therefore, The normal component of the force is, along the string,

$$T - W_{as} = m\frac{v^2}{l}$$

Here v is speed of bob

$$T - mg\cos\theta = m\frac{v^2}{l}$$

From the figure, we can observe that the tangential component W_{ps} of the gravitational force always points towards the equilibrium position i.e., the direction in which it always points opposite to the direction of displacement of the bob from the mean position. Hence, in this case, the tangential force is nothing but the restoring force. Applying Newton's second law along tangential direction, we have

$$m\frac{d^2s}{dt^2} + F_{ps} = 0 \Rightarrow m\frac{d^2s}{dt^2} = -F_{ps}$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta \qquad \text{...(1)}$$

where, s is the position of bob which is measured along the arc. Expressing arc length in terms of angular displacement i.e.,

$$s = l\theta \qquad ...(2)$$

then its acceleration, $\frac{d^2s}{dt^2} = l\frac{d^2\theta}{dt^2}$ ...(3)

Substituting equation (3) in equation (1), we get

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \qquad ...(4)$$

Because of the presence of sin 0 in the above differential equation, it is a non-linear differential equation (Here, homogeneous second order). Assume "the small oscillation approximation", sin θ ≈ θ, the above differential equation becomes linear differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \qquad ...(5)$$

This is the well-known oscillatory differential equation. Therefore, the angular frequency of this oscillator (natural frequency of this system) is

$$\omega^2 = \frac{g}{l} \qquad ...(6)$$

$$\therefore \quad \omega = \sqrt{\frac{g}{l}} \text{ in rad s}^{-1} \qquad ...(7)$$

The frequency of oscillations is $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ in Hz ...(8)

and time period of oscillations is $T = 2\pi\sqrt{\frac{l}{g}}$ in second ...(9)

Question 6.
Explain the horizontal oscillations of a spring.

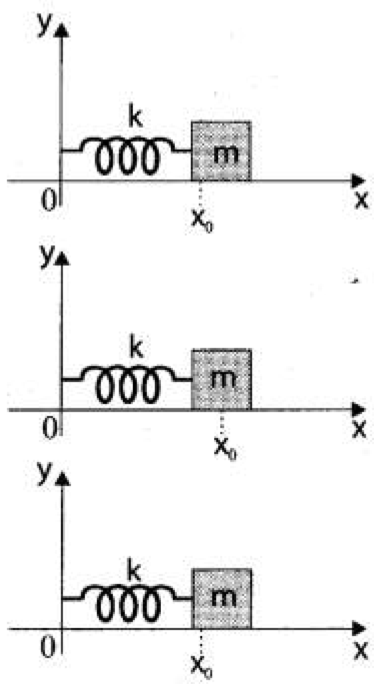
Answer:
Horizontal oscillations of a spring-mass system: Consider a system containing a block of mass m attached to a massless spring with stiffness constant or force constant or spring constant k placed on a smooth horizontal surface (frictionless surface) as shown in figure. Let x_0 be the equilibrium position or mean position of mass m when it is left undisturbed. Suppose the mass is

displaced through a small displacement x towards right from its equilibrium position and then released, it will oscillate back and forth about its mean position Let F be the restoring force (due to stretching of the spring) which is proportional to the amount of displacement of block. For one dimensional motion, mathematically, we have

$$F \propto x$$

$$F = -kx \qquad ...(1)$$

where negative sign implies that the restoring force will always act opposite to the direction of the displacement. This equation is called Hooke's law. Notice that, the restoring force is linear with the displacement (i.e., the exponent of force and displacement are unity). This is not always true; in case if we apply a very large stretching force, then the amplitude of oscillations becomes very large (which means, force is proportional to displacement containing higher powers of x) and therefore, the oscillation of the system ìs not linear and hence, it is called non-linear oscillation. We restrict ourselves only to linear oscillations throughout our discussions, which means Hooke's law is valid (force and displacement have a linear relationship).



From Newton's second law, we can write the equation for the particle executing simple harmonic motion

$$m\frac{d^2x}{dt^2} = -kx \qquad ...(1)$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad ...(2)$$

Comparing the equation with simple harmonic motion equation, we get

$$\omega^2 = \frac{k}{m} \qquad ...(3)$$

The frequency of the oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \text{ Hertz} \qquad ...(5)$$

>

and the time period of the oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \text{ seconds} \qquad ...(6)$$

Question 7.
Describe the vertical oscillations of a spring.

