PROGRESSIONS AND SERIES

2

The chapter on progressions essentially yields common-sense based questions in examinations.

Questions in the CAT and other aptitude exams mostly appear from either Arithmetic Progressions (more common) or from Geometric Progressions.

The chapter of progressions is a logical and natural extension of the chapter on Number Systems; since there is such a lot of commonality of logic between the problems associated with these two chapters. Let us start by looking at the definitions and the mathematics with regards to various kinds of Progressions and Series.

ARITHMETIC PROGRESSIONS

Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

Thus, each of the following series forms an arithmetic progression:

$$3, 7, 11, 15, \ldots$$

$$8, 2, -4, -10, \ldots$$

$$a, a + d, a + 2d, a + 3d, \ldots$$

The common difference is found by subtracting any term of the series from the next term.

That is, common difference of an *A.P.* = $(t_N - t_{N-1})$.

In the first of the above examples, the common difference is 4; in the second it is −6; in the third it is d.

If we examine the series $a, a + d, a + 2d, a + 3d, \ldots$ we notice that *in any term, the coefficient of d is always less by one than the position of that term in the series.*

Thus, the rth term of an arithmetic progression is given by $T_r = a + (r - 1)d$.

If n be the number of terms, and if L denotes the last term or the nth term, we have

$$L = a + (n - 1)d$$

To Find the Sum of the Given Number of Terms in an Arithmetic Progression

Let a denote the first term d, the common difference, and n the total number of terms. Also, let L denote the last term, and S the required sum; then

$$S = \frac{n(a+L)}{2} \qquad (1)$$

$$L = a + (n - 1)d \qquad (2)$$

$$S = \frac{n}{2} \times [2a + (n - 1)d] \qquad (3)$$

If any two terms of an arithmetical progression be given, the series can be completely determined; for this data results in two simultaneous equations, the solution of which will give the first term and the common difference.

When three quantities are in arithmetic progression, the middle one is said to be the **arithmetic mean** of the other two.

Thus, a is the arithmetic mean between $a - d$ and $a + d$. So, when it is required to arbitrarily consider three numbers in A.P. Take $a - d$, a and $a + d$ as the three numbers as this reduces one unknown, thereby, making the solution easier.

To Find the Arithmetic Mean Between any Two Given Quantities

Let a and b be two quantities and A be their arithmetic mean. Then since a, A, b, are in A.P. We must have

$$b - A = A - a$$

Each being equal to the common difference;

This gives us $A = \frac{(a+b)}{2}$

Between two given quantities, it is always possible to insert any number of terms such that the whole series thus formed shall be in A.P. The terms, thus, inserted are called the **arithmetic means**.

To Insert a Given Number of Arithmetic Means Between Two Given Quantities

Let a and b be the given quantities and n be the number of means.

Including the extremes, the number of terms will then be $n + 2$ so that we have to find a series of $n + 2$ terms in A.P., of which a is the first, and b is the last term.

Let d be the common difference;

then $b = \text{the } (n+2)^{\text{th}} \text{ term}$

$= a + (n + 1)d$

Hence, $d = \frac{(b-a)}{(n+1)}$

and the required means are

$$a+\frac{(b-a)}{n-1}, a+\frac{2(b-a)}{n+1}, \cdots a+\frac{n(b-a)}{n+1}$$

Till now we have studied A.P.s in their mathematical context. This was important for you to understand the basic mathematical construct of A.P.s. However, you need to understand that questions on A.P. are seldom solved on a mathematical basis (especially under the time pressure that you are likely to face in the CAT and other aptitude exams). In such situations, the mathematical processes for solving progressions based questions are likely to fail or at the very least, be very tedious. Hence, understanding the following logical aspects about Arithmetic Progressions is likely to help you solve questions based on A.P's. in the context of an aptitude exam.

Let us look at these issues one by one:

1. Process for finding the nth term of an A.P.

Suppose you have to find the 17th term of the

A.P. 3, 7, 11.......

The conventional mathematical process for this question would involve using the formula.

$$T_n = a + (n - 1) d$$

Thus, for the 17th term we would do

$$T_{17} = 3 + (17 - 1) \times 4 = 3 + 16 \times 4 = 67$$

Most students would mechanically insert the values for a, n and d and get this answer.

However, if you replace the above process with a thought algorithm, you will get the answer much faster.

The algorithm goes like this:

In order to find the 17th term of the above sequence, add the common difference to the first term, sixteen times.

> ***Note:*** Sixteen, since it is one less than 17.

Similarly, in order to find the 37th term of the A.P. 3, 11 …, all you need to do is add the common difference (8 in this case), 36 times.

Thus, the answer is 288 + 3 = 291.

> ***Note:*** You ultimately end up doing the same thing, but you are at an advantage since the entire solution process is reactionary.

2. Average of an A.P. and corresponding terms of the A.P.

Consider the A.P., 2, 6, 10, 14, 18, 22. If you try to find the average of these six numbers you will get: Average = (2 + 6 + 10 + 14 + 18 + 22)/6 = 12

Notice that 12 is also the average of the first and the last terms of the A.P. In fact, it is also the average of 6 and 18 (which correspond to the second and 5th terms of the A.P.). Further, 12 is also the average of the 3rd and 4th terms of the A.P.

> ***Note:*** In this A.P. of six terms, the average was the same as the average of the 1st and 6th terms. It was also given by the average of the 2nd and the 5th terms, as well as that of the 3rd and 4th terms.

We can call each of these pairs as "CORRESPONDING TERMS in an A.P.

What you need to understand is that every A.P. has an average.

And for any A.P., the average of any pair of corresponding terms will also be the average of the A.P.

If you try to notice the sum of the term numbers of the pair of corresponding terms given above:

1st and 6th (hence, 1 + 6 = 7)

2nd and 5th (hence, 2 + 5 = 7)

3rd and 4th (hence, 3 + 4 = 7)

Note: In each of these cases, the sum of the term numbers for the terms in a corresponding pair is one greater than the number of terms of the A.P.

This rule will hold true for all A.P.s.

For example, if an A.P. has 23 terms then for instance, you can predict that the 7th term will have the 17th term as its corresponding term, or for that matter the 9th term will have the 15th term as its corresponding term. (Since 24 is one more than 23 and 7 + 17 = 9 + 15 = 24.)

3. Process for finding the sum of an A.P.

Once you can find a pair of corresponding terms for anyA.P., you can easily find the sum of the A.P. by using the property of averages:

i.e., Sum = Number of terms × Average

In fact, this is the best process for finding the sum of an A.P. It is much more superior than the process of finding the sum of an A.P. using the expression $\frac{n}{2}(2a + (n - 1)d)$.

4. Finding the common difference of an A.P., given two terms of an A.P.

Suppose you were given that an A.P. had its 3rd term as 8 and its 8th term as 28. You should visualise this A.P. as

– , – , 8 , – , – , –, – , 28.

From the above figure, you can easily visualise that to move from the third term to the eighth term, (8 to 28) you need to add the common difference five times. The net addition being 20, the common difference should be 4.

Illustration: Find the sum of an A.P. of 17 terms, whose 3rd term is 8 and 8th term is 28.

Solution: Since we know the third term and the eighth term, we can find the common difference as 4 by the process illustrated above.

The total = 17 × Average of the A.P.

Our objective now shifts into the finding of the average of the A.P. In order to do so, we need to identify either the 10th term (which will be the corresponding term for the 8th term) or the 15th term (which will be the corresponding term for the 3rd term.)

Again: Since the 8th term is 28 and $d = 4$, the 10th term becomes 28 + 4 + 4 = 36.

Thus, the average of the A.P.

= Average of 8th and 10th terms

= (28 + 36)/2 = 32

Hence, the required answer is sum of the A.P. = 17 × 32 = 544.

The logic that has applied here is that the difference in the term numbers will give you the number of times the common difference is used to get from one to the other term.

For instance, if you know that the difference between the 7th term and 12th term of an A.P. is −30, you should realise that 5 times the common difference will be equal to −30 (Since 12 − 7 = 5).

Hence, $d = -6$.

Note: Replace this algorithmic thinking in lieu of the mathematical thinking of:

$$
\begin{aligned}
12^{\text{th}} \text{ term} &= a + 11d \\
7^{\text{th}} \text{ term} &= a + 6d \\
\text{Hence, difference} = -30 &= (a + 11d) - (a + 6d) \\
-30 &= 5d \\
\therefore \quad d &= -6
\end{aligned}
$$

5. Types of A.P.s: Increasing and Decreasing A.P.s.

Depending on whether 'd' is positive or negative, an A.P. can be increasing or decreasing.

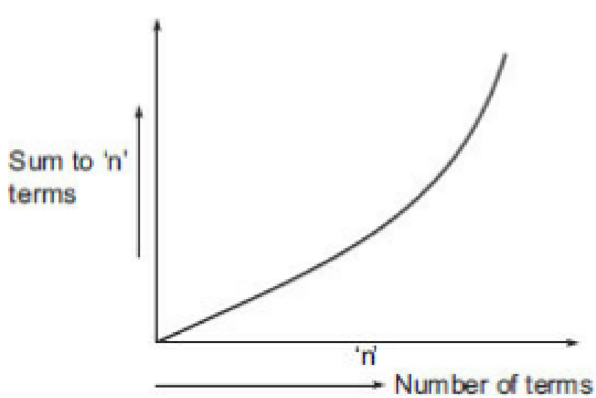
Let us explore these two types of A.P.s further:

(A) Increasing A.P.s:

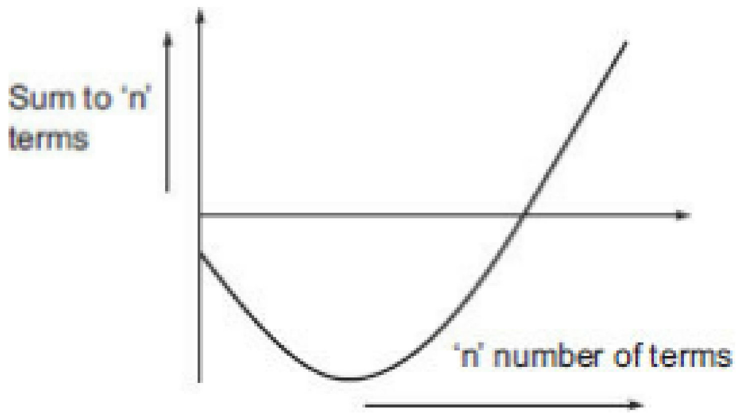
Every term of an increasing A.P. is greater than the previous term.

Depending on the value of the first term, we can construct two graphs for sum of an increasing A.P.

Case 1: When the first term of the increasing A.P. is positive. In such a case, the sum of the A.P. will show a continuously increasing graph which will look like the one shown in the figure below.



Case 2: When the first term of the increasing A.P. is negative. In such a case, the sum of the A.P. plotted against the number of terms will give the following figure:



The specific case of the sum to n_1 terms being equal to the sum to n_2 terms.

In the series case 2 above, there is a possibility of the sum to 'n' terms being repeated for 2 values of 'n'. However, this will not necessarily occur.

This issue will get clear through the following example:

Consider the following series:

Series 1: −12, −8, −4, 0, 4, 8, 12

As is evident, the sum to 2 terms and the sum to 5 terms in this case is the same. Similarly, the sum to 3 terms is the same as the sum to 4 terms. This can be written as:

$$S_2 = S_5 \text{ and } S_3 = S_4$$

In other words, the sum to n_1 terms is the same as the sum to n_2 terms.

Such situations arise for increasing A.P.s where the first term is negative. But as we have already stated that this does not happen for all such cases.

Consider the following A.P.s.

Series 2 : −8, −3, +2, + 7, + 12…

Series 3 : −13, −7, −1, + 5, + 11…

Series 4 : −12, −6, 0, 6, 12 …

Series 5 : −15, −9, −3, + 3, 9, 15 …

Series 6 : −20, −12, −4, 4 , 12, …

If you check the series listed above, you will realise that this occurrence happens in the case of series 1, series 4, series 5 and series 6 while in the case of series 2 and series 3 the same value is not repeated for the sum of the series.

A clear look at the two series will reveal that this phenomenon occurs in series which have what can be called a balance about the number zero.

Another issue to notice is that in series 4,

$$S_2 = S_3 \text{ and } S_1 = S_4$$

While in series 5,

$$S_1 = S_5 \text{ and } S_2 = S_4$$

In the first case (where '0' is part of the series), the sum is equal for two terms such that one of them is odd and the other is even.

In the second case, on the other hand (when '0' is not part of the series) the sum is equal for two terms such that both are odd or both are even.

Also notice that the sum of the term numbers which exhibit equal sums is constant for a given A.P.

Consider the following question which appeared in CAT and is based on this logic:

The sum to 12 terms of an A.P. is equal to the sum to 18 terms. What will be the sum to 30 terms for this series?

Solution : If $S_{12} = S_{18}$, $S_{11} = S_{19}$... and $S_0 = S_{30}$

But sum to zero terms for any series will always be 0. Hence, $S_{30} = 0$.

Note: The solution to this problem does not take more than ten seconds if you know this logic

(B) Decreasing A.P.s.

Similar to the cases of the increasing A.P.s, we can have two cases for decreasing A.Ps. —

Case 1: Decreasing A.P. with first term negative

Case 2: Decreasing A.P. with first term positive

I leave it to the reader to understand these cases and deduce that whatever was true for increasing A.P.s with first term negative will also be true for decreasing A.P.s with first term positive.

GEOMETRIC PROGRESSION

Quantities are said to be in geometric progression when they increase or decrease by a constant factor.

The constant factor is also called the common ratio and it is found by dividing any term by the term immediately preceding it.

If we examine the series $a, ar, ar^2, ar^3, ar^4, \ldots$

we notice that in any term the index of r is always less by one than the number of the term in the series.

If n be the number of terms and if l denote the last, or nth term, we have

$$l = ar^{n-1}$$

When three quantities are in geometrical progression, the middle one is called the geometric mean between the other two. While arbitrarily choosing three numbers in G.P., we take a/r, a and ar. This makes it easier since we come down to two variables for the three terms.

To Find the Geometric Mean Between Two Given Quantities

Let a and b be the two quantities; G the geometric mean. Then since a, G, b are in G.P.,

$$b/G = G/a$$

Each being equal to the common ratio

$$G^2 = ab$$

Hence, $G = \sqrt{ab}$

To Insert a Given Number of Geometric Means Between Two Given Quantities

Let a and b be the given quantities and n the required number of means to be inserted. In all, there will be $n + 2$ terms so that we have to find a series of $n + 2$ terms in G.P. of which a is the first and b the last.

Let r be the common ratio;

Then b = the $(n + 2)$th term $= ar^{n+1}$;

$$\therefore \quad r^{(n+1)} = \frac{b}{a}$$

$$\therefore \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} \qquad (1)$$

Hence, the required number of means are ar, ar^2, … ar^n, where r has the value found in (1).

To Find the Sum of a Number of Terms in a Geometric Progression

Let a be the first term, r the common ratio, n the number of terms, and S_n be the sum to n terms.

If $r > 1$, then

$$S_n = \frac{a(r^n - 1)}{(r - 1)} \qquad (1)$$

If $r < 1$, then

$$S_n = \frac{a(1 - r^n)}{(1 - r)} \qquad (2)$$

Note: It will be convenient to remember both forms given above for S.

Sum of an infinite geometric progression when $r < 1$

$$S_\infty = \frac{a}{(1 - r)}$$

Obviously, this formula is used only when the common ratio of the G.P. is less than one.

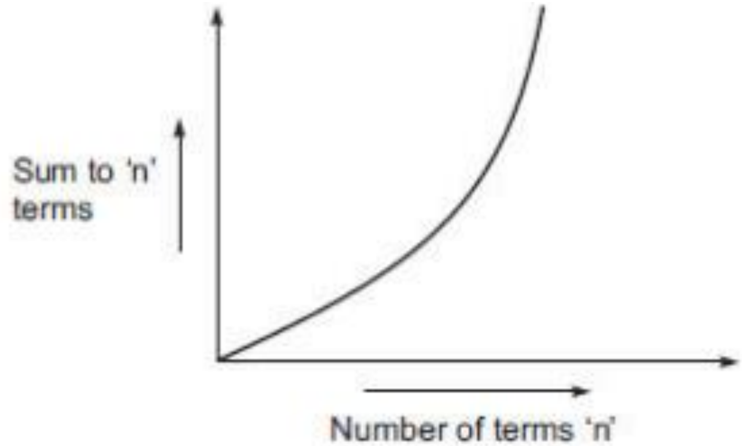
Similar to A.P.s, G.P.s can also be logically viewed. Based on the value of the common ratio and its first term, a G.P. might have one of the following structures:

(1) Increasing G.P.s type 1:

A G.P. with first term positive and common ratio greater than 1. This is the most common type of G.P.,

e.g: 3, 6, 12, 24…(A G.P. with first term 3 and common ratio 2).

The plot of the sum of the series with respect to the number of terms in such a case will appear as follows:



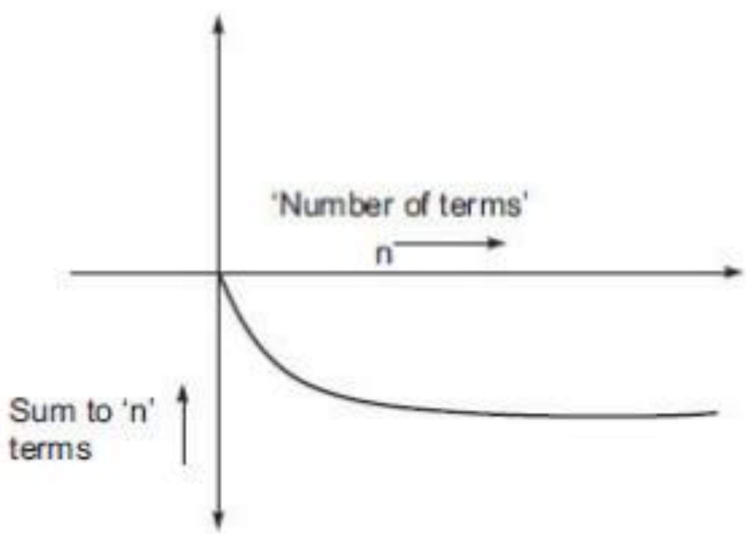
(2) Increasing G.P.s type 2:

A G.P. with first term negative and common ratio less than 1.

e.g: −8, −4,−2, −1, − ......

As you can see in this G.P., all terms are greater than their previous terms.

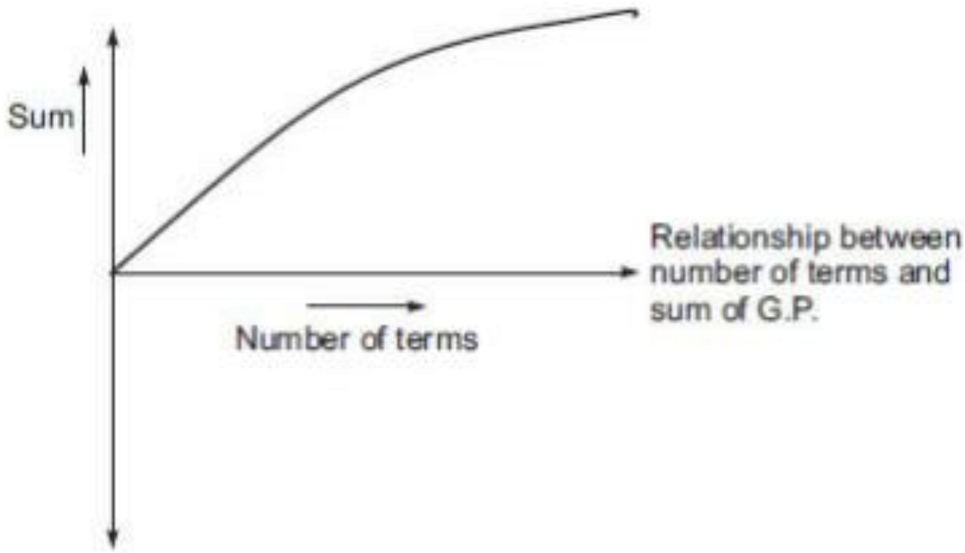
[The following figure will illustrate the relationship between the number of terms and the sum to '*n*' terms in this case]



(3) Decreasing G.P.s type 1:

These G.P.s have their first term positive and common ratio less than 1.

e.g: 12, 6, 3, 1.5, 0.75 .....

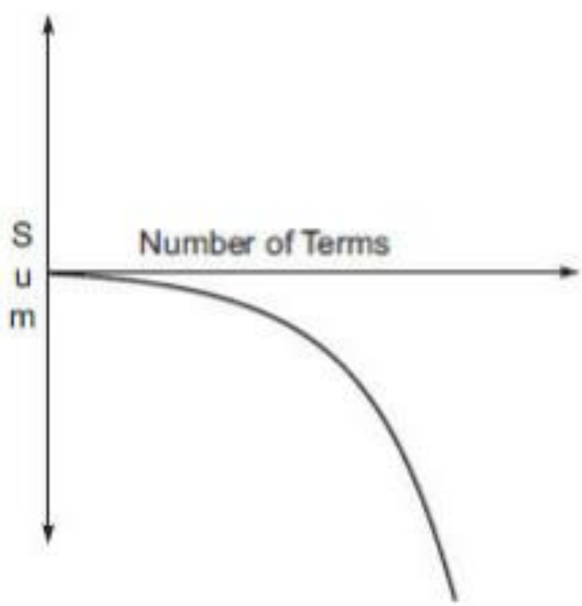


(4) Decreasing G.P.s type 2:

First term negative and common ratio greater than 1.

e.g: −2, −6, −18 ....

In this case, the relationship looks like



HARMONIC PROGRESSION

Three quantities a, b, c are said to be in harmonic progression when $a/c = \frac{(a-b)}{(b-c)}$.

In general, if a, b, c, d are in A.P. then $1/a$, $1/b$, $1/c$ and $1/d$ are all in H.P.

Any number of quantities are said to be in harmonic progression when every three consecutive terms are in harmonic progression.

The reciprocals of quantities in harmonic progression are in arithmetic progression. This can be proved as:

By definition, if a, b, c are in harmonic progression,

$$\frac{a}{c} = \frac{(a-b)}{(b-c)}$$

$$\therefore \quad a(b-c) = c(a-b),$$

dividing every term by abc, we get

$$\left[\frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a}\right]$$

which proves the proposition.

> ***Note:*** There is no general formula for the sum of any number of quantities in harmonic progression. Questions in H.P. are generally solved by inverting the terms, and making use of the properties of the corresponding A.P.

To Find the Harmonic Mean Between Two Given Quantities

Let a, b be the two quantities, H their harmonic mean; then

$1/a$, $1/H$ and $1/b$ are in A.P.;

$$\therefore \quad \frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$$

$$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

i.e. $$H = \frac{2ab}{(a+b)}$$

THEOREMS RELATED WITH PROGRESSIONS

If A, G, H are the arithmetic, geometric, and harmonic means between a and b, we have

$$A = \left(\frac{a+b}{2}\right) \qquad (1)$$

$$G = \sqrt{ab} \qquad (2)$$

$$H = \frac{2ab}{(a+b)} \qquad (3)$$

Therefore, $A \times H = \frac{(a+b)}{2} \times \frac{2ab}{(a+b)} = ab = G^2$ that is, *G is the geometric mean between A and H.*

From these results, we see that

$$A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{(a+b-2\sqrt{ab})}{2}$$

$$= \left[\frac{(\sqrt{a}-\sqrt{b})}{\sqrt{2}}\right]^2$$

which is positive if a and b are positive. Therefore, the arithmetic mean of any two positive quantities is greater than their geometric mean.

Also from the equation $G_2 = AH$, we see that G is intermediate in value between A and H; and it has been proved that $A > G$, therefore $G > H$ and $A > G > H$.

The arithmetic, geometric, and harmonic means between any two positive quantities are in descending order of magnitude.

As we have already seen in the ‘Back to School’ section of this block, there are some number series which have a continuously decreasing value from one term to the next — and such series have the property that they have what can be defined as the sum of infinite terms. Questions on such series are very common in most aptitude exams. Even though they cannot be strictly said to be under the domain of progressions, we choose to deal with them here.

Consider the following question which appeared in CAT 2003.

Find the infinite sum of the series:

$$1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \ldots\ldots$$

(a) 27/14

(b) 21/13

(c) 49/27

(d) 256/147

Solution : Such questions have two alternative widely divergent processes to solve them.

The first relies on mathematics using algebraic solving. Unfortunately, this process being overly mathematical requires a lot of writing and hence is not advisable to be used in an aptitude exam.

The other process is one where we try to predict the approximate value of the sum by taking into account the first few significant terms. (This approach is possible to use because of the fact that in such series, we invariably reach the point where the value of the next term becomes insignificant and does not add substantially to the sum). After adding the significant terms, we are in a position to guess the approximate value of the sum of the series.

Let us look at the above question in order to understand the process.

In the given series, the values of the terms are:

First term = 1

Second term = 4/7 = 0.57

Third term = 9/63 = 0.14

Fourth term = 16/343 = 0.04

Fifth term = 25/2401 = 0.01

Addition up to the fifth term is approximately 1.76.

Options (b) and (d) are smaller than 1.76 in value and hence, cannot be correct.

That leaves us with options (a) and (c).

Option 1 has a value of 1.92 approximately while option 3 has a value of 1.81 approximately.

At this point, you need to make a decision about how much value the remaining terms of the series would add to 1.76 (sum of the first 5 terms).

Looking at the pattern, we can predict that the sixth term will be

$$36/7_5 = 36/16807 = 0.002 \text{ (approx.)}$$

And the seventh term would be $49/7_6$ = 49/117649 = 0.0004 (approx.).

The eighth term will obviously become much smaller.

It can be clearly visualised that the residual terms in the series are highly insignificant. Based on this judgement, you realise that the answer will not reach 1.92 and will be restricted to 1.81. Hence, the answer will be option (c).

Try using this process to solve other questions of this nature whenever you come across them. (There are a few such questions inserted in the LOD exercises of this chapter)

Useful Results

1. If the same quantity be added to, or subtracted from, all the terms of an A.P., the resulting terms will form an A.P., but with the same common difference asbefore.
2. If all the terms of an A.P. be multiplied or divided by the same quantity, the resulting terms will form an A.P., but with a new common difference, which will be the multiplication/division of the old common difference (as the case may be).
3. If all the terms of a G.P. be multiplied or divided by the same quantity, the resulting terms will form a G.P. with the same common ratio as before.
4. If a, b, c, d, ... are in G.P., they are also in continued proportion, since, by definition,

$$a/b = b/c = c/d = \dots = 1/r$$

Conversely, a series of quantities in continued proportion may be represented by $x, xr, xr^2, \dots$

5. If you have to assume 3 terms in A.P., assume them as

   $a - d, a, a + d$ or as $a, a + d$ and $a + 2d$

   For assuming 4 terms of an A.P., we use: $a - 3d, a - d, a + d$ and $a + 3d$.

   For assuming 5 terms of an A.P., take them as:

   $a - 2d, a - d, a, a + d, a + 2d$.

   These are the most convenient in terms of problem solving.

6. For assuming three terms of a G.P., assume them as

$$a, ar \text{ and } ar2 \text{ or as } a/r, a \text{ and } ar$$

7. To find the sum of the first n natural numbers

Let the sum be denoted by S; then

$$S = 1 + 2 + 3 + \ldots\ldots + n, \text{ is given by}$$

$$S = \frac{n(n+1)}{2}$$

8. To find the sum of the squares of the first n natural numbers

Let the sum be denoted by S; then

$$S = 12 + 22 + 32 + \ldots\ldots + n2$$

This is given by : $S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}$

9. To find the sum of the cubes of the first n natural numbers.

Let the sum be denoted by S; then

$$S = 1^3 + 2^3 + 3^3 + \ldots + n^3$$

$$S = \left[\frac{n(n+1)}{2}\right]^2$$

Thus, the sum of the cubes of the first n natural numbers is equal to the square of the sum of these numbers.

10. To find the sum of the first n odd natural numbers.

$$S = 1 + 3 + 5 + \ldots + (2n - 1) \rightarrow n2$$

11. To find the sum of the first n even natural numbers.

$$S = 2 + 4 + 6 + \ldots + 2n \rightarrow n(n + 1)$$
$$= n^2 + n$$

12. To find the sum of odd numbers $\leq n$ where n is a natural number:

Case 1: If n is odd → $[(n + 1)/2]2$

Case 2: If n is even → $[n/2]2$

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13. To find the sum of even numbers $\leq n$ where n is a natural number:

Case 1: If n is even → $\{(n/2)[(n/2) + 1]\}$

Case 2: If n is odd → $[(n - 1)/2][(n + 1)/2]$

14. Number of terms in a count:

- If we are counting in steps of 1 from n_1 to n_2 including both the end points, we get $(n_2 - n_1) + 1$ numbers.

- If we are counting in steps of 1 from n_1 to n_2 including only one end, we get $(n_2 - n_1)$ numbers.

- If we are counting in steps of 1 from n_1 to n_2 excluding both ends, we get $(n_2 - n_1) - 1$ numbers.

Example: Between 16 and 25 both included there are 9 + 1 = 10 numbers.

Between 100 and 200 both excluded there are100 – 1 = 99 numbers.

- If we are counting in steps of 2 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/2] + 1$ numbers.

- If we are counting in steps of 2 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/2]$ numbers.
- If we are counting in steps of 2 from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/2] - 1$ numbers.
- If we are counting in steps of 3 from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/3] + 1$ numbers.
- If we are counting in steps of 3 from n_1 to n_2 including only one end, we get $[(n_2 - n_1)/3]$ numbers.
- If we are counting in steps of 3 from n_1 to n_2, excluding both ends, we get $[(n_2 - n_1)/3] - 1$ numbers.

Example: Number of numbers between 100 and 200 divisible by three.

Solution : The first number is 102 and the last number is 198. Hence, answer = (96/3) + 1 = 33 (since both 102 and 198 are included).

Alternately, highest number below 100 that is divisible by 3 is 99, and the lowest number above 200 which is divisible by 3 is 201.

Hence, 201 – 99 = 102 → 102/3 = 34 → Answer = 34 – 1 = 33 (Since both ends are not included).

In general

- If we are counting in steps of x from n_1 to n_2 including both the end points, we get $[(n_2 - n_1)/x] + 1$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 including only one end, we get $(n_2 - n_1)/x$ numbers.
- If we are counting in steps of "x" from n_1 to n_2 excluding both ends, we get $[(n_2 - n_1)/x] - 1$ numbers.

For instance, if we have to find how many terms are there in the series

107, 114, 121, 128 … 254, then we have

(254 – 107)/7 + 1 = 147/7 + 1 = 21 + 1 = 22 terms in the series

Of course, an appropriate adjustment will have to be made when n_2 *does not fall into the series. This will be done as follows:*

For instance, if we have to find how many terms of the series 107, 114, 121, 128 … are below 258, then we have by the formula:

(258 – 107)/7 + 1 = 151/7 + 1 = 21.57 + 1. = 22.57 This will be adjusted by taking the lower integral value = 22. → The number of terms in the series below 258.

The student is advised to try and experiment on these principles to get a clear picture.

Space for Notes

WORKED-OUT PROBLEMS

Problem 2.1 Two persons—Ramu Dhobi and Kalu Mochi have joined Donkey-work Associates. Ramu Dhobi and Kalu Mochi started with an initial salary of ₹500 and ₹640, respectively with annual increments of ₹25 and ₹20 each respectively. In which year will Ramu Dhobi start earning more salary than Kalu Mochi?

Solution The current difference between the salaries of the two is ₹140. The annual rate of reduction of this dfference is ₹5 per year. At this rate, it will take Ramu Dhobi 28 years to equalise his salary with Kalu Dhobi's salary.

Thus, in the 29th year, he will earn more.

This problem should be solved while reading and the thought process should be 140/5 = 28. Hence, answer is 29th year.

Problem 2.2 Find the value of the expression

1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms.

(a) –250

(b) –500

(c) –450

(d) –300

Solution The series (1 – 6 + 2 – 7 + 3 – 8 +……… to 100 terms) can be rewritten as:

⇒ (1 + 2 + 3 + … to 50 terms) – (6 + 7 + 8 + … to 50 terms)

Both these are A.P.'s with values of a and d as →

$a = 1, n = 50$ and $d = 1$ and $a = 6, n = 50$ and $d = 1$, respectively.

Using the formula for sum of an A.P., we get:

→ 25(2 + 49) – 25(12 + 49)

→ 25(51 – 61) = −250

Alternatively, we can do this faster by considering (1 – 6), (2 – 7), and so on as one unit or one term.

1 – 6 = 2 – 7 = … = −5. Thus, the above series is equivalent to a series of fifty −5's added to each other.

So, (1 – 6) + (2 – 7) + (3 – 8) + … 50 terms = −5 × 50 = −250

Problem 2.3 Find the sum of all numbers divisible by 6 in between 100 to 400.

Solution Here 1st term = a = 102 (which is the 1st term greater than 100 that is divisible by 6.)

The last term less than 400, which is divisible by 6 is 396.

The number of terms in the *A.P.;* 102, 108, 114…396 is given by [(396 – 102)/6] +1= 50 numbers.

Common difference = d = 6

So, S = 25 (204 + 294) = 12450

Problem 2.4 If x, y, z are in G.P., then $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ will be in

(a) A.P.

(b) G.P.

(c) H.P.

(d) Cannot be determined

Solution Go through the options.

