2 Rational Numbers

Let's learn :

- **Natural numbers :**
  - 1, 2, 3, ... are natural numbers.
  - 1 is the smallest natural number.
  - Natural numbers are also called calculative numbers. These are also called positive integers.
  - They are infinite.
- **Whole numbers :**
  - 0, 1, 2, 3, ... are whole numbers.
  - 0 (zero) is the smallest whole number.
  - They are infinite.
- **Negative integers :**
  - (–1), (–2), (–3), ... are negative integers.
  - They are infinite.
- **Integers :**
  - The group of positive integers, negative integers and zero are called integers.
  - They are infinite.
- **Rational numbers :**
  - $\frac{1}{2}, \frac{2}{5}, \frac{3}{7}, \frac{4}{8}, \ldots$ etc. are rational numbers.
  - $\left(-\frac{1}{5}\right), \left(-\frac{3}{5}\right), \left(-\frac{7}{8}\right), \left(-\frac{5}{7}\right), \ldots$ are negative rational numbers.
  - In $\frac{1}{2}$, 1 is numerator and 2 is denominator. Similarly in $\left(-\frac{3}{5}\right)$, (–3) is numerator and 5 is denominator.
- **Decimal rational numbers :**
  - 0.5, –0.2, 1.3, 2.25 are decimal rational numbers.
  - Decimal rational numbers can be shown as below

$$0.5 = \frac{5}{10} \qquad 1.3 = \frac{13}{10}$$

$$-0.2 = \frac{-2}{10} \qquad 2.25 = \frac{225}{100}$$

Let's learn new :

We know that $\frac{2}{5}$, $-\frac{4}{7}$ etc. are non-integers, -0.3, 0.7 etc. are decimal non-integers which are denoted as $-\frac{3}{10}$ and $\frac{7}{10}$ respectively as simple fractions. Similarly, number 5, 9, 0 etc are also can be denoted by writing 1 as denominators.

For example, $5 = \frac{5}{1}$, $9 = \frac{9}{1}$, $0 = \frac{0}{1}$ etc.

Therefore, any number can be denoted in the $\frac{p}{q}$ form.

- The numbers which can be in the form $\frac{p}{q}$ (where p is a zero, positive and negative integer and q is a positive integer) is called a rational number.

Therefore, group of integers and non-integers are called rational numbers.

In given rational number $\frac{p}{q}$,

- If p is a positive integer, then it is a positive rational number.

  For example, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{2}{5}$, $\frac{5}{7}$, $\frac{2}{10}$, 5, ...
- If p is a negative integer, then it is a negative rational number.

  For example, $-\frac{2}{3}$, $-\frac{5}{7}$, $-\frac{5}{8}$, $-\frac{3}{7}$, -11, -20, ...
- If p is a zero, then 0 (zero) is a rational number.

  For example, 0

Write examples of required numbers in the following Table :

Negative rational number	Positive rational number	Zero rational number

- **Representation of rational numbers on the number line :**

You have learnt to represent integers on a number line. Let us revise them.

Represent the following numbers on the number line as required :

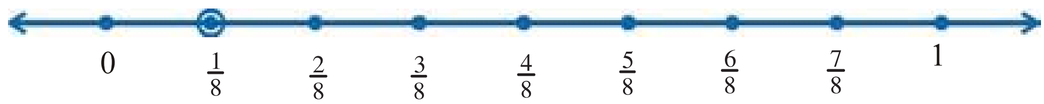
(1) Any two integers less than 3.

(2) Two integers more than (–4) and less than (–1).

(3) Integer more than 2 and less than 4.

Example 1 : Represent $\frac{1}{8}$ on the number line.

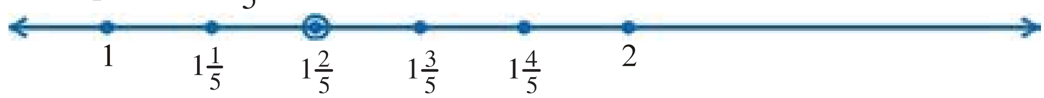
$\frac{1}{8} < 1$, therefore $\frac{1}{8}$ can be represented on the number line between 0 and 1. In the denominator of $\frac{1}{8}$, there is 8, so we will have to divide 0 to 1 in 8 equal parts on number line. His first part is $\frac{1}{8}$, second part is $\frac{2}{8}$ is represented respectively. Represent $\frac{1}{8}$ as ◉ :



Think : How many rational numbers can be represented between 0 and 1 on the number line.