Answer:
Vertical oscillations of a spring: Let us consider a massless spring with stiff ness constant or force constant k attached to a ceiling as shown in figure. Let the length of the spring before loading mass m be L. If the block of mass m is attached to the other end of spring, then the spring elongates by a length l. Let F_1 be the restoring force due to stretching of spring. Due to mass m, the gravitational force acts vertically downward. We can draw free-body diagram for this system as shown in figure. When the system is under equilibrium,
$F_1 + mg = 0$ ...(1)

But the spring elongates by small displacement /, therefore,

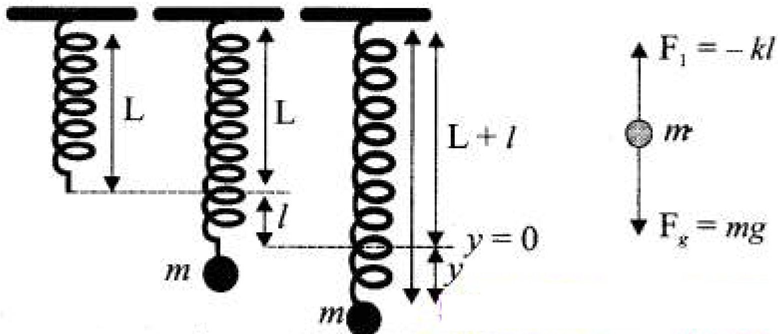
$$F_1 \propto l \Rightarrow F_1 = -kl \qquad ...(2)$$

Substituting equation (2) in equation (1), we get
$-kl + mg = 0$

$$-kl + mg = 0$$

$$mg = kl \quad \text{(or)} \quad \frac{m}{k} = \frac{l}{g} \qquad ...(3)$$

Suppose we apply a very small external force on the mass such that the mass further displaces downward by a displacement y, then it will oscillate up and down. Now, the restoring force due to this stretching of spring (total extension of spring is y +1) is



A massless spring with stiffness constant *k*

$$F_2 \propto (y + l)$$
$$F_2 = -k(y + l) = -ky - kl \quad ...(4)$$

Since, the mass moves up and down with acceleration $\frac{d^2y}{dt^2}$, by drawing the free body diagram for this case, we get

$$-ky - kl + mg = m\frac{d^2y}{dt^2} \quad ...(5)$$

The net force acting on the mass due to this stretching is

$$F = F_2 + mg$$
$$F = -ky - kl + mg \quad ...(6)$$

The gravitational force opposes the restoring force. Substituting equation (3) in equation (6), we get

$$F = -ky - kl + kl = -ky$$

Applying Newton's law, we get

$$m\frac{d^2y}{dt^2} = -ky \Rightarrow \frac{d^2y}{dt^2} = -\frac{k}{m}y \quad ...(7)$$

The above equation is in the form of simple harmonic differential equation. Therefore, we get the time period as

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ second} \quad ...(8)$$

The time period can be rewritten using equation (3)

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l}{g}} \quad ...(9)$$

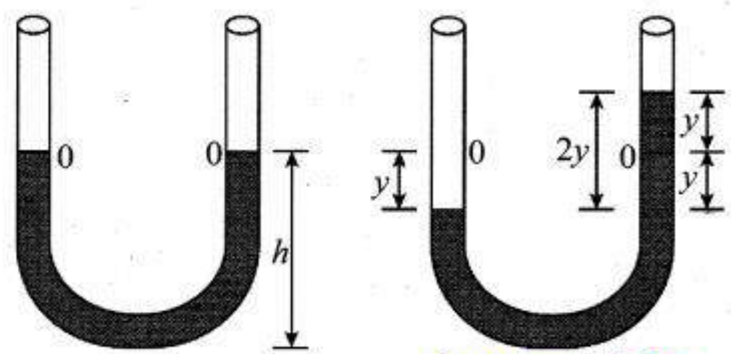
The acceleration due to gravity g can be computed from the formula

$$g = 4\pi^2\left(\frac{l}{T^2}\right) \text{ ms}^{-2} \quad ...(10)$$

Question 8.
Write short notes on the oscillations of liquid column in U-tube.