Checking option (a), the three will be in A.P. if the 2nd expression is the average of the 1st and 3rd expressions. This can be mathematically written as

$$2/(1 + \log_{10}y) = [1/(1 + \log_{10}x)] + [1/(1 + \log_{10}z)]$$

$$= \frac{[1 + (1 + \log_{10} x) + 1 + (1 + \log_{10} z)]}{[(1 + \log_{10} x)(1 + \log_{10} z)}$$

$$= \frac{[2 + \log_{10} xz]}{(1 + \log_{10} x)(1 + \log_{10} z)}$$

Applying our judgement, there seems to be no indication that we are going to get a solution.

Checking option (b),

$$[1/(1 + \log_{10}y)]^2 = [1/(1 + \log_{10}x)][1/(1 + \log_{10}z)]$$

$$= [1/(1 + \log 10(x + z) + \log_{10}xz)]$$

Again we are trapped and any solution is not in sight.

Checking option (c),

$1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *H.P.* then $1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ will be in A.P.

So, $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$ will also be in A.P.

Hence, $2 \log_{10}y = \log_{10}x + \log_{10}z$

$\Rightarrow$ $y^2 = xz$ which is given

So, (c) is the correct option.

Alternatively, you could have solved through the following process.

x, y and z are given in a G.P.

Assume $x = 1$, $y = 10$ and $z = 100$ as x, y, z are in G.P.

So, $1 + \log_{10}x = 1$, $1 + \log_{10}y = 2$ and $1 + \log_{10}z = 3$

$\Rightarrow$ Thus, we find that since 1, 2 and 3 are in A.P., we can assume that $1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ are in A.P.

the variables should be taken in such a manner that the basic restrictions put on the variables should be respected. For example, if an expression in three variables a, b and c is given and it is mentioned that $a + b + c = 0$ then the values that you assume for a, b and c should satisfy this restriction. Hence, you should look at values like 1, 2 and −3 or 2, −1, −1, etc.

This process is especially useful in the case where the question as well as the options both contain expressions. Factorisation and advanced techniques of Maths are then not required. This process will be very beneficial for students who are weak at Mathematics.

Problem 2.5 Find t_{10} and S_{10} for the following series:

$$1, 8, 15, \ldots$$

Solution This is an A.P. with first term 1 and common difference 7.

$$t_{10} = a + (n - 1)\, d = 1 + 9 \times 7 = 64$$

$$S_{10} = \frac{10 \times 65}{2} = 325$$

Problem 2.6 Find t_{18} and S_{18} for the following series:

$$2, 8, 32, \ldots$$

Solution s This is a G.P. with first term 2 and common ratio 4.

$$t_{18} = ar^{n-1} = 2.4^{17}$$

$$S_{18} = \frac{a(r^n - 1)}{r - 1} = \frac{2(4^{18} - 1)}{(4 - 1)}$$

Problem 2.7 Is the series 1, 4,… to n terms an A.P., or a G.P., or an H.P., or a series which cannot be determined?

Solution *To determine any progression, we should have at least three terms.*

If the series is an A.P., then the next term of this series will be 7.

Again, if the next term is 16, then this will be a G.P. series (1, 4, 16 …).

So, *we cannot determine* the nature of the progression of this series.

Problem 2.8 Find the sum to 200 terms of the series

1 + 4 + 6 + 5 + 11 + 6 +…

(a) 30,200

(b) 29,800

(c) 30,200

(d) None of these

Solution Spot that the above series is a combination of two A.P.s.

The 1st A.P. is (1 + 6 + 11 +…) and the 2nd A.P. is (4 + 5 + 6 +…).

Since the terms of the two series alternate, S = (1 + 6 + 11 +… to 100 terms) + (4 + 5 + 6 + … to 100 terms)

$= \frac{100[2\times 1+99\times 5]}{2}+\frac{100[2\times 4+99\times 1]}{2}$ → (Using the formula for the sum of an *A.P.*)

= 50[497 + 107] = 50[604] = 30200

Alternatively, we can treat every two consecutive terms as one.

So we will have a total of 100 terms of the nature:

(1 + 4) + (6 + 5) + (11 + 6) … → 5, 11, 17…

Now, a = 5, d = 6 and n = 100

Hence, the sum of the given series is

$$S = \frac{100}{2} \times [2 \times 5 + 99 \times 6]$$
$$= 50[604] = 30200$$

Problem 2.9 How many terms of the series −12, −9, −6,… must be taken that the sum may be 54?

Solution Here $S = 54$, $a = -12$, $d = 3$, n is unknown and has to be calculated. To do so, we use the formula for the sum of an *A.P.* and get

$$54 = \frac{[2(-12)+(n-1)3]n}{2}$$

or $108 = -24n - 3n + 3n^2$ or $3n^2 - 27n - 108 = 0$

or $n^2 - 9n - 36 = 0$, or $n^2 - 12n + 3n - 36 = 0$

$n(n - 12) + 3(n - 12) = 0 \Rightarrow (n + 3)(n - 12) = 0$

The value of n (the number of terms) cannot be negative. Hence −3 is rejected.

So we have $n = 12$

Alternatively, we can directly add up individual terms and keep adding manually till we get a sum of 54. We will observe that this will occur after adding 12 terms (In this case, as also in all cases where the number of terms is mentally manageable, mentally adding the terms till we get the required sum will turn out to be much faster than the equation-based process).

Problem 2.10 Find the sum of n terms of the series 1.2.4 + 2.3.5 + 3.4.6 + …

(a) $n(n + 1)(n + 2)$

(b) $(n(n + 1)/12)(3n^2 + 19n + 26)$

(c) $((n + 1)(n + 2)(n + 3))/4$

(d) $(n^2 (n + 1)(n + 2)(n + 3))/3$

Solution In order to solve such problems in the examination, the option-based approach is the best. Even if you can find out the required expression mathematically, it is advisable to solve through the options as this will end up saving a lot of time for you. Use the options as follows:

If we put $n = 1$, we should get the sum as 1.2.4 = 8. By substituting $n = 1$ in each of the four options, we will get the following values for the sum to 1 term:

Option (a) gives a value of: 6

Option (b) gives a value of: 8

Option (c) gives a value of: 6

Option (d) gives a value of: 8

From this check, we can reject the options (a) and (c).

Now put $n = 2$. You can see that up to 2 terms, the expression is 1.2.4 + 2.3.5 = 38.

The correct option should also give 38 if we put $n = 2$ in the expression. Since, (a) and (c) have already been rejected, we only need to check for options (b) and (d).

Option (b) gives a value of 38.

Option (d) gives a value of 80.

Hence, we can reject option (d) and get (b) as the answer.

> ***Note:*** The above process is very effective for solving questions having options. The student should try to keep an eye open for the possibility of solving questions through options. In my opinion, approximately 50–75% of the questions asked in CAT in the QA section can be solved with options (at least partially).

LEVEL OF DIFFICULTY (I)

1. How many terms are there in the A.P. 20, 25, 30,… 130?

   (a) 22

   (b) 23

   (c) 21

   (d) 24

2. Bobby was appointed to Mindworkzz in the pay scale of ₹7000–500–12,500. Find how many years he will take to reach the maximum of the scale.

   (a) 11 years

   (b) 10 years

   (c) 9 years

   (d) 8 years

3. Find the 1st term of an A.P. whose 8th and 12th terms are respectively 39 and 59.

   (a) 5

   (b) 6

   (c) 4

   (d) 3

4. A number of squares are described whose perimetres are in G.P. Then their sides will be in

(a) A.P.

(b) G.P.

(c) H.P.

(d) Nothing can be said

5. There is an A.P. 1, 3, 5…. Which term of this A.P. is 55?

(a) 27th

(b) 26th

(c) 25th

(d) 28th

6. How many terms are identical in the two A.P's. 1, 3, 5,… up to 120 terms and 3, 6, 9,… up to 80 terms?

(a) 38

(b) 39

(c) 40

(d) 41

7. Find the lowest number in an A.P. such that the sum of all the terms is 105 and greatest term is 6 times the least.

(a) 5

(b) 10

(c) 15

(d) (a), (b) & (c)

8. Find the 15th term of the sequence 20, 15, 10, …

(a) −45

(b) −55

(c) −50

(d) 0

9. A sum of money kept in a bank amounts to ₹1240 in 4 years and ₹1600 in 10 years at simple interest. Find the sum.

(a) ₹800

(b) ₹900

(c) ₹1150

(d) ₹1000

10. A number 15 is divided into three parts which are in A.P. and the sum of their squares is 83. Find the smallest number.

(a) 5

(b) 3

(c) 6

(d) 8

11. The sum of the first 16 terms of an A.P. whose first term and third term are 5 and 15 respectively is

(a) 600

(b) 765

(c) 640

(d) 680

12. The number of terms of the series 54 + 51 + 48 +… such that the sum is 513 is

(a) 18

(b) 19

(c) Both (a) and (b)

(d) 15

13. The least value of n for which the sum of the series 5 + 8 + 11… n terms is not less than 670 is

(a) 20

(b) 19

(c) 22

(d) 21

14. A man receives ₹60 for the first week and ₹3 more each week than the preceding week. How much does he earn by the 20th week?

(a) ₹1770

(b) ₹1620

(c) ₹1890

(d) ₹1790

15. How many terms are there in the G.P. 5, 20, 80, 320,… 20480?

(a) 6

(b) 5

(c) 7

(d) 8

16. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy get if he starts working on the 1st of February and finishes on the 20th of February?

(a) 2^{20}

(b) $2^{20} - 1$

(c) $2^{19} - 1$

(d) 2^{19}

17. If the fifth term of a G.P. is 81 and first term is 16, what will be the 4th term of the G.P.?

(a) 36

(b) 18

(c) 54

(d) 24

18. The seventh term of a G.P. is 8 times the fourth term. What will be the first term when its fifth term is 48?

(a) 4

(b) 3

(c) 5

(d) 2

19. The sum of three numbers in a G.P. is 14 and the sum of their squares is 84. Find the largest number.

(a) 8

(b) 6

(c) 4

(d) 12

20. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.?

(a) 4551

(b) 10091

(c) 7881

(d) 13531

21. How many natural numbers between 300 to 500 are multiples of 7?

(a) 29

(b) 28

(c) 27

(d) 30

22. The sum of the first and the third term of a geometric progression is 20 and the sum of its first three terms is 26. Find the progression.

(a) 2, 6, 18,...

(b) 18, 6, 2,...

(c) Both of these

(d) None of these

23. If a man saves ₹4 more each year than he did the year before and if he saves ₹20 in the first year, after how many years will his savings be more than ₹1000 altogether?

(a) 19 years

(b) 20 years

(c) 21 years

(d) 18 years

24. A man's salary is ₹800 per month in the first year. He has joined in the scale of 800-40-1600. After how many years will his savings be ₹64,800?

(a) 8 years

(b) 7 years

(c) 6 years

(d) Cannot be determined

25. The 4th and 10th term of a G.P. are 1/3 and 243 respectively. Find the 2nd term.

(a) 3

(b) 1

(c) 1/27

(d) 1/9

26. The 7th and 21st terms of an A.P. are 6 and −22 respectively. Find the 26th term.

(a) −34

(b) −32

(c) −12

(d) −10

27. The sum of 5 numbers in A.P. is 30 and the sum of their squares is 220. Which of the following is the third term?

(a) 5

(b) 6

(c) 8

(d) 9

28. Find the sum of all numbers in between 10–50 excluding all those numbers which are divisible by 8. (include 10 and 50 for counting.)

(a) 1070

(b) 1220

(c) 1320

(d) 1160

29. The sum of the first four terms of an A.P. is 28 and sum of the first eight terms of the same A.P. is 88. Find the sum of the first 16 terms of the A.P..

(a) 346

(b) 340

(c) 304

(d) 268

30. Find the general term of the G.P. with the third term 1 and the seventh term 8.

(a) $(2^{3/4})^{n-3}$

(b) $(2^{3/2})^{n-3}$

(c) $(2^{3/4})^{3-n}$

(d) $(2^{3/4})^{2-n}$

31. Find the number of terms of the series 1/81, −1/27, 1/9, ... 729.

(a) 11

(b) 12

(c) 10

(d) 13

32. Four geometric means are inserted between 1/8 and 128. Find the third geometric mean.

(a) 4

(b) 16

(c) 32

(d) 8

33. *A* and *B* are two numbers whose AM is 25 and GM is 7. Which of the following may be a value of *A*?

(a) 10

(b) 20

(c) 49

(d) 25

34. Two numbers *A* and *B* are such that their GM is 20% lower than their AM. Find the ratio between the numbers.

(a) 3 : 2

(b) 4 : 1

(c) 2 : 1

(d) 3 : 1

35. A man saves ₹100 in January 2014 and increases his saving by ₹50 every month over the previous month. What is the annual saving for the man in the year 2014?

(a) ₹4200

(b) ₹4500

(c) ₹4000

(d) ₹4100

36. In a nuclear power plant, a technician is allowed an interval of maximum 100 minutes. A timer with a bell rings at specific intervals of time such that the minutes when the timer rings are not divisible by 2, 3, 5 and 7. The last alarm rings with a buzzer to give time for decontamination of the technician. How many times will the bell ring within these 100 minutes and what is the value of the last minute when the bell rings for the last time in a 100 minute shift?

(a) 25 times, 89

(b) 21 times, 97

(c) 22 times, 97

(d) 19 times, 97

37. How many zeroes will be there at the end of the expression $(2!)^{2!} + (4!)^{4!} + (8!)^{8!} + (9!)^{9!} + (10!)^{10!} + (11!)^{11!}$?

(a) $(8!)^{8!} + (9!)^{9!} + (10!)^{10!} + (11!)^{11!}$

(b) 10^{101}

(c) 4! + 6! + 8! + 2 (10!)

(d) $(0!)^{0!}$

38. The 1st, 8th and 22nd terms of an A.P. are three consecutive terms of a G.P. Find the common ratio of the G.P., given that the sum of the first twenty-two terms of the A.P. is 385.

(a) Either 1 or 1/2

(b) 2

(c) 1

(d) Either 1 or 2

39. The internal angles of a plane polygon are in A.P.. The smallest angle is 100° and the common difference is 10°. Find the number of sides of the polygon.

(a) 8

(b) 9

(c) Either 8 or 9

(d) None of these

40. After striking a floor, a rubber ball rebounds (7/8)th of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 420 metres.

(a) 2940

(b) 6300

(c) 1080

(d) 3360

41. Each of the series 13 + 15 + 17 +.... and 14 + 17 + 20 +... is continued to 100 terms. Find how many terms are identical between the two series.

(a) 35

(b) 34

(c) 32

(d) 33

42. Jack and Jill were playing mathematical puzzles with each other. Jill drew a square of sides 8 cm and then kept on drawing squares inside the squares by joining the mid points of the squares. She continued this process indefinitely. Jill asked Jack to determine the sum of the areas of all the squares that she drew. If Jack answered correctly then what would be his answer?

(a) 128

(b) 64

(c) 256

(d) 32

43. How many terms of the series 1 + 3 + 5 + 7 + ... amount to 123454321?

(a) 11101

(b) 11011

(c) 10111

(d) 11111

44. A student takes a test consisting of 100 questions with differential marking is told that each question after the first is worth 4 marks more than the preceding question. If the third question of the test is worth 9 marks. What is the maximum score that the student can obtain by attempting 98 questions?

(a) 19698

(b) 19306

(c) 9900

(d) None of these

45. In an infinite geometric progression, each term is equal to two times the sum of the terms that follow. If the first term of the series is 8, find the sum of the series.

(a) 12

(b) 32/3

(c) 34/3

(d) Data inadequate

46. What is the maximum sum of the terms in the arithmetic progression 25, 24 ½, 24, ………..?

(a) 637 ½

(b) 625

(c) 662 ½

(d) 650

47. An equilateral triangle is drawn by joining the midpoints of the sides of another equilateral triangle. A third equilateral triangle is drawn inside the second one joining the midpoints of the sides of the second equilateral triangle, and the process continues infinitely. Find the sum of the perimeters of all the equilateral triangles, if the side of the largest equilateral triangle is 24 units.

(a) 288 units
(b) 72 units

(c) 36 units

(d) 144 units

48. The sum of the first two terms of an infinite geometric series is 18. Also, each term of the series is seven times the sum of all the terms that follow. Find the first term and the common ratio of the series respectively.

(a) 16, 1/8

(b) 15, 1/5

(c) 12, 1/2

(d) 8, 1/16

49. Find the 33rd term of the sequence: 3, 8, 9, 13, 15, 18, 21, 23...

(a) 93

(b) 99

(c) 105

(d) 83

50. For the above question, find the sum of the series till 33 terms.

(a) 728

(b) 860

(c) 1595

(d) 1583

51. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean of a and b then find the value of n.

52. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the harmonic mean of a and b then find the value of n.

(a) ₹1

(b) 0

(c) 1

(d) None of these

53. If a, b are two numbers such that $a, b > 0$. If harmonic mean of a, b is equal to geometric mean of a, b then what can be said about the relationship between a and b?

54. Product of 36 positive integers is 1. Their sum is ≥

55. If we have two numbers a, b. Arithmetic of a, b is 12 and H.M. is 3. Find the value of ab.

56. If $x+\frac{1}{yz}, y+\frac{1}{zx}, z+\frac{1}{xy}$ are in A.P. then x, y, z are in:

LEVEL OF DIFFICULTY (II)

1. If a times the ath term of an A.P. is equal to b times the bth term, find the (a + b)th term.

   (a) 0

   (b) $a^2 - b^2$

   (c) $a - b$

   (d) 1

2. A number 20 is divided into four parts that are in A.P. such that the product of the first and fourth is to the product of the second and third is 2 : 3. Find the largest part.

   (a) 12

   (b) 4

   (c) 8

   (d) 9

3. Find the value of the expression: 1 – 4 + 5 – 8 … to 50 terms.

   (a) –150

   (b) –75

   (c) –50

   (d) 75

4. If a clock strikes once at one o'clock, twice at two o'clock and twelve times at 12 o'clock and again once at one o'clock and so on, how many times will the bell be struck in the course of 2 days?

(a) 156

(b) 312

(c) 78

(d) 288

5. What will be the maximum sum of 44, 42, 40, ... ?

(a) 502

(b) 504

(c) 506

(d) 500

6. Find the sum of the integers between 1 and 200 that are multiples of 7.

(a) 2742

(b) 2842

(c) 2646

(d) 2546

7. If the mth term of an A.P. is $1/n$ and nth term is $1/m$, then find the sum to mn terms.

(a) $(mn - 1)/4$

(b) $(mn + 1)/4$

(c) $(mn + 1)/2$

(d) $(mn - 1)/2$

8. Find the sum of all odd numbers lying between 100 and 200.

(a) 7500

(b) 2450

(c) 2550

(d) 2650

9. Find the sum of all integers of 3 digits that are divisible by 7.

(a) 69,336

(b) 71,336

(c) 70,336

(d) 72,336

10. The first and the last terms of an A.P. are 107 and 253. If there are five terms in this sequence, find the sum of sequence.

(a) 1080

(b) 720

(c) 900

(d) 620

11. Find the value of $1 - 2 - 3 + 2 - 3 - 4 + \ldots +$ up to 100 terms.

(a) −694

(b) −626

(c) −624

(d) −549

12. What will be the sum to n terms of the series 8 + 88 + 888 +...?

(a) $\frac{8(10^n - 9n)}{81}$

(b) $\frac{8(10^{n+1} - 10 - 9n)}{81}$

(c) $8(10^{n-1} - 10)$

(d) $8(10^{n+1} - 10)$

13. If a, b, c are in G.P., then log a, log b, log c are in

(a) A.P.

(b) G.P.

(c) H.P.

(d) None of these

14. After striking the floor, a rubber ball rebounds to 4/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest if it has been gently dropped from a height of 120 metres.

(a) 540 metres

(b) 960 metres

(c) 1080 metres

(d) 1020 metres

15. If x be the first term, y be the nth term and p be the product of n terms of a G.P., then the value of p^2 will be

(a) $(xy)^{n-1}$

(b) $(xy)^{n}$

(c) $(xy)^{1-n}$

(d) $(xy)^{n/2}$

16. The sum of an infinite G.P. whose common ratio is numerically less than 1 is 32 and the sum of the first two terms is 24. What will be the third term?

(a) 2

(b) 16

(c) 8

(d) 4

17. What will be the value of $x^{1/2}.x^{1/4}.x^{1/8}$... to infinity?

(a) x^2

(b) x

(c) $x^{3/2}$

(d) x^3

18. Find the sum to n terms of the series.

1.2.3 + 2.3.4 + 3.4.5 +...

(a) $(n + 1)(n + 2)(n + 3)/3$

(b) $n(n + 1)(2n + 2)(n + 2)/4$

(c) $n(n + 1)(n + 2)$

(d) $n(n + 1)(n + 2)(n + 3)/4$

19. Determine the first term of the geometric progression, the sum of whose first term and third term is 40 and the sum of the second term and fourth term is 80.

(a) 12

(b) 16

(c) 8

(d) 4

20. Find the second term of an A.P., if the sum of its first five even terms is equal to 15 and the sum of the first three terms is equal to −3.

(a) −3

(b) −2

(c) −1

(d) 0

21. The sum of the second and the fifth term of an A.P. is 8 and that of the third and the seventh term is 14. Find the eleventh term.

(a) 19

(b) 17

(c) 15

(d) 16

22. How many terms of an A.P. must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20?

(a) 6

(b) 9

(c) 7

(d) 8

23. Four numbers are inserted between the numbers 4 and 39 such that an A.P. results. Find the biggest of these four numbers.

(a) 31.5

(b) 31

(c) 32

(d) 30

24. Find the sum of all three-digit natural numbers, which on being divided by 5, leave a remainder equal to 4.

(a) 57,270

(b) 96,780

(c) 49,680

(d) 99,270

25. The sum of the first three terms of the arithmetic progression is 30 and the sum of the squares of the first term and the second term of the same progression is 116. Find the seventh term of the progression if its fifth term is known to be exactly divisible by 14. Assume all the terms in the A.P. are positive.

(a) 36

(b) 40

(c) 43

(d) 22

26. *A* and *B* set out to meet each other from two places 165 km apart. *A* travels 15 km the first day, 14 km the second day, 13 km the third day and so on. *B* travels 10 km the first day, 12 km the second day, 14 km the third day and so on. After how many days will they meet?

(a) 8 days

(b) 5 days

(c) 6 days

(d) 7 days

27. If a man saves ₹1000 each year and invests at the end of the year at 5% compound interest, how much will the amount be at the end of 15 years?

(a) ₹21,478

(b) ₹21,578

(c) ₹22,578

(d) ₹22,478

28. If sum to n terms of a series is given by $(n + 8)$, then its second term will be given by

(a) 10

(b) 9

(c) 8

(d) 1

29. If A is the sum of the n terms of the series $1 + 1/4 + 1/16 + \ldots$ and B is the sum of $2n$ terms of the series $1 + 1/2 + 1/4 + \ldots$, then find the value of A/B.

(a) 1/3

(b) 1/2

(c) 2/3

(d) 3/4

30. A man receives a pension starting with ₹100 for the first year. Each year he receives 90% of what he received the previous year. Find the maximum total amount he can receive even if he lives forever.

(a) ₹1100

(b) ₹1000

(c) ₹1200

(d) ₹900

31. The sum of the series represented as:

$1/1 \times 5 + 1/5 \times 9 + 1/9 \times 13 \ldots.. + 1/221 \times 225$ is

(a) 28/221

(b) 56/221

(c) 56/225

(d) None of these

32. The sum of the series

$1/(\sqrt{2} + \sqrt{1}) + 1/(\sqrt{2} + \sqrt{3}) + \ldots\ldots\ldots\ldots + 1/(\sqrt{120} + \sqrt{121})$ is

(a) 10

(b) 11

(c) 12

(d) None of these

33. Find the infinite sum of the series 1/1 + 1/3 + 1/6 + 1/10 + 1/15 ....

(a) 2

(b) 2.25

(c) 3

(d) 4

34. The sum of the series 5 × 8 + 8 × 11 + 11 × 14 up to n terms will be

(a) $(n + 1)[3(n + 1)_2 + 6(n + 1) + 1] - 10$

(b) $(n + 1)[3(n + 1)_2 + 6(n + 1) + 1] + 10$

(c) $(n + 1)[3(n + 1) + 6(n + 1)_2 + 1] - 10$

(d) $(n + 1)[3(n + 1) + 6(n + 1)_2 + 1] + 10$

35. The sum of the series: ½ + 1/6 + 1/12 + 1/20 + ...1/156 + 1/182 is

(a) 12/13

(b) 13/14

(c) 14/13

(d) None of these

36. For the above question 35, what is the sum of the series if taken to infinite terms?

(a) 1.1

(b) 1

(c) 14/13

(d) None of these

Directions for Questions 37 to 39: Answer these questions based on the following information.

There are 250 integers, a_1, a_2.......a_{250}; not all of them necessarily different. Let the greatest integer of these 250 integers be referred to as Max, and the smallest integer be referred to as Min. The integers a_1 through a_{124} form sequence *A*, and the rest form sequence *B*. Each member of *A* is less than or equal to each member of *B*.

37. All values in *A* are changed in sign, while those in *B* remain unchanged. Which of the following statements is true?

(a) Every member of *A* is greater than or equal to every member of *B*

(b) Max is in *A*

(c) If all numbers originally in A and B had the same sign, then after the change of sign, the largest number of A and B is in A

(d) None of these

38. Elements of A are in ascending order, and those of B are in descending order. a_{124} and a_{125} are interchanged. Then which of the following statements is true?

(a) A continues to be in ascending order

(b) B continues to be in descending order

(c) A continues to be in ascending order and B in descending order.

(d) None of these

39. Every element of A is made greater than or equal to every element of B by adding to each element of A an integer x. Then, x cannot be less than

(a) 2^{10}

(b) The smallest value of B

(c) The largest value of B

(d) Max-Min

40. Rohit drew a rectangular grid of 529 cells, arranged in 23 rows and 23 columns, and filled each cell with a number. The numbers with which he filled each cell were such that the numbers of each row taken from left to right formed an arithmetic series and the numbers of each column taken from top to bottom also formed an arithmetic series. The seventh and the seventeenth numbers of the fifth row were 47 and 63 respectively, while the seventh and the seventeenth numbers of the fifteenth row were 53 and 77 respectively. What is the sum of all the numbers in the grid?

(a) 32798

(b) 65596

(c) 52900

(d) None of these

41. How many three digit numbers have the property that their digits taken from left to right form an arithmetic or a geometric progression?

(a) 15

(b) 36

(c) 20

(d) 42

Directions for Questions 42 to 43: These questions are based on the following data.

At Burger King'a famous fast food centre on Main Street in Pune, burgers are made only on an automatic burger making machine. The machine continuously makes different sorts of burgers by adding different sorts of fillings on a common bread. The machine makes the burgers at the rate of one burger per half a minute. The various fillings are added to the burgers in the following manner. The 1st, 5th, 9th, ... burgers are filled with a chicken patty; the 2nd, 9th, 16th, ... burgers with vegetable patty; the 1st, 5th, 9th, ... burgers with mushroom patty; and the rest with plain cheese and tomato fillings.

The machine makes exactly 660 burgers per day.

42. How many burgers per day are made with cheese and tomato as fillings?

(a) 424

(b) 236

(c) 237

(d) None of these

43. How many burgers are made with all three fillings chicken, vegetable and mushroom?

(a) 23

(b) 24

(c) 25

(d) 26

44. An arithmetic progression P consists of n terms. From the progression, three different progressions P_1 P_2 and P_3 are created such that P_1 is obtained by the 1st, 4th, 7th...... terms of P, P_2 has the 2nd, 5th, 8th,......terms of P and P_3 has the 3rd, 6th, 9th, ...terms of P. It is found that of P_1, P_2 and P_3, two progressions have the property that their average is itself a term of the original progression P. Which of the following can be a possible value of n?

(a) 20

(b) 26

(c) 36

(d) Both (a) and (b)

45. For the above question, if the common difference between the terms of P_1 is 6, what is the common difference of P?

(a) 2

(b) 3

(c) 6

(d) Cannot be determined

Directions for Questions 46 to 48:

If $S = a, b, b, c, c, c, d, d, d, d, \ldots..z, z, z$.

46. Find the number of terms in the above series.

47. Find 144th term of the above series.

48. If $a = 1, b = 3, c = 5, d = 7, \ldots.., z = 51$, then find the sum of all terms of S.

49. If $f(4x) = 8x + 1$. Then for how many positive real values of x , $f(2x)$ will be G.M. of $f(x)$ and $f(4x)$.

50. If x, y, z, w are positive real numbers such that x, y, z, w form an increasing A.P. and x, y, w form an increasing G.P. then w/x = ?

    (a) 1

    (b) 2

    (c) 3

    (d) 4

51. If x, y, z are the mth, nth and pth terms, respectively of a G.P. then $(n - p) \log x + (p - m) \log y + (m - n) \log z$ = ?

52. Find the sum of first n groups of 1 + (1 + 2) + (1 + 2 + 3) + ...

(a) $\frac{n(n+1)(n+2)}{6}$

(b) $\frac{n(n+1)(n+2)}{12}$

(c) $\frac{n(n+1)(n+2)(n+3)}{6}$

(d) None of these

53. If $A = 1 + x + x^2 + x^3 + \ldots$.. and $B = 1 + y + y^2 + y^3 + \ldots$.. and $0 < x, y < 1$, then the value of $1 + xy + x^2y^2 + x^3y^3 + \ldots$.. is

(a) $AB/(A + B)$

(b) $AB/(A + B - 1)$

(c) $(AB - 1)/(A + B)$

(d) AB

54. If all the angles of a quadrilateral are in G.P. and all the angles and the common ratio are natural numbers. Exactly two angles are acute and two are obtuse then find the largest angle.

55. The sum to 16 groups of the series (1) + (1 + 3) + (1 + 3 + 5) + (1 + 3 + 5 + 7) + ...

56. Sum of 16 terms of the series 1 + 1 + 3 + 1 + 3 + 5 + 1 + 3 + 5 + 7 + ...

57. If the sum of n terms of a progression is $2n^2 + 3$. Then which term is equals to 78?

58. Sum of 17 terms of the series $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots$..

59. Find the sum of 20 terms of the series 3 + 6 + 10 + 15 + ...

60. If $1^n + 2^n + 3^n + \ldots + x^n$ is always divisible by $1 + 2 + 3 + \ldots + x$ then n is

(a) Even

(b) Odd

(c) Multiple of 2

(d) None of these

61. Find the 12th term of the series 3, 14, 61, 252, …..

LEVEL OF DIFFICULTY (III)

1. If in any decreasing arithmetic progression, sum of all its terms, except for the first term, is equal to −36, the sum of all its terms, except for the last term, is zero, and the difference of the tenth and the sixth term is equal to −16, then what will be first term of this series?

(a) 16

(b) 20

(c) −16

(d) −20

2. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 89. Find the third term of the progression if the sum of the first and the fifth term is equal to 10.

(a) 15

(b) 5

(c) 8

(d) 10

3. Product of the fourth term and the fifth term of an arithmetic progression is 456. Division of the ninth term by the fourth term of the progression gives quotient as 11 and the remainder as 10. Find the first term of the progression.

(a) – 52

(b) – 42

(c) – 56

(d) – 66

4. A number of saplings are lying at a place by the side of a straight road. These are to be planted in a straight line at a distance interval of 10 metres between two consecutive saplings. Mithilesh, the country's greatest forester, can carry only one sapling at a time and has to move back to the original point to get the next sapling. In this manner, he covers a total distance of 1.32 km. How many saplings does he plant in the process if he ends at the starting point?

(a) 15

(b) 14

(c) 13

(d) 12

5. A geometric progression consists of 500 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio.

(a) P_2/P_1

(b) P_1/P_2

(c) $P_2 + P_1/P_1$

(d) $P_2 + P_1/P_2$

6. The sum of the first ten terms of the geometric progression is S_1 and the sum of the next ten terms (11th through 20th) is S_2. Find the common ratio.

(a) $(S_1/S_2)_{1/10}$

(b) $-(S_1/S_2)_{1/10}$

(c) $\pm \sqrt[10]{S_2/S_1}$

(d) $(S_1/S_2)_{1/5}$

7. The first and the third terms of an arithmetic progression are equal, respectively, to the first and the third term of a geometric progression, and the second term of the arithmetic progression exceeds the second term of the geometric progression by 0.25. Calculate the sum of the first five terms of the arithmetic progression if its first term is equal to 2.

(a) 2.25 or 25

(b) 2.5

(c) 1.5

(d) 3.25

8. If (2 + 4 + 6 +... 50 terms)/(1 + 3 + 5 +... n terms) = 51/2, then find the value of n.

(a) 12

(b) 13

(c) 9

(d) 10

9. (666.... n digits)^2 + (888... n digits) is equal to

(a) $(10^n - 1) \times \frac{4}{9}$

(b) $(10^{2n} - 1) \times \frac{4}{9}$

(c) $\frac{4(10^n - 10^{n-1} - 1)}{9}$

(d) $\frac{4(10^n + 1)}{9}$

10. The interior angles of a polygon are in A.P.. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

(a) 7

(b) 8

(c) 9

(d) 10

11. Find the sum to n terms of the series 11 + 103 + 1005 + …

(a) $\frac{10(10^n - 1)}{9} + 1$

(b) $\frac{10(10^n - 1)}{9} + n$

(c) $\frac{10(10^n - 1)}{9} + n^2$

(d) $\frac{10(10^n + 1)}{11} + n^2$

12. The sum of the first term and the fifth term of an *A.P.* is 26 and the product of the second term by the fourth term is 160. Find the sum of the first seven terms of this *A.P.*.

