Real Numbers

Real-Numbers

1. **Natural Numbers:** Counting numbers $1, 2, 3, 4, 5, 6, \ldots etc.$ are called Natural Numbers.

2. **Whole Numbers:** Counting numbers with 0 are whole numbers, i.e. $0, 1, 2, 3, 4, \ldots$ etc. are Whole numbers.

3. **Integers:** All natural numbers, the negatives of all natural numbers and zero are collectively known as integers, i.e. $\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$ etc. are integers.

4. **Rational Numbers:** The numbers that can be expressed in the form $\frac{p}{q}$, where p and q are integers are called rational numbers. Each rational number can be expressed either in a terminating or in a non-terminating repeating decimal form.

5. **Irrational Numbers:** The numbers which when expressed in decimal form are expressible as non-terminating and non-repeating decimals are known as irrational numbers.

6. **Euclid's Division Lemma or Euclid's Division Algorithm**

For any two given positive integer a and b there exist unique integer q and r satisfying
$a = bq + r$, where $0 \le r < b$

Here a is known as dividend, b as divisor, q as quotient and r as remainder.

7. **Fundamental Theorem of Arithmetic:** Every composite number can be expressed (factorised) as a product of primes and this factorisation is unique apart from order in which prime factors occurs.

To find the H.C.F. and L.C.M. of numbers using the method of Fundamental Theorem of Arithmetic (Prime Factorisation Method):

Express each one of the given numbers as a product of prime factors. Then,
H.C.F. = Product of the smallest powers of each common prime factor in the numbers
L.C.M. = Product of the greatest powers of each prime factor, involved in the numbers

8. **To test whether a given rational number is a terminating or a repeating decimal:**

A rational number, in the simplest form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is:

A terminating decimal if prime factorisation of q is of the form $\left(2^m \times 5^n\right)$, where m and n are non-negative integers.

(ii) A non-terminating repeating decimal if prime factorisation of q is not of the form $\left(2^m \times 5^n\right)$, where m and n are non-negative integers.

Snap Test

1. **Using Prime Factorisation method, find the H.C.F. of 9775 and 11730.**

(a) 1990 (b) 1980
(c) 1955 (d) 1985
(e) None of these

Ans. (c)

Explanation: $9775 = 52 \times 17 \times 23$

$11730 = 2 \times 3 \times 5 \times 17 \times 23$

$\therefore$ H.C.F. $(9775, 11730) = 5 \times 17 \times 23 = 1955.$

2. **Without actual division find whether the rational number $\frac{45}{37500}$ is a terminating or a non-terminating repeating decimal.**

(a) Terminating
(b) Non-terminating repeating
(c) Non-terminating non-repeating
(d) Can't be find
(e) None of these

Ans. (b)

Explanation: The given rational number is in the form $\frac{p}{q}$, where p = 41 and q = 37500.

After prime factorisation of 37500 we get:

$37500 = (2^2 \times 3 \times 5^2)$ which is not of the form $(2^m \times 5^n)$

Hence, $\frac{41}{37500}$ is a non-terminating repeating decimal.

3. **Which one of the following is equivalent to $2.\overline{357}$?**

(a) $\frac{785}{330}$ (b) $\frac{785}{333}$

(c) $\frac{785}{335}$ (d) $\frac{780}{335}$

(e) None of these

Ans. (b)

Explanation:

Let x = 2.357357357..... ... (i)
Then, 1000x = 2357.357357357.... ... (ii)
Subtracting (i) from (ii), we get:

$$999x = 2355 \Rightarrow x = \frac{2355}{999} = \frac{785}{333}$$

4. **Write the decimal number $0.5\overline{7}$ in the form of $\frac{p}{q}$ in the simplest form.**

(a) $\frac{26}{45}$ (b) $\frac{26}{40}$

(c) $\frac{45}{26}$ (d) $\frac{45}{22}$

(e) None of these

Ans. (a)

Explanation:

Let $x = 0.5\overline{7} = 0.57777$ ... (i)

Then, $10x = 5.7777...$ ... (ii)

$100x = 57.7777...$ ... (iii)

Subtracting (ii) from (iii), we get:

$$90x = 52 \Rightarrow x = \frac{52}{90} = \frac{26}{45}$$