Chapter 12

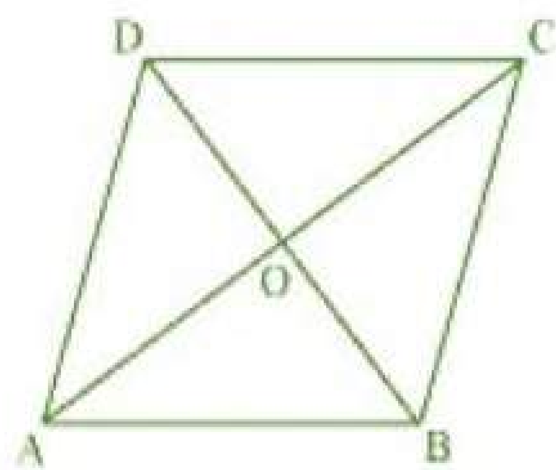
Introduction To Three Dimensional Geometry

Miscellaneous Exercise

Question 1: Three vertices of a parallelogram ABCD are A (3, –1, 2), B (1, 2, –4) and C (–1, 1, 2). Find the coordinates of the fourth vertex.

Answer 1:

The three vertices of a parallelogram ABCD are given as A (3, –1, 2), B (1, 2, –4), and C (–1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

∴ Mid-point of AC = Mid-point of BD

$= \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$

$= (1, 0, 2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$

$= \frac{x+1}{2} = 1, \frac{y+2}{2} = 0$ and $\frac{z-4}{2}$

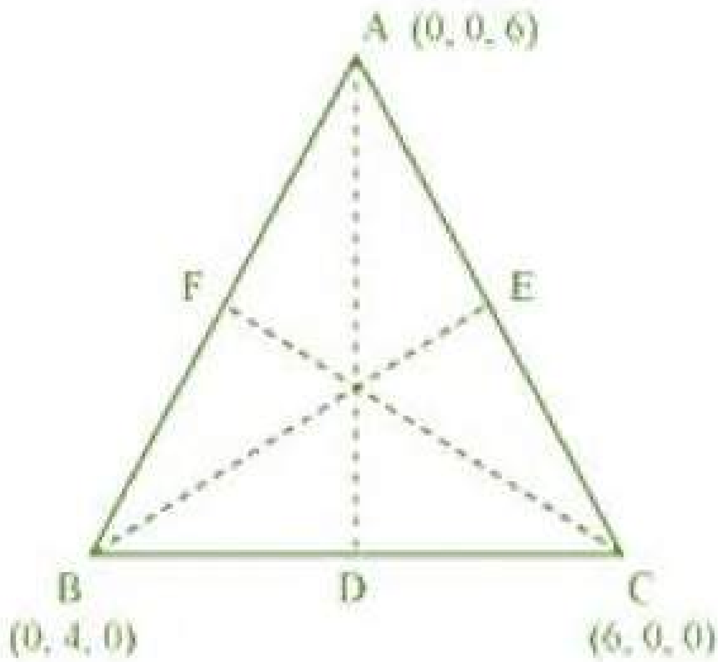
⇒ x = 1, y = –2, and z = 8

Thus, the coordinates of the fourth vertex are (1, –2, 8).

Question 2: Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Answer 2:

Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

∴Coordinates of point D $=\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$ = (3, 2, 0)

AD $= \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$

Since, BE is the median, E is the mid – point of AC

∴Coordinates of point E $= \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right) = (3, 0, 3)$