(a) 110

(b) 114

(c) 112

(d) 116

13. The sum of the third and the ninth term of an *A.P.* is 10. Find a possible sum of the first 11 terms of this *A.P.*

(a) 55

(b) 44

(c) 66

(d) 48

14. The sum of the squares of the fifth and the eleventh term of an *A.P.* is 3 and the product of the second and the fourteenth term is equal to *P*. Find the product of the first and the fifteenth term of the *A.P.*.

(a) $(58P - 39)/45$

(b) $(98P + 39)/72$

(c) $(116P - 39)/90$

(d) $(98P + 39)/90$

15. If the ratio of harmonic mean of two numbers to their geometric mean is 12 : 13, find the ratio of the numbers.

(a) 4/9 or 9/4

(b) 2/3 or 3/2

(c) 2/5 or 5/2

(d) None of these

16. Find the sum of the series $1.2 + 2.2^2 + 3.2^3 + \ldots + 100.\ 2^{100}$.

(a) $100.2^{101} + 2$

(b) $99.2^{100} + 2$

(c) $99.2^{101} + 2$

(d) None of these

17. The sequence $[x_n]$ is a *G.P.* with $x_2/x_4 = 1/4$ and $x_1 + x_4 = 108$. What will be the value of x_3?

(a) 42

(b) 48

(c) 44

(d) 56

18. If x, y, z are in G.P. and a^x, b^y and c^z are equal, then a, b, c are in

(a) A.P.

(b) G.P.

(c) H.P.

(d) None of these

19. Find the sum of all possible whole number divisors of 720.

(a) 2012

(b) 2624

(c) 2210

(d) 2418

20. Sum to n terms of the series $\log m + \log m^2n + \log m^3n^2 + \log m^4n^3 \ldots$ is

(a) $\frac{10(10^n+1)}{11}+n^2$

(b) $\log\left(\frac{m^{n+1}}{n^{n-1}}\right)^{\frac{n}{2}}$

(c) $\log\left(\frac{n^{n-1}}{m^{n+1}}\right)^{\frac{n}{2}}$

(d) $\log\left(\frac{m^n}{n^n}\right)^{\frac{n}{2}}$

21. The sum of first 20 and first 50 terms of an A.P. is 420 and 2550. Find the eleventh term of a G.P. whose first term is the same as the A.P. and the common ratio of the G.P. is equal to the common difference of the A.P.

(d) $2^{3/4}$

23. If an be the nth term of an A.P. and if $a_7 = 15$, then the value of the common difference that would make $a_2a_7a_{12}$ greatest is

(a) 3

(b) 3/2

(c) 7

(d) 0

24. If $a_1, a_2, a_3 \ldots a_n$ are in A.P., where $a_i > 0$, then what will be the value of the expression?

$1/(\sqrt{a_1} + \sqrt{a_2}) + 1/(\sqrt{a_2} + \sqrt{a_3}) + 1/(\sqrt{a_3} + \sqrt{a_4}) + \ldots$ to n terms

(a) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$

(b) $(n - 1)/(\sqrt{a_1} + \sqrt{a_n})$

(c) $(n - 1)/(\sqrt{a_1} - \sqrt{a_n})$

(d) $(1 - n)/(\sqrt{a_1} + \sqrt{a_n})$

25. If the first two terms of a *H.P.* are 2/5 and 12/13, respectively, which of the following terms is the largest term?

(a) 4th term

(b) 5th term

(c) 6th term

(d) 2nd term

26. One side of a staircase is to be closed in by rectangular planks from the floor to each step. The width of each plank is 9 inches and their heights are successively 6 inches, 12 inches, 18 inches and so on. There are 24 planks required in total. Find the area in square feet.

(a) 112.5

(b) 107

(c) 118.5

(d) 105

27. The middle points of the sides of a triangle are joined forming a second triangle. Again a third triangle is formed by joining the middle points of this second triangle and this process is repeated infinitely. If the perimeter and area of the outer triangle are P and A respectively, what will be the sum of perimeters of triangles thus formed?

(a) $2P$

(b) P_2

(c) $3P$

(d) $P_2/2$

28. In problem 27, find the sum of areas of all the triangles.

(a) $\frac{4}{5}$A

(b) $\frac{4}{3}$A

(c) $\frac{3}{4}$A

(d) $\frac{5}{4}$A

29. A square has a side of 40 cm. Another square is formed by joining the mid-points of the sides of the given square and this process is repeated infinitely. Find the perimeter of all the squares thus formed.

(a) $160(1+\sqrt{2})$

(b) $160(2+\sqrt{2})$

(c) $160(2-\sqrt{2})$

(d) $160(1-\sqrt{2})$

30. In problem 29, find the area of all the squares thus formed.

(a) 1600

(b) 2400

(c) 2800

(d) 3200

31. The sum of the first n terms of the arithmetic progression is equal to half the sum of the next n terms of the same progression. Find the ratio of the sum of the first 3n terms of the progression to the sum of its first n terms.

(a) 5

(b) 6

(c) 7

(d) 8

32. In a certain colony of cancerous cells, each cell breaks into two new cells every hour. If there is a single productive cell at the start and this process continues for 9 hours, how many cells will the colony have at the end of 9 hours? It is known that the life of an individual cell is 20 hours.

(a) $2^9 - 1$

(b) 2^{10}

(c) 2^9

(d) $2^{10} - 1$

33. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.

(a) 49637

(b) 39767

(c) 49634

(d) 39770

34. If a be the arithmetic mean and b, c be the two geometric means between any two positive numbers, then $(b^3 + c^3)/abc$ equals

(a) $(ab)^{1/2}/c$

(b) 1

(c) a^2c/b

(d) None of these

(d) $2^{10} - 1$

33. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.

(a) 49637

(b) 39767

(c) 49634

(d) 39770

34. If a be the arithmetic mean and b, c be the two geometric means between any two positive numbers, then $(b^3 + c^3) / abc$ equals

(a) $(ab)^{1/2}/c$

(b) 1

(c) a^2c/b

(d) None of these

35. If p, q, r are three consecutive distinct natural numbers then the expression $(q + r - p)(p + r - q)(p + q - r)$ is

(a) Positive

(b) Negative

(c) Non-positive

(d) Non-negative

36. If $S = \left[1+\left(-\frac{1}{3}\right)\right]\left[1+\left(-\frac{1}{3}\right)^2\right]\left[1+\left(-\frac{1}{3}\right)^4\right][1+\left(-\frac{1}{3}\right)^8$ ..... till n terms. Then S = ?

(a) $4(10^{2n} - 1)$

(b) $4/3(10^n - 1)$

(c) $2/3(10^n - 1)$

(d) None of these

37. The number 7777....77 (total 133 digits) is

(a) Divisible by 3

(b) A prime Number

(c) A composite Number

(d) None of these

38. First term of an A.P. of consecutive integers is $n^2 + 1$ (n is a positive integer). Sum of 1st $2n$ terms of the series will be

(a) $\underline{n}(2n^2 + 2n + 1)$

(b) $(2n^2 + 2n + 3)$

(c) $n(2n^2 + 2n + 3)$

(d) None of these

39. $S = \frac{1}{1!+2!} + \frac{1}{2!+3!} + \frac{1}{3!+4!} + ... + \frac{1}{19!+20!}$, then S = ?

(a) $\frac{1}{2!} - \frac{1}{21!}$

(b) $\frac{1}{2!} - \frac{1}{20!}$

(c) None of these

40. a, b, c are in H.P. and $n > 1$ then $a^n + c^n$ is

(a) Less than $2b^n$

(a) Less than $2b_n$

(b) Less than or equal to $2b_n$

(c) More than $2b_n$

(d) More than or equal to $2b_n$

41. The sum to 17 terms of the series

$\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+....$ is

42. Find the value of S if

$$S=\frac{4}{11}+\frac{44}{11^2}+\frac{444}{11^3}+\frac{4444}{11^4}+...\infty$$

43. Find the value of $S=\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+.....+\infty$

(a) 1/3

(b) 1/2

(c) 1/6

(d) None of these

44. $A = a + A_1 + A_2 + ...+ A_N + b$

$B = a + G_1 + G_2 + ...+ G_N + b$

A is the sum of $n + 2$ terms of an A.P. with first term a and last term b.

B is the sum of $n + 2$ terms of a G.P. with first term a and last term b.

Then, what can be said about the relative values of A and B?

45. $\frac{9}{2}+\frac{25}{6}+\frac{49}{12}+....+\frac{9801}{2450}$ =?

46. Two series X ($x_1, x_2, x_3, x_4, \ldots. x_n$) and Y ($y_1, y_2, y_3, y_4, \ldots.., y_n$) are in A.P., such that $x_n - y_n = n - 2$.

It is also known that $x_3 = b_5$. Find the value of $x_{99} - y_{197}$.

(a) 47

(b) 48

(c) 49

(d) 50

47. From the first 12 natural numbers, how many arithmetic progressions of 4 terms can be formed such that the common difference is a factor of the 4th term?

48. The product of 1st five terms of an increasing A.P. is 3840. If the 1st, 2nd and 4th terms of the A.P. are in G.P. Find 10th term of the series.

49. If $S = \left[1+\left(-\frac{1}{3}\right)\right]\left[1+\left(-\frac{1}{3}\right)^2\right]\left[1+\left(-\frac{1}{3}\right)^4\right]\left[1+\left(-\frac{1}{3}\right)^8 \ldots..\right]$

then S = ?

50. Let the positive numbers a, b, c, d be in A.P. Then the type of progression for the numbers abc, abd, acd, bcd is

51. Sum of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots.$ to 10 terms

ANSWER KEY

Level of Difficulty (I)

1. (b)
2. (a)
3. (c)
4. (b)
5. (d)
6. (c)
7. (d)
8. (c)
9. (d)
10. (b)
11. (d)
12. (c)
13. (a)
14. (a)
15. (c)
16. (b)
17. (c)
18. (b)
19. (a)
20. (c)
21. (a)
22. (c)

23. (a)
24. (d)
25. (c)
26. (b)
27. (b)
28. (a)
29. (c)
30. (a)
31. (a)
32. (d)
33. (c)
34. (b)
35. (b)
36. (c)
37. (d)
38. (b)
39. (c)
40. (b)
41. (d)
42. (a)
43. (d)
44. (d)
45. (a)
46. (a)
47. (d)
48. (a)

49. (b)

50. (c)

51. 0

52. −1

53. $a = b$

54. 36

55. 36

56. A.P.

Level of Difficulty (II)

1. (a)
2. (c)
3. (b)
4. (b)
5. (c)
6. (b)
7. (c)
8. (a)
9. (c)
10. (c)
11. (b)
12. (b)
13. (a)
14. (c)
15. (b)
16. (d)
17. (b)
18. (d)

19. (c)
20. (c)
21. (a)
22. (d)
23. (c)
24. (d)
25. (b)
26. (c)
27. (b)
28. (d)
29. (c)
30. (b)
31. (c)
32. (a)
33. (a)
34. (a)
35. (b)
36. (b)
37. (d)
38. (a)
39. (d)
40. (a)
41. (d)
42. (a)
43. (b)
44. (d)

45. (a)
46. 351
47. q
48. 12051
49. 0
50. (d)
51. 0
52. (a)
53. (b)
54. 192 degrees
55. 1496
56. 56
57. 20
58. 153
59. 1770
60. (b)
61. 412 – 12

Level of Difficulty (III)

1. (a)
2. (b)
3. (d)
4. (d)
5. (a)
6. (c)
7. (b)
8. (d)
9. (b)

10. (c)
11. (c)
12. (c)
13. (a)
14. (c)
15. (a)
16. (c)
17. (b)
18. (b)
19. (d)
20. (a)
21. (d)
22. (b)
23. (d)
24. (b)
25. (d)
26. (a)
27. (a)
28. (b)
29. (b)
30. (d)
31. (b)
32. (c)
33. (b)
34. (d)
35. (d)

36. (d)

37. (c)

38. (a)

39. (a)

40. (c)

41. $\frac{323}{324}$

42. 22/5

43. (c)

44. A > B

45. 9849/50

46. (b)

47. 13

48. 24

49. 3/4

50. reciprocals are in H.P

51. 126.25

Solutions and Shortcuts

Level of Difficulty (I)

1. In order to count the number of terms in the A.P., use the short cut:

   [(last term – first term)/ common difference] + 1. In this case, it would become:

   [(130 – 20)/5] +1 = 23. Option (b) is correct.

2. 7000 – 500 – 12500 means that the starting scale is 7000 and there is an increment of 500 every year. Since, the total increment required to

reach the top of his scale is 5500, the number of years required would be 5500/500 = 11. Option (a) is correct.

3. Since the 8th and the 12th terms of the A.P. are given as 39 and 59 respectively, the difference between the two terms would equal 4 times the common difference. Thus, we get $4d = 59 - 39 = 20$. This gives us $d = 5$. Also, the 8th term in the A.P. is represented by $a + 7d$, we get:

   $a + 7d = 39 \rightarrow a + 7 \times 5 = 39 \rightarrow a = 4$. Option (c) is correct.

4. If we take the sum of the sides we get the perimeters of the squares. Thus, if the side of the respective squares are $a_1, a_2, a_3, a_4 \ldots$, their perimeters would be $4a_1, 4a_2, 4a_3, 4a_4$. Since the perimeters are in G.P., the sides would also be in G.P.

5. The number of terms in a series are found by:

$$\frac{\text{Difference between first and last terms}}{\text{Common difference}} + 1$$

6. The first common term is 3, the next will be 9 (Notice that the second common term is exactly 6 away from the first common term. 6 is also the LCM of 2 and 3 which are the respective common differences of the two series.

Thus, the common terms will be given by the A.P 3, 9, 15 ….., last term.

To find the answer, you need to find the last term that will be common to the two series.

The first series is 3, 5, 7 … 239.

While the second series is 3, 6, 9 ….. 240.

Hence, the last common term is 237.

Thus, our answer becomes $\frac{237-3}{6} + 1 = 40$

7. Trying option (a),

We get least term 5 and largest term 30 (since the largest term is 6 times the least term).

The average of the A.P becomes (5 + 30)/2 = 17.5

Thus, 17.5 × n = 105 gives us:

to get a total of 105 we need n = 6, i.e. 6 terms in this A.P. That means the A.P. should look like:

5, _ , _ , _, _, 30.

It can be easily seen that the common difference should be 5. The A.P, 5, 10, 15, 20, 25, 30 fits the situation.

The same process used for option (b) gives us the A.P. 10, 35, 60. (10 + 35 + 60 = 105) and in the third option 15, 90 (15 + 90 = 105).

Hence, all the three options are correct.

8. The first term is 20 and the common difference is −5, thus the 15th term is:

20 + 14 × (−5) = −50. Option (c) is correct.

9. The difference between the amounts at the end of 4 years and 10 years will be the simple interest on the initial capital for 6 years.

Hence, 360/6 = 60 = (simple interest)

Also, the simple interest for 4 years when added to the sum gives 1240 as the amount.

Hence, the original sum must be 1000.

10. The three parts are 3, 5 and 7 since $3^2 + 5^2 + 7^2 = 83$. Since, we want the smallest number, the answer would be 3. Option (b) is correct.

11. $a = 5$, $a + 2d = 15$ means $d = 5$. The 16th term would be $a + 15d = 5 + 75 = 80$. The sum of the series would be given by:

[16/2] × [5 + 80] = 16 × 42.5 = 680. Option (d) is correct.

12. Use trial and error by using various values from the options.

If you find the sum of the series till 18 terms, the value is 513. So also for 19 terms, the value of the sum would be 513. Option (c) is correct.

13. Solve this question through trial and error by using values of n from the options:

For 19 terms, the series would be 5 + 8 + 11 + ... + 59 which would give us a sum for the series as 19 × 32 = 608. The next term (20th term of the series) would be 62. Thus, 608 + 62 = 670 would be the sum to 20 terms. It can, thus, be concluded that for 20 terms, the value of the sum of the series is not less than 670. Option (a) is correct.

14. His total earnings would be 60 + 63 + 66 + ... + 117 = ₹1770. Option (a) is correct.

15. The series would be 5, 20, 80, 320, 1280, 5120, 20480. Thus, there are a total of 7 terms in the series. Option (c) is correct.

16. Sum of a G.P. with first term 1 and common ratio 2 and number of terms 20.

$$\frac{1 \times (2^{20} - 1)}{(2 - 1)} = 2^{20} - 1$$

17. $16r^4 = 81 \rightarrow r^4 = 81/16 \rightarrow r = 3/2$. Thus, 4th term = $ar^3 = 16 \times (3/2)^3 = 54$. Option (c) is correct.

18. In the case of a G.P., the 7th term is derived by multiplying the fourth term thrice by the common ratio.

(**Note:** This is very similar to what we had seen in the case of an A.P.)

Since, the seventh term is derived by multiplying the fourth term by 8, the relationship $r^3 = 8$ must be true.

Hence, $r = 2$

If the fifth term is 48, the series in reverse from the fifth to the first term will look like:

48, 24, 12, 6, 3. Hence, Option (b) is correct.

19. Visualising the squares below 84, we can see that the only way to get the sum of 3 squares as 84 is: $2^2 + 4^2 + 8^2 = 4 + 16 + 64$. The largest number is 8. Option (a) is correct.

20. The series would be given by: 1, 5, 9… which essentially means that all the numbers in the series are of the form $4n + 1$. Only the value in Option (c) is a $4n + 1$ number and is hence, the correct answer.

21. The series will be 301, 308, …….. 497.

Hence, answer = $\frac{196}{7} + 1 = 29$.

22. The answer to this question can be seen from the options. Both 2, 6, 18 and 18, 6, 2 satisfy the required conditions- viz: G.P. with sum of first and third terms as 20. Thus, Option (c) is correct.

23. We need the sum of the series 20 + 24 + 28 to cross 1000. Trying out the options, we can see that in 20 years, the sum of his savings would be: 20 + 24 + 28 + … + 96. The sum of this series would be 20 × 58 =1160. If we remove the 20th year, we will get the series for savings for 19 years. The series would be 20 + 24 + 28 + …. 92. Sum of the series would be 1160 – 96 = 1064. If we remove the 19th year's savings, the savings would be 1064 – 92 which would go below 1000. Thus, after 19 years his savings would cross 1000. Option (a) is correct.

24. The answer to this question cannot be determined because the question is talking about income and asking about savings. You cannot solve this unless you know the value of the expenditure the man incurs over the years. Thus, "Cannot be Determined" is the correct answer.

25. Similar to what we saw in question 18,

4th term × r^6 = 10th term.

The 4th term here is 3^{-1} and the 10th term is 3^5.

Hence $3^{-1} \times r^6 = 3^5$

Gives us: $r = 3$.

Hence, the second term will be given by (fourth term/r^2)

$^1/_3 \times {}^1/_3 \times {}^1/_3 = {}^1/_{27}$

(**Note:** To go forward in a G.P., you multiply by the common ratio, to go backward in a G.P., you divide by the common ratio.)

26. $a + 6d = 6$ and $a + 20d = -22$. Solving we get $14d = -28 \rightarrow d = -2$. 26th term = 21st term + $5d = -22 + 5 \times (-2) = -32$. Option (b) is correct.

27. Since the sum of 5 numbers in A.P. is 30, their average would be 6. The average of 5 terms in A.P. is also equal to the value of the 3rd term (logic of the middle term of an A.P.). Hence, the third term's value would be 6. Option (b) is correct.

28. The answer will be given by:

[10 + 11 + 12 + ... + 50] – [16 + 24 + ... + 48]

= 41 × 30 – 32 × 5

= 1230 – 160 = 1070.

29. Think like this:

The average of the first 4 terms is 7, while the average of the first 8 terms must be 11.

Now visualize this :

Hence, $d = 4/2 = 2$

(**Note:** understand this as a property of an A.P.)

Hence, the average of the 6th and 7th terms = 15 and the average of the 8th and 9th term = 19

But this (19) also represents the average of the 16 term A.P.

Hence, required answer = 16 × 19 = 304.

30. Go through the options. The correct option should give value as 1, when $n = 3$ and as 8 when $n = 8$.

Only option (a) satisfies both conditions.

31. The series is: 1/81, −1/27, 1/9, −1/3, 1, −3, 9, −27, 81, −243, 729. There are 11 terms in the series. Option (a) is correct.

32. $1/8 \times r^5 = 128 \rightarrow r^5 = 128 \times 8 = 1024 \rightarrow r = 4$. Thus, the series would be 1/8, 1/2, 2, 8, 32, 128. The third geometric mean would be 8. Option (d) is correct.

33. AM = 25 means that their sum is 50 and GM = 7 means their product is 49. The numbers can only be 49 and 1. Option (c) is correct.

34. Trial and error gives us that for option (b):

    With the ratio 4:1, the numbers can be taken as $4x$ and $1x$. Their AM would be $2.5x$ and their GM would be $2x$. The GM can be seen to be 20% lower than the AM. Option (b) is, thus, the correct answer.

35. The total savings would be given by the sum of the series:

    100 + 150 + 200 + 650 = 12 × 375 = ₹4500. Option (b) is correct.

36. In order to find how many times the alarm rings, we need to find the number of numbers below 100 which are not divisible by 2,3, 5 or 7. This can be found by:

    100 − (numbers divisible by 2) − (numbers divisible by 3 but not by 2) − (numbers divisible by 5 but not by 2 or 3) − (numbers divisible by 7 but not by 2 or 3 or 5)

Numbers divisible by 2: up to 100 would be represented by the series 2, 4, 6, 8, 10..100 → A total of 50 numbers

Numbers divisible by 3 but not by 2: up to 100 would be represented by the series 3, 9, 15, 21…99 → A total of 17 numbers. [**Note:** use short cut for finding the number of number in this series :

[(last term – first term)/ common difference] + 1 = [(99 – 3)/6] + 1 = 16 + 1 = 17.]

Numbers divisible by 5 but not by 2 or 3: Numbers divisible by 5 but not by 2 up to 100 would be represented by the series 5, 15, 25, 35…95 → a total of 10 numbers. But from these numbers, the numbers 15, 45 and 75 are also divisible by 3. Thus, we are left with 10 – 3 = 7 new numbers which are divisible by 5 but not by 2 and 3.

Numbers divisible by 7, but not by 2, 3 or 5: Numbers divisible by 7 but not by 2 up to 100 would be represented by the series 7, 21, 35, 49, 63, 77, 91 → a total of 7 numbers. But from these numbers we should not count 21, 35 and 63 as they are divisible by either 3 or 5. Thus, a total of 7 – 3 = 4 numbers are divisible by 7 but not by 2, 3 or 5.

37. For looking at the zeroes in the expression, we should be able to see that the number of zeroes in the third term onwards is going to be very high. Thus, the number of zeroes in the expression would be given by the number of zeroes in $4 + 24^{24}$. 24^{24} has a unit digit 6. Hence, the number of zeroes in the expression would be 1. Option (d) is correct.

38. Since the sum of 22 terms of the A.P. is 385, the average of the numbers in the A.P. would be 385/22 = 17.5. This means that the sum of the first and last terms of the A.P. would be 2 × 17.5 = 35. Trial and error gives us the terms of the required G.P. as 7, 14, 28. Thus, the common ratio the G.P. can be 2 or 1/2

39. The sum of the interior angles of a polygon are multiples of 180 and are given by $(n - 1) \times 180$ where n is the number of sides of the polygon. Thus, the sum of interior angles of a polygon would be a member of the series: 180, 360, 540, 720, 900, 1080, 1260

The sum of the series with first term 100 and common difference 10 would keep increasing when we take more and more terms of the series. In order to see the number of sides of the polygon, we should get a situation where the sum of the series represented by 100 + 110 + 120… should become a multiple of 180. The number of sides in the polygon would then be the number of terms in the series 100, 110, 120 at that point.

If we explore the sums of the series represented by

100 + 110 + 120… we realise that the sum of the series becomes a multiple of 180 for 8 terms as well as for 9 terms.

It can be seen in: 100 + 110 + 120 + 130 + 140 + 150 + 160 + 170 = 1080

Or 100 + 110 + 120 + 130 + 140 + 150 + 160 + 170 + 180 = 1260.

40. The sum of the total distance it travels would be given by the infinite sum of the series:

420 × 8/1 + 367.5 × 8/1 = 3360 + 2940 = 6300. Option (b) is correct.

41. The two series till their hundredth terms are 13, 15, 17....211 and 14, 17, 20...311. The common terms of the series would be given by the series 17, 23, 29....209. The number of terms in this series of common terms would be 192/6 + 1 = 33. Option (d) is correct.

42. The area of the first square would be 64 sq cm. The second square would give 32, the third one 16 and so on. The infinite sum of the geometric progression 64 + 32 + 16 + 8... = 128. Option (a) is correct.

43. It can be seen that for the series, the average of two terms is 2, for 3 terms the average is 3 and so on. Thus, the sum to 2 terms is 2^2, for 3 terms it is 3^2 and so on. For 11111 terms, it would be 11111^2 = 123454321. Option (d) is correct.

44. The maximum score would be the sum of the series 9 + 13 + ... + 389 + 393 + 397 = 98 × 406/2 = 19894. Option (d) is correct.

45. The series would be 8, 8/3, 8/9 and so on. The sum of the infinite series would be 8/(1 – 1/3) = 8 × 3/2 = 12. Option (a) is correct.

46. The maximum sum would occur when we take the sum of all the positive terms of the series.

The series 25, 24.5, 24, 23.5, 23, ... 1, 0.5, 0 has 51 terms. The sum of the series would be given by:

$$n \times \text{average} = 51 \times 12.5 = 637.5$$

Option (a) is correct.

47. The side of the first equilateral triangle being 24 units, the first perimeter is 72 units. The second perimeter would be half of that and so on.

72, 36, 18 …

The infinite sum of this series = 72(1 – 1/2) = 144. Option (d) is correct.

48. Solve using options. Option (a) fits the situation as 16 + 2 + 2/8 + 2/64 meets the conditions of the question. Option (a) is correct.

49. The 33rd term of the sequence would be the 17th term of the sequence 3, 9, 15, 21 ….

The 17th term of the sequence would be 3 + 6 × 16 = 99.

50. The sum to 33 terms of the sequence would be:

The sum to 17 terms of the sequence 3, 9, 15, 21, …99 + the sum to 16 terms of the sequence 8, 13, 18, … 83.

The required sum would be 17 × 51 + 16 × 45.5 = 867 + 728 = 1595.

51. $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{a+b}{2}$

For n = 0 the above equality is true. So n must be 0.

52. $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\frac{2ab}{a+b}$

For n = −1 the above equality is true so n = −1.

53. According to the question $\frac{2ab}{a+b}=\sqrt{ab}$ this equality will be true only for a = b. Hence, a = b.

54. A.M. of n positive integers is always greater than or equals to G.M. of the numbers. Then according to the question,

55. $G.M.^2 = A.M. \times H.M. = 12 \times 3 = 36$.

56. We can solve this problem by checking for values. If we assume that x, y, z are in A.P. & $x = 1, y = 2, z = 3$.

$$x + \frac{1}{yz} = 1 + \frac{1}{6} = \frac{7}{6}$$

$$y + \frac{1}{zx} = 2 + \frac{1}{3} = \frac{7}{3}$$

$$z + \frac{1}{xy} = 3 + \frac{1}{2} = \frac{7}{2}$$

We can see if x, y, z are in A. P. then $x + \frac{1}{yz}, y + \frac{1}{zx}, z + \frac{1}{xy}$ are also in A.P.

Level of Difficulty (II)

1. Identify an A.P. which satisfies the given condition.

Suppose we are talking about the second and third terms of the A.P.

Then an A.P. with second term 3 and third term 2 satisfies the condition.

a times the a_{th} term = b times the b_{th} term.

In this case the value of $a = 2$ and $b = 3$.

Hence, for the $(a + b)_{th}$ term, we have to find the fifth term.

It is clear that the fifth term of this A.P. must be zero.

Check the other three options to see whether any option gives 0 when $a = 2$ and $b = 3$.

Since none of the options b, c or d gives zero for this particular value, option (a) is correct.

2. Since the four parts of the number are in *A.P.* and their sum is 20, the average of the four parts must be 5. Looking at the options for the largest part, only the value of 8 fits in, as it leads us to think of the *A.P.* 2, 4, 6, 8. In this case, the ratio of the product of the first and fourth (2 × 8) to the product of the first and second (4 × 6) are equal. The ratio becomes 2:3.

3. View: 1 – 4 + 5 – 8 + 9 – 12 . . . . . 50 terms as (1 – 4) + (5 – 8) + (9 – 12) ...... 25 terms.

   Hence, –3 + –3 + –3 . . . . 25 terms

   = 25 × – 3 = –75.

4. In the course of 2 days, the clock would strike 1 four times, 2 four times, 3 four times and so on. Thus, the total number of times the clock would strike would be:

   4 + 8 + 12 + ...48 = 26 × 12 = 312. Option (b) is correct.

5. Since this is a decreasing A.P. with first term positive, the maximum sum will occur up to the point where the progression remains non-negative.

   44, 42, 40 0

   Hence, 23 terms × 22 = 506.

6. The sum of the required series of integers would be given by 7 + 14 + 21 + ...196 = 28 × 101.5 = 2842. Option (b) is correct.

7. A little number juggling would give you 2nd term as 1/3 and 3rd term as ½ is a possible situation that satisfies the condition.

   The A.P. will become:

   1/6, 1/3, 1/2, 2/3, 5/6, 1

or in decimal terms, 0.166, 0.333, 0.5, 0.666, 0.833, 1

Sum to 6 terms = 3.5

Check the option with $m = 2$ and $n = 3$. Only Option (c) gives 3.5. Hence, it must be the answer.

8. 101 + 103 + 105 + … 199 = 150 × 50 = 7500

9. The required sum would be given by the sum of the series 105, 112, 119, …. 994. The number of terms in this series = (994 – 105)/7 + 1 = 127 + 1 = 128. The sum of the series = 128 × (average of 105 and 994) = 70336. Option (c) is correct.

10. 5 × average of 107 and 253 = 5 × 180 = 900. Option (c) is correct.

11. The first 100 terms of this series can be viewed as:

    (1 – 2 –3) + (2 – 3 –4) + ... + (33 – 34 – 35) + 34.

    The first 33 terms of the above series (indicated inside the brackets) will give an A.P.: –4, –5, –6 …. –36

    Sum of this A.P. = 33 × –20 = –660

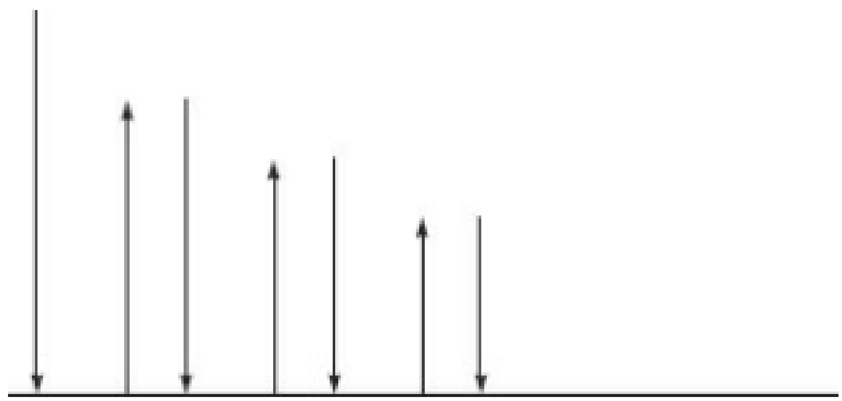
    Answer = –660 + 34 = –626

12. Solve this one through trial and error. For $n = 2$ terms the sum up to 2 terms is equal to 96. Putting $n = 2$, in the options it can be seen that for Option (b) the sum to two terms would be given by

    8 × (1000 – 10 – 18)/81 = 8 × 972/81 = 8 × 12 = 96.

13. If we take the values of a, b, and c as 10,100 and 1000 respectively, we get $\log a$, $\log b$ and $\log c$ as 1, 2 and 3 respectively. This clearly shows that the values of $\log a$, $\log b$ and $\log c$ are in A.P..

14. The path of the rubber ball is:



In the figure above, every bounce is 4/5th of the previous drop.

In the above movement, there are two infinite G.Ps. (The G.P. representing the falling distances and the G.P. representing the rising distances.)

The required answer: (Using $a/(1 - r)$ formula)

$$\frac{120}{1/5} + \frac{96}{1/5} = 1080$$

15. Solve this for a sample G.P. Let us say, we take the G.P. as 2, 6, 18, 54. x, the first term is 2, let $n = 3$ then the 3rd term $y = 18$ and the product of 3 terms $p = 2 \times 6 \times 18 = 216 = 6^3$. The value of $p^2 = 216 \times 216 = 6^6$.

Putting these values in the options we have:

Option (a) gives us $(xy)^{n-1} = 36^2$ which is not equal to the value of p^2 we have from the options.

Option (b) gives us $(xy)^n = 36^3 = 6^6$ which is equal to the value of p^2 we have from the options.

It can be experimentally verified that the other options yield values of p_2 which are different from 6_6 and hence, we can conclude that Option (b) is correct.

16. Trying to plug in values we can see that the infinite sum of the G.P. 16, 8, 4, 2… is 32 and hence, the third term is 4.

17. The expression can be written as $x^{(1/2 + 1/4 + 1/8 + 1/16.....)} = x^{\text{INFINTE SUM OF THE G.P.}} = x^1$. Option (b) is correct.

18. For $n = 1$, the sum should be 6. Option (b), (c) and (d) all give 6 as the answer.

   For $n = 2$, the sum should be 30.

   Only option (*d*) gives this value. Hence, must be the answer.