Answer:

Oscillation of liquid in a U-tube:
Consider a U-shaped glass tube which consists of two open arms with uniform cross sectional area A. Let us pour a non- viscous uniform incompressible liquid of density ρ in the U-shaped tube to a height h as shown in the figure. If the liquid and tube are not disturbed then the liquid surface will be in equilibrium position O.



U-shaped glass tube

It means the pressure as measured at any point on the liquid is the same and also at the surface on the arm (edge of the tube on either side), which balances with the atmospheric pressure. Due to this the level of liquid in each arm will be the same. By blowing air one can provide sufficient force in one arm, and the liquid gets disturbed from equilibrium position O, which means, the pressure at blown arm is higher than the other arm.

This creates difference in pressure which will cause the liquid to oscillate for a very short duration of time about the mean or equilibrium position and finally comes to rest. Time period of the oscillation is

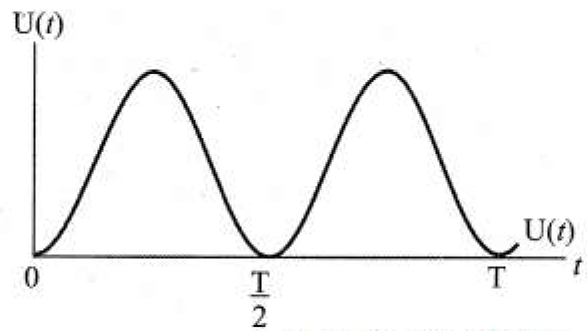
$$T = 2\pi\sqrt{\frac{l}{2g}} \text{ second}$$

Question 9.
Discuss in detail the energy in simple harmonic motion.

Answer:
Energy in simple harmonic motion:

(a) Expression for Potential Energy
For the simple harmonic motion, the force and the displacement are related by Hooke's law

$$\vec{F} = -k\vec{r}$$



Variation of potential energy with time t

Since force is a vector quantity, in three dimensions it has three components. Further, the force in the above equation is a conservative force field; such a force can be derived from a scalar function which has only one component. In one dimensional case
F = — kx …… (1)

The work done by the conservative force field is independent of path. The potential energy U can be calculated from the following expression.

$$F = -\frac{dU}{dx} \qquad ...(2)$$

Comparing (1) and (2), we get

$$-\frac{dU}{dx} = -kx$$

$$dU = kx\,dx$$

This work done by the force F during a small displacement dx stores as potential energy

$$U(x) = \int_0^x kx' \, dx' = \frac{1}{2}k(x')^2 \Big|_0^x = \frac{1}{2}kx^2 \quad ...(3)$$

From equation $\omega = \sqrt{\frac{k}{m}}$, we can substitute the value of force constant $k = m\omega^2$ in equation (3),

$$U(x) = \frac{1}{2}m\omega^2 x^2 \quad ...(4)$$

where ω is the natural frequency of the oscillating system. For the particle executing simple harmonic motion from equation y = A sift ωt, we get
x = A sin ωt

$$U(t) = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t \quad ...(5)$$

This variation of U is shown in figure.

(b) Expression for Kinetic Energy

Kinetic energy

$$KE = \frac{1}{2}mv_x^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 \quad ...(6)$$

Since the particle is executing simple harmonic motion, from equation

$$y = A \sin \omega t$$
$$x = A \sin \omega t$$

Therefore, velocity is

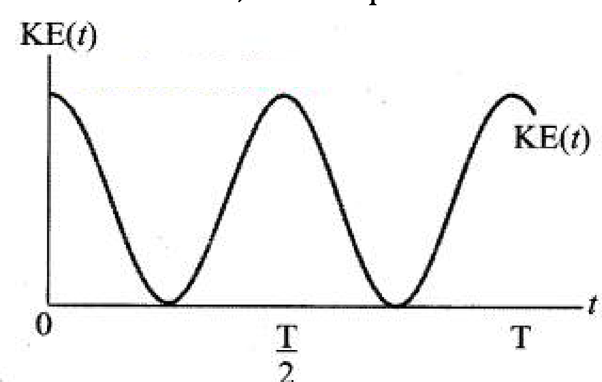
$$v_x = \frac{dx}{dt} = A\omega \cos \omega t \quad ...(7)$$

$$= A\omega\sqrt{1 - \left(\frac{x}{A}\right)^2}$$

$$v_x = \omega\sqrt{A^2 - x^2} \quad ...(8)$$

Hence, $KE = \frac{1}{2}mv_x^2 = \frac{1}{2}m\omega^2 (A^2 - x^2)$ ...(9)

$$KE = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t \quad ...(10)$$



Variation of kinetic energy with time t

This variation with time is shown in figure.