Example 2 : Represent $1\frac{2}{5}$ on the number line.

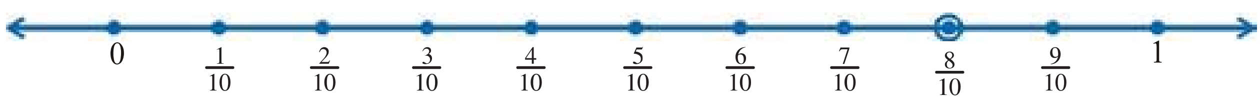
In $1\frac{2}{5}$, 1 is integer and $\frac{2}{5}$ is rational. Therefore, $1\frac{2}{5}$ will have to represent between 1 and 2. In the denominator of $\frac{2}{5}$, there is 5, so we will have to divide 1 to 2 in 5 equal parts. Here, 2 is in the numerator of $\frac{2}{5}$, therefore, on the second part we can represent $1\frac{2}{5}$.



Example 3 : Represent 0.8 on the number line.

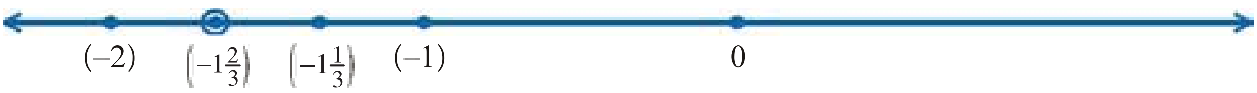
$0.8 = \frac{8}{10}$, here 8 is in numerator and 10 is in denominator.

See, by dividing 10 equal parts from 1 to 10, each part shows $\frac{1}{10}$. In the numerator of $\frac{8}{10}$ there is 8, therefore eighth point is represented as 0.8.



Example 4 : Represent $\left(-1\frac{2}{3}\right)$ on the number line.

$(-1) > \left(-1\frac{2}{3}\right) > (-2)$, therefore $\left(-1\frac{2}{3}\right)$ is to be represented between (-1) and (-2) on the number line. Because $\left(-1\frac{2}{3}\right)$ is greater than (-2) and smaller than (-1). The line segment from (-2) to (-1) is divided in three equal parts the $\left(-1\frac{2}{3}\right)$ is represented on L.H.S. of (-1) on the second point.



Think : Where to represent (-2.5) on the number line ?

- **Represent the given rational numbers on separate number lines :**

(1) $\left(-1\frac{4}{5}\right)$ (2) $2\frac{1}{4}$ (3) $\frac{4}{7}$ (4) $\left(-\frac{3}{5}\right)$ (5) 0.5 (6) (-1.5)

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- **The additive inverse negative and the multiplicative inverse (reciprocal) of rational number :**

Negative of a number : Which number should be added to $\frac{2}{5}$ to get zero result.

For every rational number always such a number exist whose addition to the given rational number results zero. These rational numbers are called negative numbers to each other.

For example, $\frac{2}{5} + \left(-\frac{2}{5}\right) = 0.$

Therefore, $\frac{2}{5}$ and $\left(-\frac{2}{5}\right)$ are negative numbers to each other.

$\frac{1}{3} + \left(-\frac{1}{3}\right) = 0$

Therefore, $\frac{1}{3}$ and $\left(-\frac{1}{3}\right)$ are negative number to each other.

$\left(-\frac{4}{7}\right) + \left(\frac{4}{7}\right) = 0$

Therefore, $\left(-\frac{4}{7}\right)$ and $\left(\frac{4}{7}\right)$ are negative number to each other.

- **Multiplicative inverse (Reciprocal) :** 3 is multiplied by such a number that the multiplication of both is 1 ?

Except zero for any number, we get such a number whose multiplication with given number results 1, then these both numbers are called reciprocal to each other.

For example, $2 \times \frac{1}{2} = 1$

Here, 2 and $\frac{1}{2}$ are reciprocal to each other.

Reciprocal of 2 is $\frac{1}{2}$ and reciprocal of $\frac{1}{2}$ is 2.

$\frac{1}{5} \times 5 = 1$

Here, $\frac{1}{5}$ and 5 are reciprocal to each other.

$\left(-\frac{4}{7}\right) \times \left(-\frac{7}{4}\right) = 1$

Here, $\left(-\frac{4}{7}\right)$ and $\left(-\frac{7}{4}\right)$ are reciprocal to each other.