19. From the facts given in the question, it is self evident that the common ratio of the G.P. must be 2 (as the sum of the 2nd and 4th term is twice the sum of the first and third term). After realising this, the question is about trying to match the correct sums by taking values from the options.

   The G.P. formed from Option (c) with a common ratio of 2 is: 8,16,32,64 and this G.P. satisfies the conditions of the problem- sum of 1st and 3rd term is 40 and sum of 2nd and 4th term is 80.

20. Since the sum of the first five even terms is 15, we have that the 2nd, 4th, 6th, 8th and 10th term of the A.P. should add up to 15. We also need to understand that these 5 terms of the A.P. would also be in an A.P. by themselves and hence, the value of the 6th term (being the middle term of the A.P.)

would be the average of 15 over 5 terms. Thus, the value of the 6th term is 3. Also, since the sum of the first three terms of the A.P. is −3, we get that the 2nd term would have a value of −1. Thus, the A.P. can be visualised as:

_, −1, _,_,_,3,....

Thus, it is obvious that the A.P. would be −2, −1, 0, 1, 2, 3. The second term is −1. Thus, Option (c) is correct.

21. Second term = $a + d$, fifth term = $a + 4d$; third term = $a + 2d$, seventh term = $a + 6d$.

Thus, $2a + 5d = 8$ and $2a + 8d = 14 \rightarrow d = 2$ and $a = -1$.
The eleventh term = $a + 10d = -1 + 20 = 19$. Thus, option (a) is correct.

22. If the difference between the seventh and the second term is 20, it means that the common difference is equal to 4. Since, the third term is 9, the *A.P.* would be 1, 5, 9, 13, 17, 21, 25, 29 and the sum to 8 terms for this *A.P.* would be 120. Thus, Option (d) is correct.

23. $5d = 35 \rightarrow d = 7$. Thus, the numbers are 4, 11, 18, 25, 32, 39. The largest number is 32. Option (c) is correct.

24. Find sum of the series:

104, 109, 114 999

Average × n = 551.5 × 180 = 99270

25. Since the sum of the first three terms of the *A.P.* is 30, the average of the *A.P.* till 3 terms would be 30/3 = 10. The value of the second term would be equal to this average and hence, the second term is 10. Using the information about the sum of squares of the first and second terms being 116,

we have that the first term must be 4. Thus, the A.P. has a first term of 4 and a common difference of 6. The seventh term would be 40. Thus, Option (b) is correct.

26. The combined travel would be 25 on the first day, 26 on the second day, 27 on the third day, 28 on the fourth day, 29 on the fifth day and 30 on the sixth day. They meet after 6 days. Option (c) is correct.

27. This is a calculation intensive problem and you are not supposed to know how to do the calculations in this question mentally. The problem has been put here to test your concepts about whether you recognise how this is a question of G.P's.. If you feel like, you can use a calculator/ computer spreadsheet to get the answer to this question.

    The logic of the question would hinge on the fact that the value of the investment of the fifteenth year would be 1000. At the end of the 15th year, the investment of the 14th year would be equal to 1000×1.05, the 13th year's investment would amount to 1000×1.05^2 and so on till the first year's investment which would amount to 1000×1.05^{14} after 15 years.

    Thus, you need to calculate the sum of the G.P.:

    $1000, 1000 \times 1.05, 1000 \times 1.05^2, 1000 \times 1.05^3$ for 15 terms.

28. Since, sum to n terms is given by $(n + 8)$,

    Sum to 1 terms = 9

    Sum to 2 terms = 10

    Thus, the 2nd term must be 1.

29. Solve this question by looking at hypothetical values for n and $2n$ terms. Suppose, we take the sum to 1 (n = 1) term of the first series and the sum to 2 terms ($2n$ = 2) of the second series, we would get A/B as 1/1.5 = 2/3.

For n = 2 and $2n$ = 4, we get A = 1.25 and B = 1.875 and A/B = 1.25/1.875 = 2/3.

Thus, we can conclude that the required ratio is always constant at 2/3 and hence, the correct option is (c).

30. We need to find the infinite sum of the G.P.: 100, 90, 81…(first term = 100 and common ratio= 0.9).

We get infinite sum of the series as 1000. Thus, Option (b) is correct.

31. Questions such as these have to be solved on the basis of a reading of the pattern of the question. The sum up to the first term is: 1/5. Up to the second term, it is 2/9 and up to the third term, it is 3/13. It can be easily seen that for the first term, second term and third term, the numerators are 1, 2 and 3 respectively. Also, for 1/5 – the 5 is the second value in the denominator of 1/1 × 5 (the first term); for 2/9 also the same pattern is followed'as 9 comes out of the denominator of the second term of series and for 3/13 the 13 comes out of the denominator of the third term of the series and so on. The given series has 56 terms and hence, the correct answer would be 56/225.

32. Solve this on the same pattern as Question 31 and you can easily see that for the first term sum of the series is $\sqrt{2}-1$, for 2 terms we have the sum as $\sqrt{3}-1$ and so on. For the given series of 120 terms the sum would be $\sqrt{121}-1 = 10$.

Option (a) is correct.

33. If you look for a few more terms in the series, the series is:

1, 1/3, 1/6, 1/10, 1/15, 1/21, 1/28, 1/36, 1/45, 1/55, 1/66, 1/78, 1/91, 1/105, 1/120, 1/136, 1/153 and so on. If you estimate the values of the individual terms, it can be seen that the sum would tend to 2 and would not be good enough to reach even 2.25.

Thus, option (a) is correct.

34. Solve this using trial and error. For 1 term, the sum should be 40 and we get 40 only from the first option when we put n = 1. Thus, option (a) is correct.

35. For this question too, you would need to read the pattern of the values being followed. The given sum has 13 terms.

It can be seen that the sum to 1 term = 1/2

Sum to 2 terms = 2/3

Sum to 3 terms = 3/4

Hence, the sum to 13 terms would be 13/14.

36. The sum to infinite terms would tend to 1 because we would get (infinity)/(infinity + 1).

37. All members of A are smaller than all members of B. In order to visualise the effect of the change in sign in A, assume that A is {1, 2, 3…124} and B is {126, 127…250}. It can be seen that for this assumption of values neither options (a), (b) or (c) is correct.

38. If elements of A are in ascending order, a_{124} would be the largest value in A. Also a_{125} would be the largest value in B. On interchanging a_{124} and a_{125}, A continues to be in ascending order, but B would lose its descending order arrangement since a_{124} would be the least value in B. Hence, option *a* is correct.

39. Since the minimum is in A and the maximum is in B, the value of x cannot be *less than Max-Min.*

40. It is evident that the whole question is built around arithmetic progressions. The 5th row has an average of 55, while the 15th row has an average of 65. Since even column-wise each column is arranged in an *A.P.,* we can conclude the following:

1st row – average 51 – total = 23 × 51

2nd row – average 52 – total = 23 × 52.....

23rd row – average 73 – total = 23 × 73

The overall total can be got by using averages as:

23 × 23 × 62 = 32798

41. The numbers forming an *A.P.* would be:

123, 135, 147, 159, 210, 234, 246, 258, 321, 345, 357, 369, 432, 420, 456, 468, 543, 531, 567, 579, 654, 642, 630, 678, 765, 753, 741, 789, 876, 864, 852, 840, 987, 975, 963 and 951. A total of 36 numbers.

If we count the G.P's., we get:

124, 139, 248, 421, 931, 842'a total of 6 numbers.

Hence, we have a total of 42 three digit numbers where the digits are either A.P.s or G.P's..

Thus, Option (d) is correct.

42. Total burgers made = 660

Burgers with chicken and mushroom patty = 165 (Number of terms in the series 1, 5, 9...657)

Burgers with vegetable patty = 95 (Number of terms in the series 2, 9, 16, ...660)

Burgers with chicken, mushroom and vegetable patty = 24 (Number of terms in the series 9, 37, 65....653)

Required answer = 660 – 165 – 95 + 24 = 424

43. From the above question, we have 24 such burgers.

44. The key to this question is what you understand from the statement- 'for two progressions out of P_1, P_2 and P_3, the average is itself a term of the original progression P.' For option (a) which tells us that the progression P has 20 terms, we can see that P_1 would have 7 terms, P_2 would have 7 terms and P_3 would have 6 terms. Since, both P_1 and P_2 have an odd number of terms, we can see that for P_1 and P_2 their 4th terms (being the middle terms for an *A.P.* with 7 terms) would be equal to their average. Since, all the terms of P_1, P_2 and P_3 have been taken out of the original *A.P. P*, we can see that for P_1 and P_2 their average itself would be a term of the original progression P. This would not occur for P_3 as P_3 has an even number of terms. Thus, 20 is a correct value for n.

45. Since, P_1 is formed out of every third term of P, the common difference of P_1 would be three times the common difference of P. Thus, the common difference of P would be 2.

46. S consists one a, two b's, three c's and so on. So total number of terms = 1 + 2 + 3 + ...+ 26 = $\frac{26}{2}(1+26) = 13 \times 27 = 351$.

47. For 16^{th} alphabet total number of terms of S = 136.

So 144^{th} term of S will be 17^{th} alphabet, which is q.

48. Let S' be the sum of all terms of the series S then according to the question:

$$S' = 1a + 2b + 3c + 4d + 5e + \ldots + 26z = 1.1 + 2.3 + 3.5 + 4.7 + \ldots + 26.\,51 = \sum_{n=26} n(2n-1) = \sum_{n=26} 2n^2 - n = 12051$$

49. If $f(4x) = 8x + 1$ then $f(x) = 2x + 1$ & $f(2x) = 4x + 1$

$(4x + 1)^2 = (8x + 1)(2x + 1)$

$x = 0$

So for no positive value of x, $f(2x)$ is the G.M. of $f(x)$, $f(4x)$.

50. If $y = x + d$, $z = x + 2d$, $w = x + 3d$ then

$(x + d)^2 = x(x + 3d)$ or $d = x$

$w/x = 4x/x = 4$

51. Let the G.P. be 1, 3, 9, 27, 81,...

Let $m = 2$, $n = 3$, $p = 5$ then $(n - p) \log x + (p - m) \log y + (m - n) \log z = (3 - 5)\log 3 + (5 - 2)\log 9 + (2 - 3)\log 81 = -2\log 3 + 6 \log 3 - 4\log 3 = 0$

52. Go through options.

Option (a) $\frac{n(n+1)(n+2)}{6}$ for $n = 1$, sum = 1

For n = 2, Sum = 4

So option (a) is correct.

53. $A = 1/(1 - x)$ and $B = 1/(1 - y)$

$x = (A - 1)/A$ and $y = (B - 1)/B$

$1 + xy + x^2y^2 + x^3y^3 + \ldots =$

$$\frac{1}{1-xy} = \frac{1}{1-\frac{A-1}{A}.\frac{B-1}{B}} = \frac{AB}{A+B-1}$$

54. Let the angles are x, xr, xr^2, xr^3

$x + xr + xr^2 + xr^3 = 360°$.

The angles are 24°, 48°, 96°, 192°. Largest angle = 192°

55. $S = 1 + 4 + 9 + 16 + \ldots$

$$S_n = \sum n^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$S_{16} = \frac{1}{6}.16.17.33 = 1496$$

56. $S_{16} = 1 + (1 + 3) + (1 + 3 + 5) + (1 + 3 + 5 + 7) + (1 + 3 + 5 + 7 + 9) + 1$

$$S_{16} = \frac{1}{6}5(5+1)(2.5+1)+1 = 56.$$

57. Going through the trial and error = $2(20)^2 + 3 - 2(19)^2 - 3 = 78$

58. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots = (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + \ldots + (15 - 16)$
$(15 + 16) + 17^2 = -(1 + 2 + 3 + 4 + \ldots + 16) + 17^2 = 289 - 136 = 153$

59. 3 + 6 + 10 + 15 + ... (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) +..... = 1 + (1 + 2) + (1 + 2 + 3)+ (1 + 2 + 3 + 4) + ... −1

Required sum = Sum of 21 terms of series 1 + (1 + 2) + (1 + 2 + 3)+ ,.... –1

$$= \sum_{n=21} \frac{(n+1)n}{2} - 1 = 1771 - 1 = 1770$$

60. If we take $x = 3$ and $n = 1$ then $1^1 + 2^1 + 3^1 = 6$ is divisible by 1 + 2 + 3 = 6.

But for $n = 2$

$1^2 + 2^2 + 3^2 = 14$ is not divisible by 6 again for $n = 3$, $1^n + 2^n + 3^n$ divisible by 6 and so on. So for every odd value of n, $1^n + 2^n + 3^n + \ldots + x^n$ is always divisible by $1 + 2 + 3 + \ldots + x$.

61. $3 = 4^1 - 1$

$14 = 4^2 - 2$

$61 = 4^3 - 3$

$252 = 4^4 - 4$

So 12th term = $4^{12} - 12$

Level of Difficulty (III)

1. Since the difference between the tenth and the sixth terms is –16, the common difference would be –4. Using a trial and error approach with the options, we can see that if we take the first term as 16, we will get the series 16,12,8,4,0,–4,–8,–12,–16,–20. We can see that both the conditions given in the question are met by this series. Hence, the first term would be 16.

2. Any sub-part of an A.P. is also an A.P.. Thus, the third term would be the average of the first and the fifth term. Hence, the third term would be 5.

3. A factor search for factor pairs of 456 give us the following possibilities. 1,456; 2,228; 3,152; 4,114; 6,76; 8,57; 12,38 and 19,24. A check of the conditions given in the problem, tells us that if we take 12 as the 4th term and 38 as the 5th term, we would get the series till 9 terms as: –66, –40, –14, 12, 38, 64, 90, 116, 142. In this series, we can see that the division of the 9th term by the 4th term gives us a quotient of 11 and a remainder of 10. Hence, the required first term is –66.

4. The distances covered by him (to and fro) would be 20, 40, 60, 80, 100, 120, 140, 160, 180, 200 and 220 to get a total distance of 1.32 km. There are 11 terms in this A.P. However, he must also have planted one sapling at the starting point and hence the number of saplings planted would be 12.

5. The answer would directly be P_2/P_1. Assume a series having a few number of terms, e.g. 1, 2, 4, 8, 16, 32. The value of P_2 here = 42, while P_1 = 21. The common ratio can be seen to be P_2/P_1 = 2.

6. Solve on the same pattern as above. The correct answer is option (c).

7. Solve this question using options. The average of the sum of the first 5 terms of the A.P. can be used to get the value of the third term of the A.P. If we try to use the options, in option 2, if the sum of the first 5 terms is 2.5, the third term must be 2.5/5 = 0.5. This means our A.P. is 2,1.25,0.5,–0.25,–1. The corresponding G.P. with the same 1st and 3rd terms is 2,1,0.5,0.25… The condition for the second term is also matched here.

8. Use the options to get the answer. For $n = 10$, we get the required ratio as 51/2.

9. For 1 digit, the sum would be $6^2 + 8$, for 2 digits, the sum would be $66^2 + 88$ and so on. Checking the options gives us option (b) as the correct answer.

10. The sum of the interior angles of any polygon of n sides is given by $(n - 2) \times 180$. This needs to match the sum of the A.P. 120 + 125 + 130 + ...n terms. For $n = 9$, we get the two sums equal and hence option (c) is correct.

11. Solve this one using options to check the correct answer.

12. Tracing the second and fourth terms through factor pairs of 160, we get the numbers that fit in the requirements of the problem as: 7, 10, 13, 16, 19, 22, 25. The sum of the series is 112.

13. The third and ninth terms of an 11 term A.P. are a pair of corresponding terms of the A.P. Hence, their average would be the average of the A.P. This gives us the required sum of the A.P. as $11 \times 5 = 55$.

14. $(a + 4d)^2 + (a + 10d)^2 = 3 \rightarrow a^2 + 14ad + 58d^2 = 1.5$. Also, $(a + d)(a + 13d) = P \rightarrow a^2 + 14ad + 13d^2 = P$. Further, we need to find the value of $a^2 + 14ad$ (product of the first and fifteenth terms of the A.P.). From the above two equations, we get that $45d^2 = \frac{3}{2} - P \rightarrow 13d^2 = \frac{(39 - 26P)}{90}$

15. Solve using options. The values in option (a) give you the required 12:13 ratio between the HM and the GM. Hence, option (a) is correct.

16. Solve this based on pattern of the options. The given series has 100 terms. For $n = 100$, the options can be converted as:

Option (a) = $n \times 2^{(n+1)} + 2$. This means that for 1 term, the sum should be $1 \times 2^2 + 2 = 6$. But we can see that for 1 term, the series has a sum of only $1 \times 2 = 2$. Hence, this option can be rejected. Option (c) satisfies the conditions.

Option (b) = $(n - 1) \times 2^n + 2$. For 1 term, this gives us a value of 2. For 2 terms, this gives us a value of 6, which does not match the actual value in the question. Hence, this option can be rejected.

17. Since the ratio of the second to the fourth term is given as ¼, we can conclude that the common ratio of the G.P. is 2. Also, $a + 8a = 108 \rightarrow 9a = 108 \rightarrow a = 12$. Thus, $x_3 = 48$.

18. You can try to fit in values to get the correct answer. a, b, c would be in G.P. If we take x, y and z as 1, 2, 4, we get a, b, c as 4, 2 and 1, respectively to keep a^x, b^y and c^z equal.

19. The prime factors of 720 are: $2^4 \times 3^2 \times 5^1$. The required sum of factors would be: $(1 + 2 + 2^2 + 2^3 + 2^4)(1 + 3 + 3^2)(1 + 5) = 31 \times 13 \times 6 = 2418$.

20. Check the options to get option (a) as the correct answer.

21. The first term of the given A.P. is 2 and the common difference is also 2. Thus, the 11th term of the G.P. = $2 \times 2^{10} = 2048$.

22. The minimum value of y would occur when all the three values are equal. Thus, $y^3 = 4 \rightarrow y = 2^{2/3}$.

23. For the product to be the maximum, since the sum of a_2, a_7 and a_{12} would be fixed; we would need to keep each of the three numbers as equal. Thus, the value of the common difference would be 0.

24. Solve based on patterns and options as discussed for question 16 above.

25. The corresponding A.P. would be 2.5, 1.0833,... This gives us a common difference of −1.4166. From the third term onwards, the A.P. and its reciprocal H.P. would both become negative. Hence, the largest term of the H.P. is the second term itself.

26. The series of plank sizes would be: 0.75 × 0.5, 0.75 × 1, 0.75 × 1.5....0.75 × 12. The sum of this A.P. is 112.5.

27. Each subsequent triangle would have the sum of sides halved from the previous triangle. Thus, the sum of the perimeters would be given by $P + P/2 + P/4 + P/8 + \ldots$. till infinite terms. Hence, the sum of all the perimeters of the infinite triangles would be $2P$.

28. The areas would be $A + A/4 + A/16 + \ldots$till infinite terms. The infinite sum of all the perimeters would be $4A/3$.

29. The first perimeter is 160, the second one is $\frac{160}{\sqrt{2}}$, the third one would be 80 and so on till infinite terms. The infinite sum would be equal to $160(2+\sqrt{2})$.

30. The areas would consecutively get halved. So, the first area being 1600, the next one would be 800, then 400 and so on till infinite terms. Thus, the infinite sum would be 3200.

31. If the sum of the first n terms is 'x', the sum of the next n terms is given as '$2x$' (as defined in the problem). Naturally, the sum of the next n terms would be '$3x$' (When you add the same number of terms of an A.P. consecutively, you get another A.P.). Thus, the required ratio is $6x/x = 6$.

32. Since, the problem says that the cell breaks into two new cells, it means that the original cell no longer exists. Hence, after 1 hour there would be 2^1 cells, after 2 hours there would be 2^2 cells and so on. After 9 hours there would be 2^9 cells. Hence, Option (c) is correct.

33. We need the sum of the A.P.: 101, 104, 107,... 497 = 133 × 299 = 39767.

34. Solve by taking values and checking with the options. If we take the numbers as 1 and 8, we would get $a = 4.5$ and b and c as 2 and 4 respectively.

    Then $(b^3 + c^3)/abc = 2$. None of the first three options gives us a value equal to 2. Hence, the correct answer is option (d).

35. This can be checked using any values of p, q, r. If we try with 1,2,3, we get the value as 0. If we try values of p, q, r as 2,3,4, we get the expression as positive. Hence, we conclude that the expression's value would always be non-negative.

36. For $n = 1$ sum = 2/3

    For $n = 2$ sum = 20/27

    For $n = 3$ sum = 1640/2187

    None of the options matches these numbers and hence Option (d) is correct.

37. 77777…7777 = $7(10^{132} + \ldots + 10^2 + 10^1 + 1) = \frac{7(10^{133}-1)}{10-1} = \frac{7(10^{133}-1)}{9}$

$10^{133} - 1$ is divisible by 9. Hence the given number is a composite number. Option (c) is correct.

38. $S = n^2 + 1 + n^2 + 2 + \ldots + n^2 + 2n = 2n^3 + n(2n + 1) = n(2n^2 + 2n + 1)$

Option (a) is correct.

Alternative method: Suppose $n = 2$ then 1^{st} term of the series will be $2^2 + 1 = 5$. Now we want to find the sum of first $2n = 4$ terms. First 4 terms of the series will be 5, 6, 7, 8. Sum = 26.

If we put $n = 2$ in the above options then only option (a) satisfies.

39. m^{th} term of the series = $\frac{1}{m! + (m+1)!}$

$$= \frac{1}{(m+1)!} - \frac{1}{(m+2)!}$$

$$S = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} + \ldots\ldots + \frac{1}{20!} - \frac{1}{21!} = \frac{1}{2!} - \frac{1}{21!}.$$

Option (a) is correct.

40. $\frac{a^n + c^n}{2} > \left(\frac{a+c}{2}\right)^n$, if n does not lie between 0 and 1.

But we know that A.M. > H.M.

b is the H.M. of a and c.

$$\frac{a+c}{2} > b$$

As $n > 1$ $\left(\frac{a+c}{2}\right)^n > b^n$

$$\frac{a^n + c^n}{2} > \left(\frac{a+c}{2}\right)^n > b^n$$

$a_n + c_n > 2b_n$. Option (c) is correct.

41. $S = \frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + ...$

$$= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + ... + \left(\frac{1}{17^2} - \frac{1}{18^2}\right)$$

$$= 1 - \frac{1}{18^2} = \frac{323}{324}$$

42. $S = \frac{4}{11} + \frac{44}{11^2} + \frac{444}{11^3} + \frac{4444}{11^4} + ...$

$$\frac{S}{11} = \frac{4}{11^2} + \frac{44}{11^3} + \frac{444}{11^4} + ...$$

$$S - \frac{S}{11} = \frac{4}{11} + \frac{40}{11^2} + \frac{400}{11^3} + ...$$

$$\frac{10S}{11} = \frac{4}{11}\left(1 + \frac{10}{11} + \frac{100}{11^2} + ...\right)$$

$$\frac{10S}{11} = \frac{4}{11}\left(\frac{1}{1 - \frac{10}{11}}\right) = 4$$

S = 44/10 = 22/5

43. $S = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \infty$

$$S = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + ... - \frac{1}{\infty}\right)$$

$$= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

44. We can easily solve these problems by considering suitable values.

Let G.P. be 1, 2, 4, 8, 16, 32 and A.P. be 1, 7.2, 13.4, 19.6, 25.8, 32.

$A = 99, B = 63$. So $A > B$.

45. $4+\frac{1}{2}+4+\frac{1}{6}+4+\frac{1}{12}+\ldots..4+\frac{1}{2450}$

$$= 4+\left(1-\frac{1}{2}\right)+4+\left(\frac{1}{2}-\frac{1}{3}\right)+4+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots..$$

$$+4+\left(\frac{1}{49}-\frac{1}{50}\right)$$

$$= 49\times 4+\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\ldots..+\frac{1}{49}-\frac{1}{50}\right)$$

$$= 196+\frac{49}{50}=\frac{9849}{50}$$

46. Let the common difference of the series X be d_1 and that of y be d_2.

Since $x_n - y_n = n - 2$, $x_1 - y_1 = -1$ or $y_1 = x_1 + 1$

$x_3 = y_5$

$x_1 + 2d_1 = y_1 + 4d_2$

$x_1 + 2d_1 = x_1 + 1 + 4d_2$

$2d_1 - 4d_2 = 1$

$x_{99} - y_{197} = x_1 + 98d_1 - y_1 - 196d_2 = -1 + 49(2d_1 - 4d_2) = -1 + 49 = 48.$

Option (b) is correct.

47. If a be the 1^{st} term and d be the common difference of the A.P., the 4^{th} term of the series will be $a + 3d$. If $a + 3d$ is divisible by d then a should be divisible by d. Hence the cases are:

$d = 1, a = 1, 2, 3, 4, 5, 6, 7, 8, 9$

$d = 2, a = 2, 4, 6$

$d = 3, a = 3$

So the required answer is 9 + 3 + 1 = 13

48. If $a - 2d$ be the first term and d be the common difference of A.P., then according to the question:

$(a - 2d)(a - d)a(a + d)(a + 2d) = 3840$ (1)

$$\frac{a-d}{a-2d} = \frac{a+d}{a-d}$$

$$d(3d - a) = 0$$

$$a = 3d$$

By putting $a = 3d$ in equation 1 we get:

$d \times 2d \times 3d \times 4d \times 5d = 3840$

By solving we get $d = 2$ and $a = 6$

10th term = 6 + 9.2 = 24.

49. Let $x = -1/3$

$S = (1 + x)(1 + x2)(1 + x4)\ldots. (1 - x)S = (1 - x)(1 + x)(1 + x2)(1 + x4)\ldots$

$(1 - x)S = (1 - x2)(1 + x2)(1 + x4)(1 + x8)\ldots.. (1 - x)S = (1 - x4)(1 + x4)(1 + x8)$ .....

Since $x < 0$ and $|x| < 1$ so the value of RHS would be equal to 1.

$(1 - x) S = 1$ or $S = 1/(1 - x)$ or $1/(1 -(-1/3)) = 3/4$.

50. a, b, c, d are in A.P.

$\frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ are also in A.P.

$\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$ are also in A.P.

Hence, their reciprocals are in H.P.

51. n^{th} term $= \dfrac{\sum n^3}{\sum(2n-1)} = \dfrac{1}{4}(n^2+2n+1)$

Sum of n terms of the given series = $\dfrac{1}{4}\left(\dfrac{1}{6}n(n+1)(2n+1)+n(n+1)+n\right)$

For n = 10, the required sum = $\dfrac{1}{4}[505] - 126.25$

SERIES

Questions based on mixed and infinite series are quite common in the CAT and XAT exams. You would also see a fair presence of the same in other Manangement entrance exams like IIFT, SNA.P. and NMAT. After getting the process to solve the infinite series summation of geometric progression, we need to understand the process to calculate the summation of mixed series using different methods. If you know the right approach to solve these types of series questions, then it would take hardly a few seconds to solve such questions. Let's now take a look at the most commonly asked questions in various exams.

Example 1: Find the value of S, where S = 4/3 + 10/9 + 28/27 + 82/81 + ...99 terms.

Solution: To solve these kinds of questions, we need to break the series in the below mentioned manner:

S = 4/3 + 10/9 + 28/27 + 82/81 + ...

S = 1 + 1/3 + 1 + 1/9 + 1 + 1/27 + 1 + 1/81 +....

S = [1 + 1 + 1 +.....99 terms] + [1/3 + 1/9 + 1/27 + 1/81 +... 99 terms]

$$S = 99 + \frac{\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^{99}\right]}{\left[1-\frac{1}{3}\right]} = 99 + \frac{1}{2}\left[1-\left(\frac{1}{3}\right)^{99}\right]$$

Example 2: Find the value of S, where

$$S = \frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\dots$$

Solution: When the numerators of all the terms are the same, the following process is adopted:

$$S=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\dots=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\dots$$

$$-\frac{1}{\infty}=1-\frac{1}{\infty}=1$$

What we have done here is first rewrite each term by re-writing its' denominator. Next, we break each term as the difference of two terms such that the second part of each term would get cancelled with the first part of the next term. In such a manner, only the first and the last term would remain at the end.

Example 3: Find the value of S, where

$$S=1+\frac{1}{11}+\frac{3}{11^2}+\frac{5}{11^3}+\frac{7}{11^4}+\dots$$

Solution: When the numerators of the series are forming an arithmetic progression and the denominator form a geometric series, then we can solve the series in the following manner.

$$S=1+\frac{1}{11}+\frac{3}{11^2}+\frac{5}{11^3}+\frac{7}{11^4}+\dots \qquad (1)$$

Multiply the given series by 1/11

$$\left(\frac{1}{11}S\right)=\frac{1}{11}+\frac{1}{11^2}+\frac{3}{11^3}+\frac{5}{11^4}+\dots \qquad (2)$$

Equation 1-Equation 2 gives:

$$\frac{10S}{11}=1+[\frac{2}{11^2}+\frac{2}{11^3}+\frac{2}{11^4}+\frac{2}{11^5}+\ldots]=1+\frac{2}{11^2}\left(\frac{1}{1-\frac{1}{11}}\right)=\frac{56}{55}$$

Note: The series in the bracket in the above expression is an infinite geometric progression with common ratio as 1/11.

$$S=\frac{56}{55}\times\frac{11}{10}=\frac{56}{50}=\frac{28}{25}.$$

PRACTICE EXERCISE

1. S = 3/2 + 5/4 + 9/8 + 17/16 + ... 99 terms

   S = ?

   (a) $100-\frac{1}{2^{99}}$

   (b) $99-\frac{1}{2^{99}}$

   (c) $10-\frac{1}{2^{99}}$

   (d) None of these

2. Find the value of S, where

   $$S=1+\frac{1}{11}+\frac{3}{11^2}+\frac{5}{11^3}+\frac{7}{11^4}+\ldots$$

3. $S=1+\frac{2}{11}+\frac{5}{11^2}+\frac{10}{11^3}+\frac{17}{11^4}+\ldots$

   What is the value of S?

   (a) 154/125

(b) 154/135

(c) 145/125

(d) None of these

4. $M=\left(1+\left(-\frac{2}{3}\right)\right)\left(1+\left(-\frac{2}{3}\right)^2\right)\left(1+\left(-\frac{2}{3}\right)^4\right)\left(1+\left(-\frac{2}{3}\right)^8\right)\ldots$

..infinite terms

Find the value of M.

5. If $a_1 = 1/(2 \times 5)$, $a_2 = 1/(5 \times 8)$, $a_3 = 1/(8 \times 11)$, ……, then a_1, + a_2, + a_3, + ……….. a_{100} is

(a) 25/151

(b) 1/2

(c) 1/4

(d) 111/55

6. Find the value of S, where

$$S=\left(\frac{\sqrt{3}}{\sqrt{3}+3}+\frac{\sqrt{5}}{\sqrt{15}+5}+\frac{\sqrt{7}}{\sqrt{35}+7}+\ldots+\frac{\sqrt{25}}{\sqrt{575}+25}\right)$$

7. Find the value of S, where

$$S = \frac{1}{1!+2!}+\frac{1}{2!+3!}+\frac{1}{3!+4!}+\ldots+\frac{1}{19!+20!}$$

(a) $2\left(\frac{1}{2!}-\frac{1}{20!}\right)$

(b) $1-\frac{1}{20!}$

(c) $\frac{1}{2!}-\frac{1}{19!}$

(d) None of these

8. What is the value of the following expression?

$$\frac{1}{(6^2-1)} + \cdots\cdots\cdots + \frac{1}{(20^2-1)}$$

(a) $\frac{9}{19}$

(b) $\frac{10}{19}$

(c) $\frac{10}{21}$

(d) $\frac{11}{21}$

9. Find the sum of the following series:

$$\frac{2}{1!}+\frac{3}{2!}+\frac{6}{3!}+\frac{11}{4!}+\frac{18}{5!}+\cdots\cdots$$

(a) 3e

(b) 3e − 1

(c) 3(e − 1)

(d) 3e + 1

10. Evaluate $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\ldots+\frac{1}{99^2-1}$

(a) $\frac{49}{100}$

(b) $\frac{49}{200}$

(c) $\frac{51}{200}$

(d) $\frac{51}{200}$

11. The value of 1/6 + 1/12 + 1/20 +.................. 1/90 is equal to

(a) 0.4

(b) 0.7

(c) 0.9

(d) 0.75

12. Find the value of S, where

$$S = \frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots\frac{1}{9900}$$

13. Find the value of S, where

$$S = \frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\ldots\frac{1}{195}$$

(a) 7/15

(b) 6/7

(c) 6/13

(d) None of these

14. Find the value of S, where

$$S = \frac{1}{12}+\frac{1}{60}+\frac{1}{140}+\frac{1}{196}+\ldots+\frac{1}{1020}$$

15. Find the value of S, where

ANSWER KEY

1. (a)
2. 28/25
3. (a)
4. 3/5
5. (a)
6. 2
7. (d)
8. (c)
9. (c)
10. (a)
11. (a)
12. 0.99
13. (a)
14. 2/17
15. 9/28
16. 4/33

Solutions and Shortcuts

1. S = 3/2 + 5/4 + 9/8 + 17/16 + ….. 99 terms

   S = (1 + ½) + (1 + ¼) + (1 + 1/8) + (1 + 1/16) + ….. 99 terms

   S = (1 + 1 + 1 + …. 99 terms) + (1/2 + ¼ + 1/8 + 1/16 + ….. 99 terms)

   $$S = 99 + (\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + 99 \text{ terms})$$

$$S = 99 + \frac{\left(\frac{1}{2}\right)\left(1-\frac{1}{2^{99}}\right)}{1-\frac{1}{2}} = 99+1-\frac{1}{2^{99}} = 100-\frac{1}{2^{99}}.$$

2. $S = 1+\frac{1}{11}+\frac{3}{11^2}+\frac{5}{11^3}+\frac{7}{11^4}+\ldots \quad (1)$

$$\left(\frac{1}{11}S\right) = \frac{1}{11}+\frac{1}{11^2}+\frac{3}{11^3}+\frac{5}{11^4}+\ldots \quad (2)$$

Subtracting (2) from (1) gives us:

$$\frac{10S}{11} = 1+\frac{2}{11^2}+\frac{2}{11^3}+\frac{2}{11^4}+\frac{2}{11^5}+\ldots = 1+\frac{2}{11^2}\left(\frac{1}{1-\frac{1}{11}}\right) = \frac{56}{55}$$

$$S = \frac{56}{55}\times\frac{11}{10} = \frac{56}{50} = \frac{28}{25}.$$

3. $S = 1+\frac{2}{11}+\frac{5}{11^2}+\frac{10}{11^3}+\frac{17}{11^4}+\ldots \quad (1)$

Multiplying by 1/11, we get:

$$S/11 = \frac{1}{11}+\frac{2}{11^2}+\frac{5}{11^3}+\frac{10}{11^4}+\ldots \quad (2)$$

$$S-\frac{S}{11} = \frac{10S}{11} = 1+\frac{1}{11}+\frac{3}{11^2}+\frac{5}{11^3}+\frac{7}{11^4}+\ldots \quad (3)$$

Again multiplying by 1/11, we get:

$$\frac{10}{11^2}S = \frac{1}{11}+\frac{1}{11^2}+\frac{3}{11^3}+\frac{5}{11^4}+\frac{7}{11^5}+\ldots \quad (4)$$

Subtracting (4) from (3):

$$\frac{10}{11}\left(\frac{10}{11}S\right) = 1+\frac{2}{11^2}+\frac{2}{11^3}+\frac{2}{11^4}+\ldots$$

$$\frac{100}{121}S = 1 + \frac{2}{11^2}\left(\frac{1}{1-\frac{1}{11}}\right) = \frac{56}{55}$$

$S = 154/125$

Hence, option (a) is correct.