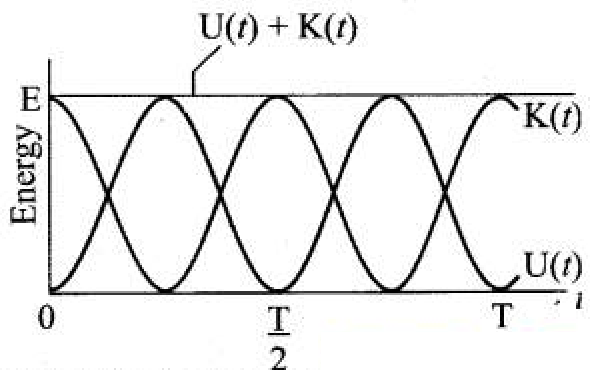
(c) Expression for Total Energy

Total energy is the sum of kinetic energy and potential energy
E = KE + U ...(11)

$$E = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

Hence, cancelling x^2 term,

$$E = \frac{1}{2}m\omega^2 A^2 = \text{constant} \qquad ...(12)$$



Both kinetic energy and potential energy vary but total energy is constant

Alternatively, from equation (5) and equation (10), we get the total energy as

$$E = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t + \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$

$$= \frac{1}{2}m\omega^2 A^2 (\sin^2 \omega t + \cos^2 \omega t)$$

From trigonometry identity, $(\sin^2 \omega t + \cos^2 \omega t) = 1$

$$E = \frac{1}{2}m\omega^2 A^2 = \text{constant}$$

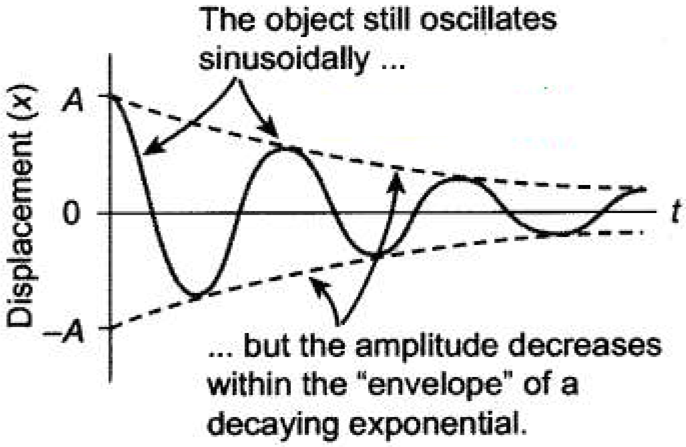
which gives the law of conservation of total energy.
Thus the amplitude of simple harmonic oscillator, can be expressed in terms of total energy.

$$A = \sqrt{\frac{2E}{m\omega^2}} = \sqrt{\frac{2E}{k}} \qquad ...(13)$$

Question 10.
Explain in detail the four different types of oscillations.

Answer:
Free oscillations: When the oscillator is* allowed to oscillate by displacing its

position from equilibrium position, it oscillates with a frequency which is equal to the natural frequency of the oscillator. Such an oscillation or vibration is known as free oscillation or free vibration. In this case, the amplitude, frequency and the energy of the vibrating object remains constant.



Damped harmonic oscillator-amplitude decreases as time increases

Examples:

1. Vibration of a tuning fork.
2. Vibration in a stretched string.
3. Oscillation of a simple pendulum.
4. Oscillations of a spring-mass system.

Damped oscillations: During the oscillation of a simple pendulum (in previous case), we have assumed that the amplitude of the oscillation is constant and also the total energy of the oscillator is constant. But in reality, in a medium, due to the presence of friction and air drag, the amplitude of oscillation decreases as time progresses. It implies that the oscillation is not sustained and the energy of the SHM decreases gradually indicating the loss of energy.