BE = $\sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$

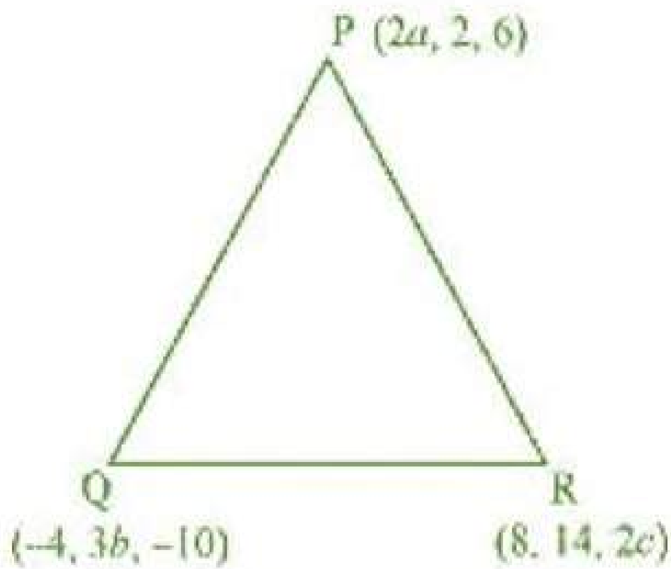
Since, CF is the median, F is the mid – point of AB

∴Coordinates of point F = $\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0, 2, 3)$

Length of CF = $\sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

Thus, the lengths of the medians of ΔABC are $7, \sqrt{34}$, and 7.

Question 3: If the origin is the centroid of the triangle PQR with vertices P (2a, 2, 6), Q (–4, 3b, – 10) and R (8, 14, 2c), then find the values of a, b and c.



It is known that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), are.

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Therefore, coordinates of the centroid of ΔPQR

$= \left(\frac{2a-4+8}{3}, \frac{2+3b+17}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

It is given that origin is the centroid of ΔPQR

= (0, 0, 0) = $\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

= $\frac{2a+4}{3} = 0, \frac{3b+16}{3} = 0$ and $\frac{2c-4}{3} = 0$

= a = - 2, b = - $\frac{16}{3}$ and c = 2

Thus, the respective values of a, b, and c are – 2, $-\frac{16}{3}$, and 2.

Question 4: Find the coordinates of a point on y-axis which are at a distance of 5√2 from the point P (3, –2, 5).

Answer 4:

If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero.

Let A (0, b, 0) be the point on the y-axis at a distance of $5\sqrt{2}$ from point P (3, –2, 5).

Accordingly, AP = $5\sqrt{2}$

AP2 = 50

= $(3 - 0)^2 + (-2 - b)^2 + (5 - 0)^2 = 50$

= $9 + 4 + b^2 + 4b + 25 = 50$

= $b^2 + 4b - 12 = 0$

= $b^2 + 6b - 2b - 12 = 0$

= b (b + 6) – 2 (b + 6) = 0

= (b + 6) (b – 2) = 0

= b = - 6 or 2

Thus, the coordinates of the required points are (0, 2, 0) and (0, –6, 0).

Question 5: A point R with x-coordinate 4 lies on the line segment joining the point P (2, –3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[Hint suppose R divides PQ in the ratio k: 1. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$]

Answer 5:

The coordinates of points P and Q are given as P (2, –3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio k:1.

Hence, by section formula, the coordinates of point R are given by

$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$

It is given that the x-coordinate of point R is 4

$= \frac{8k+2}{k+1} = 4$

$= 8k + 2 = 4k + 4$

$= 4k = 2$

$= k = \frac{1}{2}$

Therefore, the coordinates of point R are $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right)$

Question 6: If A and B be the points (3, 4, 5) and (–1, 3, –7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Answer 6:

The coordinates of points A and B are given as (3, 4, 5) and (–1, 3, –7) respectively.

Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$PA^2 = (x - 3)^2 + (y - 4)^2 + (z - 5)^2$

$= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z$

$= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50$

$PB^2 = (x + 1)^2 + (y - 3)^2 + (z + 7)^2$

$= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59$

Now, if $PA^2 + PB^2 = k^2$, then

$(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50) + (x^2 + 2x + y^2 - 6y + z^2 + 14z + 59) = k^2$

$= 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$

$= 2\,(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$

$= x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2-109}{2}$

Thus, the required equation is $= x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2-109}{2}$