4. $\left(1-\left(-\frac{2}{3}\right)\right)M = \left(1-\left(-\frac{2}{3}\right)\right)\left(1+\left(-\frac{2}{3}\right)\right)\left(1+\left(-\frac{2}{3}\right)^2\right)\left(1+\left(-\frac{2}{3}\right)^4\right)\left(1+\left(-\frac{2}{3}\right)^8\right)\ldots$

$$\frac{5}{3}M = \left(1-\left(-\frac{2}{3}\right)^2\right)\left(1+\left(-\frac{2}{3}\right)^2\right)\left(1+\left(-\frac{2}{3}\right)^4\right)\left(1+\left(-\frac{2}{3}\right)^8\right)\ldots$$

$$\frac{5}{3}M = \left(1-\left(-\frac{2}{3}\right)^4\right)\left(1+\left(-\frac{2}{3}\right)^4\right)\left(1+\left(-\frac{2}{3}\right)^8\right)\left(1+\left(-\frac{2}{3}\right)^{16}\right)\ldots$$

The expression on the RHS can be rewritten as: $\left(1-\left(-\frac{2}{3}\right)^{\infty}\right)$

$\left(-\frac{2}{3}\right)^{\infty}$ can be approximated as 0.

Hence, $\frac{5}{3}M = 1$ OR $M = 3/5$

5. $a_1 = \frac{1}{2\times5} = \frac{1}{3}\left(\frac{1}{2}-\frac{1}{5}\right)$

$$a_2 = \frac{1}{5\times8} = \frac{1}{3}\left(\frac{1}{5}-\frac{1}{8}\right)$$

.

.

.

.

$$a_{100} = \frac{1}{299\times302} = \frac{1}{3}\left(\frac{1}{299}-\frac{1}{302}\right)$$

The required sum = $a_1 + a_2 + a_3 + a_4 + a_5 \ldots\ldots + a_{100}$

$$= \frac{1}{3}\left(\frac{1}{2}-\frac{1}{5}\right)+\frac{1}{3}\left(\frac{1}{5}-\frac{1}{8}\right)+\ldots\ldots+\frac{1}{3}\left(\frac{1}{299}-\frac{1}{302}\right)=$$

$$= \frac{1}{3}\left(\frac{1}{2}-\frac{1}{302}\right)=\frac{1}{3}\times\frac{300}{604}=\frac{100}{604}=\frac{25}{151}$$

Hence, option (a) is correct.

6. $$S=\left(\frac{\sqrt{3}}{\sqrt{3}+3}+\frac{\sqrt{5}}{\sqrt{15}+5}+\frac{\sqrt{7}}{\sqrt{35}+7}+\ldots+\frac{\sqrt{25}}{\sqrt{575}+25}\right)$$

$$S=\left(\frac{\sqrt{3}}{\sqrt{3}\left(1+\sqrt{3}\right)}+\frac{\sqrt{5}}{\sqrt{5}\left(\sqrt{3}+\sqrt{5}\right)}+\frac{\sqrt{7}}{\sqrt{7}(\sqrt{5}+\sqrt{7}})+\ldots+\frac{\sqrt{25}}{\sqrt{25}(\sqrt{23}+\sqrt{25})}\right)$$

$$S=\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots+\frac{1}{\sqrt{23}+\sqrt{25}}$$

$$S=\frac{\sqrt{3}-1}{3-1}+\frac{\sqrt{5}-\sqrt{3}}{5-3}+\frac{\sqrt{7}-\sqrt{5}}{7-5}+\ldots+\frac{\sqrt{25}-\sqrt{23}}{\sqrt{25}+\sqrt{23}}$$

$$=\frac{\sqrt{25}-1}{2}=\frac{5-1}{2}=2.$$

7. Any expression of the sort: $1/[(n! + (n + 1)!]$ can be rewritten as

$$=\frac{1}{n!(n+2)}=\frac{(n+1)}{(n+2)!}=\frac{1}{(n+1)!}-\frac{1}{(n+2)!}$$

Put n = 1, 2, 3, ...19 to get:

$$\left[\frac{1}{2!}-\frac{1}{3!}\right]+\left[\frac{1}{3!}-\frac{1}{4!}\right]+\ldots+\left[\frac{1}{19!}-\frac{1}{20!}\right]=\frac{1}{2!}-\frac{1}{20!}$$

Hence, option (d) is correct.

8. The series can be simplified as

$$\frac{1}{(2^2-1)}+\frac{1}{(4^2-1)}+\frac{1}{(6^2-1)}+ \dots\dots\dots\dots + \frac{1}{(20^2-1)}$$

$$=\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+ \dots\dots\dots\dots + \frac{1}{399}$$

$$=\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+ \dots\dots\dots\dots + \frac{1}{19\times21}$$

$$=\frac{1}{2}\left(1-\frac{1}{3}\right)+\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)+\frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}\right)+ \dots\dots\dots\dots$$

$$+\frac{1}{2}\left(\frac{1}{19}-\frac{1}{21}\right)$$

$=\frac{1}{2}-\frac{1}{42}=\frac{10}{21}$. Hence, option (c) is correct.

9. $S=\frac{2}{1!}+\frac{3}{2!}+\frac{6}{3!}+\frac{11}{4!}+\frac{18}{5!}+\dots$

Let T_n be the n_{th} term of S.

We can observe that $T_n=\frac{n^2-2n+3}{n!}$

$$S=\sum_{n=1}^{\infty}\frac{n^2}{n!}-\frac{2n}{n!}+\frac{3}{n!}$$

$S=\sum\frac{n^2}{n!}-\sum\frac{2n}{n!}+\sum\frac{3}{n!}$. Evaluating these components individually, we see that:

$$\sum\frac{n^2}{n!}=\sum\frac{n}{(n-1)!}=\sum\frac{n-1+1}{(n-1)!}=\sum_{n=2}^{\infty}\frac{1}{(n-2)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}=e+e=2e$$

$$\left(Since, e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots\infty\right)$$

$$\sum_{n=1}^{\infty}\frac{2n}{n!}=2\sum_{n=1}^{\infty}\frac{1}{(n-1)!}=2e$$

$$\sum_{n=1}^{\infty}\frac{3}{n!}=3\sum_{n=1}^{\infty}\frac{1}{(n)!}=3(e-1)$$

$$S=\sum_{n=1}^{\infty}\frac{n^2}{n!}-\frac{2n}{n!}+\frac{3}{n!}=2e-2e+3e-3=3(e-1)$$

Hence, option (c) is correct.

10. $S=\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\ldots+\frac{1}{99^2-1}$

$$=\frac{1}{2\times4}+\frac{1}{4\times6}+\frac{1}{6\times8}+\ldots+\frac{1}{98\times100}$$

$$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{4}-\frac{1}{6}+\frac{1}{6}-\frac{1}{8}+\ldots+\frac{1}{98}-\frac{1}{100}\right)$$

$=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{100}\right)=\frac{49}{200}$. Hence, option (a) is correct.

11. Consider S = 1/6 + 1/12 + 1/20........... + 1/90

= 1/2 × 3 + 1/3 × 4 + 1/4 × 5 + ... + 1/9 × 10

= 1/2 – 1/3 + 1/3 – 1/4 + 1/4 – 1/5 + ... + 1/9 – 1/10

= 1/2 – 1/10 = 0.4. Hence, option (a) is correct.

12. $S=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\ldots\frac{1}{9900}=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ \ldots+\frac{1}{99}-\frac{1}{100}=1-\frac{1}{100}=0.99$

13. $S=\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\ldots\frac{1}{195}=[\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\ldots]$

$$=\frac{1}{2}\left[1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}+\ldots+\frac{1}{13}-\frac{1}{15}\right]$$

$$=\frac{1}{2}\times\left[1-\frac{1}{15}\right]=\frac{7}{15}$$

Hence, option (a) is correct.

14. $S=\frac{1}{12}+\frac{1}{60}+\frac{1}{140}+\frac{1}{252}+\ldots+\frac{1}{1020}=\frac{1}{4}\left[\frac{4}{12}+\frac{4}{60}+\right.$

$\left.\frac{4}{140}+\frac{4}{252}+\ldots+\frac{4}{1020}\right]=\frac{1}{4}\left[\frac{1}{2}-\frac{1}{6}+\frac{1}{6}-\frac{1}{10}+\frac{1}{10}-\frac{1}{14}\right.$

$\left.+\frac{1}{14}-\frac{1}{18}+\ldots+\frac{1}{30}-\frac{1}{34}\right]=\frac{1}{4}\left[\frac{1}{2}-\frac{1}{34}\right]=\frac{1}{4}\times\frac{32}{68}=\frac{8}{68}$

$=\frac{2}{17}$

15. $S=\frac{1}{4}+\frac{1}{28}+\frac{1}{70}+\frac{1}{130}+\ldots+\frac{1}{700}=\frac{1}{3}\left[\frac{3}{4}+\frac{3}{28}+\frac{3}{70}+\right.$

$\left.\ldots+\frac{3}{700}\right]=\frac{1}{3}\left[1-\frac{1}{4}+\frac{1}{4}-\frac{1}{7}+\frac{1}{7}-\frac{1}{10}+\ldots+\frac{1}{25}-\frac{1}{28}\right]$

$=\frac{1}{3}\left[1-\frac{1}{28}\right]=\frac{9}{28}$

16. $S=\frac{1}{9}+\frac{1}{153}+\frac{1}{425}+\frac{1}{825}=\frac{1}{8}\left[\frac{8}{9}+\frac{8}{153}+\frac{8}{425}+\frac{8}{825}\right]$

$=\frac{1}{8}\left[1-\frac{1}{9}+\frac{1}{9}-\frac{1}{17}+\frac{1}{17}-\frac{1}{25}+\frac{1}{25}-\frac{1}{33}\right]=\frac{1}{8}\left[1-\frac{1}{33}\right]$

$=\frac{4}{33}$

Note on how to think in the questions 11 to 16: Observe the progression of the denominators and how the denominators can be constructed as a product of 2 numbers. For example, in question 11: ½ + 1/6 + 1/12, we can see that 2 = 1 × 2, 6 = 2 × 3, 12 = 3 × 4, 20 = 4 × 5 and so on. Hence, we rewrite the expression as: 1/2 × 3 + 1/3 × 4 + 1/4 × 5 + ... + 1/9 × 10 = 1/2 – 1/3 + 1/3 – 1/4 + 1/4 – 1/5+.......+1/9 – 1/10 and solve. Question 12 is solved on a similar process and pattern.

In question 13, however, we are given the expression

$$\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\ldots\frac{1}{195}.$$

In such an expression, the denominators being 3, 15, 35 and so on, they can be visualised as: 3 = 1 × 3; 15 = 3 × 5; 35 = 5 × 7 and so on. We can observe that the difference between the multipliers is 2 for each denominator. In this case, when we rewrite the expression as: $\left[\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots\right]$, the next transformation requires as ½ to multiply the expression. Hence, we can rewrite this as:

$$\frac{1}{2}\left[1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}+\ldots+\frac{1}{13}-\frac{1}{15}\right]$$

Similarly, in question 16, looking through the denominators, we see that:

9, 153, 425, etc., are the denominators and hence they can be written as: 9 = 1 × 9; 153 = 9 × 17; 425 = 17 × 25 and so on. Hence, when we do the last step in the question, we use a 1/8 to multiply the expression as we see in:

$$\frac{1}{8}\left[1-\frac{1}{9}+\frac{1}{9}-\frac{1}{17}+\frac{1}{17}-\frac{1}{25}+\frac{1}{25}-\frac{1}{33}\right]$$

TRAINING GROUND FOR BLOCK I

HOW TO THINK ON PROBLEMS ON BLOCK I?

Problem 1: A number x is such that it can be expressed as $a + b + c = x$ where a, b, and c are factors of x. How many numbers below 200 have this property?

(a) 31

(b) 32

(c) 33

(d) 5

Solution: In order to think about this question, you need to think about whether the number can be divided by the initial numbers below the square root like 2, 3, 4, 5… and so on.

Let us say, if we think of a number that is not divisible by 2, in such a case if we take the number to be divisible by 3, 5 and 7, then the largest factors of x that we will get would be $\frac{x}{3}, \frac{x}{5}$ and $\frac{x}{7}$.

Even in this situation, the percentage value of these factors as a percentage of x would only be: for $\frac{x}{3}$ = 33.33%, $\frac{x}{5}$ = 20% and $\frac{x}{7}$ = 14.28%

Hence, if we try to think of a situation where $a = \frac{x}{3}$, $b = \frac{x}{5}$, and $c = \frac{x}{7}$ the value of $a + b + c$ would give us only (33.33% + 20% + 14.28%) = 67.61% of x, which is not equal to 100% $\frac{x}{7}$.

Since the problem states that $a + b + c = x$, the value of $a + b + c$ should have added up to 100% of x.

The only situation, for this to occur would be if

$a = \frac{x}{2}$ = 50% of x $b = \frac{x}{3}$ = 33.33% of x and

$c = \frac{x}{6}$ = 16.66% of x

This means that 2, 3 and 6 should divide x necessarily. In other words x

should be a multiple of 6.The multiples of 6 below 200 are 6, 12, 18 ... 198, a total of 33 numbers.

Hence, the correct answer is *c*.

Problem 2. Find the sum of all 3 digit numbers that leave a remainder of 3 when divided by 7.

(a) 70821

(b) 60821

(c) 50521

(d) 80821

Solution: In order to solve this question, you need to visualise the series of numbers, which satisfy this condition of the remainder 3 when divided by 7.

The first number in 3 digits for this condition is 101 and the next will be 108, followed by 115, 122 ...

This series would have its highest value in 3 digits as 997.

The number of terms in this series would be 129 (using the logic that for any A.P., the number of terms is given by $\frac{D}{d} + 1$).

Also the average value of this series is the average of the first and the last term i.e. the average of 101 and 997. Hence, the required sum = 549 × 129 = 70821.

Hence, the correct option is (a).

Problem 3. How many times would the digit 6 be used in numbering a book of 639 pages?

(a) 100

(b) 124

(c) 150

(d) 164

Solution: In order to solve this question, you should count the digit 6 appearing in units digit, separately from the instances of the digit 6 appearing in the tens place and appearing in the hundreds place.

When you want to find out the number of times 6 appears in the unit digit, you will have to make a series as follows: 6, 16, 26, 36 … 636.

It should be evident to you that the above series has 64 terms because it starts from 06, 16, 26 … and continues till 636. The digit 6 will appear once in the unit digit for each of these 64 numbers.

Next you need to look at how many times the digit 6 appears in the ten's place.

In order to do this, we will need to look at instances when 6 appears in the tens place. These will be in 6 different ranges 60s, 160s, 260s, 360s, 460s, 560s and in each of these ranges there are 10 numbers each with exactly one instance of the digit 6 in the tens place, a total of 60 times.

Lastly, we need to look at the number of instances where 6 appears in the hundreds place. For this, we need to form the series 600, 601, 602, 603 … 639. This series will have 40 numbers each with exactly one instance of the digit 6 appearing in the hundreds place.

Therefore, the required answer would be 64 + 60 + 40 = 164.

Hence, option (d) is correct.

Problem 4. A number written in base 3 is 1001001001001001001. What will be the value of this number in base 27?

(a) 999999

(b) 900000

(c) 989999

(d) 888888

The number can be visualised as:

1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
3^{17}	3^{16}	3^{15}	3^{14}	3^{13}	3^{12}	3^{11}	3^{10}	3^9	3^8	3^7	3^6	3^5	3^4	3^3	3^2	3^1	3^0

Now in the base 27, we can visualise this number as:

—	—	—	—	—	—
27^5	27^4	27^3	27^2	27^1	27^0

When you want to write this number in base 27 the unit digit of the number will have to account for the value of 32 in the above number. Since the unit digit of the number in base 27 will correspond to 27^0 we would have to use 27^0, 9 times to make a value 3^2. The number would now become:

—	—	—	—	—	9
27^5	27^4	27^3	27^2	27^1	27^0

Similarly, the number of times that we will have to use 27^1 in order to make the value of 3^5 (or 243) will be $\frac{243}{27}$ = 9. Hence the second last digit of the number will also be 9.

Similarly, to make a value of 3^8 using 27^2, the number of times we will have to use 27^2 will be given by $\frac{3^8}{27^2} = \frac{3^8}{3^6}$ = 9. The number would now look as:

—	—	—	9	9	9
27^5	27^4	27^3	27^2	27^1	27^0

It can be further predicted that each of subsequent 1's in the original number will equal 9 in the number, which is being written in base 27. Since the original number in base 3 has 18 digits, the left most '1' in that number will be covered by a number corresponding to 27^5 in the new number (i.e., 3^{15}). Hence the required number will be a 6-digit number 999999. Hence, option (a) is correct.

Problem 5. Let $x = 1640$, $y = 1728$ and $z = 448$. How many natural numbers are there that divide at least one amongst x, y, z?

(a) 47

(b) 48

(c) 49

(d) 50

Solution: 1640 can be prime factorised as $2^3 \times 5^1 \times 41^1$. This number would have a total of 16 factors.

Similarly, 1728 can be prime factorised as $2^6 \times 3^3$. Hence, it would have 28 factors. While $448 = 2^6 \times 7^1$ would have 14 factors. Thus, there are a total of (16 + 28 + 14) = 58 factors amongst x, y and z.

However, some of these factors must be common between x, y, z. Hence, in order to find the number of natural numbers that would divide at least one amongst x, y, z, we will need to account for double and triple counted numbers amongst these 58 numbers (by reducing the count by 2 for each triple counted number and by reducing the count by 1 for each double counted number).

It can be seen from these 2 standard forms of the numbers that the highest common factors of these 2 numbers is 8. Hence, there is no new number to be subtracted for double counting in this case.

The case of 1640 and 448 is similar because $1640 = 2^3 \times 5^1 \times 41^1$ while $448 = 2^6 \times 7^1$ and HCF = 2^3 and hence they will not give any more numbers as common factors apart from 1, 2, 4 and 8.

Thus, there is no need of adjustment for the pair 1640 and 448.

Finally when we look for 1728 and 448, we realise that the HCF = 2^6 = 64 and hence, the common factors between 448 and 1728 are 1, 2, 4, 8, 16, 32, 64. But we are looking for factors which are common for 1728 and 448 but not common to 1640 to estimate the double counting error for this case.

Hence, we can eliminate the number 1, 2, 4, 8 from this list and conclude that there are only 3 numbers 16, 32 and 64 that divide both 1728 and 448 but do not divide 1640.

If we subtract these numbers once each, from the 50 numbers, we will end up with $50 - 3 \times 1 = 47$.

The complete answer can be visualised as $16 + 28 + 14 - 4 \times 2 - 3 \times 1 = 47$.

Hence, option (a) is correct.

Problem 6. How many times will the digit 6 be used when we write all the six digit numbers?

(a) 5,50,000

(b) 5,00,000

(c) 4,50,000

(d) 4,00,000

Solution: When we write all 6-digit numbers, we will have to write all the numbers from 100000 to 999999, a total of 9 lac numbers in 6 digits without omitting a single number. There will be a complete symmetry and balance in the use of all the digits. However, the digit 0 is not going to be used in the leftmost place.

Using this logic, we can visualise that when we write 9 lakh, 6-digit numbers, the units place, tens place, hundreds place, thousands place, ten thousands place and lakh place ' Each of these places will be written 9 lakh times. Thinking about the units place, we can think as follows: In writing, the units place 9 lakh times (once for every number)

we will be using the digit 0, 1, 2, 3 … 9 an equal number of times. Hence, any particular digit like 6 would get used in the units digit a total number of 90000 times (9 lakh/10). The same logic will continue for tens, hundreds, thousands and ten thousands, i.e. the digit 6 will be used (9 lakh/10) = 90000 times in each of these places (Note here that we are dividing by 10 because we have to equally distribute 9 lakh digits amongst the ten digits 0, 1, 2, 3,4, 5, 6, 7, 8, 9).

Finally, for the lakh place since "0" is not be used in the lakh place as it is the leftmost digit of the number, the number of times the digit 6 will be used would be 9 lakh/9 = 1 lakh. Hence, the next time you are solving the problem of this type, you should solve directly using $\frac{9\text{ lakh}}{9}+\frac{9\text{ lakh}}{10}\times 5$ = 5,50,000.

Hence, option (a) is the correct answer.

REVIEW CAT SCAN

REVIEW CAT Scan 1

1. In 1936, I was as old as the number formed by the last two digits of my year of birth. Find the date of birth of my father who is 25 years older to me.

(a) 1868

(b) 1893

(c) 1902

(d) 1900

(e) Can not be determined

2. Find the total number of integral solutions of the equation $(407)x - (ddd)y = 2589$, where '*ddd*' is a three-digit number.

(a) 0

(b) 1

(c) 2

(d) 3

(e) Can not be determined

3. Find the digit at the ten's place of the number $N = 7^{281} \times 3^{264}$.

(a) 0

(b) 1

(c) 6

(d) 5

(e) None of these

4. Raju went to a shop to buy a certain number of pens and pencils. Raju calculated the amount payable to the shopkeeper and offered that amount to him. Raju was surprised when the shopkeeper returned him ₹24 as balance. When he came back home, he realised that the shopkeeper had actually transposed the number of pens with the number of pencils. Which of the following is certainly an invalid statement?

(a) The number of pencils that Raju wanted to buy was 8 more than the number of pens.

(b) The number of pens that Raju wanted to buy was 6 less than the number of pencils.

(c) A pen cost ≤ ₹4 more than a pencil.

(d) None of these.

5. HCF of 384 and a^5b^2 is $16ab$. What is the correct relation between a and b?

(a) $a = 2b$

(b) $a + b = 3$

(c) $a - b = 3$

(d) $a + b = 5$

6. In ancient India, 0 to 25 years of age was called Brahmawastha and 25 to 50 was called Grahastha. I am in Grahastha and my younger brother is also in Grahastha such as the difference in our ages is 6 years and both of our ages are prime numbers. Also twice my brother's age is 31 more than my age. Find the sum of our ages.

(a) 80

(b) 68

(c) 70

(d) 71

7. Volume of a cube with integral sides is the same as the area of a square with integral sides. Which of these can be the volume of the cube formed by using the square and its replicas as the 6 faces?

(a) 19683

(b) 512

(c) 256

(d) Both (a) and (b)

REVIEW CAT Scan 2

1. Let A be a two-digit number and B be another two-digit number formed by reversing the digits of A. If $A + B +$ (Product of digits of the number A) = 145, then what is the sum of the digits of A?

(a) 9

(b) 10

(c) 11

(d) 12

2. When a two-digit number N is divided by the sum of its digits, the result is Q. Find the minimum possible value of Q.

(a) 10

(b) 2

(c) 5.5

(d) 1.9

3. A one-digit number, which is the ten's digit of a two digit number *X*, is subtracted from *X* to give *Y* which is the quotient of the division of 999 by the cube of a number. Find the sum of the digits of *X*.

(a) 5

(b) 7

(c) 6

(d) 8

4. After Yuvraj hit 6 sixes in an over, Geoffery Boycott commented that Yuvraj just made 210 runs in the over. Harsha Bhogle was shocked and he asked Geoffery which base system was he using? What must have been Geoffery's answer?

(a) 9

(b) 2

(c) 5

(d) 4

5. Find the ten's digit of the number 7^{2010}.

(a) 0

(b) 1

(c) 2

(d) 4

6. Find the HCF of 481 and the number '*aaa*' where '*a*' is a number between 1 and 9 (both included).

(a) 73

(b) 1

(c) 27

(d) 37

7. The number of positive integer valued pairs (x, y), satisfying $4x - 17y = 1$ and $x < 1000$ is

(a) 59

(b) 57

(c) 55

(d) 58

REVIEW CAT Scan 3

1. Let a, b, c be distinct digits. Consider a two digit number '*ab*' and a three digit number '*ccb*', both defined under the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$, then the value of b is

(a) 1

(b) 0

(c) 5

(d) 6

2. The remainder when 7^{84} is divided by 342 is

(a) 0

(b) 1

(c) 49

(d) 341

3. Let x, y and z be distinct integers; x and y are odd and positive, and z is even and positive. Which one of the following statements can not be true?

(a) $(x - z)^2 y$ is even

(b) $(x - z)y^2$ is odd

(c) $(x - y)y$ is even

(d) $(x - y)^2 z$ is even

4. A boy starts adding consecutive natural numbers starting with 1. After some time, he reaches a total of 1000 when he realises that he has made the mistake of double counting 1 number. Find the number double counted.

(a) 44

(b) 45

(c) 10

(d) 12

5. In a number system, the product of 44 and 11 is 1034. The number 3111 of this system, when converted to decimal number system, becomes

(a) 406

(b) 1086

(c) 213

(d) 691

6. Ashish is given ₹158 in one rupee denominations. He has been asked to allocate them into a number of bags such that any amount required between ₹1 and ₹158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required?

(a) 11

(b) 12

(c) 13

(d) None of these

7. Find the number of 6-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 once such that the 6-digit number is divisible by its unit digit.

(a) 648

(b) 528

(c) 728

(d) 128

REVIEW CAT Scan 4

1. Which is the highest 3-digit number that divides the number 11111....1(27 times) perfectly without leaving any remainder?

(a) 111

(b) 333

(c) 666

(d) 999

2. $W1, W2, \dots, W7$ are 7 positive integral values such that by attaching the coefficients of +1, 0 and −1 to each value available and adding the resultant values, any number from 1 to 1093 (both included) could be formed. If $W1, W2, \dots, W7$ are in ascending order, then what is the value of W3?

(a) 10

(b) 9

(c) 0

(d) 1

3. What is the unit digit of the number $63^{25} + 25^{63}$?

(a) 3

(b) 5

(c) 8

(d) 2

4. Find the remainder when $(2222^{5555} + 5555^{2222})$ is divided by 7.

(a) 1

(b) 0

(c) 2

(d) 5

5. What is the number of nines used in numbering a 453 page book?

(a) 86

(b) 87

(c) 84

(d) 85

6. How many four digit numbers are divisible by 5 but not by 25?

   (a) 2000

   (b) 8000

   (c) 1440

   (d) 9999

7. The sum of two integers is 10 and the sum of their reciprocals is 5/12. What is the value of larger of these integers?

   (a) 7

   (b) 5

   (c) 6

   (d) 4

REVIEW CAT Scan 5

1. Saurabh was born in 1989. His elder brother Siddhartha was also born in the 1980's such that the last two digits of his year of birth form a prime number P. Find the remainder when (P^2 + 11) is divided by 5.

   (a) 0

   (b) 1

   (c) 2

   (d) 3

2. The HCF of x and y is H. Find the HCF of $(x - y)$ and $(x^3 + y^3)/(x^2 - xy + y^2)$.

(a) $H - 1$

(b) H2

(c) H

(d) $H + 1$

3. Four bells toll together at 9:00 A.M. They toll after 7, 8, 11 and 12 seconds, respectively. How many times will they toll together again in the next three hours?

(a) 3

(b) 5

(c) 6

(d) 9

4. What power of 210 will exactly divide 142?

(a) 22

(b) 11

(c) 34

(d) 33

5. Find the total numbers between 122 and 442 that are divisible by 3 but not by 9.

(a) 70

(b) 71

(c) 72

(d) 73

6. If 146! is divisible by $6n$, then find the maximum value of n.

(a) 74

(b) 75

(c) 76

(d) 70

7. If we add the square of the digit in the tens place of a positive two-digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number.

(a) 18

(b) 39

(c) 49

(d) 28

REVIEW CAT Scan 6

1. Find the smallest natural number n such that $(n + 1)n[(n - 1)!]$ is divisible by 990.

(a) 2

(b) 4

(c) 10

(d) 11

2. If x, y and z are odd, even and odd, respectively, then $(x^2 - yz^2 + y^3)$ and $(x^2 + y^2 + z^2)$ are respectively

(a) Odd and odd

(b) Even and odd

(c) Odd and even

(d) Odd and odd

3. A two digit number n has its digits reversed to form another two digit number M. What could be the unit digit of M, if product of M and N is 574?

(a) 1

(b) 3

(c) 6

(d) 9

4. For what relation between b and c is the number $abcacb$ divisible by 7, if $b > c$?

(a) $b + c = 7$

(b) $b = c + 7$

(c) $2bc = 7$

(d) $c = 7b$

5. What is the remainder when a^6 is divided by $(a + 1)$?

(a) $a + 1$

(b) a

(c) 0

(d) 1

6. What is the last digit of 62^43^54^65^76^87?

(a) 2

(b) 4

(c) 6

(d) 8

7. $N = 99^3 - 36^3 - 63^3$, how many factors does N have?

(a) 51

(b) 96

(c) 128

(d) 192

REVIEW CAT Scan 7

1. Find the highest power of 2 in 1! + 2! + 3! + 4! .... 600!

(a) 1

(b) 494

(c) 0

(d) 256

2. 100! is divisible by 160^n...what is the maximum integral value of n?

(a) 19

(b) 24

(c) 26

(d) 28

3. What is the sum of the digits of the decimal form of the product 2^{999} * 5^{1001}?

(a) 2

(b) 4

(c) 5

(d) 7

4. What is the remainder when 1*1 + 11*11 + 111*111 + 1111*1111 +......+ (2001 times 1)*(2001 times 1) is divided by 100 ?

(a) 99

(b) 22

(c) 01

(d) 21

5. What is the remainder when 789456123 is divided by 999?

(a) 123

(b) 369

(c) 963

(d) 189

6. What is the total number of the factors of 16!

(a) 2016

(b) 1024

(c) 3780

(d) 5376

7. Find the sum of the first 125 terms of the sequence 1, 2, 1, 3, 2, 1, 4, 3, 2, 1, 5, 4, 3, 2....

(a) 616

(b) 460

(c) 750

(d) 720

REVIEW CAT Scan 8

1. Umesh purchased a Tata Nano recently, but the faulty car odometre of Tata Nano proceeds from digit 4 to digit 6, always skipping the digit 5, regardless of position. If the odometre now reads 003008 (starting with 000000), how many km has the Nano actually travelled?

(a) 2100

(b) 1999

(c) 2194

(d) 2195

2. What is the number of consecutive zeroes in the end of 1000!?

(a) 248

(b) 249

(c) 250

(d) 251

3. Mr. Ramlal lived his entire life during the 1800s. In the last year of his life, Ramlal stated: Once I was x years old in the year x^2. He was born in the year

(a) 1822

(b) 1851

(c) 1853

(d) 1806

4. Find the unit's digit of LCM of $13^{501} - 1$ and $13^{501} + 1$.

(a) 2

(b) 4

(c) 5

(d) 8

5. If you were to add all odd numbers between 1 and 2007 (both inclusive), the result would be

(a) A perfect square

(b) divisible by 2008

(c) Multiple of 251

(d) all of these

6. Find the remainder when $971(30^{99} + 61^{100}) * (1148)^{56}$ is divided by 31.

(a) 25

(b) 0

(c) 11

(d) 21

7. What is the remainder when 2^{100} is divided by 101?

(a) 1

(b) 100

(c) 0

(d) 99

REVIEW CAT Scan 9

1. Find the last two digits of 2^{134}.

(a) 04

(b) 84

(c) 24

(d) 64

2. Find the remainder when $(10^3 + 9^3)^{1000}$ is divided by 12^3.

(a) 01

(b) 11

(c) 1001

(d) 1727

3. The number of factors of the number 3000 are

(a) 16

(b) 32

(c) 24

(d) 28

4. If *N*! has 73 zeroes at the end, then find the value of *N*.

(a) 295

(b) 300

(c) 290

(d) Not possible

5. Let a, b, c be distinct digits. Consider a two digit number '*ab*' and a three digit number '*ccb*', both defined under the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$, then the value of b is:

(a) 1

(b) 0

(c) 5

(d) 6

6. The remainder when 7^{84} is divided by 342 is:

(a) 0

(b) 1

(c) 49

(d) 341

7. If $n = 1 + x$, where x is the product of four consecutive positive integers, then which of the following is/are true?