If any number is multiplied with zero (0), the result is zero (0). Therefore, the reciprocal of zero (0) does not exist.

Practice 2

1. Write additive inverse of the given numbers :

(1) $\left(-\frac{3}{4}\right)$ (2) 0 (3) (-2) (4) $\frac{13}{20}$ (5) (-0.7)

(6) 0.8 (7) 0.01 (8) $1\frac{2}{3}$ (9) $\frac{2}{5}$ (10) $\frac{7}{8}$

2. Write multiplicative inverse (reciprocal) of the given numbers :

(1) (-1) (2) $\left(-\frac{5}{8}\right)$ (3) $\frac{1}{8}$ (4) $\frac{2}{5}$ (5) (-0.7)

(6) 0.8 (7) 0.01 (8) $1\frac{3}{2}$ (9) $\frac{2}{5}$ (10) $\frac{7}{8}$

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- **Properties for addition and multiplicative of rational numbers :**

We have learnt about properties of addition and multiplication of integers. Let us now learn about properties of addition and multiplication of rational numbers.

- **Properties for addition of rational numbers :**

Note the result by adding the following rational numbers :

No.	Addition	Result	Properties
(1)	$\frac{4}{7}+\left(-\frac{2}{3}\right)=...$ $\left(-\frac{1}{4}\right)+\frac{3}{8}=...$	Is resulting number a rational number ?	**Closure property :** The addition of any two rational numbers is a rational number.
(2)	$\frac{4}{7}+\left(-\frac{2}{3}\right)=...$ $\left(-\frac{2}{3}\right)+\frac{4}{7}=...$	How result is obtained when order is changed.	**Commutative property :** Two rational numbers can be added in any order but the result is same.
(3)	$\left[\left(-\frac{3}{4}\right)+\frac{1}{2}\right]+\frac{2}{6}=...$ $\left(-\frac{3}{4}\right)+\left[\frac{1}{2}+\frac{2}{6}\right]=...$	How result is obtained when group is changed ?	**Associative property :** For any three rational numbers if in the group of any two numbers, third number is added, the result is same.
(4)	$\left(-\frac{3}{4}\right)+0=...$ $0+\frac{2}{3}=...$	How result is obtained when addition is done with zero (0) ?	**Existence of identity element :** For addition of a rational number and zero, we get the same rational number. Therefore, zero (0) is the identity element for addition.
(5)	$\left(-\frac{7}{17}\right)+\frac{7}{17}=...$ $\frac{3}{5}+\left(-\frac{3}{5}\right)=...$	What is the result when two opposite rational numbers are added ?	For any rational number there always exist an opposite number such that addition of both number is zero.

- **Properties for multiplication of rational numbers :**

Note the result by multiplication of following rational numbers :

No.	Addition	Result	Properties
(1)	$0 \times \frac{5}{9} = ...$ $\left(-\frac{3}{2}\right) \times \frac{2}{6} = ...$	Is resulting number a rational number ?	**Closure property :** The addition of any two rational numbers is a rational number.
(2)	$\left(-\frac{2}{5}\right) \times \frac{10}{3} = ...$ $\frac{10}{3} \times \left(-\frac{2}{5}\right) = ...$	How result is obtained when order of numbers are changed.	**Commutative property :** When two rational numbers are multiplied in any order then the result is same.
(3)	$\left[\left(-\frac{1}{3}\right)\times\frac{3}{4}\right]\times\frac{6}{7} = ...$ $\left(-\frac{1}{3}\right)\times\left[\frac{3}{4}\times\frac{6}{7}\right] = ...$	How result is obtained when group is changed ?	**Associative property :** For any three rational numbers if in the group of any two number, the third number is multiplied, the result is same.
(4)	$\left(-\frac{4}{9}\right) \times 1 = ...$ $\frac{3}{7} \times 1 = ...$	How result is obtained when any number is multiplied with one ?	**Existence of identity element :** Multiplication of any rational number and 1 is always the same rational number. Therefore, 1 is the identity element for multiplication.
(5)	$\frac{3}{5} \times \frac{5}{3} = ...$ $\left(-\frac{1}{2}\right) \times \left(-\frac{2}{1}\right) = ...$	What is the result when two inverse (reciprocal) rational numbers are multiplied ?	For any rational number there always exist a reciprocal number such that multiplication of both number is one.