The energy lost is absorbed by the surrounding medium. This type of oscillatory motion is known as damped oscillation. In other words, if an oscillator moves in a resistive medium, its amplitude goes on decreasing and the energy of the oscillator is used to do work against the resistive medium. The motion of the oscillator is said to be damped and in this case, the resistive force (or damping force) is proportional to the velocity of the oscillator.

Examples:

1. The oscillations of a pendulum (including air friction) or pendulum oscillating inside an oil filled container.
2. Electromagnetic oscillations in a tank circuit.
3. Oscillations in a dead beat and ballistic galvanometers.

Maintained oscillations: While playing in swing, the oscillations will stop aft er a few cycles, this is due to damping. To avoid damping we have to supply a push to sustain oscillations. By supplying energy from an external source, the amplitude of the oscillation can be made constant. Such vibrations are known as maintained vibrations.

Example: The vibration of a tuning fork getting energy from a battery or from external power supply.

Forced oscillations: Any oscillator driven by an external periodic agency to overcome the damping is known as forced oscillator or driven oscillator. In this type of vibration, the body executing vibration initially vibrates with its natural frequency and due to the presence of external periodic force, the body later vibrates with the frequency of the applied periodic force. Such vibrations are known as forced vibrations.

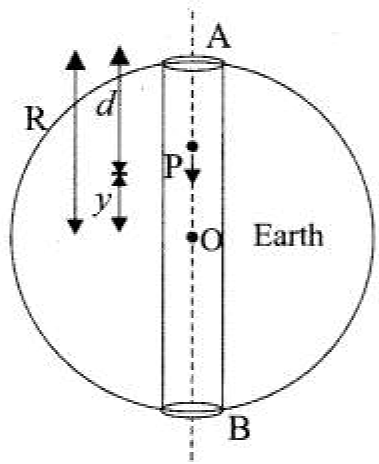
Example: Sound boards of stringed instruments.

Numerical Problems

Question 1.

Consider the Earth as a homogeneous sphere of radius R and a straight hole is bored in it through its centre. Show that a particle dropped into the hole will execute a simple harmonic motion such that its time period is

$$T = 2\pi\sqrt{\frac{R}{g}}$$



Answer:

Oscillations of a particle dropped in a tunnel along the diameter of the earth. Consider earth to be a sphere of radius R and centre O. A straight tunnel is dug along the diameter of the earth. Let 'g' be the value of acceleration due to gravity at the surface of the earth.

Suppose a body of mass 'm' is dropped into the tunnel and it is at point R i.e., at a depth d below the surface of the earth at any instant.
If g' is acceleration due to gravity at P.

then $$g' = g\left(1-\frac{d}{R}\right) = g\left(\frac{R-d}{R}\right)$$

If y is distance of the body from the centre of the earth, then

$$R - d = y$$

$$\therefore \quad g' = g\left(\frac{y}{R}\right)$$

Force acting on the body a point P is

$$F = -mg' = -\frac{mg}{R}y \text{ i.e., } F \propto y$$

Negative sign indicates that the force acts in the opposite direction of displacement.

Thus the body will execute SHM with force constant, $k = \frac{mg}{R}$

The period of oscillation of the body will be $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/R}}$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

Question 2.
Consider a simple pendulum of length l = 0.9 m which is properly placed on a trolley rolling down on a inclined plane which is at θ = 45° with the horizontal. Assuming that the inclined plane is frictionless, calculate the time period of oscillation of the simple pendulum.

Answer:
Length of the pendulum l = 0.9 m
Inclined angle θ = 45°

Time period of a simple pendulum $T = 2\pi\sqrt{\frac{l}{g'}}$

$$g' = g\cos\theta$$

$$T = 2\pi\sqrt{\frac{l}{g\cos\theta}} = 2\times 3.14\sqrt{\frac{0.9}{9.8\times\cos 45^\circ}} = 6.28\times\sqrt{0.1298}$$

$$T = 2.263\text{s}$$

Question 3.
A piece of wood of mass m is floating erect in a liquid whose density is p. If it is slightly pressed down and released, then executes simple harmonic motion.