A. n is odd

B. n is prime

C. n is a perfect square

(a) A and C only

(b) A and B only

(c) A only

(d) None of the above.

ANSWER KEY

REVIEW CAT Scan 1

1. (b)
2. (a)
3. (c)
4. (d)
5. (d)

6. (a)
7. (d)

REVIEW CAT Scan 2

1. (c)
2. (d)
3. (a)
4. (d)
5. (d)
6. (d)
7. (a)

REVIEW CAT Scan 3

1. (a)
2. (b)
3. (a)
4. (c)
5. (a)
6. (d)
7. (a)

REVIEW CAT Scan 4

1. (d)
2. (b)
3. (c)
4. (b)
5. (d)
6. (c)
7. (c)

REVIEW CAT Scan 5

1. (a)
2. (c)
3. (b)
4. (a)
5. (b)
6. (d)
7. (c)

REVIEW CAT Scan 6

1. (c)
2. (c)
3. (a)
4. (b)
5. (d)
6. (a)
7. (b)

REVIEW CAT Scan 7

1. (c)
2. (a)
3. (d)
4. (c)
5. (b)

6. (d)
7. (c)

REVIEW CAT Scan 8

1. (c)
2. (b)
3. (d)
4. (b)
5. (d)
6. (b)
7. (a)

REVIEW CAT Scan 9

1. (b)
2. (a)
3. (b)
4. (d)
5. (a)
6. (b)
7. (a)

TASTE OF THE EXAMS—BLOCK I

CAT

1. Let a, b, c be distinct digits. Consider a two digit number 'ab' and a three digit number 'ccb', both defined under the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$, then the value of b is: **(CAT 1999)**

   (a) 1

   (b) 0
   (c) 5

   (d) 6

2. The remainder when 7^{84} is divided by 342 is: **(CAT 1999)**

   (a) 0

   (b) 1

   (c) 49

   (d) 341

3. If $n = 1 + x$, where x is the product of four consecutive positive integers, then which of the following is/are true? **(CAT 1999)**

   A. n is odd B. n is prime

   C. n is a perfect square

   (a) A and C only

   (b) A and B only

(c) A only

(d) None of these

4. For two positive integers a and b, define the function h(a, b) as the greatest common factor (gcf) of a, b. Let A be a set of n positive integers. $G(A)$, the gcf of the elements of set A is computed by repeatedly using the function h. The minimum number of times h is required to be used to compute G is: **(CAT 1999)**

(a) $12n$

(b) $(n - 1)$

(c) n

(d) None of these

5. If n_2 = 123456787654321, what is n?(CAT 1999)

(a) 12344321

(b) 1235789

(c) 11111111

(d) 1111111

Directions for Questions 6 to 8: These questions are based on the situation given below.

There are 50 integers $a_1, a_2, \ldots, a_{50}$, not all of them are necessarily different. Let the greatest integer of these 50 integers be referred to as G

and the smallest integer be referred to as L. The integers a_1–a_{24} form sequence S_1, and the rest form sequence S_2. Each member of S_1 is less than or equal to each member of S_2.

6. If we change the sign of all values in S_1, while those in S_2 remain unchanged, which of the following statements is true? **(CAT 1999)**

(a) Every member of S_1 is greater than or equal to every member of S_2.

(b) G is in S_1

(c) If all numbers originally in S_1 and S_2 had the same sign, then after the change of sign, the largest number of S_1 and S_2 will be in S_1.

(d) None of these

7. Elements of S_1 are in ascending order, and those of S_2 are in descending order, a_{24} and a_{25} are interchanged. Then, which of the following statements is true? **(CAT 1999)**

(a) S_1 continues to be in ascending order

(b) S_2 continues to be in descending order

(c) S_1 continues to be in ascending order and S_2 in descending order

(d) None of these

8. Every element of S_1 is made greater than or equal to every element of S_2 by adding to each element of S_1 an integer x. Then, x cannot be less than **(CAT 1999)**

(a) 210

(b) The smallest value of S_2

(c) The largest value of S_2

(d) $(G - L)$

9. Let D be a recurring decimal of the form, $D = 0.\ a_1\ a_2\ a_1\ a_2\ a_1\ a_2$ ...., where digits a_1 and a_2 lie between 0 and 9. Further, at most one of them is zero. Then which of the following numbers necessarily produces an integer, when multiplied by D. **(CAT 2000)**

(a) 18

(b) 108

(c) 198

(d) 288

10. Let S be the set of integers x such that: **(CAT 2000)**

(i) $100 \leq x \leq 200$

(ii) x is odd

(iii) x is divisible by 3 but not by 7

How many elements does S contain?

(a) 16

(b) 12

(c) 11

(d) 13

11. Let x, y and z be distinct integers, that are odd and positive. Which one of the following statements cannot be true? **(CAT 2000)**

(a) xyz^2 is odd.

(b) $(x - y)^2 z$ is even.

(c) $(x + y - z)^2 (x + y)$ is even.

(d) $(x - y)(y + z)(x + y - z)$ is odd.

12. Let S be the set of prime numbers greater than or equal to 2 and less than 100. [Multiply all elements of S. With how many consecutive zeros will the product end?] **(CAT 2000)**

(a) 1

(b) 4

(c) 5

(d) 10

13. Let $N = 1421 \times 1423 \times 1425$, what is the remainder when N is divided by 12? **(CAT 2000)**

(a) 0

(b) 9

(c) 3

(d) 6

14. The integers 34041 and 32506 when divided by a three-digit integer n leave the same remainder. What is n? **(CAT 2000)**

(a) 289

(b) 367

(c) 453

(d) 307

15. Let $N = 55^3 + 17^3 - 72^3$, N is divisible by: **(CAT 2000)**

(a) both 7 and 13

(b) both 3 and 13

(c) both 17 and 7

(d) both 3 and 17

16. *ABCDEFGH* is a regular octagon. *A* and *E* are opposite vertices of the octagon. A frog starts jumping from vertex to vertex, beginning from *A*. From any vertex of the octagon except *E*, it may jump to either of the two adjacent vertices. When it reaches *E*, the frog stops and stays there. Let a_n be the number of distinct paths of exactly n jumps ending in *E*. Then, what is the value of a_{2n-1}? **(CAT 2000)**

(a) Zero

(b) Four

(c) $2n - 1$

(d) Cannot be determined

17. Convert the number 1982 from base 10 to base 12. The result is: **(CAT 2000)**

(a) 1182

(b) 1912

(c) 1192

(d) 1292

18. Let x, y and z be distinct integers, x and y are odd and positive, and z is even and positive. Which one of the following statements can not be true? **(CAT 2001)**

(a) $(x - z)^2 y$ is even

(b) $(x - z)y^2$ is odd

(c) $(x - y)y$ is even

(d) $(x - y)^2 z$ is even

19. A boy starts adding consecutive natural numbers starting with one. After reaching a total of 1000, he realises that he has made the mistake of double counting one number. Find the number double counted. **(CAT 2001)**

(a) 44

(b) 45

(c) 10

(d) 12

20. x and y are real numbers satisfying the conditions $2 < x < 3$ and $-7 < y < -1$. Which of the following expressions will have the least value? **(CAT 2001)**

(a) x^2y

(b) xy^2

(c) $5xy$

(d) None of these

21. In a number system the product of 44 and 11 is 1034. The number 3111 of this system, when converted to the decimal number system, becomes **(CAT 2001)**

(a) 406

(b) 1086

(c) 213

(d) 691

22. Raju has 128 boxes with him. He can put a minimum of 120 oranges and a maximum of 144 in a box. Then the least number of boxes which will have the same number of oranges is: **(CAT 2001)**

(a) 5

(b) 103

(c) 3

(d) 6

23. Three friends, returning from a movie, stopped to eat at a restaurant. After dinner, they paid their bill and noticed a bowl of mints at the front counter. Sita took one-third of the mints, but returned four because she had a momentary pang of guilt. Fatima then took one-fourth of what was left but returned three for similar reasons. Eswari then took half of the remainder but threw two back into the bowl. The bowl had only 17 mints left when the raid was over. How many mints were originally in the bowl? **(CAT 2001)**

(a) 38

(b) 31

(c) 41

(d) None of these

24. Anita had to do a multiplication. Instead of taking 35 as one of the multipliers, she took 53. As a result, the product went up by 540. What is the new product? **(CAT 2001)**

(a) 1050

(b) 540

(c) 1440

(d) 1590

25. In a four-digit number, the sum of the first two digits is equal to that of the last two digits. The sum of the first and last digits is equal to the third digit. Finally, the sum of the second and fourth digits is twice the sum of the other two digits. What is the third digit of the number? **(CAT 2001)**

(a) 5

(b) 8

(c) 1

(d) 4

26. A red light flashes three times per minute and a green light flashes five times in two minutes at regular intervals. If both lights start flashing at

the same time, how many times do they flash together in each hour? **(CAT 2001)**

(a) 30

(b) 24

(c) 20

(d) 60

27. Ashish is given ₹158 in one rupee denominations. He has been asked to allocate them into a number of bags such that any amount required between ₹1 and ₹158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required? **(CAT 2001)**

(a) 11

(b) 12

(c) 13

(d) None of these

28. For a Fibonacci sequence, from the third term onwards, each term in the sequence is the sum of the previous two terms in that sequence. If the difference of squares of seventh and sixth terms of this sequence is 517, what is the tenth term of this sequence? **(CAT 2001)**

(a) 147

(b) 76

(c) 123

(d) Cannot be determined

29. In some code, letters a, b, c, d and e represent numbers 2, 4, 5, 6 and 10. We don't know which letter represents which number. Consider the following relationships: **(CAT 2001)**

(i) $a + c = e$

(ii) $b - d = d$

(iii) $e + a = b$

Which statement below is true?

(a) $b = 4, d = 2$

(b) $a = 4, e = 6$

(c) $b = 6, e = 2$

(d) $a = 4, c = 6$

30. m is the largest positive integer such that $n > m$. Also, it is known that $n^3 - 7n^2 + 11n - 5$ is positive. Then, the possible value for m is: **(CAT 2001)**

(a) 4

(b) 5

(c) 8

(d) None of these

31. Let b be a positive integer and $a = b^2 - b$. If $b \geq 4$, then $a^2 - 2a$ is divisible by **(CAT 2001)**

(a) 15

(b) 20

(c) 24

(d) None of these

32. A change making machine contains 1 rupee, 2 rupee and 5 rupee coins. The total number of coins is 300. The amount is ₹960. If the number of 1 rupee coins and the number of 2 rupee coins are interchanged, the value comes down by ₹40. The total number of 5 rupee coins is: **(CAT 2001)**

(a) 100

(b) 140

(c) 60

(d) 150

33. $7^{6n} - 6^{6n}$, where n is an integer > 0, is divisible by **(CAT 2002)**

(a) 13

(b) 127

(c) 559

(d) None of these

Directions for Questions 35 and 37: Answer the questions independently of each other.

34. After the division of a number successively by 3, 4 and 7, the remainders obtained are 2, 1 and 4 respectively What will be the remainder if 84 divides the number? **(CAT 2002)**

(a) 80

(b) 76

(c) 41

(d) 53

35. Three pieces of cakes weighing 4 1/2 lbs, 6 3/4 lbs and 7 1/5 lbs respectively are to be divided into parts of equal weights Further, each part must be as heavy as possible If one such part is served to each guest, then what is the maximum number of guests that could be entertained? **(CAT 2002)**

(a) 54

(b) 72

(c) 20

(d) None of these

36. At a bookstore, "MODERN BOOK STORE" is flashed using neon lights The words are individually flashed at intervals of 2 1/2, 4 1/4, 5 1/8 seconds respectively and each word is put off after a second The least time after which the full name of the bookstore can be read again is: **(CAT 2002)**

(a) 49.5 seconds

(b) 73.5 seconds

(c) 1744.5 seconds

(d) 855 seconds

37. When 2^{256} is divided by 17, the remainder would be **(CAT 2002)**

(a) 1

(b) 16

(c) 14

(d) None of these

38. A child was asked to add first few natural numbers (that is, 1 + 2 + 3 …) as long as his patience permitted As he stopped, he gave the sum as 575 When the teacher declared the result wrong the child discovered he had missed one number in the sequence during addition The number he missed was: **(CAT 2002)**

(a) less than 10

(b) 10

(c) 15

(d) more than 15

39. A car rental agency has the following terms If a car is rented for 5 hours or less, the charge is 60 per hour or ₹12 per kilometre, whichever is more On the other hand, if the car is rented for more than 5 hours, the charge is ₹50 per hour or ₹7.50 per kilometre whichever is more Akil rented a car from this agency, drove it for 30 kilometres and ended up paying ₹300 For how many hours did he rent the car? **(CAT 2002)**

(a) 4

(b) 5

(c) 6

(d) None of these

40. Shyam visited Ram on vacation In the mornings, they both would go for yoga In the evenings they would play tennis To have more fun, they indulge, only in one activity per day, i.e., either they went for yoga or played tennis each day There were days when they were lazy and stayed

home all day long There were 24 mornings when they did nothing, 14 evenings when they stayed at home, and a total of 22 days when they did yoga or played tennis For how many days did Shyam stay with Ram? **(CAT 2002)**

(a) 32

(b) 24

(c) 30

(d) None of these

Directions for Questions 41 and 42: Answer these questions based on the information given below:

A boy is asked to put in a basket one mango when ordered 'One', one orange when ordered 'Two', one apple when ordered 'Three' and is asked to take out from the basket one mango and an orange when ordered 'Four' A sequence of orders is given as:

1 2 3 3 2 1 4 2 3 1 4 2 2 3 3 1 4 1 1 3 2 3 4

41. How many total oranges were in the basket at the end of the above sequence? **(CAT 2002)**

(a) 1

(b) 4

(c) 3

(d) 2

42. How many total fruits will be in the basket at the end of the above order sequence? **(CAT 2002)**

(a) 9

(b) 8

(c) 11

(d) 10

Directions for Questions 43 and 44: Answer the questions independently of each other.

43. A rich merchant had collected many gold coins He did not want anybody to know about them One day, his wife asked, “How many gold coins do we have?” After pausing a moment, he replied, “Well! If I divide the coins into two unequal numbers, then 48 times the difference between the two numbers equals the difference between the squares of the two numbers.” The wife looked puzzled Can you help the merchant’s wife by finding out how many coins the merchant has? **(CAT 2002)**

(a) 96

(b) 53

(c) 43

(d) None of these

44. On a straight road XY, 100 metres long, five heavy stones are placed two metres a part, beginning at the end X A worker, starting at X, has to transport all the stones to Y, by carrying only one stone at a time The minimum distance he has to travel (in metres) is: **(CAT 2002)**

(a) 860

(b) 422

(c) 744

(d) 844

45. If there are 10 positive real numbers $n_1 < n_2 < n_3 \ldots < n_{10}$ How many triplets of these numbers $(n_1, n_2, n_3), (n_2, n_3, n_4), \ldots$ can be generated such that in each triplet the first number is always less than the second number, and the second number is always less than the third number? **(CAT 2002)**

(a) 45

(b) 90

(c) 120

(d) 180

46. Davji Shop sells samosas in boxes of different sizes The samosas are priced at ₹2 per samosa up to 200 samosas For every additional 20 samosas, the price of the whole lot goes down by 10 paise per samosa What should be the maximum size of the box that would maximise the revenue? **(CAT 2002)**

(a) 240

(b) 300

(c) 400

(d) None of these

47. Three travellers are sitting around a fire, and are about to eat a meal One of them has five small loaves of bread; the second has three small loaves of bread The third has no food, but has eight coins He offers to pay for some bread They agree to share the eight loaves equally among them-

selves and the third traveller will pay eight coins for his share of the eight loaves All loaves were the same size The second traveller (who had three loaves) suggests that he be paid three coins and that the first traveller be paid five coins The first traveller says that he should get more than five coins How much should the first traveller get? **(CAT 2002)**

(a) 5

(b) 7

(c) 1

(d) None of these

48. A piece of string is 40 centimetres long It is cut into three pieces The longest piece is 3 times as long as the middle-sized piece and the shortest piece is 23 centimeters shorter than the longest piece Find the length of the shortest piece. **(CAT 2002)**

(a) 27

(b) 5

(c) 4

(d) 9

49. Mayank, Mirza, Little and Jaspal bought a motorbike for $60.00 Mayank paid one half of the sum of the amounts paid by the other boys Mirza paid one third of the sum of the amounts paid by the other boys; and Little paid one fourth of the sum of the amounts paid by the other boys How much did Jaspal have to pay? **(CAT 2002)**

(a) 15

(b) 13

(c) 17

(d) None of these

50. The owner of a local jewellery store hired three watchmen to guard his diamonds, but a thief still got in and stole some diamonds On the way out, the thief met each watchman, one at a time To each he gave half of the diamonds he had then, and two more besides that He escaped with one diamond How many did he steal originally? **(CAT 2002)**

(a) 40

(b) 36

(c) 25

(d) None of these

51. If x, y and z are real numbers such that: $x + y + z = 5$ and $xy + yz + zx = 3$, what is the largest value that x can have? **(CAT 2002)**

(a) 5/3

(b) $\sqrt{19}$

(c) 13/3

(d) None of these

52. Number S is obtained by squaring the sum of digits of a two digit number D If difference between S and D is 27, then the two digit number D is: **(CAT 2002)**

(a) 24

(b) 54

(c) 34

(d) 45

53. If three positive real numbers x, y, z satisfy $y - x = z - y$ and $xyz = 4$, then what is the minimum possible value of y? **(CAT 2003)**

(a) 21/3

(b) 22/3

(c) 21/4

(d) 23/4

54. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, ... 9, such that the first digit of the code is not zero The code, hand-written on a slip, can however potentially create confusion when read upside down—for example, the code 91 may appear as 16 How many codes are there for which no such confusion can arise? **(CAT 2003)**

(a) 80

(b) 78

(c) 71

(d) 69

55. Consider the sets $Tn = \{n, n + 1, n + 2, n + 3, n + 4\}$, where $n = 1, 2, 3, ...,$ 96 How many of these sets contain 6 or any integral multiple thereof (i.e., any one of the numbers 6, 12, 18, ...)? **(CAT 2003)**

(a) 80

(b) 81

(c) 82

(d) 83

56. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7? **(CAT 2003)**

(a) 666

(b) 676

(c) 683

(d) 777

57. Let x and y be positive integers such that x is prime and y is composite. Then, **(CAT 2003)**

(a) $y - x$ cannot be an even integer

(b) xy cannot be an even integer.

(c) $(x + y)/x$ cannot be an even integer

(d) None of the above statements is true.

58. Let $n > 1$ be a composite integer such that $\sqrt{n}$ is not an integer Consider the following statements: **(CAT 2003)**

A: n has a perfect integer-valued divisor, which is greater than 1 and less than $\sqrt{n}$

B: n has a perfect integer-valued divisor, which is greater than $\sqrt{n}$ but less than n

Then,

(a) Both A and B are false

(b) A is true but B is false

(c) A is false but B is true

(d) Both A and B are true

59. Let a, b, c, d, and e be integers such that $a = 6b = 12c$, and $2b = 9d = 12e$ Then which of the following pairs contain a number that is not an integer? **(CAT 2003)**

(a) $(a/27, b/e)$

(b) $(a/36, c/e)$

(c) $(a/12, bd/18)$

(d) $(a/6, c/d)$

60. If a, $a + 2$, and $a + 4$ are prime numbers, then the number of possible solutions for a is: **(CAT 2003)**

(a) 1

(b) 2

(c) 3

(d) more than 3

61. What is the remainder when 4^{96} is divided by 6? **(CAT 2003)**

(a) 0

(b) 2

(c) 3

(d) 4

62. Using only 2, 5, 10, 25 and 50 paise coins, what will be the minimum number of coins required to pay exactly 78 paise, 69 paise, and ₹ 1.01 to three different persons?**(CAT 2003)**

(a) 19

(b) 20

(c) 17

(d) 18

63. Each family in a locality has at most two adults, and no family has fewer than 3 children Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families Then, the minimum possible number of families in the locality is: **(CAT 2004)**

(a) 4

(b) 1

(c) 2

(d) 3

64. If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms? **(CAT 2004)**

(a) 0

(b) –1

(c) 1

(d) Not unique

65. On January 1, 2004 two new societies, S_1 and S_2 are formed, each with n members. On the first day of each subsequent month, S_1 adds b members while S_2 multiplies its current number of members by a constant factor r. Both the societies have the same number of members on July 2, 2004. If $b = 10.5n$, what is the value of r? **(CAT 2004)**

(a) 2.0

(b) 1.9

(c) 1.8

(d) 1.7

66. Suppose n is an integer such that the sum of the digits of n is 2, and $10^{10} < n < 10^{11}$. The number of different values for n is **(CAT 2004)**

(a) 11

(b) 10

(c) 9

(d) 8

67. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is **(CAT 2004)**

(a) 4

(b) 15

(c) 0

(d) 18

68. Consider the sequence of numbers $a_1, a_2, a_3, \ldots\ldots$ to infinity where a_1 = 81.33 and $a_2 = -19$ and $a_j = a_{j-1} - a_{j-2}$ for $j \geq 3$. What is the sum of the first 6002 terms of this sequence? **(CAT 2005)**

(a) –100.33

(b) –30.00

(c) 62.33

(d) 119.33

69. If $x = (16^3 + 17^3 + 18^3 + 19^3)$, then x divided by 70. This leaves a remainder of: **(CAT 2005)**

(a) 0

(b) 1

(c) 69

(d) 35

70. If $R = (30^{65} - 29^{65})/(30^{64} + 29^{64})$, then **(CAT 2005)**

(a) $0 < R \leq 0.1$

(b) $0.1 < R \leq 0.5$

(c) $0.5 < R \leq 1.0$

(d) $R > 1.0$

71. Let $n! = 1 \times 2 \times 3 \times \ldots \times n$ for integer $n > 1$. If $p = (1 \times 1!) + (2 \times 2!) + (3 \times 3!) + \ldots + (10 \times 10!)$, then $p + 2$ when divided by $11!$ leaves a remainder of: **(CAT 2005)**

(a) 10

(b) 0

(c) 7

(d) 1

72. The digits of a three-digit number A are written in the reverse order to form another three-digit number B. If $B > A$ and $B - A$ is perfectly divisible by 7, then which of the following is necessarily true? **(CAT 2005)**

(a) $100 < A < 299$

(b) $106 < A < 305$

(c) $112 < A < 311$

(d) $118 < A < 317$

73. The rightmost non-zero digit of the number 30^{2720} is: **(CAT 2005)**

(a) 1

(b) 3

(c) 7

(d) 9

74. For a positive integer n, let p_n denote the product of the digits of n, and s_n denote the sum of the digits of n. The number of integers between 10 and 1000 for which $p_n + s_n = n$ is: **(CAT 2005)**

(a) 81

(b) 16

(c) 18

(d) 9

75. Let S be a set of positive integers such that every element n of S satisfies the conditions **(CAT 2006)**

A. $1000 \leq n \leq 1200$

B. Every digit in n is odd

Then, how many elements of S are divisible by 3?

(a) 9

(b) 10

(c) 11

(d) 12

76. If $x = -0.5$, then which of the following has the smallest value? **(CAT 2006)**

(a) $2^{1/x}$

(b) $1/x$

(c) $1/x^2$

(d) $2x$

(e) $1/\sqrt{-x}$

77. Which among $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $6^{1/6}$ and $12^{1/12}$ is the largest? **(CAT 2006)**

(a) $2^{1/2}$

(b) $3^{1/3}$

(c) $4^{1/4}$

(d) $6^{1/6}$

(e) $12^{1/12}$

78. A group of 630 children is arranged in rows for a group photograph session. Each row contains three fewer children than the row in front of it. What number of rows is not possible? **(CAT 2006)**

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

79. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square Which of the following can possibly be one of these four numbers? **(CAT 2006)**

(a) 21

(b) 25

(c) 41

(d) 67

(e) 73

80. Consider the set S = 1, 2, 3, ; .., 1000 How many arithmetic progressions can be formed from the elements of *S* that start with l and end with 1000 and have at least 3 elements? **(CAT 2006)**

(a) 3

(b) 4

(c) 6

(d) 7

(e) 8

Directions for Questions 81 and 82: Answer questions 81 and 82 on the basis of the information given below:

An airline has a certain free luggage allowance and charges for excess luggage at a fixed rate per kgs Two passengers, Raja and Praja have 60 kgs of luggage between them, and are charged ₹1200 and ₹2400 respectively for excess luggage Had the entire luggage belonged to one of them, the excess luggage charge would have been ₹5400.

81. What is the weight of Praja's luggage? **(CAT 2006)**

(a) 20 kgs

(b) 25 kgs

(c) 30 kgs

(d) 35 kgs

(e) 40 kgs

82. What is the free luggage allowance? **(CAT 2006)**

(a) 10 kgs

(b) 15 kg

(c) 20 kg

(d) 25 kg

(e) 30 kg

Directions for Questions 83 to 104: Answer each question independently.

83. When you reverse the digits of the number 13, the number increases by 18 How many other two-digit numbers increase by 18 when their digits are reversed? **(CAT 2006)**

(a) 5

(b) 6

(c) 7

(d) 8

(e) 10

84. The number of employees in Obelix Menhir Co. is a prime number and is less than 300. The ratio of the number of employees who are graduates and above, to that of employees who are not, can possibly be: **(CAT 2006)**

(a) 101:88

(b) 87:100

(c) 10:111

(d) 85:98

(e) 97:84

85. Consider the set $S = \{1, 2, 3, 4, ..., 2n+1\}$, where n is a positive integer larger than 2007. Define X as the average of the odd integers in S and Y as the average of the even integers in S. What is the value of $X - Y$? **(CAT 2007)**

(a) 0

(b) 1

(c) $(1/2)n$

(d) $(n + 1)/2n$

(e) 2008

86. Suppose you have a currency, named Miso, in three denominations: 1 Miso, 10 Misos and 50 Misos. In how many ways can you pay a bill of 107 Misos? **(CAT 2007)**

(a) 17

(b) 16

(c) 18

(d) 15

(e) 19

87. A confused bank teller transposed the rupees and paise when he cashed a cheque for Shailaja, giving her rupees instead of paise and paise instead of rupees. After buying a toffee for 50 paise, Shailaja noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount? **(CAT 2007)**

(a) Over ₹13 but less than ₹14

(b) Over ₹7 but less than ₹8

(c) Over ₹22 but less than ₹23

(d) Over ₹18 but less than ₹19

(e) Over ₹4 but less than ₹5

88. What are the last two digits of 72008? **(CAT 2008)**

(a) 21

(b) 61

(c) 01

(d) 41

(e) 81

89. A shop stores x kg of rice. The first customer buys half this quantity plus half a kg of rice. The second customer buys half the remaining quantity plus half a kg of rice. The third customer also buys half the remaining quantity plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x? **(CAT 2008)**

(a) $2 \leq x \leq 6$

(b) $5 \leq x \leq 8$

(c) $9 \leq x \leq 12$

(d) $11 \leq x \leq 14$

(e) $13 \leq x \leq 18$

90. The number of common terms in the two sequences 17, 21, 25,……,417 and 16, 21, 26,…., 466 is **(CAT 2008)**

(a) 78

(b) 19

(c) 20

(d) 77

(e) 22

91. The integers 1, 2, …….., 40 are written on a blackboard. The following operation is then repeated 39 times: in each repetition, any two numbers say a and b, currently on the blackboard are erased and a new number $a + b - 1$ is written. What will be the number left on the board at the end? **(CAT 2008)**

(a) 820

(b) 821

(c) 781

(d) 819

(e) 780

92. Three consecutive positive integers are raised to the first, second and third powers respectively and then added. The sum so obtained is a perfect square, whose square root equals the total of the three original integers. Which of the following best describes the minimum, say m, of these three integers? **(CAT 2008)**

(a) $1 \le m \le 3$

(b) $4 \le m \le 6$

(c) $7 \le m \le 9$

(d) $10 \le m \le 12$

(e) $13 \le m \le 15$

93. The seed of any positive integer n is defined as follows: **(CAT 2008)**

Seed $(n) = n$, if $n < 10$ = seed $(s(n))$, otherwise,

where $s(n)$ indicates the sum of digits of n. For example,

seed (7) = 7, seed (248) = seed(2 + 4 + 8) = seed (14) = seed (1 + 4) = seed (5) = 5, etc. How many positive integers n, such that $n < 500$, will have seed $(n) = 9$?