No.	Addition	Result	Properties
(6)	$\left(-\frac{3}{5}\right) \times \left(-\frac{3}{4}\right) + \frac{2}{3} = \ldots$	Can the distribution of multiplication of rational numbers over addition is possible ?	**The distributive property :** The distribution of multiplication of rational numbers over addition is possible.

Exercise

1. Fill in the blanks :

(1) (–5) is an integer but it is not a .... number. (rational, natural)

(2) Zero is an integer but it is not considered in .... number. (rational, natural)

(3) $\frac{5}{7}$ is a/an .... (integer, rational number)

(4) In $\frac{3}{8}$, ... is numerator and ...... is denominator. (8, 3, 83)

(5) In $2\frac{3}{8}$, ... is an integer and ...... is a rational number. $\left(8, \frac{3}{8}, 2\right)$

2. Represent the following number in $\frac{p}{q}$ form :

(1) $2\frac{1}{7}$ (2) 0.6 (3) $\left(-3\frac{1}{4}\right)$ (4) 0 (5) 28

3. Write which properties are used in the following rational numbers :

(1) $\frac{2}{7} + \left(-\frac{7}{4}\right) = \left(-\frac{7}{4}\right) + \frac{2}{7}$

(2) $\left(-\frac{1}{8}\right) \times \frac{1}{7} = \frac{1}{7} \times \left(-\frac{1}{8}\right)$

(3) $\frac{1}{2} \times \left(\frac{1}{3} \times \frac{1}{4}\right) = \left(\frac{1}{2} \times \frac{1}{3}\right) \times \frac{1}{4}$

(4) $\frac{1}{5} \times \left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{1}{5} \times \frac{1}{2}\right) + \left(\frac{1}{5} \times \frac{1}{3}\right)$

(5) $\left(-\frac{3}{5}\right) + 0 = \left(-\frac{3}{5}\right)$

(6) $\frac{1}{4} + \left(\frac{1}{6} + \frac{1}{3}\right) = \left(\frac{1}{4} + \frac{1}{6}\right) + \frac{1}{3}$

Answers

Practice 2

1. (1) $\frac{3}{4}$ (2) 0 (3) 2 (4) $\left(-\frac{13}{20}\right)$ (5) 0.7

(6) (–0.8) (7) (–0.01) (8) $\left(-1\frac{2}{3}\right)$ (9) $\left(-\frac{2}{5}\right)$ (10) $\left(-\frac{7}{8}\right)$

2. (1) (-1) (2) $\left(-\frac{8}{5}\right)$ (3) 8 (4) $\frac{5}{2}$ (5) $\frac{10}{7}$ (6) $\frac{10}{8}$ (7) $\frac{100}{1}$ (8) $\frac{2}{5}$ (9) $\frac{5}{2}$ (10) $\frac{8}{7}$

Exercise

1. (1) natural (2) rational (3) rational number (4) 3, 8 (5) 2, $\frac{3}{8}$

2. (1) $\frac{15}{7}$ (2) $\frac{6}{10}$ (3) $\left(-\frac{13}{4}\right)$ (4) $\frac{0}{1}$ (5) $\frac{28}{1}$

3. (1) Commutative property for addition (2) Commutative property for multiplication
(3) Associative property for multiplication (4) Distributive property
(5) Existence of identity element for addition (6) Associative property for addition

For information only

- **Irrational Numbers :**

$\sqrt{4} = 2$ is an integer, so it is also a rational.

$\sqrt{1.69} = 1.3$ is also a rational number.

$\sqrt[3]{8} = 2$ is also a rational number.

These each can be denoted in $\frac{p}{q}$ form. Additionally each rational number can be written as finite decimal or infinite repeating decimal.

For example, $\frac{1}{5} = 0.2$, $\frac{1}{8} = 0.125$, $\frac{1}{3} = 0.333...$, $\frac{1}{7} = 0.1428571428571$

Here, 0.333 and 0.1428571428571... can be written as $0.\dot{3}$ and $0.\dot{1}4285\dot{7}$ respectively.

Here, for 0.123456789101112... there is no possibility of finite and repeating. These numbers are called irrational numbers. Therefore, the numbers whose decimal form is infinite and non-repeating are called irrational numbers.

For example, $\sqrt{2}$, $\sqrt{3}$, π, $\frac{\sqrt{3}}{3}$

- **Real Numbers :**

The set which is formed by collection of rational and irrational numbers, is called the set of Real numbers. They are denoted by R.