Show that its time period of oscillation is $T = 2\pi\sqrt{\frac{m}{Ag\rho}}$

Answer:
Spring factor of liquid (k) = $A\rho g$
Inertia factor of piece of wood = m

$$\text{Time period } T = 2\pi\sqrt{\frac{\text{Intertia factor}}{\text{Spring factor}}}\ ;\ T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Question 4.
Consider two simple harmonic motion along x and y-axis having same frequencies but different amplitudes as x = A sin (ωt + φ) (along x axis) and y = B sin ωt (along y axis).

Then show that $\frac{x^2}{A^2}+\frac{y^2}{B^2}-\frac{2xy}{AB}\cos\phi=\sin^2\phi$ **and also discuss the special cases when**

(*a*) $\phi=0$ (*b*) $\phi=\pi$ (*c*) $\phi=\frac{\pi}{2}$ (*d*) $\phi=\frac{\pi}{2}$ and $A=B$ (*e*) $\phi=\frac{\pi}{4}$

Note: when a particle is subjected to two simple harmonic motion at right angle to each other the particle may move along different paths. Such paths are called Lissajous figures.

Answer:

(a) $y=\frac{B}{A}x$, equation is a straight line passing through origin with positive slope.

(b) $y=-\frac{B}{A}x$, equation is a straight line passing through origin with negative slope.

(c) $\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$, equation is an ellipse whose center is origin. A2 B2

(d) $x^2+y^2=A^2$, equation is a circle whose center is origin.

(e) $\frac{x^2}{A^2}+\frac{y^2}{B^2}-\frac{2xy}{AB}\frac{1}{\sqrt{2}}=\frac{1}{2}$, equation is an ellipse which (oblique ellipse which means tilted ellipse)

Question 5.

Show that for a particle executing simple harmonic motion

(a) the average value of kinetic energy is equal to the average value of potential energy.

(b) average potential energy = average kinetic energy = 1/2 (total energy)

[**Hint:** average kinetic energy = <kinetic energy> = $\frac{1}{T}\int_0^T$ (Kinetic energy) dt and

average Potential energy = <Potential energy> = $\frac{1}{T}\int_0^T$ (Potential energy) dt]

Answer 5:

Suppose a particle of mass m executes SHM of period T. The displacement of the particles at any instant t is given by y = A sin ωt

$$\text{Velocity } v = \frac{dy}{dt} = \omega A \cos \omega t$$

$$\text{Kinetic energy } E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$

$$\text{Potential energy, } E_P = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$

(*a*) Average K.E. over a period of oscillation,

$$E_{K_{av}} = \frac{1}{T}\int_0^T E_K dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t\, dt$$

$$= \frac{1}{2T}m\omega^2 A^2 \int_0^T \left(\frac{1+\cos 2\omega t}{2}\right) dt$$

$$= \frac{1}{4T}m\omega^2 A^2 \left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{1}{4T}m\omega^2 A^2 T$$

$$\boxed{E_{K_{av}} = \frac{1}{4}m\omega^2 A^2}$$

(*b*) Average P.E. over a period of oscillation

$$E_{P_{av}} = \frac{1}{T}\int_0^T E_P\, dt = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t\, dt$$

$$= \frac{1}{2T}m\omega^2 A^2 \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$$

$$= \frac{1}{4T}m\omega^2 A^2 \left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{1}{4T}m\omega^2 A^2 T$$

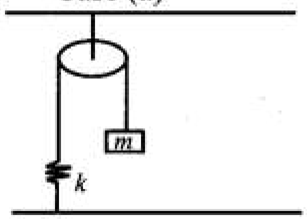
$$\boxed{E_{P_{av}} = \frac{1}{4}m\omega^2 A^2}$$

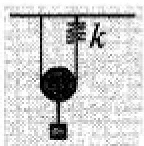
Question 6.

Compute the time period for the following system if the block of mass m is slightly displaced vertically down from Its equilibrium position and then released. Assume that the pulley is light and smooth, strings and springs are light.

Answer:

Case (a)





Case (a): Pulley is fixed rigidly here. When the mass displace by y and the spring will also stretch by y. Therefore, F = T = ky

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Case (b): Mass displace by y, pulley also displaces by y. T = 4ky

$$T = 2\pi\sqrt{\frac{m}{4k}}$$