(a) 39

(b) 72

(c) 81

(d) 108

(e) 55

94. If a and b are integers of opposite signs such that $(a + 3)^2 : b^2 = 9 : 1$ and $(a - 1)^2 : (b - 1)^2 = 4 : 1$, then the ratio $a^2 : b^2$ is **(CAT 2017)**

(a) 9 : 4

(b) 81 : 4

(c) 1 : 4

(d) 25 : 4

95. If $x + 1 = x^2$ and $x > 0$, then $2x^4$ is **(CAT 2017)**

(a) $6 + 4\sqrt{5}$

(b) $3 + 5\sqrt{5}$

(c) $5 + 3\sqrt{5}$

(d) $7 + 3\sqrt{5}$

96. The number of solutions (x, y, z) to the equation $x - y - z = 25$, where x, y, and z are positive integers such that $x \leq 40$, $y \leq 12$, and $z \leq 12$ is **(CAT 2017)**

(a) 101

(b) 99

(c) 87

(d) 105

97. For how many integers n, will the inequality $(n - 5)(n - 10) - 3(n - 2) \leq 0$ be satisfied? **(CAT 2017)**

98. If the square of the 7th term of an arithmetic progression with positive common difference equals the product of the 3rd and 17th terms, then the ratio of the first term to the common difference is **(CAT 2017)**

(a) 2 : 3

(b) 3 : 2

(c) 3 : 4

(d) 4 : 3

99. Let $a_1, a_2, \ldots\ldots.. a_{3n}$ be an arithmetic progression with $a_1 = 3$ and $a_2 = 7$. If $a_1 + a_2 + \ldots. + a_{3n} = 1830$, then what is the smallest positive integer 'm' such that $m \times (a_1 + a_2 + \ldots. + a_n) > 1830$? **(CAT 2017)**

(a) 8

(b) 9

(c) 10

(d) 11

100. If the product of three consecutive positive integers is 15600 then the sum of the squares of these integers is **(CAT 2017)**

(a) 1777

(b) 1785

(c) 1875

(d) 1877

101. Let a_1, a_2, a_3, a_4, a_5 be a sequence of five consecutive odd numbers. Consider a new sequence of five consecutive even numbers ending with $2a_3$. If the sum of the numbers in the new sequence is 450, then a_5 is **(CAT 2017)**

102. How many different pairs (a, b) of positive integers are there such that $a \leq b$ and $\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$? **(CAT 2017)**

103. An infinite geometric progression $a_1, a_2, a_3 \ldots$ has the property that $a_n = 3(a_{n+1} + a_{n+2} + \ldots.)$ for every $n \geq 1$. If the sum $a_1 + a_2 + a_3 + \ldots. = 32$, then a_5 is **(CAT 2017)**

(a) 1/32

(b) 2/32

(c) 3/32

(d) 4/32

104. If a_1 = 1/2 × 5, a_2 = 1/5 × 8, a_3 = 1/8 × 11, ……, then a_1 + a_2 + a_3 + ………. a_{100} is **(CAT 2017)**

(a) 25/151

(b) 1/2

(c) 1/4

(d) 111/55

105. While multiplying three real numbers, Ashok took one of the numbers as 73 instead of 37. As a result, the product went up by 720. Then the minimum possible value of the sum of squares of the other two numbers is **(CAT 2018)**

106. If $u_2 + (u - 2v - 1)_2 = -4v\,(u + v)$, then what is the value of $u + 3v$? **(CAT 2018)**

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) 0

(d) –1/4

107. Let x, y, z be three positive real numbers in a geometric progression such that $x < y < z$. If $5x$, $16y$, and $12z$ are in an arithmetic progression then the common ratio of the geometric progression is **(CAT 2018)**

108. The number of integers x such that $0.25 < 2^x < 200$, and $2^x + 2$ is perfectly divisible by either 3 or 4, is **(CAT 2018)**

109. Given that $x^{2018} y^{2017} = \frac{1}{2}$ and $x^{2016} y^{2019} = 8$ the value of $x^2 + y^3$ is **(CAT 2018)**

(a) 37/4

(b) 31/4

(c) 35/4

(d) 33/4

110. Let $a_1, a_2, ..., a_{52}$ be positive integers such that $a_1 < a_2 < ... < a_{52}$. Suppose, their arithmetic mean is one less than the arithmetic mean of $a_2, a_3, ..., a_{52}$. If $a_{52} = 100$, then the largest possible value of a_1 is **(CAT 2018)**

(a) 48

(b) 20

(c) 45

(d) 23

111. Let $t_1, t_2, \ldots$ be real numbers such that $t_1 + t_2 + ... + t_n = 2n^2 + 9n + 13$, for every positive integer $n \geq 2$. If $t_k = 103$, then k equals **(CAT 2018)**

112. If N and x are positive integers such that $N^N = 2^{160}$ and $N^2 + 2^N$ is an integral multiple of 2^x, then the largest possible x is **(CAT 2018)**

113. The smallest integer n for which $4^n > 17^{19}$ holds, is closest to **(CAT 2018)**

(a) 33

(b) 39

(c) 37

(d) 35

114. The smallest integer n such that $n^3 - 11n^2 + 32n - 28 > 0$ is **(CAT 2018)**

115. How many two-digit numbers, with a non-zero digit in the units place, are there, which are more than thrice the number formed by inter-changing the positions of its digits? **(CAT 2018)**

(a) 6

(b) 8

(c) 7

(d) 5

116. If the sum of squares of two numbers is 97, then which one of the following cannot be their product? **(CAT 2018)**

(a) 48

(b) 64

(c) −32

(d) 16

117. The value of the sum $7 \times 11 + 11 \times 15 + 15 \times 19 + \ldots\ldots + 95 \times 99$ is **(CAT 2018)**

(a) 80707

(b) 80730

(c) 80751

(d) 80773

118. If $A = \{6^{2n} - 35n - 1; n = 1, 2, 3....\}$ and $B = 35(n - 1); n = 1, 2, 3, ...$ then which of the following is true? **(CAT 2018)**

(a) Neither every member of A is in B, nor is every member of B in A

(b) Every member of A is in B, and at least one member of B is not in A

(c) Every member of B is in A

(d) At least one member of A is not in B

119. If the population of a town is p in the beginning of any year then it becomes $3 + 2p$ in the beginning of the next year. If the population in the beginning of 2019 is 1000, then the population in the beginning of 2034 will be **(CAT 2019)**

(a) $(1003)^{15} + 6$

(b) $(977)^{15} - 3$

(c) $(1003) \times 2^{15} - 3$

(d) $(977) \times 2^{14} + 3$

120. The product of two positive numbers is 616. If the ratio of the difference of their cubes to the cube of their difference is 157 : 3, then the sum of the two numbers is **(CAT 2019)**

(a) 50

(b) 85

(c) 95

(d) 58

121. If $a_1 + a_2 + a_3 + ... + a_n = 3(2^{n+1} - 2)$, then a_{11} equals **(CAT 2019)**

122. If $a_1, a_2, a_3, \ldots$ are in A.P., then $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$ is equal to **(CAT 2019)**

(a) $\frac{n}{\sqrt{a_1}+\sqrt{a_{n-1}}}$

(b) $\frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$

(c) $\frac{n}{\sqrt{a_1}-\sqrt{a_{n+1}}}$

(d) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_{n-1}}}$

123. If m and n are integers such that $(\sqrt{2})^{19} \times 3^4 \times 4^2 \times 9^m \times 8^n = 3^n \times 16^m \times (\sqrt[4]{64})$, then m is **(CAT 2019)**

124. Let a, b, x, y be real numbers such that $a^2 + b^2 = 25$, $x^2 + y^2 = 169$ and $ax + by = 65$. If $k = ay - bx$, then **(CAT 2019)**

(a) $k = 0$

(b) $k > 5/13$

(c) $k = 5/13$

(d) $0 < k \leq 5/13$

125. Let a_1, a_2 be integers such that $a_1 - a_2 + a_3 - a_4 + \ldots\ldots + (-1)^{n-1} a_n = n$, for $n \geq 1$. Then $a_{51} + a_{52} + \ldots\ldots + a_{1023}$ equals **(CAT 2019)**

(a) –1

(b) 1

(c) 0

(d) 10

126. How many factors of $2^4 \times 3^5 \times 10^4$ are perfect squares which are greater than 1? **(CAT 2019)**

127. How many pairs of positive integers (m, n) satisfy the equation $m^2 + 105 = n^2$ **(CAT 2019)**

128. In a six-digit number, the sixth, that is, the rightmost digit is the sum of the first three digits, the fifth digit is the sum of first two digits, the third digit is equal to the first digit, the second digit is twice the first digit and the fourth digit is the sum of fifth and sixth digits. Then, the largest possible value of the fourth digit is **(CAT 2019)**

129. The number of common terms in the two sequences: 15, 19, 23, 27, ….. , 415 and 14, 19, 24, 29, ….. , 464 is **(CAT 2019)**

(a) 20

(b) 18

(c) 21

(d) 19

130. If $(2n + 1) + (2n + 3) + (2n + 5) + .. + (2n + 47) = 5280$, then what is the value of $1 + 2 + 3 + .. + n$? **(CAT 2019)**

ANSWER KEY

1. (a)
2. (b)
3. (a)
4. (b)
5. (c)
6. (d)
7. (a)
8. (d)
9. (c)
10. (d)
11. (d)
12. (a)
13. (c)
14. (d)
15. (d)
16. (a)
17. (c)
18. (a)
19. (c)
20. (c)
21. (a)
22. (d)
23. (d)
24. (d)
25. (a)
26. (a)

27. (d)

28. (c)

29. (b)

30. (b)

31. (d)

32. (b)

33. (a)

34. (d)

35. (d)

36. (b)

37. (a)

38. (d)

39. (c)

40. (c)

41. (d)

42. (c)

43. (d)

44. (a)

45. (c)

46. (b)

47. (b)

48. (c)

49. (b)

50. (b)

51. (b)

52. (b)

53. (b)

54. (b)

55. (a)

56. (b)

57. (d)

58. (d)

59. (d)

60. (a)

61. (d)

62. (a)

63. (d)

64. (a)

65. (a)

66. (a)

67. (c)

68. (c)

69. (a)

70. (d)

71. (d)

72. (b)

73. (a)

74. (d)

75. (a)

76. (b)

77. (b)

78. (d)

79. (c)

80. (d)

81. (d)

82. (b)

83. (b)

84. (e)

85. (a)

86. (c)

87. (d)

88. (c)

89. (b)

90. (c)

91. (c)

92. (a)

93. (e)

94. (d)

95. (d)

96. (b)

97. 11

98. (a)

99. (b)

100. (d)

101. 51

102. 3

103. (c)

104. (a)

105. 40

106. (d)

107. 5/2

108. 5

109. (d)

110. (d)

111. 24

112. 10

113. (b)

114. 8

115. (a)

116. (b)

117. (a)

118. (b)

119. (c)

120. (a)

121. 6144

122. (b)

123. –12

124. (a)

125. (b)

126. 44

127. 4

128. 7

129. (a)

130. 4851

Solutions

1. It is self evident that the value of b can only be 1, 5 or 6 for $(ab)^2 = ccb$

   Given that $ccb > 300$, we need to start from squares of numbers greater than 17.

   $21^2 = 441$ will satisfy the given conditions.

2. $7^{84}/342 = (7^3)^{28}/342 = 343^{28}/342$ remainder 1. Option (b).

3. If x is the product of 4 consecutive positive integers, it must be even Thus, $(x + 1)$ has to be odd Also, by trial and error, it can be seen that $(x + 1)$ need not be prime, but it is always a perfect square Thus, option (a) is correct.

4. The function $h(a, b)$ is such that it takes two values as it's input and returns one value (the GCF of the two given values). Thus, there is a reduction of one number when we use the function 'h' once. Hence, if we have to reduce the set A containing 'n' elements into one element (as defined by $G(A)$) we would need to use the function 'h' $n - 1$ times.

5. The squares of numbers containing only ones have a pattern which can be judged from the following:

   $11^2 = 121$

   $111^2 = 12321$

   $1111^2 = 1234321$

   Thus, 123456787654321 would be the square of 11111111. Hence, we mark option (c) as the correct answer.

6. In order to get to the correct option in this question, you need to try to disprove each of the options by thinking of possible values for the elements in S_1 and S_2.

Options (a), (b) and (c), all would not be true in case we were to take the elements in S_1 to be 1 – 24, while the elements in S_2 as 25 – 50. Then, if we change the signs of each element of S1, we will get these values as –1, –2,... –24. It can be seen that neither of the first three option statements would be true i.e., we would not have every member of S_1 greater than every member of S_2 (as stated in option (a)), we would not have G in S_1 (as stated in option (b)), and we would not have the largest number between S_1 and S_2 in S_1 (as stated in option (c)). Thus, option (d) is correct.

7. Let the elements in S_1 be 1, 2, 3, ..., 24 and the elements in S_2 be 50, 49, 48, ..., 27, 26, 25. Then after interchanging a_{24} and a_{25}, S_1 would have (1, 2, 3, 4, ..., 22, 23 and 50), while S_2 would have (24, 49, 48, 47, ..., 28, 27, 26, 25). It is obvious that S_1 would continue to be in ascending order, while S_2 would not continue to be in the descending order. Thus, option (a) is correct.

8. It is obvious that since every element of S_1 has to be made equal to or greater than every element of S_2, L would have to be made greater than or equal to G. For this the value of x cannot be less than, $G - L$. Thus, option (d) is correct.

9. Numbers which have 2 digits recurring after the decimal are related to denominators which have 99 as a factor. From the options only 198 is related to 99 and is hence the correct answer.

10. We first need to find odd numbers between 100 and 200 such that they are divisible by 3. This is given by the set: 105, 111, 117, ..., 195 (a total of 16 such numbers). However, we do not have to count all these numbers as some of these would be divisible by 7 and we need to remove those before we can conclude our count. From the above list 105, 147 and 189 are three such numbers which are divisible by 7. Hence, the required answer is 16 – 3 = 13.

11. Let the numbers be 3, 5, 7. We can see easily that xyz^2 is odd as it is an Odd × Odd × odd situation. Thus, option (a) is necessarily true.

    Also $(x - y)^2 \times z$ = Even × Odd = Even. Thus option (b) is also true. $(x + y - z)^2 (x + y)$ = Odd × Even = Even. Thus option (c) is also true.

    The last option represents Even × Even × Odd situation and hence should always give us an even value. Thus, the fourth option need not be true always.

12. The set of prime numbers would have only 1 multiple of 2 (2 itself) and one five (in 5 itself). Thus, the product would have only 1 zero.

13. 1421 × 1423 × 1425 /12 → 5 × 7 × 9 /12 = 315/12 → remainder 3.

14. The difference between 32506 and 34041 is 1535. The number which would leave the same remainder with both these numbers must necessarily be a factor of 1535. From the given options only 307 is a factor of 1535.

15. N would be divisible by 3 because it can be rewritten as: $(55 + 17)(55^2 + \ldots + 17^2) - 72^3 = 72m - 72^3$ which would be divisible by 3. [Using $a^3 + b^3 = (a + b)(a^2 + \ldots + b^2)$

N would be divisible by 17 because it can be rewritten as:

$17^3 + (55 - 7^2)(55^2 + ... + 72^2) = 17^3 - 17y$ which would be divisible by 17.

16. The point E is at an even number of steps away from point A. Thus, $a_{2n-1} = 0$.

17. The highest power of 12 in 1982 is $12^3 = 1728$. Thus, the number would be a 4 digit number.

1728	144	12	1
1	1	9	2

Thus the number is 1192.

18. To solve this question, go through the options.

Option (a) gives us: Odd × Odd ≠ even. Hence cannot be true and is the correct answer.

The other options need not be checked since we have reached the correct option already.

19. The sum of the first 10 natural numbers are 55, that of the 11th to 20th natural numbers are 155 and so on. In order to find the number added twice, we need to reach the last triangular number below 1000. This can be got by adding 55 + 155 + 255 + 355 + 41 + 42 + 43 + 44 = 990. Hence, the number added twice must be 10, i.e., option (c).

20. x^2y and $5xy$ are both negative. Amongst them 5xy will be the smaller value. Hence, option (c) is correct.

21. The base can only be 5, 6, 7, 8 or 9. Testing for base 5, we can see that this is true in base 5. Thus, we need the value of 3111 in base 5. The value would be $3 \times 125 + 1 \times 25 + 1 \times 5 + 1 \times 1 = 406$.

22. He has to have at least six boxes with the same number of oranges in it. The logic of this question comes from the pigeon hole principle—where we take the ratio 128/25 and take the least integer value greater than this ratio. Note, 25 comes from the different values of oranges that you can put in the boxes. Hence, we would get six as the answer.

23. Such questions have to be solved using reverse thinking. So start thinking about the last person, Eswari must have seen 30 mints (only in such a case would you get 17 mints left after taking half and then returning 2 to the bowl.) For Eswari to see 30 mints, it must be the case that after Fatima took one-fourth of what she saw, there must have been 27 mints left and when she put 3 back, Eswari would have seen 30 mints.

    Further, for Fatima to see 36 mints, Sita must have seen 48 mints to start with—as to leave 36 after taking one-third of the mints she sees and then giving back 4 the only starting point possible is 48.

24. Since she increases one part of the product by 18 and the result in the answer is an increase of 540, she must have been multiplying the number by 30. Hence, the new product would be given by $53 \times 30 = 1590$.

25. If the number is abcd, then $a + b = c + d$, $a + d = c$ and $b + d = 2(a + c)$. Thus, $b + d$ should be even. Thus either both b and d are even or both are odd. From the first expression, since both b and d are of the same nature—a and c should also have the same nature (both even or both odd).

From this point it is best to move through informed trial and error solution. Try to take c as 5 and fit in the remaining values. Note the following while doing this thinking. If $c = 5$ then, $b + d$ should be a minimum of 12. All we need to do is find 1 value which satisfies all conditions.

So if we take $b + d$ as 12, $a + c$ should be 6. The number would be $1b5d$. Trying to fit in value in this case we would get 1854 satisfying all conditions. Hence, (a) is correct.

26. The red light would flash every 20 seconds, the green light every 24 seconds. They would flash together every 2 minutes and hence 30 times in an hour.

27. In order to do this he should allocate an independent power of 2 in every bag. Thus, the first bag should contain Re, 1 the second, ₹2 the third 4 ₹8, 16, 32, 64. Using these he can form any value from 1 — 127. The last bag should contain the remaining ₹31 as we can add any combination of the above to 31 to get all values between 128 to 158.

28. Since 517 is a composite number, the two consecutive terms of the sequence (say a and b) should be such that their values should obey the relationship $(a - b)(a + b) = 517$. 517 can be written as 11×47. Thus we need two numbers whose difference is 11 and sum is 47. 18 and 29 satisfy this condition. Thus, the series can be written from the sixth term onwards as:18, 29, 47, 76, 123.

29. If $b - d = d$, it must be the case that b is 10 and d is 5. Then we get $e = 6$ and using the other equations we would get that $a = 4$ as follows:

$A + c = e$ means either 2 + 4 = 6 or 4 + 2 = 6 (in order). Since $e + a = b$ it must follow that 6 + 4 = 10 and hence $a = 4$ and $e = 6$. Option (b) satisfies.

Note: we rule out $b = 4$ and $d = 2$, since the other variables (a, c and e) cannot be fitted with values from 5, 6 and 10 such that $a + c = e$.

30. The expression becomes positive when $n > 5$. Hence, the largest possible value of m is 5.

31. Using trial and error all the options can get eliminated. At $b = 11$, the resultant value of the expression is not divisible by any of the three numbers.

32. The second statement means that there are 40 one rupee coins less than two rupee coins. Using this information the question can be easily solved through the options. The given conditions are satisfied at 140 five-rupee coins.

33. Suppose we take the value of n as 1, we would get $(7^6 – 6^6) = (7^3 – 6^3)(7^3 + 6^3)(7 – 6)(7^2..) \times (7 + 6)(7^2....)$. It can be clearly seen that this is divisible by 13.

    You can easily visualise that even if we were to take the value of n as 2, there would always be a (7 + 6) component in the simplification.

    In general: $(7^{6n} – 6^{6n}) = (7^{3n} – 6^{3n})(7^{3n} + 6^{3n}) = (7 – 6)(7^{3n – 1}..) \times (7 + 6)(7^{3n – 1}....)$

34. From the options, the only number which gives successive remainders as 2, 1 and 4 when divided successively by 3, 4 and 7 respectively is 53. Hence, the correct answer is 53.

35. We need to find the HCF of 4.5, 6.75 and 7.2. Or 9/2, 27/4 and 36/5.

HCF of Numerators/ LCM of denominators = 9/20 = 0.45.

Dividing the cakes into these sizes, the number of pieces we would get would be:

10 + 15 + 16 = 41 pieces—and hence 41 guests.

The answer would be—None of these.

36. To solve this question, you would need to find the LCM of 7/2, 21/4, 49/8.

LCM of Numerators/ HCF of denominators = 147/2 = 73.5.

37. $2^{256} = (2^4)^{64} = 16^{64}$. $16^{64}/17$ would give us a remainder of 1 (since 16/17 leaves a remainder –1, and when the power is even the remainder becomes +1).

38. In order to find the number missed, we need to find the least triangular number above 575. This can be done by 55 (sum of the first 10 natural numbers) + 155 (sum of the next ten natural numbers) + 255 (Sum of natural numbers from 21 to 30) + 31 + 32 + 33 + 34 = 595.

39. He can only pay ₹300 if the car is rented for 6 hours @ ₹50 per hour.

40. The answer would be given by 22 + 8 and is quite easy to work out. We need to understand that since there are 22 days when they play tennis or do yoga, in each of these 22 days there would be either a free morning or evening. This would account for a total of 22 free mornings/evenings. Also, the total number of free morning/evenings is 24 + 14 = 38. This means that 16 free mornings/evenings are still available—which would mean 8 days when they did nothing.

41. There are 6 twos and 4 fours. Hence, there would be 2 oranges in the basket.

42. 19 – 4 × 2 = 11

43. This question requires us to use the principle of difference of squares:

We know that $x^2 - y^2 = (x - y)(x + y)$. Hence, $x + y$ should be 48.

44. 100 + 196 + 192 + 188 + 184 = 860

45. In this type of question, the key is to look at a systematic way of counting the number of instances. So with 1 & 2 as the first 2 numbers, we can get 8 sets (by varying the third value from 3 to 10), Similarly, with 1 & 3, we will get 7 pairs, with 1 & 4 six pairs, and so on till 1 & 9, we would get 1 pair. (A total of 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36). Similarly for the first number to be starting with 2, we would get 1 + 2 + 3 + ... + 7 = 28 sets. Similarly, for sets starting with 3, there would be 21 sets, for 4, 15 sets, for sets starting with 5, 10 sets, for sets starting with 6, 6 sets, for sets starting with 7, 3 sets and for those starting with 8, 1 set. Thus, there would be a total of 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 120.

Alternately, you can also solve this using Combinations. The correct answer would be ${}_{10}C_3$, since you only have to select 3 numbers from 10 distinct numbers. There is no arrangement to be done.

46. It can be easily seen that the revenues at different values would be:

200 × 2, 220 × 1.9, 240 × 1.8, 260 × 1.7, 280 × 1.6, 300 × 1.5 and 320 × 1.4. The value goes up till 300 × 1.5 and then reduces. Hence, option (b) is correct.

47. The price per piece of bread would be 3 coins as the third traveller is paying 8 coins for his 2.66 loaves. Also the contribution of the first traveller is 2.33 loaves, while that of the second is only 0.33 loaves. Hence, the first traveller should get $2.33 \times 3 = 7$.

48. Solve using options. The required conditions are met by 4, 27 and 9. Hence, option (c) is correct.

49. From the statements, Mayank paid 20, Mirza paid 15, and Little paid 12. Thus Jaspal paid 13.

50. Since he is left with one diamond at the end, to the third watchman he must have reached with six diamonds, given him half (3) and two more (total 5) and be left with one diamond. With the same thought pattern you can solve the remaining part of the question.

The thought process would go as this:

$1 \rightarrow 6 \rightarrow 16 \rightarrow 36$.

51. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz = 25 \rightarrow x^2 + y^2 + z^2 = 19 \rightarrow$ maximum value of x can be the square root of 19 if we take y^2 and z^2 both to be 0.

52. Solve using options. The conditions are met for 54.

53. The expression $y - x = z - y$ means that x, y and z are in arithmetic progression, y being the arithmetic mean between x and z. This also means that for y to have the minimum value given that xyz is equal to 4, x, y and z should be equal. Thus, the minimum value of y is $4^{1/3} = 2^{2/3}$.

54. The codes which will create a confusion would be:

16 and 91, 18 and 81, 19 and 61, 66 and 99, 68 and 89, 86 and 98: A total of 12 codes which will have confusion. Hence out of 90 two-digit codes, 78 would have no confusion.

55. $Tn = \{n, n + 1, n + 2, n + 3, n + 4\}$ where $n = 1, 2, \ldots\ldots 96$ So we have, $T1 = \{$ 1 to 5$\}$, $T2 = \{$2 to 6$\}$ ..$T6 = \{$6 to 10 $\}$, $T7 = \{$7 to 11$\}$ and so on. Clearly, only the sets $T1$, $T7$, $T13$,.....$T91$ would not have 6 in it. These are 16 sets out of 96. The other 80 would have no 6 in them. Thus, the correct answer would be option (a).

56. We need to find the sum of the arithmetic series: 10, 17, 24, 31, ..., 94.

The sum would be given by: number of terms x average = $13 \times 52 = 676$.

57. It can be seen clearly that $y - x$ would be even, if we take both of them as odd, so the first option can be rejected. Similarly, if we take x as 2, we would get the product as even—thus, option (b) can also be rejected. Considering the (c) option we can see that $(x + y)/x = 1 + y/x$ should be even. For this y/x should be odd. We can see this occurring at values like $y = 9$ and $x = 3$. Hence, even this option can be rejected. This leaves us only with the fourth option as the correct answer.

58. If N is a composite integer and is not a perfect square, it would mean that n would have at least one pair of factors (apart from $1 \times N$) such that one of this pair of factors would be user than the square root of N, while the other would be greater than the square root of N. This means that both statements A and B have to be true. Thus, option (d) is correct.

59. A sample set of numbers that satisfy the situation is:

$a = 216, b = 36, c = 18, d = 8, e = 6$

It can be seen that each of $a/27$, b/e, $a/36$, c/e, $a/12$, $bd/18$ and $a/6$ are integers. The only expression which is not an integer is c/d. Thus, option (d) is correct.

60. The only possible solution is for the set of numbers 3,5,7. Option (a) is correct.

61. $4^{96}/6 = 2 \times 4^{95}/3$ → remainder of this expression would be 2. But, we would need to multiply this by 2 to get the actual remainder. Thus, the answer would be 4—option (d) is correct.

62. 78 = 50 + 10 + 10 + 2 + 2 + 2 + 2 7 coins

69 = 50 + 10 + 5 + 2 + 2 5 coins

₹1.01 = 50 + 25 + 10 + 10 + 2 + 2 + 2 7 coins

Thus, a total 19 coins would be required option (a).

63. Two families can have 4 adults, 3 boys and 2 girls, however, the number of girls has to be greater than the number of families. Hence, the given constraints are not met at 2 families. With 3 families, we can have 6 adults, 5 boys and 4 girls. Hence, option (d) is correct.

64. Since $S_{11} = S_{19}$, it means that the sum of the 12th to 19th term of the *AP* would be zero. These terms represent an *AP* with an average of 0. It can also be seen that the average of the 12th to 19th terms can be derived out of the average of the middle terms (15th and 16th terms). So, the average of the 15th and 16th term of the *AP* would be equal to the average of the 12th to 19th terms of the *AP* = 0.

65. Since the value of b is given as $10.5n$, the Society S_1 would have 64n members on July 2, 2004. At the same time, S_2 also has $64n$ members (as given in the question). Hence,

$n \times r_6 = 64n \rightarrow n = 2$

Thus, option (a) is the correct answer.

66. n is an integer greater than $10^{10} = 10000000000$ but less than 10^{11}. (So, it is an 11 digit number). The sum of digits of n can be equal to 2 only if, (a) the number starts with 1 and contains 9 zeroes and one 1 in the remaining 10 places (This can occur in 10 ways as 11000000000, 10100000000… 10000000001) OR (b) the number starts with 2 and has 10 zeroes—which is the case in only 1 number, 20000000000.

Hence, a total of 11 such numbers are possible option (a).

67. $(15^{23} + 23^{23}) = (15+23)(15^{22}+........+23^{22})$. This number would be divisible by 19. Hence, the remainder would be 0 option (c).

68. $a_3 = a_2 - a_1$ ---- gives $a_3 = -100.33$; $a_4 = a_3 - a_2 = -81.33$, $a_5 = a_4 - a_3 = 19$, $a_6 = a_5 - a_4 = 100.33$, $a_7 = 81.33$, $a_8 = -19$, $a_9 = -100.33$. We can see that there is a cyclicity of 6 in the value of the terms and the addition of the first six terms equals 0. $81.33 - 19 - 100.33 - 81.33 + 19 + 100.33 = 0$.
So, the sum of the first 6000 terms would also be 0. Thus, the sum to 6002 terms would be the sum of the 6001st and the 6002nd terms, which would be the same as the sum of the first two terms of the sequence. Thus, the answer would be $81.33 - 19 = 62.33$ option (c).

69. The value can be written as: $(16 + 17 + 18 + 19)(.....) = 70x$. Hence, the number would be divisible by 70 and the remainder would be 0, option (a).

70. The numerator would be of the form: $(30 - 29)(30^{64} + ... + 29^{64})$. Hence, the value of R would definitely be greater than 1. Hence, $R > 1$ is the correct answer option (d).

71. The value of $p = 1 \times 1! + 2 \times 2! + 3 \times 3! + ... + 10 \times 10! = 11! - 1$. Hence, $p + 2 = 11! + 1$.

When divided by 11!, the remainder would be 1.

option (d)

72. When we reverse a three digit number, the new number formed differs from the original number by a multiple of 99. Also, the value of the multiple of 99 that we have, is decided by the difference between the hundred digit and the units digit. This would be easier to understand and explain using an example—321 becomes 123, and the difference of 198 is arrived at by 99×2 (where 2 comes out of the difference between 3 and 1—the hundreds digit and the units digit).

Thus, in this question, when we see that the difference between B and A is given as a multiple of 7, we realise that the units digit of A must be 7 more than the hundreds digit. Thus, the first number possible is 108 (reversed 801) and the last number possible would be 299 (reversed 992). Only option (b) contains both these values.

73. The rightmost non zero digit of 30^{2720} would be given by the units digit of 3^{2720} = units digit of 3^{4n} = Units digit of $3^4 = 1$, option (a).

74. The only numbers for which this would be true would be 19, 29, 39, 49, 59, 69, 79, 89 and 99. It would not be true for any three digit number. Hence, the right answer is 9 option (d).

75. The numbers 1113, 1119, 1131, 1137, 1155, 1173, 1179, 1191 and 1197 would satisfy the given conditions. Option (a) is correct.

76. The first, third, fourth and fifth options are all positive. Obviously, it has to be option (b), as it is the only negative value in the (5) options.

Solving time is nearly 5–15 seconds.

77. Solve by taking approximate values:

We know that $2^{1/2} = 1.41$

$3^{1/3}$ will be greater than 1.41 as 1.41 × 1.41 × 1.41 = 2 × 1.41 which would be lower than 3.

$4^{1/4} = 1.41$ again as 1.41 × 1.41 × 1.41 × 1.41 = 2 × 2 = 4.

$6^{1/6}$ will definitely be lower than 1.4 as it can be seen that 1.4 × 1.4 = 1.96

So, 1.4 × 1.4 × 1.4 × 1.4 × 1.4 × 1.4 would be closer to 8 than 6.

Similarly, we can see that $12^{1/12}$ would also be lower than 1.4. So, option (b) is the largest value.

78. **Thought Process:**

The number of people in the respective rows will form an *AP* with a common difference of –3.

In this case, we have to find which number of rows is not possible. For this, take it option by option.

Use the principle that for an *AP* the sum is given by n × average.

For 3 rows—the average of the *AP* would be 210. And, this would also be the value of the middle term (when there are 3 rows, the average of the *AP* is given by the middle term). We can thus form an *AP* of 3 terms with middle term 210 and common difference–3. Thus, it is possible to arrange the children in 3 rows.

For 4 rows—the average would be 630/4 = 157.5. Since, there will be two middle terms in this case, the *AP* can be easily formed with the middle terms as 159 and 156 (so that they average 157.5 with a common difference of –3). Thus, it is possible to arrange the children in 4 rows.

For 5 rows—the average of the *AP* would be 630/5 = 126. And, this would also be the value of the middle term (as when there are 5 rows, the average of the *AP* is given by the middle term). We can thus form an *AP* of 5 terms with middle term 126 and common difference –3. Thus it is possible to arrange the children in 5 rows.

For 6 rows—The average would be 630/6 = 105. Since, there will be two middle terms in this case. the *AP* would have to be formed with the two middle terms as 106.5 and 103.5 (so that they average 105 with a common difference of –3). Thus, it is not possible to arrange the children in 6 rows as the value of the terms in the *AP* would not be in integers.

Hence, we will mark option (d).

Maximum solution time: 45 – 60 seconds in case you know the principle of middle terms of an *AP*.

79. Thought process: The sum has to be divisible by 10. This would occur only if the numbers end in 7, 9, 1 and 3.

Option (a): If 21 has to be one of the numbers, the sum would have to be 17 + 19 + 21 + 23 = 20 × 4 (treating the series as an *AP*) = 80. When divided by 10, this does not leave a perfect square.

Option (b): 25 is not possible as a value which would be part of the 4 numbers, as the sum would never end in 0.

Option (c): The numbers would be 37, 39, 41 and 43. $40 \times 4 = 160$. When divided by 10, we get 16 as the value—giving us a perfect square as required. Hence, this is correct.

80. We need to find arithmetic progressions with first term 1 and last term 1000.

In order to do this, we would need to find the number of factors of 999 —which is $37 \times 3^3 \rightarrow 2 \times 4 = 8$ factors. However, the factor 999 cannot be used. Thus, there will be 7 factors option (d).

81. What is the weight of Praja's luggage?

(a) 20 kg

(b) 25 kg

(c) 30 kg

(d) 35 kg

(e) 40 kg

82. What is the free luggage allowance?

(a) 10 kg

(b) 5 kg

(c) 20 kg

(d) 25 kg

(e) 30kg

Note: These were the options in the original CAT paper. The correct answer of 15 kg was missing from the options.

Start from the second question. From the given information, it is pretty clear that the extra luggage for Praja is twice the extra luggage for Raja. This means that, when the two of them take their luggage separately, after reducing the free luggage from 60 kgs, whatever remains has to be divided into three parts and two of them have to be carried by Praja and one by Raja.

This is because, if Raja and Praja were to both carry their luggage separately, the total free luggage would be thrice the free luggage allowance of one of them.

Also, when only one person carried the luggage—the amount of extra luggage would be 50% higher than the extra luggage, when both are carrying their luggage separately.

From the options, it is clear that:

Option (a) is not possible as when both carry their luggage separately, extra luggage = 40 kgs. However, when only 1 carries all the luggage, the extra luggage would be 50 kgs. But from 40 to 50, we do not have a 50% increase. Hence, the option can be rejected.

Repeat the same thought process for Option (b). 50 to 55 not a 50% increase.

Option (c): 20 to 40 not a 50% increase.

Option (d): 10 to 35 not a 50% increase.

Option (e): 0 to 30 not a 50% increase.

Obviously the question is wrong. If you were to try to solve this through an equation:

$1.5(60 - 2x) = 60 - x \rightarrow 2x = 30$ and thus $x = 15$. But the options did not contain this. A lot of students got stuck to this question, but the fact of the matter is that you should have been able to exit this question within a maximum of 1 – 2 minutes.

With a free luggage allowance of 15 kgs, Raja should have had $15 + y$ and Praja $15 + 2y$ giving a total of the two as 60. Thus, $30 + 3y = 60$ gives us $y = 10$. Hence, Praja = 35 kgs. Hence, option (d) is correct.

83. The numbers would be 24, 35, 46, 57, 68 and 79. Hence, 6 numbers. Option (b) is the correct answer.

84. It is obvious that only option (e) (97 + 84 = 181) gives us a prime number. Hence, option (e) is correct.

85. How did you react to this question? The ideal solution pattern in this question is on the basis of pattern recognition. We can try to get this done with a value of n as say 2. So, we have the series {1, 2, 3, 4, 5}.

Then, $X = 9/3 = 3$ and $Y = 3$. Thus, $X - Y = 0$. Converting the options for $n > 2007$, we get the options changing to:

(a) 0

(b) 1

(c) 1

(d) $\frac{3}{4}$

(e) 2

The first and second options obviously have nothing to do with the value of n. Note that the third and fourth options created above, have been created on the basis of n as 2. For the fifth option, 2008 is equivalent to the lowest value of n we can take. So for $n = 2008$, if we get the value as 2008, for $n = 2$, we should get a value of 2.

If you want to be sure, you can also take the value of n as 3, in which case the numbers would be:

{1, 2, 3, 4, 5, 6,7}. In this case also, the values of X and Y will be equal to 4 and $X - Y = 0$.

A little bit of logical quantitative thinking here can also tell you that there will be no difference between, the values of X and Y ever. Thus, option (a) is correct.

86. Thought Process:

Deduction 1: If you were to use 2, 50 miso notes, you can only pay the remaining 7 misos through 1 miso notes.

Deduction 2: If you were to use only 1, 50 miso note, you could use 10 miso notes in 6 different ways (from 0 to 5).

Deduction 3: If you want to avoid 50 miso notes, you could use 10 miso notes in 11 different ways (from 0 to 10).

Hence, the required answer is 1 + 6 + 11 = 18 option (c).

Maximum solution time: 45 seconds.

87. **Thought Process: for this question**

Deduction 1: Question Interpretation: The solution language for this question requires you to think about what possible amount could be such that when it's rupees and paise value are interchanged, the resultant value is 50 paise more than thrice the original amount.

Deduction 2: Option checking process:

Armed with this logic, suppose we were to check for option (a) i.e., The value is above ₹22 but below ₹23. This essentially means that the amount must be approximately between ₹22.66 to ₹22.69. (We get the paise amount to be between 66 to 69 based on the fact that the relationship between the Actual Amount, x and the transposed amount y is: y – 50 paise = $3x$. Hence, values below 22.66 and values above 22.70 are not possible.

→ From this point onwards, we just have to check whether this relationship is satisfied by any of the values between ₹22.66 to ₹22.69.

Also, realise the fact that in each of these cases, the paise value in the value of the transposed amount y would be 22. Thus, $3x$ should give us the paise value as 72 (since we have to subtract 50 paise from the value of 'y' in order to get the value of $3x$).

→ This also means that the unit digit of the paise value of $3x$ should be 2.

→ It can be clearly seen that none of the numbers 66, 67, 68 or 69 when multiplied by 3 give us a units digit of 2. Hence, this is not a possible answer.

Checking for option (b) in the same fashion:

Hence, we check for the check amount to be 18.56. Transposition of the rupee and paise value would give us 56.18. When you subtract 50 paise from this you would get 55.68, which also happens to be thrice 18.56. Hence, the correct answer is option (d).

88. Well, the solution depends on the fact that $7^4 = 2401$ gives the last two digits as 01. Thus, for every $4n$ power of 7, the last two digits would always be 01.

Hence, the required answer would be 01 option (c).

Solving time: If you knew this logic: 5–10 seconds

If you had to discover this logic: 30–40 seconds (by looking at the pattern of the powers of 7)

89. This question is based on odd numbers as only with an odd value of x would you keep getting integers if you halved the value of rice and took out another half a kg from the shop store.

From the options, let us start from the second option. (**Note:** In such questions, one should make it a rule to start from one of the middle options as the normal realisation we would get from checking one option would have been that more than one option gets removed if we have not picked up the correct option- as we would normally know whether the correct answer needs to be increased from the value we just checked or it should be decreased.)

Thus, trying for $x = 7$ according to the second option, you would get

$7 \rightarrow 3 \rightarrow 1 \rightarrow 0$ (after three customers).

This means that $5 \le x \le 8$ is a valid option for this question. Also, since the question is definitive about the correct range, there cannot be two ranges. Hence, we can conclude that option (b) is correct. (**Note:** The total solving time for this question should not be more than 30 seconds. Even, if you are not such an experienced solver through options, and you had to check (b)–(c) options in order to reach the correct option, you would still need a maximum of 90 seconds.)

90. 7, 21, 25, ..., 417 and 16, 21, 26, ..., 466 would have common terms as:

21, 41, 61, ..., 401. The number of such terms would be 20 given by [(401 – 21)/20] + 1 = 20. Hence, option (c) is correct.

91. Well what are we doing? Every time we combine two numbers in the set, we replace it by adding the two and subtracting 1 from it. So, if there are 4 numbers say 1, 2, 3 and 4, our answer would be:

1 2 3 4
2 3 4 (After combining 1 & 2 in the row above)

4 4 (After combining 2 & 3 in the row above)

7 (After combining 4 & 4 in the row above)

Notice that what we are doing here is adding the numbers and subtracting 1 for every iteration. So for numbers from 1 to 40, we would get the sum of 1 to 40 → 55 + 155 + 255 + 355 = 820 and subtract 39 from that (as there would be 39 iterations that would leave us with only 1 number if we start with 40 numbers). Hence, 820 – 39 = 781 is the correct answer option (c).

92. Trial and error gives us the feasibility of $3^1 + 4^2 + 5^3 = 144$ which is the required perfect square. Note that at $m = 1$ and $m = 2$, we do not get a perfect square as the value of the addition. Hence, option (a) which contains the value of $m = 3$ is the correct answer.

93. The first number to have a seed of 9 would be number 9 itself.

The next number whose seed would be 9 would be 18, then 27 and you should recognise that we are talking about numbers which are multiples of 9. Hence, the number of such numbers would be the number of numbers in the Arithmetic Progression: 9, 18, 27, 36, 45, ... 495 = [(495 – 9)/9] + 1 = 55 such numbers. Hence, we will mark option (e).

94. $(a + 3)^2 : b^2 = 9 : 1$ and $(a - 1)^2 : (b - 1)^2 = 4 : 1$

$(a + 3) : b = \pm 3, \frac{a-1}{b-1} = \pm 2$

Four cases are possible:

$a + 3 = 3b, a - 1 = 2b - 2; (a, b) = (3, 2)$ (Rejected, since the integers a, b are of opposite signs)

$a + 3 = 3b, a - 1 = -2b + 2; a, b$ are non integers. (Rejected)

$a + 3 = -3b, a - 1 = 2b - 2; a, b$ are non integers. (Rejected)

$a + 3 = -3b, a - 1 = -2b + 2 ; a = 15, b = -6.$

$a^2 : b^2 = \left(\frac{15}{6}\right)^2 = 25:4$

95. $x^2 - x - 1 = 0$ on solving we get: $x = \frac{1+\sqrt{5}}{2}$

$2x^4 + 2(x + 1)^2 = 2x^2 + 4x + 2 = 2x + 2 + 4x + 2 = 6x + 4 = 6 \times \frac{1+\sqrt{5}}{2} + 4 = 7 + 3\sqrt{5}$

96. $x - y - z = 25$

$x = 25 + y + z$

Maximum possible value of $y + z = 12 + 12 = 24$, Minimum possible value of $y + z = 1 + 1 = 2$.

For $y + z = 2$, $x = 25 + 2 = 27$. $(x, y, z) = (27, 1, 1)$ one value.

For $y + z = 3$, $x = 25 + 3 = 28$, $(x, y, z) = (28, 1, 2), (25, 2, 1)$. Two possible values.

For $y + z = 4$, $x = 25 + 4 = 29$, $(x, y, z) = (29, 1, 3), (29, 3, 1), (29, 2, 2)$. Three possible values.

Similarly for $y + z = 5$, Four possible values.

For $y + z = 6$, five possible values.

⋮

⋮

For $y + z = 13$, twelve possible values.

For $y + z = 14$, $x = 39$, eleven possible values.

For $y + z = 15$, $x = 40$, ten possible values.

The number of solutions = 1 + 2 + 3 + 4 + 5 + ….. + 12 + 11 + 10 = 99.

97. $(n - 5)(n - 10) - 3(n - 2) \leq 0$

$n^2 - 15n + 50 - 3n + 6 \leq 0$

$n^2 - 18n + 56 \leq 0$

$n^2 - 14n - 4n + 56 \leq 0$

$n \in [4, 14]$

Total 11 values are possible.

98. Let 'a' and 'd' are the first term and the common difference of the A.P.

$[a + 6d]^2 = (a + 2d)(a + 16d)$

$a^2 + 36d^2 + 12ad = a^2 + 18ad + 32d^2$

$4d^2 = 6ad$, $a : d = 2 : 3$

99. Common difference = 7 – 3 = 4.

$a_1 + a_2 + ... + a_{3n} = 1830$,

3 + 7 + 11 + ...$3n$ terms = 1830.

$\frac{3n}{2}[2.3 + (3n - 1)4] = 1830$

$\frac{3n}{2}[2 + 12n] = 1830$

$3n(1 + 6n) = 1830 = 30 \times 61$

$3n = 30$ or $n = 10$.

$a_1 + a_2 + ... + a_n = 3 + 7 + 11 + ...10$ terms = 210

$210m > 1830$

$m > 8.7$. Hence, the smallest integer value of $m = 9$.

100. $15600 = 24 \times 25 \times 26$

Sum of squares = $24^2 + 25^2 + 26^2 = 1877$

101. Five consecutive even numbers ending with $2a_3$. Hence, the numbers are $2a_3 - 8, 2a_3 - 6, 2a_3 - 4, 2a_3 - 2, 2a_3$.

Required sum = $2a_3 - 8 + 2a_3 - 6 + 2a_3 - 4 + 2a_3 - 2 + 2a_3 = 450$

$10a_3 - 20 = 450$ or $a_3 = 47$. Since, we are looking for 5 consecutive odd integers, we get $a_5 = 51$

102. You can experimentally verify that there would be only three pairs of values that would satisfy this- viz: (18, 18) ; (12, 36) and (10, 90). No other pair would satisfy this condition.

103. Let the first term be 'a' and the common ratio be 'r'.

$a_n = 3(a_{n+1} + a_{n+2} + ...)$

$ar_{n-1} = 3(ar_n + ar_{n+1} + ...)$

$ar_{n-1} = 3\left(\frac{ar^n}{1-r}\right)$ or $3r = 1 - r$ or $r = \frac{1}{4}$

$a_1 + a_2 + a_3 + \ldots = 32$

$a + ar + ar_2 + \ldots = 32$

$$\frac{a}{1-r} = 32$$

$a = 24$

$a_5 = ar_4 = 24 ◊ (1/4)_4 = 3/32$

104. $a_1 = \frac{1}{2 \times 5} = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{5}\right)$

$$a_2 = \frac{1}{5 \times 8} = \frac{1}{3}\left(\frac{1}{5} - \frac{1}{8}\right)$$

⋮

⋮

$$a_{100} = \frac{1}{299 \times 302} = \frac{1}{3}\left(\frac{1}{299} - \frac{1}{302}\right)$$

The required sum = $a_1 + a_2 + a_3 + a_4 + a_5 \ldots + a_{100}$ =

$$\frac{1}{3}\left(\frac{1}{2} - \frac{1}{5}\right) + \frac{1}{3}\left(\frac{1}{5} - \frac{1}{8}\right) + \ldots.. + \frac{1}{3}\left(\frac{1}{299} - \frac{1}{302}\right)$$

$$= \frac{1}{3}\left(\frac{1}{2} - \frac{1}{302}\right) = \frac{1}{3} \times \frac{300}{604} = \frac{100}{604} = \frac{25}{151}$$

105. Let other two numbers are 'a' and 'b'.

$73 \times a \times b - 37 \times a \times b = 720$

$36 \times a \times b = 720$

$a \times b = 20$

To minimise the sum of squares of two numbers 'a' must be equal to 'b'.

$a = b = \sqrt{20}$

Required sum = $a2 + b2 = 20 + 20 = 40$.

106. $(u - 2v - 1)2 + u2 \rightarrow u2 + 4v2 + 1 - 4uv + 4v - 2u + u2 = -4vu - 4v2$

$2u2 + 8v2 + 1 + 4v - 2u = 0$

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots\ldots + \frac{1}{\sqrt{a_n} + \sqrt{a_{n-1}}}$$

Hence, option (d) is correct.

107. $16y = \frac{5x + 12z}{2}$ or $32y = 5x + 12z$

x, y and z are a/r, a, ar, respectively.

$$32a = \frac{5a}{r} + 12ar$$

$$32 = \frac{5}{r} + 12r$$

$12r^2 - 32r + 5 = 0$

On solving the above equation, we get $r = 1/6, 5/2$

As $x < y < z$, the Geometric Progression has to be an increasing GP. $r = 5/2$ is the only possible value that satisfies this requirement. Hence, 5/2 is the correct answer.

108. $0.25 < 2^x < 200$ for $-2 < x < 8$. Between these values, for ($x = 0, 1, 2, 4, 6$), $2^x + 2$ would be divisible by 3 or 4. Hence, there are total five integers.

109. $\frac{x^{2018}y^{2017}}{x^{2016}y^{2019}} = \frac{1}{16}$ or $\frac{x^2}{y^2} = \frac{1}{16}$ or $\frac{x}{y} = \frac{1}{4}$

$y = 4x$

Put $y = 4x$ in $x^{2018}\,y^{2017} = \frac{1}{2}$ or $x^{2018}\,(4x)^{2017} = \frac{1}{2}$ or $x^{4035} = \frac{1}{2^{4035}}$ or $x = \frac{1}{2}$

Consequently, $y = 2$

Value of $x^2 + y^3 = \left(\frac{1}{2}\right)^2 + 8 = \frac{33}{4}$. Hence, option (d) is correct.

110. Let 'x' is the average of all the 52 positive integers $a_1, a_2, \ldots\ldots, a_{52}$.

$a_1 + a_2 + a_3 + \ldots.. + a_{52} = 52x$ (1)

Therefore, the average of $a_2, a_3, a_4, a_5, \ldots.. a_{52} = x + 1$

Or $a_2 + a_3 + a_4 + a_5 + \ldots.. a_{52} = 51(x + 1)$ (2)

From (1) and (2), we get:

$a_1 = x - 51$. a_1 will be maximum when 'x' is maximum. The maximum value of 'x' will happen if the numbers from a_{52}, a_{51}, a_{50} are taken as 100, 99, 98 etc. Thus, the numbers from a_1 to a_{52} would be 49, 50, 51,..100 in order to maximise the value of x. With these values, the average of these 52 numbers turns out as 74.5. So, the maximum value of a_1 would be $74.5 - 51 = 23.5$. But since a_1 is an integer, its value should turn out to be 23. Hence, option (d) is correct.

111. Let $S_n = t_1 + t_2 + ... + t_n$

$t_k = S_k - S_{k-1} = 2k^2 + 9k + 13 - 2(k-1)^2 - 9(k-1) - 13 = 4k + 7 = 103$

$k = 24$

112. $N^N = 2^{160}$ is possible only for $N = 32$

For $N^2 + 2^N = 32^2 + 2^{32} = 2^{10} + 2^{32}$ to be an integral multiple of 2^x, then the maximum possible value of $x = 10$.

113. $4^n > 17^{19}$

$4^{38} = 16^{19} < 17^{19}$. Hence, for $n = 38$, $4^n < 17^{19}$

Hence, option (a), (c) and (d) are incorrect. Only option (b) is correct.

114. $n^3 - 11n^2 + 32n - 28 > 0$. You would need to do a trial-and-error with values to check for the smallest value of 'n' where the expression becomes positive. The first thing that should be evident, should be that at n=0, the expression is negative and for negative values of n, the value of the expression would always be negative (since each of the four terms of the expression would be negative). So, your check needs to start with positive integral values of x. Checking by inserting values of n as 1, 2, 3,...we see that at $n = 7$, the value becomes equal to 0. And at $n = 8$ we get: $n^3 - 11n^2 + 32n - 28 = 36$

Hence, the smallest possible integer is 8.

115. Let the number be 'ab' where b is not equal to 0.

According to the question: $(10a + b) > 3(10b + a)$

$7a > 29b$

The above condition is possible when $a = 5$ and $b = 1$. $a = 6$ and $b = 1$. $a = 7$ and $b = 1$. $a = 8$, $b = 1$, $a = 9$ and $b = 1, 2$. Hence, six such numbers are possible. Option (a) is correct.

116. Let a and b are the given numbers.

According to the question: $a^2 + b^2 = 97$

$$a^2 + b^2 - 2ab = 97 - 2ab$$
$$(a - b)^2 = 97 - 2ab$$

$97 - 2ab$ must be non-negative, since the LHS is non-negative. This is not possible for $ab = 64$.

117. $S = 7 \times 11 + 11 \times 15 + 15 \times 19 + \ldots\ldots, + 95 \times 99^{th}$ term of the series:

$$S = \sum_{n=1}^{n=23} (4n+3)\times(4n+7) =$$

$$\sum_{n-1}^{n=23} 16n^2 + 40n + 21 = \frac{16\times23\times24\times47}{6} + 40\times\frac{23\times24}{2} + 21 \times 23 = 80707$$

Hence, option (a) is correct.

118. The value of A for $n = 1$, would be 0 and for $n = 2$ would be 1225. B, on the other hand is the set of all multiples of 35 (starting from 0). It can also be observed that every member of A would be a multiple of 35, but obviously not all multiples of 35. Hence, clearly every member of A is in B, while every member of B is not in A.

119. Let the population of the town in 2019 is ‘*p*’. Population in 2020 (2nd year) is 3 + 2 × 1000 = 1003 × 2^1 – 3 = 2003

Population in 2021 (3rd year) = 3 + 2 × 2003 = (1003) × 2^2 – 3

Population in 2022 (4th year) = (1003) × 2^3 – 3

Similarly, population in 2034 will be (1003) × 2^{15} – 3. Hence, option (c) is correct.

120. Do a quick factor search of 616.

1 × 616; 2 × 308; 4 × 154; 7 × 88; 8 × 77; 11 × 56; 14 × 44; 22 × 28

The pairs to consider are: 22 & 28 (for option a); 77 and 8 for option (b); 7 and 88 for option (c); and 44 and 14 for option (d). Also one needs to understand that the targeted ratio of 157 : 3 has a value of 52.33.

For option (a), we can see that difference of cubes $28^3 - 22^3 = (28 - 22)(28^2 + 28 \times 22 + 22^2)$, while the cube of the difference = 6^3. So the effective thought would go as follows:

$$\frac{(28-22)(28^2+28\times 22+22^2)}{6\times 6\times 6} = \frac{784+616+484}{36} = \frac{1884}{36}$$

= 52.33

Hence, this option is correct.

Just for your information, the mathematical process goes like this:

Let the numbers are ‘*a*’ and ‘*b*’.

$ab = 616 = 2^3 \times 7 \times 11$

$$\frac{a^3 - b^3}{(a-b)^3} = \frac{157}{3}$$

$3a^3 - 3b^3 = 157a^3 - 157b^3 - 157 \times 3 \times ab(a-b)$

$154a^3 - 154b^3 + 3 \times 616 \times 157(a-b) = 0$

$a^3 - b^3 + 3 \times 4 \times 157(a-b) = 0$

$(a-b)(a^2 + b^2 + ab) + 3 \times 4 \times 157(a-b) = 0$

$a^2 + b^2 + ab = 3 \times 4 \times 157$

$a^2 + b^2 + 2ab = 3 \times 4 \times 157 + ab$

$= 3 \times 4 \times 157 + 616 = 2500$

$(a+b)^2 = 2500$ or $a + b = 50$

Hence, option (a) is correct.

121. $S_n = a_1 + a_2 + a_3 + \ldots + a_n$

$a_n = S_n - S_{n-1}$

$a_{11} = S_{11} - S_{10} = 3 \times (2^{12} - 2) - 3 \times (2^{11} - 2) = 3 \times (2^{12} - 2^{11}) = 3 \times 2^{11} = 6144.$

122. Let the common difference of the AP be d.

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} = \frac{1}{\sqrt{a_1} + \sqrt{a_2}} \times \frac{\sqrt{a_2} - \sqrt{a_1}}{\sqrt{a_2} - \sqrt{a_1}} = \frac{\sqrt{a_2} - \sqrt{a_1}}{d}$$

$$\frac{1}{\sqrt{a_2} + \sqrt{a_3}} = \frac{\sqrt{a_3} - \sqrt{a_2}}{d}$$

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots\ldots + \frac{1}{\sqrt{a_n} + \sqrt{a_{n-1}}}$$

$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{d} + \frac{\sqrt{a_3}-\sqrt{a_2}}{d} + \ldots.. + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d}$$

$$= \frac{\sqrt{a_n}-\sqrt{a_1}}{d}$$

$$\frac{\sqrt{a_n}-\sqrt{a_1}}{d} = \frac{\sqrt{a_n}-\sqrt{a_1}}{d} \times \frac{\sqrt{a_n}+\sqrt{a_1}}{\sqrt{a_n}+\sqrt{a_1}}$$

$$= \frac{a_n - a_1}{d\left(\sqrt{a_n}+\sqrt{a_1}\right)} = \frac{n-1}{\left(\sqrt{a_n}+\sqrt{a_1}\right)}$$

Hence, option (b) is correct.

123. $(\sqrt{2})^{19} \times 3^4 \times 4^2 \times 9^m \times 8^n = 3^n \times 16^m \times (\sqrt[4]{64})$

$\sqrt{2} \times 2^9 \times 3^4 \times 2^4 \times 3^{2m} \times 2^{3n} = 3^n \times 2^{4m} \times 2 \times \sqrt{2}$

$2^{13+3n} \times 3^{4+2m} = 2^{4m+1} \times 3^n$

On comparing RHS with LHS, we get:

$13 + 3n = 4m + 1$ and $4 + 2m = n$

On solving the above two equations, we get: $m = -12$, $n = -20$

124. $a^2 + b^2 = 25$ and $x^2 + y^2 = 169$ $(a^2 + b^2)(x^2 + y^2) = a^2x^2 + b^2y^2 + a^2y^2 + b^2x^2$

$= 25 \times 169 = 5^2 \times 13^2$ (1)

$ax + by = 65$ & $(ax + by)^2 = a^2x^2 + b^2y^2 + 2abxy$

$= 65^2 = 13^2 \times 5^2$ (2)

equation (1) – equation (2):

$a^2y^2 + b^2x^2 - 2abxy = 0$

$(ay - bx)^2 = 0$ *or* $ay - bx = 0$

Hence, option (a) is correct.

125. $a_1 - a_2 = 2, a_1 - a_2 + a_3 = 3$

On solving the above two equations, we get $a3 = 1$.

$a_1 - a_2 + a_3 - a_4 = 4, a_4 = -1$

Similarly, $a_n = 1$ for odd value of n and $a_n = -1$ for even value of n.
$a_{51} + a_{52} + ... + a_{1023} = 1 - 1 + 1 - 1 + ... + 1 = 1$. Hence, option (b) is correct.

126. $2^4 \times 3^5 \times 10^4 = 2^4 \times 3^5 \times 2^4 \times 5^4 = 2^8 \times 3^5 \times 5^4$

For the factor to be a perfect square, all the prime factors should be even powers of the number.

In 2^8, the factors which are perfect squares are $2^0, 2^2, 2^4, 2^6, 2^8$. Five values.

In 3^5 the factors which are perfect squares are $3^0, 3^2, 3^4$. Three values.

In 5^4 the factors which are perfect squares are $5^0, 5^2, 5^4$. Three values.

Number of perfect squares which are greater than $1 = 5 \times 3 \times 3 - 1 = 44$

127. $n^2 - m^2 = (n - m)(m - n) = 105$

We need to find factor pairs of 105 and check for values of n and m. Thus, for

$105 \times 1 = 105 \rightarrow n - m = 1, n + m = 105; n = 53, m = 52$

$35 \times 3 = 105 \rightarrow n - m = 3, n + m = 35; n = 19, m = 16$

$21 \times 5 = 105 \rightarrow n - m = 5, n + m = 21; n = 13, m = 8$

$15 \times 7 = 105 \rightarrow n - m = 7, \text{n} + m = 15; n = 11, m = 4$

Thus, only four possible pairs can be found.

128. Let the six digit number is *pqrstu*.

$u = p + q + r, t = p + q, r = p, q = 2p, s = t + u$

Therefore, $s = 2p + 2q + r = 2p + 4p + p = 7p$.

p cannot be 0 as the number is a six-digit number. p cannot be *1* as s would become a two-digit number. Therefore, $p = 1$ and $s = 7$. Hence, the correct answer is 7.

129. First common term = 19

Common difference = LCM of 5, 4 = 20. We will need to find the number of terms of the series, 19, 39, 59,…399. The number of terms in the series is: 380/20 + 1 = 20.

Hence, option (a) is correct.

130. $(2n + 1) + (2n + 3) + (2n + 5) + \ldots + (2n + 47) = 5280$

There are total $\frac{47-1}{2}+1 = 24$ terms in the above series. $(2n + 1) + (2n + 3) + (2n + 5) + \ldots + (2n + 47) = 5280$

$48n + (1 + 3 + 5 + \ldots\ldots + 47) = 5280$

$48n + 576 = 5280$

On solving, we get $n = 98$

$1 + 2 + 3 + \ldots. + 98 = \frac{98 \times 99}{2} = 4851.$

BLOCK II
AVERAGES AND MIXTURES

Chapter 3 – **Averages**

Chapter 4 – **ligations and Mixtures**

BACK TO SCHOOL...

- **Relevance of Average**

Average is one of the most important mathematical concepts that we use in our day-to-day life. In fact, even the most non-mathematical individuals regularly utilise the concept of averages on a day-to-day basis.

So, we use averages in all the following and many more instances.

- How a class of students fared in an exam is assessed by looking at the average score
- What is the average price of items purchased by an individual
- A person might be interested in knowing his average telephone expenditure, electricity expenditure, petrol expenditure, etc.
- A manager might be interested in finding out the average sales per territory or even the average growth rate month-to-month.
- Clearly there can be immense application of averages that you might be able to visualise on your own

- **Meaning of an Average**

An average is best seen as a representative value which can be used to represent the value of the general term in a group of values.

For instance, suppose that a cricket team had 10 partnerships as follows:

1st wicket 28	2nd wicket 42
3rd wicket 112	4th wicket 52
5th wicket 0	6th wicket 23
7th wicket 41	8th wicket 18
9th wicket 9	10th wicket 15

On adding the ten values above, we get a total of 340—which gives an average of 34 runs per wicket, i.e. the average partnership of the team was 34 runs.

In other words, if we were to replace the value of all the ten partnerships by 34 runs, we would get the same total score. Hence, 34 represents the average partnership value for the team.

Suppose, in a cricket series of five matches between two countries, you are given that Team A had an average partnership of 58 Runs per wicket while Team B had an average partnership of 34 runs per wicket. What conclusion can you draw about the performance of the two teams, given that both the teams played five complete test matches?

Obviously, Team B would have performed much worse than Team A: For that matter, if I tell you that the average daytime high temperature of Lucknow was 18° C for a particular month, you can easily draw some kind of conclusion in your mind about the month we could possibly be talking about.

Thus, you should realise that the beauty of averages lies in the fact that it is one single number that tells you a lot about the group of numbers—hence, it is one number that represents an entire group of numbers.

But one of the key concepts that you need to understand before you move into the chapters of this block is the concept of WEIGHTED AVERAGES.

As always, the concept is best explained through an example.

Suppose I had to buy a shirt and a trouser and let us say that the average cost of a shirt was ₹1200 while that of a trousers was ₹900.

In such a case, the average cost of a shirt and a trouser would be given by (1200 + 900)/2 = 1050.

This can be visualised on the number line as:

(midpoint) = answer

As you can easily see in the figure, the average occurs at the midpoint of the two numbers.

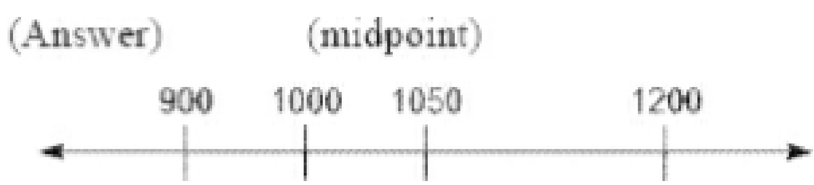
Now, let us try to modify the situation:

Suppose, I were to buy 2 trousers and 1 shirt. In such a case, I would end up spending (900 + 900 + 1200) = ₹3000 in buying a total of 3 items. What would be my average in this case?

Obviously, 3000/3 = ₹1000!!

Clearly, the average has shifted!!

On the number line, we could visualise this as follows:



It is clearly visible that the average has shifted towards 900 (which was the cost price of the trousers—the larger purchased item).

In a way, this shift is similar to the way a two pan weighing balance shifts on weights being put on it. The balance shifts towards the pan containing the larger weight.

Similarly, in this case, the correct average (1000) is closer to 900 than it is to 1200. This has happened because the number of elements in the group of average 900 is greater than the number of elements in the group having average 1200. Since, this is very similar to the system of weights, we call this as a weighted average situation.

At this stage, you should realise that weighted averages are not solely restricted to two groups. We can also come up with a weighted average situation for three groups (although in such a case the representation of the weighted average on the number line might not be so easily possible.) In fact, it is the number line representation of a weighted average situation that is defined as alligation (when 2 groups are involved).

CAT Scan 1

1. X's age is 1/10th of Y's present age. Y's age will be thrice of Z's age after ten years. If Z's eighth birthday was celebrated two years ago, then the present age of X must be

   (a) 5 years

   (b) 10 years

   (c) 15 years

   (d) 20 years

2. Dravid was twice as old as Rahul 10 years back. How old is Rahul today if Dravid will be 45 years old 15 years hence?

(a) 20 years

(b) 10 years

(c) 30 years

(d) None of these

3. A demographic survey of 100 families in which two parents were present revealed that the average age A, of the oldest child, is 15 years less than ½ the sum of the ages of the two parents. If *X* represents the age of one parent and *Y* the age of the other parent, then which of the following is equivalent to A?

(a) $\frac{X+Y}{2} - 15$

(b) $\frac{X+Y}{2} + 15$

(c) $\frac{X+Y}{2} - 15$

(d) $X + Y - 7.5$

4. If 10 years are subtracted from the present age of Randy and the remainder divided by 12, then you would get the present age of his grandson Sandy. If Sandy is 19 years younger to Sundy whose age is 24, then what is the present age of Randy?

(a) 80 years

(b) 70 years

(c) 60 years

(d) None of these

5. Two groups of students, whose average ages are 15 years and 25 years, combine to form a third group whose average age is 23 years. What is the ratio of the number of students in the first group to the number of students in the second group?

(a) 8 : 2

(b) 2 : 8

(c) 4 : 6

(d) None of these

6. A year ago, Mohit was four times his son's age. In six years, his age will be 9 more than twice his son's age. What is the present age of the son?

(a) 10 years

(b) 9 years

(c) 20 years

(d) None of these

7. In 1952, I was as old as the number formed by the last two digits of my birth year. When I mentioned this interesting coincidence to my grandfather, he surprised me by saying that the same applied to him also.

The difference in our ages is:

(a) 40 years

(b) 50 years

(c) 60 years

(d) None of these

CAT Scan 2

8. The average age of three boys is 18 years. If their ages are in the ratio 4:5:9, then the age of the youngest boy is

   (a) 8 years

   (b) 9 years

   (c) 12 years

   (d) 16 years

9. "I am eight times as old as you were when I was as old as you are", said a man to his son. Find out their present ages if the sum of their ages is 75 years.

   (a) 40 years and 35 years

   (b) 56 years and 19 years

   (c) 48 years and 27 years

   (d) None of these

10. My brother was 3 years of age when my sister was born, while my mother was 26 years of age when I was born. If my sister was 4 years of age when I was born, then what was the age of my father and mother respectively when my brother was born?

    (a) 35 years, 33 years

    (b) 35 years, 29 years

(c) 32 years, 23 years

(d) None of these

11. Namrata's father is now four times her age. In five years, he will be three times her age. In how many years, will he be twice her age?

(a) 5 years

(b) 20 years

(c) 25 years

(d) 15 years

12. A father is twice as old as his daughter. Twenty years back, he was seven times as old as the daughter. What are their present ages?

(a) 24, 12

(b) 44, 22

(c) 48, 24

(d) None of these

13. The present ages of three persons are in the proportion of 5:8:7. Eight years ago, the sum of their ages was 76. Find the present age of the youngest person.

(a) 20

(b) 25

(c) 30

(d) None of these

14. The average age of a class is 14.8 years. The average age of the boys in the class is 15.4 years and that of girls is 14.4 years. What is the ratio of boys to girls in the class?

(a) 1 : 2

(b) 3 : 2

(c) 2 : 3

(d) None of these

CAT Scan 3

15. In an organisation, the daily average wages of 20 illiterate employees is decreased from ₹25 to ₹10, thus the average salary of all the literate (educated) and illiterate employees is decreased by ₹10 per day. The number of educated employees working in the organisation are

(a) 15

(b) 20

(c) 10

(d) 25

16. Mr.Akhilesh Bajpai while going from Lucknow to Jamshedpur covered half the distance by train at the speed of 96 km/hr, half the rest of the distance by his scooter at the speed of 60 km/hr and the remaining distance at the speed of 40 km/hr by car. The average speed at which he completed his journey is

(a) 64 km/hr

(b) 56 km/hr

(c) 60 km/hr

(d) 36 km/hr

17. There are four types of candidates in MINDWORKZZ preparing for the CAT. The number of students of Engineering, Science, Commerce and Humanities is 400, 600, 500 and 300 respectively and the respective percentage of students who qualified the CAT is 80%, 75%, 60% and 50%, respectively overall percentage of successful candidates in our institute is

(a) 67.77%

(b) 66.66%

(c) 68.5%

(d) None of these

18. Mr. Jagmohan calculated the average of 10 'Three digit numbers'. But due to mistake, he reversed the digits of a number and thus, his average increased by 29.7. The difference between the unit digit and hundreds digit of that number is

(a) 4

(b) 3

(c) 2

(d) can not be determined

Directions for Questions 19 and 20: Answer the questions based on the following information.

Production pattern for number of units (in cubic feet) per day.

Days	1	2	3	4	5	6	7
Numbers of units	150	180	120	250	160	120	150

For a truck that can carry 2,000 cubic feet, hiring cost per day is ₹1,000. Storing cost per cubic feet is ₹5 per day. Any residual material left at the end of the seventh day has to be transferred.

19. If all the units should be sent to the market, then on which days should the trucks be hired to minimise the cost:

(a) 2nd , 4th , 6th , 7th

(b) 7th

(c) 2nd , 4th , 5th , 7th

(d) None of these

20. If the storage cost is reduced to ₹0.9 per cubic feet per day, then on which day/days, should the truck be hired?

(a) 4th

(b) 7th

(c) 4th and 7th

(d) None of these

21. A 20% ethanol solution is mixed with another ethanol solution, say, *S* of unknown concentration in the proportion 1:3 by volume. This mixture is then mixed with an equal volume of 20% ethanol solution. If the resultant mixture is a 31.25% ethanol solution, then the unknown concentration of *S* is

(a) 52%

(b) 50%

(c) 55%

(d) 48%

ANSWER KEY

CAT Scan 1

1. (a)
2. (a)
3. (a)
4. (b)
5. (b)
6. (b)
7. (b)

CAT Scan 2

8. (c)
9. (c)
10. (d)
11. (b)
12. (c)
13. (b)
14. (c)

CAT Scan 3

15. (c)
16. (a)
17. (a)

18. (b)
19. (a)
20. (b)
21. (b)